Math 473 HW 2

Instructions: show all steps to get full credits. Assemble Matlab code and results and turn in together with written parts. No late homework is accepted in any case.

1. **Weak derivative.** Let $f(x) = -|x|$. Show that f is not differentiable at 0 and find its weak derivative.

2. Read paper "Nonlinear total variation based noise removal algorithms" by L. Rudin and S. Osher and E. Fatemi posted on Canvas and write an one page summary. The summary should provide enough information to understand the main ideas of the paper. This part should be typed using Latex or Word.

3. **Tiknov and TV in image inpainting.**

   Compare Tikonov and TV on inpainting. Starting with a ground truth image I, we simulate an image u_0 with missing intensities by zeroing out part of the intensities of I randomly. The following code does the simulation. Set D (the complement of matrix "Mask") as the set of pixels (2D locations) without missing intensity.

```
I = imread('cameraman.tif');
I = double(I); % change I from unit8 to double precision format
% normalize I to intensity [0,1] for easier parameter selection
I = (I-min(I(:)))/(max(I(:))-min(I(:)));
[m,n] = size(I);
% amount of removed pixels.
perc = .7;
% random mask, Mask==1 for removed pixels
Mask = zeros(m,n);
Pick = randperm(m*n); Pick = Pick(1:round(perc*m*n));
Mask(Pick) = 1;
u0 = I;
u0(Mask == 1) = 0;
```

   We have learned two models to recover the missing data:

   Tikhonov inpainting model: $\min_u \int_\Omega |\nabla u|^2, s.t., u = u_0$ in D

   and TV inpainting model: $\min_u \int_\Omega |\nabla u|, s.t., u = u_0$ in D

   Use cameraman (imread('cameraman.tif')) image as the test image.

   3.1 Implement the two models and compare their performances using peak signal noise ration (PSNR) and relative errors, another two popular quantitative measures. Explain your observation. Let $X, \bar{X}$ be the ground truth and an estimate of it respectively, PSNR is defined as $10 \log_{10}\left(\frac{MAX_X^2}{MSE}\right)$ with mean squared error (MSE) between $m \times n$ X and $\bar{X}$

defined as $\frac{1}{mn}\sum_{i,j}(X_{ij} - \bar{X}_{ij})^2$ and MAX_X represent the maximum intensity of X. The relative error is defined as $\frac{\|X-\hat{X}\|_F}{\|X\|_F}$ with $\|\cdot\|_F$ represents Frobenius norm.

3.2 Observe performances of the two models when "perc", the ratio of missing pixels, increases to 0.80 and 0.90.

3.3 Test on the cameraman image with scratch and noise. Download scratch.mat which contains a 0-1 mask with locations with 0 intensity corresponding to scratches. Zero out intensity of I at scratch to get J and add white (zero mean) Gaussian noise with standard deviation $\sigma = 0.1$ using $J + \sigma * randn(size(J))$. Note, when there is noise, the fidelity term is only integrated over the domain D. You need to solve $\min_u \int_\Omega |\nabla u| + \frac{\lambda}{2}\int_D \|u - u_0\|_2^2$ and $\min_u \int_\Omega |\nabla u|^2 + \frac{\lambda}{2}\int_D \|u - u_0\|_2^2$ for TV and Tikonov respectively. The parameter λ depends on the standard deviation of the noise and needs to be tuned. Try several λ values $\lambda = 10^d, d = -2, -1, 0, 1, 2$ etc. d values vary with the standard deviation σ of the added noise. You may choose a different pool of parameters for λ if a better result is achieved. List a table to compare PSNR and relative error as λ varies. Choose and display the result corresponding to the largest PSNR or smallest relative error.

3.4 Test on the cameraman image with texts superimposed, and try to remove the texts. Download text.mat which contains a 0-1 mask with 0 intensity corresponding to texts. Revise the intensity of u_0 at locations with texts as 0.92 to simulate an image with texts superimposed on it.