HANDOUT 6

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- Exam resource on course website!

1. Compute

$$\begin{bmatrix} -1 & 3 & 0 \end{bmatrix} \left(\begin{bmatrix} 3 & -5 & 1 \\ 6 & -10 & 2 \\ 0 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & -4 & 0 \\ -6 & 10 & 2 \\ 0 & 5 & -1 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Solution: 21

2. Find the inverse of the matrix $\begin{bmatrix} 3 & 2 \\ 7 & 4 \end{bmatrix}$.

Solution: $\begin{bmatrix} -2 & 1 \\ 7/2 & -3/2 \end{bmatrix}$.

3. Let $A = \begin{bmatrix} 3 & -2 & -5 \\ 2 & -1 & -3 \\ -4 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & -1 \\ -2 & 1 & 3 \\ 3 & -1 & 2 \end{bmatrix}$. Find $A + B, A^T, AB, A^{-1}, B^{-1}, (AB)^{-1}$.

Solution:

(1) $A + B = \begin{bmatrix} 4 & -2 & -6 \\ 0 & 0 & 0 \\ -1 & -1 & 3 \end{bmatrix}$

(2) $A^\top = \begin{bmatrix} 3 & 2 & -4 \\ -2 & -1 & 0 \\ -5 & -3 & 1 \end{bmatrix}$

(3) $AB = \begin{bmatrix} -8 & 3 & -19 \\ -5 & 2 & -11 \\ -1 & -1 & 6 \end{bmatrix}$

(4) $A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & -2 & -1 \\ -10 & 17 & 1 \\ 4 & -8 & -1 \end{bmatrix}$

(5) $B^{-1} = \frac{1}{6} \begin{bmatrix} 5 & 1 & 1 \\ 13 & 5 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

(6) $(AB)^{-1} = \frac{1}{18} \begin{bmatrix} -1 & -1 & -5 \\ -41 & 67 & -7 \\ -7 & 11 & 1 \end{bmatrix}$

There also exists the formula for the inverse of 3×3 matrix.
See https://mathworld.wolfram.com/MatrixInverse.html
It require more knowledge about the determinant of a matrix.