

I. Binary Method

I used binary method found the roots of the following four functions, the iterating times and the times of postcondition ϵ are nearly in linear relationship. The converging speed is 1, not so fast.

I-a $x^{-1} - \tan x$ on $[0, \frac{\pi}{2}]$

postcondition ϵ and δ	α	iterating times
10^{-5}	0.860335521730643	17
10^{-6}	0.860333274688473	21
10^{-7}	0.860333555568744	24
10^{-8}	0.860333590678778	27
10^{-9}	0.860333589215860	30
10^{-10}	0.860333589032996	33
10^{-11}	0.860333589021567	37
10^{-12}	0.860333589019424	41
10^{-13}	0.860333589019334	44
10^{-14}	0.860333589019379	45

I-b $x^{-1} - 2^x$ on $[0, 1]$

postcondition ϵ and δ	α	iterating times
10^{-5}	0.641181945800781	17
10^{-6}	0.641185760498047	18
10^{-7}	0.641185760498047	18
10^{-8}	0.641185745596886	26
10^{-9}	0.641185744665563	30
10^{-10}	0.641185744490940	34
10^{-11}	0.641185744505492	36
10^{-12}	0.641185744504583	40
10^{-13}	0.641185744504980	44
10^{-14}	0.641185744504988	47

I-c $2^{-x} + e^x + 2 \cos x - 6$ on $[1, 3]$

postcondition ϵ and δ	α	iterating times
10^{-5}	1.829383850097656	18
10^{-6}	1.829382896423340	21
10^{-7}	1.829383611679077	23
10^{-8}	1.829383604228497	28
10^{-9}	1.829383601434529	31
10^{-10}	1.829383601958398	35
10^{-11}	1.829383601936570	38
10^{-12}	1.829383601933841	41
10^{-13}	1.829383601933841	41
10^{-14}	1.829383601933849	48

I-d $(x^3 + 4x^2 + 3x + 5) / (2x^3 - 9x^2 + 18x - 2)$ on $[0, 4]$

postcondition ϵ and δ	α	iterating times
10^{-5}	0.117881774902344	19
10^{-6}	0.117877006530762	22
10^{-7}	0.117876589298248	26
10^{-8}	0.117876566946507	29
10^{-9}	0.117876566015184	32
10^{-10}	0.117876566771884	36
10^{-11}	0.117876566793711	39
10^{-12}	0.117876566794621	42
10^{-13}	0.117876566795360	46
10^{-14}	0.117876566795310	49

II. Newton's Method

I used Newton's Method found the roots near 4.5 and 7.7 of the following function. It can be found that the Newton's Method converges so fast that the root can be found only in iterating times in unit digits.

II-a $x = \tan x$, $x_0 = 4.5$

postcondition ϵ	α	iterating times
10^{-5}	4.493409655013248	2
10^{-6}	4.493409457909247	3
10^{-7}	4.493409457909247	3
10^{-8}	4.493409457909247	3
10^{-9}	4.493409457909247	3
10^{-10}	4.493409457909247	3
10^{-11}	4.493409457909247	3
10^{-12}	4.493409457909064	4
10^{-13}	4.493409457909064	4
10^{-14}	4.493409457909064	4

II-b $x = \tan x$, $x_0 = 7.7$

There is a problem that when $\epsilon = 10^{-14}$, the iteration does not stop until the iteration times out of the postcondition. I think the problem happens because the finite digits of cpp program, the x_n is so close to the root that after $f(x_n)$ is rounded up in double, $f(x_n) = f(x_{n+1})$ and $f(x_n) > 10^{-14}$, that may be why the iteration will not stop.

postcondition ϵ and δ	α	iterating times
10^{-5}	7.725251836938464	4
10^{-6}	7.725251836938464	4
10^{-7}	7.725251836938464	4
10^{-8}	7.725251836938464	4
10^{-9}	7.725251836938464	4
10^{-10}	7.725251836938464	4
10^{-11}	7.725251836937707	5
10^{-12}	7.725251836937707	5
10^{-13}	7.725251836937707	5
10^{-14}	7.725251836937707	100

III. Secant Method

I used Secant Method found the root of the following functions. It can be found that the convergent speed of Secant Method is also very fast. But it is slower than Newton's Method.

III-a $\sin\left(\frac{x}{2}\right) - 1$ with $x_0 = 0, x_1 = \frac{\pi}{2}$

postcondition ϵ and δ	α	iterating times
10^{-5}	2.221441469079183	1
10^{-6}	2.579569691806805	2
10^{-7}	3.134268450825327	11
10^{-8}	3.138795060236532	13
10^{-9}	3.140932231503763	16
10^{-10}	3.141340394802018	18
10^{-11}	3.141533103369373	21
10^{-12}	3.141569907429061	23
10^{-13}	3.141583965313214	25
10^{-14}	3.141590602603975	28

III-b $e^x - \tan x$ with $x_0 = 0, x_1 = 1.4$

postcondition ϵ and δ	α	iterating times
10^{-5}	1.159924363529063	1
10^{-6}	1.121423255162759	3
10^{-7}	1.306326943021557	13
10^{-8}	1.306326943021557	13
10^{-9}	1.306326943021557	13
10^{-10}	1.306326940423042	14
10^{-11}	1.306326940423042	14
10^{-12}	1.306326940423042	14
10^{-13}	1.306326940423042	14
10^{-14}	1.306326940423042	14

III-c $x^3 - 12x^2 + 3x + 1$ with $x_0 = 0, x_1 = -0.5$

postcondition ϵ and δ	α	iterating times
10^{-5}	-0.108108108108108	1
10^{-6}	-0.182578753220966	3
10^{-7}	-0.188685400608274	6
10^{-8}	-0.188685400608274	6
10^{-9}	-0.188685400608274	6
10^{-10}	-0.188685403446523	7
10^{-11}	-0.188685403446523	7
10^{-12}	-0.188685403446523	7
10^{-13}	-0.188685403446523	7
10^{-14}	-0.188685403446523	7