

# CIS 419/519: Homework 5

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Although the solutions are entirely my own, I consulted with the following people and sources while working on this homework: *YuchenSun, JunfanPan*  
<https://stackoverflow.com/questions/50994504/how-to-put-figure-between-items-on-enumerate-list>

## 1 Logical Functions with Neural Nets

a.

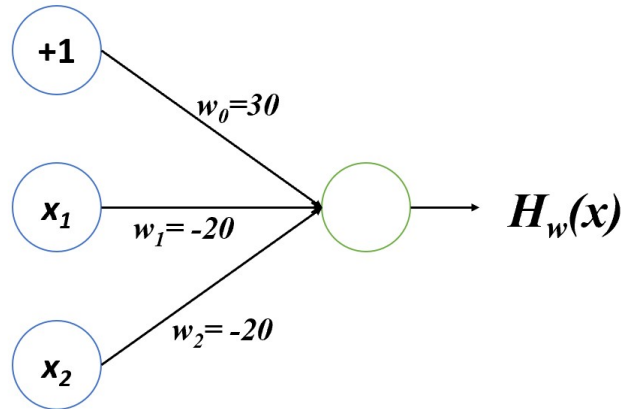


Figure 1: NAND

Truth Table		
$x_1$	$x_2$	$H(x)$
0	0	$\sigma(30) = 1$
0	1	$\sigma(10) = 1$
1	0	$\sigma(10) = 1$
1	1	$\sigma(-10) = 0$

Table 1: NAND

b.

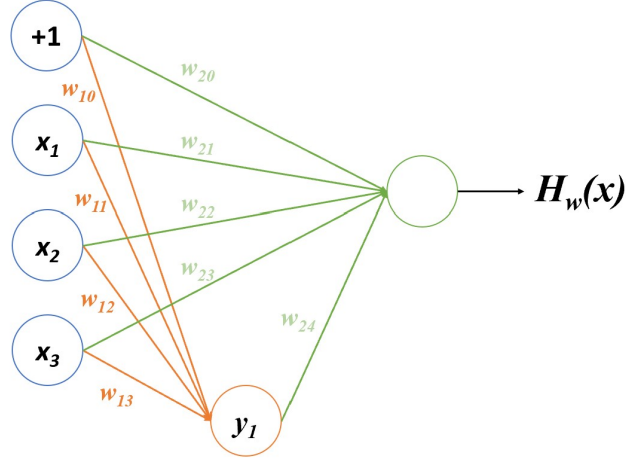


Figure 2: NAND

Truth Table								
$w_{10}$	$w_{11}$	$w_{12}$	$w_{13}$	$w_{20}$	$w_{21}$	$w_{22}$	$w_{23}$	$w_{24}$
-30	20	20	20	-10	20	20	20	-40

Table 2: Weight table

Truth Table				
$x_1$	$x_2$	$x_3$	$y_1$	$H(x)$
0	0	0	$\sigma(-30) = 0$	$\sigma(-10) = 0$
0	0	1	$\sigma(-10) = 0$	$\sigma(10) = 1$
0	1	0	$\sigma(-10) = 0$	$\sigma(10) = 1$
0	1	1	$\sigma(-10) = 0$	$\sigma(10) = 1$
1	0	0	$\sigma(10) = 1$	$\sigma(-10) = 0$
1	0	1	$\sigma(10) = 1$	$\sigma(-10) = 0$
1	1	0	$\sigma(10) = 1$	$\sigma(-10) = 0$
1	1	1	$\sigma(30) = 1$	$\sigma(10) = 1$

Table 3: Parity

## 2 Calculating Backprop by Hand

In the following solution, superscript  $l$  denotes the number of layer and subscript  $p$  denotes the number's position in its matrix.

$z_p^l$  denotes the dot product of weights and last layer's output, and  $a_p^l$  denotes the output of current layer, which is after the activation function.

Since the hidden layer uses a sign function and the output layer uses a sigmoid function, the output can be expressed as follows:

$$a^2 = \text{Sigmoid}(z^2) = \frac{1}{1 + e^{-z^2}} = \frac{1}{1 + e^{-\mathbf{W}^2 \mathbf{a}^1}} = \frac{1}{1 + e^{-\mathbf{W}^2 \cdot \text{Sign}(\mathbf{W}^1 \mathbf{X})}}$$

where

$$a^1 = \text{Sign}(z_1) = \text{Sign}(\mathbf{W}^1 \mathbf{X}) = \text{Sign}\left(\begin{bmatrix} 0.1 & 0.2 \\ -0.4 & 0.3 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix}\right) = \text{Sign}\left(\begin{bmatrix} 1.3 \\ -0.8 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

and

$$a^2 = \frac{1}{1 + e^{-\mathbf{W}^2 a^1}} = \frac{1}{1 + e^{-\begin{bmatrix} 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}}} = \frac{1}{1 + e^{0.1}} = 0.475$$

To calculate the output gradient with respect to each of the weights, we can use the chain rule. Every part in the partial derivative chain is calculated and listed in:

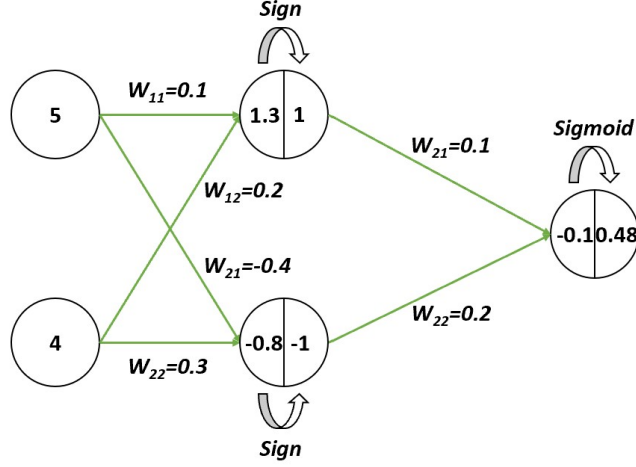


Figure 3: Forward Propagation

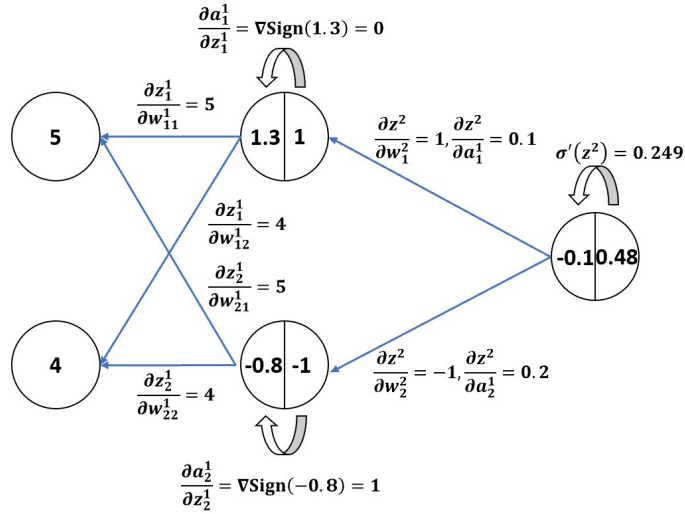


Figure 4: Backpropagation

$$\frac{da^2}{d\mathbf{W}^2} : \begin{cases} \frac{da^2}{\partial w_{21}^2} = \frac{\partial a^2}{\partial z^2} \frac{\partial z^2}{\partial w_1^2} = 0.249 \cdot 1 = 0.249 \\ \frac{da^2}{\partial w_{21}^2} = \frac{\partial a^2}{\partial z^2} \frac{\partial z^2}{\partial w_2^2} = 0.249 \cdot (-1) = -0.249 \end{cases}$$

$$\frac{da^2}{d\mathbf{W}^1} : \begin{cases} \frac{da^2}{\partial w_{11}^1} = \frac{\partial a^2}{\partial z^2} \frac{\partial z^2}{\partial a_1^1} \frac{\partial a_1^1}{\partial z_1^1} \frac{\partial z_1^1}{\partial w_{11}^1} = 0.249 \cdot 0.1 \cdot 0 \cdot 5 = 0 \\ \frac{da^2}{\partial w_{12}^1} = \frac{\partial a^2}{\partial z^2} \frac{\partial z^2}{\partial a_1^1} \frac{\partial a_1^1}{\partial z_1^1} \frac{\partial z_1^1}{\partial w_{12}^1} = 0.249 \cdot 0.1 \cdot 0 \cdot 4 = 0 \\ \frac{da^2}{\partial w_{21}^1} = \frac{\partial a^2}{\partial z^2} \frac{\partial z^2}{\partial a_2^1} \frac{\partial a_2^1}{\partial z_2^1} \frac{\partial z_2^1}{\partial w_{21}^1} = 0.249 \cdot 0.2 \cdot 1 \cdot 5 = 0.249 \\ \frac{da^2}{\partial w_{22}^1} = \frac{\partial a^2}{\partial z^2} \frac{\partial z^2}{\partial a_2^1} \frac{\partial a_2^1}{\partial z_2^1} \frac{\partial z_2^1}{\partial w_{22}^1} = 0.249 \cdot 0.2 \cdot 1 \cdot 4 = 0.199 \end{cases}$$

So we finally get

$$\frac{da^2}{d\mathbf{W}^2} = \begin{bmatrix} 0.249 \\ -0.249 \end{bmatrix}, \quad \frac{da^2}{d\mathbf{W}^1} = \begin{bmatrix} 0 & 0 \\ 0.249 & 0.199 \end{bmatrix}$$

### 3 Neural Nets in SuperTuxKart