ECS 60 **Written Homework #1: Series, Proofs, and Complexity (46 points)** **Winter 2017**

Due Wednesday, January 18th, 4:00 pm in 2131 Kemper
For inductive proofs, you must state when you use your inductive hypothesis in your proof.

1. (2 points) Let F_i be the Fibonacci numbers, where $F_0 = 1$, $F_1 = 1$, $F_2 = 2$, $F_3 = 3$, $F_4 = 5$ …. Prove $\sum_{i=1}^{N-2} F_i = F_N - 2$, where $N > 2$ using induction. (Lipschutz, HW 3)

2. (2 points) Use induction to prove that for all natural numbers $x > 1$ and n, $x^n - 1$ is divisible by $x - 1$. (Heileman, p.415)

3. (2 points) Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$. (Lipshutz)

4. (2 points) Prove by induction that the sum of the first n odd positive integers is n^2, i.e., $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.

5. (2 points) Prove that $\sum_{k=0}^{n} k2^k = (n-1)2^{n+1} + 2$. (Heileman, p. 415)

6. (2 points) Assuming a and b are arbitrary constants, and $0 < a < 1 < b$, order the following functions by growth rate:
   $lg\ n$, $log(log\ n)$, $n\ logn$, n^b, n^a, $\frac{1}{n}$, $\frac{1}{\log n}$, n^n, b^n, 1, $n^{log\ n}$, b^{b^n}, $\frac{1}{b^n}$. (Heileman p. 27)

7. (2 points) The number of operations executed by algorithms A and B is $8n\log n$ and $2n^2$, respectively. Determine an n_0 such that A < B for $n > n_0$. (Goodrich, p.185)

8. (4 points) Prove that $x^2 + 3x - 10$ is $\Theta(N^2)$. This actually involves two proofs. (Lipshutz HW 3)

9. (4 points, 2 points each) Assuming that $f_1(n)$ is $O(g_1(n))$ and $f_2(n)$ is $O(g_2(n))$: (Drozdek, p.71)
   a. Find a counterexample to refute that $f_1(n) - f_2(n)$ is $O(g_1(n) - g_2(n))$ by supplying f_1, f_2, g_1, and g_2.
   b. Find a counterexample to refute that $f_1(n) / f_2(n)$ is $O(g_1(n) / g_2(n))$ by supplying f_1, f_2, g_1, and g_2.

10. (2 points) Show that $log(n + 1) = O(log\ n)$. (Heileman p.28)

11. (12 points, 2 points each) Find the computational complexity for the following code fragments: (a Nyhoff p. 364, b-e Drozdek pp. 72-73, f Weiss, p. 72)

```
a. for(int x = 1, count = 0, i = 0; i < n; i++)
   {
     for(int j = 0; j <= x; j++)
      count++;
     x = 2;
   }
b. for(int count = 0, i = 0; i < n; i++)
     for(int j = 0; j < n; j++)
       count++;

c. for(int count = 0, i = 0; i < n; i++)
     for(int j = 0; j < i; j++)
       count++;

d. for(int count = 0, i = 1; i < n; i *= 2)
     for(int j = 0; j < n; j++)
       count++;

e. for(int count = 0, i = 1; i < n; i * = 2)
     for(int j = 0; j < i; j++)
       count++;
```

```
f. for(int count = 0, i = 0; i < n * n; i++)
     if( i % n == 0)
       for(int j = 0; j < i; j++)
         count++;
```

12. (4 points, 2 points each) Let $p(x)$ be a polynomial of degree n, that is, $p(x) = \sum_{i=0}^{n} a_i x^i$. (Goodrich, p. 190)
   a. Describe a simple $O(n^2)$ time method for computing $p(x)$.
   b. Now consider a rewriting of $p(x)$ as
      $p(x) = a_0 + x(a_1 + x(a_2 + x(a_3 + \ldots + x(a_{n-1} + xa_n) \ldots)))$, which is known as Horner's method. Using the big-Oh notation, characterize the number of arithmetic operations this method executes.

13. (6 points, 2 points each) Evaluate the following sums: (Weiss, p. 47)
   a. $\sum_{i=0}^{\infty} \frac{1}{4^i}$
   b. $\sum_{i=0}^{\infty} \frac{i}{4^i}$
   c. $\sum_{i=0}^{\infty} \frac{i^2}{4^i}$

Sources of questions:

Thomas H. Cormen, Charles E. Leiserson, and Ronald L. Rivest, *Introduction to Algorithms*, New York, New York, McGraw-Hill, 1990.

Adam Drozdek, *Data Structures and Algorithms in C++, Second Edition*, Pacific Grove, CA, Brooks/Cole, 2001.

Michael T. Goodrich, Roberto Tamassia, and David Mount, *Data Structures & Algorithms*, *Second Edition*, Hoboken, NJ, John Wiley & Sons, 2011.

Gregory L. Heileman, *Data Structures, Algorithms, and Object Oriented Programming*, New York, NY, McGraw-Hill, 1996.

Seymour Lipschutz, *Schaum's Outline of Theory and Problems of Discrete Mathematics, 3rd ed.*, NewYork, NY, McGraw-Hill, 2007.

Larry R. Nyhoff, *C++: An Introduction to Data Structures*, Upper Saddle River, NJ, 1999.

Mark Weiss, *Data Structures and Algorithm Analysis in C++, Fourth Edition*, New York, NY, Pearson Education, 2014.