1. A solid disk with mass $M = 6.0\ kg$ and radius $R = 2\ m$ is initially at rest. A constant tangential force $F = 100\ N$ is applied to the rim of the disk for $t = 6.0\ s$. Friction is negligible.
    a. What is the angular acceleration of the disk?
    b. What is the angular velocity of the disk after 4.0 seconds?
    c. How much work is done by the force during this time?
    d. What is the rotational kinetic energy of the disk after 4.0 seconds?
2. A uniform rod of length $L = 3.0\ m$ and mass $M = 6.0\ m$ is pivoted at one end and held horizontally. It is then released from rest.
    a. What is the angular acceleration of the rod immediately after release?
    b. What is the linear acceleration of the rod's free end immediately after release?
    c. What is the angular speed of the rod as it passes through the vertical position?
3. A hoop (ring) and a solid sphere both have mass $M = 2.0\ kg$ and radius $R = 3.0\ m$. They are released from rest at the top of an inclined plane of height $h = 2.0\ m$. They roll without slipping.
    a. Calculate the final speed of each object at the bottom of the incline.
    b. Which object reaches the bottom first?
4. A particle of mass $m = 0.5\ kg$ moves in a circle of radius $r = 1\ m$ with an angular speed of $\omega = 10\ rad/s$.
    a. What is the magnitude of its angular momentum?
    b. If the angular speed is doubled, what happens to the angular momentum?
    c. If a torque is applied to bring the particle to rest in 2 seconds, what is the magnitude of the torque?
5. A uniform solid cylinder of mass $M = 4.0\ kg$ and radius $R = 0.5\ m$ rolls without slipping down an inclined plane inclined at $\theta = 45^{\circ}$.
    a. What is the acceleration of the cylinder down the incline?
    b. What is the frictional force acting on the cylinder?
    c. If the cylinder starts from rest, how long does it take to roll 5 meters down the incline?
6. A turntable rotates at an initial angular speed $\omega_0 = 20\ rad/s$. It experiences a constant angular acceleration of $\alpha = -4\ rad/s^2$.
    a. How long does it take for the turntable to stop?
    b. How many revolutions does it make before coming to rest?

7. A thin rod of length $L = 1.5\ m$ and mass $M = 8\ kg$ rotates about an axis through its center and perpendicular to its length. A small mass $M = 1\ kg$ is attached to one end.
    a. What is the new moment of inertia of the system?
    b. If the rod is rotating at $\omega = 9\ rad/s$, what is the angular momentum of the system?
8. A uniform solid sphere of mass $M = 7.0\ kg$ and radius $R = 0.5\ m$ is initially at rest at the top of an inclined plane that makes an angle $\theta = 60^{\circ}$ with the horizontal. The inclined plane has a vertical height $h = 7.0\ m.$ The sphere rolls without slipping down the incline.
    a. Derive an expression for the linear acceleration a of sphere of the sphere as it rolls down the incline in terms of $g\ and\ \theta$.
    b. Calculate the time t it takes for the sphere to reach the bottom of the incline.
    c. Determine the rotational kinetic energy K_{rot} of the sphere when it reaches the bottom.
    d. If the sphere encounters a rough horizontal surface after descending the incline and comes to rest after rolling a distance of $d = 10\ m$, calculate the average frictional force f acting on the sphere.
9. A disk of mass $M = 5.0\ kg$ and radius $R = 0.8\ m$ rotates about an axis through its center with an initial angular velocity of $\omega_0 = 15\ rad/s.$ A small piece of putty of mass $m = 1\ kg$ is dropped vertically onto the disk and sticks to it at a distance $r = 0.3\ m$ from the center.
    a. Calculate the initial angular momentum $L_{initial}$ of the system before the putty sticks to the disk.
    b. Determine the angular velocity ω_{final} of the system after the putty sticks to the disk.
    c. Calculate the kinetic energy of the system before and after the putty sticks. Explain the reason for any difference in kinetic energy.
    d. If instead the putty hits the disk and bounces off without sticking, how would this affect the disk's angular velocity?

rotational kinematics

1. A solid disk with a moment of inertia $I = \frac{1}{2}MR^2$, $M = 3.0\ kg$ and $R = 1\ m$. s initially at rest. A constant net torque of $\tau = 10.0\ N \cdot m$ is applied to the disk.
   a. What is the angular acceleration α of the disk?
   b. What is the angular velocity ω of the disk after $t = 5.0\ s$.
   c. Through what angle θ (in radians) does the disk rotate during this time?
2. A uniform rod of length $L = 1.5\ m$ and $M = 4.0\ kg$ is hinged at one end and initially held horizontally. It is then released and allowed to rotate downward under the influence of gravity.
   a. What is the angular acceleration α of the rod immediately after it is released?
   b. What is the angular velocity ω of the rod as it passes through the vertical position?
3. A figure skater with arms extended spins at an angular velocity of $\omega_1 = 5.0\ rad/s$. She then pulls her arms in, reducing her moment of inertia from $I_1 = 5.0\ kg \cdot m^2$ to $I_2 = 2.0\ kg \cdot m^2$.
   a. What is her new angular velocity ω_2?
   b. By how much does her rotational kinetic energy change?
4. A solid sphere of radius $R = 0.10\ m$ and mass $M = 0.50\ kg$ starts from rest at the top of an incline of height $h = 2.0\ m$ and rolls without slipping to the bottom.
   a. What is the linear speed v of the sphere at the bottom of the incline?
   b. What is the sphere's angular speed ω\omegaω at the bottom?
5. A uniform circular platform of radius $R = 2.0\ m$ and mass $M = 150\ kg$ is free to rotate about a vertical axis through its center. A technician applies a force $F = 200\ N$ tangentially at the edge of the platform for $t = 7.0$.
   a. What is the angular acceleration α produced by the force?
   b. What is the angular velocity ω of the platform after $7.0\ s$.
6. A wheel with a moment of inertia $I = 0.80\ kg \cdot m^2$ is spinning at an angular velocity of $\omega = 20\ rad/s$. A brake is applied to the wheel, exerting a constant frictional torque of $\tau =- 3.0\ N \cdot m$. (the negative sign indicates that the torque opposes the rotation).

torque

1. A uniform rectangular door has a mass of $M = 40\ kg$ and a width of $W = 1.0\ m$ the door rotates about its hinges along one side. A force $F = 30\ N$ is applied perpendicular to the door at a point $d = 1\ m$ from the hinges.
    a. What is the torque τ produced by the force about the hinges?
    b. If the door starts from rest, what is its angular acceleration α ? Assume the door can be modeled as a uniform rod rotating about one end.
2. A seesaw consists of a uniform plank of length $L = 6.0\ m$ and mass $M = 15\ kg$ pivoted at its center. A child of mass $m_1 = 25\ kg$ sit at one end of the seesaw.
    a. Where should a second child of mass $m_2 = 35\ kg$ sit to balance the seesaw?
    b. If the second child sits $1.5\ m$ from the pivot on the opposite side, what is the net torque on the seesaw?
3. A flywheel in the form of a solid disk has a mass $M = 50\ kg$ and a radius $R = 1\ m$. A constant torque $r = 25\ N \cdot m$ is applied to the flywheel.
    a. What is the angular acceleration α of the flywheel?
    b. How much time t is required for the flywheel to reach an angular velocity of $\omega = 300\ rad/s$ starting from rest?
    c. What is the angular momentum L of the flywheel at $\omega = 300\ rad/s$?
4. A uniform beam of length $L = 6.0\ m$ and mass $M = 80\ kg$ is hinged at one end and held at an angle of $\theta = 30^{\circ}$ above the horizontal by a cable attached at the other end and anchored to a wall above the hinge point.
    a. What is the torque due to the weight of the beam about the hinge?
    b. If the cable makes an angle of $\phi = 45^{\circ}$ with the beam, what is the tension T in the cable required to keep the beam in equilibrium?
5. A uniform ladder of length $L = 10\ m$ and mass $M = 20\ kg$ leans against a frictionless vertical wall, making an angle of $\theta = 60^{\circ}$ with the ground. The coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$.
    a. What is the torque due to the ladder's weight about the point where the ladder touches the ground?
    b. what is the maximum distance up the ladder that a person of mass $m = 100\ kg$ can climb before the ladder begins to slip?

rotational inertia

1. A uniform rod of mass M and length L is pivoted at one end and can rotate freely about the pivot.
   a. Using the definition of rotational inertia and standard formulas, find the moment of inertia I of the rod about the pivot.
   b. How would your answer change if the rod were pivoted about its center instead?
2. A solid disc of mass M and radius R has a known rotational inertia $I_{center} = \frac{1}{2}MR^2$ about its center.
   a. Use the parallel-axis theorem to find the moment of inertia of the disc about an axis tangent to its rim and parallel to the original axis.
   b. If the disc's mass doubles but its radius stays the same, how does the moment of inertia about the tangent axis change?
3. Two small masses m_1 and m_2, are attached at the ends of a massless rod of length L. The rod is rotated about an axis through one end and perpendicular to its length.
   a. Find the rotational inertia of the system about that end.
   b. If the axis of rotation is moved to the midpoint of the rod, find the new moment of inertia.
4. A solid sphere of mass M and radius R has $I_{soild} = \frac{2}{5}MR^2$. A hollow spherical shell of the same mass and radius has $I_{hollow} = \frac{2}{3}MR^2$.
   a. Explain why the hollow sphere has a larger rotational inertia than the solid sphere.
   b. Explain physically why a hollow sphere's mass distribution leads to a larger moment of inertia compared to a solid sphere of the same mass and radius.
5. A rod of length L and mass M has a linear mass density that varies linearly from one end to the other. Specifically, $\lambda(x) = \lambda_0(1 + \frac{x}{L})$, where $x = 0$ at the left end. The rod is to be rotated about the left end, perpendicular to its length.
   a. Set up the integral for the moment of inertia of the rod.
   b. Perform the integral and express your answer in terms of M and L.

6. A wheel with a known moment of inertia I experiences a net torque τ.
    a. Write down the equation that relates torque, moment of inertia, and angular acceleration α.
    b. If the wheel's moment of inertia doubles while the same torque is applied, how does the angular acceleration change?
7. A small, uniform rod of mass m and length L is attached at its midpoint to the center of a uniform disk of mass M and radius R. The combined object is rotated about the disk's center axis, perpendicular to the plane of the disk.
    a. Find the total moment of inertia of the system.
    b. If the rod is instead attached to the rim of the disk (end of the rod touching the disk's circumference), find the new total moment of inertia.
8. A uniform, solid cylinder of mass M and radius R is mounted so that it can rotate freely about its central axis (the axis going through its center and perpendicular to its circular faces). A small point mass m is then attached at a distance d from the cylinder's center, on its flat face (at the cylinder's rim if $d = R$).
    a. Derive an expression for the total rotational inertia of the system about the cylinder's central axis.
    b. If the point mass is now moved so that it is located at a point on the lateral surface of the cylinder (still distance d from the central axis, but sticking out along the curved surface), determine the new rotational inertia. Use appropriate theorems (such as the parallel axis theorem) as needed.
9. A uniform rod of length L and mass M is pivoted about one end, and is initially free to rotate in a horizontal plane about that pivot. A small sphere of mass m is placed at a distance $\frac{L}{2}$ from the pivot along the rod.
    a. Find the moment of inertia of the combined rod-sphere system about the pivot.
    b. Now, suppose the small sphere is moved out to the very end of the rod (at a distance L from the pivot). Determine how the moment of inertia changes.
    c. Discuss how the angular acceleration of the system, under a given torque, would compare before and after moving the mass m from $L/2$ to L.

rotational kinetic energy and work

1. A uniform disk of mass M and radius R is spinning with an initial angular speed ω_0. A constant torque τ is applied for a time interval t in the direction of the disk's rotation.
   a. Determine the final angular speed ω_f.
   b. Find the total work done on the disk by this torque.
   c. Calculate the change in rotational kinetic energy of the disk.
2. A solid cylindrical pulley of mass M and radius R is initially at rest. A rope is wound around it, and you pull the rope horizontally with a constant force F over a length L, causing the pulley to rotate freely about its central axis.
   a. Find the angular acceleration α of the pulley.
   b. Determine the final angular velocity ω after pulling the rope through distance L.
   c. Calculate the work done by the applied force and show how it relates to the final rotational kinetic energy of the pulley.
3. A horizontal turntable (a uniform disk) of moment of inertia I rotates with angular speed ω_0. A small lump of clay (mass m) is dropped onto the rim of the turntable and sticks.
   a. Determine the final angular speed ω_f of the system.
   b. Find the loss in rotational kinetic energy due to the inelastic collision.
   c. If an external torque is applied to restore the system to its original angular speed ω_0, calculate the work that must be done.
4. A rigid body with a known moment of inertia I about a fixed axis starts from rest. A constant torque τ acts on it for a period of time t.
   a. Derive an expression for the angular velocity ω as a function of time.
   b. Find the total work done by the torque.
   c. Express the rotational kinetic energy after time t in terms of τ and t.
5. A spinning flywheel of moment of inertia I and angular speed ω_0 suddenly has a second identical, initially stationary flywheel attached to it coaxially. They stick together and rotate as one body.
   a. Determine the final angular speed ω_f.
   b. Find the work lost to internal energy (e.g., heat, deformation)during the attachment.
   c. Compare the initial and final rotational kinetic energies.

6. A torque $\tau(\theta) = a\theta^2$ (a is a constant) acts on a rigid body that starts from rest. The torque depends on the angular position θ.
    a. Write down the rotational work-energy theorem and apply it to find the work done in rotating the body from $\theta = 0$ to $\theta = \Theta$.
    b. Express the resulting angular velocity $\omega(\Theta)$ in terms of I, a, and Θ.
    c. Discuss how the rotational kinetic energy changes as a function of Θ.
7. A solid sphere of mass M and radius R rolls without slipping down an incline of height h.
    a. Determine its translational and rotational kinetic energies at the bottom of the incline.
    b. Confirm that energy is conserved and show how the height h sets the final rotational kinetic energy.
    c. If the sphere is replaced by a hoop of the same mass and radius, compare the final rotational kinetic energies of both objects.
8. A wheel of moment of inertia I is spinning at an angular speed ω_0. A frictional torque τ_f (constant in magnitude) acts to slow it down until it stops.
    a. Determine the angular deceleration α.
    b. Find the time t_f it takes to come to rest.
    c. Calculate the total work done by the frictional torque and confirm that this equals the loss in rotational kinetic energy.
9. Two gears are in contact: a smaller gear (moment of inertia I_s) spinning at angular speed ω_s meshes with a larger gear (moment of inertia I_L), initially at rest. Due to frictionless, ideal contact, angular momentum about their axis is conserved as they reach a common angular speed ω_f.
    a. Determine ω_f.
    b. Calculate the initial and final rotational kinetic energies.
    c. Discuss the work that must be done by an external agent to restore the system to the initial total kinetic energy (if any).

10. A motor applies a constant power P to a rotating shaft with moment of inertia I. The shaft starts from rest and speeds up under this constant power input.
    a. Find the torque $\tau(\omega)$ as a function of the angular velocity ω.
    b. Integrate over time to find $\omega(t)$.
    c. Determine the total work done by the motor in bringing the shaft up to a certain angular velocity ω_f and verify that this matches the increase in rotational kinetic energy.

rotational power

1. A flywheel of moment of inertia I starts from rest and a constant torque τ.
    a. Find the angular acceleration α of the flywheel.
    b. Express the instantaneous power P delivered to the flywheel as a function of its angular velocity ω.
    c. Determine the power at the instant when the flywheel's angular velocity reaches ω_0.
2. A motor provides constant power P to a solid cylinder of moment of inertia I mounted on a frictionless axle. The cylinder starts from rest.
    a. Derive an expression for the angular velocity ω as a function of time t.
    b. Find the torque $\tau(\omega)$ acting on the cylinder as ω increases.
    c. Determine the work done on the cylinder after it reaches angular speed ω_f.
3. A wind turbine rotor of moment of inertia I is spinning at a constant angular speed ω . The wind applies a constant torque τ that balances out a frictional torque τ_f.
    a. Determine the net power input to the turbine.
    b. If the wind speed increases slightly, causing ω to increase at a rate α, find how the net power changes.
    c. Discuss how the rotational kinetic energy changes during this acceleration.
4. A rotating machine experiences a torque $\tau(\omega) = a\omega^2$, where a is a constant.
    a. Write an expression for the rotational power $P(\omega)$.
    b. If the machine starts from rest and is subject to this torque, find how the angular velocity ω changes with time.
    c. Determine how the instantaneous power changes as the machine speeds up.

5. A belt drives a pulley of radius R with tension T in the belt on one side and no slip. The pulley's moment of inertia is I, and it spins with angular velocity ω.
    a. Find the torque on the pulley due to the tension T.
    b. Determine the rotational power delivered to the pulley by the belt.
    c. If the pulley accelerates from ω_1 to ω_2 in time Δt, find the average power over this interval.
6. A spinning wheel of moment of inertia I is initially rotating with angular velocity ω_0. A constant external torque τ is applied for a time t.
    a. Find the change in angular momentum.
    b. Determine the final angular velocity ω_f.
    c. Calculate the instantaneous power at the end of the time interval using the torque and the final angular velocity, and discuss how this relates to the rate of change of rotational kinetic energy.
7. A generator's rotor of moment of inertia I is spun by a turbine at a steady angular velocity ω. Frictional forces cause a constant opposing torque τ_f.
    a. Find the mechanical power input required to keep the rotor spinning at a constant ω.
    b. If the generator produces electrical power P_{out} and you know the frictional torque τ_f, determine the mechanical (input) to electrical (output) efficiency.
    c. Discuss what happens to the required input power if ω is increased.
8. A factory machine consists of two connected flywheels: Flywheel A and B. Flywheel A has a moment of inertia $I_A = 8\ kg \cdot m^2$ and is connected via a massless, frictionless axle to Flywheel B, which has a moment of inertia $I_B = 2\ kg \cdot m^2$ A constant torque τ is applied to Flywheel A, causing both flywheels to accelerate together.
    a. If the system reaches an angular velocity of $\omega = 30.0\ rad/s$ in $t = 15.0\ s$, calculate the magnitude of the applied torque τ.
    b. Determine the average power delivered to the system over the 15-second interval.
    c. If friction causes an opposing torque of $\tau_f = 50.0\ N \cdot m$ to act on Flywheel B, how does this affect the required applied torque τ to achieve the same angular acceleration? Calculate the new torque.

linear and rotational conservation of energy

1. A simple pendulum consists of a mass $m = 2\ kg$ attached to a string of length $L = 1.50\ m$. The pendulum is pulled to an angle of 30° from the vertical and released from rest.
   a. Calculate the maximum speed of the mass as it passes through the lowest point of its swing.
   b. Determine the tension in the string at the lowest point of the swing.
2. A roller coaster car of mass $m = 500\ kg$ starts from rest at the top of a frictionless hill of height $h = 20.0\ m$. It then descends and ascends a second hill of height $h^{'}$.
   a. Determine the speed of the car at the bottom of the first hill.
   b. Calculate the maximum height $h^{'}$ of the second hill that the car can reach.
3. Two cars, Car A and Car B, collide inelastically on a frictionless, straight road. Car A has a mass of $m_A = 1500\ kg$ and is moving east at $v_A = 20\ m/s$. Car B has a mass of $m_B = 1000\ kg$ and is moving west at $v_B = 10\ m/s$. After the collision, the cars stick together.
   a. Determine the velocity of the combined mass immediately after the collision.
   b. Calculate the loss in kinetic energy due to the collision.
4. Two billiard balls of equal mass $m = 0.16\ kg$ collide elastically on a frictionless table. Ball 1 is moving east at $v_1 = 2\ m/s$, and Ball 2 is moving west at $v_2 = -1\ m/s$. After the collision, Ball 1 moves west at $v_1^{'} = -0.5\ m/s$.
   a. Determine the velocity $v_2^{'}$ of Ball 2 after the collision.
   b. Verify that kinetic energy is conserved in this collision.
5. A solid cylinder of mass $m = 5\ kg$ and radius $R = 0.5\ m$ rolls without slipping down an inclined plane of height $h = 10\ m$.
   a. Determine the linear speed of the cylinder's center of mass at the bottom of the incline.
   b. Calculate the angular speed of the cylinder at the bottom.(Assume no energy is lost to friction.)

6. A disk of mass $m = 2\ kg$ and radius $R = 0.3\ m$ is spinning with an angular speed $\omega = 50.0\ rad/s$. A small mass $m' = 0.5\ kg$ is placed at the rim of the disk and sticks to it.
   a. Determine the new angular speed of the disk after the mass sticks to it.
   b. Calculate the rotational kinetic energy before and after the mass is added. How much energy is lost in the process?
7. A flywheel with a moment of inertia $I = 10\ kg \cdot m^2$ is spun up from rest to an angular speed of $\omega = 100\ rad/s$. The flywheel is then connected to a generator that extracts energy from it until its angular speed drops to $\omega = 80\ rad/s$.
   a. Calculate the total energy stored in the flywheel at $\omega = 100\ rad/s$.
   b. Determine the amount of energy extracted by the generator as the flywheel slows down to $\omega = 80\ rad/s$.
8. A rotating wheel of mass $m = 10\ kg$ and radius $R = 0.4\ m$ is spinning at $\omega = 20\ rad/s$. A small mass $m' = 1\ kg$ collides and sticks to the rim of the wheel.
   a. Determine the new angular speed of the wheel after the collision.
   b. Calculate the change in rotational kinetic energy due to the collision.

rotational momentum

1. A figure skater is spinning with her arms extended. She has a moment of inertia of $6\ kg \cdot m^2$ and is rotating at an angular velocity of $4\ rad/s$. She pulls her arms in, reducing her moment of inertia to $2\ kg \cdot m^2$.
   a. What is her new angular velocity after pulling her arms in?
   b. By what factor does her rotational speed increase?
2. A solid cylinder of mass m and radius R is rolling without slipping down an incline of height h.
   a. Derive an expression for the cylinder's angular velocity at the bottom of the incline.
   b. What fraction of its potential energy is converted into rotational kinetic energy?
3. A uniform rod of length L and mass M is pivoted at one end and is initially at rest. A constant torque τ is applied perpendicular to the rod.
   a. What is the angular acceleration of the rod?
   b. How long does it take for the rod to reach an angular velocity of ω?

4. Two ice skaters, Skater A and Skater B, are initially at rest facing each other on a frictionless ice surface. Skater A has a mass of $50\ kg$ and Skater B has a mass of $70\ kg$. They push off against each other, and Skater A moves away with a velocity of $3\ m/s$.
   a. What is the velocity of Skater B after they push off?
   b. Calculate the angular momentum of the system about the point where they started pushing off.
5. A bicycle wheel of mass $2\ kg$ and radius $0.3\ m$ is spinning with an angular velocity of $20\ rad/s$. The rider applies brakes that exert a torque of $3\ N \cdot m$ opposite to the wheel's rotation.
   a. What is the angular deceleration of the wheel?
   b. How long does it take for the wheel to come to rest?
6.
   a. Calculate the moment of inertia of a solid sphere of mass M and radius R about an axis passing through its center.
   b. Compare it with the moment of inertia of a hollow spherical shell of the same mass and radius about the same axis.
7. A spinning disk with a moment of inertia of $I_1 = 0.4\ kg \cdot m^2$ is rotating at $\omega = 8\ rad/s$. A second disk with moment of inertia $I_2 = 0.3\ kg \cdot m^2$ is placed on top of the first disk and eventually reaches the same angular velocity as the first disk due to friction.
   a. What is the final common angular velocity of the two disks?
   b. How much rotational kinetic energy is lost in the process?
8. A projectile is launched horizontally with speed v from the top of a frictionless turntable of radius R. The projectile lands r meters from the center of the turntable and sticks to it.
   a. Determine the angular velocity of the system (turntable plus projectile) after the collision.
   b. If the turntable has a moment of inertia I, express the final angular velocity in terms of I, v, R, and r.

9. A bicycle wheel of mass M and radius R is spinning with angular velocity ω. The wheel is held so that its axis is horizontal. A torque τ is applied to tilt the axis of the wheel at a rate of $\dot{\theta}$ (theta dot).
    a. Relate the applied torque τ to the rate of change of the tilt angle $\dot{\theta}$.
    b. If the torque is doubled, how does the rate of change of the tilt angle change?
10. A spinning top has a moment of inertia I about its symmetry axis and is spinning with angular velocity ω. The gravitational torque causes the top to precess with an angular velocity φ.
    a. Derive the relationship between φ, I, ω, the mass m of the top, gravitational acceleration g, and the distance r from the pivot to the center of mass.
    b. If the spin rate ω is increased, what happens to the precession rate φ?

linear and rotational conservation of momentum

1. Two ice skaters, Skater A and Skater B, are initially at rest on a frictionless ice surface. Skater A has a mass of $50\ kg$, and Skater B has a mass of $70\ kg$. They push off against each other.
    a. What are the velocities of Skater A and Skater B after they push off?
    b. Calculate the total linear momentum of the system before and after the push.
2. A spinning disk with a moment of inertia $I_1 = 0.8\ kg \cdot m^2$ is rotating at an angular velocity of $\omega_1 = 6\ rad/s$. A second disk with moment of inertia $I_2 = 0.5\ kg \cdot m^2$. is placed on top of the first disk and sticks to it due to friction.
    a. What is the final common angular velocity of the two disks?
    b. Determine the total angular momentum before and after the collision.
3. A figure skater of mass $m = 60\ kg$ is spinning with her arms extended, giving her a moment of inertia of $I_1 = 4\ kg \cdot m^2$ and an angular velocity of $\omega_1 = 3\ rad/s$. She pulls her arms in, reducing her moment of inertia to $I_2 = 2\ kg \cdot m^2$.
    a. What is her new angular velocity after pulling her arms in?
    b. If she moves her arms inward by $0.5\ m/s$ linearly while doing so, calculate the rate of change of her angular momentum.
4. A stationary artillery shell of mass $10\ kg$ explodes into two fragments. Fragment A has a mass of $6\ kg$ and Fragment B has a mass of $4\ kg$.

a. If Fragment A moves east with a velocity of $15\ m/s$, what is the velocity of Fragment B?

b. Verify the conservation of linear momentum for the system.

5. A solid cylinder of mass $m = 5\ kg$ and radius $R = 0.3\ m$ is rolling without slipping on a horizontal surface with a linear velocity $v = 4\ m/s$.
   a. Calculate the angular velocity of the cylinder.
   b. If a frictional force causes the cylinder to come to rest after traveling $20\ m$, determine the work done by friction.
6. A gun of mass $3\ kg$ fires a bullet of mass $0.05\ kg$ with a muzzle velocity of $400\ m/s$.
   a. Calculate the recoil velocity of the gun.
   b. Determine the kinetic energy of the gun after the shot.
7. A uniform rod of length $2\ m$ and mass $4\ kg$ is pivoted freely at one end. Initially, the rod is at rest. A force $F = 10\ N$ is applied perpendicular to the other end of the rod for $3\ s$.
   a. Calculate the angular impulse delivered to the rod.
   b. Determine the angular velocity of the rod after the force is applied.
8. A ball of mass $0.5\ kg$ moving at $10\ m/s$ collides elastically with a stationary spinning wheel of mass $2\ kg$ and radius $0.5\ m$ which can rotate freely about its central axis. The wheel has a moment of inertia $I = 0.4\ kg \cdot m^2$. (After the collision, the ball bounces back with a velocity of $4\ m/s$)
   a. Determine the angular velocity of the wheel after the collision.
   b. Verify the conservation of linear and angular momentum for the system.
9. A satellite of mass $500\ kg$ is orbiting the Earth at a constant speed of $7800\ m/s$ in a circular orbit with a radius of $7000\ km$.
   a. Calculate the linear momentum of the satellite.
   b. Determine the angular momentum of the satellite about the center of the Earth.
10. A rotating merry-go-round has a moment of inertia of $I_1 = 200\ kg{\cdot}m^2$ and is spinning at $\omega_1 = 2\ rad/s$. A child of mass $30\ kg$ standing on the edge ($radius = 3\ m$) walks towards the center at a speed of $1\ m/s$.
    a. Calculate the final angular velocity of the merry-go-round when the child reaches the center.
    b. Discuss the conservation of angular momentum in this scenario.

11. A projectile of mass $m = 2\ kg$ is launched horizontally from the end of a rigid, massless rod of length $L = 1.5\ m$ that is fixed to a pivot. Just before launch, the rod is rotating with an angular velocity of $\omega = 10\ rad/s$.
    a. Determine the linear momentum of the projectile at the moment of launch.
    b. Calculate the angular momentum of the projectile about the pivot point at the moment of launch.
12. Two disks, Disk A and Disk B, are on a frictionless surface. Disk A has a mass of $3\ kg$, radius $0.2\ m$, and is spinning at $\omega_1 = 12\ rad/s$. Disk B has a mass of $0.2\ m$, radius $0.2\ m$, and is initially at rest. The two disks collide and stick together without slipping.
    a. What is the final angular velocity of the combined system?
    b. Calculate the total angular momentum before and after the collision.
13. A sprinter of mass $70\ kg$ is initially at rest on a stationary rotating platform with a moment of inertia $I = 50\ kg{\cdot}m^2$. The sprinter then sprints off tangentially with a velocity of $8\ m/s$.
    a. Determine the angular velocity of the platform after the sprinter starts sprinting.
    b. Discuss the conservation principles involved in this scenario.
14. A wind turbine blade with a moment of inertia $I = 150\ kg{\cdot}m^2$ is initially stationary. A gust of wind exerts a torque of $\tau = 75\ N{\cdot}m$ on the blade for $10\ s$.
    a. Calculate the angular momentum imparted to the blade.
    b. Determine the final angular velocity of the blade after the torque is applied.
15. A uniform wheel of mass $5\ kg$ and radius $0.4\ m$ is rolling without slipping on a horizontal surface with a linear velocity of $6\ m/s$.
    a. Calculate the wheel's angular momentum about its center.
    b. If the wheel collides with a stationary object and comes to a stop after rolling $2\ m$, determine the work done by the object on the wheel.

rotational impulse

1. A uniform wheel of mass $10\ kg$ and radius $0.5\ m$ is initially at rest. A constant torque $\tau = 20\ N{\cdot}m$ is applied to the wheel for $5\ s$.
    a. Calculate the angular impulse delivered to the wheel.
    b. Determine the final angular velocity of the wheel after the torque is applied.

2. A door with a moment of inertia $I = 15\ kg{\cdot}m^2$ is initially stationary. A force $F = 30\ N$ is applied perpendicularly at the edge of the door for $2\ s$. The door is $1\ m$ wide.
    a. Calculate the angular impulse imparted to the door.
    b. Find the angular velocity of the door after the force is applied.
3. A baseball bat of mass $1.2\ kg$ and length $1\ m$ is swung about its end. During a time interval of $0.05\ s$, a force $F = 100\ N$ is applied perpendicular to the bat $0.9\ m$ from the pivot.
    a. Calculate the angular impulse delivered to the bat.
    b. Determine the final angular velocity of the bat assuming it starts from rest.
4. A spinning disk with a moment of inertia $I_1 = 2\ kg{\cdot}m^2$ is rotating at an angular velocity of $\omega_1 = 10\ rad/s$.A second disk with moment of inertia $I_2 = 1\ kg{\cdot}m^2$ is gently placed on top of the first disk and brought to rotate with it over a time interval of $\Delta t = 0.2\ s$. The torque applied during this process is constant.
    a. Calculate the angular impulse required to bring the second disk up to the rotation of the first disk.
    b. Determine the final common angular velocity of the two disks after the impulse.
5. A rotating cylinder with a moment of inertia $I = 5\ kg{\cdot}m^2$ is spinning at an angular velocity of $\omega = 8\ rad/s$. A hammer applies a tangential force of $F = 50\ N$ at a radius of $0.4\ m$ for $0.1\ s$.
    a. Calculate the angular impulse delivered by the hammer.
    b. Determine the change in angular velocity of the cylinder after the hammer strike.
6. A merry-go-round with a moment of inertia $I = 200\ kg{\cdot}m^2$ is rotating at an angular velocity of $\omega = 1.5\ rad/s$. A child pushes against the merry-go-round with a tangential force of $F = 30\ N$ for $3\ seconds$ at a radius of $2\ m$.
    a. Calculate the angular impulse provided by the child.
    b. Determine the new angular velocity of the merry-go-round after the push.

7. A wind turbine blade with a moment of inertia $I = 10\ kg\cdot m^2$ is initially at rest. A gust of wind exerts a variable torque on the blade that can be approximated as a constant torque $\tau = 25\ N\cdot m$ for a duration of $4\ s$.
   a. Calculate the angular impulse delivered to the blade.
   b. Find the angular velocity of the blade after the gust of wind.
8. A spinning top with a moment of inertia $I = 0.05\ kg\cdot m^2$ is rotating at an angular velocity of $\omega = 100\ rad/s$. A small force is applied tangentially at the top of the spinning top for $0.02\ s$, resulting in an angular impulse of $\Delta L = 1\ N\cdot m\cdot s$.
   a. Determine the new angular velocity of the top after the impulse.
   b. Discuss whether the top speeds up or slows down as a result of the impulse.
9. A robotic arm with a moment of inertia $I = 8\ kg\cdot m^2$ is initially stationary. To perform a task, the arm needs to reach an angular velocity of $\omega = 3\ rad/s$. A motor applies a constant torque τ for $t = 2\ s$.
   a. Calculate the required angular impulse to achieve the desired angular velocity.
   b. Determine the magnitude of the torque τ that the motor must apply.
10. A flywheel with a moment of inertia $I = 12\ kg\cdot m^2$ is rotating at $\omega = 5\ rad/s$. A sudden spike in torque delivers an angular impulse of $\Delta L = 30\ N\cdot m\cdot s$ over a very short time.
   a. Calculate the change in angular velocity of the flywheel due to the torque spike.
   b. Find the new angular velocity of the flywheel after the impulse.
11. A rotating system with a moment of inertia $I = 50\ kg\cdot m^2$ is spinning at $\omega = 20\ rad/s$. A braking mechanism applies a constant torque opposite to the rotation, providing an angular impulse of $\Delta L = -500\ N\cdot m\cdot s$over a certain time period.
   a. Determine the final angular velocity of the system after the braking torque is applied.
   b. Calculate the time duration if the braking torque is $\tau = -100\ N\cdot m$.
12. A turntable with a moment of inertia $I = 3\ kg\cdot m^2$ is spinning at $\omega = 12\ rad/s$. A musician gives a quick rotational kick by applying a torque of $\tau = 18\ N\cdot m$ for $0.5\ s$.
   a. Calculate the angular impulse imparted by the musician's kick.
   b. Determine the new angular velocity of the turntable after the kick.

13. A rotational launch system uses a spinning arm with a moment of inertia $I = 4\ kg{\cdot}m^2$. To launch a projectile, a torque $\tau = 40\ N{\cdot}m$ is applied for $0.25\ s$.
    a. Calculate the angular impulse delivered to the spinning arm.
    b. Find the angular velocity of the spinning arm after the torque is applied.
14. A rotating platform with a moment of inertia $I = 25\ kg{\cdot}m^2$ is spinning at $\omega = 2\ rad/s$. A person pushes tangentially with a force $F = 10\ N$ for $3\ s$ at a radius of $1.5\ m$.
    a. Calculate the angular impulse provided by the person's push.
    b. Determine the resulting change in angular velocity of the platform.
15. A rotational pendulum consists of a rod of length $2\ m$ and mass $3\ kg$, pivoted at one end. The moment of inertia I about the pivot is $I = (1/3)\ m\ L^2$. A sudden torque $\tau = 9\ N{\cdot}m$ is applied for $0.4\ s$.
    a. Calculate the angular impulse delivered to the pendulum.
    b. Find the angular velocity of the pendulum immediately after the torque is applied.
16. In a manufacturing machine, a rotating shaft with a moment of inertia $I = 10\ kg{\cdot}m^2$ is spinning at $\omega = 15\ rad/s$. To adjust its speed, a torque $\tau = -50\ N{\cdot}m$ is applied for $0.3\ seconds$.
    a. Calculate the angular impulse exerted by the torque.
    b. Determine the new angular velocity of the shaft after the torque is applied.
17. A small satellite with a moment of inertia $I = 0.2\ kg{\cdot}m^2$ is spinning in space at $\omega = 30\ rad/s$. To change its spin rate, a thruster applies a torque of $\tau = -6\ N{\cdot}m$ for $2\ s$.
    a. Calculate the angular impulse delivered by the thruster.
    b. Find the new angular velocity of the satellite after the impulse.
18. A wind turbine rotor with a moment of inertia $I = 100\ kg{\cdot}m^2$ is initially stationary. A gust of wind applies a varying torque that can be approximated as a constant torque $\tau = 50\ N{\cdot}m$ over a period of $\Delta t = 10\ seconds$.
    a. Calculate the angular impulse delivered to the rotor.
    b. Determine the angular velocity of the rotor after the gust.
19. A rotational launchpad has a moment of inertia $I = 20\ kg{\cdot}m^2$ and is spinning at $\omega = 4\ rad/s$. To launch a payload, a torque $\tau = 80\ N{\cdot}m$ is applied for $0.25\ s$.

a. Calculate the angular impulse imparted to the launchpad.
b. Find the final angular velocity of the launchpad after the torque is applied.