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MATH 185 – HW2

Professor Xu

Q1.

a)

Table I : Binomial simulation with p=0.3

| | Size Parameter | | | |
|---|---|---|---|---|
| | 1 | 2 | 5 | 10 |
| Mean | 0.00 | 0.25 | 0.32 | 0.32 |
| Sample Variance | N/A | 0.125 | 0.052 | 0.017 |
| $E(\hat{p})$ | 0.3 | | | |
| $Var(\hat{p})$ | 0.21 | 0.105 | 0.042 | 0.021 |

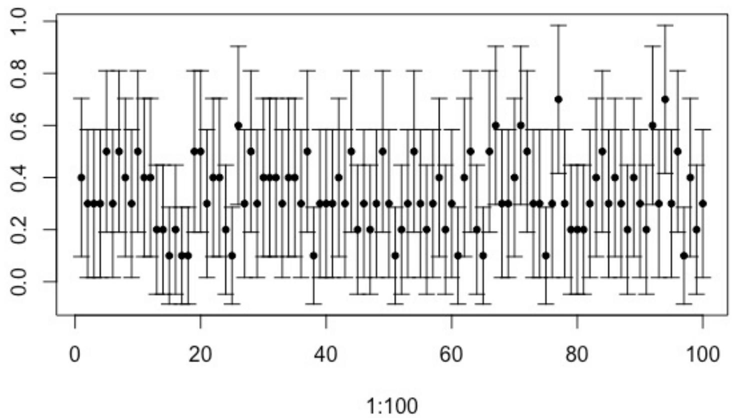
**Note.** Sample variance does not apply when there is only 1 observation, requires at least 2

b)

We observe the difference between the means decreases (though weakly, since size of 10 and 5 gives the same mean) as sample size gets larger. And the difference between variances decreases in absolute terms, since the variance is inversely related to the sample size n.

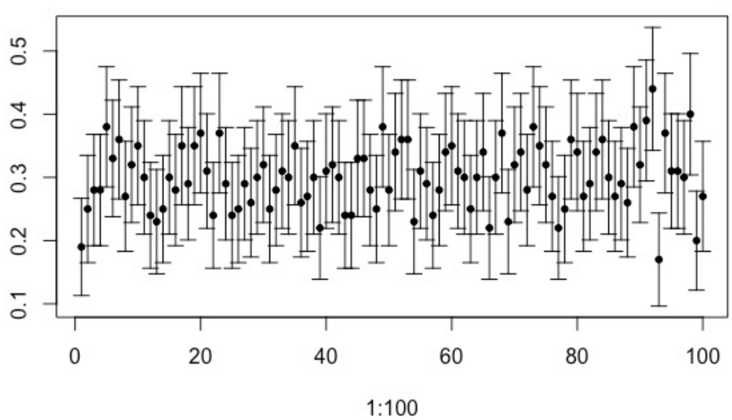
Q2.

a)




Coverge porbability = 88%

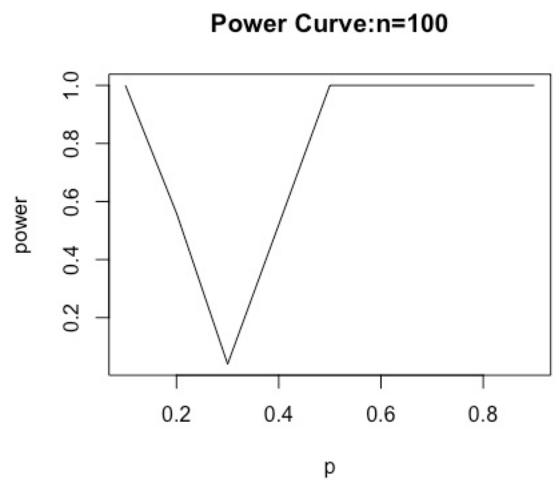
b)



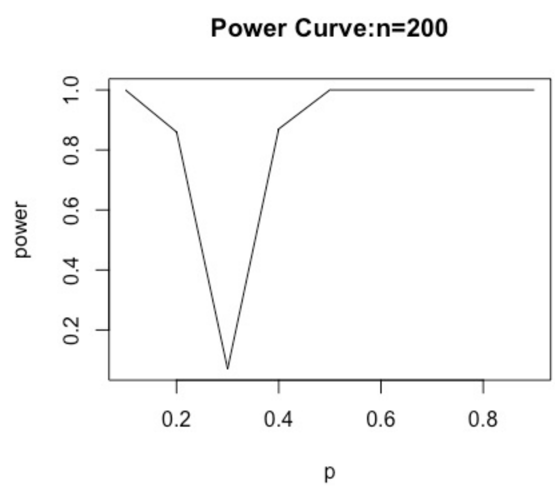

Coverage probability = 95%

c) we have better coverage probability, that's closer to the theoretical value with larger n. That's supposed to be due to the improved asymptotic normality when n gets larger.

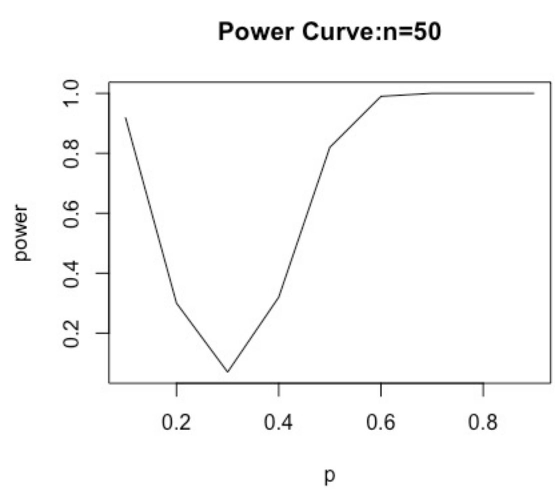
Q3.



Power Curve:n=100
power
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
p

Power Curve:n=200
power
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
p

Power Curve:n=50
power
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
p

Q4.

a)

$$P(X_i = k) = \frac{e^{-\lambda}\lambda^k}{k!}\text{, } want\ to\ find\ \max(L(\lambda))$$
$$equivalently,\ to\ find\ \max(\ln L(\lambda))$$
$$L(\lambda; X_1,\ X_2,\ ...,\ X_n) = \prod_{i=1}^{n} P(x_i;\lambda)$$
$$\ln(L(\lambda)) = \ln\prod_{i=1}^{n} P(x_i;\lambda)$$
$$\ln\prod_{i=1}^{n}\frac{e^{-\lambda}\lambda^{x_i}}{x_i!} = \sum_{i=1}^{n}\ln\frac{e^{-\lambda}\lambda^{x_i}}{x_i!} = \sum_{i=1}^{n}\ln e^{-\lambda} + \sum_{i=1}^{n}\ln\lambda^{x_i} - \sum_{i=1}^{n}\ln x_i\ (*)$$
$$maximization\ with\ respect\ to\ \lambda\ by\ letting\ \frac{d(\ln L(\lambda))}{d\lambda} = 0$$
$$differentiate\ each\ term\ in\ (*),\ we\ have: \frac{d\ln e^{-\lambda}}{d\lambda}\sum_{i=1}^{n}1 + \frac{d\ln\lambda}{d\lambda}\sum_{i=1}^{n}x_i - 0 = 0$$
$$-1\sum_{i=1}^{n}1 + \frac{1}{\lambda}\sum_{i=1}^{n}x_i = 0$$
$$-n + \frac{1}{\lambda}n\bar{x} = 0$$
$$finally\ MLE: \hat{\lambda} = \bar{x}$$
$$we\ got\ the\ first\ derivative\ , differentiate\ again\ to\ get: \frac{d^2\ln L}{d\lambda^2} = -\frac{1}{\lambda^2}\sum_{i=1}^{n}x_i$$
$$observed\ fisher\ information\ by\ definition: \hat{Var}(\hat{\lambda}) = -\{-\frac{1}{\lambda^2}\sum_{i=1}^{n}x_i\}^{-1} = \frac{n}{\bar{x}}$$

b)

Normal Approximation 95% CI: (8.874, 10.605)

Variance-stabilized 95% CI: (8.894, 10.624)

c)

Normal Approximation 95% CI coverage rate: 95.6%

Variance-stabilization 95% CI coverage rate: 95.8%

For a given simulation trial, i.e. a fixed sample mean, the lengths are the same by their definitions since the widths are computed in the same way. In my simulation, the overage rate is slightly higher for the variance-stabilized interval, but both are close to the theoretical 95%.