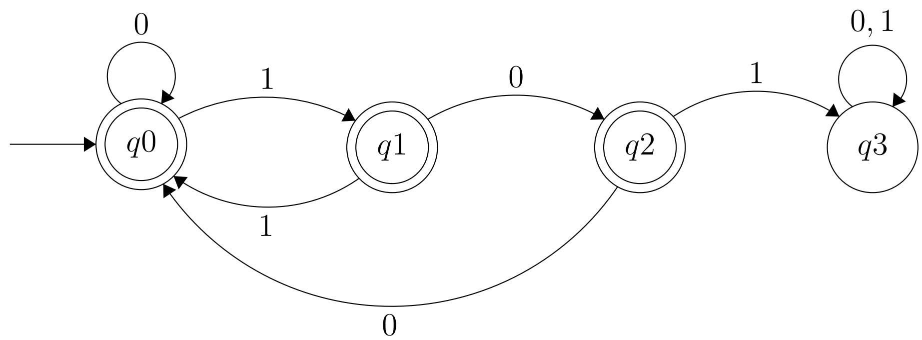
Question 1:

1.1 State diagram of DFA

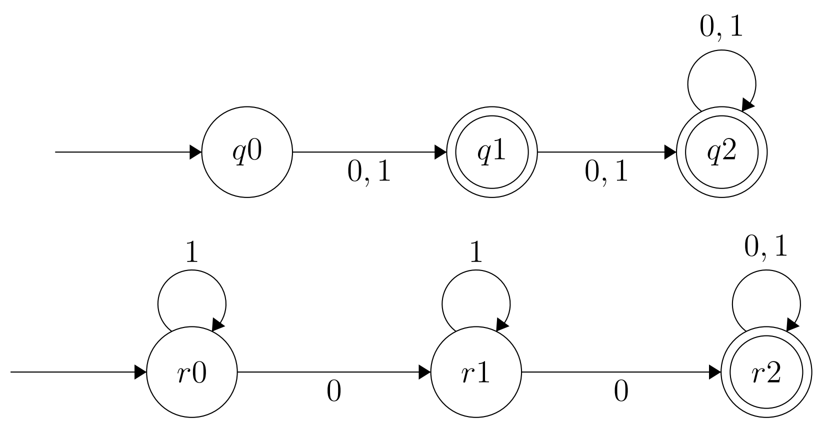


1.2 Full formal specifications of the Machine

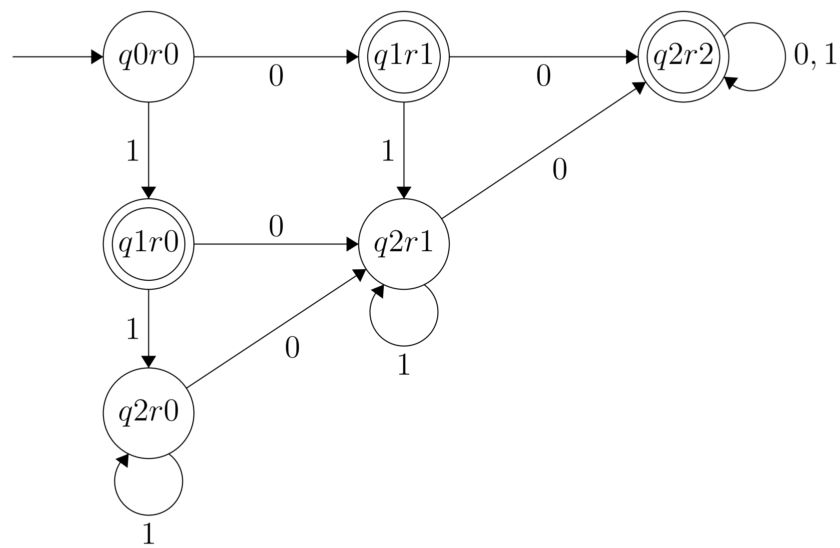
$Q = \{q_0, q_1, q_2, q_3\}$
$\Sigma = \{0, 1\}$
$\delta : Q \text{ x } \Sigma \to Q$
$\delta(q_0, 0) = q_0$
$\delta(q_0, 1) = q_1$
$\delta(q_1, 0) = q_2$
$\delta(q_1, 1) = q_0$
$\delta(q_2, 0) = q_0$
$\delta(q_2, 1) = q_3$
$\delta(q_3, 0) = q_3$
$\delta(q_3, 1) = q_3$
$q_0 \in Q$: initial state
$F = q_0, q_1, q_2$

Question 2:

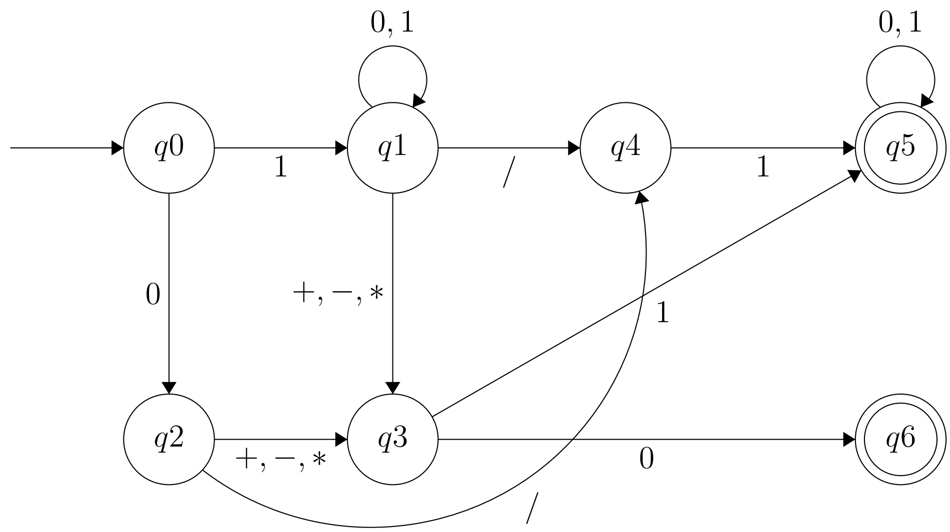
2.1 Two simpler languages



2.2 Combine together



Question 3:



Question 4:

1. From the question, we already know that the complement of $L_{0>1}$: $\overline{L}$ is regular if $L_{0>1}$ is regular, so we can use this to prove that $L_{0>1}$ is not a regular language by contradiction by using $\overline{L}$

2. Assume $\overline{L}$ is regular

3. Let n be the pumping length given by the pumping lemma

4. Choose $\omega = 0^n 1^{n+k}$, $k \geq 0$

5. Let $w = xyz \in \overline{L}$ and $|w| \geq n$ and w follows pumping lemma as well

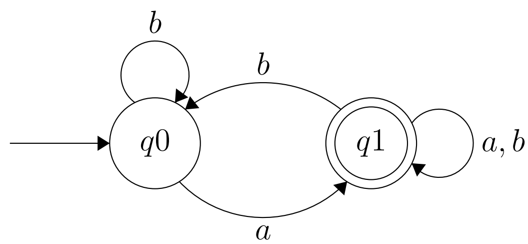
6. Since $|xy| \leq n$ so no matter how we split the string, x will consist of only 0's and so will y

7. Take $k = 1$, $i = 3$ as example, $w = xy^i z = xyyyz$, since y consists at least one 0, then no matter how we divide string, 0's will always be more than 1's, for example: $w = 00111$, when $y = 0$, w becomes 0000111 and this does not meet the requirement

8. So $w \notin L_{\leq}$ and thus we can conclude $\overline{L}$ is not regular, so $L_{0>1}$ is also not regular

Question 5:

5.1



5.2

1. Assume $L_{\leq}$ is regular
2. Let n be the pumping length given by the pumping lemma
3. Let $w = xyz \in L_{\leq}$ and $|w| \geq n$ and w follows pumping lemma as well
4. Let w be the string ababb, and divide in substring x, y, z, we know $\forall i \geq 0, xy^i z \in L_{\leq}$
5. $|y| \geq 1$, and no matter how we divide x, y, z, the result string does not meet the requirement, for example: Let $i = 2$, when $y = a$, $w = abaabb$, in which a's are more than succeeding b's, when $y = ab$, $w = abababb$, still more a's than succeeding b's
6. So $w \notin L_{\leq}$ and thus we can conclude $L_{\leq}$ is not regular

Question 6:

1. Assume L_* is regular
2. Let n be the pumping length given by the pumping lemma
3. Let $w = xyz \in L_*$ and $|w| \geq n$ and w follows pumping lemma as well
4. Let w be the string 10*10=100, and divide in substring x, y, z, we know $\forall i \geq 0, xy^i z \in L_*$
5. $|y| \geq 1$, and no matter how we divide x, y, z, the equation is always incorrect, for example: Let $i = 2$, when $y = 10$, 10*1010=100 is incorrect, when $y = 1$, 10*110=100 is also incorrect
6. So $w \notin L_*$ and thus we can conclude L_* is not regular