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Question 1:

- Let $\dot{D}$ be a DFA that generates strings set $\dot{S}$ such that all strings contain ω
- According to Theorem 4.3, we can decide whether a regular expression generates a given string
- According to Theorem 4.4, E_{DFA} is also decidable
- Construct a DFA A such that $L(A) = L(D) \cap L(\dot{D})$
- So we use these two Theorems and construct a TM such that it can decide whether ω is a substring of $\dot{D}$ and A is not empty
- If the machine accepts, we accept
- Therefore the language is decidable

Question 2:

- According to Lemma 2.27, a PDA P has an equivalent CFG
- So we can construct CFG G such that it is equivalent to PDA P
- According to Theorem 4.8, E_{CFG} is decidable
- So we construct TM T and give $\langle C \rangle$
- P is not empty and does accept some strings, so there is only one accept state, and therefore:
- If T accepts, reject, else accept
- Therefore the language is decidable

Question 3:

3.1

- According to the given information, We know that g and f are individually Turing-simulable, so both of them satisfy the rule 1 and 2.
- So for b' where $b' = f(b)$ and $b' \in B$, TM also halts.
- And by this logic we add another layer where $g \circ f = g(f(b)) = g(b') = b''$, this one also halts.
- So the composition of two Turing-simulable binary functions is also Turing-simulable

3.2

- $f(b') = b$ then $f^{-1}(b) = b'$
- Construct a TM and give $\langle m, b \rangle$
- If it halts with output b' such that $|b'| \leq m$, accept, else reject
- So the language is decidable

Question 4:

- Assume END_1 is decidable with a TM A
- Construct A' such that it is a TM to decide A
- Give the TM A a string s, if it is a binary string ending in a 1, accept, else reject
- Then we use A' with given input A,s, if A' accpets, accept, else reject
- However according to Theorem 4.11, A_{TM} is undecidable so the last step cannot stand
- There is a contradiction and therefore END_1 is undecidable

Question 5:

- Assume $BLANK_{TM}$ is decidable with a TM B
- Construct B' such that it is a TM to decide B
- Give the TM B a string s, for each operation, replace the char with an empty blank, and at the end of all operations, if s is empty, accept, else reject
- Then we use B' with given input B,s, if B' accpets, accept, else reject
- However according to Theorem 4.11, A_{TM} is undecidable so the last step cannot stand
- There is a contradiction and therefore $BLANK_{TM}$ is undecidable

Question 6:

6.1

$S \rightarrow 0S0|1S1|0|1|\varepsilon$
This is the grammar for the set of all binary palindrome strings

6.2

- A palindrome string s can be written as ww^R
- Contruct a TM A to map D to L, given input string ww^R and check if w is a palindrome
- So we successfully map two languages, and we already knew D is not context-free
- Therefore Claim 1 is false.

6.3

- Construct a TM B to map L to D, given input string ww and check if w is a palindrome
- So we successfully map two languages, and we already knew L is context-free
- Therefore Claim 2 is also false.