Momentum & Impulse

1. A $2.0\ kg$ object moves on a frictionless surface. It experiences a net force given by $F(t) = 6t\ (in\ newtons)$, for $0 \leq t \leq 4.0s$.
    a. Find the impulse delivered to the object over this 4.0 s interval.
    b. If the object starts from rest, determine its final velocity at $t = 4.0s$.
2. A force on a $0.50\ kg$ particle is described by the function $F(x) = 2x$ (x is in meters). The particle moves in one dimension from $x = 0$ to $x = 3$ m.
    a. Find the work done by this force.
    b. By relating work and kinetic energy, determine the change in momentum if the particle starts from rest at $x = 0$.
3. A billiard ball of mass m moving at v_0 in the +x direction collides elastically with an identical ball initially at rest. After the collision, the first ball moves off at an angle θ to the original direction; the second ball moves at some angle ψ.
    a. Write down the momentum conservation equations in the x and y directions.
    b. Show that energy is conserved.
    c. Solve for the final speeds of both balls in terms of v_0 and θ.
4. A $3.0\ kg$ block slides with an initial speed of $8.0\ m/s$ across a horizontal surface ($\mu_k = 0.20$). It collides inelastically with a stationary 5.0 kg block, and they stick together.
    a. Use the work-energy theorem (or frictional impulse) to find the speed of the $3.0\ kg$ block right before impact (taking friction into account).
    b. Determine the combined final velocity of the stuck-together blocks immediately after the collision.
    c. Compute the impulse delivered by the friction on the $3.0\ kg$ block from start to collision.
5. A bullet of mass m traveling at speed v is fired into a stationary wooden block of mass M suspended as a pendulum. The bullet embeds in the block, and the system rises to a height h.
    a. Derive an expression for the initial speed v of the bullet in terms of $m,\ M,\ h\ and\ g$.

    b. Discuss how momentum and energy conservation principles apply in this two-stage process (collision and rise).
6. A $1200\ kg$ car traveling at $25\ m/s$ crashes into a wall and comes to a stop in $0.6\ s$.
    a. Find the impulse on the car during the crash.
    b. Determine the average force exerted on the car by the wall.
    c. If the collision time were halved, discuss the effect on the average force.
7. Consider a rocket of initial mass M (including fuel) that expels mass at a constant rate φ with exhaust velocity v_{ex} relative to the rocket.
    a. Derive the rocket equation for the rocket's velocity as a function of the expelled mass.
    b. Discuss how conservation of momentum applies to systems with changing mass.
8. A $0.045\ kg$ golf ball is initially at rest and is struck by a golf club, leaving the club with a speed of $60\ m/s$. The collision lasts $4.0 \times 10^{-3}\ s$.
    a. Calculate the impulse delivered to the ball.
    b. Find the average force exerted by the club on the ball.
    c. If the collision time were doubled, would the impulse change? Would the average force change?
9. A $0.3\ kg$ kg object's momentum changes from $(4i + 1j)\ kg\ m/s$ to $(1i - 3j)\ kg\ m/s$ over a period of $0.50\ s$.
    a. Determine the impulse vector acting on the object.
    b. Calculate the average net force (in vector form).
10. A $0.20\ kg$ rubber ball is dropped from rest at a height of $2.0\ m$. It strikes the floor and rebounds to $1.2\ m$. The collision lasts $0.060\ s$.
    a. Calculate the speed just before the ball hits the floor and just after it leaves the floor.
    b. Find the magnitude of the change in momentum.
    c. Determine the average force exerted by the floor on the ball.

Center of Mass

1. Two masses, $m_1 = 2.0\ kg$ and $m_2 = 5.0\ kg$, line on the x-axis at $x_1 = 0$ and $x_2 = 8.0\ m$, respectively.
   a. Find the center of masses of the system.
   b. If m_1 moves to $x = 2.0\ m$ and m_2 moves to $x = 6.0\ m$ (at the same instant), demonstrate that the **center of mass** of the system will not necessarily be at the midpoint of $2.0\ m$ and $6.0\ m$. Explain why.
2. A rod of length L has a mass per unit length given by $\lambda(x) = \lambda_0(1 + x)$, where x is measured from one end of the rod.
   a. Express the total mass M in terms of λ_0 and L.
   b. Find the center of mass location in terms of L.
   c. Discuss how the result differs from the center of mass of a uniform rod.
3. Four masses are placed at the corners of a rectangle of sides a and b.
   - m_1 at $(0, 0)$
   - m_2 at $(a, 0)$
   - m_3 at (a, b)
   - m_4 at $(0, b)$

   a. Find the coordinates (x_{CM}, y_{CM}) of the center of mass.
   b. Discuss whether the center of mass must lie inside the rectangle for arbitrary values of m_1, m_2, m_3, m_4.
4. Two blocks of masses $M_1 and\ M_2$ slide toward one another on a frictionless track. They collide and stick together. Prove mathematically (using Newton's laws or direct momentum arguments) that the center of mass maintains constant velocity before, during, and after the collision, provided there are no external horizontal forces.

5. A projectile of total mass M is launched and follows a parabolic trajectory under gravity (no air resistance). At the peak of its flight, it explodes into two fragments $m_1 and\ m_2$ such that $m_1 + m_2 = M$.
    a. Show that the center of mass of the two fragments continues on the original parabolic path.
    b. Explain why external forces (gravity only) determine the motion of the center of mass regardless of the internal explosion.
6. Treat Earth (mass M_E) and the Moon (M_M) as point masses separated by a center-to-center distance R.
    a. Determine the position of the center of mass of the Earth–Moon system measured from Earth's center.
    b. Show that, given realistic values of $M_E\ and\ M_M$, the center of mass lies inside Earth's radius. Explain the physical meaning of this location.
7. A uniform chain of length L and total mass M is initially coiled on a frictionless table. One end is pulled off the edge so that the chain slides off under the influence of gravity. At any instant, a length x of the chain hangs off the table.
    a. Find the speed of the chain as a function of x.
    b. Calculate the acceleration of the chain's center of mass.
8. A seesaw of negligible mass has length $4.0\ m$ and is pivoted at its center. A $30\ kg$ child sits at the left end, and a $20\ kg$ child sits somewhere to the right so that the seesaw is balanced.
    a. Find how far from the pivot the $20\ kg$ child must sit.
    b. Find the position of the center of mass of the two-child system, measured from the pivot.
9. A $70\ kg$ person stands at one end of a $100\ kg$ boat of length $4.0\ m$ floating on a lake (negligible friction). The person walks to the other end of the boat.
    a. Prove that the center of mass of the (boat + person) system does not move.
    b. Calculate how far the boat itself moves in the water, assuming the person walks at a constant speed.