Commonality and Risk Drivers of Liquidity in the FX Market: Evidence from High-Frequency Data During the Global Financial Crisis

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1 Introduction

Market liquidity is an essential aspect of financial markets, particularly in the foreign exchange (FX) market, which had a size of over 8 trillion US dollars in 2022[1]. The importance of liquidity in the FX market has been underscored by the financial crisis and by studies such as Brunnermeier et al. (2008). The FX market plays a vital role in ensuring efficiency and arbitrage conditions in other asset markets (Pasquariello 2014). Furthermore, it is distinct from other asset markets in terms of limited transparency, the heterogeneity of participants, and market fragmentation (Karnaukh et al. 2015). Additionally, FX exchange rates are frequently linked to central bank policies and actions, making the study of FX liquidity all the more important.

In this project I have reviewed important literature in FX market as well as market microstructure, and then test the theory based on tick-by-tick FX data from Reuters ranging from January 2008 to December 2009 for three currency pairs AUD/USD, USD/CAD, USD/JPY. The FX market liquidity for each trading day is measured as daily averages of (1) bid-ask spread (2) effective spread (3) price impact. Based on these measures, I perform principal component analysis (PCA) to extract the common component to represent the market-wide liquidity, which exhibits a high correlation with the noise measure (Hu et al. 2013). I also identify the jumps every day using the methodology of Andersen et al. (2007). The findings of the analysis are as follows: First, the FX market liquidity decreases with funding liquidity, with this effect being more significant in times of financial crisis. Second, there is a strong comovement across exchange rates, and commonality in FX liquidity increases

[1] See ”Triennial Central Bank Survey of foreign exchange and Over-the-counter (OTC) derivatives markets in 2022”, Bank for International Settlements, Oct 2022.

when the market becomes more stressed. Finally, FX market liquidity decreases with jump risk. Several specification checks are performed, and the conclusions remain robust.

The remainder of the paper is organized as follows: section 2 reviews related literature and develops hypotheses, section 3 describes the dataset, section 4 discusses the liquidity measure, section 5 analyses the commonality across exchange rates, section 6 explores the relationship between funding liquidity and market liquidity, section 7 studies jump risk, section 8 concludes.

2 Hypotheses Development

Brunnermeier & Pedersen (2009) develop a model that links market liquidity and trader's funding liquidity. Their model shows that the co-reinforcement of market liquidity and funding liquidity, under specific conditions, can result in the occurrence of liquidity spirals. When the economy becomes more uncertain, funding liquidity is decreasing, making it harder for investors and institutions to get the funding they need, resulting in lower market liquidity. If investors are then forced to sell off their assets, prices can deviate from their fundamental values, leading to even greater losses and further reduction in funding liquidity. And this creates a self-reinforcing cycle of declining liquidity. Thus, there are following two hypothesis:

H1:FX market liquidity decreases with funding liquidity.

H2:Negative impact of funding constraints on liquidity amplifies during times of crisis.

Their model implies that there are common components across different securities determining the assets liquidity as when there is a decrease in traders' funding liquidity, the liquidity of the assets also decreases. Thus, the following hypothesis comes:

H3:There are comovements of market liquidity across various exchange rates.

According to Hameed et al. (2010), there is a positive relationship between market volatility and liquidity commonality. Their study also provides evidence supporting contagion effects across assets following negative market shocks, primarily due to supply effects. Similarly, Karolyi et al. (2012) examine time-series commonality in liquidity and find similar results. Karnaukh et al. (2015) show that commonality in FX liquidity tends to increase during periods of tighter funding constraints and higher global risks. Thus, the fourth hypothesis is:

H4:The commonality in FX liquidity increases in distressed market conditions.

Another commonly accepted fact is that bid-ask spreads are positively influenced by return volatility, due to the increased adverse selection and inventory risk (Stoll 1978). Tauchen & Zhou (2011) find that jump (price discontinuity) risk could capture some long-run macroeconomic risk similarly examined by Bansal & Yaron (2004), and jumps are negatively correlated with the market liquidity. The presence of high-frequency jumps in the market may allow arbitragers to exploit arbitrage opportunities, which can potentially expose liquidity providers to higher trading risks by trading at stale quotes. Such ”toxic” arbitrage opportunities may significantly impact market liquidity Foucault et al. (2017). Therefore, the fifth hypothesis is:

H5:FX market liquidity decreases with jump risks.

Based on the article’s progression, the testing sequence begins with hypotheses 3 and 4 in section 5, followed by hypotheses 1 and 2 in section 6, and finally, hypothesis 5 in section 7.

3 Data

In this project I use tick-by-tick FX data from Reuters ranging from January 2008 to December 2009 for three currency pairs AUD/USD, USD/CAD, USD/JPY. For each order submitted, This dataset comprises the following information for each order submitted: (1) the currency pair, (2) the order type (limit or market), (3) the direction and price for the market order (buy or sell), (4) the best ask and bid price for the limit order, and (5) the precise time at which an order is submitted, with an accuracy of one-hundredth of a second. After data cleaning[2], the high-frequency dataset can provide an accurate evaluation not only of liquidity in the FX market, but also other variables of interest such as returns and volatilities.

4 Liquidity Measure

For each currency pair i on day t, I use three different measures of illiquidity:

1. daily averages of quoted bid-ask spread

$$IL^s = (P^A - P^B)/P^M \tag{1}$$

   where P^A, P^B and P^M denote the ask price, bid price and mid-quote price.

2. daily averages of effective spread

[2]there several outlier data points in the original datasets. For instance, in the line 21369-21370 of -2009-03-JPY=D3.csv, the transaction price is 990.12 while the ask-bid mid is around 99; in line 127716 - 127718 of -2009-01-CAD=D3.csv, the bid price is 0.00001. These outliers date are removed from the dataset. Furthermore, some limit orders in the original datasets have one of the ask or bid price set to zero while the other is normal, these orders are also excluded from my sample.

$$IL^e = \begin{cases} (P^T - P^M_{before})/P^M_{before} & \text{for buyer-initiated trades} \\ (P^M_{before} - P^T)/P^M_{before} & \text{for seller-initiated trades} \end{cases} \tag{2}$$

where P^T is the transaction price and P^M_{before} is the prevailing mid-quote price.

3. daily averages of price impact IL^p:

   It reflects how much the exchange rate would be influenced by the order flow (Kyle 1985). The Kyle's Lambda is a widely used metric of market illiquidity, and it indicates the degree to which a market maker hedges against potential losses resulting from trading with informed traders. In the dataset there is no trading volume, so instead of estimating the Kyle's Lambda, here I calculate the change in the mid-quote after the transaction, divided by mid-quote and multiply it by the buy-sell indicator[3]:

$$IL^p = \begin{cases} (P^M_{after} - P^M_{before})/P^M_{before} & \text{for buyer-initiated trades} \\ (P^M_{before} - P^M_{after})/P^M_{before} & \text{for seller-initiated trades} \end{cases} \tag{3}$$

   where P^M_{after} is the next mid-quote price after the transaction.

Table 1 describes the summary statistics of daily illiquidity estimates measured by quoted spread, effective spread and price impact. The findings indicate that the quoted spread and effective spread for USD/JPY are considerably higher than those for the other two pairs, which could be attributed to poor data quality. For instance, on March 19, 2009, there are only 474 observations, and the quoted spread is 103.82 bps. On the other hand, the data quality for AUD/USD and USD/CAD is much better. For this reason, only comparing the pairs AUD/USD and USD/CAD is meaningful although I suppose that liquidity for USD/JPY should be higher. The price impact may better capture the market illiquidity since it solely calculates the change in mid-quote and does not depend on spread calculation. Depending on the currency pairs, it ranges from 0.145 bps to 0.222 bps. The fact that the price impact for USD/JPY is the lowest among the three currency pairs confirms the common belief that USD/JPY is the most liquid. The market liquidities for AUD/USD and USD/CAD are close. The effective spreads, which are less than half of the quoted spreads, suggest that there is substantial trading occurring within the quoted prices. The average log returns are relatively small over the time period but the standard variation is very high, implying the strong volatility of exchange rates during the crisis.

[3]For robustness check, I also calculate the change in the mid-quote over a period of one minute following the transaction as in (Foucault et al. 2017). Due to time constraint and computational cost, I only calculate the USD /JPY put it in the Figure B1, which has the similar trend with my simple price impact measure as shown in Figure 1.

Table 1: **Summary statistics for liquidity measures**

	mean	std	min	25%	50%	75%	max
Quoted spread (in bps)							
AUD/USD	2.938	1.426	1.583	1.972	2.302	3.329	8.242
USD/CAD	3.137	1.264	1.898	2.368	2.598	3.420	14.391
USD/JPY	16.617	22.848	2.529	5.202	8.332	17.829	217.327
Effective spread (in bps)							
AUD/USD	0.983	0.394	0.594	0.710	0.806	1.116	2.618
USD/CAD	0.972	0.288	0.681	0.803	0.863	1.035	3.547
USD/JPY	4.642	6.288	-43.808	1.687	2.896	5.504	55.273
Price impact (in bps)							
AUD/USD	0.222	0.081	0.134	0.167	0.186	0.256	0.692
USD/CAD	0.193	0.047	0.097	0.161	0.182	0.207	0.441
USD/JPY	0.145	0.515	0.000	0.021	0.036	0.078	5.811
Log returns (in bps)							
AUD/USD	1.002	133.130	-594.191	-56.596	13.636	76.676	718.691
USD/CAD	1.060	90.651	-371.023	-54.947	0.000	47.543	346.271
USD/JPY	-1.122	91.385	-669.070	-49.939	1.511	51.358	550.680

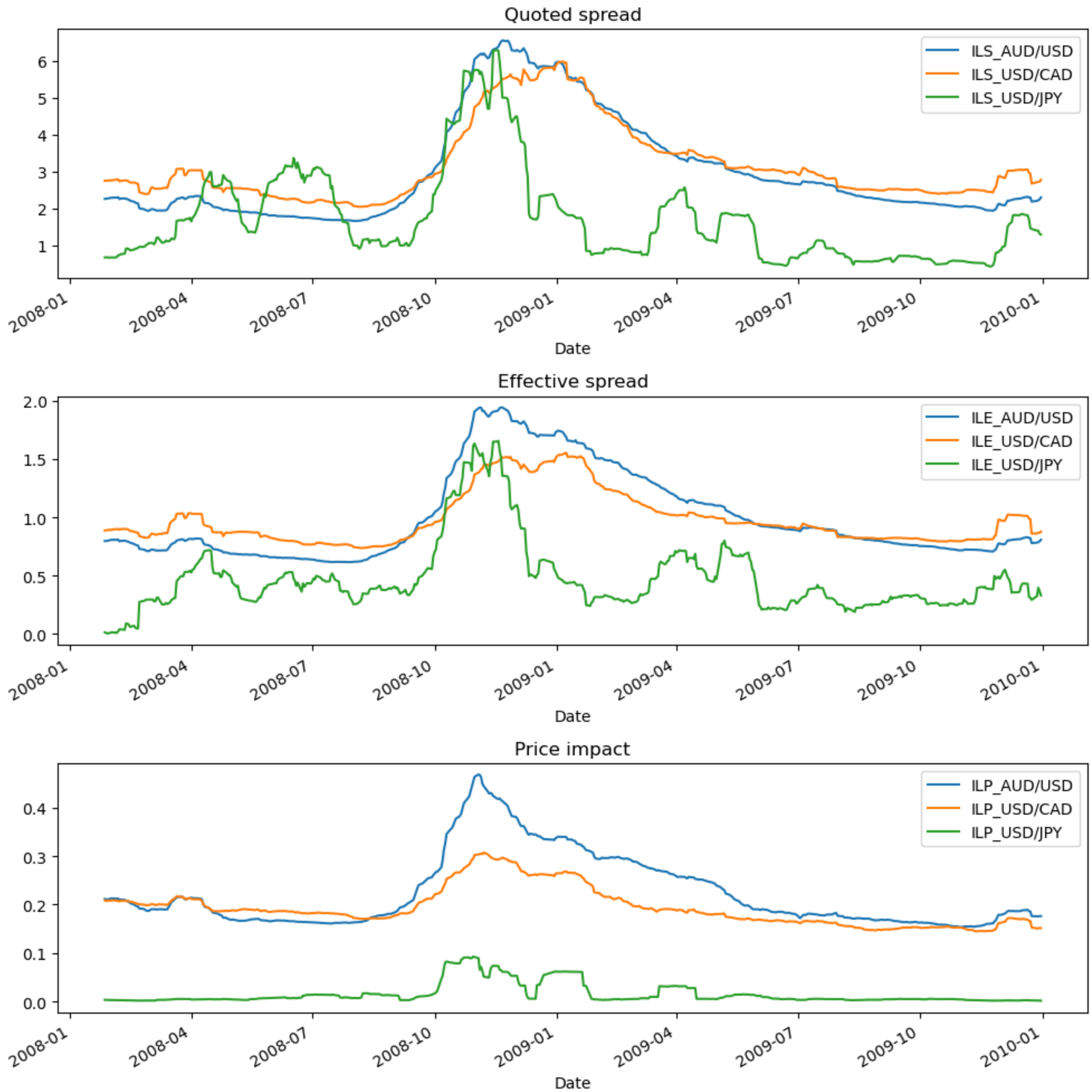
Notes: This table shows the summary statistics for the daily illiquidity estimates for three currency pairs (AUD/USD,USD/CAD,USD/JPY) from January 2008 to December 2009. The upper part is the quoted spread IL^s, the middle part is the effective spread IL^e, and the lower part shows the price impact IL^p.

Figure 1 shows the time series of the daily illiquidity estimates [4]. All measures in all three pairs exhibit the similar time trend. The increase in FX illiquidity during the first quarter of 2008 was primarily caused by the collapse of Bear Stearns in March of that year. During the second quarter of 2008, liquidity began to increase again as investors believed that the worst of the crisis was over and started investing in the FX market once more. Additionally, central banks worldwide implemented policies to protect the financial system, which helped to boost liquidity. However, the last quarter of 2008 saw a significant increase in market illiquidity due to the collapse of Lehman Brothers, which triggered widespread uncertainty and turmoil in the financial markets. This decline in market liquidity the effects of the bankruptcy on market sentiment. In 2009, the

[4] Since there are massive jumps, here I plot a rolling average of liquidity measure sampled from every 5 days. The value for USD/JPY has been divided by 10 for better display.

market liquidity increased steadily and slowly as central banks continued to implement measures to stabilize the financial system and restore confidence in the markets. While there is a strong correlation in the movement of liquidity across different foreign exchange rates, the level of liquidity varies across the cross-section of rates. In the next section I will further analyze the commonality in liquidity across different foreign exchange rates.

Figure 1: **Time series of liquidity measures (rolling)**



Notes: This figure shows the daily illiquidity estimates for three currency pairs (AUD/USD,USD/CAD,USD/JPY) from January 2008 to December 2009. To avoid the massive jumps, I plot a rolling average of liquidity measure sampled from every 5 days. The unit of measure is basis points. The value for USD/JPY has been divided by 10 for better display. The upper graph shows the quoted spread IL^s, the middle graph shows the effective spread IL^e, and the lower graph shows the price impact IL^p.

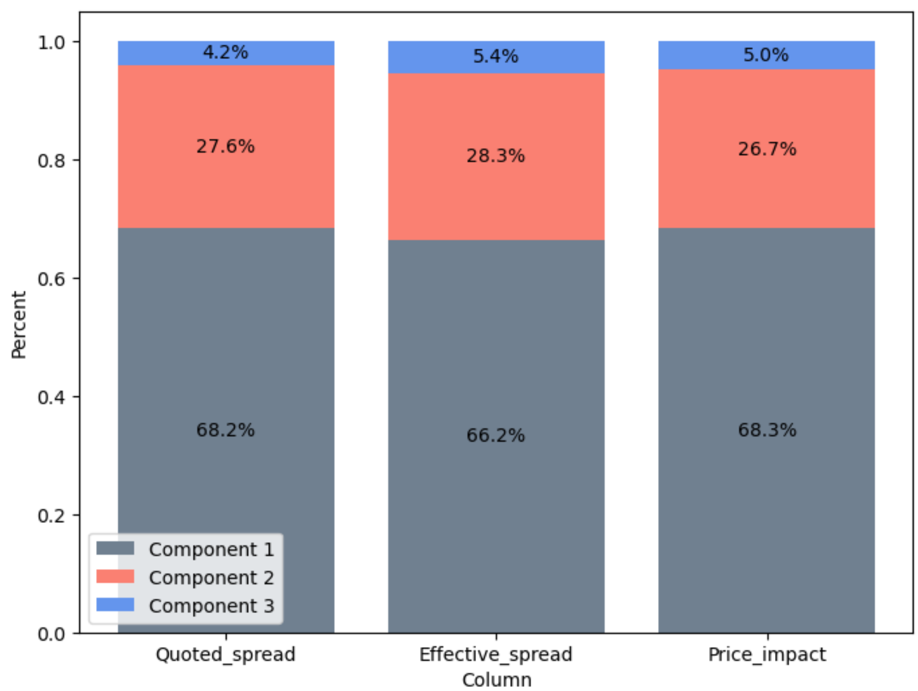
5 Commonality in FX Liquidity

5.1 Principal Component Analysis

Follow Korajczyk & Sadka (2008) and Mancini et al. (2013), here I use principal component analysis (PCA) to extract the common factor to represent the market-wide liquidity. Realizing the cross-sectional difference across exchange rates, first I standardize the the dataset based on the time-series mean and standard derivation. For each liquidity measure, the first three principal components are computed across exchange rates ,and then the first principal component is then selected to represent the market-wide liquidity, denoted by $IL^s_{M,PCA}$, $IL^e_{M,PCA}$, $IL^p_{M,PCA}$.

Figure 2 shows the proportion of variance explained by each principal component for each liquidity measure. The first component explains between 66% and 68% of the variation, and this is slightly lower than the results from Mancini et al. (2013), probably because they fit the PCA with more exchange rates.

Figure 2: **PCA: commonality in liquidity measures**



Notes: This figure plots the proportion of variance explained by each principal component for each daily standardized liquidity measure. The gray part of the bar denotes the goodness of fit using the first principal component, the pink part of the bar denotes increase in the goodness of fit by including the second principal component, and the blue part of the bar denotes increase in the goodness of fit by adding the third principal component.

For comparison, I also calculate the cross-sectional average of liquidity measure at the individual currency pair level. Due to the speciality of USD/JPY, I only average the other two currency pairs and get $IL^s_{M,A}$, $IL^e_{M,A}$, $IL^p_{M,A}$. Hu et al. (2013) develop a liquidity measure from the US. Treasury market, and find that instead of focusing on specific assets, their noise measure could capture the level of liquidity in the aggregate financial market. For this reason, their liquidity measures are also collected to check the correlations[5]. Table 2 reports the correlations between market-wide liquidity measures. The lowest correlation is 0.73 and all values are significant in 1% level, implying a strong correlations between different market-wide measures.

Table 2: **Correlations between market-wide liquidity measures**

	$IL^s_{M,PCA}$	$IL^e_{M,PCA}$	$IL^p_{M,PCA}$	$IL^s_{M,A}$	$IL^e_{M,A}$	$IL^p_{M,A}$	$Noise\_bp$
$IL^s_{M,PCA}$	1.00***						
$IL^e_{M,PCA}$	0.98***	1.00***					
$IL^p_{M,PCA}$	0.86***	0.88***	1.00***				
$IL^s_{M,A}$	0.96***	0.96***	0.84***	1.00***			
$IL^e_{M,A}$	0.96***	0.97***	0.87***	0.99***	1.00***		
$IL^p_{M,A}$	0.89***	0.90***	0.96***	0.88***	0.91***	1.00***	
$Noise\_bp$	0.84***	0.83***	0.73***	0.88***	0.87***	0.78***	1.00***

Notes: This table shows the correlations between market-wide liquidity measures. From the left to right are: PCA-based quoted spread, PCA-based effective spread, PCA-based price impact, average quoted spread, average effective spread, average price impact, and noise measure (Hu et al. 2013). The average measures are calculated based on AUD/USD and USD/CAD. The sample period is from January 2008 to December 2009. ***, ** and * denote significant level 1%, 5% and 10% level respectively.

5.2 FX Market-wide Liquidity and Currency Liquidity

To test how the market-wide liquidity could represent each individual exchange rate liquidity, I construct a regression:

$$IL_{i,t} = \alpha_i + \beta_i IL_{M,t} + \epsilon_{i,t} \tag{4}$$

where i denotes the currency pair. To avoid endogeneity issue, here $IL_{M,t}$ denotes the PCA-based market-wide illiquidity estimated by excluding the currency pair i[6] Table 3 reports the estimated β from equation 4 for each

[5] Avaiable at https://en.saif.sjtu.edu.cn/junpan/

[6] If including currency pair i, the left side of equation 4 is contained in the right side.

currency pair and each liquidity measure. The R^2 for exchange rate USD/JPY is low mainly due to the poor data quality. All estimated β are statistically significant and positive, meaning strong comovements of market liquidity, and this confirms the fourth prediction in Brunnermeier & Pedersen (2009) Chapter 6 and also my hypothesis 3.

During the financial crisis, the liquidity of AUD/USD is highly sensitive to market liquidity, whereas that of USD/JPY is the least sensitive. The sensitivity of currency liquidity to changes in market-wide FX liquidity tends to be higher for currencies with high interest rates, and this is in line with Mancini et al. (2013). This observation can be attributed to the funding constraints faced by carry traders during the crisis. When speculators are close to their funding constraints, they are forced to unwind their carry trades (Brunnermeier et al. 2008) . Figure 3 displays the time series of the cumulative log returns for three currency pairs and the market liquidity[7]. It is important to note that the declining trend in USD/JPY represents JPY appreciation, while the declining trend in AUD/USD indicates AUD depreciation. Figure 3 is consistent with the theory of Brunnermeier et al. (2008) that low-interest rate currencies such as JPY appreciate during low liquidity periods and the unwinding of carry trades, while high-interest rate currencies like AUD depreciate due to supply and demand pressures.

Table 3: **Individual liquidity and market-wide liquidity**

	AUD/USD			USD/CAD			USD/JPY		
	ILS	ILE	ILP	ILS	ILE	ILP	ILS	ILE	ILP
β	0.69***	0.67***	0.66***	0.58***	0.56***	0.59***	0.23***	0.21***	0.25***
	(0.02)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)
Adj. R^2	0.60	0.55	0.56	0.46	0.42	0.48	0.10	0.08	0.11
N	525	525	525	525	525	525	525	525	525

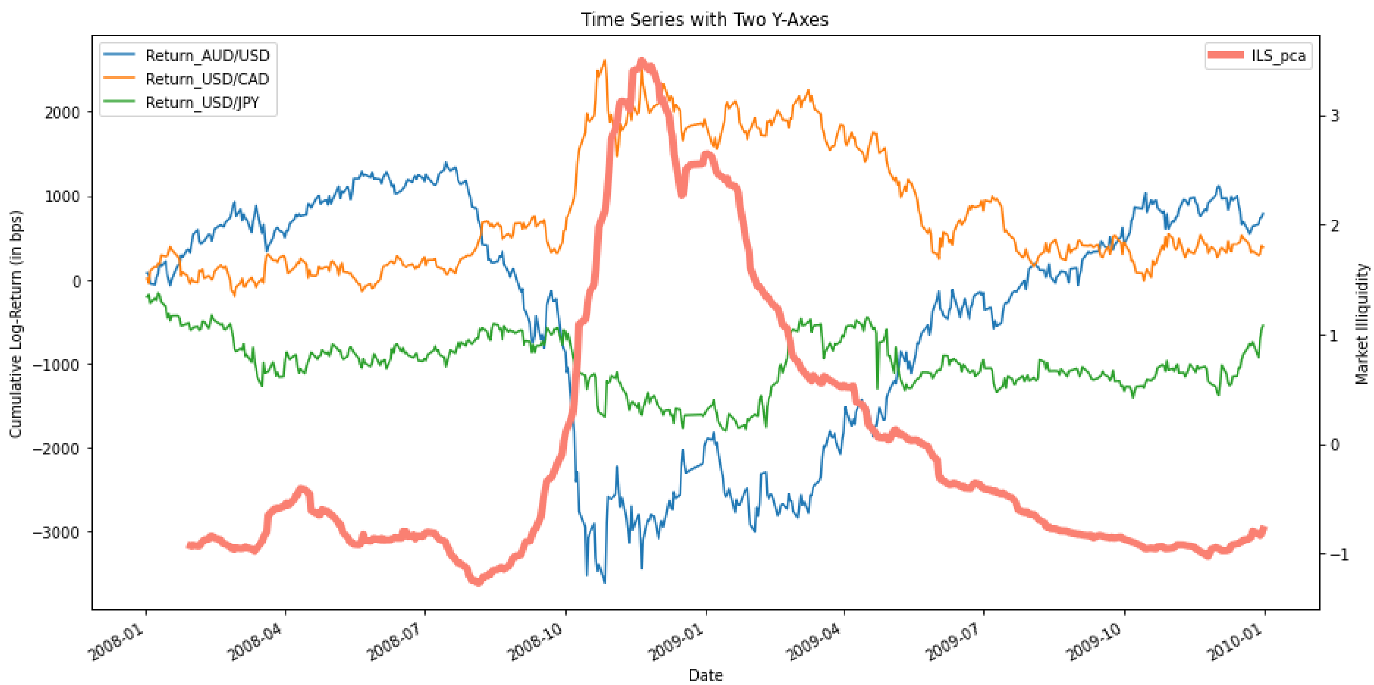
Notes: This table shows the estimated β from equation 4 for each currency pair and each liquidity measure: ILS denotes the quoted spread, ILE denotes teh effective spread, ILP denotes the price impact. There are 525 observations for each specification. The sample period is from January 2008 to December 2009. ***, ** and * denote significant level 1%, 5% and 10% level respectively.

5.3 FX Commonality in Distressed Market

To better derive the time-series properties of FX commonality, I extend equation 4 by adding the market stress variable MD_t measured by Chicago Board Options Exchange Volatility Index (VIX) and the TED spread

[7]Other liquidity measures from Table 2 can also be used as they exhibit strong correlations.

Figure 3: **Cumulative returns and market liquidity**



Notes: The cumulative log returns for three currency pairs (AUD/USD, USD/CAD, USD/JPY) are shown as thin lines in basis points. For each trading day, the first-observed mid-quote price and the last-observed mid-quote price are used to obtain the daily log-returns. The upward tendency of USD/CAD means the depreciation of CAD, same as USD/JPY, ontrast with AUD/USD. The thick line shows the PCA-based quoted spread. To avoid large jumps, a rolling average of liquidity measures sampled every 5 days is plotted. The sample period is from January 2008 to December 2009.

[8](Karnaukh et al. 2015). VIX captures the implied volatility of S&P 500 and is thought to describe the investor's fear and market uncertainty. TED spread is the difference between the LIBOR inter-bank market interest rates and the risk free rates, and describes the credit risk.

$$IL_{i,t} = \alpha_i + \beta_i IL_{M,t} + \gamma_i IL_{M,t} \cdot MD_t + \epsilon_{i,t} \tag{5}$$

Table 4 shows the results for equation 5. In most cases, γ is significantly positive, suggesting that commonality in FX liquidity increases in distressed market, and this is consistent with the results of Karnaukh et al. (2015), also similar to the findings of Hameed et al. (2010) and Karolyi et al. (2012) in stock market. My hypothesis 4 is confirmed. It shows that the spillover effect of illiquidity due to supply shocks is a significant factor in transmitting negative market shocks across all FX market.

[8]Both datasets are available in Fred Economic Database.

Table 4: **FX commonality in distressed market**

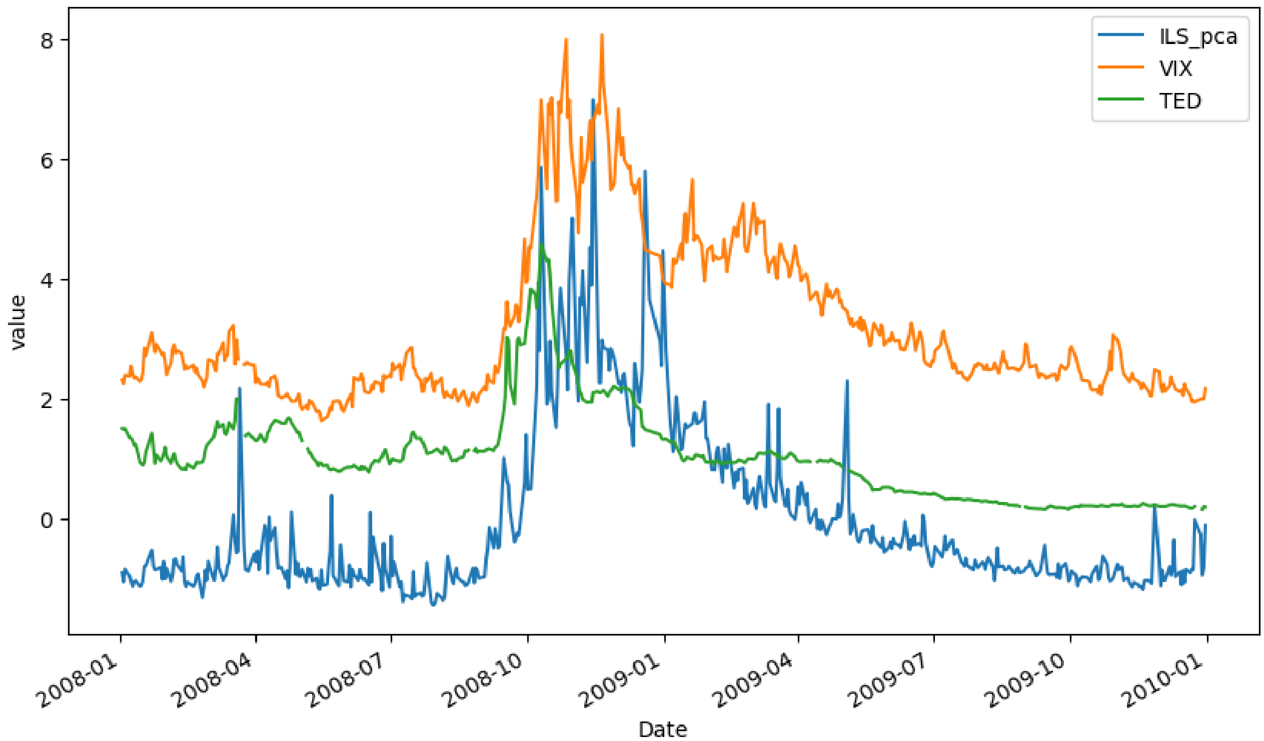
market stress variable $MD : VIX$									
	AUD/USD			USD/CAD			USD/JPY		
	ILS	ILE	ILP	ILS	ILE	ILP	ILS	ILE	ILP
β	0.30***	0.30***	0.30***	0.21***	0.27***	0.34***	0.16**	0.14**	0.32***
	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.07)	(0.06)	(0.06)
γ	0.05***	0.05***	0.05***	0.04***	0.03***	0.03***	0.01*	0.01**	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)
Adj. R^2	0.89	0.90	0.85	0.72	0.69	0.58	0.14	0.15	0.13
N	490	490	490	490	490	490	490	490	490
market stress variable $MD : TEDspread$									
	AUD/USD			USD/CAD			USD/JPY		
	ILS	ILE	ILP	ILS	ILE	ILP	ILS	ILE	ILP
β	0.70***	0.71***	0.40***	0.56***	0.53***	0.28***	0.11**	0.15**	0.23***
	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.03)	(0.05)
γ	0.20***	0.24***	0.54***	-0.02	0.08**	0.68***	0.49***	0.34***	0.11
	(0.04)	(0.04)	(0.05)	(0.04)	(0.04)	(0.04)	(0.06)	(0.06)	(0.09)
Adj. R^2	0.67	0.67	0.65	0.58	0.60	0.69	0.25	0.20	0.13
N	490	490	490	490	490	490	490	490	490

Notes: This table shows the estimated β and γ from equation 5 for each currency pair and each liquidity measure: ILS denotes the quoted spread, ILE denotes teh effective spread, ILP denotes the price impact. The upper panel reports the results of using VIX as the market stress variable and the bottom panel reports TED spread instead. There are 490 observations for each specification (Some are dropped due to the availability of VIX/TED data. The sample period is from January 2008 to December 2009. ***, ** and * denote significant level 1%, 5% and 10% level respectively.

6 Funding Liquidity and Market Liquidity

Inspired by Brunnermeier et al. (2008), VIX and the TED spread are used to measure the funding constraints. Figure 4 shows the time series of market-wide FX market illiquidity based on PCA and funding liquidity measured by VIX and TED spread. It shows clear comovements and during the financial crisis, all indexes are significantly larger than normal.

Figure 4: **Funding liquidity and market liquidity**



Notes: This figure shows the market liquidity (measured by PCA-based quoted spread) and funding liquidity (measured by VIX and TED spread). The value for VIX has been divided by 10. To better compare the jumps, here I do not plot the rolling average. The sample period is from January 2008 to December 2009.

To test the liquidity spirals theory, construct a linear model:

$$IL_t = \eta_t + \gamma_1 Il_{t-1} + \gamma_2 VIX_{t-1} + \gamma_3 TED_{t-1} + \mu_t \tag{6}$$

The parameters γ_2 and γ_3 are of interests as they measure how funding constraints influence market liquidity. Table 5 reports the results of different specifications of equation 6. In the full sample case, both lagged VIX and lagged TED spread have significantly negative impact on current market liquidity, even controlling for the lagged liquidity measure. An one standard deviation increase in the VIX could decrease the next day's liquidity by 0.03, and one standard deviation increase in the TED spread could decrease the next day's liquidity by 0.08, and these effects are significant at 1% level. These results show that funding constraints could negatively influence the FX liquidity. My hypothesis 1 is confirmed.

In column (5)-(8) I show the results with sample from January 2008 to mid-September 2008, which corresponds to the occurrence of the Lehman bankruptcy and is considered as the symbol of the financial crisis. These results suggest that the impact of lagged VIX and lagged TED on the dependent variable, although weaker, is still statistically significant in most cases. This observation is in line with the theoretical prediction of Brunnermeier & Pedersen (2009), which posits that during periods of financial crisis, funding constraints become more binding and, as a result, have a greater influence on market-wide liquidity. My hypothesis 2 is confirmed. Table B1 reports the results of using other proxies of market-wide liquidity measure in Table 2 and shows the conclusion is robust.

Table 5: **Funding liquidity and market liquidity**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	-2.94***	-1.17***	-2.91***	-0.96***	-1.61***	-1.41***	-1.70***	-0.70***
	(0.08)	(0.08)	(0.08)	(0.12)	(0.20)	(0.11)	(0.19)	(0.20)
IL_{t-1}				0.66***				0.56***
				(0.04)				(0.07)
VIX_{t-1}	0.09***		0.08***	0.03***	0.03***		0.02*	0.01
	(0.00)		(0.00)	(0.00)	(0.01)		(0.01)	(0.01)
TED_{t-1}		1.04***	0.20***	0.08**		0.49***	0.41***	0.15*
		(0.06)	(0.05)	(0.04)		(0.09)	(0.09)	(0.09)
R-squared Adj.	0.75	0.36	0.76	0.86	0.08	0.15	0.16	0.39
N	489	489	489	489	178	178	178	178

Notes: This table shows the results of different specifications of equation 6. The dependent variable is the daily market-wide liquidity index (IL_t). The table reports the results based on PCA-based quoted spread, while the tests for other measures are presented in the Appendix. Columns (1)-(4) display the results using the full sample period from January 2008 to December 2009, while columns (5)-(8) focus on the sample period from January 2008 to mid-September 2008. In some dates VIX or TED spread are not available and these dates are dropped (9 observations in total). Standard errors in parentheses. ***, ** and * denote significant level 1%, 5% and 10% level respectively.

7 Jump Risk

If we zoom in to look at Figure 3, then return curve is not smooth and there are many jumps. I employ the jump detection approach proposed by Andersen et al. (2007) to estimate the intraday jumps at a 5-minute frequency [9]. I opted for this frequency as it strikes a balance between preserving the information richness of high-frequency data and mitigating the microstructure distortions that could arise from more frequent sampling, as documented by (Ait-Sahalia et al. 2005).

[9]The theory of their methodology is outlined in the appendix.

After dividing each day to 5-minute intervals, I calculate the daily realized volatility for day t:

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2 \tag{7}$$

where M denotes the number of intervals, and in this case $M = 24 \times 60 \div 5 = 288$. The realized bipower variation (Barndorff-Nielsen & Shephard 2004) is calculated as:

$$BV_t = \frac{\pi}{2} \sum_{j=2}^{M} |r_{t,j}||r_{t,j-1}| \tag{8}$$

As M approaches infinity, estimation of the contribution of jumps to the underlying price process is consistent:

$$RV_t - BV_t \rightarrow \sum_{t-1<s\leq t} J_s^2 \tag{9}$$

To prevent the squared jumps from being negative in the sample estimation, the measure of jumps in truncated at zero:

$$J_t^2 = max[RV_t - BV_t, 0] \tag{10}$$

In Figure 5, the time series of realized volatility and jumps estimated from equations 7 and 10 are presented in standard deviation form. These series exhibit a significant degree of autocorrelation, which is confirmed by the Ljung-Box statistics for up to tenth-order serial correlation reported in Table B2. The autocorrelation for AUD/USD and USD/CAD is more prominent, and these series show a distinct pattern: higher during the financial crisis compared to the normal period. In contrast, USD/JPY does not show the same pattern, possibly due to data quality issues.

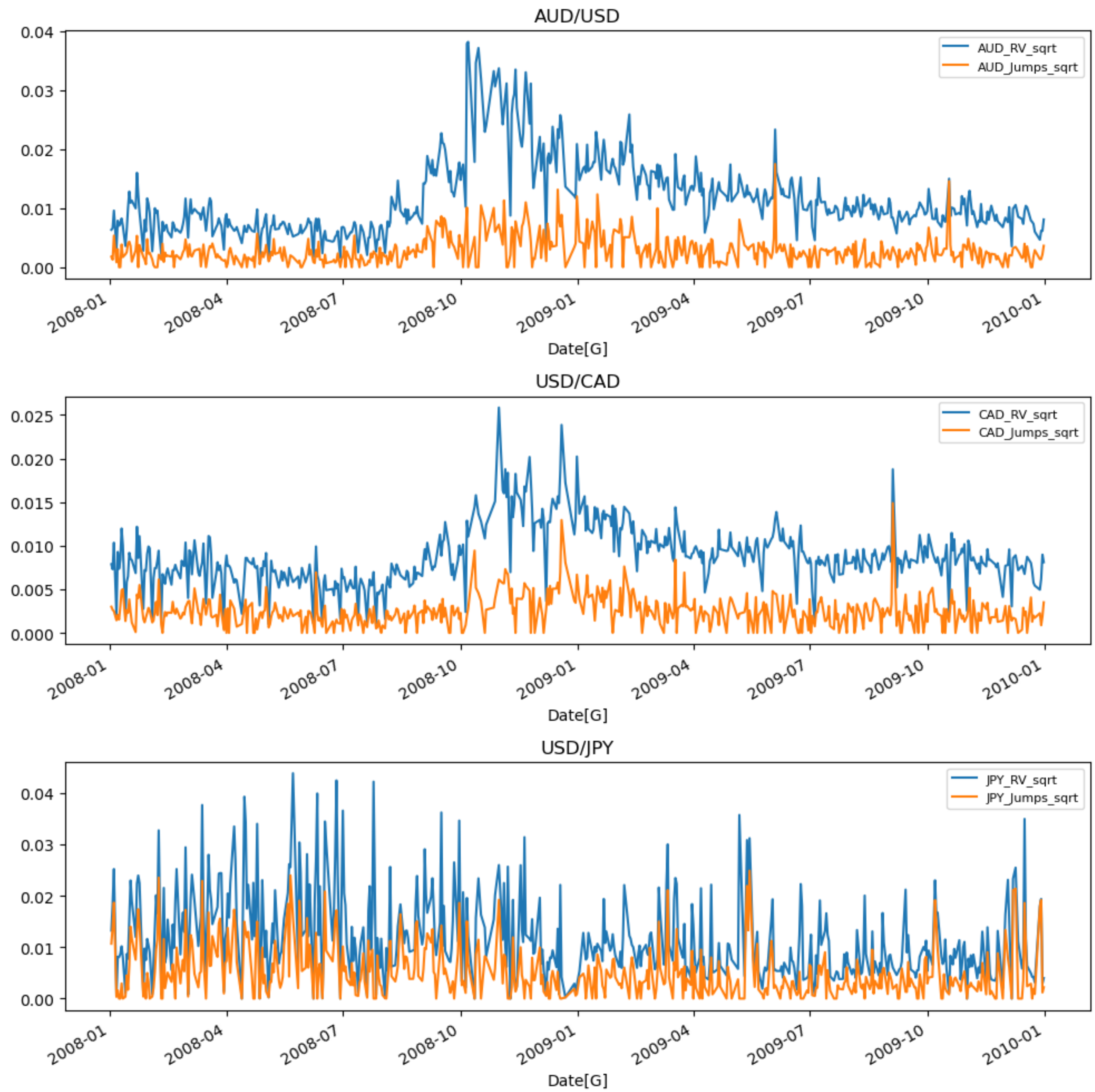
To examine the how the jumps influence market liquidity, for each currency pair i at day t, I run the following regression:

$$IL_{i,t} = \kappa_t + \theta_1 J_{i,t-1}^2 + +\zeta_t \tag{11}$$

The results of regression 11 is shown in Table 6[10]. Excluding USD/JPY, all the coefficients θ_1 are statistically significant and positive. For example, one standard deviation increase in the jump could increase the effective spread of AUD/USD by 1.66. Overall, the jumps are negatively correlated with the market liquidity. My hypothesis 5 is confirmed. High frequency trader may take advantage of these jumps due to their transaction speed advantage, and thus the liquidity provider would bear the pick-off risk. To compensate the adverse selection cost, they may charge a higher bid-ask spread.

[10]Here I consider the Jumps $J_{i,t}$ as the dependent variable. In Table B3 I show the results of using $J_{i,t}^2$ instead of $J_{i,t}$ as the dependent variable. And the conclusion remains robust.

Figure 5: **Daily realized volatility and jumps**



Notes: This figure shows time series of the realized volatility and jumps estimated using equations 7 and 10 for the period spanning January 2008 to December 2009, in standard deviation form. The topmost figure represents AUD/USD, followed by USD/CAD in the middle, and USD/JPY at the bottom.

Table 6: **Market liquidity and jump risk**

	ILSAUD	ILEAUD	ILPAUD	ILSCAD	ILECAD	ILPCAD	ILSJPY	ILEJPY	ILPJPY
$J_{AUD,t-1}$	1.66***	1.43***	0.50***						
	(0.16)	(0.17)	(0.14)						
$J_{CAD,t-1}$				1.81***	1.99***	0.08			
				(0.26)	(0.28)	(0.24)			
$J_{JPY,t-1}$							-0.06	0.01	0.00
							(0.04)	(0.05)	(0.05)
R-squared	0.19	0.13	0.02	0.09	0.10	0.00	0.00	0.00	0.00
R-squared Adj.	0.19	0.13	0.02	0.09	0.09	-0.00	0.00	-0.00	-0.00
N	488	488	488	488	488	488	488	488	488

Notes: This table shows the results of different specifications of equation 11. Jumps are estimated based on equation 10. The liquidity measures are quoted spread IL^s, effective spread IL^e and price impact IL^p. There are 488 observations from January 2008 to December 2009. Standard errors in parentheses. ***, ** and * denote significant level 1%, 5% and 10% level respectively.

8 Conclusion

Based on the high-frequency FX data, I provide an in-depth study of the liquidity in the FX market which has three main findings: first, the liquidity of the FX market decreases as funding liquidity decreases, and this effect is more pronounced during times of financial crisis; second, there is a high degree of comovement among exchange rates, and the commonality in FX liquidity increases during periods of market stress; third, the liquidity of the FX market is negatively impacted by jump risk. Given the time limit and also the data availability, there are some questions unanswered in this project: for example, how liquidity risk is priced in the assets, and how changes in liquidity impact the real economy. Nonetheless, the study provides valuable insights for market participants and policymakers, and highlight the importance of monitoring and managing liquidity risk in the FX market, particularly during times of financial stress.

A. High-Frequency Data, Bipower Variation and Jumps

In this appendix I will describe the jump detection methodology of Andersen et al. (2007). Let $p(t)$ be the logarithmic exchange rate at time t, and it follows the continuous-time jump diffusion process:

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + J(t)dq(t) \tag{12}$$

where $W(t)$ denotes a standard Brownian motion, $\mu(t)$ represents a continuous and locally bounded variation process, $\sigma(t)$ denotes a strictly positive stochastic volatility process, $J(t)$ represents the jump size. and $q(t)$ is a counting process with a time-varying intensity $\lambda(t)$. Given equation 12, the quadratic variation of the cumulative return is calculated as:

$$[r, r]_t = \int_0^t \sigma^2(s)ds + \sum_{1<s\leq t} J^2(s) \tag{13}$$

Define the daily realized volatility for day t:

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2 \tag{14}$$

As $M \to \infty$, the realized volatility converges to the increment of the quadratic variation process:

$$RV_t \to \int_{t-1}^t \sigma^2(s)ds + \sum_{t-1<s\leq t} J^2(s) \tag{15}$$

The realized bipower variation (Barndorff-Nielsen & Shephard 2004) is calculated as:

$$BV_t = \frac{\pi}{2}\sum_{j=2}^{M} |r_{t,j}||r_{t,j-1}| \tag{16}$$

As $M \to \infty$,

$$BV_t \to \int_{t-1}^t \sigma^2(s)ds \tag{17}$$

Therefore, estimation of the contribution of jumps to the underlying price process is consistent:

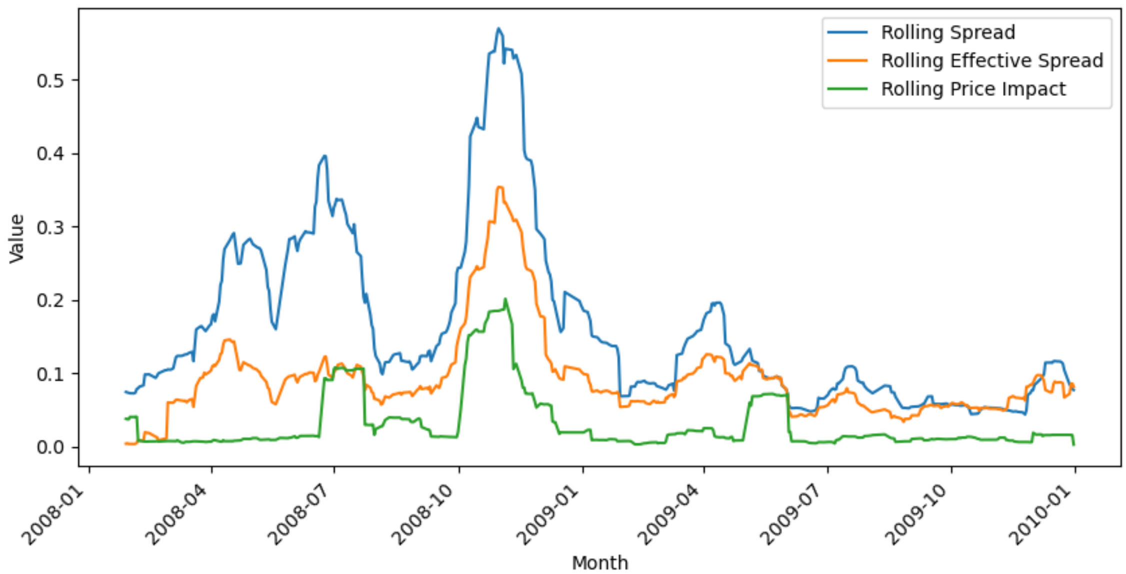
$$RV_t - BV_t \to \sum_{t-1<s\leq t} J_s^2 \tag{18}$$

To prevent the squared jumps from being negative in the sample estimation, the measure of jumps in truncated at zero:

$$J_t^2 = max[RV_t - BV_t, 0] \tag{19}$$

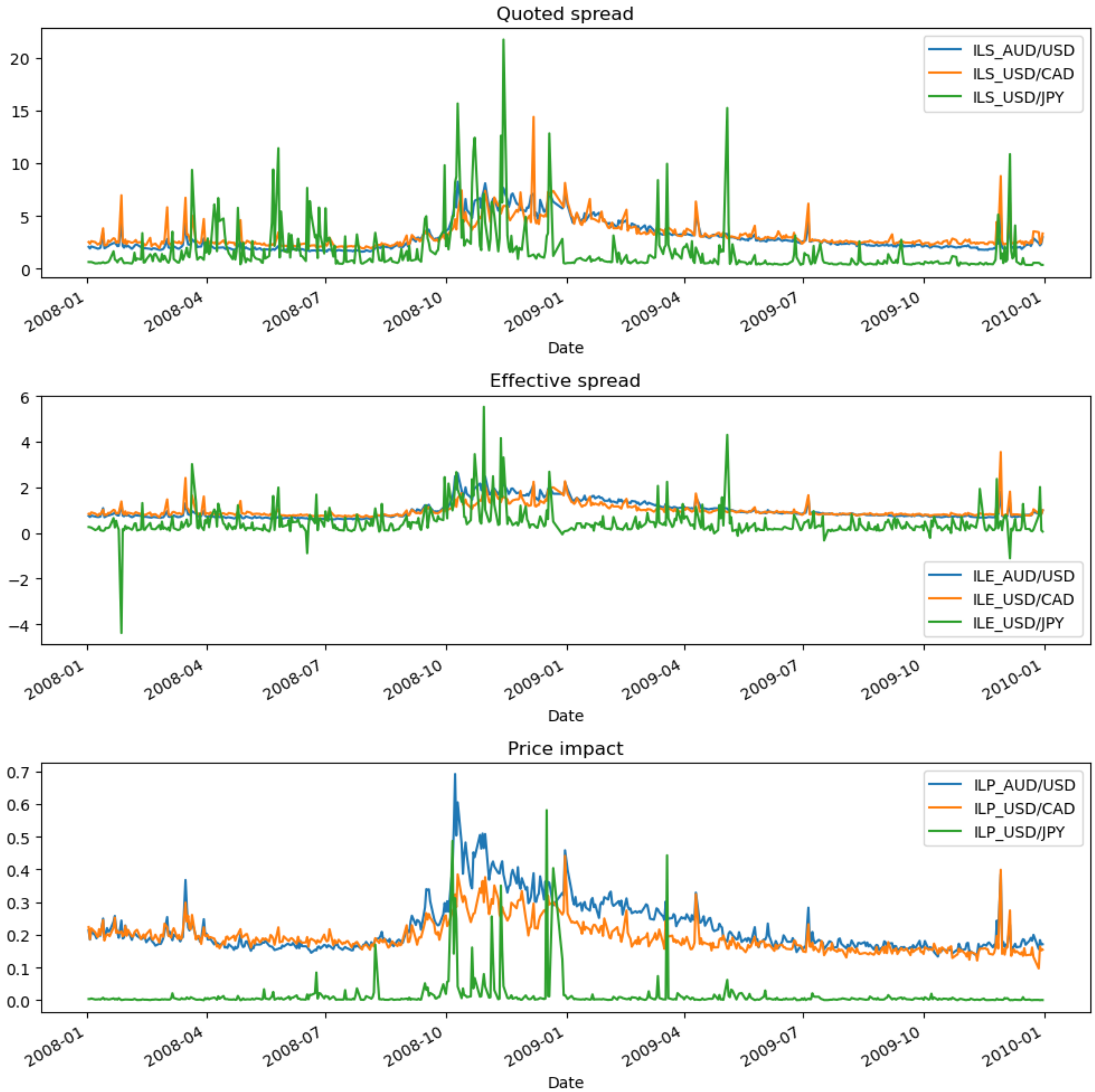
B. Figures and Tables

Figure B1: **Time series of liquidity measures for JPY/USD**



Notes: This figure shows the daily illiquidity estimates for exchange rate (USD/JPY) from January 2008 to December 2009. To avoid the massive jumps, I plot a rolling average of liquidity measure sampled from every 5 days. The unit of measure is basis points. Here the price impact is calculated as the change in the mid-quote over a period of one minute following the transaction, the same price impact measure in (Foucault et al. 2017).

Figure B2: **Time series of liquidity measures (not-rolling)**



Notes: This figure shows the daily illiquidity estimates for three currency pairs (AUD/USD,USD/CAD,USD/JPY) from January 2008 to December 2009. The unit of measure is basis points. The value for USD/JPY has been divided by 10 for better display. The upper graph shows the quoted spread IL^s, the middle graph shows the effective spread IL^e, and the lower graph shows the price impact IL^p.

Table B1: **Funding liquidity and market liquidity: alternative proxies**

	$ILS_{pca,t}$	$ILE_{pca,t}$	$ILP_{pca,t}$	$ILS_{a,t}$	$ILE_{a,t}$	$ILP_{a,t}$	$Noise_{bp,t}$
VIX_{t-1}	0.03***	0.03***	0.02***	0.02***	0.00***	0.00***	0.01**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
TED_{t-1}	0.08**	0.08**	0.38***	0.02	0.01	0.01***	0.10***
	(0.04)	(0.04)	(0.05)	(0.02)	(0.01)	(0.00)	(0.03)
$ILS_{pca,t-1}$	0.66***						
	(0.04)						
$ILE_{pca,t-1}$		0.66***					
		(0.03)					
$ILP_{pca,t-1}$			0.58***				
			(0.04)				
$ILS_{a,t-1}$				0.79***			
				(0.03)			
$ILE_{a,t-1}$					0.75***		
					(0.03)		
$ILP_{a,t-1}$						0.64***	
						(0.04)	
$Noise_{bp,t-1}$							0.97***
							(0.01)
R-squared Adj.	0.86	0.87	0.82	0.92	0.91	0.88	0.99
N	489	489	489	489	489	489	489

Notes: This table shows the results of different specifications of equation 6. The dependent variable is the daily market-wide liquidity index measured using different proxies as shown in Table 2: PCA-based quoted spread, PCA-based effective spread, PCA-based price impact, average quoted spread, average effective spread, average price impact, and noise measure (Hu et al. 2013). In some dates VIX or TED spread are not available and these dates are dropped (9 observations in total). The sample period is from January 2008 to December 2009. Standard errors in parentheses. ***, ** and * denote significant level 1%, 5% and 10% level respectively.

Table B2: **Ljung-Box test for daily realized volatility and jumps**

Realized volatility						
	AUD/USD		USD/CAD		USD/JPY	
Lag	statistic	pvalue	statistic	pvalue	statistic	pvalue
1	339.287	0.0	206.591	0.0	34.600	0.0
2	624.891	0.0	395.159	0.0	41.195	0.0
3	914.851	0.0	572.140	0.0	62.983	0.0
4	1193.691	0.0	742.236	0.0	86.503	0.0
5	1475.783	0.0	928.433	0.0	122.940	0.0
6	1753.708	0.0	1130.420	0.0	133.718	0.0
7	2010.819	0.0	1292.436	0.0	136.345	0.0
8	2252.203	0.0	1450.689	0.0	146.253	0.0
9	2496.560	0.0	1592.738	0.0	153.821	0.0
10	2730.776	0.0	1755.393	0.0	161.334	0.0

Jumps						
	AUD/USD		USD/CAD		USD/JPY	
Lag	statistic	pvalue	statistic	pvalue	statistic	pvalue
1	38.282	0.0	31.274	0.0	47.771	0.0
2	66.657	0.0	49.325	0.0	49.761	0.0
3	95.374	0.0	71.882	0.0	62.085	0.0
4	110.872	0.0	80.005	0.0	70.348	0.0
5	136.835	0.0	91.111	0.0	82.786	0.0
6	163.817	0.0	104.248	0.0	91.095	0.0
7	192.098	0.0	110.005	0.0	92.236	0.0
8	201.724	0.0	124.781	0.0	93.660	0.0
9	219.539	0.0	143.877	0.0	97.312	0.0
10	234.039	0.0	168.152	0.0	98.819	0.0

Notes: The table displays the outcomes of the Ljung-Box test conducted on the daily realized volatility and jumps (in standard deviation) for up to a tenth-order serial correlation. The test results for realized volatility are shown in the upper panel for three currency pairs, and the bottom panel shows the results for jumps for the same currency pairs. For each currency pair, the left column reports the statistics and the right column reports the p-value. The sample period is from January 2008 to December 2009.

Table B3: **Market liquidity and jump risk (in standard deviation)**

	ILSAUD	ILEAUD	ILPAUD	ILSCAD	ILECAD	ILPCAD	ILSJPY	ILEJPY	ILPJPY
$J^2_{AUD,t-1}$	0.02***	0.02***	0.01***						
	(0.00)	(0.00)	(0.00)						
$J^2_{CAD,t-1}$				0.02***	0.02***	-0.00			
				(0.00)	(0.00)	(0.00)			
$J^2_{JPY,t-1}$							-0.00	0.00	0.00
							(0.00)	(0.00)	(0.00)
R-squared	0.21	0.14	0.02	0.14	0.12	0.00	0.00	0.00	0.00
R-squared Adj.	0.21	0.14	0.02	0.13	0.12	-0.00	-0.00	0.00	-0.00
N	488	488	488	488	488	488	488	488	488

Notes: This table shows the results of different specifications of equation 11. Jumps are estimated based on equation 10. Here the dependent variable is the square root of Jumps. The liquidity measures are quoted spread IL^s, effective spread IL^e and price impact IL^p. There are 488 observations from January 2008 to December 2009. Standard errors in parentheses. ***, ** and * denote significant level 1%, 5% and 10% level respectively.

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