I. Run the subroutines on the function $\frac{1}{1+x^2}$.

I-a Plot the polynomials.

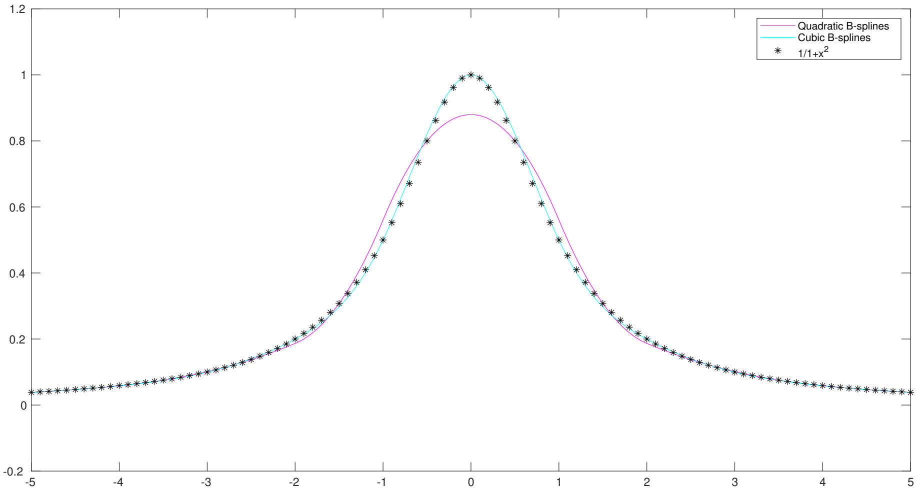

Figure 1: Interpolating f by quadratic and cubic cardinal B-splines.

I-b Consider the $E_s(x)$.

The $E_s(x)$ of quadratic B-spline interpolants.

x	$E_s(x)$
-3.5	0.000000000000000
-3	0.001418382673699
-0.5	0.000000000000000
0	0.120237781803927
0.5	0.000000000000000
3	0.001418382673699
3.5	0.0000000000000000

The $E_s\,(x)$ of cubic B-spline interpolants.

x	$E_s(x)$
-3.5	0.000669568235464
-3	0.000000000000000
-0.5	0.020528884666179
0	0.000000000000000
0.5	0.020528884666179
3	0.000000000000000
3.5	0.000669568235464

We can find that some errors close to machine precision. The reason is that the quadratic B-splines interpolant is interpolated by f on $t_i = i - \frac{11}{2}$, so on -3.5,-0.5,0.5,3.5, the interpolant is absolutely equal to f. Similarly, quadratic B-splines interpolant is interpolated by f on $t_i = i - 6$, so the error is definitly 0 on -3,0,3.

Although at some points like -3,3, the error of quadratic B-splines interpolants is small , its maximal error is far larger than that of cubic B-splines. So the cubic B-splines is more accurate.

II. Plot the heart shape.

I choose the complete cubic spline to interpolate the heart shape function. Since complete cubic spline gurantee the smoothness that $f'\,(a) = s'\,(f;a)$ and $f'\,(b) = s'\,(f;b)$.

I change the function to $y = \frac{2}{3}(\sqrt{3-t^4} + sign(t) * t), x = t$ and $y = \frac{2}{3}\left(-\sqrt{3-t^4} + sign(t) * t\right)$ and $x = \mid t \mid, \left(t \in [0, \sqrt[4]{3}]\right)$, I take $\frac{n}{2}$ knots for each of the y function. So we can plot the function by taking n knots.

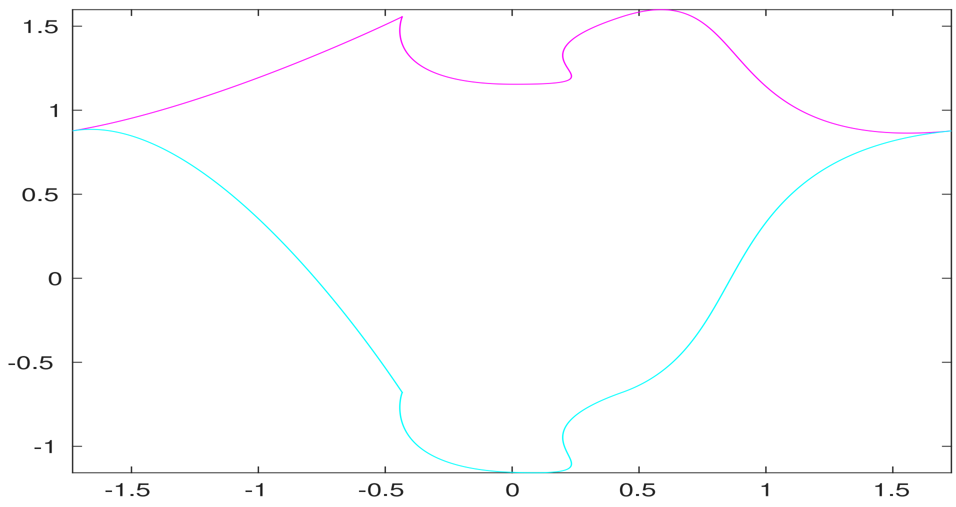

Figure 2: The interpolants of Heart shape function when n = 10.

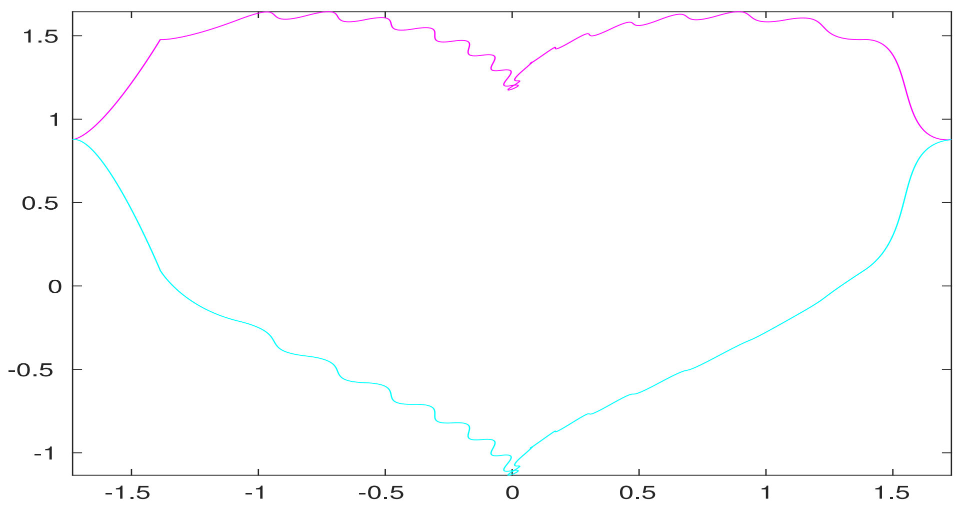

Figure 3: The interpolants of Heart shape function when n = 40.

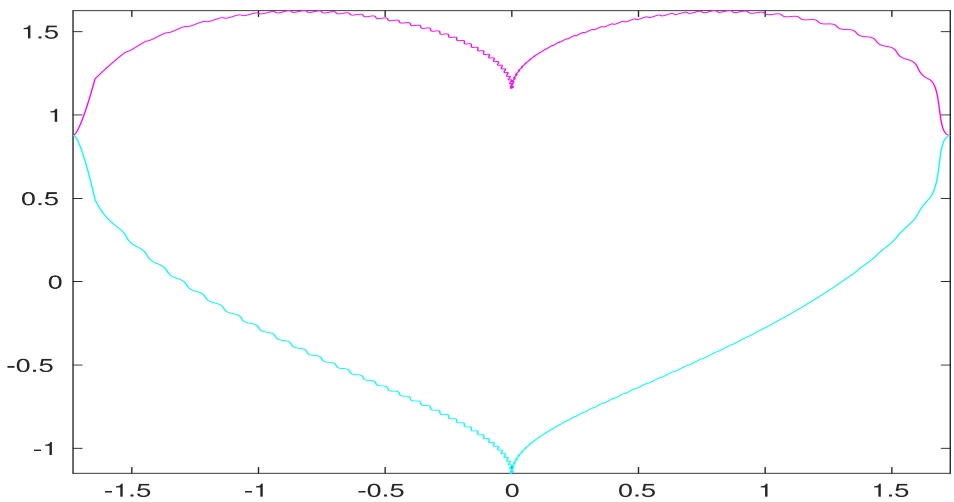

Figure 4: The interpolants of Heart shape function when n = 160.