Cumulative prospect theory and stock returns

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Abstract

Recent studies find that cumulative prospect theory can explain stock returns in the cross-section. We find that the explanatory power varies with the estimated probability weights from the empirical pricing kernel. Allowing time-varying probability weights strengthens the support for the cumulative prospect theory in explaining stock returns beyond existing return predictors and time-varying stock characteristics. A conditional strategy based on time-varying probability weights significantly improves the performance of the unconditional strategy.

Keywords: cumulative prospect theory, time-varying probability weighting, TK anomaly

JEL Codes: G40, G12, G14

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1 Introduction

Investor's risk attitudes are fundamental in explaining stock returns. The prospect theory of (Kahneman and Tversky, 1979) and (Tversky and Kahneman, 1992) extends the expected utility framework to accommodate several empirical findings in asset pricing. As a direct application of the cumulative prospect theory, Barberis et al. (2016) evaluate stocks based on the prospect theory (Tversky-Kahneman, or TK) values of their past return distributions and demonstrate that stocks with higher TK values earn lower subsequent returns, evidence of investors tilting towards this type of stocks. Among the various components in the cumulative prospect theory, Barberis et al. (2016) find that probability weighting, which captures investors' attitudes towards the tails of a return distribution, contributes most to the stock return prediction. In this paper, we study stock returns with different TK values and find that the return of the long-short TK portfolio is highly volatile yet predictable using the estimated probability weights. Our finding highlights the importance of time-varying tail preference in the cumulative prospect theory.

Recent studies have focused on the micro-foundation of probability weighting and suggested probability weighting could be time-varying and predictable (Frydman and Jin, 2023). To the extent that the aggregate demand for stocks with high TK values is time-varying, variables that correlate with such demand should predict excess returns of these stocks. One example is the estimated probability weighting function from the empirical pricing kernel. The empirical pricing kernel captures a representative preference, and the implied aggregate attitude toward tail events could vary by aggregating heterogeneous preferences and expectations. We estimate the probability weighting function from the returns of S&P 500 index options and find considerable time variations in the estimates, with periods of investors overweighting or underweighting the tail events. The overweight of tails is a common assumption when the prospect theory is applied to understand stock returns. However, using the implied probability weight from the S&P 500 index options, Polkovnichenko and Zhao (2013) show that investors do not always overweight tail events. At some periods, investors may even underweight tails when relatively complacent, perceiving those rare events as improbable. Variations in probability weights could induce time-varying demand for stocks with high TK values, generating a time-variation in the TK return predictability. Similar evidence exists for the demand for active portfolio management. Polkovnichenko et al. (2019) find that the implied probability weighting function covaries with investor demand for mutual funds that can outperform in the tails of the return distribution.

Our first result demonstrates the variation in the stock return predictability with TK values. Within the framework of the prospect theory, we construct the long-short TK portfolio following Barberis et al. (2016) from 1996 to 2020 and compute monthly returns for value-weighted and equal-weighted portfolios. Table 1 reports the average returns for the whole sample and sub-samples when investors over- and under-weigh tail events, where the probability weights are estimated from the empirical pricing kernel. We find that the performance of the long-short TK portfolio is significantly different between the two sub-samples, with the average return much stronger in the overweight subsample.[1]

[Insert Table 1 here]

Furthermore, we find the TK portfolio return prediction of probability weights persists for longer horizons. Table 2 shows the predictive power of the indicator for overweight periods for the long-short TK portfolio returns over the 3-, 6-, and 12-month horizons. The predictive power is statistically and economically significant, and the estimated coefficient and the adjusted R-square increase with forecasting horizons.

[Insert Table 2 here]

We next investigate how the variation in investors' attitudes toward tail events could affect the demand and expected return of stocks with different TK values. Conceptually,

[1]While the average return of equal-weighted portfolio is large and statistically significant in the whole sample, the value-weighted portfolio is statistically insignificant. Part of this weaker performance is due to the small sample period as opposed to the post-1963 sample in Barberis et al. (2016).

investor risk attitudes from the expected utility theory and the prospect theory can jointly affect stock valuations and portfolio choices, and stock characteristics in the portfolios sorted on the TK values can affect returns through channels distinct from probability weighting. As such, we illustrate the effect of time-varying probability weights in an equilibrium framework of Barberis et al. (2021). We calculate the model-predicted CAPM alpha spread for the long-short TK portfolio and demonstrate a sign change in the spreads from positive to negative as investors vary risk attitudes from over-weighing to under-weighing tail events.

We conduct additional empirical analyses to draw the contrast in the TK premium between overweight and underweight periods. We investigate whether the variation in the long-short TK portfolio return is the result of exposures to common risk factors or time-varying stock characteristics. We also conduct Fama-MacBeth regressions to control for known predictors of stock returns, including capital gains overhang (CGO) and other lottery-related stock characteristics. Our findings suggest that time-varying probability weights provide a robust and distinct channel to explain the variation in long-short TK portfolio returns. As an application of our findings, we propose a conditional trading strategy that substantially outperforms the original unconditional TK strategy. For the value-weighted returns, our strategy raises the Sharpe ratio from 0.21 for the original strategy to 0.50. Our findings are robust to alternative estimation methods for the probability weights.

This paper is related to several strands of literature. First, this paper adds to the literature on applying the prospect theory to explain stock returns and stock anomalies, such as Barberis and Huang (2008), Barberis et al. (2016), and Barberis et al. (2021). In contrast to the constant probability weights in these studies, we allow the probability weights to be time-varying to reflect the fluctuation in the risk attitudes of the representative agent towards tail events, and we study its asset pricing implications. Our findings not only strengthen the empirical support for the prospect theory but also highlight the importance of the variation in aggregate demands in the prospect theory.

Several studies investigate the time-series variation in asset returns related to the prospect theory. Kumar (2009) finds lottery demand is stronger during economic downturns. Baele et al. (2018) apply the cumulative prospect theory to simultaneously explain low average returns of both out-of-the-money put and out-of-the-money call options. They combine probability weighting and time-varying equity return volatility to match the time-series pattern of the variance premium. In contrast to their finding that time-varying probability weights are not crucial in explaining the dynamics of variance premium, we find that time-varying probability weights are essential to explain the dynamics of stock returns. Our paper is also related to but different from Liu et al. (2020), which focus on the variation of probability weights around earnings announcements. Chen et al. (2021) use Internet search volume for lottery to capture gambling sentiment shifts. An et al. (2020) find that several lottery-related anomalies are state-dependent and are stronger among stocks in which investors have lost money. Barberis et al. (2021) also include capital overhanging (CGO) in their model to capture prior trading gains or losses. We find that the explanatory power of the time-varying probability weights for the long-short TK portfolio return is little affected by CGO.

This paper is also related to the literature on the estimation of probability weighting functions, including Tversky and Kahneman (1992), Gonzalez and Wu (1999), and Prelec (1998) which use laboratory experiments, and Polkovnichenko and Zhao (2013) which use prices from stock indices and index options.

The rest of the paper is organized as follows. In Section 2, we discuss the conceptual framework and the model's implications. Section 3 describes the data and variables and how we estimate the parameter. In Section 4, we empirically test our hypotheses. We do further analysis in Section 5 and conclude in Section 6.

2 The Conceptual Framework

Our aim is to study how time-varying probability weighting preferences affect the asset pricing implication of the cumulative prospect theory. In this section, we review the construction of portfolio based on the prospect theory (Barberis et al., 2016) and the Barberis et al. (2021) model that incorporates the prospect theory in an equilibrium structure.

2.1 Portfolio construction based on prospect theory

Probability weighting is one of the two important ingredients in the prospect theory. The original version of the prospect theory is proposed by Kahneman and Tversky (1979) as an alternative model of the expected utility in explaining people's decision under risk. To incorporate the cumulative functional (rank dependent) and allow for risky prospects with more than two outcomes, Tversky and Kahneman (1992) develop an extended version, the cumulative prospect theory, which is the one we use in this paper[2].

[insert Figure 1 here]

To calculate the value of any given gamble under the cumulative prospect theory and the expected utility framework, investors have to consider both the utility of outcomes and their probabilities. Under the expected utility framework, the utility is typically a function of the final wealth, differentiable and concave everywhere, and the probabilities in calculating the expected utility are objective. On the contrary, under the cumulative prospect theory, the utility function (also called value function) depends on both gains and losses. It is kinked at the origin, concave for gains but convex for losses, with investors being more sensitive to losses than gains. In addition, the probabilities in weighting different outcomes are transformed from the objective probabilities. Figure 1 demonstrates a typical value function and probability weighting function. Depending on the attitude of investors on tail events, the probability weighting function can be inverse S-shaped if investors overweight

[2]See Barberis et al. (2016) section 1 for a simple but more detailed description.

tails ($\delta < 1$), linear if investors use objective probabilities ($\delta = 1$), and S-shaped if investors underweight tails ($\delta > 1$).

In applying the prospect theory to stock returns, Barberis et al. (2016) suggest to calculate the prospect theory value (TK value) for each stock using monthly returns in excess of the market over the past five years. First, they sort the sixty excess returns in increasing order, which starts with the most negative returns through to the most positive returns. Second, they give each return equal weight in the distribution, so each return has a physical probability of 1/60. To illustrate this, suppose that m of these returns are negative, while $n = 60 - m$ of these returns are positive. Label the most negative return as r_{-m}, the second most negative one as r_{-m+1}, and so on, through to r_n, the most positive return, where r is a monthly return in excess of the market. The stock's historical return distribution is then

$$(r_{-m}, \frac{1}{60}; r_{-m+1}, \frac{1}{60}; ...; r_{-1}, \frac{1}{60}; r_1, \frac{1}{60}; ...; r_{n-1}, \frac{1}{60}; r_n, \frac{1}{60}). \tag{1}$$

Third, calculate the TK value for each stock-month using value function and probability weighting function

$$\begin{aligned} TK \equiv \ & \textstyle\sum_{i=-m}^{-1} v(r_i) \left[w^- \left(\frac{i+m+1}{60} \right) - w^- \left(\frac{i+m}{60} \right) \right] \\ & + \textstyle\sum_{i=1}^{n} v(r_i) \left[w^+ \left(\frac{n-i+1}{60} \right) - w^+ \left(\frac{n-i}{60} \right) \right], \end{aligned} \tag{2}$$

where $v(.)$ is the value function with the form

$$v(x) = \begin{cases} x^\alpha, & x \geq 0 \\ -\lambda(-x)^\alpha, & x < 0 \end{cases} \tag{3}$$

and w^+ and w^- are the probability weighting function with the form

$$w^+(P) = \frac{P^\phi}{(P^\phi + (1-P)^\phi)^{1/\phi}}, \quad w^-(P) = \frac{P^\delta}{(P^\delta + (1-P)^\delta)^{1/\delta}}. \tag{4}$$

They use the parameters estimated by Tversky and Kahneman (1992), namely,

$$\alpha = 0.88, \quad \lambda = 2.25$$
$$\phi = 0.69, \quad \delta = 0.61.$$

Lastly, at the start of each month, they sort stocks into deciles based on TK values and construct value-weighted and equal-weighted portfolios within each decile for the following month. A long-short strategy portfolio, which is adopted in this paper, is to long the low-TK decile portfolio and short the high-TK decile portfolio.

2.2 Model and implication

Barberis et al. (2021) present a model that incorporates all elements of the prospect theory and find the model can explain a majority of well-known anomalies in the cross-sectional stock returns. In this subsection, we briefly describe their model and show its implication of time-varying probability weighting preferences.

Consider an economy with three dates, $t = -1$, 0, and 1, where investors make decision at date 0. There is a risk-free asset with gross per-period return R_f and N risky assets with gross per-period return $\tilde{R}_i$ for risky asset i. Let $\tilde{R} = (\tilde{R}_1, ..., \tilde{R}_N)'$ be the return vector and has cumulative distribution function $P(\tilde{R})$. The vector of expected returns on the risky assets is $\bar{R} = (\bar{R}_1, ..., \bar{R}_N)$ and the covariance matrix of returns is Σ.

Assume investors in the economy are identical in their preferences, wealth at time -1 (W_{-1}), and wealth at time 0 (W_0). Let the fraction of time-0 wealth that an investor allocates to risky asset i is Θ_i, and the allocation vector is $\Theta = (\Theta_1, ..., \Theta_N)'$. Thus, wealth at time 1 is

$$\tilde{W_1} = W_0((1 - 1'\Theta)R_f + \Theta'\tilde{R}). \tag{5}$$

At date 0, each investor maximizes the following objective function to determine the optimal asset allocation:

$$\begin{aligned}
&\max_{\Theta_1,\ldots,\Theta_N} E(\tilde{W}_1) - \tfrac{\gamma}{2} Var(\tilde{W}_1) + b\textstyle\sum_{i=1}^N V(\tilde{G}_i) \\
&= \max_{\Theta_1,\ldots,\Theta_N} W_0((1-1'\Theta)R_f + \Theta'\bar{R}) - \tfrac{\gamma}{2} W_0^2 \Theta'\Sigma\Theta + b\textstyle\sum_{i=1}^N V(\tilde{G}_i),
\end{aligned} \tag{6}$$

where

$$\tilde{G}_i = W_0\Theta_i(\tilde{R}_i - R_f) + W_{-1}\Theta_{i,-1}g_i. \tag{7}$$

In equation (6), the first two terms capture the traditional mean-variance preference and the third term captures both prospect theory and narrow framing. γ measures the aversion to portfolio risk and b determines the weight of prospect theory in decision making. G_i is the potential gain or loss on asset i, which is the sum of potential future gain or loss from asset i, i.e., $W_0\Theta_i(\tilde{R}_i - R_f)$, and the prior gain or loss in investor's holdings of asset i up to time 0, i.e., $W_{-1}\Theta_{i,-1}g_i$. $V(\tilde{G}_i)$ is the cumulative prospect theory value of this potential gain or loss, incorporating loss aversion, diminishing sensitivity, and probability weighting, and the last term in equation (6) sums over $V(\tilde{G}_i)$ across all asset i.

For $\Theta_i > 0$, $V(\tilde{G}_i)$ can be written as

$$\begin{aligned}
&-\lambda W_0^\alpha \textstyle\int_{-\infty}^{R_f-\Theta_{i,-1}g_i/\Theta_i} (\Theta_i(R_f - R_i) - \Theta_{i,-1}g_i)^\alpha dw(P(R_i)) \\
&\quad -W_0^\alpha \textstyle\int_{R_f-\Theta_{i,-1}g_i/\Theta_i}^{\infty} (\Theta_i(R_i - R_f) + \Theta_{i,-1}g_i)^\alpha dw(1 - P(R_i)).
\end{aligned} \tag{8}$$

Let $\Theta_{M,R} = \sum_{i=1}^N \Theta_{M,i}$, where $\Theta_{M,i}$ is the market value of asset i divided by the total market value of all traded assets. Define $\theta_i = \Theta_i/\Theta_{M,R}$, $\theta_{M,i} = \Theta_{M,i}/\Theta_{M,R}$, and $\theta_{i,-1} = \Theta_{i,-1}/\Theta_{M,R}$. With a few assumptions, Barberis et al. (2021) show a bounded rationality with heterogeneous holdings equilibrium structure that can help to quantitatively predict the cross-section of average returns when investors evaluate risk according to prospect theory[3]:

$$
\begin{aligned}
&\theta_i \left(\mu_i + \tfrac{\nu\zeta_i}{\nu-2} - R_f\right) - \tfrac{\hat{\gamma}}{2}(\theta_i^2\sigma_i^2 + 2\theta_i(\beta_i\sigma_M^2 - \theta_{M,i}\sigma_i^2)) \\
&-\lambda\hat{b}\int_{-\infty}^{R_f-\theta_{i,-1}g_i/\theta_i}(\theta_i(R_f - R_i) - \theta_{i,-1}g_i)^{\alpha}dw(P(R_i)) \\
&-\hat{b}\int_{R_f-\theta_{i,-1}g_i/\theta_i}^{\infty}(\theta_i(R_i - R_f) + \theta_{i,-1}g_i)^{\alpha}dw(1 - P(R_i)),
\end{aligned}
\tag{9}
$$

where μ_i, ζ_i, and ν are parameters governing location, asymmetry, and degree of freedom, respectively, assuming one-dimensional GH skewed t distribution for stock returns. $\hat{\gamma} = \gamma W_0 \Theta_{M,R}$ and $\hat{b} = bW_0^{\alpha-1}\Theta_{M,R}^{\alpha-1}$. As discussed in their paper, if stocks in each anomaly decile are assumed to be identical, one can calculate the expected return of stocks in each decile using five model inputs: their average beta, average standard deviation, average skewness, average capital gain overhang, and the total market capitalization of the decile. In their paper, the probability weighting parameters ϕ (for right tail) and δ (for left tail) described in Section 2.1 are both set to be equal to 0.65, and the value function parameters (α, λ) are set to be equal to (0.7, 1.5) to reflect the recent findings in the literature. In our paper, we assume the same probability weighting parameter for left tail and right tail, denoted by δ, and we vary the value of δ to study its implications for asset prices.

We sort all stocks in deciles based on the estimated TK value and calculate the above mentioned model inputs[4]. We set the capital gain overhang input in all deciles to the average market return, which is a common reference point for when investors evaluate expected returns. On the contrary, if we use the capital gain overhang calculated in each decile as the paper uses, the difference between low-TK decile and high-TK decile in our sample period will be fifty percent, attributing almost all the prediction power of TK to capital gain overhang[5]. We also vary parameter values for $\hat{\gamma}$ and $\hat{b}$ to examine whether the model

[3]See Barberis et al. (2021) section 2 for more details.

[4]We follow Barberis et al. (2021) strictly to calculate beta and market capitalization, but use past 5 years monthly returns to calculate standard deviation and skewness rather than use cross-sectional returns as in the paper. We argue that time-series volatility and skewness are better measures for a typical stock in the anomaly decile from the sense of picking stocks. However, we also admit that backward-looking method has its own disadvantages. See Section 3 for more discussion about the choice in their paper.

[5]The low-TK decile has average capital gain overhang of thirty-four percent, which is too high for a typical stock return to be treated as right tail.

implication is sensitive to the weights of risk aversion and prospect theory.

[insert Figure 2 here]

To see how the value of δ affects the CAPM adjusted alpha spread of the TK long-short portfolio, we vary δ from 0.55 to 2.50. Figure 2 shows that the alpha spread in general decreases with δ. Importantly, when $\delta \leq 1$, the long-short TK portfolio has positive alphas, whereas when $\delta > 1$, the long-short portfolio alpha tends to be close to zero or become negative. This result is consistent with the finding in Barberis et al. (2016): when investors overweight tails, high-TK stocks are overvalued by individuals so that, on average, they earn lower returns in subsequent months than low-TK stock. When investors underweight tails, however, the TK anomaly disappears or changes sign.

3 Data and estimation of probability weights

3.1 Data

We obtain stock return and accounting data from CRSP and Compustat, respectively. Market return is the value-weighted return including distribution (vwretd) from CRSP. S&P 500 index return is the return on S&P composite index (sprtrn) from CRSP. Our universe contains all common stocks that are publicly traded on NYSE, AMEX, and NASDAQ from January 1996 to December 2020[6]. Consistent with Barberis et al. (2016), we require stocks to have at least five years of monthly return observations. We obtain monthly risk-free rate, Fama-French 3 factors (MktRf, SMB, HML), and Carhart momentum factor (UMD) from French Data Library[7]. The Pastro and Stambaugh (2003)'s liquidity factor is downloaded from WRDS. Cyclically adjusted price earnings ratio (CAPE) is obtained from Robert Shiller's homepage[8]. Variance risk premium (VRP) is from Hao Zhou's personal homepage[9].

[6] We start our sample from January 1996 because of the availability of option data to estimate δ.

[7] Website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[8] Website: http://www.econ.yale.edu/ shiller/data.htm.

[9] Website: https://sites.google.com/site/haozhouspersonalhomepage.

The main variable for stock return prediction in the cross section is TK. It is the prospect theory value defined in equation (2). We use past five years monthly return to calculate it following Barberis et al. (2016).

For model inputs, Beta, standard deviation, and skewness are the average of the firm-level corresponding variables within each TK decile over the whole sample period. Firm-level Beta is CAPM beta computed using daily returns over the following year, following Barberis et al. (2021). Firm-level standard deviation and skewness are calculated using past five years monthly returns and are annualized assuming returns are independent. The spreads of these inputs are the difference between low-TK and high-TK deciles (i.e., decile 1 minus decile 10). Capital gain overhang is the average previous year market return over whole sample period and is the same for all deciles. Market capitalization is the average previous-month ratio of the decile market capitalization over the total market capitalization over the whole sample period.

For time-series tests, past n to m month market excess return is the difference between past-n and past-m month market cumulative return and risk-free cumulative return. Standardized physical moments are estimated using past 5 years monthly S&P 500 index returns. Standardized risk-neutral moments are calculated using S&P 500 index options with 30-days maturity (see Section 3.2 for data source). GFC is a dummy variable for the 2008 global financial crisis, which equals 1 if the month falls between October 2008 and March 2009 and 0 otherwise.

We include several common control variables in Fama-MacBeth tests. Beta is calculated from monthly returns over the previous five years, following Fama and French (1992). Size is the log market capitalization at the end of the previous month. Bm is the log of book value of equity scaled by market value of equity, computed following Fama and French (1992) and Fama and French (2008). When the book value of equity is missing from Compustat, we use data from Davis et al. (2000); observations with negative book value are removed. Mom is the cumulative return from the start of month t-12 to the end of month t-2. Cgo is the capital gain overhang in month t-1, following Barberis et al. (2021). It is computed as $(P_i - R_i)/R_i$, where P_i is the stock's current price and R_i is investors' average purchase price. This definition is slightly different from Grinblatt and Han (2005), but it is a more precise match for the capital gain variable in the cumulative prospect theory model. Rev is the return in month t-1. Illiq is Amihud (2002)'s measure of illiquidity, scaled by 10^5. Lt rev is the cumulative return from the start of month t-60 to the end of month t-13. Ivol is idiosyncratic return volatility, as in Ang et al. (2006). Max and Min are the maximum and the negative of the minimum daily returns in month t-1, as in Bali et al. (2011). Skew is the skewness of monthly returns over the previous five years. Eiskew is expected idiosyncratic skewness, as in Boyer et al. (2010). Coskew is coskewness, computed as in Harvey and Siddique (2000) using five years of monthly returns.TK, Mom, Rev, Ivol, Max, and Min are scaled up by 100. The sample period runs from March 1996 to December 2020[10]. All variables are winsorized at 1 and 99 percent in each month, and TK is standardized after the winsorization.

3.2 Estimation of probability weighting function

We obtain the S&P 500 index option (symbol SPX) data from OptionMetrics and the S&P 500 index return from CRSP. We extract the needed information for option from the implied volatility surface.

The estimation of probability weighting parameter, δ, includes two parts: the risk-neutral distribution and the physical distribution. Through the stochastic discount factor, Polkovnichenko and Zhao (2013) show that the probability weighting function can be related to the risk-neutral and physical distributions in the following way. Let $P(.)$ and $p(.)$ be the physical distribution function and its density, $Q(.)$ and $q(.)$ be the risk-neutral distribution function and its density, and $G(.)$ be the probability weighting function. For a specific P_0

[10] It starts from March 1996 since the benchmark three-month moving average δ starts from March 1996

with corresponding gross return R_0 such that $P(R_0) = P_0$, we have

$$G(P_0) = c\int_0^{R_0} \frac{q(R)}{u^{'}(R)}dR, \tag{10}$$

where $u^{'}(R)$ is the marginal utility and $c = (\int_0^{\infty}(q(R)/u^{'}(R))dR)^{-1}$ is the normalizing constant. Apply equation (10) to the prospect theory described in Section 2.1, where $v(.)$ is the value function[11], w^+ and w^- are the probability weighting function for positive returns and negative returns starting from the two tails, respectively, and $r_i = R_i - 1$ is the net return. For a specific P_i with corresponding to positive net return r_i such that $P(r_i) = P_i$ and a specific P_{-j} with corresponding to negative net return r_{-j} such that $P(r_{-j}) = P_{-j}$[12], we have

$$\begin{aligned} w^+(P_i) &= k\textstyle\sum_{l=i}^{n} \frac{q(R_l)}{v^{'}(r_l)} \\ w^-(P_{-j}) &= k\textstyle\sum_{l=-m}^{-j} \frac{q(R_l)}{v^{'}(r_l)} \end{aligned} \tag{11}$$

where $k = (\sum_{l=-m}^{n} q(R_l)/v^{'}(r_l))^{-1}$ is the normalizing constant.

For risk-neutral distribution, we follow the procedure of Polkovnichenko and Zhao (2013)[13]. To match monthly returns, we use the S&P 500 index option with 30 days to expiration[14]. First, we estimate the risk-neutral moments based on Bakshi and Madan (2000) and Bakshi et al. (2003). Define the $(T-t)$ period log return as $R_t(T) = ln(F_T(T)/F_t(T))$. Then, we can compute the risk-neutral moments, $\mu_{R,n} = E_t^Q[R_t^n(T)]$, from the out-of-money call and put prices. Second, we estimate the risk-neutral density from the moments through Gram-Charlier series expansion (GCSE), which is a semi-parameter method. We calculate both A-type GCSE following Jarrow and Rudd (1982) and C-type GCSE following Rompolis and Tzavalis (2008), and then pick the best estimate based on the information criteria,

[11]Note that we use a different pair of parameter value for the value function in benchmark following Barberis et al. (2021), namely, $\alpha = 0.7$ and $\lambda = 1.5$, to reflect the findings in recent literature (see Walasek et al. (2018) and Chapman et al. (2018)). We also use the values in Tversky and Kahneman (1992) for robustness test.

[12]The return distribution follows equation (1)

[13]See Section 3 and Appendix B in their paper for the procedure and technique details

[14]Note that the monthly return from CRSP is simple return while the estimation in the following procedure uses log return, so one may need to unify them when using the distribution.

such as the Akaike information criterion (AIC) or Schwarz criterion (SC), as the guidance for a nonparametric method, the constrained local polynomial method. Finally, we use the constrained local polynomial method to estimate the risk-neutral density. It is estimated by taking derivatives of call price function with respect to strike based on the idea that the risk-neutral density is the scaled second derivative of the call option price with respect to the call strike price (Breeden and Litzenberger, 1978).The procedure follows Aït-Sahalia and Duarte (2003) but when implementing it, we estimate the call price function using the GCSE method rather than simulating data from the Black and Scholes formula with implied volatility modeled as a parametric function of moneyness.

For the physical distribution to be consistent with the prospect theory, we use past five years monthly returns with equal probability, which is the same distribution as equation (1). Forward-looking approaches, such as the GARCH model, may not be appropriate in this circumstance, because as described in Barberis et al. (2016), the model is supposed to be used by individual investors rather than professional economists. As a robustness check, we also try GARCH(1,1) for the physical distribution. The trend of moving average δ is close to the benchmark while its level deviates. Since we only need to separate high and low δ periods, the results based on alternative estimation methods are very similar.

We use pairs of transformed probability $w^+(P_i)$ ($w^-(P_{-j})$) and physical probability P_i (P_{-j}) in each month to fit equation (4). As in Barberis et al. (2021), we assume $\phi = \delta$ so that we can use all sixty pairs of probabilities[15]. We use the nonlinear least-squares estimator to estimate the value of δ[16].

$$\min_{\delta} \sum_{i=-m}^{n} \left[w(P_i) - \frac{P_i^\delta}{(P_i^\delta + (1-P_i)^\delta)^{1/\delta}} \right]^2. \tag{12}$$

Since the true preference should not change much from one month to another, we use 3-month

[15]For robustness, we relax this assumption by estimating the parameters for the two tails separately and find similar results.

[16]Use nl command in Stata with initial level 0.65. The initial level, 0.65, is picked so that the estimation is as close to experimental level as possible. One may try other initial level that possibly lead to different estimation, but need to restrict specific level of R-square to obtain reasonable estimation

moving average to reduce the effect from high-frequency measurement errors.[17]

The time series of the estimated δ is shown at the bottom panel of Figure 3. There are three time periods in which δ is generally greater than 1: November 2004 to March 2006, March 2012 to July 2015, and June 2016 to January 2018, corresponding to the periods where the "representative" investor does not overweight tails. We can also see that δ drops quickly in March 2020, which is consistent with the narrative in the media that investors reassess the tail probabilities during the COVID-19 crisis.

To compare with the estimated δ, we plot the time series of cumulative gross returns of the value-weighted long-short TK portfolio at the top panel of Figure 3. The cumulative gross return reaches over 400% in earlier years before gradually declining until 2020. The poor performance in the later sample appears not caused by the 2008 financial crisis. Comparing the two panels in Figure 3 shows that the time periods with $\delta < 1$ are often accompanied with rising returns, lending support to the prediction of the cumulative prospect theory.

[insert Figure 3 here]

4 Empirical analysis

In this section, we formally test how time-varying probability weight δ affects the stock return prediction by TK values. As explained in Section 2, investors overweight tails when $\delta < 1$ and underweight tails when $\delta > 1$. Our main hypothesis is that the predictive power of TK values for stock returns is stronger during the overweight periods.

4.1 Decile portfolio performance

In this section, we test our hypothesis using the portfolio approach. Following Barberis et al. (2016), we sort stocks into deciles based on TK values at the start of each month from March 1996 to December 2020. We then compute both value-weighted and equal-weighted

[17] The results are similar with 6- or 12-month moving averages or no moving average.

average return of each TK decile over the next month, generating a time-series of monthly returns for each TK decile. Using δ estimated in Section 3.2, we divide the time series of monthly returns into two subsample periods: the overweight periods (months with $\delta < 1$) and the underweight periods (months with $\delta > 1$).[18] We calculate the average return of each decile over the whole sample, the overweight period, and the underweight period. In Table 3, we report the average return in excess of the risk-free rate, the four-factor alpha following Carhart (1997) (the return adjusted by the Fama and French (1993) three factors and a momentum factor), and the five-factor alpha for each decile (the return adjusted by the Fama and French (1993) three factors, the momentum factor, and the Pastro and Stambaugh (2003) liquidity factor). We also report the difference in the return and alphas between the lowest TK decile and the highest TK decile in the rightmost column (low-minus-high portfolio).

[insert Table 3 here]

Barberis et al. (2016) show that the average return of the long-short TK portfolio is positive and stronger for the equal-weighted portfolios. In Table 1, we find significant positive returns for the equal-weighted portfolios and statistically insignificant returns for the value-weighted portfolios in our sample period. The statistical insignificance of the value-weighted portfolio return is likely due to the small sample size, because its economic magnitude is sizable at 0.56% per month. However, during the overweight periods, the value-weighted average return is twice as large and becomes statistically significant (at the 10% level for excess returns and 5% level for four-factor and five-factor adjusted returns). On the other hand, the value-weighted average return during the underweight periods is significantly negative. The difference in the average return between the two periods is about four (two) times of the whole-sample average return for the value-weighted (equal-weighted) portfolios.

Figure 4 plots the four-factor alphas of the TK decile portfolios in Table 3. According

[18] We do not have months with $\delta = 1$ in our sample. In Section 5.1 and Section 4.3, we further divide the sample into three periods using 0.9 and 1.1 as the cutoff values for δ.

to Barberis et al. (2016), the alphas should decline monotonically from the low TK portfolio to the high TK portfolio. Figure 4 shows that for the value-weighted portfolios, although the predicted decreasing trend does not exist during the whole sample periods, it does show up during the overweight periods. In contrast, the average alpha increases with TK values during the underweight periods. The pattern is similar for the equal-weighted portfolios, although the negative TK-alpha relation is more prominent than the value-weighted portfolios.

[insert Figure 4 here]

4.2 Variations in the long-short TK portfolio returns

We now investigate whether the time-series variation in the long-short TK portfolio return is due to exposures to common risk factors or time-varying stock characteristics.

One possible source of time-series variations in the long-short TK portflio return is the exposures to common risk factors. In Table 4, we regress the long-short TK portfolio return onto a constant and a dummy variable that equals one if the month is probability overweighting, and then we gradually include Carhart (1997)'s four factors (MktRf, SMB, HML, and UMD), GFC, and the interaction terms between the factors and probability overweighting dummy. These interaction terms could capture the dependence of factor exposures on probability overweighting, and the coefficient on the dummy variable measures the difference in the TK premium between overweight and underweight periods. For both value-weighted (Panel A) and equal-weighted (Panel B) long-short portfolios, the estimates for the overweight dummy variable remain robust in magnitudes (more than 1.7% per month) and statistical significance. In Specification (6), the coefficients on the interaction terms are statistically insignificant, indicating that the effect of conditional factor exposures is weak.

[insert Table 4 here]

Another source of time-series variations in the long-short TK portfolio return is the time-varying stock characteristics in the long and short portfolios. As illustrated in the model in

Section 2.2, the differences between the low- and high-TK decile returns are determined by the difference in firms' characteristics between these two deciles. In Table 5, we include the differences in market beta, return volatility, and skewness, as well as variance risk premium (Baele et al. (2018)), into the regression in explaining the long-short TK portfolio return. The coefficient estimate of overweight dummy variable remains significant after we include these additional control variables. This result shows that time-varying probability weights, not time-varying stock characteristics, drive the results from the previous subsection.

[insert Table 5 here]

4.3 Fama-MacBeth regressions

We now examine the effect of time-varying probability weights using Fama-MacBeth regressions to control for known predictors of stock returns in Table 6. In the first step, we run cross-sectional regressions of excess returns on stocks' TK values and a list of stock characteristics that are known to predict stock returns. In the second step, we regress the TK coefficients from the first step in three specifications: a constant for whole sample period (Whole); a constant and an overweight dummy indicating $\delta < 1$ (Over by 1); a constant, an overweight dummy indicating $\delta < 0.9$ and a middle dummy indicating $0.9 \leq \delta \leq 1.1$ (Over by 0.9 and 1.1). The introduction of the middle regime in the last specification is intended to control the effect of measurement errors in δ, although it reduces the number of observations in both over- and under-weight periods. We report the estimate of the constant for the whole sample as benchmark and the estimates on overweight dummy variables.

[insert Table 6 here]

As in Barberis et al. (2016), the results of whole sample indicate that TK value has significant return predictive power after controlling for the known predictors in our sample periods. In terms of stock characteristics, prior one-month returns (Rev) weaken the coefficient on TK, whereas lottery-like features (IVOL, Maxret, and Minret) and the reference point in prospect theory value function (CGO) all increase the economic magnitude of the TK coefficient. In summary, the return predictive power of TK values remain in economic magnitude and statistical significance after controlling the known stock return predictors.

To examine the effect of the time-varying probability weights on TK premium, we focus on the coefficients on the overweight dummy variable. During the overweight periods, the economic magnitudes of the TK coefficient is larger than those in the whole sample. Furthermore, a lower cutoff value for δ for probability overweighting generates a larger TK premium, consistent with the prediction of the prospect theory. Despite the variation in the TK coefficient across specifications, the estimate of the overweight dummy remains significant. This finding indicates that the time-series variations in TK portfolio returns cannot be explained by firm characteristics. For example, CGO has been shown in An et al. (2020) that several lottery-related anomalies are state-dependent and stronger among stocks where investors have lost money. Barberis et al. (2021) also include CGO in their model to capture prior trading gains or losses. Our results indicate that the variation in the TK coefficients is not explained by stock's CGO.

5 Further analysis

5.1 An augmented investment strategy

Section 4.1 shows that the TK return prediction has opposite signs between overweight periods and underweight periods, making the average value-weighted return in whole sample periods insignificant. In this section, we take advantage of this pattern and propose an augmented investment strategy.

The basic idea of this alterantive strategy is to flip the long and short positions of the original TK strategy when investors underweight tails. That is, we take a long position in low TK stocks and a short position in high TK stocks during overweighting periods, and a long position in high TK stocks and a short position in low TK stocks during underweighting periods. Considering the measurement error in δ which can result in excessive portfolio rebalancing, we introduce an inaction region corresponding to the periods during which δ is between 0.9 and 1.1. Specifically, we begin with the original long-short TK strategy positions in March 1996 when $\delta \leq 1$. We keep the same position signs until $\delta > 1.1$, when we flip the positions, i.e., long in high-TK stocks and short in low-TK stocks. We keep these opposite positions until δ falls below 0.9, when we switch back to the original TK positions. The switching continues until the end of the sample.

[insert Table 7 here]

We summarize the performances of the original TK strategy and our augmented TK strategy in Table 7. While the standard deviation and skewness are comparable between these two strategies, the average return is substantially higher in our augmented strategy, leading to an improvement in Sharpe ratio from 0.21 to 0.50 for value-weighted returns and from 0.54 to 0.74 for equal-weighted returns. The improvement is similar when we adjust the returns using the four-factor model or five-factor model.

[insert Figure 5 here]

Figure 5 compares the performance of these two investment strategies by plotting the cumulative return of both value-weighted (left panels) and equal-weighted (right panels) portfolios. For the value-weighted portfolios, the original strategy consistently generates negative returns in the past 15 years, whereas the cumulative return of the augmented strategy demonstrates an upward trend. An \$1 invested in the augmented strategy in 1996 is turned to \$16 at the end of 2021. Interestingly, there is a sharp decline in the cumulative return for both original and augmented strategies in 2008, but this is likely due to factor exposures because the large decline disappears after we control the four factors. Figure 5 also shows that the performance of the equal-weighted augmented strategy almost overlaps with that of the original TK strategy in the first fifteen years of the sample period but diverges over time after that. This result is consistent with the estimated δ being below 1.1 in the earlier years, as shown in Figure 3. It is also striking to observe that, in the past ten years, the cumulative return of the augmented strategy quadruples while the original strategy generates little return.

5.2 Robustness of probability weights

Our main finding depends on the estimated probability overweighting parameter δ. To check the robustness of the time-series results, in Panel A of Table 8, we examine whether δ using different estimating methods affects the difference in the long-short TK portfolio return between overweight and underweight periods. In Panel B, we study the impact of the number of months used to smoothing preferences on our main finding.

[insert Table 8 here]

First, the literature following Tversky and Kahneman (1992) usually uses a two-parameter model for the probability weighting function, rather than a single-parameter model, to separately control for the curvature and elevation of the function[19]. We can interpret the curvature as reflecting how much an investor discriminates between probabilities (the magnitude of overweight or underweight) and the elevation as how attractive the chance domain of prospect is to the investor (the crossing point between the curve and 45-degree line). In Panel A, we use Gonzalez and Wu (1999)'s linear in log odds probability weighting function to replace the Tversky and Kahneman (1992) probability weighting function to estimate the parameter for curvature, which differentiates overweight and underweight periods. Panel A shows that although we have fewer months in overweight, the difference in the average return is close to the benchmark result for both value-weighted and equal-weighted portfolios. This result indicates that the alternative estimation model for the probability function does not substantially change our conclusion.

[19]See, for example, Prelec (1998), Gonzalez and Wu (1999), , Zhang and Maloney (2012), etc.

Second, we examine the effect of the value function parameters in Panel A. In Section 3.2, we use $\alpha = 0.7$ and $\lambda = 1.5$ in the benchmark parameterization. Here, we set $\alpha = 0.88$ and $\lambda = 2.25$, which are the values used in Tversky and Kahneman (1992). Again, Panel A shows that the results are quantitatively similar to the benchmark result.

Third, we also relax the assumption of equal probability weighting parameters for left and right tails in Panel A. We find that while they have similar trend, the parameter of left tail is generally smaller than the parameter of right tail. To have a meaningful number of months in each group, we use only right tails to differentiate overweight and underweight periods. Panel A shows that the difference in average returns between these two periods still holds its economic magnitude and statistical significance.

In Panel B of Table 8, we test how different number of months in the smoothing process of δ affects our conclusion. We find that even without smoothing (i.e., 1-month), the main result still holds and the difference in the average value-weighted return is economically large although not statistically significant. On the other hand, the difference in equal-weighted average return is significant. When we increase the number of months in the moving average to six and twelve, the results are close to the benchmark result. These findings suggest that although measurement errors in δ may weaken the difference in the TK premium between overweight and underweight periods, our main conclusion is not sensitive to alternative moving average parameters.

5.3 Sources of time variation in probability weighting

We have shown that time-varying probability weighting preferences affect the return prediction of TK by differentiating overweight and underweight periods. In this section, we explore the sources of time variation in probability weighting and study whether it can be captured by other related variables in the literature.

We focus on the dummy variable for overweight, which equals 1 if $\delta \leq 1$ and 0 otherwise. In the first three columns of Table 9, we regress the overweight dummy onto the past market excess returns. The result shows that past 1 month and past 2 to 12 month market excess returns have significantly negative relationship with overweight, while the past 2 to 60 month market excess returns and overweight are not significantly correlated. This result indicates that only recent good stock performance may boost investors' overweighting on tail events. Columns (4) to (6) study the effect of higher return moments of the physical and risk-neutral distributions. When all these higher moments are included, the coefficients of physical variance, risk-neutral skewness, and risk-neutral kurtosis are significantly negative while the coefficient of physical kurtosis is positive and marginally significant. The negative sign for physical variance indicates that investors tend not to overweight when the market is volatile, consistent with their aversions to lotteries. The positive sign for physical kurtosis implies that investors overweight tails when small-probability events happen more frequently. It seems counter-intuitive that risk-neutral skewness and kurtosis have negative relationship with overweight preferences since one would expect investors to overweight tails when the risk premium on the third and fourth moments are high. One potential explanation is that out-of-money options, which mainly reflect the third and fourth risk-neutral moments, are illiquid when implied volatility is low. As a result, the risk-neutral skewness and kurtosis contain information about not only the risk premium but also liquidity premium, and it is empirically difficult to disentangle these two premiums. In Column (7), we includes all return moments in the same regression. The coefficients of past market excess returns are no longer significant, indicating that the effect of the first moments are spanned by those higher moments. In Column (8), we also include previous month CAPE ratio and GFC dummy. The coefficient on the previous month CAPE is not significant, suggesting that overweight preferences are not associated with whether stocks are relatively expensive or cheap. GFC is negatively correlated with overweight preferences, meaning that after controlling for return moments, the 2008 global financial crisis tends to stimulate investors to underweight tails. It is also worth noting that adjusted R-squares are below 0.5 in all specifications, suggesting that the estimated probability overweight contains additional information beyond these variables.

[insert Table 9 here]

6 Conclusion

Probability weighting preferences in the prospect theory are empirically time-varying and the implied long-short TK portfolio return is different between overweight and underweight periods. We use option data to estimate the time-varying preference based on the relation among risk-neutral distribution, physical distribution, and probability weighting function. We find that in the post-1996 US sample, the return predictive power of TK value is weaker than the post-1963 sample in Barberis et al. (2016). However, after differentiating overweight periods from underweight periods, the TK return predictive power improves substantially in overweight periods but becomes insignificant or even switches sign in underweight periods.

Our results support the asset pricing implications of the prospect theory. Moreover, our findings highlight the importance of time variation in probability weighting preferences in later research on prospect theory.

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Table 1
TK long-short portfolio performance

	Whole	Over	Under	Over-Under
	Panel A. Value-weighted			
Mean	0.56	1.42*	-0.93*	2.35**
	(1.03)	(1.78)	(-1.73)	(2.44)
# of months	298	190	108	
	Panel B. Equal-weighted			
Mean	1.16***	2.01***	-0.34	2.35***
	(2.69)	(3.20)	(-0.86)	(3.16)
# of months	298	190	108	

The table reports the average monthly return (in percent) in the whole (Whole), overweight (Over), and underweight (Under) samples for both equal-weighted (EW) and value-weighted (VW) long-short TK portfolios. TK portfolios are formed following Barberis et al. (2016). A month is classified as overweight (Over) if $\delta \leq 1$ in that month. A month is classified as underweight (Under) if $\delta > 1$ in that month. Over-Under is the difference between Over and Under. t-statistics, in parentheses, are based on the heteroskedasticity-consistent standard errors of White (1980).

Table 2
Long-horizon return prediction

	Panel A. Value-weighted			Panel B. Equal-weighted		
	(1)	(2)	(3)	(4)	(5)	(6)
	3-month	6-month	12-month	3-month	6-month	12-month
Over	0.070***	0.118**	0.238**	0.078***	0.151***	0.316***
	(2.59)	(2.28)	(2.28)	(3.01)	(2.73)	(2.80)
Constant	-0.028*	-0.044	-0.083*	-0.013	-0.018	-0.036
	(-1.70)	(-1.54)	(-1.68)	(-1.04)	(-0.73)	(-0.81)
Observations	296	293	287	296	293	287
Adjusted R-square	0.033	0.045	0.085	0.053	0.066	0.107

The table reports the result for the long-horizon prediction of the long-short TK portfolio return using overweight dummy (Over), which equals 1 if $\delta \leq 1$ and 0 otherwise. t-statistics, in parentheses, are Newey-West adjusted with n+3 lags where n is the number of months in the long-horizon return.

Table 3
Decile portfolio analysis

			P1 Low TK	P2	P3	P4	P5	P6	P7	P8	P9	P10 High TK	TK Low-high portfolio
Excess return	VW	Whole	1.217	0.818	0.778	0.858	0.969	0.727	0.890	0.771	0.769	0.653	0.564
			(1.85)	(1.61)	(1.95)	(2.38)	(2.94)	(2.51)	(3.31)	(3.09)	(3.20)	(2.34)	(1.03)
		Over	1.901	1.158	1.060	1.067	1.108	0.802	0.938	0.797	0.809	0.485	1.416
			(1.97)	(1.57)	(1.83)	(2.01)	(2.31)	(1.91)	(2.39)	(2.19)	(2.35)	(1.22)	(1.78)
		Under	0.015	0.218	0.282	0.490	0.723	0.595	0.804	0.727	0.697	0.949	-0.935
			(0.02)	(0.42)	(0.69)	(1.41)	(2.14)	(1.99)	(2.97)	(2.80)	(2.57)	(2.95)	(-1.73)
	EW	Whole	1.859	1.223	0.994	1.061	1.005	1.012	0.984	0.876	0.859	0.702	1.158
			(3.02)	(2.72)	(2.60)	(3.02)	(3.14)	(3.47)	(3.63)	(3.44)	(3.44)	(2.11)	(2.69)
		Over	2.573	1.535	1.170	1.171	1.068	1.069	1.022	0.847	0.872	0.564	2.010
			(2.86)	(2.35)	(2.11)	(2.31)	(2.31)	(2.55)	(2.64)	(2.34)	(2.46)	(1.16)	(3.20)
		Under	0.602	0.676	0.683	0.867	0.893	0.911	0.916	0.929	0.837	0.944	-0.342
			(1.00)	(1.47)	(1.69)	(2.29)	(2.56)	(2.82)	(2.94)	(3.10)	(2.81)	(2.77)	(-0.86)
Four-alpha alpha	VW	Whole	0.382	-0.064	0.054	0.194	0.287	0.074	0.304	0.154	0.168	-0.115	0.496
			(1.02)	(-0.26)	(0.33)	(1.46)	(2.28)	(0.83)	(3.37)	(1.88)	(2.01)	(-1.41)	(1.28)
		Over	0.904	0.154	0.271	0.344	0.419	0.142	0.381	0.218	0.280	-0.208	1.112
			(1.72)	(0.45)	(1.20)	(1.86)	(2.41)	(1.13)	(2.96)	(1.91)	(2.50)	(-1.98)	(2.07)
		Under	-0.700	-0.296	-0.372	-0.087	-0.008	-0.014	0.168	0.001	-0.126	0.040	-0.740
			(-1.97)	(-1.15)	(-2.14)	(-0.56)	(-0.05)	(-0.14)	(1.81)	(0.01)	(-1.31)	(0.30)	(-1.81)
	EW	Whole	1.131	0.486	0.261	0.342	0.309	0.353	0.345	0.239	0.202	-0.097	1.228
			(3.10)	(2.88)	(2.31)	(3.76)	(3.97)	(5.48)	(5.31)	(3.68)	(2.94)	(-0.85)	(3.62)
		Over	1.633	0.646	0.327	0.359	0.302	0.357	0.352	0.194	0.208	-0.300	1.933
			(3.27)	(2.73)	(2.06)	(2.77)	(2.75)	(4.00)	(3.95)	(2.23)	(2.18)	(-1.92)	(4.11)
		Under	0.037	0.109	0.095	0.262	0.232	0.285	0.247	0.287	0.154	0.190	-0.153
			(0.10)	(0.65)	(0.88)	(2.66)	(2.79)	(4.54)	(3.90)	(3.00)	(1.78)	(1.69)	(-0.41)
Five-alpha alpha	VW	Whole	0.360	-0.115	0.024	0.185	0.301	0.070	0.291	0.157	0.153	-0.127	0.488
			(0.95)	(-0.45)	(0.15)	(1.42)	(2.29)	(0.80)	(3.19)	(1.91)	(1.85)	(-1.53)	(1.25)
		Over	0.879	0.063	0.245	0.336	0.435	0.134	0.359	0.225	0.251	-0.238	1.117
			(1.63)	(0.17)	(1.07)	(1.85)	(2.36)	(1.09)	(2.73)	(1.96)	(2.25)	(-2.16)	(2.03)
		Under	-0.709	-0.308	-0.429	-0.095	0.002	-0.020	0.160	-0.004	-0.132	0.043	-0.752
			(-1.99)	(-1.20)	(-2.71)	(-0.61)	(0.02)	(-0.19)	(1.68)	(-0.03)	(-1.35)	(0.32)	(-1.84)
	EW	Whole	1.082	0.440	0.223	0.325	0.296	0.338	0.335	0.224	0.184	-0.093	1.175
			(3.10)	(2.76)	(2.13)	(3.73)	(3.86)	(5.36)	(5.20)	(3.42)	(2.74)	(-0.82)	(3.60)
		Over	1.531	0.549	0.255	0.325	0.269	0.323	0.324	0.157	0.164	-0.307	1.838
			(3.22)	(2.49)	(1.72)	(2.61)	(2.44)	(3.69)	(3.63)	(1.78)	(1.78)	(-2.00)	(4.08)
		Under	0.047	0.112	0.086	0.258	0.236	0.284	0.248	0.291	0.161	0.208	-0.161
			(0.12)	(0.65)	(0.81)	(2.56)	(2.86)	(4.57)	(4.01)	(3.12)	(1.91)	(1.81)	(-0.44)

The table reports the average monthly excess return and monthly alpha (in percent) in the whole (Whole), overweight (Over), and underweight (Under) samples for both equal-weighted (EW) and value-weighted (VW) TK portfolios. A month is classified as overweight (Over) if $\delta \leq 1$ in that month. A month is classified as underweight (Under) if $\delta > 1$ in that month. Each month, all stocks are sorted into deciles based on TK values. For each decile from P1 (low TK) to P10 (high TK), we report the average excess return, four-factor alpha following Carhart (1997), and five-factor alpha (Carhart four-factor model augmented by Pastro and Stambaugh (2003) liquidity factor). The sample runs from March 1996 to December 2020. t-statistics, in parentheses, are based on the heteroskedasticity-consistent standard errors of White (1980).

Table 4
Factor exposures

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Value-weighted						
Over	0.024**	0.024**	0.021***	0.017***	0.016**	0.017***
	(2.37)	(2.54)	(2.85)	(2.72)	(2.51)	(2.70)
MktRf		0.820***	0.678***	0.312***	0.337***	0.304**
		(4.91)	(5.21)	(2.74)	(2.87)	(2.18)
SMB			1.051***	1.129***	1.129***	1.108***
			(5.72)	(9.27)	(9.20)	(4.88)
HML			0.953***	0.567***	0.598***	0.672***
			(4.95)	(3.69)	(3.89)	(2.75)
UMD				-0.936***	-0.924***	-0.648***
				(-10.83)	(-10.28)	(-4.94)
GFC					0.032	0.032
					(1.07)	(1.04)
MktRf*Over						0.031
						(0.16)
SMB*Over						0.022
						(0.08)
HML*Over						-0.074
						(-0.25)
UMD*Over						-0.307*
						(-1.89)
Constant	-0.009	-0.015**	-0.014***	-0.006	-0.006	-0.007*
	(-1.47)	(-2.53)	(-3.49)	(-1.41)	(-1.47)	(-1.74)
Observations	298	298	298	298	298	298
Adjusted R-square	0.011	0.168	0.349	0.563	0.564	0.560
Panel B. Equal-weighted						
Over	0.024***	0.024***	0.022***	0.019***	0.018***	0.019***
	(2.78)	(2.95)	(2.98)	(3.02)	(2.85)	(3.02)
MktRf		0.523***	0.459***	0.162	0.198*	0.191
		(3.94)	(4.14)	(1.51)	(1.81)	(1.62)
SMB			0.505**	0.568***	0.569***	0.648***
			(2.59)	(4.45)	(4.47)	(3.33)
HML			0.536***	0.222	0.269	-0.025
			(3.15)	(1.32)	(1.62)	(-0.10)
UMD				-0.762***	-0.743***	-0.493***
				(-7.17)	(-7.40)	(-2.82)
GFC					0.048	0.050
					(1.41)	(1.50)
MktRf*Over						0.006
						(0.03)
SMB*Over						-0.066
						(-0.27)
HML*Over						0.371
						(1.17)
UMD*Over						-0.281
						(-1.39)
Constant	-0.003	-0.007*	-0.007*	0.000	-0.000	-0.002
	(-0.76)	(-1.69)	(-1.77)	(0.00)	(-0.10)	(-0.40)
Observations	298	298	298	298	298	298
Adjusted R-square	0.020	0.122	0.198	0.428	0.433	0.435

The table reports the factor exposures of the long-short TK portfolio. We includes overweight dummy (Over), Fama and French (1993) three factors (MktRf, SMB, HML), Carhart (1997) momentum factor (UMD), 2008 financial crisis dummy (GFC), and interaction terms between factors and overweight dummy. Over dummy equals 1 if $\delta \leq 1$ and 0 otherwise. GFC equals 1 if the month falls between October 2008 and March 2009 and 0 otherwise. t-statistics, in parentheses, are Newey-West adjusted with four lags.

Table 5
Time-varying probability weights vs characteristics

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Value-weighted						
Over	0.024**	0.025**	0.019*	0.022**	0.026**	0.021*
	(2.37)	(2.35)	(1.90)	(2.28)	(2.44)	(1.84)
Beta spread		0.010				0.016
		(0.61)				(0.93)
Std Dev spread			-0.065**			-0.134**
			(-2.18)			(-2.11)
Skewness spread				-0.052		0.069
				(-1.31)		(0.75)
VRP					-0.000	-0.000
					(-0.89)	(-0.95)
Constant	-0.009	-0.012	-0.000	-0.022*	-0.006	0.026
	(-1.47)	(-1.63)	(-0.05)	(-1.73)	(-0.73)	(0.72)
Observations	298	298	298	298	298	298
Adjusted R-square	0.011	0.010	0.021	0.012	0.026	0.042
Panel B. Equal-weighted						
Over	0.024***	0.025***	0.021**	0.023***	0.026***	0.022**
	(2.78)	(2.66)	(2.33)	(2.70)	(2.84)	(2.19)
Beta spread		0.010				0.014
		(0.68)				(0.74)
Std Dev spread			-0.040			-0.100*
			(-1.39)			(-1.72)
Skewness spread				-0.024		0.063
				(-0.69)		(0.70)
VRP					-0.000	-0.000
					(-0.89)	(-0.92)
Constant	-0.003	-0.006	0.002	-0.010	-0.000	0.026
	(-0.76)	(-1.04)	(0.39)	(-0.90)	(-0.08)	(0.72)
Observations	298	298	298	298	298	298
Adjusted R-square	0.020	0.020	0.024	0.018	0.035	0.047

The table reports the result of the long-short TK portfolio return precition by time-varying probability weights and firm characteristics. Over is the overweight dummy, which equals 1 if $\delta \leq 1$ and 0 otherwise. Beta, Std Dev, and Skewness are the averages of corresponding firm-level variables within each TK decile following Barberis et al. (2021). Firm-level Beta is calculated using daily returns over the subsequent year. Firm-level Std Dev and Skewness are calculated using monthly returns over the past five years. Spread is the difference between low-TK decile and high-TK decile. VRP is the previous month variance risk premium from https://sites.google.com/site/haozhouspersonalhomepage. t-statistics, in parentheses, are Newey-West adjusted with four lags.

Table 6
Fama-MacBeth regression analysis

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Whole	-0.277*	-0.249**	-0.288***	-0.220**	-0.229**	-0.262**	-0.269**	-0.269*	-0.231**	-0.266**
	(-1.80)	(-2.06)	(-2.58)	(-2.10)	(-2.18)	(-2.35)	(-2.40)	(-1.81)	(-2.17)	(-2.40)
Over by 1	-0.606**	-0.518***	-0.535***	-0.431***	-0.417**	-0.340**	-0.337*	-0.402*	-0.319*	-0.343**
	(-2.43)	(-2.71)	(-3.10)	(-2.62)	(-2.53)	(-1.96)	(-1.94)	(-1.72)	(-1.87)	(-1.98)
Over by 0.9 and 1.1	-0.748**	-0.650***	-0.675***	-0.552***	-0.541***	-0.601***	-0.601***	-0.652**	-0.603***	-0.605***
	(-2.40)	(-2.78)	(-3.17)	(-2.71)	(-2.65)	(-2.66)	(-2.65)	(-2.22)	(-2.71)	(-2.70)
Variables in first step										
TK	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Beta	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
Size	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
Bm	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
Mom	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
Cgo	N	N	Y	Y	Y	Y	Y	Y	Y	Y
Rev	N	N	N	Y	Y	Y	Y	Y	Y	Y
Illiq	N	N	N	N	Y	Y	Y	Y	Y	Y
Lt rev	N	N	N	N	N	Y	Y	Y	Y	Y
Ivol	N	N	N	N	N	Y	Y	Y	Y	Y
Max	N	N	N	N	N	N	Y	Y	Y	Y
Min	N	N	N	N	N	N	Y	Y	Y	Y
Skew	N	N	N	N	N	N	N	Y	N	N
Eiskew	N	N	N	N	N	N	N	N	Y	N
Coskew	N	N	N	N	N	N	N	N	N	Y

The table reports the results of Fama-MacBeth regressions. The dependent variable is percentage return. In the first step, at each month, we regress excess returns on firm characteristics. In the second step, we regress the coefficients from the first step on a constant for whole sample period (Whole), or on a constant and overweight dummy (Over by 1), or on a constant, an overweight dummy, and middle dummy (Over by 0.9 and 1.1). For Over by 1, overweight dummy equals 1 if $\delta \leq 1$ and 0 otherwise. For Over by 0.9 and 1.1, overweight dummy equals 1 if $\delta < 0.9$ and 0 otherwise and middle dummy equals 1 if $0.9 \leq \delta \leq 1.1$ and 0 otherwise. TK is the prospect theory value of a stock's historical return distribution, following Barberis et al. (2016). Beta is calculated from monthly returns over the previous five years, following Fama and French (1992). Size is the log market capitalization at the end of the previous month. Bm is the book value of equity scaled by market value of equity, computed following Fama and French (1992) and Fama and French (2008); when the book value of equity is missing from Compustat, we use data from Davis et al. (2000); observations with negative book value are removed. Mom is the cumulative return from the start of month t-12 to the end of month t-2. Cgo is the capital overhang in month t-1, following Barberis et al. (2021). It is computed as $(P_i - R_i)/R_i$, where P_i is the stock's current price and R_i is investors' average purchase price. It is slightly different from Grinblatt and Han (2005), but is a more precise match for the capital gain variable in the cumulative prospect theory model. Rev is the return in month t-1. Illiq is Amihud (2002)'s measure of illiquidity, scaled by 10^5. Lt rev is the cumulative return from the start of month t-60 to the end of month t-13. Ivol is idiosyncratic return volatility, as in Ang et al. (2006). Max and Min are the maximum and the negative of the minimum daily returns in month t-1, as in Bali et al. (2011). Skew is the skewness of monthly returns over the previous five years. Eiskew is expected idiosyncratic skewness, as in Boyer et al. (2010). Coskew is coskewness, computed as in Harvey and Siddique (2000) using five years of monthly returns. TK, Mom, Rev, Ivol, Max, and Min are scaled up by 100. The sample period runs from March 1996 to December 2020. All variables are winsorized at 1 and 99 percent in each month, and TK is standardized after the winsorization. t-statistics, in parentheses, are Newey-West adjusted with four lags.

Table 7
Comparison between original TK strategy and augmented TK strategy

Portfolio	Mean	Std	Sharpe	Skewness	four-factor alpha	five-factor alpha
Original Value-weighted	0.068	0.328	0.206	0.505	0.060	0.059
	(1.03)				(1.28)	(1.25)
Augmented Value-weighted	0.162	0.325	0.498	0.467	0.171	0.170
	(2.48)				(3.12)	(3.07)
Original Equal-weighted	0.139	0.257	0.540	0.539	0.147	0.141
	(2.69)				(3.62)	(3.60)
Augmented Equal-weighted	0.189	0.255	0.743	0.526	0.205	0.198
	(3.70)				(4.65)	(4.62)

The table compares the performance of orginal TK strategy and an augmented TK strategy for both equal-weighted (EW) and value-weighted (VW) portfolios. The original TK strategy takes a long position in low-TK stocks and a short position in high-TK stocks for all months. For the augmented TK strategy, we begin with the original long-short TK strategy positions in March 1996 when $\delta \leq 1$. We keep the same position sign until $\delta > 1.1$, when we flip the positions, i.e., long in high-TK stocks and short in low-TK stocks. We keep these opposite positions until δ falls below 0.9, when we switch back to the original TK positions. The switching continues until the end of the sample. t-statistics, in parentheses, are based on the heteroskedasticity-consistent standard errors of White (1980).

Table 8
Robustness

	Value-weighted				Equal-weighted			
	Whole	Over	Under	Over - Under	Whole	Over	Under	Over - Under
Panel A. Different estimating method								
GW two-parameter model								
Mean	0.56	1.82*	-0.73	2.54**	1.16***	2.53***	-0.25	2.78***
	(1.03)	(1.90)	(-1.46)	(2.36)	(2.69)	(3.37)	(-0.66)	(3.31)
# of months	298	151	147		298	151	147	
TK with alternative value function parameters								
Mean	0.56	1.69*	-0.59	2.28**	1.16***	2.47***	-0.19	2.67***
	(1.03)	(1.75)	(-1.22)	(2.11)	(2.69)	(3.26)	(-0.54)	(3.17)
# of months	298	151	147		298	151	147	
Right tail only								
Mean	0.56	1.04	-1.02	2.06**	1.16***	1.70***	-0.63	2.33***
	(1.03)	(1.53)	(-1.44)	(2.11)	(2.69)	(3.19)	(-1.17)	(3.07)
# of months	298	229	69		298	229	69	
Panel B. Different moving average								
1-month								
Mean	0.55	1.12	-0.44	1.56	1.18***	1.87***	0.01	1.86**
	(1.01)	(1.40)	(-0.85)	(1.64)	(2.77)	(2.97)	(0.01)	(2.48)
# of months	300	190	110		300	190	110	
6-month								
Mean	0.52	1.46*	-1.13**	2.60***	1.11**	2.11***	-0.64	2.75***
	(0.94)	(1.81)	(-2.17)	(2.70)	(2.56)	(3.35)	(-1.58)	(3.67)
# of months	295	188	107		295	188	107	
12-month								
Mean	0.65	1.39*	-0.84	2.23**	1.19***	2.14***	-0.71*	2.86***
	(1.16)	(1.77)	(-1.49)	(2.31)	(2.71)	(3.47)	(-1.72)	(3.84)
# of months	289	193	96		289	193	96	

The table reports the robustness checks on the average monthly return (in percent) in the whole (Whole), overweight (Over), and underweight (Under) samples for both equal-weighted (EW) and value-weighted (VW) long-short TK portfolios In panel A, we estimate δ using different methods. The GW two-parameter model follows Gonzalez and Wu (1999). TK with alternative value function parameters follows the same method as benchmark but changes the value function parameters to (0.88, 2.25) as in Tversky and Kahneman (1992). Right tail only follows the same method as benchmark but estimates parameters separately on the two tails and uses the δ estimated from right tails. In panel B, we follows the same method as benchmark but changes the moving average horizon to 1-, 6-, and 12-month respectively. t-statistics, in parentheses, are based on the heteroskedasticity-consistent standard errors of White (1980).

Table 9
Sources of time variation in probability weighting preferences

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Past 1 month market excess return	-1.005**	-0.927**	-0.837**				0.238	-0.283
	(-2.03)	(-2.48)	(-2.25)				(0.58)	(-0.69)
Past 2-12 month market excess return		-0.879***	-1.091***				-0.308	-0.329
		(-2.98)	(-3.18)				(-0.98)	(-1.16)
Past 2-60 month market excess return			0.168				0.046	-0.154
			(1.28)				(0.42)	(-1.14)
Standardized physical variance				-0.062		-0.113**	-0.102*	-0.140***
				(-0.71)		(-2.39)	(-1.72)	(-2.97)
Standardized physical skewness				-0.054		0.033	0.038	0.040
				(-0.67)		(0.73)	(0.74)	(0.79)
Standardized physical kurtosis				0.117		0.088*	0.090*	0.133**
				(1.29)		(1.88)	(1.86)	(2.35)
Standardized risk-neutral variance					-0.034	-0.069	-0.081	0.057
					(-0.46)	(-0.85)	(-1.20)	(0.72)
Standardized risk-neutral skewness					-0.194***	-0.155**	-0.155**	-0.097
					(-2.67)	(-2.01)	(-2.22)	(-1.56)
Standardized risk-neutral kurtosis					-0.452***	-0.467***	-0.458***	-0.335***
					(-4.74)	(-5.10)	(-5.43)	(-4.54)
CAPE								0.015
								(1.61)
GFC								-1.036***
								(-2.70)
Constant	0.645***	0.713***	0.638***	0.638***	0.638***	0.638***	0.635***	0.358*
	(8.53)	(12.62)	(8.20)	(9.45)	(11.65)	(13.81)	(8.43)	(1.71)
Observations	298	298	298	298	298	298	298	298
Adjusted R-square	0.006	0.091	0.124	0.119	0.344	0.427	0.430	0.487

The table analyzes the sources of time variation in probability weighting preferences. The dependent variable is an overweight dummy variable that equals 1 if $\delta \leq 1$ and 0 otherwise. Past 1 month market excess return is the market excess return in previous month. Past 2-12 month market excess return is the difference between past 2 to 12 month market cumulative return and risk-free cumulative return. Past 2-60 month market excess return is the difference between past 2 to 60 month market cumulative return and risk-free cumulative return. Standardized physical moments are calculated using past five years monthly S&P 500 index return. Standardized risk-neutral moments are calculated using S&P 500 index option with 30-days maturity. CAPE is previous month cyclically adjusted price earnings ratio from http://www.econ.yale.edu/ shiller/data.htm. GFC is a dummy variable control for 2008 financial crisis, which equals 1 if the month falls between October 2008 and March 2009 and 0 otherwise. t-statistics, in parentheses, are Newey-West adjusted with twelve lags.

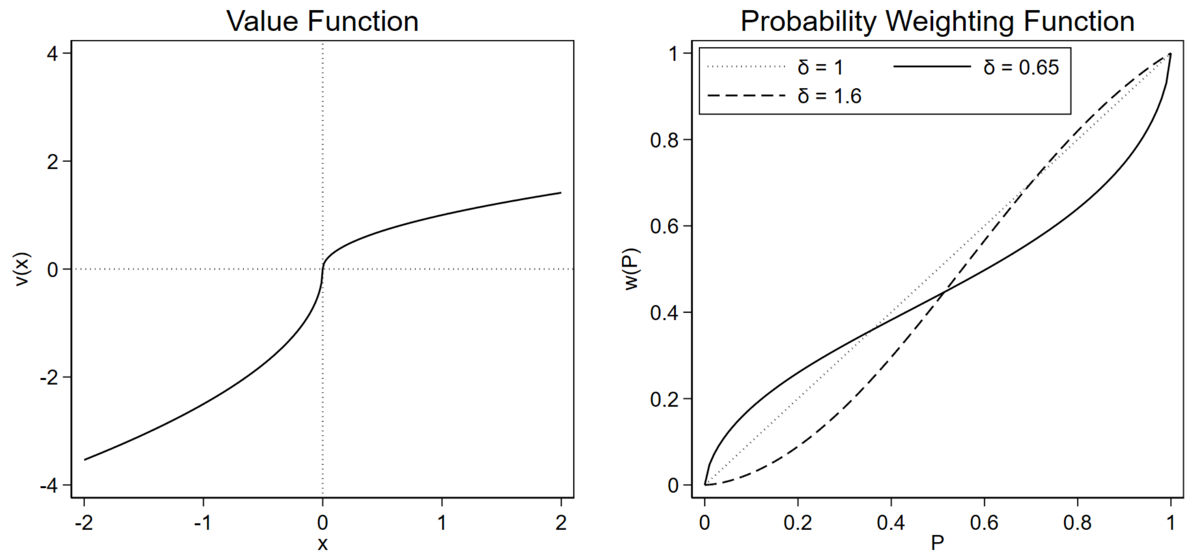

Figure 1
The prospect theory value function and probability weighting function
The left panel plots the value function proposed by Tversky and Kahneman (1992) as part of their cumulative prospect theory, namely, $v(x) = x^{\alpha}$ for $x \geq 0$ and $v(x) = -\lambda(-x)^{\alpha}$ for $x < 0$, for $\alpha = 0.5$ and $\lambda = 2.5$. The right panel plots the probability weighting function they propose, namely, $w(P) = P^{\delta}/(P^{\delta} + (1-P)^{\delta})^{1/\delta}$, for three different values of δ. The dotted line corresponds to $\delta = 1$, the solid line to $\delta = 0.65$, and the dash line to $\delta = 1.6$.

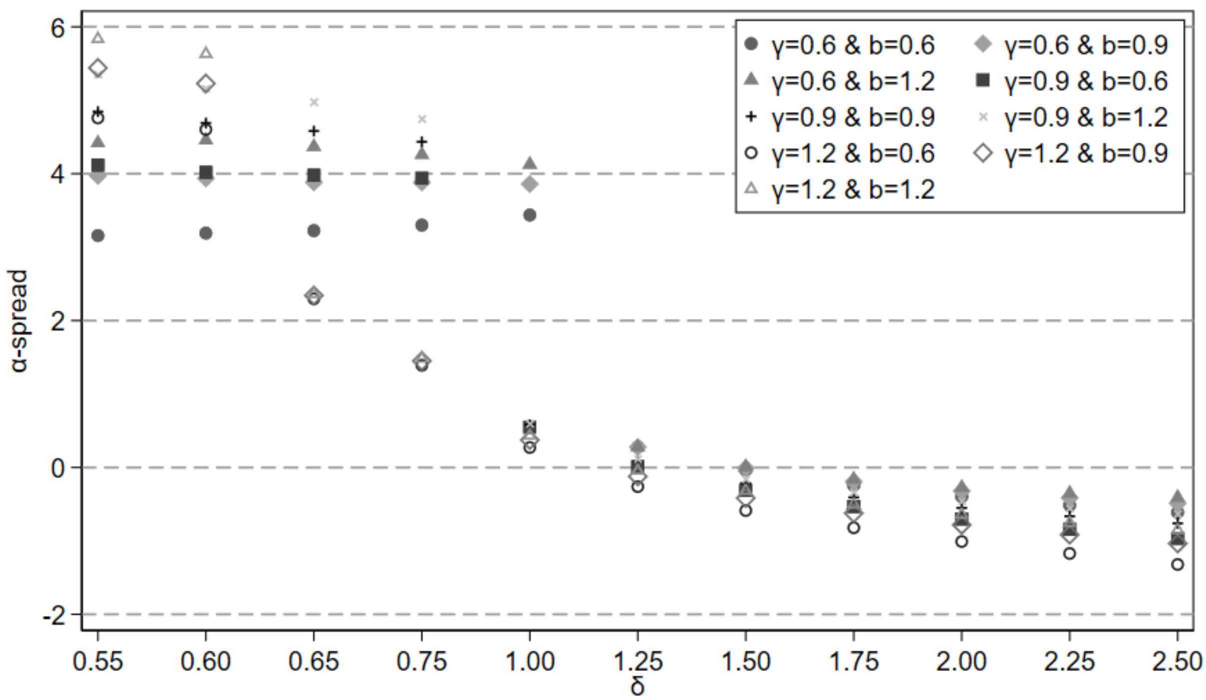

Figure 2
Model-predicted alpha spread with respect to δ
The figure plots the model-predicted alpha spread against δ under different parameterizations. The model follows Barberis et al. (2021). The probability weighting parameter, δ, is set between 0.55 and 2.50. The weight of risk aversion in the investor's objective function, γ ($\hat{\gamma}$ in equation (9)), is set to 0.6, 0.9, or 1.2. The weight of prospect theory in the investor's objective function, b ($\hat{b}$ in equation (9)), is set to 0.6, 0.9, or 1.2.

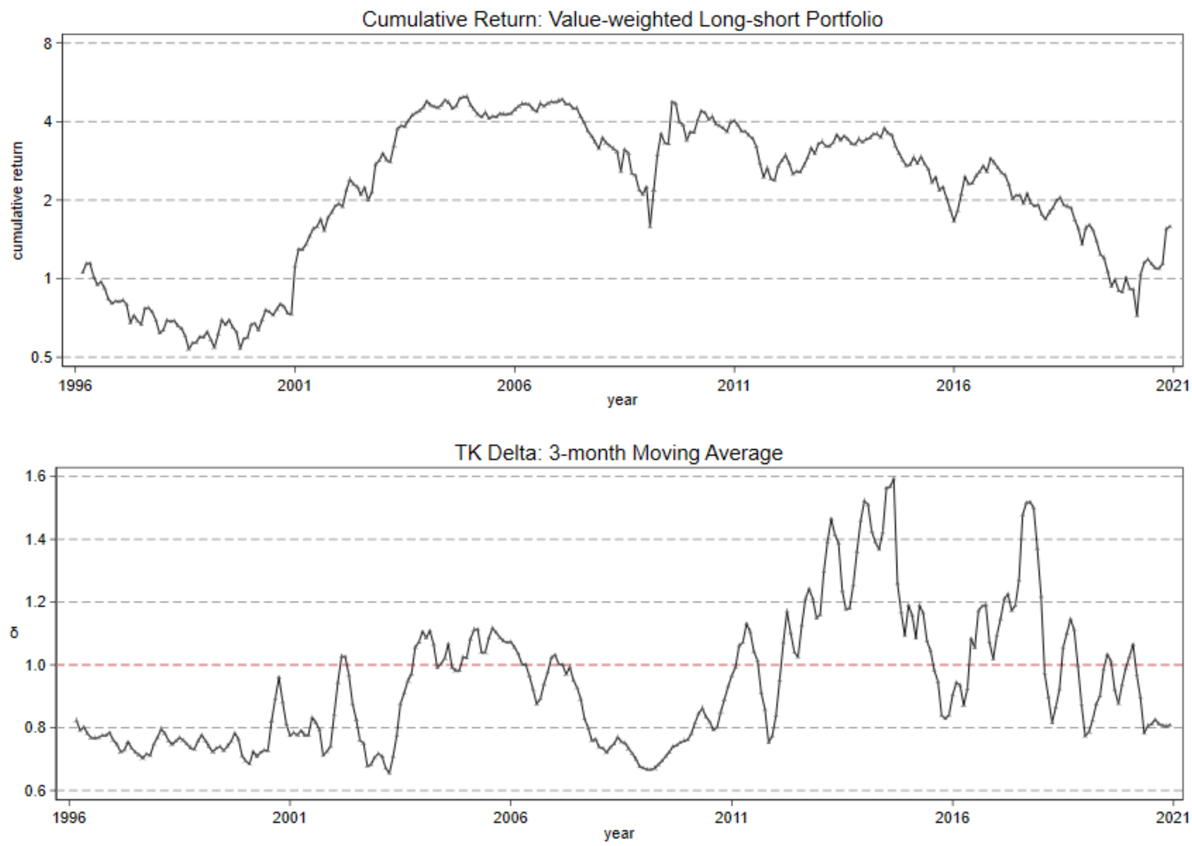

Figure 3
Performance of value-weighted long-short TK portfolio and empirical TK delta
The top panel plots the time series of the cumulative gross return of value-weighted long-short TK portfolio that starts from the beginning of March 1996. TK portfolios are formed following Barberis et al. (2016). The bottom panel plots the time series of 3-month moving average of probability weighting parameter (δ), estimated by the method described in Section 3.2. δ is estimated at the beginning of each month.

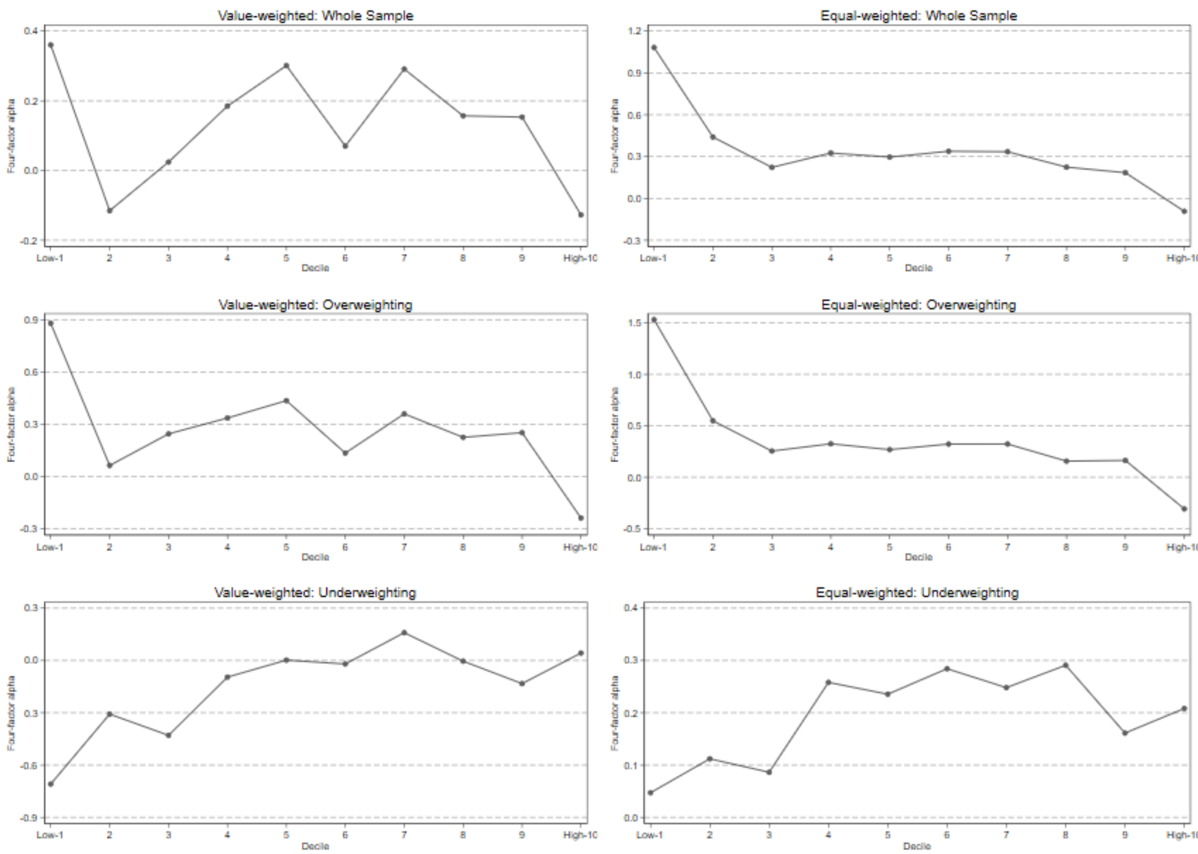

Figure 4
Performance of TK deciles

The figure plots the four-factor alpha of TK decile portfolios in the whole sample, the overweight sample ($\delta \leq 1$), and underweight sample ($\delta > 1$), for both value-weighted (left panels) and equal-weighted (right panels) portfolios. The TK decile portfolios are formed following Barberis et al. (2016). The vertical axis is the monthly alpha (in percent) and the horizontal axis marks the decile portfolio number, from decile 1 (low TK) on the left to decile 10 (high TK) on the right.

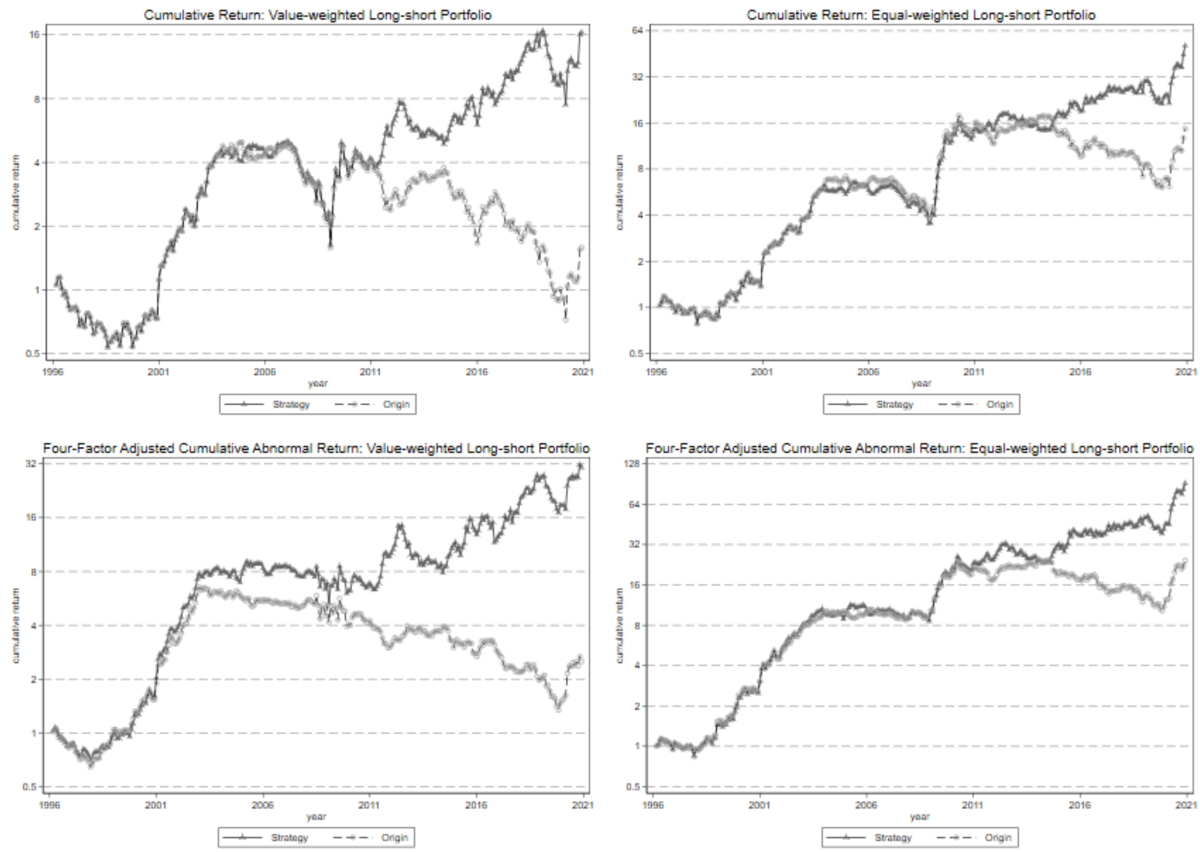

Figure 5
Performance of original and augmented TK strategies
The figure plots the time series of the cumulative gross return of value-weighted (left panels) and equal-weighted (right panels) long-short original and augmented TK strategies. The top panels are for excess returns; the bottom panels are for four-factor adjusted abnormal returns, where four factors are MKT, SMB, HML, and UMD. In each panel, we present performance of both the original TK strategy following Barberis et al. (2016) and the augmented TK strategy. For the augmented TK strategy, we begin with the original long-short TK strategy positions in March 1996 when $\delta \leq 1$. We keep the same position sign until $\delta > 1.1$, when we flip the positions, i.e., long in high-TK stocks and short in low-TK stocks. We keep these opposite positions until δ falls below 0.9, when we switch back to the original TK positions. The switching continues until the end of the sample. The sample runs from March 1996 to December 2020.