HUDM 4122
Homework #7 key

1. A speedy oil and lubrication chain surveyed its stores as to the time needed to process a customer. Six customers from each of its 17 stores were selected at random and monitored. The processing times had a mean of 11 minutes with a standard deviation of 5.8 minutes. Find a 95% CI for the mean time needed to process a customer.

   N=6*17= 102 (large sample, use z, assume $\sigma = s;\ \sigma_{\bar{X}} = s_{\bar{X}}$)

   $$\sigma_{\bar{X}} = s_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{5.8}{\sqrt{102}} = 0.574$$

   95% CI = $\bar{X} \pm z_{\alpha/2}(\sigma_{\bar{X}}) = 11 \pm (1.96)(0.574) = 11 \pm 1.13$ = **[9.87, 12.13]**

2. Pyrometric cones of different composition are supplied to the ceramic industry to use in measuring temperatures in kilns. Different cones are designed to melt at specific temperatures. Suppose a random sample of 40 No. 17 cones had melting temperatures with a mean of 1845° F and a standard deviation of 54° F. Find a 95% CI for the mean melting temperature of the No. 17 cones.

   N=40 (large sample, use z, assume $\sigma = s;\ \sigma_{\bar{X}} = s_{\bar{X}}$)

   $$\sigma_{\bar{X}} = s_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{54}{\sqrt{40}} = 8.54$$

   95% CI = $\bar{X} \pm z_{\alpha/2}(\sigma_{\bar{X}}) = 1845 \pm (1.96)(8.54) = 1845 \pm 16.74$ = **[1828.26, 1861.74]**

3. The pH levels of a stream below a power dam were measured on 10 occasions and found to have a mean pH of 7.9 with a standard deviation of 0.5. Find a 99% CI for the mean pH level of this stream.

   N=10 (use t)

   $$s_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{0.5}{\sqrt{10}} = 0.158$$

   99% CI = $\bar{X} \pm t_{df=n-1=9}^{\alpha/2=0.005}(s_{\bar{X}}) = 7.9 \pm (3.25)(0.158) = 7.9 \pm 0.514$ = **[7.386, 8.414]**

4. A random sample of 16 university students are questioned on how much they spend monthly for their personal entertainment. If $\overline{X}$ = \$56.45 and s = \$14.50, find a 90% CI for the mean expenditure for entertainment using the *t* distribution. What assumption is necessary?

   N=16 (use t)

   $$s_{\overline{X}} = \frac{s}{\sqrt{n}} = \frac{14.50}{\sqrt{16}} = 3.625$$

   90% CI = $\overline{X} \pm t^{\alpha/2=0.05}_{df=n-1=15}(s_{\overline{X}}) = 56.45 \pm (1.753)(3.625) = 56.45 \pm 6.355$ = **[50.095, 62.805]**

Approximately normal distribution assumption is necessary.

5. In your state, possible scores on the written part of the driver's license exam range from 0 to 20. The department's records indicate that the mean score on this test is 11.2, and that the scores are roughly normally distributed. They do not keep information on the standard deviation of scores on the test. A random sample of 12 applicants are given a new experimental version of the preparation booklet to study before taking the test. After studying the new version, they each take the test. Their scores are given below.

   X = 10 7 17 12 20 15 14 8 12 16 19 11

   a. Estimate the mean and variance for the new version of the booklet (i.e. calculate X and s^2).

      $$\overline{X} = \frac{1}{n}\sum X_i = 13.42$$

      $$s^2 = \frac{1}{n-1}\sum\left(X_i - \overline{X}\right)^2 = 17.18 \rightarrow s = \sqrt{17.18} = 4.14 \quad s_{\overline{X}} = \frac{s}{\sqrt{n}} = \frac{4.14}{\sqrt{12}} = 1.20$$

   b. Using the t distribution, construct a 95% C.I. for the mean of people studying the new booklet.

      95% CI = $\overline{X} \pm t^{\alpha/2=0.025}_{df=n-1=11}(s_{\overline{X}}) = 13.42 \pm (2.201)(1.20) = 13.42 \pm 2.64$ = **[10.78, 16.06]**