Cfrm 425 final project (Change it later)

Ethan Kinnan, Nicolaas Jefferson, Jiatong Gao, Chengyu Gong

March 11, 2023

Abstract

Predicting asset prices has been one of the principal challenges in finance since markets and trade started to develop. In both professional and academic environments predicting these prices has been extensively explored. Studies on equities(stocks) prices have resulted in a couple of different models. We focused on three. The first and oldest model is the CAPM model, based on the risk-free rate, beta, and market risk premium. In 1993 Fama and French built their Three Factor Model that expanded on the CAPM model by looking at company size(market capitalization) and value(Book-to-Market ratio, aka B/M). In 2013 they further expanded upon the model and released their Five Factor Model, which also included profitability(Operating Profit as a Percentage of Revenue) and investment(Growth in Book Equity).

We believe the Five Factor Model will perform better than the Three Factor Model and CAPM, especially in uncommon scenarios like micro-caps and firms with unusually low profitability.

Our research is based on the research done by Fama and French in their 2013 paper on the Five Factor Model. We start with the three different models:

CAPM :

$$R_{t,i} = \alpha + \beta R_{t,MKT} + \varepsilon_t$$

Three Factor Model:

$$R_{t,i} = \alpha + \beta_{MKT} R_{t,MKT} + \beta_{SMB} R_{t,SMB} + \beta_{HML} R_{t,HML} + \varepsilon_t$$

Five Factor Model :

$$R_{t,i} = \alpha + \beta_{MKT} R_{t,MKT} + \beta_{SMB} R_{t,SMB} + \beta_{HML} R_{t,HML} + \beta_{RMW} R_{t,RMW} + \beta_{CMA} R_{t,CMA} + \varepsilon_t$$

1) The MKT(Market) coefficient represents the overall market risk premium. The market return minus the risk-free rate.

2) The SMB(Small minus Big) coefficient corresponds to the difference in returns between a portfolio of small-capitalization stocks and large-capitalization stocks.

3) The HML(High minus Low) coefficient corresponds to the difference between a portfolio of high Book-to-Market(B/M) portfolio of low B/M stocks. High B/M stocks are normally referred to as value stocks, while low B/M stocks are commonly referred to as growth stocks.

4) The RMW(Robus minus Weak) coefficient corresponds to the difference between a portfolio of highly profitable firms minus a portfolio of firms with low profitability. Here profitability refers to the firm's net profit margin.

5) The CMA(Conservative minus Aggressive) coefficient corresponds to the difference between a portfolio of firms that invest very conservatively and return most of their capital to shareholders vs. a portfolio of firms that invests very aggressively and returns little to none of their capital to shareholders.

1 Regression Analysis

1.1 Portfolios based on Size

Looking at the coefficients of the Three and Five-Factor models across the deciles, we can see a clear drop in the value of the SMB coefficient as size(market cap) increases.
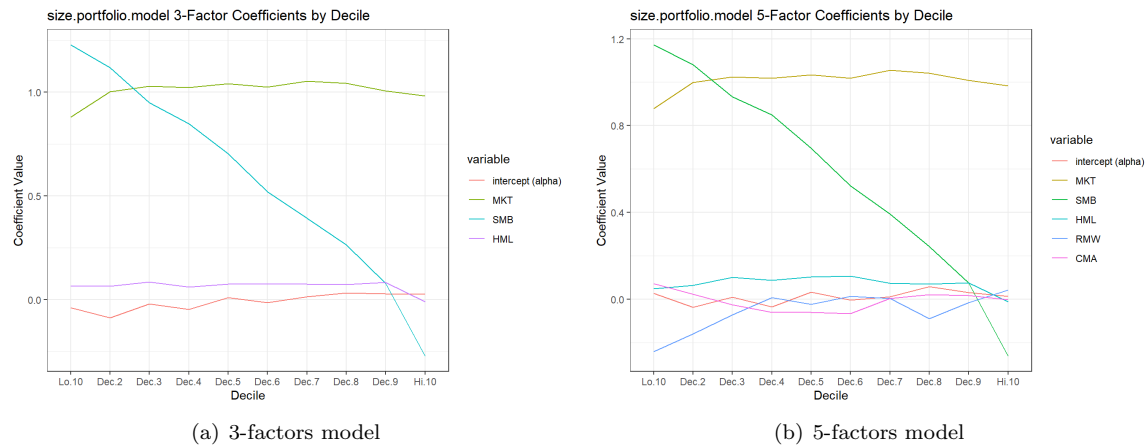
(a) 3-factors model   (b) 5-factors model

Figure 1: Coefficient by decile of portfolio sorted by size

In both plots, we see the MKT coefficient stay about the same, which, combined with the declining SMB coefficient, shows us the excess returns that smaller businesses generate on average. The size of the dropoff in the SMB coefficient also provides initial evidence that business size plays a significant role in the returns of stocks. At the same time, in the plot of the coefficients for the Five Factor Model, we see an interesting rise in the RMW factor as we get to the upper deciles of the size-based portfolios. While we don't have information on the significance of this rise, this does correspond with the idea that larger companies are more developed with better economies of scale and, therefore, more profitable. This is also seen in the factor correlation matrix, where SMB and RMW have one of the strongest correlations at -0.30143.

1.2 Portfolios based on Book-to Market

The Three and Five Factor models show a steeply increasing slope for the HML coefficient across portfolios based Book-to-Market ratios(B/M). Interestingly, we also see a noticeable increase in the SMB coefficient, especially as we reach the deciles with high B/M ratios.

(a) 3-factors model   (b) 5-factors model

Figure 2: Coefficient by decile of portfolio sorted by book-to-market

Unlike in the size-based portfolios, the MKT coefficient is slightly upward-sloping in both models as we approach the top deciles. In the Five Factor model, there is a pretty clear downward slope in the profitability coefficient RMW and the investment coefficient CMA. The fundamental explanation for these movements is not as clear. The coefficient plots do, however, clearly show the return premium of value stocks. The upward movement of the SMB coefficient also suggests that companies with high B/M ratios tend to be smaller. This can also be seen in the correlation matrix. While not as strong as

the SMB and RMW correlation, the correlation between HML and SMB was 0.193, suggesting some relationship.

1.3 Portfolios based on Profitability

The RMW coefficient followed a similar pattern to HML, having a steep, increasing slope as the operating profitability increased.

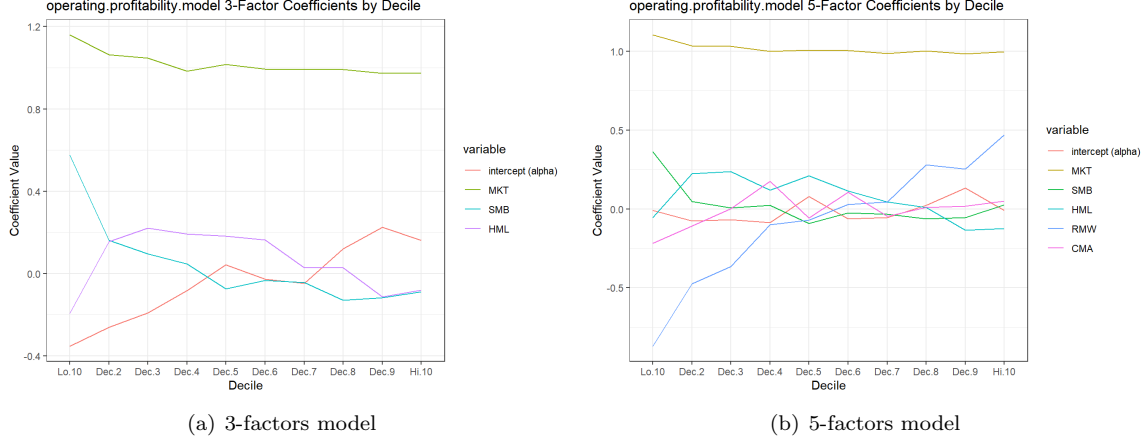(a) 3-factors model

(b) 5-factors model

Figure 3: Coefficient by decile of portfolio sorted by profitability

Because the RMW factor is not in the Three Factor Model, it is interesting to see how the other coefficients behave. SMB has a clear downward slope, especially as we get out of the lowest deciles, further suggesting that smaller firms tend to be less profitable. In the Three Factor Model, operating profitability is the most reliable indicator of alpha, with a steeply increasing intercept as profitability increases. This makes sense because high-profit margins tend to indicate well-run businesses in favorable markets. Unlike the other portfolios, there is not a drastic change in coefficients for the Five Factor Model, suggesting that profitability is independent of other factors. In the correlation matrix, however, we see that RMW has a correlation of about -0.3 with both MKT and RMW. The most likely explanation for the negative correlation is that profitability is the main factor driving these returns, not size or market risk.

1.4 Portfolios based on Investment

The CMA coefficient reversed the pattern seen in the HML and RMW coefficients, decreasing sharply as firm investment increased.

(a) 3-factors model
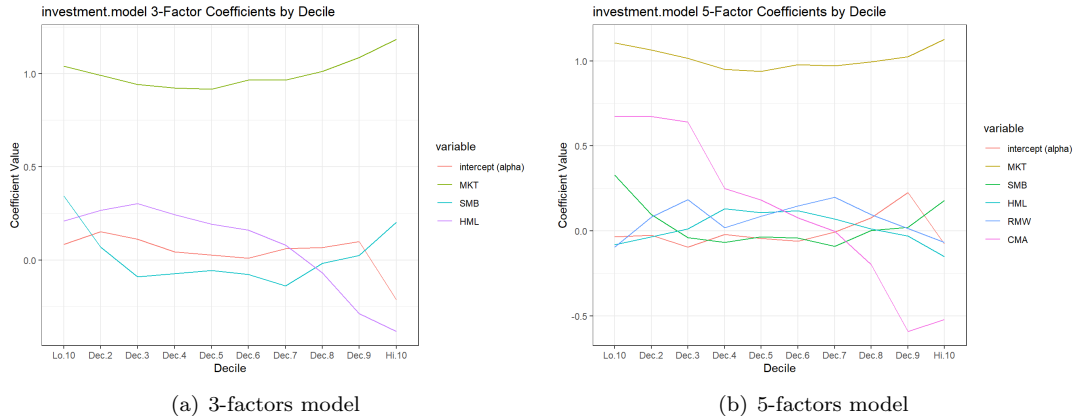

(b) 5-factors model

Figure 4: Coefficient by decile of portfolio sorted by investment

Like the RMW coefficient, CMA was not a factor in the Three Factor model. This again allows us to view how the other coefficients responded to changes in firm investment. HML was the most notable coefficient in the Three Factor model, dropping sharply as firm investment increased. The SMB coefficient makes a bit of a U shape, higher at the extremes. This makes sense because smaller businesses tend to have more freedom in their capital deployment, so will probably have either abnormally low or high levels of investment. In the Five Factor model, we don't see much change in any of the coefficients. CMA and HML do share the largest correlation at 0.58, which explains why the movement of the HML coefficient in the Three Factor model was very similar to the CMA coefficient in the Five Factor Model.

2 R squared and comparison of statistical significance

2.1 Portfolios based on Size

	Lo.30	Med.40	Hi.30
5-factor vs. CAPM	0.267	0.093	0.013
5-factor vs 3-factor	0.002	0.000	0.000

Table 1: 3 portfolios sorted by size

	Lo.20	Qnt.2	Qnt.3	Qnt.4	Hi.20
5-factor vs. CAPM	0.298	0.191	0.104	0.034	0.02
5-factor vs 3-factor	0.004	0.000	0.000	0.000	0.00

Table 2: 5 portfolios sorted by size

	Lo.10	Dec.2	Dec.3	Dec.4	Dec.5	Dec.6	Dec.7	Dec.8	Dec.9	Hi.10
5-factor vs. CAPM	0.321	0.260	0.207	0.176	0.131	0.082	0.048	0.026	0.005	0.034
5-factor vs 3-factor	0.007	0.003	0.001	0.000	0.000	0.000	0.000	0.001	0.000	0.000

Table 3: 10 portfolios sorted by size

The difference in R-squared values in between the Five Factor Model and CAPM and the Five Factor Model and Three Factor Model for portfolios based on size. The difference in R-Squared values between the two models decreases as company size increases. For the three portfolios, the difference at the lower 30% is 0.267, and the difference at top 30% is 0.013. For the ten portfolios, the difference for the lowest decile is 0.321, and 0.034 for the highest decile. This can be seen in the plow below. We selected 10 portfolios to plot, which can help us explore how these models performed at various extremes.

(a) R square for all models
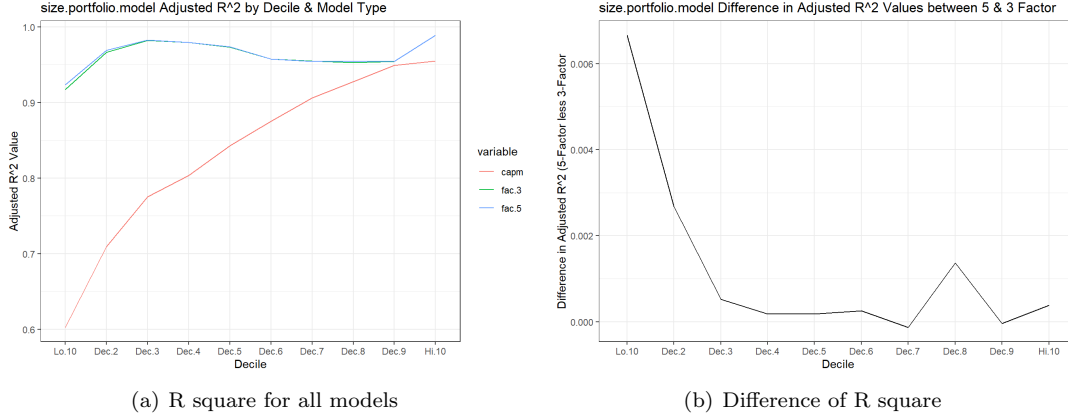
(b) Difference of R square

Figure 5: R square values and difference across models

It's easy to see that the curve is the change in gap between CAPM and the factor-based models, i.e., the difference in R-Squared values decreases as we approach the upper deciles. Since the portfolios are formed on size this shows that the Five Factor and Three Factor Models are superior to CAPM in explaining the returns of micro-caps and smaller companies. The difference between the Five Factor and Three Factor Models were minimal across all deciles.

2.2   Portfolios based on Book-to-Market ratios:

	Lo.30	Med.40	Hi.30
5-factor vs. CAPM	0.034	0.048	0.196
5-factor vs 3-factor	0.005	0.003	0.002

Table 4: 3 portfolios sorted by book-to-market ratio

	Lo.20	Qnt.2	Qnt.3	Qnt.4	Hi.20
5-factor vs. CAPM	0.051	0.009	0.050	0.146	0.204
5-factor vs 3-factor	0.005	0.006	0.003	0.000	0.003

Table 5: 5 portfolios sorted by book-to-market ratio

	Lo.10	Dec.2	Dec.3	Dec.4	Dec.5	Dec.6	Dec.7	Dec.8	Dec.9	Hi.10
5-factor vs. CAPM	0.076	0.015	0.008	0.014	0.032	0.078	0.109	0.169	0.177	0.226
5-factor vs 3-factor	0.008	0.007	0.007	0.005	0.002	0.005	0.000	0.001	0.002	0.004

Table 6: 10 portfolios sorted by book-to-market ratio

In our regression, the difference between the Five Factor Model and the CAPM model were larger for the higher Book-to-Market portfolios than for the lower Book-to-Market portfolios.
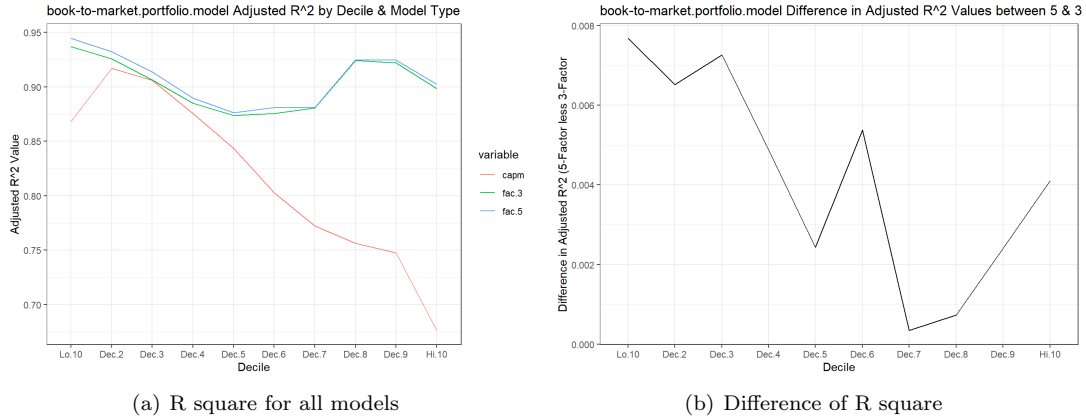
(a) R square for all models   (b) Difference of R square

Figure 6: R square values and difference across models

From the plot, although choppy, we still can see that in the lowest decilse, the gap between the Five and Three Factor Models and CAPM was relatively small . However as we move to the upper deciles the gap between Five and Three Factor Models and CAPM grows significantly. Because these portfolios are sorted by book-to-market ratio, we can say that for growth stocks with low Book-to-Market ratios the Factor models don't perform significantly better. However as we approach the upper extremes of high value stocks the R-Squared value for CAPM drops significantly while the factor models hold relatively steady. This shows us that the Factor models are superior at the upper extreme where more of a companies return comes from it's Book-to-Market ratio as opposed to it's market exposure.

2.3   Portfolios based on Profitability

	Lo.30	Med.40	Hi.30
5-factor vs. CAPM	0.074	0.021	0.038
5-factor vs 3-factor	0.046	0.004	0.017

Table 7: 3 portfolios sorted by profitability

	Lo.20	Qnt.2	Qnt.3	Qnt.4	Hi.20
5-factor vs. CAPM	0.069	0.058	0.020	0.010	0.055
5-factor vs 3-factor	0.041	0.022	0.004	0.007	0.020

Table 8: 5 portfolios sorted by profitability

	Lo.10	Dec.2	Dec.3	Dec.4	Dec.5	Dec.6	Dec.7	Dec.8	Dec.9	Hi.10
5-factor vs. CAPM	0.083	0.067	0.087	0.036	0.023	0.018	0.019	0.007	0.048	0.055
5-factor vs 3-factor	0.035	0.039	0.044	0.007	0.004	0.005	0.008	0.006	0.024	0.014

Table 9: 10 portfolios sorted by profitability

The differences in R-Squared between the Five Factor Model and CAPM was 0.078, 0.099 and 0.142 in the lowest portfolios based on profitability. The differences between the Five and Three Factor Models and was 0.061, 0.070 and 0.072 in the lowest portfolios and 0.024, 0.027 and 0.042 in the highest portfolios. When comparing the extremely low and high profitability based portfolios. With the plot below It's clear that the difference between Five Factor Model and the other two models.

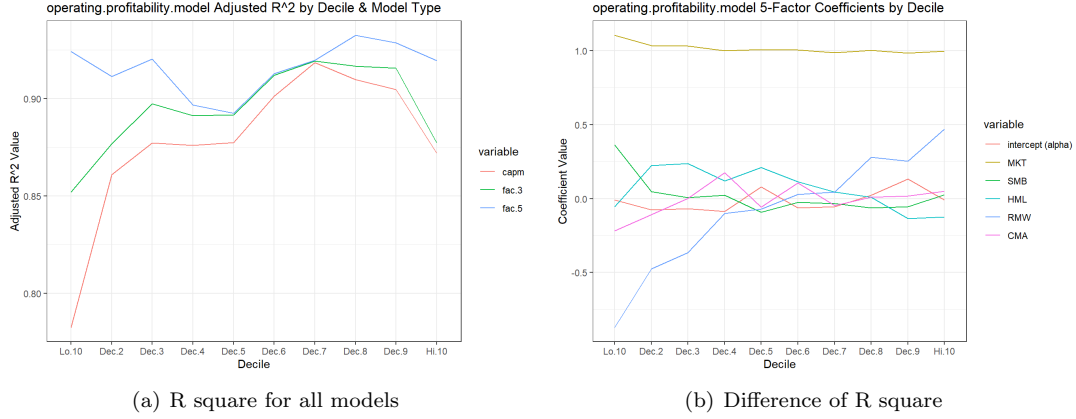(a) R square for all models   (b) Difference of R square

Figure 7: R square values and difference across models

The plot is a U-shape centered around the 5th decile . We can see that for the lowest couple deciles the difference of R square values between the Five Factor Model and Three Factor Model is notable. They start to converge around the middle but after 7 deciles the difference between these two models increases again. The difference of R-squared values in higher deciles is still smaller than in the lower deciles. Because these portfolios are sorted by profitability it means for the lower profitability business, the Five Factor Model does better than the Three Factor Model and CAPM. This is also true for the businesses that are extremely profitable. This supports the analysis done when analyzing the coefficients suggesting that the profitability factor does play an important part in predicting a stocks return. The convergence in the middle shows us the market average which is where CAPM performs almost as well as the Five Factor Model.

2.4 Portfolios based on Investment

	Lo.30	Med.40	Hi.30
5-factor vs. CAPM	0.078	0.007	0.032
5-factor vs 3-factor	0.061	0.000	0.024

Table 10: 3 portfolios sorted by investment

	Lo.20	Qnt.2	Qnt.3	Qnt.4	Hi.20
5-factor vs. CAPM	0.099	0.029	0.012	0.010	0.037
5-factor vs 3-factor	0.070	0.011	0.000	0.006	0.027

Table 11: 5 portfolios sorted by investment

	Lo.10	Dec.2	Dec.3	Dec.4	Dec.5	Dec.6	Dec.7	Dec.8	Dec.9	Hi.10
5-factor vs. CAPM	0.142	0.050	0.043	0.021	0.015	0.012	0.001	0.023	0.024	0.048
5-factor vs 3-factor	0.072	0.035	0.023	0.006	0.001	0.001	0.001	0.016	0.013	0.042

Table 12: 10 portfolios sorted by investment

Looking at the difference R-squared values between models difference across the decile, we can see that the difference between the Five Factor Model and CAPM is 0.074, 0.069, and 0.083 in the lowest sections and 0.038 ,0.55 and 0.055 in the highest sections when we sort portfolios based on firm investment.

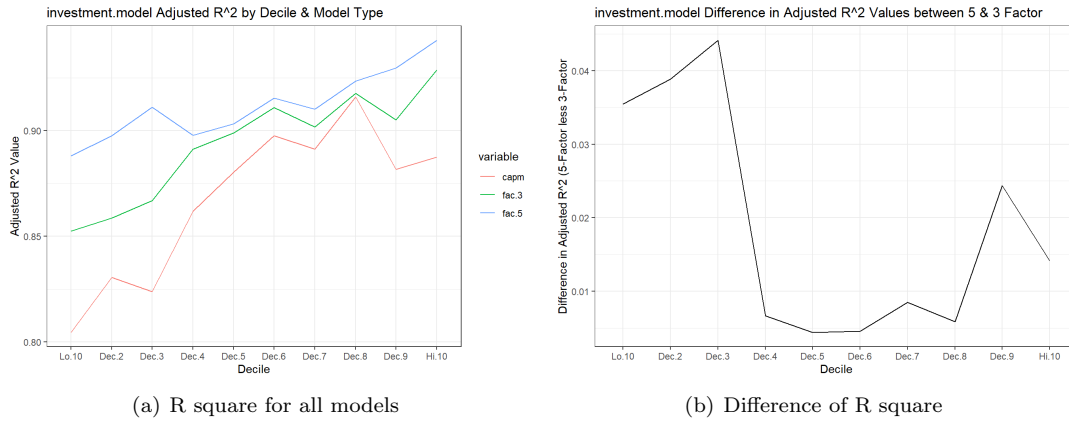(a) R square for all models   (b) Difference of R square

Figure 8: R square values and difference across models

The plot of the differences in R-squared values between Five Factor and Three Factor model is nearly a U shape. When firm investment increases, the difference between the R-squared for the Three and Five Factor becomes smaller. An extreme decrease happens between 3 decile and 4 decile. Although the difference increases between 8 deciles and 9 deciles, it is still smaller than the difference in the lower deciles. Because these portfolios are sorted by investment, it means for the lower investment business the Five Factor model is better than the Three Factor Model and by extension CAPM for predicting the returns of firms with extremely high rates of investment. The differences ,however,are smaller than for profitability.

3 Deeper Analysis

3.1 Influence of 2008 crisis

After 2008 crisis people more interested in lower part. The reason may be In a low-interest rate environment, the performance of portfolios may be influenced by changes in market conditions and economic environment. For instance, low interest rates may lead investors to seek higher risk, higher return investments rather than safer, more stable investments. This may cause changes in the weights of certain stocks and assets in the portfolio, affecting the performance and R-squared values. Therefore, analyzing the performance of portfolios in a low-interest rate environment can provide valuable information on market trends and investment decision-making.

3.2 Numbers of factors

Because smaller, less profitable companies and capital are more volatile, so if we use a smaller number of factors to fit the model, the fit is not as good as if we use a larger number of factors to fit the model.

4 Conclusion

In conclusion, the Five Factor Model outperformed the Three Factor Model and the CAPM in most portfolios, especially in portfolios containing micro-cap stocks and firms with low profitability. These results suggest that the inclusion of profitability and investment factors can improve the prediction of stock returns. However, it is essential to note that no model can perfectly predict stock prices, and the results should be interpreted with caution. To find more accruate result, we need to find more factors which influence the r-square values

The regression analysis of portfolios based on size, book-to-market ratios, profitability, and investment provide insights into the relationship between stock returns and these factors. The results indicate that smaller businesses tend to generate higher returns than larger companies. Value stocks

also tend to outperform growth stocks, and high-profitability firms generate higher returns than low-profitability companies. In terms of investment, there is a negative relationship between investment and stock returns. Furthermore, the Five Factor and Three Factor models perform better than the CAPM in explaining the returns of micro-caps and smaller companies. The difference between the Five Factor and Three Factor models was minimal across all deciles, while the difference between the Five Factor Model and the CAPM was larger for higher Book-to-Market portfolios than for lower Book-to-Market portfolios.

The weights of stocks and assets in portfolios can vary based on performance and market trends, and R-squared values can be influenced by portfolio composition, market conditions, and economic environment. In a low-interest rate environment, investors may seek higher risk, higher return investments, which can affect portfolio performance and R-squared values. Analyzing portfolio performance in such an environment can provide valuable insights for investment decision-making.