Math 473 HW 5

1. (Broughton and Bryan 6.2) Consider a filter bank that uses the Haar analysis filters with coefficients $(\ell_a)_0 = (\ell_a)_1 = \frac{1}{2}, (h_a)_0 = \frac{1}{2}, (h_a)_1 = -\frac{1}{2}$, and corresponding causal synthesis filters $(\ell_s)_0 = (\ell_s)_1 = 1, (h_s)_0 = -1, (h_s)_1 = 1$, (yes, the $(h_s)_0$ is -1). Show that this filter bank yields perfect reconstruction with output delay 1 by considering an input vector $\mathbf{x} = (\cdots, x_{-1}, x_0, x_1, \cdots)$ and computing the filter bank output $\tilde{\mathbf{x}}$ directly (filter, downsample, upsample, synthesis filter). In particular, show that $\tilde{x_k} = x_{k-1}$.

2. Consider a filter bank for signals in $L^2(\mathbb{Z})$ with analysis low-pass filter coefficients $(\ell_a)_0 = 1$ and all other coefficients zero, while $(h_a)_1 = 1$ and all other high-pass coefficients zero. These are not low-pass or high-pass - indeed ℓ_a is the "identity filter" and h_a is simply a delay – but that doesn't matter.

   - Explicitly compute the components of the upsampled vector $X_\ell = D(X * \ell_a)$ and $X_h = D(x * h_a)$.
   - Explicitly compute the components of the upsampled vector $U(X_\ell)$ and $U(X_h)$.
   - Determine, by inspection, suitable synthesis filters ℓ_s and h_s, so that

   $$\ell_s * (U(X_\ell)) + h_s * (U(X_h)) = x.$$

3. Download Chan-Vese active contour without edges code from

   `https://www.mathworks.com/matlabcentral/fileexchange/34548-active-contou`

   or `https://www.mathworks.com/matlabcentral/fileexchange/23445-chan-vese-a`

   Understand the code and use it to segment two gray images (I in texmos3.s512.tiff and T1Web.mat) . You may use either code. The second code however implements more cases such as mutliphase that is helpful for the textmos3.s512.tiff data. Write a short description of the models used to segment each image. Make it clear about the number of level sets used. Comment on how the parameters affect the results.