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0.1 Prove that $3n^2 + 10n + 729 = O(n^2)$:

In order to prove so, we need to show that there exists n_0 and c such that $3n^2 + 10n + 729 \leq cn^2$, for all $n > n_0$. In our case, we will use $n_0 = 9$ and $c = 22$. Since $n > 1$ we can upper bound $10n$ by $10n^2$ and thus we get:

$$3n^2 + 10n + 729 < 3n^2 + 10n^2 + 729 = 13n^2 + 729$$

Now we use the fact that for $n > n_0 = 9$ we have $9n^2 > 729$, thus:

$$13n^2 + 729 < 22n^2$$

Thus there exists n_0 and c such that $3n^2 + 10n + 729 \leq n^2$, for all $n > n_0$

0.2 Prove that $3n^2 + 10n + 729 = O(n^3)$:

In order to prove so, we need to show that there exists n_0 and c such that $3n^2 + 10n + 729 \leq cn^3$, for all $n > n_0$. In our case, we will use $n_0 = 3$ and $c = 22$. Since $n > 1$ we can upper bound $3n^2$ by $3n^3$ and $10n$ by $10n^3$ and thus we get:

$$3n^2 + 10n + 729 < 3n^2 + 10n^2 + 729 < 3n^3 + 10n^3 + 729 = 13n^3 + 729$$

Now we use the fact that for $n > n_0 = 3$ we have $9n^3 > 729$, thus:

$$13n^3 + 729 < 22n^3$$

Thus there exists n_0 and c such that $3n^2 + 10n + 729 \leq n^3$, for all $n > n_0$

0.3 Prove that $3n^2 + 10n + 729 = \Omega(n)$:

In order to prove so, we need to show that there exists n_0 and c such that $3n^2 + 10n + 729 \geq cn$, for all $n > n_0$. In our case, we will use $n_0 = 1$ and $c = 1$. Since $n > 1$ we can lower bound $3n^2$ by $3n$ and thus we get:

$$3n^2 + 10n + 729 > 3n + 10n + 729$$

Now we use the fact that for $n > n_0 = 1$ we have $13n > n$, thus:

$$3n + 10n + 739 > n$$

Thus there exists n_0 and c such that $3n^2 + 10n + 729 \geq n$, for all $n > n_0$

0.4 Prove that $3n^2 + 10n + 729 = \Omega(n^2)$:

In order to prove so, we need to show that there exists n_0 and c such that $3n^2 + 10n + 729 \geq cn^2$, for all $n > n_0$. In our case, we will use $n_0 = 1$ and $c = 1$. Since $n > 1$ we can lower bound $10n$ by 10 and thus we get:

$$3n^2 + 10n + 729 > 3n^2 + 10 + 729$$

Now we use the fact that for $n > n_0 = 1$ we have $3n^2 > n^2$, thus:

$$3n^3 + 10 + 729 > n^2$$

Thus there exists n_0 and c such that $3n^2 + 10n + 729 \geq n^2$, for all $n > n_0$

0.5 Which is best for each:

I think $3n^2 + 10n + 729 = O(n^2)$ and $3n^2 + 10n + 729 = \Omega(n^2)$ are better for upper bound and lower bound because if you draw all the five lines ($f(n)$, $O(n^2)$, $O(n^3)$, $\Omega(n)$, $\Omega(n^2)$)in the x-y coordinate axis, you can see that when the n gets bigger, the $O(n^3)$ and $\Omega(n)$ start to go far from other 3 lines and this distance continues to grow bigger. If you give n a big enough value, $O(n^3)$ becomes much bigger than the other 4 and $\Omega(n)$ becomes much smaller than the other 4 so I think the best two are $O(n^2)$ and $\Omega(n^2)$.