Oscillations

1. A 3 kg block is attached to a horizontal spring with a spring constant of 75 N/m. The block is pulled 0.2 m from its equilibrium position and released from rest on a frictionless surface.
    a. Calculate the angular frequency and period of the oscillation.
    b. Determine the maximum speed of the block during its motion.
2. A simple pendulum has a length of 0.8 meters. Assuming small oscillations and neglecting air resistance, calculate the period of the pendulum on Earth where $g = 9.8\, m/s^2$.
3. A 0.5 kg mass is hung vertically from a spring, causing it to stretch by 0.1 m at equilibrium. The mass is then pulled down an additional 0.05 m and released from rest.
    a. Calculate the spring constant k.
    b. Determine the total mechanical energy of the system during oscillation.
4. A harmonic oscillator experiences a damping force proportional to its velocity. The amplitude of the oscillation decreases to 60% of its initial value after 5 complete cycles. What is the damping constant (b) if the mass of the oscillator is 2 kg and the spring constant is 200 N/m?

    (hint)The damping factor per cycle can be found using $A = A_0 e^{\frac{-bt}{2m}}$, where t is the total time for 5 cycles.
5. A system with mass $m = 5\ kg$, and spring constant is $k = 500\ N/m$ is subjected to a driving force $F(t) = F_0 cos(\omega_d t)$, where $F_0 = 10\ N$.
    a. Find the natural frequency (ω_0) of the system.
    b. Determine the amplitude of steady-state oscillations when the driving frequency ω_d equals the natural frequency.
6. A uniform rod of length $L = 2\ m$ and mass $M = 5\ kg$ is pivoted about one end and swings freely in a vertical plane under the influence of gravity.
    a. Calculate the moment of inertia of the rod about the pivot point.
    b. Determine the period of small oscillations.
7. A mass-spring system oscillates with a maximum displacement of 0.3 m and a maximum speed of 2 m/s.
    a. Calculate the angular frequency of the system.
    b. Determine the spring constant if the mass is 1.5 kg.

8. A small object is attached to the end of a light rod, forming a simple pendulum of length $L = 3.0\ m$. A uniform rod of the same length and total mass equal to that of the small object is used as a physical pendulum.
    a. Which pendulum has a longer period of oscillation?
    b. Calculate the period difference between the two pendulums.
9. A horizontal spring with a force constant $k = 350\ N/m$ has a block of mass $m = 4\ kg$ attached to it. The block oscillates with amplitude $A = 0.5\ m$. While the block is passing through equilibrium, a lump of clay of mass $m_c = 3\ kg$ is dropped vertically onto it and sticks.
    a. Determine the new amplitude of oscillation after the collision.
    b. Calculate the percentage change in mechanical energy due to the collision.
10. Two oscillations are described by $x_1(t) = Acos(\omega t)$ and $x_2(t) = Acos(\omega t + \phi)$. If these two oscillations are superimposed, the resultant amplitude varies with the phase difference ϕ.
    a. Derive an expression for the resultant amplitude.
    b. For $A = 1\ m$ and $\phi = \frac{\pi}{4}$, calculate the resultant amplitude.