# A Regularized Linear Dynamical System Framework for Multivariate Time Series Analysis

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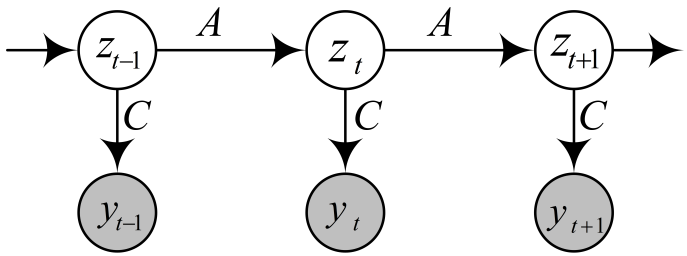
# Problem & Solution

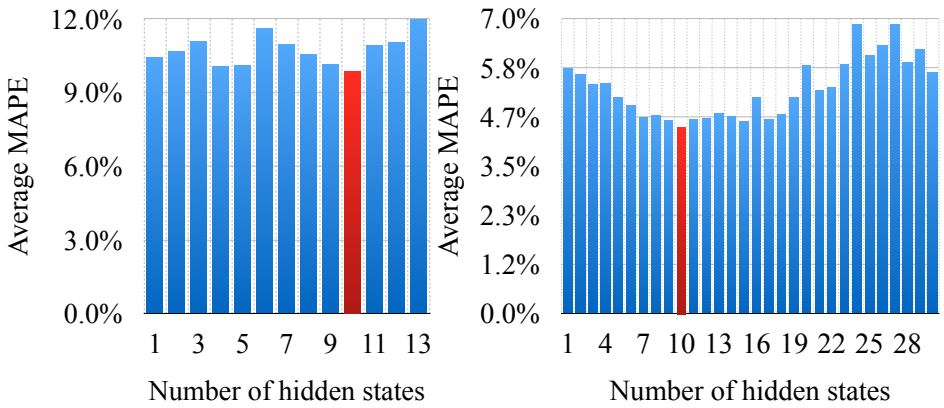
**Standard linear dynamical system(LDS):**



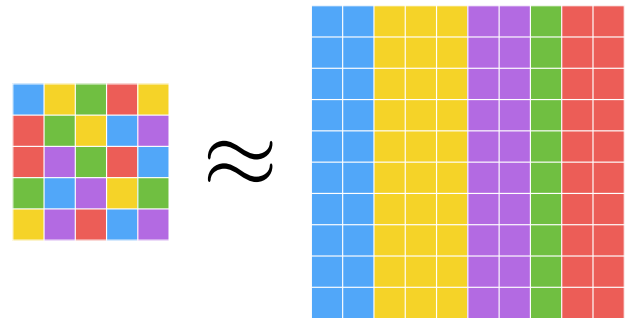

$$\mathbf{z}_t = A\mathbf{z}_{t-1} + \mathbf{e}_t; \quad \mathbf{y}_t = C\mathbf{z}_t + \mathbf{v}_t$$
$$\mathbf{e}_t \sim \mathcal{N}(\mathbf{0}, Q);\ \mathbf{v}_t \sim \mathcal{N}(\mathbf{0}, R);\ \mathbf{z}_1 \sim \mathcal{N}(\boldsymbol{\xi}, \Psi)$$

**Problem:** If the dimension of the hidden space is large, it may cause overfitting of the LDS model.

**Overfiting phenomena:**



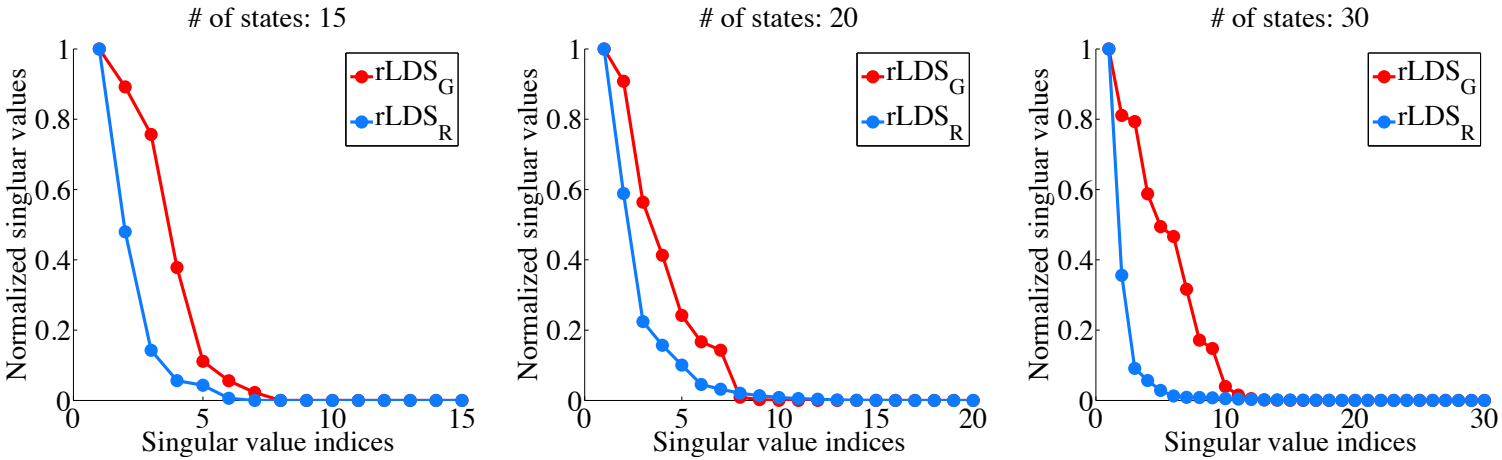

**Solution: low-rank approx.**




**5x5 matrix ("optimal" *A*)** **10x10 matrix ("a low-rank approximation" of *A*)**

# Experimental Results

## Recovery of the intrinsic hidden space dimensionality on synthetic data




## Improved prediction performance on real-world clinical data

| | Training Size: 50 | | | Training Size: 400 | | |
|---|---|---|---|---|---|---|
| # of states | 10 | 20 | 30 | 10 | 20 | 30 |
| EM | 6.28 | 17.24 | 23.98 | **4.43** | 5.91 | 5.72 |
| SubspaceID | 6.55 | 6.99 | 7.44 | 6.10 | 6.16 | 6.27 |
| StableLDS | 6.54 | 6.99 | 7.40 | 6.10 | 6.16 | 6.27 |
| rLDS$_{\mathcal{G}}$ | 4.98 | 4.97 | **4.86** | 4.51 | **4.25** | **4.35** |
| rLDS$_{\mathcal{R}}$ | **4.65** | **4.95** | 5.01 | 4.65 | 4.46 | 4.67 |