Appendix: Training and Application of Modular Linear Power Flow Model

The basic idea of the modular linear power flow model is first to split the wide operating conditions into several regions based on the k-means method, and then the best-fit linear power flow models with minimized errors are segmented trained for each region. A classifier is trained based on the divided sub-regions, which aims to select the applicable segmented linear model for each scenario regarding the scenario-based method. The applicable segmented linear models for different scenarios are dynamically selected and may be different. The individual sensitivity factor $\mathbf{\Gamma}$ for various scenarios is calculated based on the Jacobian matrix derived from different applicable segmented models. It suggests that the sensitivity factor $\mathbf{\Gamma}$ may be different for different scenarios. The process is illustrated in more detail below.

Since the power injections of multiple scenarios directly affect the operation boundary margins reserved for uncertainty and the violation probability of chance constraints, operating conditions in this paper are considered to consist of power injections, which can be shown as follows:

$$h_{OC}\left(\mathbf{P},\mathbf{Q}\right), \tag{1}$$

where the power injections $\mathbf{P}$ and $\mathbf{Q}$ include the bus load, the output of traditional generators, forecasted generation of renewable energy sources (RESs), and the uncertainty of RES generation.

Various segmented linear models use different state variable functions for Taylor expansion. This paper uses the linear power flow model with individual independent variable function $\varphi_i(v_i) = v_i^{k_i}$ for different buses and unified function $\phi(\theta) = \theta_{ij}$, which is shown as follows:

$$PF_{ij}^{L} = g_{ij}\left(\frac{v_i^{k_i}-1}{k_i}-\frac{v_j^{k_j}-1}{k_j}\right)-b_{ij}\theta_{ij}+P_{ij,loss}^{L}, \tag{2}$$

$$QF_{ij}^{L} = -b_{ij}\left(\frac{v_i^{k_i}-1}{k_i}-\frac{v_j^{k_j}-1}{k_j}\right)-g_{ij}\theta_{ij}+Q_{ij,loss}^{L}. \tag{3}$$

To improve the linearization accuracy, the network losses $P_{ij,loss}^{L}$ and $Q_{ij,loss}^{L}$ are considered in (2)-(3), which are modeled as follows:

$$\begin{aligned} P_{ij,loss,L} &= g_{ij}\frac{v_{i,0}^{H}-v_{j,0}^{H}}{v_{i,0}^{H}+v_{j,0}^{H}}\left(v_{i,sL}-v_{j,sL}\right)+\frac{1}{2}g_{ij}\left(v_{i,0}^{H}-v_{j,0}^{H}\right)^2 \\ &\quad +g_{ij}\theta_{ij,0}^{H}\theta_{ij}+\frac{1}{2}g_{ij}\left(\theta_{ij,0}^{H}\right)^2, \end{aligned} \tag{4}$$

$$\begin{aligned} Q_{ij,loss,L} &= -b_{ij}\frac{v_{i,0}^{H}-v_{j,0}^{H}}{v_{i,0}^{H}+v_{j,0}^{H}}\left(v_{i,sL}-v_{j,sL}\right)-\frac{1}{2}b_{ij}\left(v_{i,0}^{H}-v_{j,0}^{H}\right)^2 \\ &\quad -b_{ij}\theta_{ij,0}^{H}\theta_{ij}-\frac{1}{2}b_{ij}\left(\theta_{ij,0}^{H}\right)^2, \end{aligned} \tag{5}$$

where $(v_{i,0}^{H}, \theta_{ij,0}^{H})$ is the hot-start initial point from historical data. $v_{i,sL}$ is linearization of v_i^2, which is expressed as a linear function with $v_i^{k_i}$ and formulated as follows:

$$\begin{aligned} v_{i,sL} &\overset{define}{=} v_{i,0}^2+2v_{i,0}\left.\frac{\partial v_i}{\partial v_i^{k_i}}\right|_{v_i=v_{i,0}}\left(v_i^{k_i}-v_{i,0}^{k_i}\right) \\ &= v_{i,0}^2+2v_{i,0}\frac{v_i^{k_i}-v_{i,0}^{k_i}}{k_i v_{i,0}^{k_i-1}}. \end{aligned} \tag{6}$$

v_0 in (6) is the initial point of the Taylor expansion.

After dividing operating conditions into several regions, the segmented linear power flow models with minimized errors can be trained for each region. The segmented linear models are derived by optimizing the independent variables $\varphi_i(v_i) = v_i^{k_i}$ and $\phi(\theta) = \theta_{ij}$ based on historical data. The following optimization model is used to select the independent variable with minimized linearization error of each region and then obtain specific linear power flow models.

$$\begin{aligned} \min_{k_i}\ obj &= \sum_{(i,j)\in\Omega_L}\left(\left|PF_{ij}^{L}-PF_{ij}^{AC}\right|+\left|QF_{ij}^{L}-QF_{ij}^{AC}\right|\right) \\ &\quad +\sum M_1\xi_1+\sum M_2\xi_2+\sum M_3\xi_3 \end{aligned} \tag{7}$$

$$\text{Decision varaible: } k_i,\ i=1,2,\ldots\ldots,NB \tag{8}$$

$$P_{ij,loss}^{L}+\xi_1\geq 0,\ Q_{ij,loss}^{L}+\xi_1\geq 0,\ \xi_1\geq 0 \tag{9}$$

$$VBRL_{ij}-\xi_2\leq v_{cp,L}\leq VBRU_{ij}+\xi_3,\ \xi_2,\xi_3\geq 0 \tag{10}$$

$$v_{cp,L}\overset{define}{=} v_{i,0}v_{j,0}+\frac{v_{j,0}\left(v_i^{k_i}-v_{i,0}^{k_i}\right)}{k_i v_{i,0}^{k_i-1}}+\frac{v_{i,0}\left(v_j^{k_j}-v_{j,0}^{k_j}\right)}{k_j v_{j,0}^{k_j-1}}, \tag{11}$$

$$Linear\ Power\ Flow: (2)-(3), \tag{12}$$

where PF_{ij}^{L}/QF_{ij}^{L} and $PF_{ij}^{AC}/QF_{ij}^{AC}$ are the linear and AC branch power flow, respectively, $v_{cp,L}$ is the linearization of $v_i v_j$, $VBRU_{ij}$ and $VBRL_{ij}$ are the upper and lower bounds of $v_{cp,L}$ after tightening. ξ_1, ξ_2, and ξ_3 are the nonnegative slack variables, and M is a large positive number. v_i and v_j represent the voltage magnitudes from historical data. The exponent k_i is the decision variable. Ω_L is the set of branches. NB is the number of buses. The segmented linear models for given regions with minimized errors are obtained by solving the optimization problem (7)-(12).

Support vector machine (SVM) as a common classifier is used to divide the operating conditions and determine the applicable segmented linear model according to the output class of the classifier. The historical operating conditions of the power grid are regarded as the input training samples of the classifier. The results of divided sub-regions are regarded as training labels. The trained classifier can automatically place a new operating condition (scenario) into the proper class, and it indicates the applicable segmented linear model.

Finally, the individual sensitivity factors can be calculated according to different segmented linear power flow models for CCOPF modeling.