Math 473 HW 4

1. Let x, y be two 1D signals of the same length, it is know that the circular convolution of x, y is $x * y = M_y x$ where M_y is a circulant matrix. Prove that $M_x M_y = M_{x*y}$.

2. l_1 sparsity based deconvolution: Let X be a 2D image, H be a 2D kernel, one can show that the vectorized version of $X * H$ (circulant convolution using period boundary conditions for X) can be represented as Ax with x the vectorized version of X, A a block circulant and circulant block (BCCB) matrix. One can prove that this matrix A is diagnoalizable under Fourier transform, i.e., $A = F\Lambda F^{-1}$ where F is the Fourier transform matrix.

   Download data PartPhantomEg.mat. It contains a sparse image X with few non-zero intensities.

   Simulate a blurred and noisy image J from the clean and sharp image X using

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n = 5;
sigma = 1;
psf = fspecial('gauss',[n n],sigma); % Gaussian PSF
load PartPhantomEg.mat X;
[nr nc] = size(X);
P = zeros(nr,nc);
P(1:n,1:n) = psf; % zero pad psf
ctr = [ceil(n/2) ceil(n/2)]; % center of psf
S = fft2(circshift(P,1-ctr));
J = ifft2(S.*fft2(X));
J = real(J);
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   Note that psf is separable and P is a zero padding extension of it. The upper left corner of P is psf. Since convolution theorem is used to simulate the blurry data, circular convolution is done, i.e., periodic boundary condition is used.

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c = 0.01;
E = randn(size(J));
E = E/norm(E,'fro');
B = J + c*E*norm(J,'fro');
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   Let b denote the vectorized version of B and assume this is the only given data, then we can find an approximation to x by solving the following inverse problem using ADMM: $\min_x \frac{1}{2}\|Ax - b\|_2^2 + \lambda\|x\|_1$ as x is sparse. Note the parameter λ should be tuned. Assume the result after reshaping to a matrix is $\hat{\lambda}$, one can compare this approximation to the ground truth X by calculating SNR and MSE. Try five values of different magnitude order and list a table to compare the SNR and MSE as the value of λ varies. Choose and display the result corresponding to the largest SNR or the smallest MSE.