Time Variations and Covariations in the Expectation and Volatility of Stock Market Returns

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ABSTRACT

This article investigates empirically the comovements of the conditional mean and volatility of stock returns. It extends the results in the literature by demonstrating the role of the commercial paper–Treasury yield spread in predicting time variation in volatility. The conditional mean and volatility exhibit an asymmetric relation, which contrasts with the contemporaneous relation that has been tested previously. The volatility leads the expected return, and this time series relation is documented using offset correlations, short-horizon contemporaneous correlations, and a vector autoregression. These results bring into question the value of modeling expected returns as a constant function of conditional volatility.

THE TIME SERIES PROPERTIES of the expectation and volatility of stock returns have recently attracted much attention in the financial economics literature. Empirical evidence suggests that variables such as yields and yield spreads in the corporate and Treasury bond markets, and dividend yields have predictive power for returns (Breen, Glosten, and Jagannathan (1989), Fama and French (1989), Kandel and Stambaugh (1989, 1990), and Keim and Stambaugh (1986)). This explanatory power over different time periods and return horizons leads some researchers to conclude that there is significant time variation in expected returns over the business cycle. See, for example, Fama and French (1989, p. 23). In addition, significant time variation in the volatility of returns has been documented using these and other economic variables (Schwert (1989) and Kandel and Stambaugh (1989, 1990)).

A natural extension of this research is to examine the covariation between the mean and volatility of returns. On a market-wide level, strong intuition suggests that risk and return should be positively related.[1] Consequently, researchers have searched for both a positive relation between expected returns and the conditional volatility of returns and a negative relation

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[1]For example, Campbell (1987) develops a specialization of the intertemporal CAPM in which the expected excess real return is approximately proportional to the variance of the return. Of course, if international capital markets are integrated, the appropriate market index is the world market.

515

between unanticipated volatility and realized returns. This latter effect arises if unanticipated increases in volatility increase required returns and cause a corresponding decline in price. Yet, prior empirical investigations into the contemporaneous correlation between the first two moments of stock market returns yield decidedly mixed results (Campbell (1987), French, Schwert, and Stambaugh (1987), Glosten, Jagannathan, and Runkle (1993), and Harvey (1991)). For example, French, Schwert, and Stambaugh (1987) find a statistically significant positive relation between expected returns and anticipated volatility only when using a generalized autoregressive conditional heteroscedasticity model with mean effects (GARCH-M). Other models yield negative and insignificant relations. Glosten, Jagannathan, and Runkle (1993), using a modified GARCH-M model, conclude that there is a negative relation or no relation at all between expected returns and anticipated volatility. Using international data, Chan, Karolyi, and Stulz (1992) conclude that the expected return on the U.S. market is not related to its own conditional variance, but that it is positively related to the conditional covariance with a foreign index.

At the same time, Backus and Gregory (1988) demonstrate that negative and even nonmonotonic relations are consistent with equilibrium. Abel (1988), Glosten, Jagannathan, and Runkle (1988), and Gennotte and Marsh (1993) also point out that equilibrium does not imply a positive relation between the first two moments of returns. Nevertheless, descriptive empirical models such as ARCH-M and its generalizations, which impose a linear relation between the conditional expectation and conditional volatility of returns, continue to be applied to stock return data. One recent variation on this approach is used by Campbell and Hentschel (1992); they model expected returns as a linear function of the variance of stock dividend news rather than of the variance of the returns.

Given the theoretical results cited above, it is primarily an empirical question as to whether the conditional first and second moments of equity returns are positively related. However, the mixed empirical results in the literature indicate that inferences are sensitive to the way in which the moments and the relation between them are modeled. In this article, a number of empirical strategies are adopted to minimize concern about this issue. First, both the conditional expected return and the conditional volatility of returns are estimated using a set of empirically proven and theoretically justified conditioning variables. Increasing the explained variation of the conditional moments of stock returns is one potentially important step in estimating their time variation and covariation. Second, no functional form is imposed on the relation between the mean and volatility of returns. Third, both the contemporaneous and noncontemporaneous relation between the moments are considered. While, unconditionally, the expected return may be weakly (positively or negatively) related to the contemporaneous level of anticipated volatility, as the existing literature suggests, it does not follow that there is no systematic relation between the conditional moments. In fact, the empirical analysis in this article demonstrates a weak unconditional

contemporaneous relation but a strong noncontemporaneous relation between the conditional volatility and the conditional expected return. Specifically, volatility appears to lead expected returns over the course of the business cycle.

The analysis employs four explanatory variables—the yield spread between Baa-rated and Aaa-rated corporate debt, the commercial paper–Treasury yield spread, the one-year Treasury yield, and the dividend yield on the S&P 500. The commercial paper-Treasury spread receives special attention for two reasons. First, this variable is the primary predictor of stock market volatility, even though it is not used extensively in the finance literature.[2] Second, the macroeconomics literature identifies this spread as an excellent predictor of real activity (Stock and Watson (1989) and Bernanke (1990)). The estimation of both the conditional mean and the conditional volatility of stock market excess returns employs the four explanatory variables in a linear framework. All have significant predictive power for one or both of the conditional moments, even for monthly returns. The fitted moments both show cyclical variation, but the cycles do not coincide.

This asymmetry and the offset in the cycles of the conditional moments are evident when looking at correlations between the conditional mean and lagged values of the conditional volatility versus correlations between the conditional mean and led values of the conditional volatility. Previous studies focus on the contemporaneous correlation between the moments, and we find that this quantity is negative, which is consistent with results in Campbell (1987) and Glosten, Jagannathan, and Runkle (1993). More important, however, are the positive correlation between the conditional mean and lagged values of the conditional volatility and the strong negative correlation between the conditional mean and led values of the conditional volatility. Moreover, the contemporaneous correlation between the moments is not stable over time but instead varies from large positive to large negative values when measured over seventeen-month horizons.

A vector autoregression estimated on the fitted moments illustrates their time series behavior. For monthly returns, lagged conditional volatility is positively related to future expected returns, yet lagged expected returns are negatively related to future volatility. Although the former effect is potentially consistent with a positive contemporaneous relation between the conditional moments, an impulse-response analysis shows that the system actually generates the opposite result. Together, these results cast doubt on the wisdom of modeling expected returns as a constant linear function of the conditional volatility of returns, and they present additional stylized facts to be explained by theoretical models of equity pricing.

The remainder of the article is organized as follows. Section I briefly discusses the choice of the explanatory variables in the context of the literature on return predictability. Section II describes the data. Section III uses the four financial variables to estimate the conditional first and second

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[2] Kairys (1991) uses changes in the commercial paper yield to predict returns.

moments of excess stock market returns. The value-weighted index is considered for monthly, quarterly, and annual holding periods. Motivated by the behavior of the fitted time series, an offset cyclical relation is proposed, and its implications are assessed. Section IV analyzes the correlations between the fitted moment series. Section V estimates and analyzes a vector autoregression on the fitted moments. Section VI considers the significance and robustness of the results via subperiod analysis and bootstrapping experiments. Finally, Section VII concludes the article.

I. Explanatory Variables

A number of financial variables have shown good predictive power for the mean and volatility of equity returns, and these variables provide the starting point for the empirical analysis that follows. Four explanatory variables are employed for estimating both the conditional mean and conditional volatility of returns: the Baa-Aaa corporate bond yield spread, the commercial paper–Treasury yield spread, the one-year Treasury yield, and the dividend yield on the S&P 500. These variables are selected from the large universe of possible regressors primarily because of their demonstrated predictive power.[3] This method of data selection leads to a natural concern about potential data-snooping biases,[4] although this concern is somewhat mitigated by international evidence and a theoretical motivation for the choice of these or similar explanatory variables.

The exception to this selection criterion based on predictive power for stock returns is the commercial paper–Treasury spread, which is included because of its recently demonstrated ability to forecast future, real economic activity (Stock and Watson (1989) and Bernanke (1990)). Specifically, the spread is negatively related to future output and income. Bernanke (1990) argues that this forecasting power arises from the spread's ability to proxy for the stance of monetary policy. He concludes that, in the process of tightening monetary policy, the Federal Reserve induces corporations to substitute commercial paper for bank loans, which increases borrowing costs, reduces investment, and causes an increase in the commercial paper-Treasury spread. Kashyap, Stein, and Wilcox (1993) provide empirical support for this explanation by

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[3]See, for example, Keim and Stambaugh (1986) and Fama and French (1989), who use the yield spread between low-grade and high-grade corporate debt or between low-grade corporate debt and Treasury bonds to predict returns. The spread is found to be positively and significantly related to future returns. Schwert (1989) shows that, even in the presence of other variables, this quality spread is positively related to future stock market volatility. Fama and French (1989) and Keim and Stambaugh (1986), among others, use the dividend yield to document significant time variations in expected returns. They find that expected returns are positively related to the level of the dividend yield. Campbell (1987) and Schwert (1989), among others, use some measure of the level of nominal interest rates to predict the conditional moments of stock market returns. They find that future returns are negatively related to interest rates, and that the volatility of returns is positively related to interest rates.

[4]See Foster and Smith (1992) and Lo and MacKinlay (1990) for a discussion of data snooping and its implications.

examining the financing mix used by corporations in relation to the level of various interest rates. The effects on investment and output of moving from internal financing (bank loans) to external financing (commercial paper) may be amplified by the existence of agency costs (Bernanke and Gertler (1989) and Gertler (1991)). In their model, if firms must increase their borrowing from outside sources, then asymmetry of information between the borrowers and lenders leads to increases in borrowing costs and reduced investment. Gertler, Hubbard, and Kashyap (1990) provide empirical evidence of this phenomenon.

We employ the spread between the yields on six-month commercial paper and three-month Treasury bills as our measure of this spread. The fact that the maturities of the two securities differ has little impact, judging from periods for which the six-month Treasury bill yield is available. Adding the spread between six-month Treasury bills and three-month Treasury bills does not qualitatively change any of the empirical results, and the coefficient on this variable is not significant. In Campbell's (1987) study the spread between six-month and one-month securities has significant predictive power only for stock volatility, and only in a single subperiod. Fama and French (1989) employ the spread between long-term and short-term securities; it has significant predictive power for returns, but the same variable adds no information to the regressions in this article.

II. The Data

The data for the four explanatory variables—the Baa-Aaa spread, the commercial paper–Treasury spread, the one-year Treasury yield, and the dividend yield—are from Citibase, the Citibank economic database. Citibase's sources are Moody's Investor Service, *Moody's Bond Survey*, and the U.S. Department of the Treasury, *Treasury Bulletin* for corporate bond yields; the Board of Governors of the Federal Reserve, *Selected Interest Rates and Bond Prices*, for Treasury yields and commercial paper yields; and *Standard and Poor's Outlook* for dividend yield data. All data are monthly and cover the period April 1953 to March 1989 (432 observations).[5]

In addition to the four explanatory variables, the analysis uses monthly and daily returns on the value-weighted market portfolio from the CRSP data files. The monthly returns cover the period May 1953 to April 1989, while the daily returns start on July 2, 1962 and run through April 28, 1989. Taking natural logarithms of one plus the return and subtracting the natural logarithm of one plus the monthly Treasury yield, as reported by Ibbotson Associates, converts the monthly simple returns to continuously compounded,

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[5] The data are actually monthly averages of daily yield data. The averaged data has the advantage of reducing noise and the disadvantage of being slightly stale. No attempt is made to assess the magnitude of these effects. The sample period is chosen to coincide approximately with the postwar sample period used in the literature, i.e., the sample begins after the 1951 Fed-Treasury Accord.

excess returns. Longer horizon excess returns are constructed by summing the monthly returns.

The analysis uses daily return data to compute estimates of the standard deviation of returns for monthly, quarterly, and annual holding periods. The conditional variance of the continuously compounded, periodic return, assuming that daily returns are mean zero and conditionally uncorrelated,[6] is

$$\text{Var}_t[r_{t,t+\tau}] = \sum_{n=1}^{N(t)} \text{E}_t[r_n^2]. \tag{1}$$

where $N(t)$ is the number of trading days in the period t to $t + \tau$, r_n is the daily, continuously compounded return for day n, $r_{t,t+\tau}$ is the return from t to $t + \tau$, and $\text{Var}_t[\cdot]$ and $\text{E}_t[\cdot]$ are the variance and expectation conditional on information available at time t. Let $s_{t,t+\tau}$ denote a measure of the ex post variation in daily returns:

$$s_{t,t+\tau}^2 = \sum_{n=1}^{N(t)} r_n^2. \tag{2}$$

The projection of $s_{t,t+\tau}^2$ onto the time t information set yields a consistent estimator of the conditional variance of the periodic return.

III. Linear Estimation of the Conditional Moments of Returns

A. The Model and Estimation Techniques

For the purposes of estimation consider a model that assumes the conditional first and second moments to be linear function of the conditioning variables as follows:

$$\text{E}_t[r_{t,t+\tau}] = X_t \beta, \tag{3}$$

$$\text{SD}_t[r_{t,t+\tau}] = \text{SD}_t[r_{t,t+\tau} - X_t \beta] = X_t \gamma, \tag{4}$$

where X_t is the vector of conditioning variables, and $\text{SD}_t[\cdot]$ is the standard deviation conditional on information at time t. A linear specification is chosen for simplicity and can be considered an approximation to the true, unknown functional form.[7] The choice of modeling the conditional moments separately as functions of the predetermined variables is made for two reasons. First, it imposes little structure on the relation between the moments, although the conditional volatility can certainly be added to the set of explanatory variables in equation (3) to test for a significant, contemporaneous, linear relation.[8] Second, it exploits information in the conditioning variables in

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[6] Mean adjustments and corrections for first-order autocovariance have no qualitative effect on the results. Therefore, the simple formulation, which French, Schwert, and Stambaugh (1987) and Schwert (1989) employ, is used throughout this article.

[7] Nonparametric approaches to the estimation problem, including kernel estimation and Taylor approximations, appear to provide little additional information.

[8] This analysis is presented in Section III.B.

estimating both the mean and volatility. This specification can be contrasted, for example, with the GARCH-M technique employed in French, Schwert, and Stambaugh (1987), which imposes a linear relation between the conditional moments, and their autoregressive integrated moving average (ARIMA) model of volatility, which uses only past levels of volatility as conditioning variables.

Given the linear specification, two approaches are used to estimate the conditional moments. The first approach uses only the return data and does not use the volatility estimates from the daily data. It consists of estimation of the system

$$r_{t,t+\tau} = X_t\beta + u_{t+\tau} \qquad u_{t+\tau} \sim N(0, \sigma_{t+\tau}^2) \qquad (5)$$

$$\sqrt{\pi/2}\,|\hat{u}_{t+\tau}| = X_t\gamma + \varepsilon_{t+\tau}, \qquad (6)$$

where $\sqrt{\pi/2}\,|\hat{u}_{t+\tau}|$ is an unbiased estimate of the standard deviation of $u_{t+\tau}$ assuming normality.[9] Consider the estimation problem for monthly returns when there is no problem with overlapping data. A two-step OLS technique, of first regressing returns on the explanatory variables and then regressing the transform of the fitted residuals on the same variables, produces consistent coefficient estimates, but incorrect standard errors. A simultaneous, generalized method of moments (GMM) estimation (Hansen (1982)) of the full system provides asymptotically correct and heteroscedasticity-consistent standard errors, and it permits us to make inferences about the joint significance of the full set of coefficient estimates. Consequently, we employ this technique for the monthly data using the moment conditions

$$E\begin{bmatrix} (r_{t,t+\tau} - X_t\beta)X_t' \\ \left(\sqrt{\pi/2}\,|r_{t,t+\tau} - X_t\beta| - X_t\gamma\right)X_t' \end{bmatrix} = 0. \qquad (7)$$

The estimation is exactly identified with parameter estimates identical to those from OLS.

A second approach to estimating the linear specification uses daily returns to estimate the volatility, following the methodology of French, Schwert, and Stambaugh (1987), and Schwert (1989) and as discussed in Section II. The equation

$$s_{t,t+\tau} = X_t\gamma + \varepsilon_{t+\tau} \qquad (8)$$

is estimated by GMM using the moment conditions

$$E[(s_{t,t+\tau} - X_t\gamma)X_t'] = 0, \qquad (9)$$

where $s_{t,t+\tau}$ is the estimate of the monthly standard deviation from daily

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[9] One could easily consider alternative specifications for the volatility such as $\hat{u}_{t+\tau}^2$ or $\ln(\hat{u}_{t+\tau}^2)$; however, the assumption of normality provides a convenient interpretation of the transformation of the absolute value of the residual as an estimate of the standard deviation. The empirical results are qualitatively similar for all three specifications.

returns from equation (2). A variant on equation (8),

$$s_{t,t+\tau} = \alpha s_{t-\tau,t} + X_t\gamma + \varepsilon_{t+\tau}, \tag{10}$$

attempts to exploit any autoregressive conditional heteroscedasticity that may exist in the return data by including a lagged value of the volatility in the conditional volatility regression. The estimation procedure using daily data has the advantage of exploiting additional information in daily returns. Its major disadvantages are that it implicitly imposes constant conditional expected daily returns over the return horizon and that daily data may contain measurement errors due to phenomena such as bid-ask bounce and nonsynchronous trading.

The approaches above provide asymptotically correct standard errors for monthly data. For longer return horizons, overlapping data are available, and these data introduce serial correlation in the regression errors. For all quarterly and annual regressions, heteroscedasticity-consistent standard errors are calculated using the methodology in Hodrick (1992).

B. Empirical Results

Table I reports the results from the estimation of the conditional first and second moments of excess returns on the CRSP value-weighted index for horizons of one month, one quarter, and one year, using the model in equations (5) and (6).[10] The results indicate that expected returns for monthly and quarterly horizons are positively related to the default spread and the dividend yield but negatively related to the one-year Treasury yield. For annual returns, the default spread is no longer significant at the 5 percent level. These conclusions are consistent with the empirical evidence in Fama and French (1989), although the inclusion of all the variables appears to improve the fit relative to previous studies. The χ^2 statistics, which test whether the coefficients are significantly different from zero, are uniformly large, leaving little doubt of return predictability. One interesting result is that, although the R^2s increase dramatically for longer horizons, the χ^2 statistics decline, suggesting that the monthly returns actually present stronger evidence against a hypothesis of unpredictable returns than do quarterly or annual returns.

The volatility of returns is also predictable and, again, the shorter horizons show stronger evidence of this predictability. The conditional volatility is positively related to the commercial paper–Treasury spread, especially for monthly and quarterly returns, while longer horizons indicate a positive relation between volatility and the one-year Treasury yield. The former result is new, while the latter is consistent with Campbell (1987) and Glosten, Jagannathan, and Runkle (1993). Multicollinearity might reduce the significance of individual coefficients, and, in fact, the estimates for the coefficients on the one-year Treasury yield and the default spread have a correlation of about -0.7 using the monthly data. Nevertheless, a hypothesis that the

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[10] Results for the CRSP equal-weighted index are very similar to those reported in Table I.

Table I

Estimation of Conditional First and Second Moments of Returns

Regressions of monthly, quarterly, and annual, continuously compounded, excess stock returns and volatilities for the CRSP value-weighted index on lagged explanatory variables for the periods May 1953 to April 1989, July 1953 to April 1989, and April 1954 to April 1989, respectively. The conditioning variables are the Baa-Aaa spread (DEF), the commercial paper-Treasury spread (CP), the one-year Treasury yield (1YR), and the dividend yield (DIV). The model is given in equations (5) and (6). Heteroscedasticity-consistent standard errors are in parentheses. Quarterly and annual regressions use overlapping data, and standard errors are computed using the methodology in Hodrick (1992). DW is the Durbin-Watson statistic. The $\chi^2(4)$ statistic tests the hypothesis that all the coefficients except the constant are zero. The number in brackets is the probability that a $\chi^2(4)$ will exceed the value of the statistic.

	Explanatory Variables				Adj.			
Constant	DEF	CP	1YR	DIV	R^2	R^2	DW	$\chi^2(4)$
			Mean					
			Monthly					
−1.820	2.295**	−0.313	−0.469**	0.794**	0.074	0.082	1.991	39
(1.003)	(0.618)	(0.720)	(0.086)	(0.269)				[0.00]
			Quarterly					
−5.991*	5.817**	−0.242	−1.270*	2.430**	0.176	0.184	0.764	34
(2.863)	(1.777)	(1.921)	(0.256)	(0.821)				[0.00]
			Annual					
−28.037**	9.806	3.970	−3.410**	10.546**	0.412	0.417	0.249	26
(10.537)	(5.974)	(3.928)	(0.933)	(2.908)				[0.00]
			Volatility					
			Monthly					
1.884**	−0.044	2.070**	0.104	−0.029	0.099	0.107	2.071	34
(0.734)	(0.500)	(0.476)	(0.067)	(0.217)				[0.00]
			Quarterly					
4.979**	−0.364	2.466**	0.274*	−0.378	0.059	0.067	1.263	19
(1.777)	(1.007)	(0.793)	(0.136)	(0.579)				[0.00]
			Annual					
16.197**	−1.559	0.277	0.822**	−1.830	0.053	0.062	0.635	8.6
(3.340)	(2.435)	(1.548)	(0.332)	(1.110)				[0.07]

*Significant at the 5 percent level.
**Significant at the 1 percent level.

coefficients on both these variables are zero cannot be rejected at standard significance levels. Although in most cases neither the Baa-Aaa spread nor the dividend yield enter significantly in the volatility regressions, both are strongly significant in univariate regressions.[11] This fact indicates that, although they both possess predictive power, the commercial paper–Treasury spread and the one-year Treasury yield are better predictors.[12]

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[11] In the interests of saving space, the univariate regression results are not reported.

This initial empirical evidence is consistent with the theoretical motivations behind the choice of the explanatory variables.[13] The default spread and dividend yield may proxy for variations in the risk premium, and they are positively related to expected returns. The Treasury yield picks up changes in inflationary expectations, and its negative relation to expected returns is consistent with explanations based on money demand theory in Fama (1981) and Kaul (1987). The fact that the inclusion of a measure of inflation, such as the producer price index, instead of the Treasury yield produces similar results further supports these explanations. To the extent that the financial predictor variables proxy for underlying macroeconomic factors, one would also expect them to be related to conditional volatility. The significance of the commercial paper–Treasury spread introduces the possibility that monetary policy may also play an important role in determining return volatility.

A final interesting feature of the empirical results is that the residuals from the two moment equations have a contemporaneous correlation of −0.17. A negative relation between unanticipated returns and unanticipated volatility is consistent with a positive relation between expected returns and conditional volatility, although the former does not imply the latter as pointed out in Glosten, Jagannathan, and Runkle (1988). Consider that an unanticipated increase in volatility may cause agents to increase their required returns; in turn, this increase in expected returns causes a drop in current price and an unanticipated negative return.

Estimations of the volatility of returns using daily data provide empirical results very similar to those discussed above. The most noticeable difference is that the coefficient on the default spread in the volatility regression is significant at the 1 percent level for monthly and quarterly horizons. This result is consistent with results in Schwert (1989) and with the univariate regressions. The only other important difference is that the residuals in the volatility equation for monthly data show first-order autocorrelation, which affects the standard errors; therefore, the regressions are reestimated with the inclusion of lagged volatility as an additional independent variable. Table II reports the results, and the coefficients on lagged volatility appear in the column labeled S. For monthly data the coefficient on lagged volatility is significant at the 1 percent level, and its inclusion eliminates the first-order autocorrelation in the residuals. Inferences about the other coefficients remain virtually unchanged, and the yield variables continue to have significant predictive power.

One important point to note from Tables I and II is that the coefficients in the mean regressions differ markedly from those in the volatility regressions. These results suggest that the conditional mean is not proportional to the contemporaneous conditional volatility and, further, that these two moments may not be positively related. One way to test the sign and significance of the contemporaneous relation between the conditional moments is to include the

[12] The results are robust to including more lags of the independent variable, and the volatility regressions are insensitive to the specification of the conditional mean.

Table II
Estimation of Conditional Volatility of Returns Using Daily Data

Regressions of monthly, quarterly, and annual CRSP value-weighted return volatilities (estimated using daily data) on lagged explanatory variables for the periods August 1962 to April 1989, December 1962 to April 1989, and June 1964 to April 1989. The conditioning variables are lagged volatility (S), the Baa-Aaa spread (DEF), the commercial paper–Treasury spread (CP), the one-year Treasury yield (1YR), and the dividend yield (DIV). The model is given in equation (10). Heteroscedasticity-consistent standard errors are in parentheses. Quarterly and annual regressions use overlapping data, and standard errors are computed using the methodology in Hodrick (1992). DW is the Durbin-Watson statistic. The $\chi^2(5)$ statistic tests the hypothesis that all the coefficients except the constant are zero. The number in brackets is the probability that a $\chi^2(5)$ will exceed the value of the statistic.

	Explanatory Variables					Adj. R^2	R^2	DW	$\chi^2(5)$
Constant	S	DEF	CP	1YR	DIV				
				Volatility					
				Monthly					
1.261*	0.289**	0.577**	1.055**	0.084	−0.231	0.311	0.321	2.091	158
(0.591)	(0.090)	(0.198)	(0.198)	(0.057)	(0.273)				[0.00]
				Quarterly					
3.204*	0.200**	1.021*	2.098**	0.247	−0.746	0.341	0.352	0.633	183
(1.604)	(0.043)	(0.516)	(0.411)	(0.129)	(0.691)				[0.00]
				Annual					
10.087*	0.165	1.306	2.657**	0.739*	−2.114	0.263	0.276	0.150	58
(4.567)	(0.132)	(2.247)	(0.621)	(0.329)	(1.691)				[0.00]

*Significant at the 5 percent level.
**Significant at the 1 percent level.

estimated conditional volatility as a regressor in the mean regression as follows:

$$r_{t,t+1} = \delta\hat{\sigma}_{t,t+1} + X_t\beta + u_{t+1}, \tag{11}$$

where $\hat{\sigma}_{t,t+1}$ is the measure of volatility based on time t information. When $\hat{\sigma}_{t,t+1}$ is the estimated volatility from equation (6), using monthly, excess returns, the estimated coefficient $\hat{\delta}$ is -0.97 with a standard error of 0.60, suggesting, if anything, a negative relation. A test of whether the measure of the conditional volatility subsumes all the information in the time t information set can easily be rejected at the 1 percent level.[14] Alternative measures of conditional volatility also lead to insignificant coefficients without diminishing the predictive power of the other explanatory variables.

Sections IV and V examine the relation between the mean and volatility more closely, but it is instructive to look first at the fitted conditional moment series. Figure 1 shows a graph of the fitted conditional mean and volatility based on monthly data and the coefficients in Table I. Business cycle peaks

[13] See the articles cited in Section I for a more extensive discussion of these issues.
[14] The test of $H_0: \beta = 0$ gives a $\chi^2(4)$ test statistic with a value of 29.
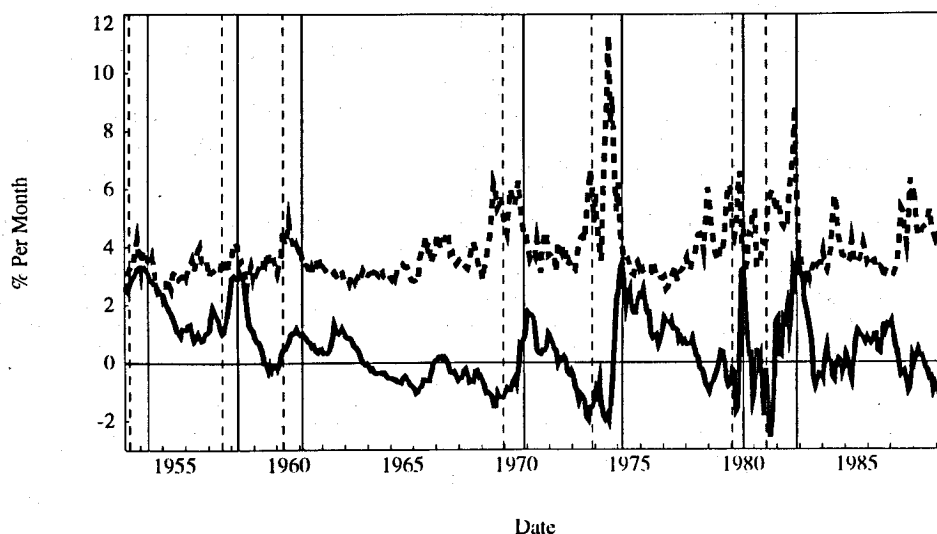
Date

Figure 1. Fitted conditional moments of the monthly value-weighted return. The fitted conditional mean (*solid line*) and volatility (*dashed line*) of the continuously compounded, monthly, excess return on the value-weighted index based on coefficient estimates in Table I for the period May 1953 to April 1989. Business cycle peaks are marked by *dashed lines*, business cycle troughs are marked by *solid lines*.

and troughs are marked by *dashed* and *solid vertical lines*, respectively. Results based on the coefficients in Table II look very similar. The fitted moments for longer horizons, appropriately adjusted for the length of the holding period, also exhibit the same time series patterns.

The graph suggests that the conditional moments exhibit cyclical behavior. The two series are highly autocorrelated, with first-order autocorrelations of 0.93 and 0.86 for the mean and volatility respectively. The expected return seems to reach a maximum at the trough of the business cycle and reach a minimum before, or at, the peak of the business cycle. Expected returns appear to decrease during economic expansions and increase during economic contractions. In contrast, the conditional volatility appears to reach a maximum earlier in the business cycle, at or slightly after the peak in the cycle, and to reach a minimum just after the business cycle trough. Of course, any attempts to link the conditional moments to the business cycle must be interpreted with caution. There are only seven cycles within the sample period, and the peaks and troughs of the cycle are determined using ex post data. Nevertheless, the overall picture is of a volatility cycle that leads the expected return cycle.

C. Implications of Offset Cycles

To get a better picture of the effect of offset cycles in the mean and volatility of returns, consider the schematic representation in Figure 2. This

figure shows smooth symmetric cycles for both conditional moments, where the volatility leads the expected return by one quarter of a cycle. Note that a single cycle can be divided up into four regions where the contemporaneous correlation between the moments varies from positive to negative. If an estimate of this contemporaneous correlation were taken over the full cycle, then it would be close to zero. The correlation between the conditional expected return and lagged values of the conditional volatility increases as the offset increases from zero to one quarter of a cycle. From an offset of one quarter of a cycle to one half of a cycle the correlation decreases again to zero. The pattern of correlations between the mean and led values of the volatility is similar, except that it is negative for offsets between zero and one half of a cycle. The magnitude of this negative correlation again reaches a maximum for an offset of one quarter of a cycle.

   The correlation patterns implied by the offset cycles in Figure 2 differ from those implied by coincident cycles. First, coincident cycles imply symmetric offset correlations, i.e., the correlation between the conditional mean and the conditional volatility lagged by a fixed number of months equals the correlation between the conditional mean and the conditional volatility led by an equal number of months. Second, cycles aligned in time imply positive and stable contemporaneous correlations measured over short horizons. An alternative hypothesis, that the conditional moments are independent, implies
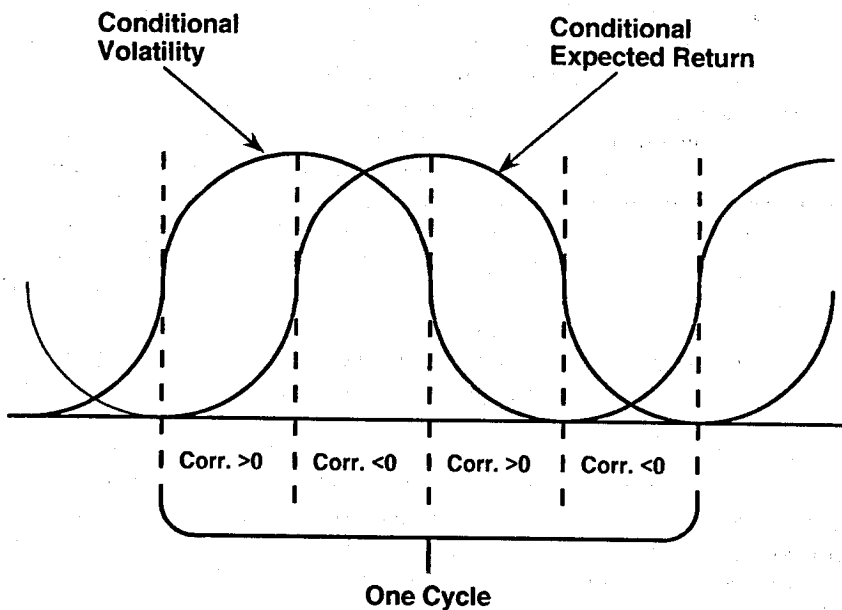

Figure 2. A schematic representation of the conditional first and second moments of returns. A schematic representation of the proposed cyclical variation in the conditional mean and the conditional volatility of returns and the implied contemporaneous correlations between the moments.

that both offset correlations and contemporaneous correlations measured over short horizons should be zero.

A second approach to understanding the relative time series properties of the two conditional moment series is a vector autoregressive analysis. If the moments exhibit independent variation, then lagged values of one moment should not appear significantly in the equation for the other moment. In other words, the off-diagonals in the coefficient matrices of an estimated vector autoregression should be insignificantly different from zero. If the cycles coincide, then the coefficient matrices should exhibit symmetry. If the cycles are offset, as in Figure 2, then one might expect the off-diagonals to vary across the two equations.

IV. Correlations Between the Conditional Moments of Returns

A. Estimation Techniques

The empirical results in Section III.*B* tell us a great deal about the individual time series properties of the estimated conditional moments of returns. Although the plot of these moments in Figure 1 also conveys information about their joint time series properties, this aspect of the data deserves further investigation. Previous studies (French, Schwert, and Stambaugh (1987), Campbell (1987), Kandel and Stambaugh (1990), and Glosten, Jagannathan, and Runkle (1993)) focus on the contemporaneous correlation between the moments in an attempt to assess if a positive relation exists. This article expands the examination to look at correlations between conditional expected returns and lagged values of the conditional volatility, correlations between conditional expected returns and led values of the conditional volatility, and the contemporaneous correlation measured over short horizons. As discussed above, the offset cyclical variation represented in Figure 2 has implications for these correlations. Specifically, offset correlations should exhibit asymmetry, and contemporaneous correlations measured over short horizons should exhibit significant time variation. These predictions are investigated in this section.

Estimated correlations are easily calculated from the fitted series based on the coefficient estimates in Section III.*B*. Let $m_{t,t+1}$ and $v_{t,t+1}$ denote the estimated conditional mean and volatility from time t to $t + 1$ based on information at time t. Then the estimates of the correlations between $m_{t,t+1}$ and $v_{t+s,t+1+s}$, varying s from -24 months to 24 months, give a good picture of the relative time series behavior of the conditional moments. Short-horizon, contemporaneous correlations are calculated over 17-month rolling periods. The choice of horizon is somewhat arbitrary, but 17 months balances the need for a sufficiently long period to get reasonably accurate estimates with the need for a short enough period to pick up time variations over the length of a business cycle. Results look similar for horizons ranging from 11 to 23 months. Note that the coefficients are not estimated separately over each

17-month period, but are constant over the sample. The contemporaneous correlations are calculated by simply selecting a 17-month time period and calculating the sample correlation.

While calculating these correlation estimates is simple, assessing their statistical significance poses greater difficulty. The offset correlations for different values of s are highly correlated because of the autocorrelation in the fitted moments, so the joint significance of a series of estimates cannot be treated as a set of independent tests. The same logic applies to the rolling contemporaneous correlations, with the additional caveat that overlapping data is used.[15] A further concern is that the fitted moments are themselves estimates, based on the estimated coefficients from the initial regressions. In order to calculate asymptotically correct standard errors, the correlations are estimated jointly with the original estimation using GMM.[16] This technique has the added benefit of providing consistent estimates of the covariance matrix of the calculated correlations, permitting statistical tests of the significance of the correlation patterns. The issue of the small sample properties of the estimates is postponed until the bootstrapping analysis in Section VI.*B*.

B. *Empirical Results*

The results reported in this section focus on the conditional moments of the continuously compounded, monthly, excess returns on the value-weighted index. These moments are estimated over the period May 1953 to April 1989 using the model in equations (5) and (6), and Table I reports the coefficient estimates. Similar patterns are observed for longer horizons and using volatilities estimated from daily data. The *bar chart* at the top of Figure 3 shows the correlations between the conditional expected return and led and lagged values of the conditional volatility. The numbers on the x-axis refer to the number of months by which the fitted conditional volatility lags or leads the expected return. For example, above the number -12 is the correlation between $m_{t,t+1}$ and $v_{t-12,t-11}$. The contemporaneous correlation over the full sample is reported above the number 0. The standard errors of these correlations are shown by *solid lines*.

Note that this figure illustrates only the time series properties of the two fitted moment series. It does not mean that volatility has predictive power for returns (or vice versa) over and above the explanatory variables already used. For all noncontemporaneous correlations, the two moments are conditioned on different information sets.

The *bar chart*, however, provides a fairly dramatic demonstration of time series behavior. The contemporaneous correlation between the conditional mean and conditional volatility of returns is -0.34, with a standard error of

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[15] Richardson (1993) makes this same point in the context of tests of stock return autocorrelations at different horizons.

[16] The estimates are identical to those from computing simple correlations of the fitted moments. GMM serves only to provide appropriate standard errors and covariances between the estimates.
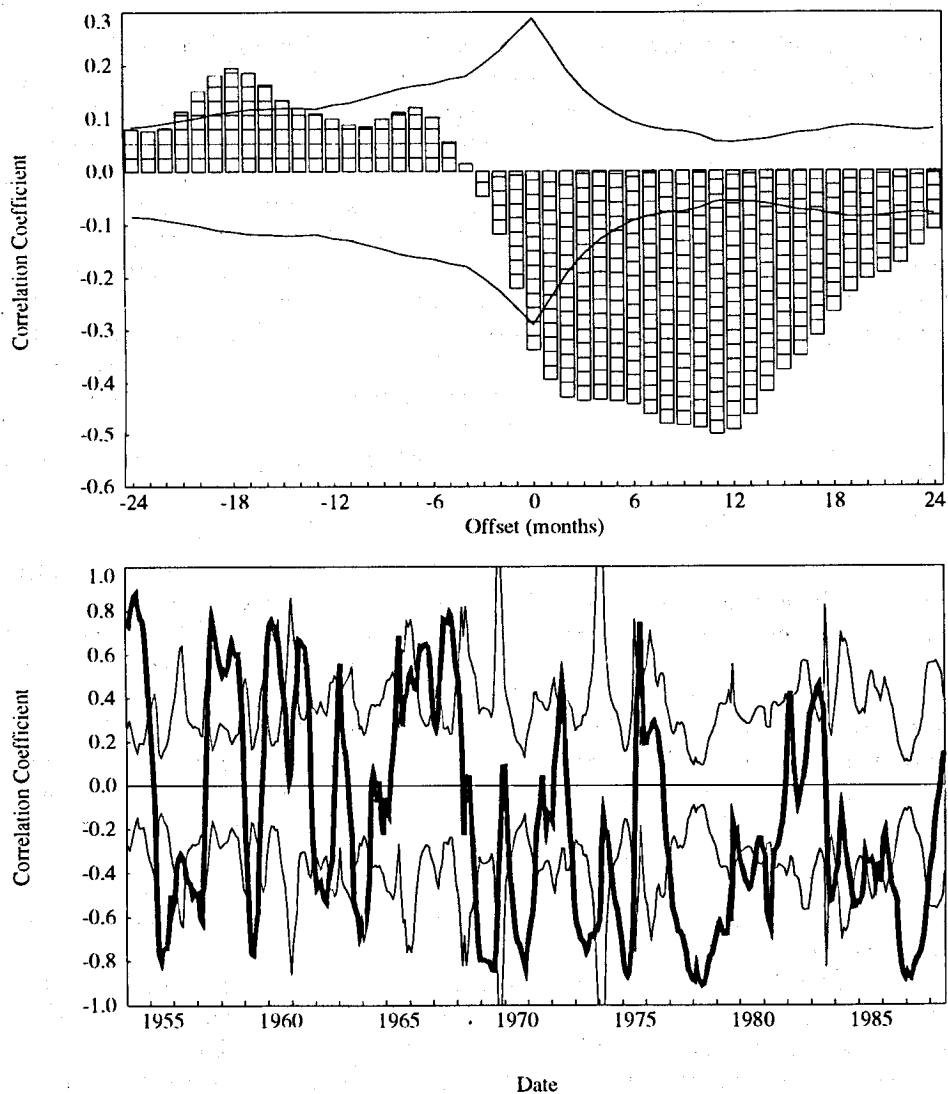
The Journal of Finance

Figure 3. Correlations between the conditional mean and conditional volatility of returns. Correlations based on the monthly, continuously compounded, excess returns on the value-weighted index. The model for the moments is equations (5) and (6), and the coefficient estimates are from Table I. *Top*: correlations between the fitted conditional expected return and lagged and led values of the conditional volatility (*thin solid lines* represent standard errors). *Bottom*: rolling, 17-month, contemporaneous correlations between the conditional first and second moments (*thin solid lines* represent standard errors). Business cycle peaks are marked by *dashed lines*, business cycle troughs are marked by *solid lines*.

0.29. The correlations between the expected return and all leads of the conditional volatility up to 24 months are negative, and the minimum value is −0.50 (standard error 0.06) at an offset of 11 months. In contrast, the correlations of the expected returns with lagged values of the conditional volatility are all positive after an offset of 3 months and reach a maximum of 0.20 (standard error 0.11) at an offset of 18 months. Although the contemporaneous correlation is insignificantly different from zero, many of the other individual estimates are significantly positive or negative. It is difficult, however, to assess their joint significance from the graph, because the underlying series and the correlation estimates are highly correlated. Nevertheless, the pattern confirms the results of Section III.*B* that the cycle for volatility tends to lead the expected return cycle by a period of between 1 and 2 years. The distance of 29 months between the minimum and maximum correlation is an estimate of the average length of half a cycle.

The simplest test for the significance of the apparent asymmetric pattern is a joint test of the null hypothesis that the correlation at a given lag equals the correlation for the same length lead. More formally, we wish to test

$$H_0: \rho(-1) = \rho(1), \qquad \rho(-2) = \rho(2), \dots \rho(-24) = \rho(24), \qquad (12)$$

where $\rho(s)$ is the correlation between $m_{t,t+1}$ and $v_{t+s,t+s+1}$. This hypothesis is a set of 24 restrictions, giving a $\chi^2(24)$ test statistic under the null hypothesis. The test statistic has a value of 53.4 with a corresponding *p*-value of 0.001, so the null hypothesis of symmetry can be rejected at all standard significance levels. The strength of this rejection is attributable, at least in part, to the fact that the correlation estimates tend to be positively correlated. As a result, differences in the correlations are strong evidence of asymmetry. In turn, this asymmetry is evidence against a positive contemporaneous relation between expected returns and conditional volatility. The fact that volatility leads expected returns should not be interpreted as evidence that agents require compensation for volatility and that markets are reacting with some delay. The apparent 7- to 18-month offset between the two conditional moments exceeds any reasonable delay that may exist.

The plot at the bottom of Figure 3 shows the rolling, 17-month, contemporaneous correlations between the fitted moments. The date on the *x*-axis refers to the date at the middle of the 17-month period, so the correlation reported above the date January 1970 is for the period May 1969 to September 1970. The rolling correlation exhibits a large degree of time variation, fluctuating from 0.88 to −0.91. The standard errors (*thin solid lines*) also fluctuate widely from 0.09 to 1.74. The rolling correlations average, over the full sample, to approximately the value reported for the contemporaneous correlation in the bar chart, and systematic variation is again consistent with noncoincident cycles for the two conditional moments.

A natural test of time variation is to test if the contemporaneous correlation is equal over each 17-month rolling horizon. Unfortunately, such a test involves estimating two means, two variances and a correlation for each of

the 416 overlapping periods and inverting the covariance matrix for the 416 correlation estimates. A more computationally feasible strategy, which discards some information, is to test for equality over a subset of the correlations. Specifically, we test the null hypothesis

$$H_0: \rho(1/54) = \rho(6/55) = \cdots = \rho(1/88), \tag{13}$$

where $\rho(1/54)$ is the 17-month correlation centered at January 1954, $\rho(6/55)$ is centered 17 months later at June 1955, etc. This formulation uses all the data in 17-month, nonoverlapping intervals, with the exception of the last 7 months, which are discarded.[17] The null hypothesis is a set of 24 restrictions with a $\chi^2(24)$ test statistic. The statistic has a value of 487.9 with a corresponding p-value of 0.000, so the null hypothesis of a constant contemporaneous correlation can be rejected at all standard significance levels. These results cast doubt on the practice of modeling expected returns as a constant multiple of volatility. To the extent that these variables are related, the relation appears to change dramatically over time.

V. A Vector Autoregressive Analysis of the Conditional Moments

A different technique for assessing the time series properties of the expectation and volatility of returns involves estimating a vector autoregression (VAR) on these fitted moments. The model is

$$X_t = A_0 + \sum_{l=1}^{q} A_l X_{t-l} + \varepsilon_t, \tag{14}$$

$$X_t = \begin{bmatrix} m_{t-1,t} \\ v_{t-1,t} \end{bmatrix},$$

where $m_{t-1,t}$ and $v_{t-1,t}$ are the fitted mean and volatility respectively from $t-1$ to t, and q is the order of the VAR. We will only consider the continuously compounded, monthly, excess return. The moments are estimated over the period May 1953 to April 1989 using the model in equations (5) and (6), which is estimated jointly with the VAR in equation (14). This joint estimation ensures that the standard errors in the VAR account for the estimation error in the fitted moments. Recall that offset cyclical behavior in the conditional moments may be reflected in asymmetry in the off-diagonals of the coefficient matrices A_l. In addition, the correlation between the innovations in the conditional moments also provides evidence as to whether these moments move together over time.

There is no theoretically motivated choice of lag length for the VAR, and the final determination depends on considerations of parsimony and goodness-of-fit. The standard criterion for determining the appropriate order is to

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[17] Similar results are obtained for different sets of correlations.

Table III

VAR(1) Estimation of the Fitted Conditional Expectation and Volatility of Returns

A VAR(1) estimation of the conditional moments of monthly, continuously compounded, excess returns for the CRSP value-weighted index for the period June 1953 to April 1989. $m_{t-1,t}$ and $v_{t-1,t}$ are the fitted mean and volatility from the estimation of equations (5) and (6). The VAR model is given in equation (14). Heteroscedasticity-consistent standard errors are in parentheses.

	Explanatory Variables			Adj. R^2	R^2
	Constant	$m_{t-2,t-1}$	$v_{t-2,t-1}$		
$m_{t-1,t}$	0.459	0.977**	0.118*	0.895	0.895
	(0.239)	(0.038)	(0.058)		
$v_{t-1,t}$	(0.743**	−0.107*	0.824**	0.759	0.760
	(0.258)	(0.045)	(0.067)		

*Significant at the 5 percent level.
**Significant at the 1 percent level.

choose the lag length that minimizes the sum of squared residuals plus a penalty function that increases in the number of parameters. We employ the Bayesian estimation criterion function as suggested in Geweke and Reese (1981), and this function is minimized for a VAR of order 1. Table III presents results for the estimation of a VAR(1). As expected, given the high autocorrelation in the fitted moment series, the coefficients on the lagged dependent variables are positive and close to 1. Although the coefficients on the lag of the other moment are smaller in magnitude, they appear significantly in both equations. Of greater interest, the lagged conditional mean has a negative coefficient in the volatility equation, while lagged volatility has a positive coefficient in the mean equation. A test of the equality of these coefficients ($\chi^2(1)$ under the null) has a value of 15.5 with a p-value of 0.00, so the hypothesis that the off-diagonals are equal can be rejected at all standard levels of significance. Finally, the two series of residuals have a correlation of −0.25.

The fact that the unanticipated movements in the conditional mean and conditional volatility tend to be in opposite directions contrasts with the intuition of a positive relation between risk and return. However, there is evidence of the volatility feedback effect considered in French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1992). In particular, the positive coefficient on lagged volatility in the mean equation suggests that increases in volatility are associated with subsequent increases in expected returns. Note, however, that increases in expected returns are associated with subsequent decreases in volatility, as indicated by the negative coefficient on the lagged mean in the volatility equation. This second effect should not be confused with predictive asymmetry in which *realized* stock returns are negatively correlated with future volatility (Black (1976)).

To further assess the implications of the estimated VAR(1) for the time series properties of the expectation and volatility of returns, we conduct an analysis of the effects of shocks to the moments on future levels of the moments. The long-run, steady state, mean of the VAR(1) is $(I - A_1)^{-1}A_0$, which equals $[0.415 \quad 3.964]'$ in this case. To account for the negative correlation between the innovations, the two processes are orthogonalized using a Cholesky decomposition of the covariance matrix of the errors. In particular, if $\text{Var}(\varepsilon_t) = \Sigma$, where ε_t is the error vector for the process X_t, as described in equation (14), then define the upper triangular matrix Ω such that $\Omega'\Omega = \Sigma$. Further, define the orthogonalized process X_t^*, which has error vector u_t, such that $\text{Var}(u_t) = I$. In this case, $\Omega'X_t^* = X_t$ and $\text{Var}(\Omega'u_t) = \Sigma$. We can now think of analyzing independent shocks of magnitude 1 to the errors of the orthogonalized processes. Instead, note that a shock of magnitude 1 to the first equation gives an impulse vector $[\sigma_1 \quad \rho\sigma_2]'$ to the untransformed processes, where σ_1, σ_2, and ρ are defined in terms of the elements of Σ.

Assuming that both moments are initially at their long-run means, Figure 4 shows the effect of the two impulse vectors $[0.395 \quad -0.131]'$ and $[-0.097 \quad 0.533]'$ on the VAR(1), where the *solid* and *dashed lines* represent the conditional mean and volatility respectively. These two vectors correspond to a shock of magnitude 1 to the process used as the basis for the orthogonalization, where, in the first case, the conditional mean is used as the basis, and, in the second case, the conditional volatility is used as the basis. The effects are displayed as deviations from the long-run mean for a period of 47 months after the initial shock.

The graph at the top of Figure 4 shows that the positive shock to the conditional mean dies out after approximately 22 months, with a half-life of approximately 8 months. It magnifies the negative shock to the conditional volatility, which peaks at 5 months and dies out after approximately 30 months. After the initial impact, both the moments decrease until month 5, and they then move in opposite directions as they converge to the steady state. One of the most striking features of the graph is that the conditional mean and conditional volatility are on opposite sides of their long-run averages. In other words, this innovation generates high expected returns and low volatilities, or vice versa for a shock of the opposite sign.

In contrast, a positive shock to the conditional volatility (bottom of Figure 4) results in a positive movement in the conditional mean, which tends to offset the initial negative shock. Consequently, after the shock, the moments move in opposite directions. The increase in the mean has an additional effect on the volatility, causing it to overshoot its long-run average. The feedback effect through the mean also causes volatility to revert to its long-run average level much faster than the conditional mean, giving a half-life of about 4 months. This estimate falls between the estimates from Poterba and Summers (1986) and Campbell and Hentschel (1992). Overall, the VAR estimation and the impulse-response analysis indicate decidedly asymmetric effects, suggesting that the cycles for the conditional expectation and volatility of returns are offset rather than coincident.
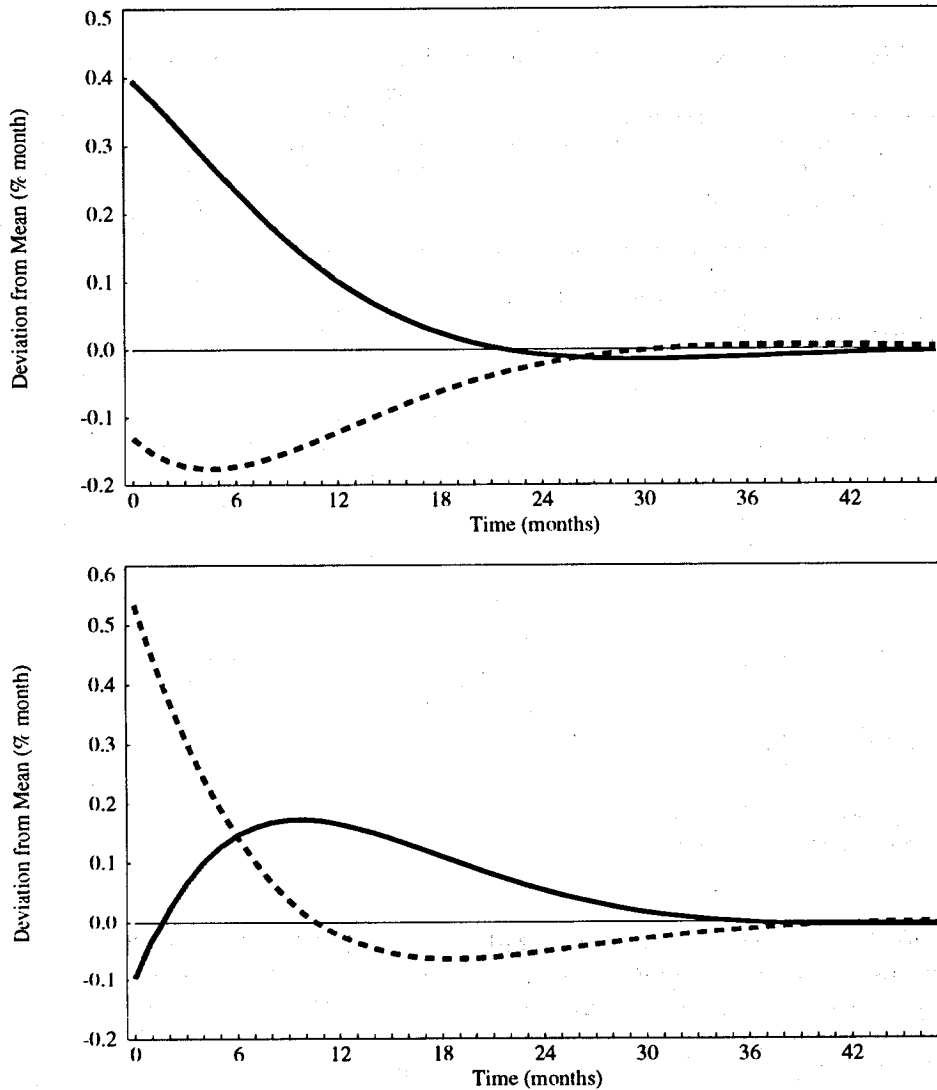
Figure 4. Impulse-response analysis based on VAR(1) estimation of the conditional moments. The response of the conditional expectation and volatility of returns to shocks to these conditional moments. The *solid* and *dashed lines* represent the conditional mean and conditional volatility respectively. Monthly changes are based on the VAR(1) estimation in Table III. *Top*: impulse vector [0.395   −0.131]′. *Bottom*: impulse vector [−0.097   0.533]′.

VI. Robustness and Significance

A. Subperiod Analysis

Table IV considers the estimation of the first and second moments of returns for the two 18-year subperiods May 1953 to April 1971 and May 1971

Table IV

Subperiod Estimation of Conditional 1st and 2nd Moments of Monthly Returns

Regressions of monthly, continuously compounded, excess stock returns and volatilities for the CRSP value-weighted index on lagged explanatory variables for the two subperiods June 1953 to April 1971 and May 1971 to April 1989. The conditioning variables are the Baa-Aaa spread (DEF), the commercial paper-Treasury spread (CP), the one-year Treasury yield (1YR), and the dividend yield (DIV). The model is given in equations (5) and (6). Heteroscedasticity-consistent standard errors are in parentheses. DW is the Durbin-Watson statistic.

Explanatory Variables					Adj.		
Constant	DEF	CP	1YR	DIV	R^2	R^2	DW
			Mean				
			5/53–4/71				
0.234	3.095**	−0.941	−0.502*	0.255	0.065	0.082	1.904
(1.980)	(1.127)	(1.232)	(0.224)	(0.406)			
			5/71–4/89				
−3.741*	2.306**	−0.118	−0.607**	1.460**	0.080	0.097	2.054
(1.694)	(0.772)	(0.876)	(0.169)	(0.561)			
			Volatility				
			5/53–4/71				
0.842	−0.286	1.266	0.286	0.228	0.046	0.064	2.033
(1.446)	(0.918)	(0.760)	(0.159)	(0.294)			
			5/71–4/89				
2.010	0.095	2.237**	−0.021	0.130	0.101	0.118	2.061
(1.379)	(0.639)	(0.523)	(0.119)	(0.467)			

* Significant at the 5 percent level.
** Significant at the 1 percent level.

to April 1989. For this analysis, only the results for the monthly, excess returns are reported, and the model is given in equations (5) and (6). The subperiods are estimated simultaneously using GMM, so that the stability of the coefficients can be tested.

In both subperiods, the coefficients in the mean regressions on the Baa-Aaa spread and the one-year Treasury yield are significant and of the same sign as in the full sample. The dividend yield only appears significantly in the second subperiod, but again the signs of the coefficients are the same as in the full sample. A $\chi^2(5)$ test of the equality of the coefficients across the two subperiods has a value of 4.9. Consequently, the hypothesis of stable coefficients cannot be rejected. For the volatility regressions, the coefficient estimates are consistent with the results for the full sample. The coefficient on the commercial paper–Treasury spread is significant in the second subperiod and has the same sign in both subperiods. A $\chi^2(5)$ statistic of 3.6 indicates that the stability of the coefficients across the subperiods cannot be rejected.

The offset correlation patterns are similar for the two subperiods, with negative correlations for leads of the volatility and positive correlations for

lags of the volatility. The major difference between the two subperiods is that the correlations are lower for lagged volatility and more negative for led volatility in the first subperiod. The rolling, 17-month, contemporaneous correlations reveal the same shifts, with negative correlations more prevalent in the first subperiod. Overall, some evidence of instability across the two subperiods exists, but the major result, that the cycle for volatility appears to lead the expected return cycle, is preserved.

B. Bootstrapping

One method to assess the significance of the covariation patterns in Figure 3 and the small sample properties of the estimates in Table I is to use bootstrapping (Efron (1982, Chap. 5)). The parametric bootstrapping procedure involves drawing a series of 432 random normal variates, u_t^*, from a normal distribution, $N(0, \sigma_t^*)$.[18] These random numbers, a vector of coefficients β^*, and the data on the original explanatory variables are then used to construct a series of returns using the model in equation (5), i.e., $r_{t,t+1}^* = X_t \beta^* + u_{t+1}^*$. The full system in equations (5) and (6) is then reestimated using the same procedure used to generate the estimates in Table I, and correlations are computed.

The key variables in the bootstrapping are the coefficient vector β^* and the volatility of the errors σ_t^*. β^* is set equal to the coefficient estimates for the monthly, excess returns in Table I. $\sigma_t^* = X_t \gamma^*$, where γ^* is chosen to match the coefficient estimates from the volatility estimation in Table I. Since the form of the conditional heteroscedasticity is assumed to match the model, we would expect to see correlation patterns close to those in Figure 3.

Table V reports the means and standard deviations of the coefficient estimates and R^2s based on 1,000 replications. The standard deviations are cross-sectional standard deviations, across the 1,000 replications. As expected, the average coefficients from the mean equation are very close to the actual parameters used to generate the data. The cross-sectional standard deviations are also close to the original standard errors. The returns are conditionally heteroscedastic, and the estimation picks up this heteroscedasticity well, with average coefficients and standard deviations consistent with the results from the data. R^2 values are close to those from the estimation.

The main purpose of the bootstrapping experiment is to assess the significance and robustness of the correlation estimates in small samples. Figure 5 shows the average offset correlations and rolling contemporaneous correlations over the 1,000 replications in a format similar to Figure 3. The *thin solid lines* in the *bar chart* and the *line graph* are the cross-sectional standard deviations of these correlation estimates.

Looking first at the correlation between the expected return and lagged and led values of the conditional volatility, the pattern of the average

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[18] Nonparametric bootstrapping, where the residuals from the original estimation are resampled, produces similar results.

Table V
Bootstrapping Results

Results from a parametric bootstrap estimation of the model in equations (5) and (6) based on 1,000 replications. The row "True" reports the actual coefficients used to generate the data, and the row "Est." reports the mean estimated coefficients with standard deviations in parentheses.

	Constant	DEF	CP	1YR	DIV	R^2
			Explanatory Variables			
			Mean			
True	−1.820	2.295	−0.313	−0.469	0.794	0.082
Est.	−1.774	2.220	−0.302	−0.461	0.787	0.094
	(0.955)	(0.657)	(0.756)	(0.096)	(0.243)	
			Volatility			
True	1.884	−0.044	2.070	0.104	−0.029	0.107
Est.	2.019	−0.046	1.940	0.107	−0.049	0.115
	(0.738)	(0.485)	(0.544)	(0.071)	(0.179)	

correlations from the bootstrapping is essentialy indistinguishable from the actual correlations. The standard deviations are also very similar to the standard errors computed previously. The correlation pattern is extremely robust across the replications. In every case, the maximum correlation occurs for a lagged or contemporaneous value of the conditional volatility and the minimum occurs for a led or contemporaneous value of the volatility. On average, the maximum correlation occurs at a lag of 15 months, and it ranges from 0 months to 20 months. On average, the minimum occurs at a lead of 7 months, and it ranges from 0 months to 18 months. The average time between the maximum and minimum is 22 months, and the average difference between the correlations is 0.70. The distance between them is never less than 11 months or more than 30 months, and the difference between the maximum and the minimum is never less than 0.38 or more than 1.15. The reason for the robustness of the results, in spite of the high standard deviations, is that the correlation estimates are highly correlated. As mentioned earlier, there is a positive correlation between correlation estimates that are close together, but there is also a negative correlation for estimates that are further away. For example, the correlation between neighboring estimates averages 0.98 while, for estimates that are separated by 24 months, the correlation averages −0.47. Turning to the rolling correlations, the pattern resembles the pattern from the data, but the magnitudes of both the estimates and the standard deviations are lower. Correlation estimates close to each other are positively correlated as expected, which makes the large swings in the contemporaneous correlations observed in the data more remarkable.
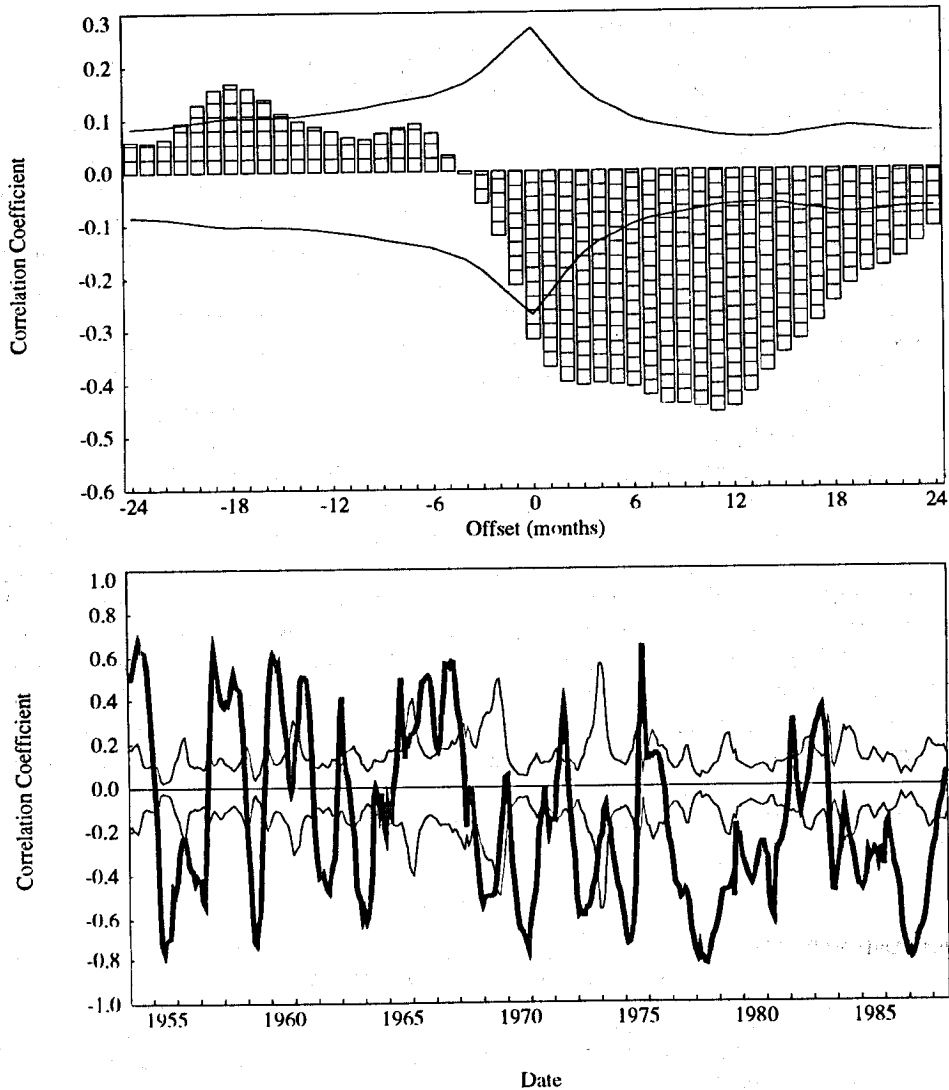
Figure 5. **Correlations from bootstrapping experiments**. Average correlations between the fitted first and second conditional moments of monthly returns on the value-weighted index based on 1,000 replications of parametric bootstrapping. *Top*: mean correlations between the fitted conditional first and second moments (*bars*) and their standard deviations (*solid lines*). *Bottom*: mean 17-month rolling correlations (*solid line*) and their standard deviations (*thin solid lines*).

VII. Conclusion

This article uses four financial variables to estimate the conditional first and second moments of stock market returns. The combination of the Baa-Aaa spread, the commercial paper–Treasury spread, the one-year Treasury yield, and the dividend yield provides convincing empirical evidence of predictable

variation in both returns and their volatility. For horizons of one month, one quarter, and one year, joint tests of whether the coefficients on all the conditioning variables are zero can easily be rejected at the 1 percent level. The fitted conditional moments from a linear model show time variation, but the conditional volatility apparently leads the expected return. Asymmetric correlation patterns between the expected return and lagged and led values of the conditional volatility and systematic time variation in the contemporaneous correlations between the moments estimated over short horizons support the hypothesis of offset cyclical variation. The asymmetry in the relation between the first and second moments is also evident in a vector autoregression estimated using the fitted values. Lagged volatility is positively related to future expected returns, but lagged expected returns are negatively related to future volatility. Moreover, innovations in the two conditional moments are negatively correlated.

These empirical results bring into doubt the value and validity of focusing on the contemporaneous relation between expected returns and volatility at the market level. This relation is apparently nonstationary, and, furthermore, provides limited information about how the moments covary over time. As a result, imposing a constant linear relation between the mean and volatility may lead to erroneous inferences. The more complex time series behavior uncovered in this study presents a challenge for both empirical and theoretical models of the relation between risk and return. From a theoretical perspective, the excellent predictive power of the commercial paper-Treasury spread for stock volatility indicates that monetary policy may be important in generating the observed covariation patterns. Further investigation of these issues is clearly warranted.

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