Redshit-space distortions

Coordinates

- Until now we have calculated $\delta_{dm}(\mathbf{x},\eta)$
- In Fourier space $\delta_{dm}(\mathbf{k},\eta) = \int d^3\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}}\, \delta_{dm}(\mathbf{x},\eta)$
- The **position** of a galaxy $\mathbf{x}$ is given by its direction and its distance $\mathbf{x} = (\mathbf{n}, r)$ with $r = \eta_0 - \eta$
- In a survey we do not measure r but we measure the **redshift** z
- We calculate the radial distance from the redshift.
- Photons travel on **null geodesics**

$$1 + z = \frac{a_0}{a} = \frac{1}{a}$$

$$dr = -d\eta = -\frac{d\eta}{da}\frac{da}{dz}dz = \frac{1}{a'}\frac{1}{(1+z)^2}dz = \frac{a}{\mathcal{H}}dz$$

Coordinates

- Radial distance $r(z) = \int_0^z dz' \frac{1}{(1+z')\mathcal{H}(z')}$

- The relation depends on the **cosmology** (from SNe, CMB).

- **Problem**: the above relation between redshift and radial distance is only correct in a **homogeneous** universe, where the redshift is entirely due to the expansion of the universe:

$$1 + z = \frac{1}{a}$$

- In a universe with **fluctuations**, the redshift is affected by other **effects**.

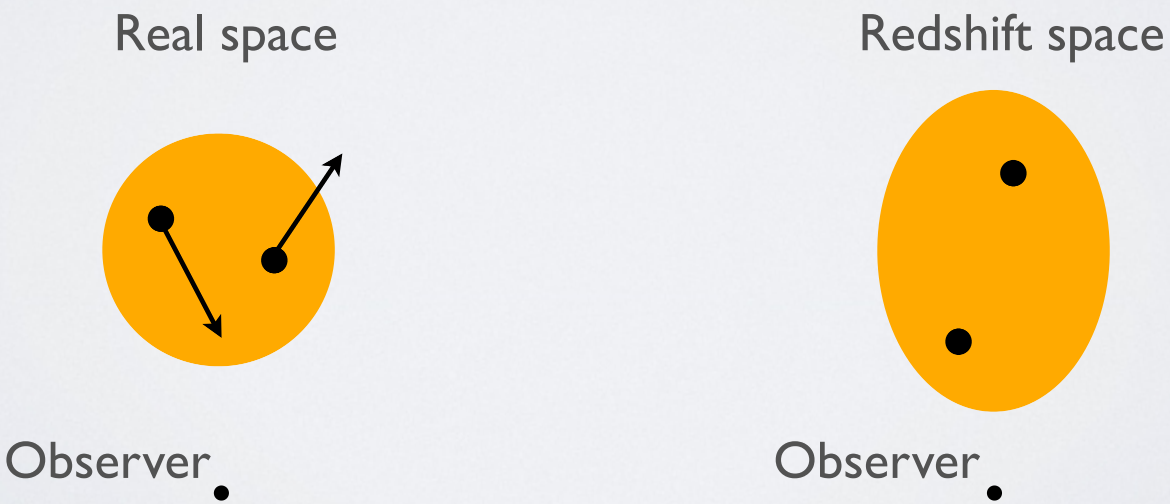
Doppler effect

- **Inhomogeneous** universe: galaxies are attracted towards over-dense regions.
- The **motion** of galaxies with respect to us induces a **Doppler** shift.
- We use the relation $r(z)=\int_0^z dz'\frac{1}{(1+z')\mathcal{H}(z')}$ and infer a slightly **wrong position**.
- Consequence: this changes the **observed large-scale structure**, e.g. the shape over-densities.

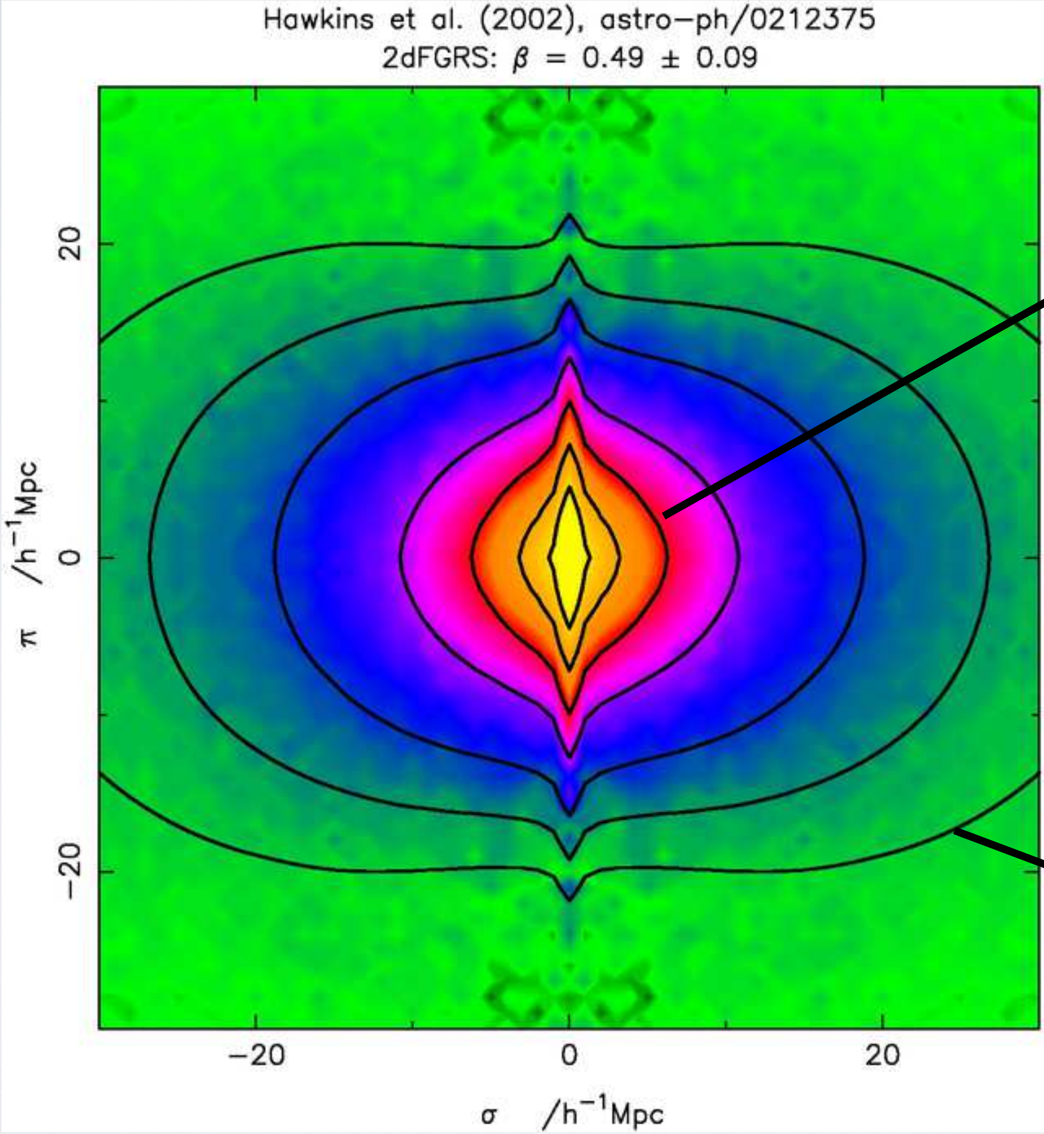
Distortions

Linear regime: over-densities are **squeezed** along the line of sight.

Non-linear regime: virialised objects (clusters) are **elongated**, fingers of god effect.



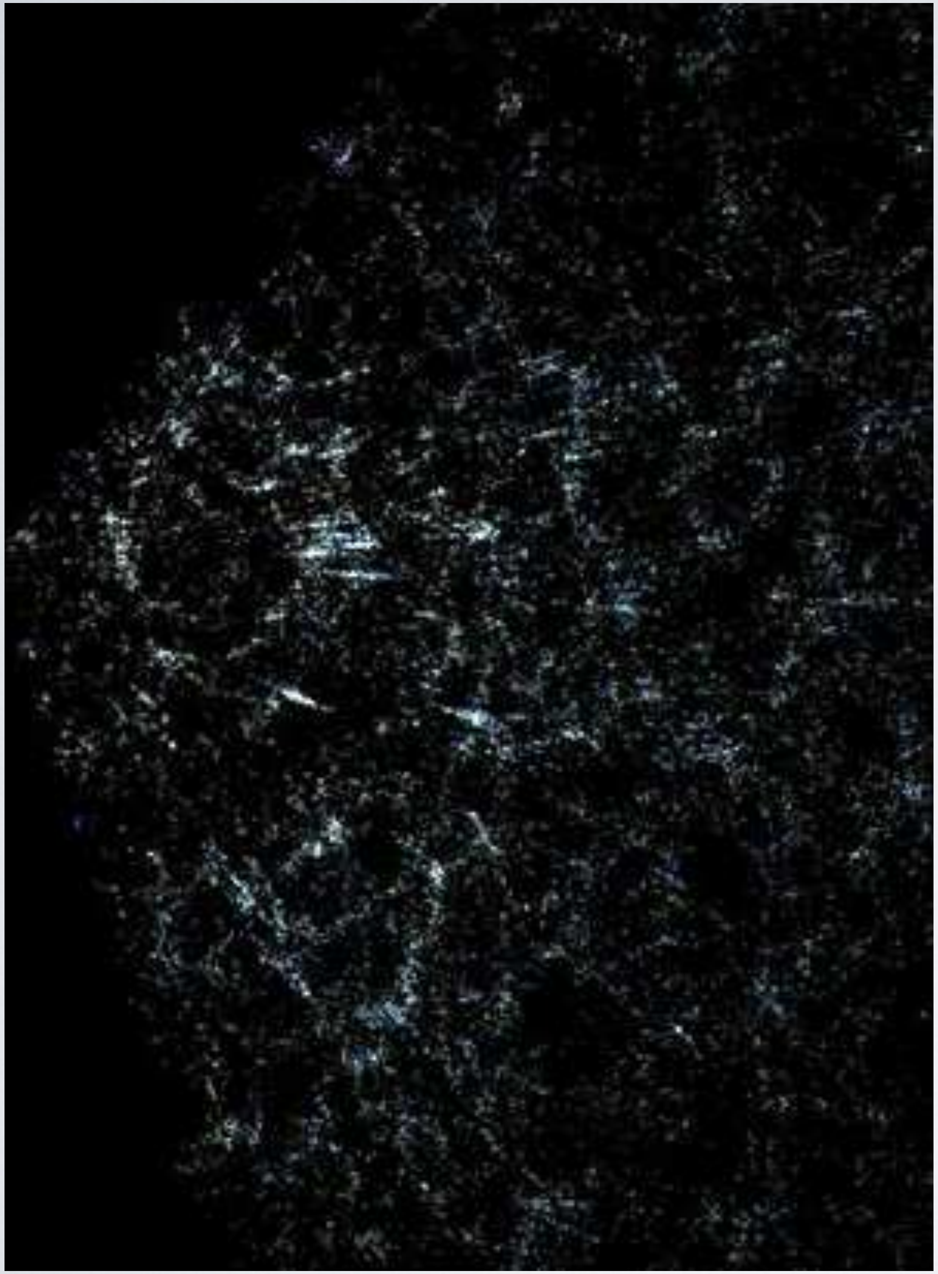
Contours



non-linear elongation

linear squeeze

Fingers of god



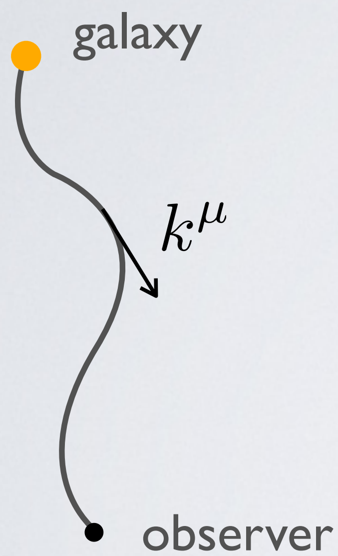
Fingers of God in a portion of the Sloan Digital Sky Survey.
Image from the Cosmus Open Source Science Outreach project.

The **non-linear** effect can be seen by eyes.

The **linear** redshift-space distortion is statistically detectable in the correlation function.

Redshift

$$1 + z = \frac{\nu_S}{\nu_O} = \frac{E_S}{E_O} = \frac{(k^\mu u_\mu)_S}{(k^\mu u_\mu)_O}$$



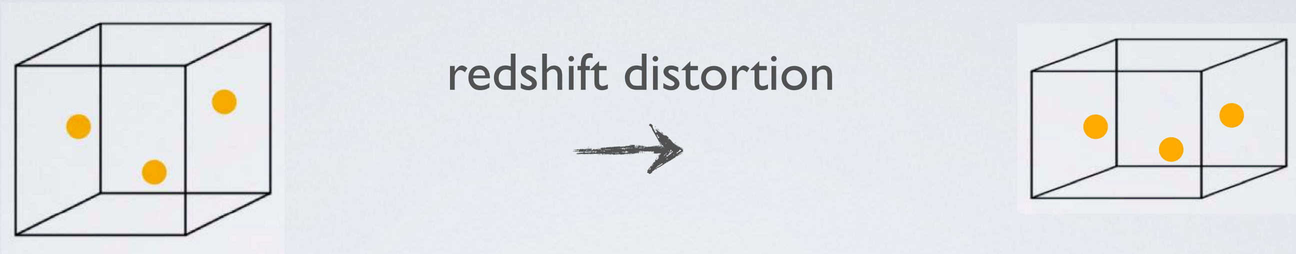
k^μ photon momentum

u^μ 4-velocity of the source and observer

Galaxy distribution

How do the redshift fluctuations affect the observation of δ ?

We extract the number of galaxies per **volume element**.



Number of galaxies is **conserved**: $\rho(\mathbf{x}_{\rm obs})\, d^3\mathbf{x}_{\rm obs} = \rho(\mathbf{x})\, d^3\mathbf{x}$

$$\bar\rho\big(1+\delta_{\rm obs}\big)\, d^3\mathbf{x}_{\rm obs} = \bar\rho\big(1+\delta\big)\, d^3\mathbf{x}$$

The change in $\delta_{\rm obs}$ is due to the change from $\mathbf{x}$ to $\mathbf{x}_{\rm obs}$

Only the **radial coordinate** is affected by redshift perturbations

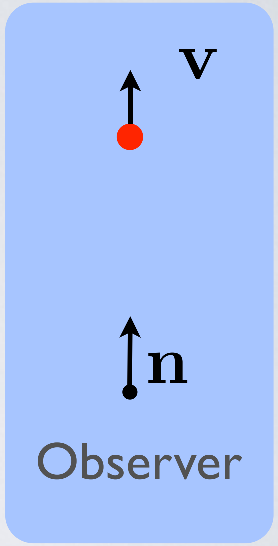
$$r_{\rm obs} = r(z) = r(\bar z + \delta z) \simeq r(\bar z) + \frac{\partial r}{\partial \bar z}\,\delta z$$

Galaxy over-density

$$r_{\rm obs} = r + \frac{\partial r}{\partial \bar z}\,\delta z \qquad\qquad \frac{\partial r}{\partial \bar z} = \frac{1}{(1+\bar z)\mathcal{H}}$$

We keep only the Doppler contribution: $r_{\rm obs} = r + \frac{1}{\mathcal{H}}\mathbf{v}\cdot\mathbf{n}$

Jacobian: $$\frac{\partial r_{\rm obs}}{\partial r} = 1 + \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n}) + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\mathbf{v}\cdot\mathbf{n}$$



Neglecting the second term:

$$(1+\delta_{\rm obs})\left[1 + \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n})\right]d^3\mathbf{x} = (1+\delta)d^3\mathbf{x}$$

$$\boxed{\delta_{\rm obs} = \delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n})}$$

Kaiser (1987)

Galaxy over-density

$$r_{\rm obs} = r + \frac{\partial r}{\partial \bar z}\,\delta z \qquad\qquad \frac{\partial r}{\partial \bar z} = \frac{1}{(1+\bar z)\mathcal{H}}$$

We keep only the **Doppler** contribution: $r_{\rm obs} = r + \frac{1}{\mathcal{H}}\mathbf{v}\cdot\mathbf{n}$

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Neglecting the second term:

$$(1+\delta_{\rm obs})\left[1+\frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n})\right]d^3\mathbf{x} =$$

We see a distorted distribution of galaxies because we are not able to measure distances directly. How problematic is that?

$$\boxed{\delta_{\rm obs} = \delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n})}$$

Kaiser (1987)

Interest of redshift distortions

Redshift distortions provides an opportunity to measure **peculiar velocities**. Galaxies move according to dark matter inhomogeneities $\rightarrow$ another way of mapping the matter distribution.

We already know the peculiar velocities from **conservation** equation:

$$\delta' = kv$$

- We want to **test** this equation
- Velocities measure directly the **evolution** of the density. More sensitive to modified gravity.
- Peculiar velocities are not sensitive to **bias**:

$$\delta = b \cdot \delta_{dm} \qquad \text{but} \qquad v = v_{dm}$$

Anisotropy

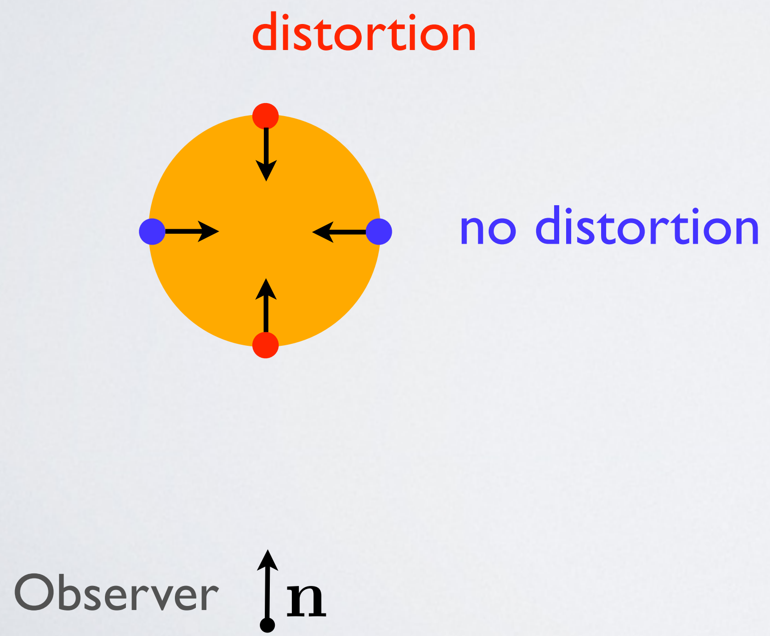
How do we measure redshift distortions and separate velocities from density?

The velocity part is **anisotropic**

line of sight

$$\delta_{\rm obs} = \delta - \frac{1}{\mathcal{H}}\partial_r(\mathbf{v}\cdot\mathbf{n})$$

We expect differences along and transverse to the **line-of-sight.**



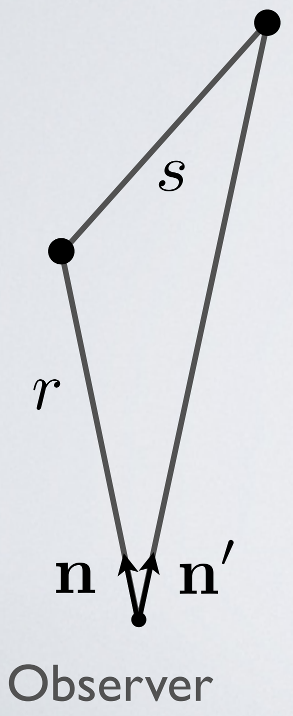
We can detect this anisotropy **statistically** in the correlation function.

Anisotropy

Two-point correlation function:

$$\xi = \left\langle \left(\delta(\mathbf{x},\eta) - \frac{1}{\mathcal{H}}\partial_r \mathbf{v}(\mathbf{x},\eta)\cdot\mathbf{n}\right)\left(\delta(\mathbf{x}',\eta') - \frac{1}{\mathcal{H}}\partial_{r'} \mathbf{v}(\mathbf{x}',\eta')\cdot\mathbf{n}'\right)\right\rangle$$

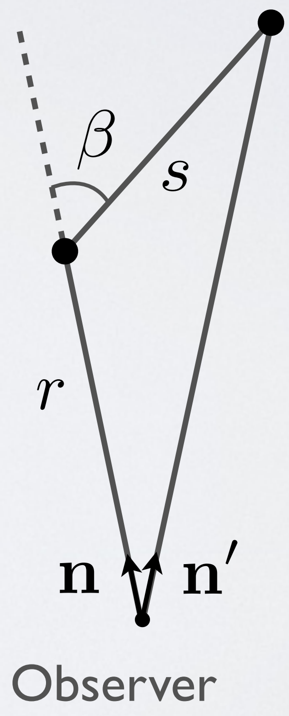
Without distortion: $\xi(s, r)$



Depends on:

- separation
- distance of the pair

With distortion: $\xi(s, r, \beta)$



Additional dependence on orientation:

max signal: $\beta = 0,\ \pi$

min signal: $\beta = \dfrac{\pi}{2}$

Anisotropy

Two-point correlation function:

$$\xi = \left\langle \left(\delta(\mathbf{x},\eta) - \frac{1}{\mathcal{H}}\partial_r \mathbf{v}(\mathbf{x},\eta)\cdot\mathbf{n}\right)\left(\delta(\mathbf{x}',\eta') - \frac{1}{\mathcal{H}}\partial_{r'} \mathbf{v}(\mathbf{x}',\eta')\cdot\mathbf{n}'\right)\right\rangle$$

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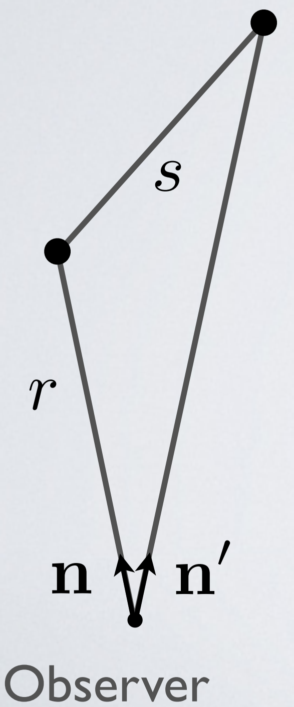

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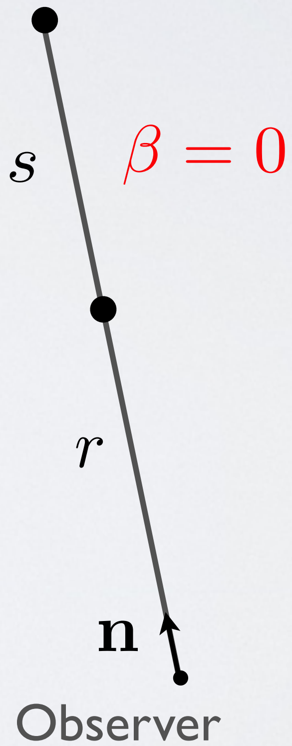

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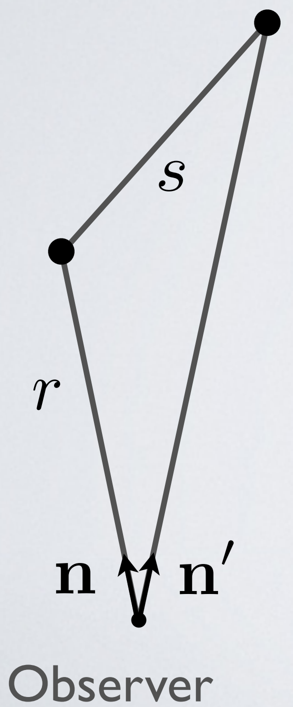
Anisotropy

Two-point correlation function:

$$\xi = \left\langle \left(\delta(\mathbf{x}, \eta) - \frac{1}{\mathcal{H}} \partial_r \mathbf{v}(\mathbf{x}, \eta) \cdot \mathbf{n} \right) \left(\delta(\mathbf{x}', \eta') - \frac{1}{\mathcal{H}} \partial_{r'} \mathbf{v}(\mathbf{x}', \eta') \cdot \mathbf{n}' \right) \right\rangle$$

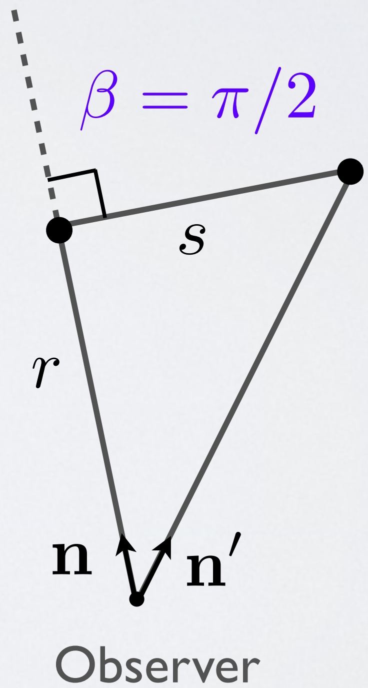
Without distortion: $\xi(s, r)$



Depends on:

- separation
- distance of the pair

With distortion: $\xi(s, r, \beta)$



Additional dependence on orientation:

max signal: $\beta = 0, \ \pi$

min signal: $\beta = \frac{\pi}{2}$

Result

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta) \right\}$$

Hamilton (1992)

$$\mu_\ell(s) = \frac{A}{2\pi^2}\int \frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta^2(k)\, j_\ell(k\cdot s)$$

sets the shape of the correlation as a function of separation.

other terms: ♦ cross-terms **density-velocity** proportional to f

♦ **velocity-velocity** terms proportional to f^2

with $f = \frac{a}{D_1}\frac{d}{da}D_1$

Result

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta)\right\}$$

growth function

Hamilton (1992)

primordial amplitude

$$\mu_0(s) = \frac{A}{2\pi^2}\int\frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta^2(k)\, j_0(k\cdot s)$$

sets the shape of the correlation as a function of separation.

transfer function

separation

other terms: ♦ cross-terms **density-velocity** proportional to f

♦ **velocity-velocity** terms proportional to f^2

with $$f = \frac{a}{D_1}\frac{d}{da}D_1$$

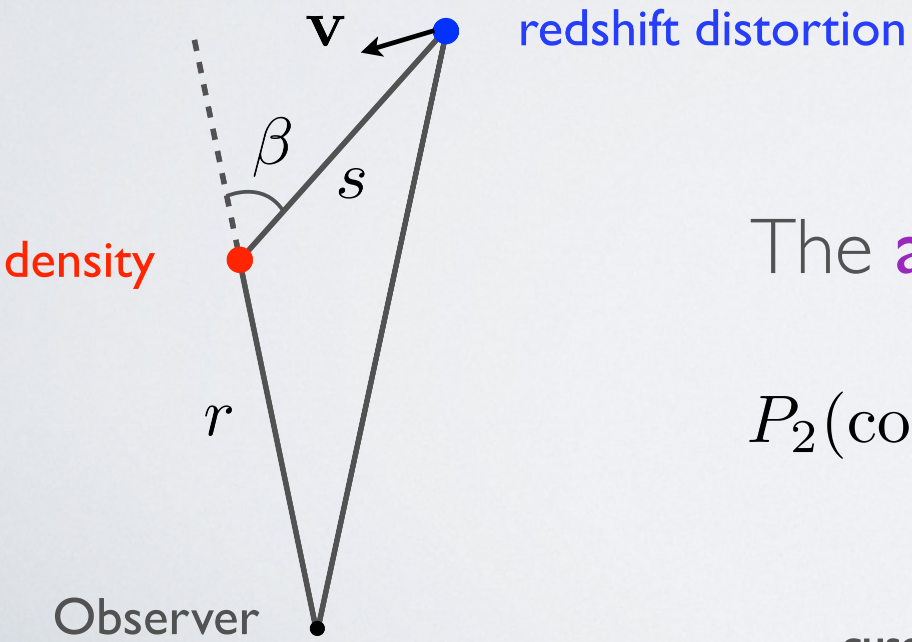
Result

$$\xi = D_1^2\left\{\left(1+\frac{2f}{3}+\frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3}+\frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta)\right\}$$

Hamilton (1992)

$$\mu_2(s) = \frac{A}{2\pi^2}\int\frac{dk}{k}\left(\frac{k}{H_0}\right)^{n_s-1} T_\delta^2(k)\, j_2(k\cdot s)$$

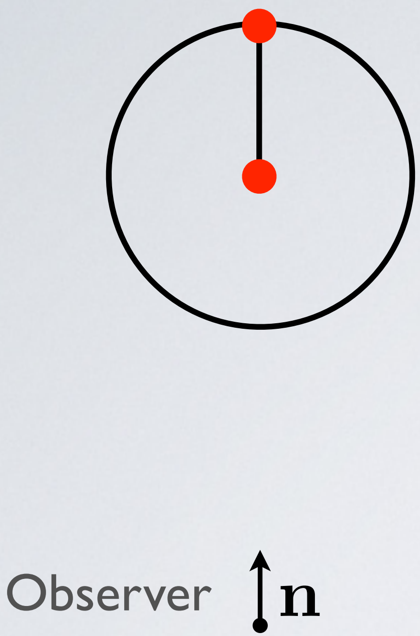
slightly different dependence in separation than the density

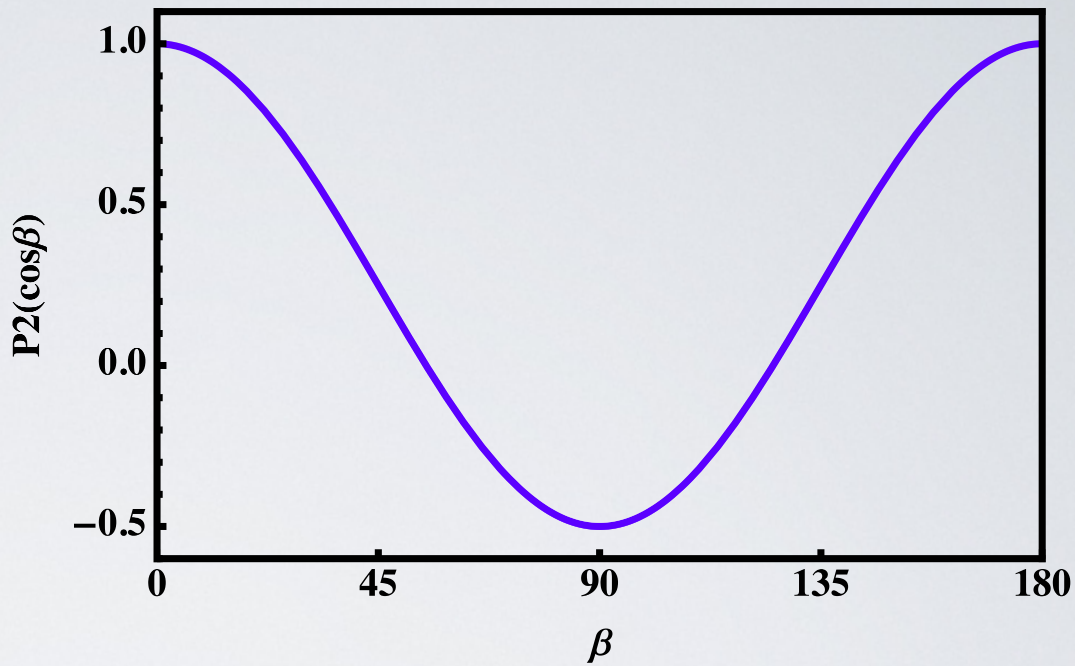


The angular dependence is given by:

$$P_2(\cos\beta) = \frac{3}{2}\cos^2\beta - \frac{1}{2}$$

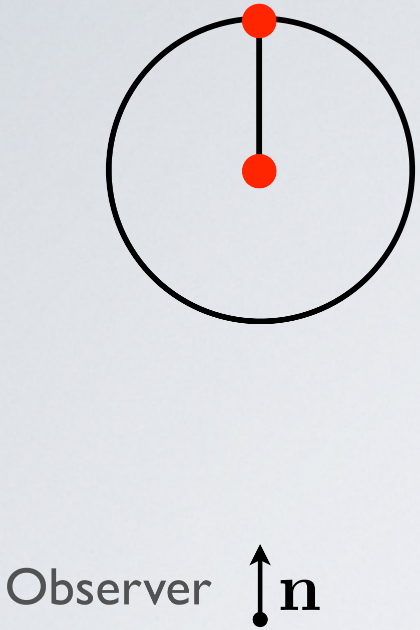
Quadrupole dependence

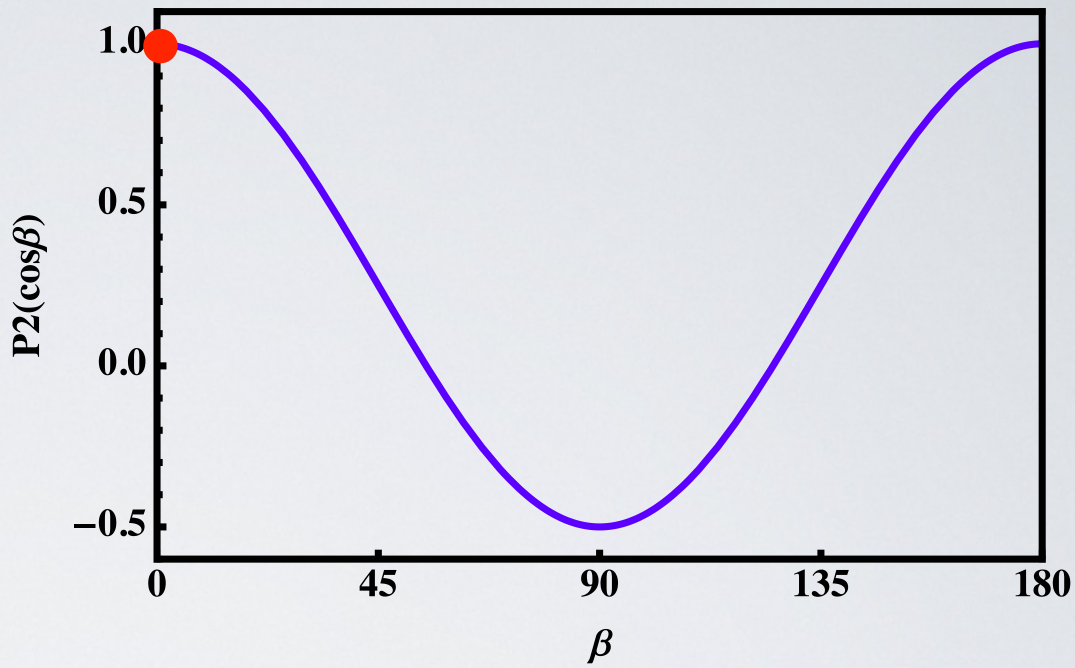




- The amplitude of the correlation function is modulated by $P_2(\cos\beta)$
- The quadrupole is **negative**: $-\dfrac{4f}{3}$

Quadrupole dependence





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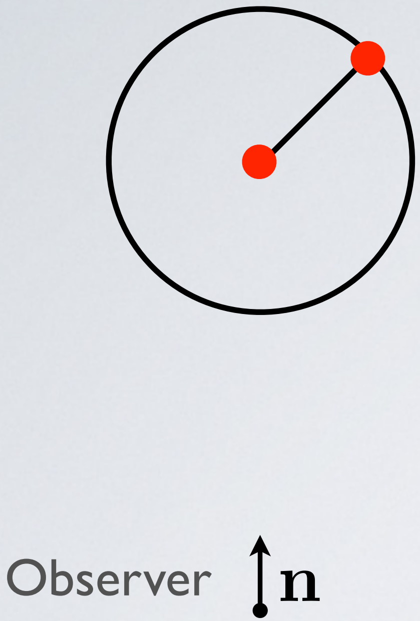

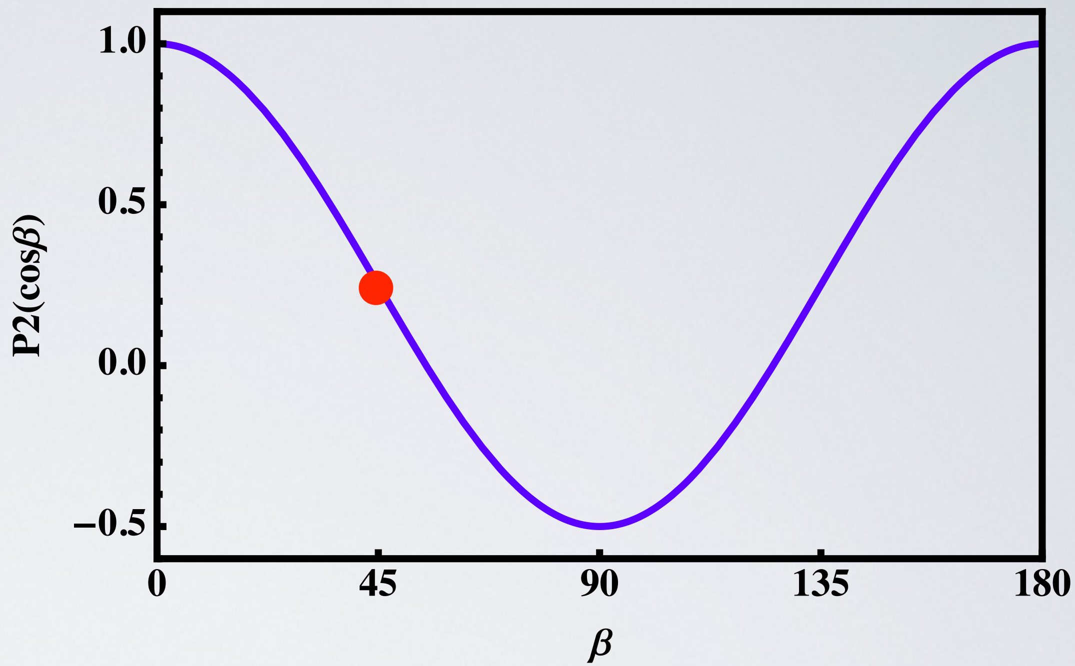

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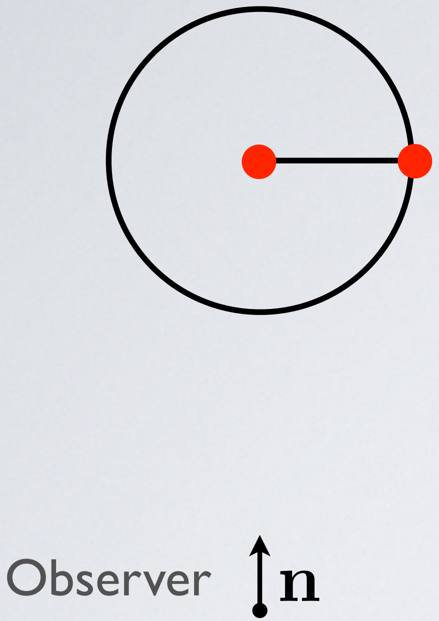

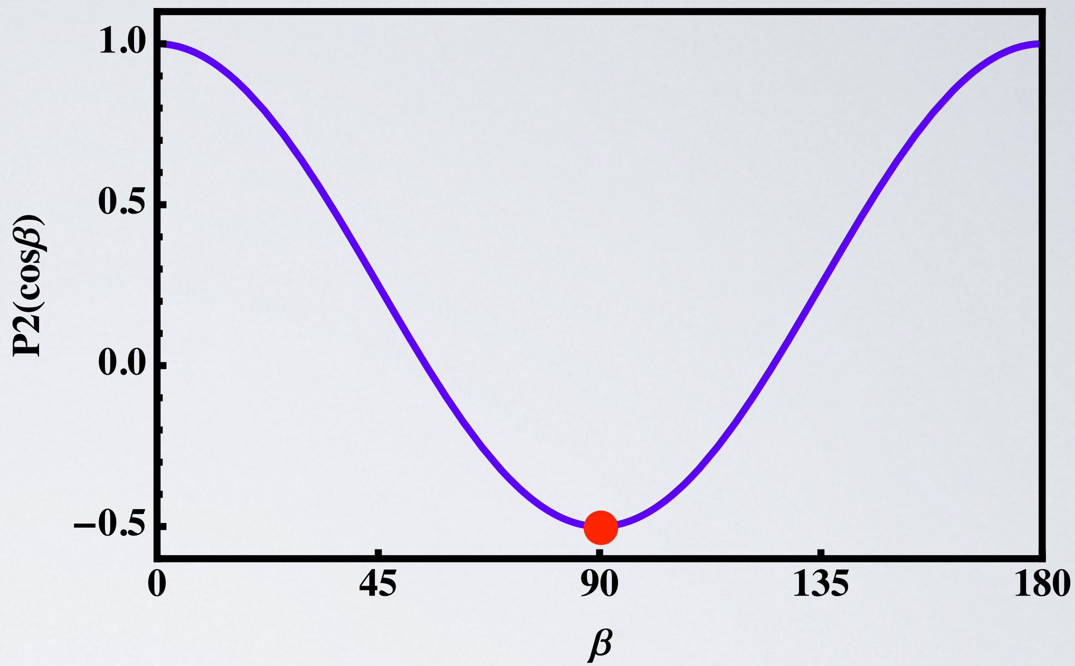

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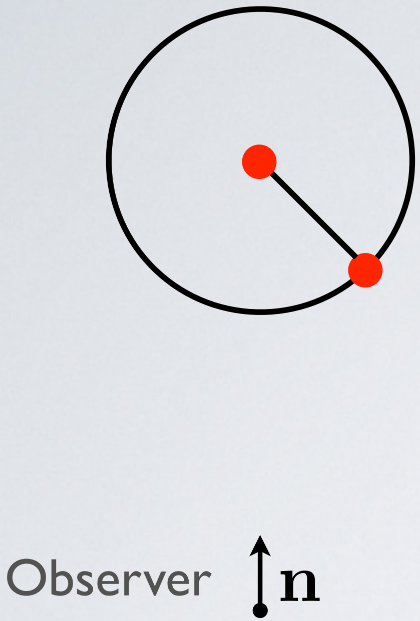
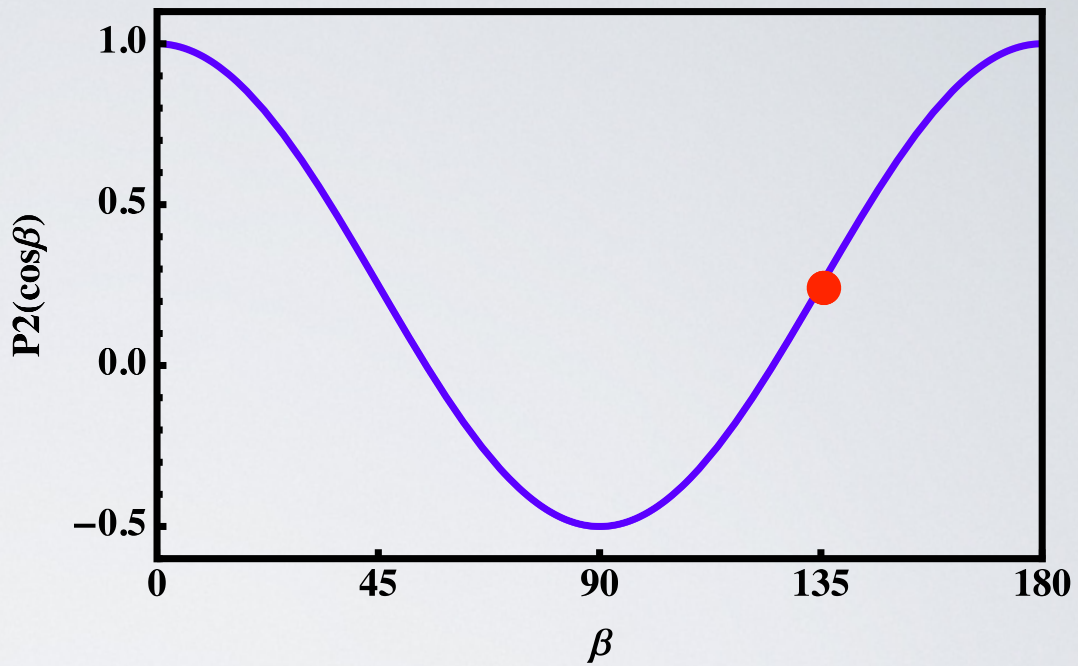
Quadrupole dependence





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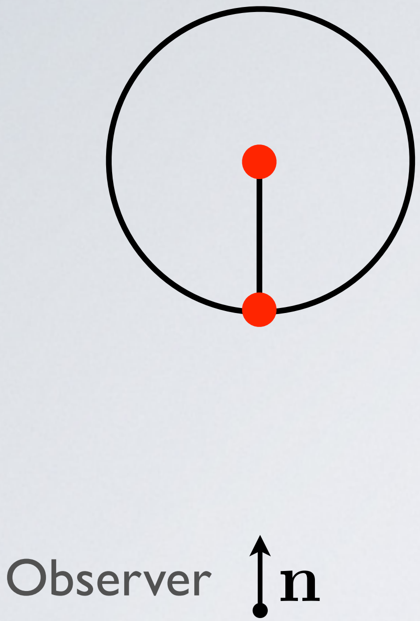
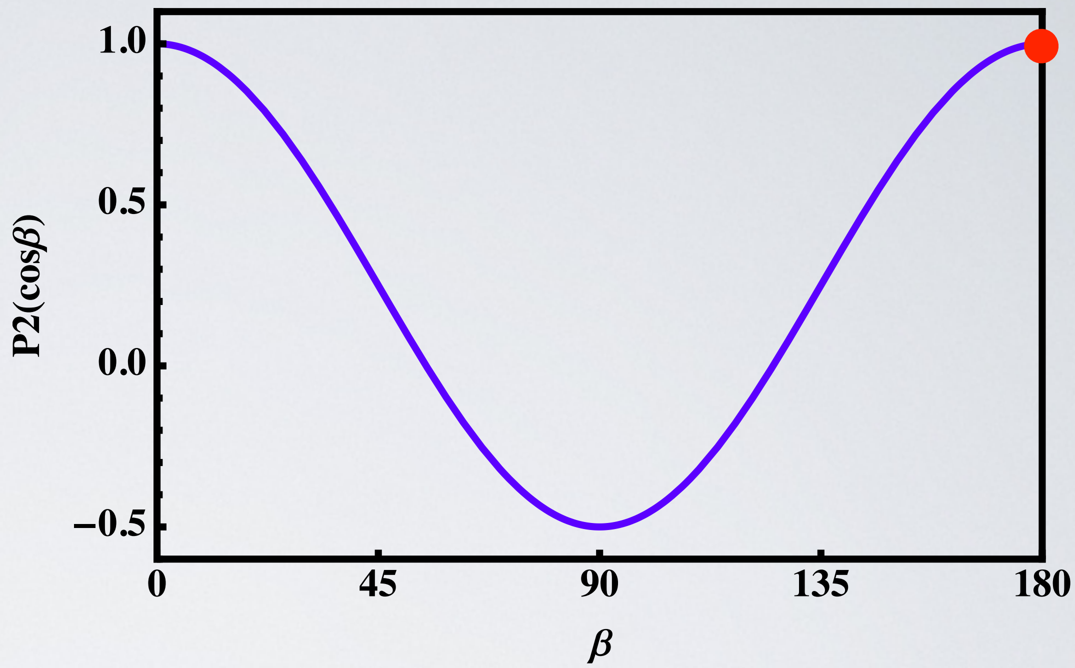
Quadrupole dependence





- The amplitude of the correlation function is modulated by $P_2(\cos\beta)$
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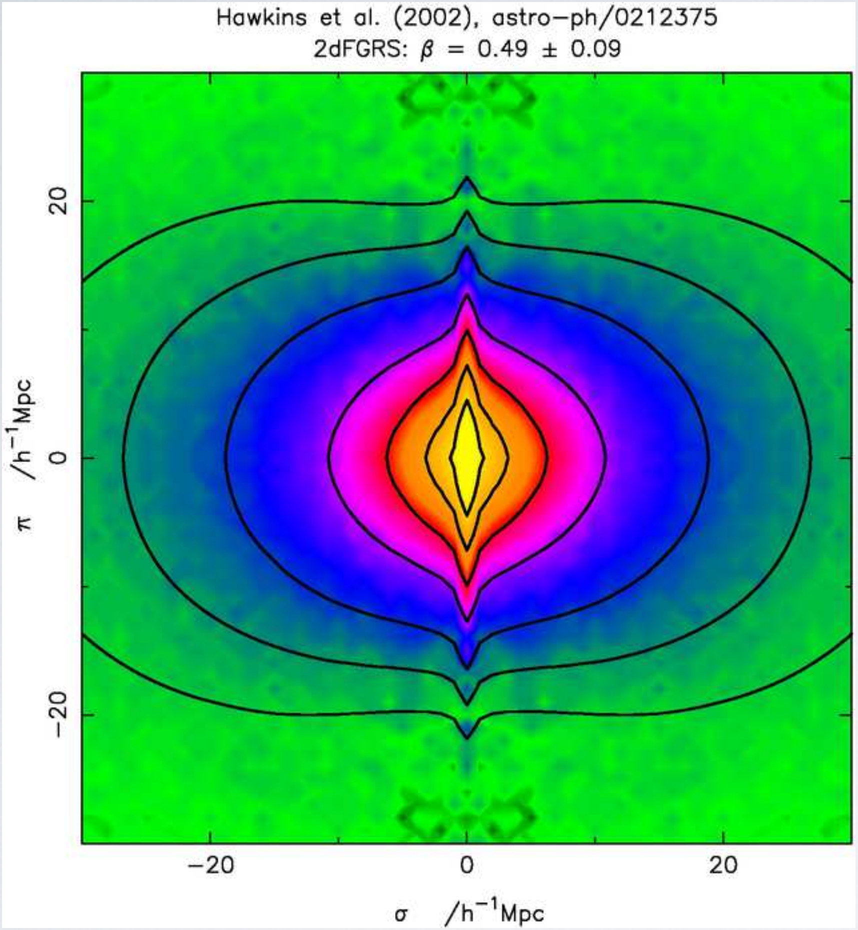
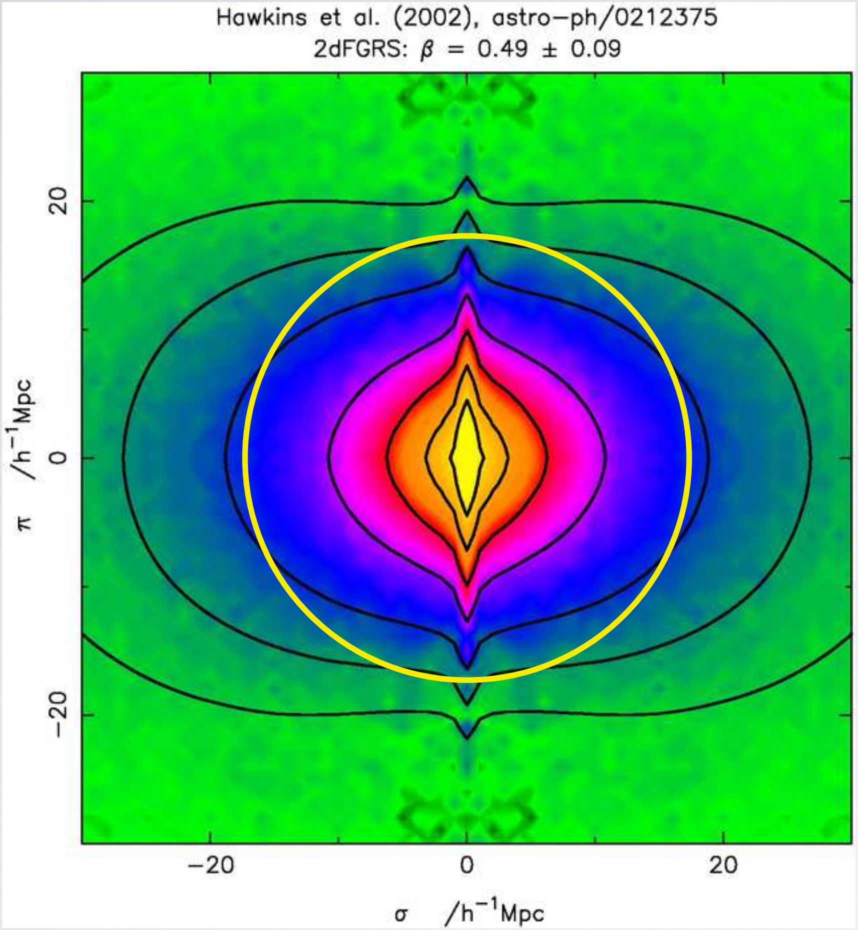
Negative quadrupole



Redshift distortions increase the **gradient** along the line-of-sight.

At a given separation, the correlation is **stronger transverse** to the line-of-sight than along the line-of-sight → negative quadrupole.

Negative quadrupole



Redshift distortions increase the **gradient** along the line-of-sight.

At a given separation, the correlation is **stronger transverse** to the line-of-sight than along the line-of-sight → negative quadrupole.

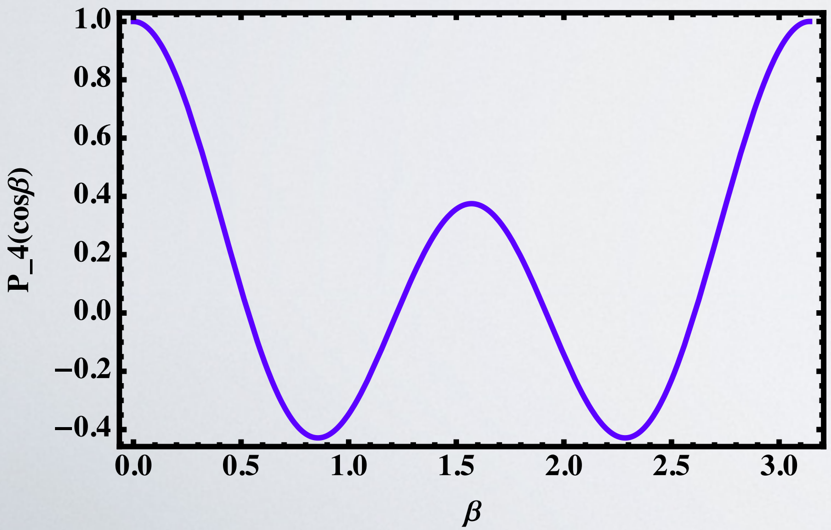
Hexadecapole dependence

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta) \right.$$
$$\left. + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta) \right\}$$

Hamilton (1992)

Other terms: velocity-velocity correlations contribute to the quadrupole and generate an **hexadecapole**.

$$P_4(\cos\beta) = \frac{1}{8}\left[35\cos^4\beta - 30\cos^2\beta + 3\right]$$



Maximum at $\beta = 0$ and π

Velocity-density decreases monotonically.

Velocity-velocity have a complicated structure due to a combination of $\cos^2\beta$

Multipoles extraction

How can we **separate** redshift distortions from density?

We can use the particular **angular dependence** of the terms.

We **average** over all orientations: $\frac{1}{2}\int_{-1}^{1} d\mu\, \xi(s,r,\mu)$

$$\int_{-1}^{1} d\mu\; P_2(\mu) = 0 \quad \text{and} \quad \int_{-1}^{1} d\mu\; P_4(\mu) = 0$$

→ extract the monopole

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s) P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s) P_4(\cos\beta) \right\}$$

Multipoles extraction

$$\xi = D_1^2 \left\{ \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right)\mu_0(s) - \left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)P_2(\cos\beta) + \frac{8f^2}{35}\mu_4(s)P_4(\cos\beta) \right\}$$

Hamilton (1992)

- To extract the **quadrupole** we weight by $P_2(\mu)$

$$\frac{5}{2}\int_{-1}^{1} d\mu\, \xi(s,r,\mu)P_2(\mu) = -D_1^2\left(\frac{4f}{3} + \frac{4f^2}{7}\right)\mu_2(s)$$

- To extract the **hexadecapole** we weight by $P_4(\mu)$

$$\frac{9}{2}\int_{-1}^{1} d\mu\, \xi(s,r,\mu)P_4(\mu) = D_1^2\,\frac{8f^2}{35}\,\mu_4(s)$$

Measure f

Bias

Fluctuations in the number of **galaxies** are biased with respect to the **dark matter** fluctuations: $\delta = b \cdot \delta_{dm}$

$$\xi = D_1^2 \left\{ \left(b^2 + \frac{2bf}{3} + \frac{f^2}{5} \right) \mu_0(s) - \left(\frac{4bf}{3} + \frac{4f^2}{7} \right) \mu_2(s) P_2(\cos\beta) + \frac{8f^2}{35} \mu_4(s) P_4(\cos\beta) \right\}$$

The monopole and quadrupole are affected by bias, but the hexadecapole is not. This reflects the fact that the **velocities** are **not biased** $v = v_{dm}$

By measuring all multipoles we can measure both b and f

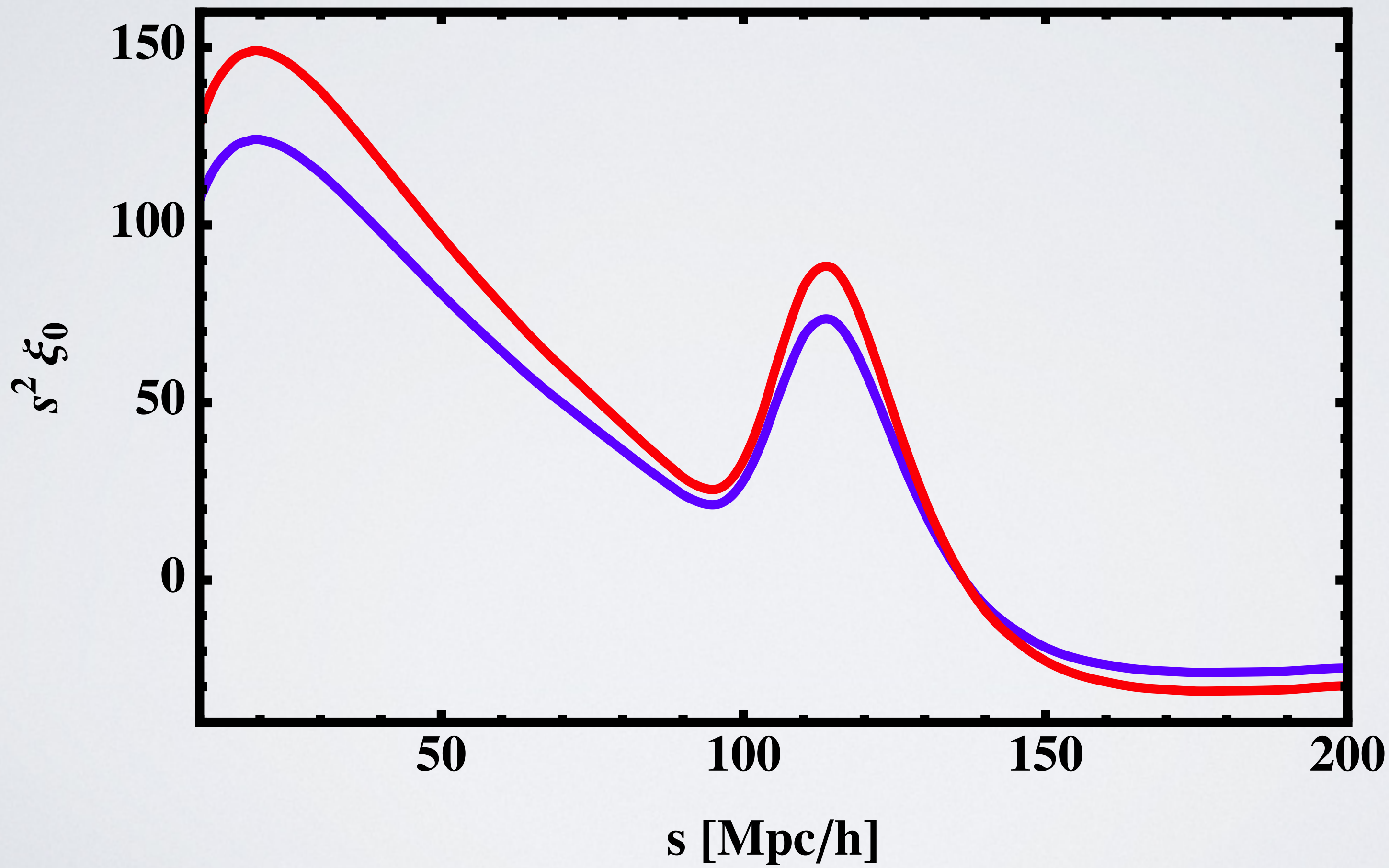
Results: monopole

without redshift distortions

$$\xi_0 = D_1^2 \; b^2 \, \mu_0(s)$$

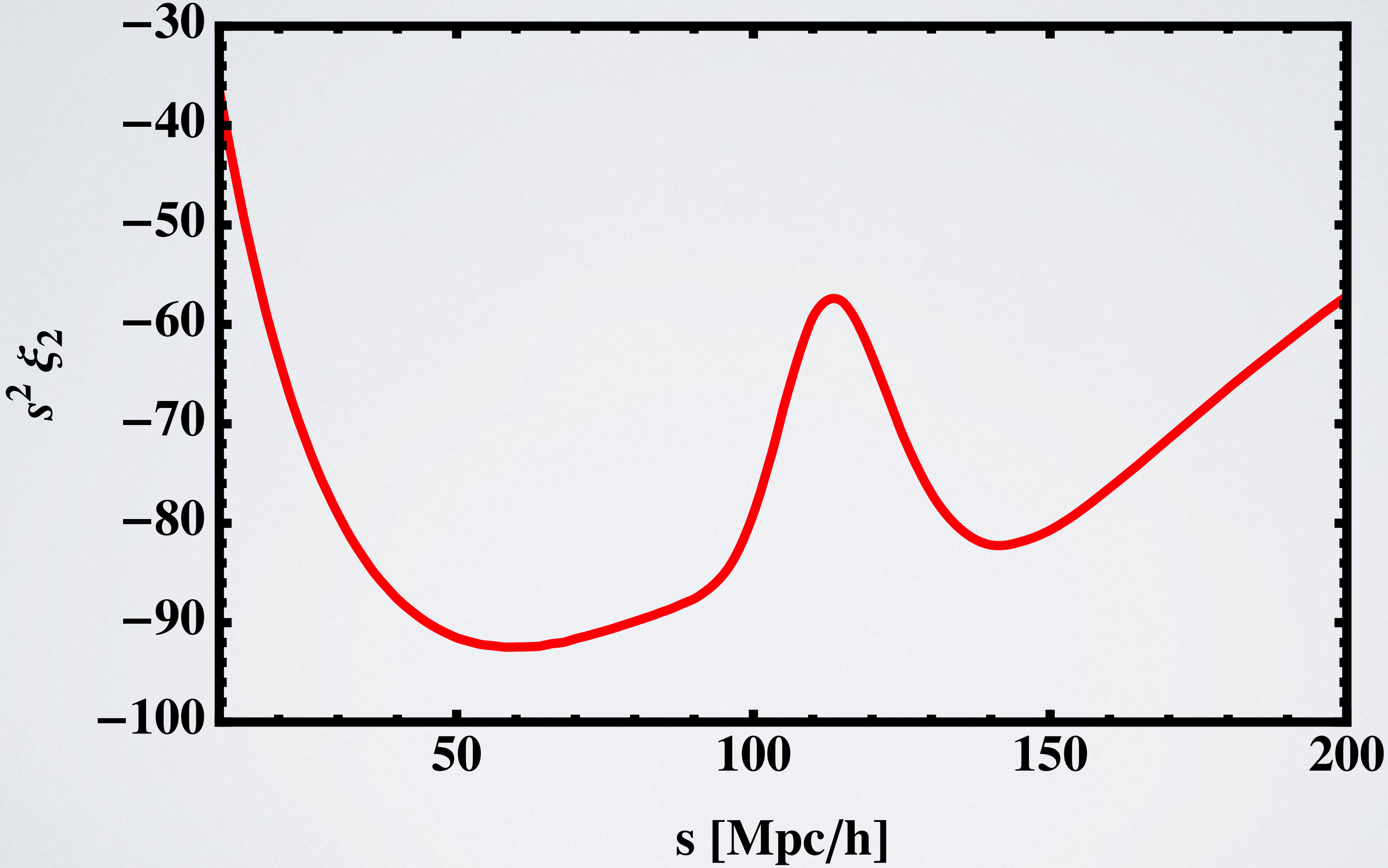
with redshift distortions

$$\xi_0 = D_1^2 \left(b^2 + \frac{2bf}{3} + \frac{f^2}{5} \right) \mu_0(s)$$



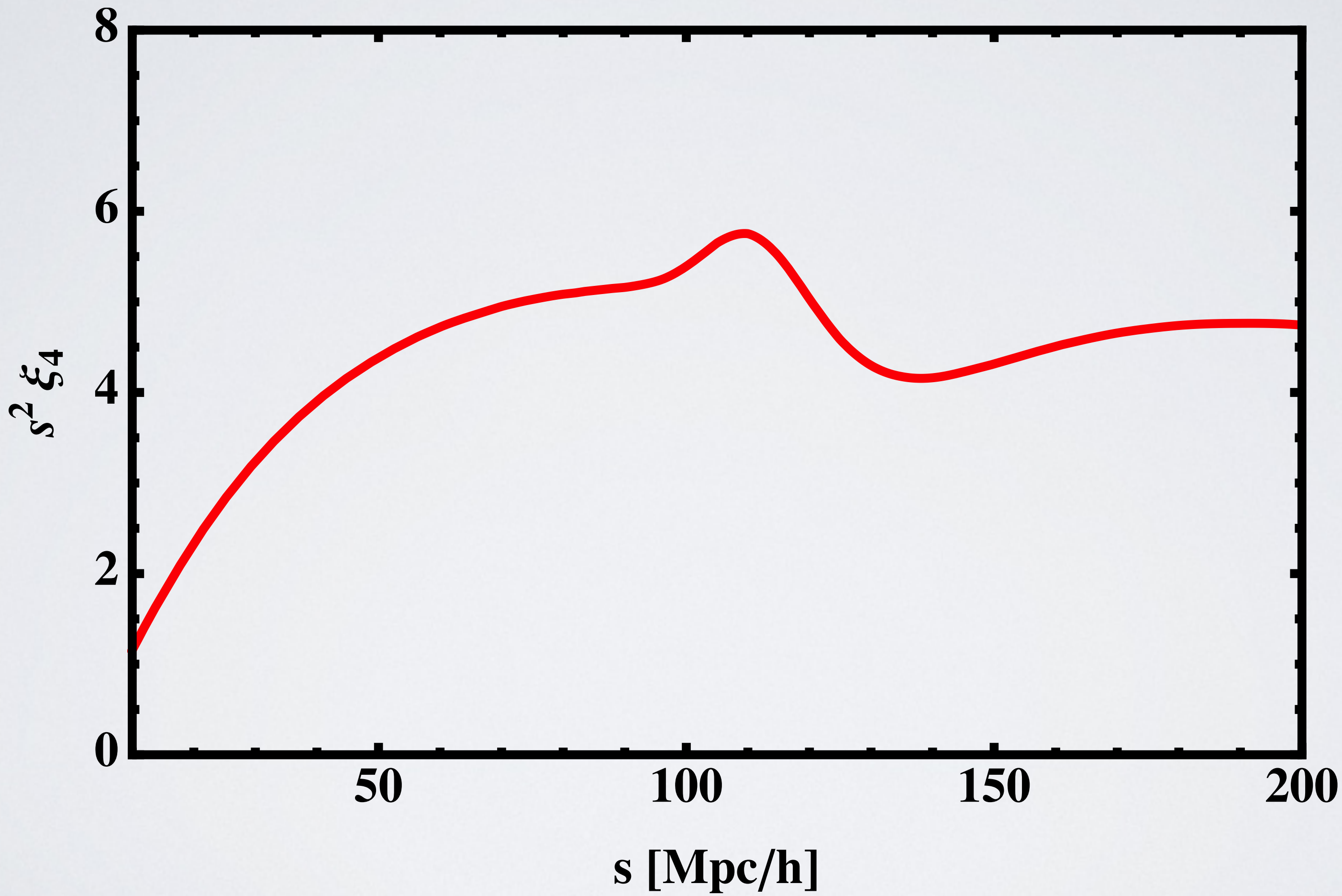
Results: quadrupole

$$\xi_2 = -D_1^2\left(\frac{4bf}{3}+\frac{4f^2}{7}\right)\mu_2(s)$$



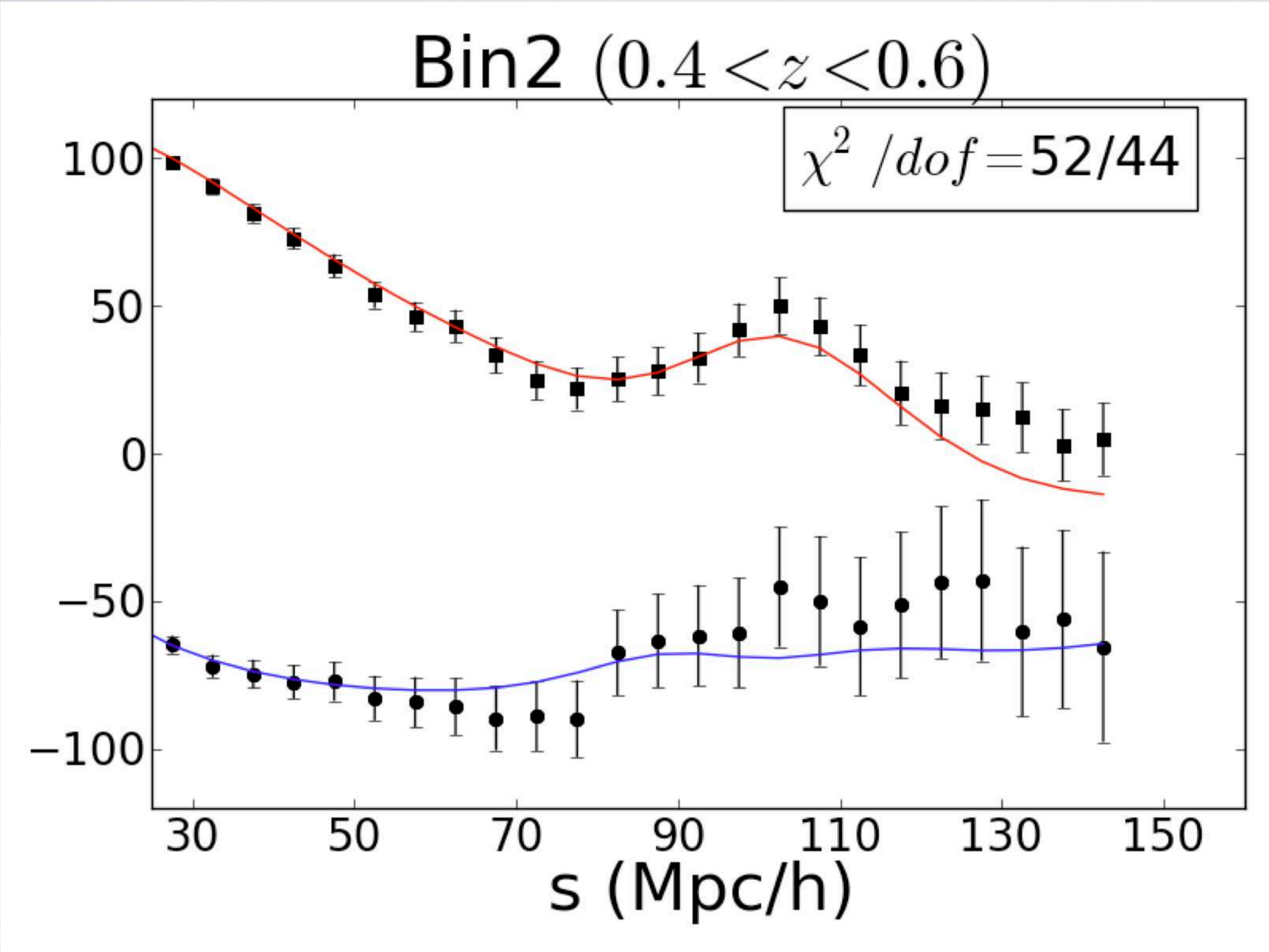
Results: hexadecapole

$$\xi_4 = D_1^2 \, \frac{8 f^2}{35} \, \mu_4(s)$$



BOSS results

Satpathy et. al., arXiv:1607.03148



monopole $\times s^2$

quadrupole $\times s^2$

Fourier space

Effect of redshift distortions on the power spectrum.

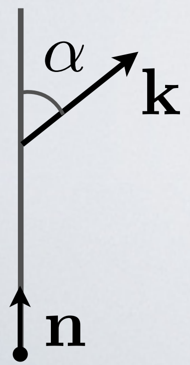
$$\delta_{\rm obs}(\mathbf{k},\eta) = \Big(1+(\hat{\mathbf{k}}\cdot\mathbf{n})^2 f\Big)\delta(\mathbf{k},\eta) \qquad\qquad P_\delta(k)\,\delta_D(\mathbf{k}+\mathbf{k}')$$

$$\langle\delta_{\rm obs}(\mathbf{k},\eta)\delta_{\rm obs}(\mathbf{k}',\eta)\rangle = \Big(1+(\hat{\mathbf{k}}\cdot\mathbf{n})^2 f\Big)\Big(1+(\hat{\mathbf{k}}'\cdot\mathbf{n}')^2 f\Big)\langle\delta(\mathbf{k},\eta)\delta(\mathbf{k}',\eta)\rangle$$

Distant observer approximation: $\mathbf{n}=\mathbf{n}'$

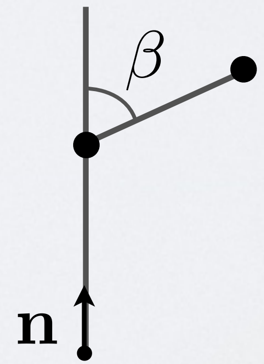
$$\boxed{P_\delta^{\rm obs}(k,\eta,\cos\alpha) = \left(1+\cos^2(\alpha)\,f^2\right)^2 P_\delta(k,\eta)} \qquad\qquad \mathbf{n}\cdot\hat{\mathbf{k}} = \cos\alpha$$

Fourier space



Breaking of isotropy:
the power spectrum depends on the direction of the Fourier mode.

Real space



Breaking of isotropy:
the correlation function depends on the orientation of the pair.

Multipole expansion

- We rewrite the cosine in terms of **Legendre polynomial** (orthogonal basis)

$$P_\delta^{\rm obs}(k,\eta,\cos\alpha) = \left\{1 + \frac{2f}{3} + \frac{f^2}{5} + \left(\frac{4f}{3} + \frac{4f^2}{7}\right)P_2(\cos\alpha) \right.$$
$$\left. + \frac{8f^2}{35}P_4(\cos\alpha)\right\}P_\delta(k,\eta)$$

- The density power spectrum **factorises out**.

- In the correlation function this was not the case: different dependence in the separation due to the spherical Bessel functions.

Multipole extraction

- monopole

$$P_\delta^{\mathrm{obs}\,0}(k,\eta)=\frac{1}{2}\int_{-1}^{1}d\mu\; P_\delta^{\mathrm{obs}}(k,\eta,\mu)=\left(1+\frac{2f}{3}+\frac{f^2}{5}\right)P_\delta(k,\eta)$$

- quadrupole

$$P_\delta^{\mathrm{obs}\,2}(k,\eta)=\frac{5}{2}\int_{-1}^{1}d\mu\, P_2(\mu)\, P_\delta^{\mathrm{obs}}(k,\eta,\mu)=\left(\frac{4f}{3}+\frac{4f^2}{7}\right)P_\delta(k,\eta)$$

- hexadecapole

$$P_\delta^{\mathrm{obs}\,4}(k,\eta)=\frac{9}{2}\int_{-1}^{1}d\mu\, P_4(\mu)\, P_\delta^{\mathrm{obs}}(k,\eta,\mu)=\frac{8f^2}{35}P_\delta(k,\eta)$$

We can measure $f\sigma_8$

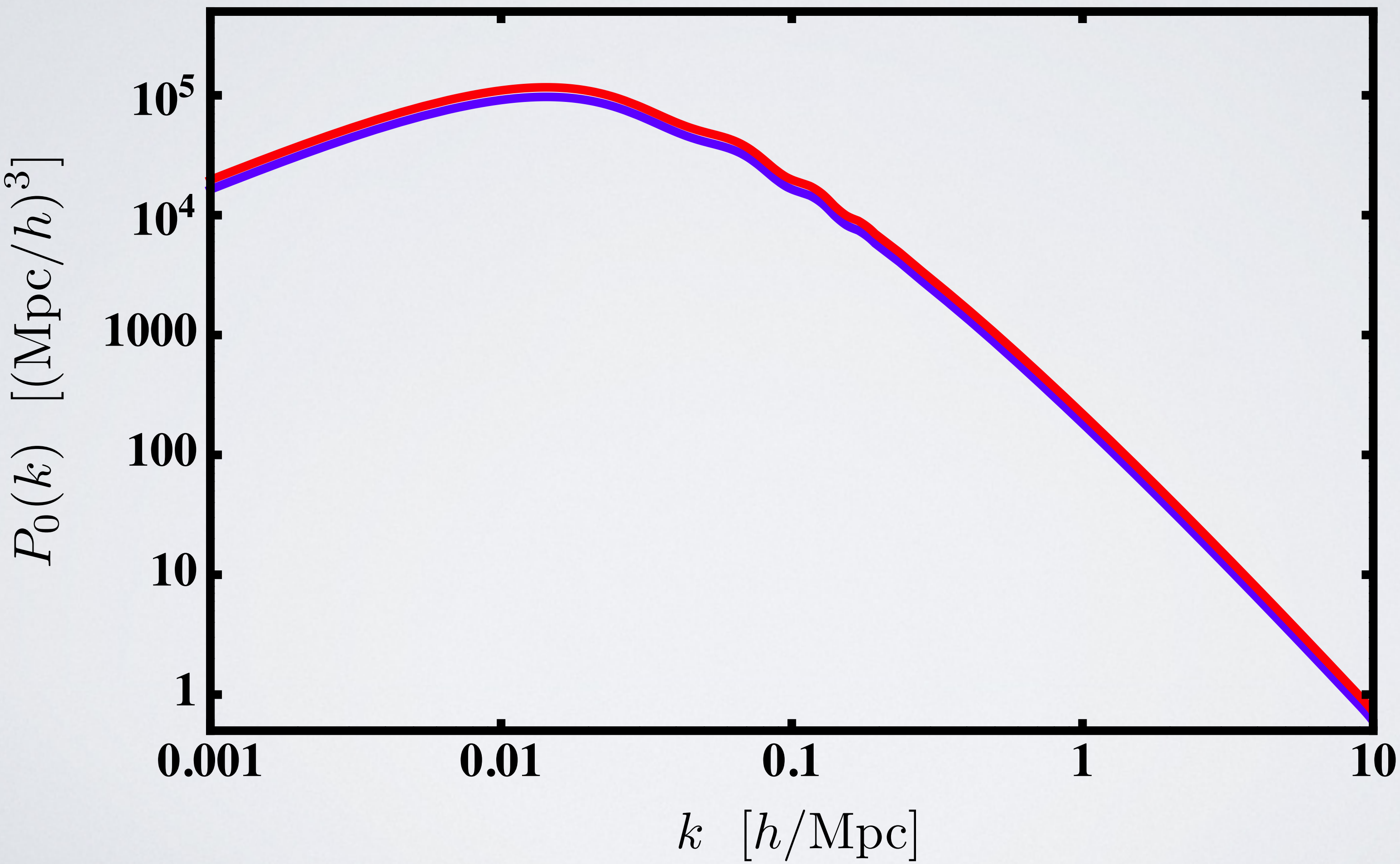
Results: monopole

without redshift distortions

$$b^2 P_\delta(k,\eta)$$

with redshift distortions

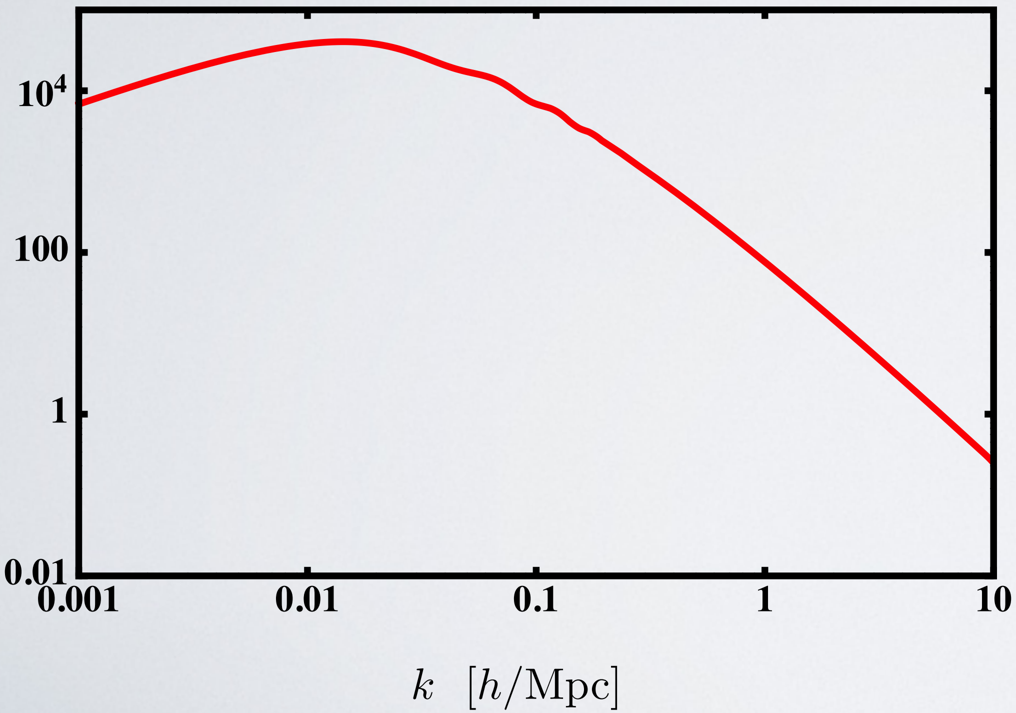
$$\left(b^2 + \frac{2bf}{3} + \frac{f^2}{5}\right) P_\delta(k,\eta)$$



Results: quadrupole and hexadecapole

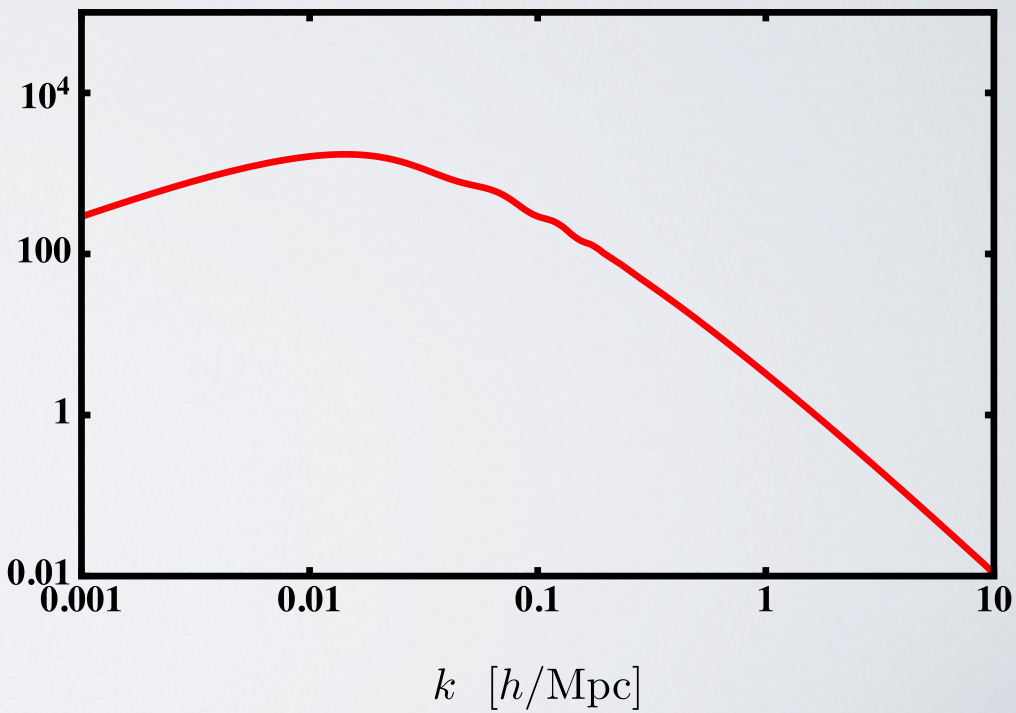
quadrupole

$$\left(\frac{4bf}{3}+\frac{4f^2}{7}\right)P_\delta(k,\eta)$$

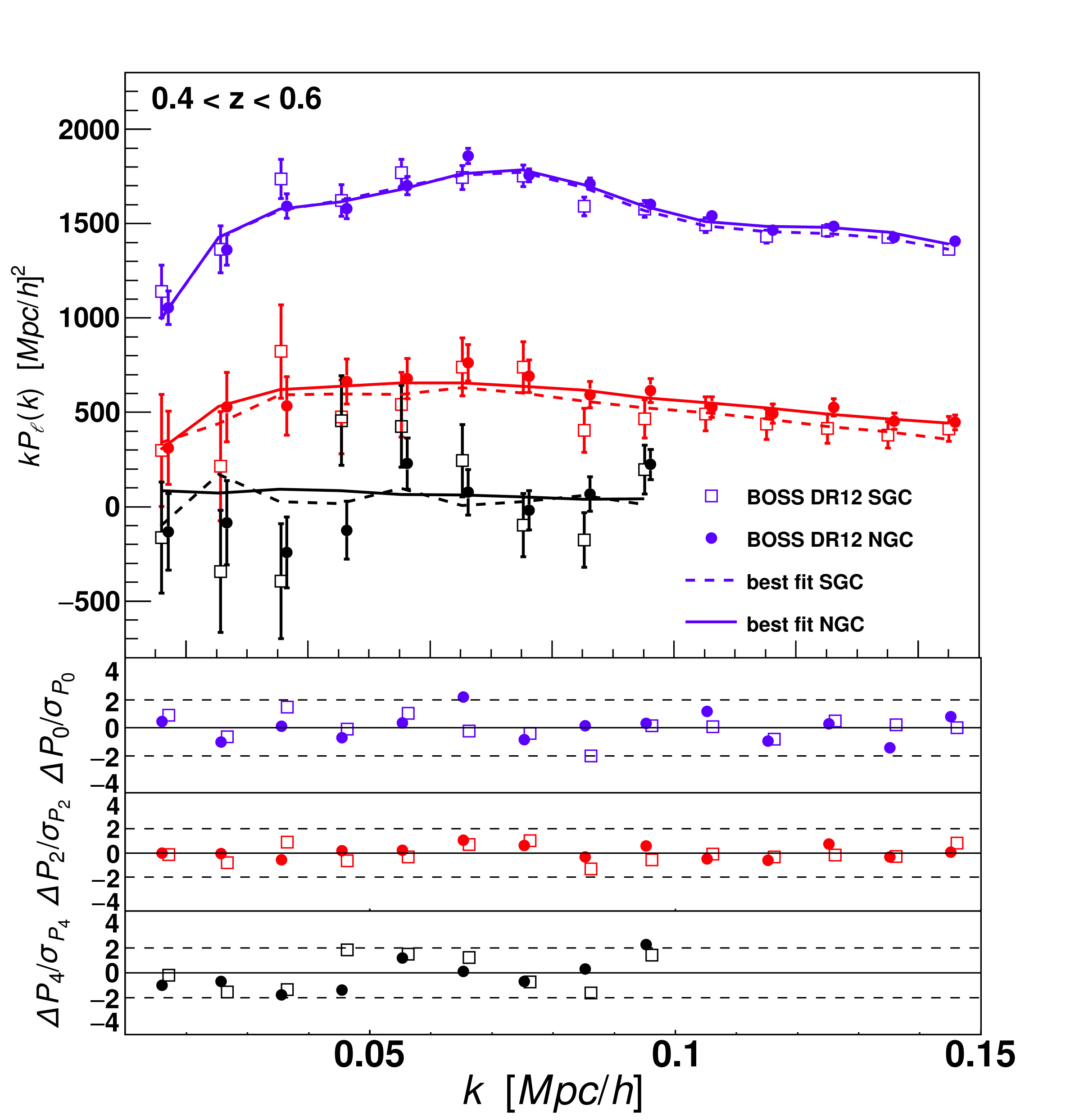


hexadecapole

$$\frac{8f^2}{35}P_\delta(k,\eta)$$



BOSS results



F. Beutler et al, arXiv:1607.03150

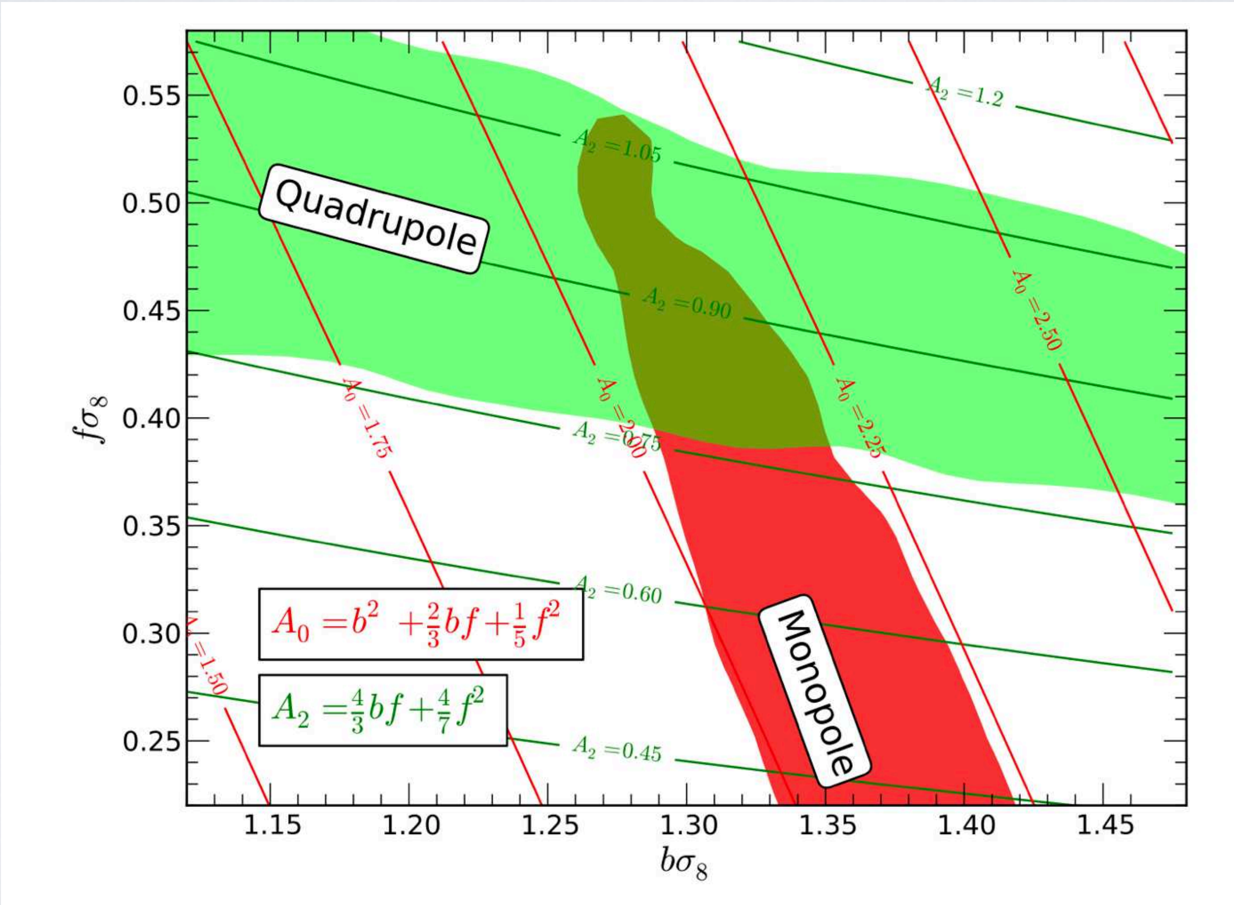
Growth evolution

Which kind of **constraints** can we obtain from redshift distortions?

The monopole and quadrupole allow to measure $f\sigma_8$ and $b\sigma_8$

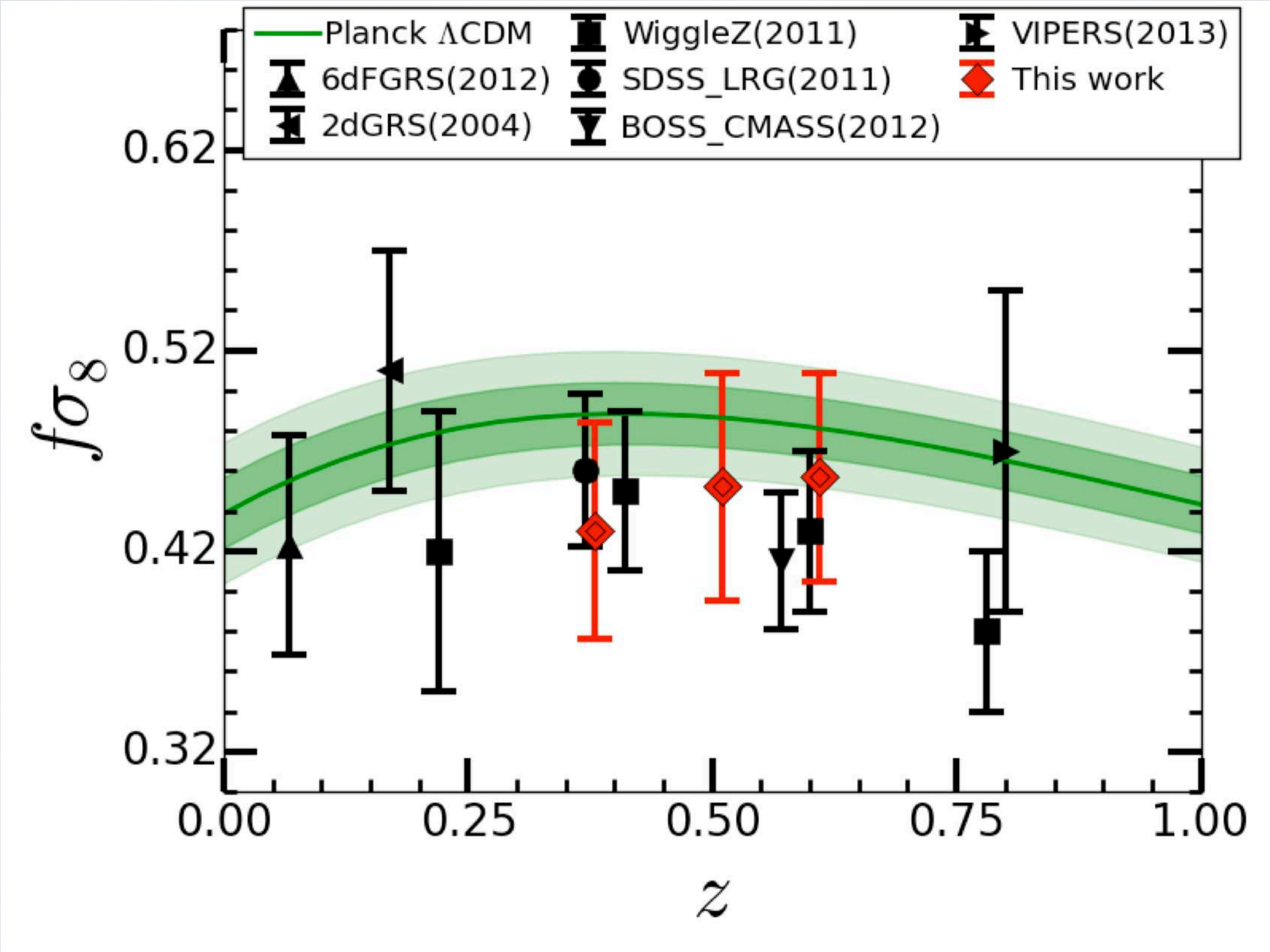
amplitude of P

L. Samushia et al, arXiv:1312.4899

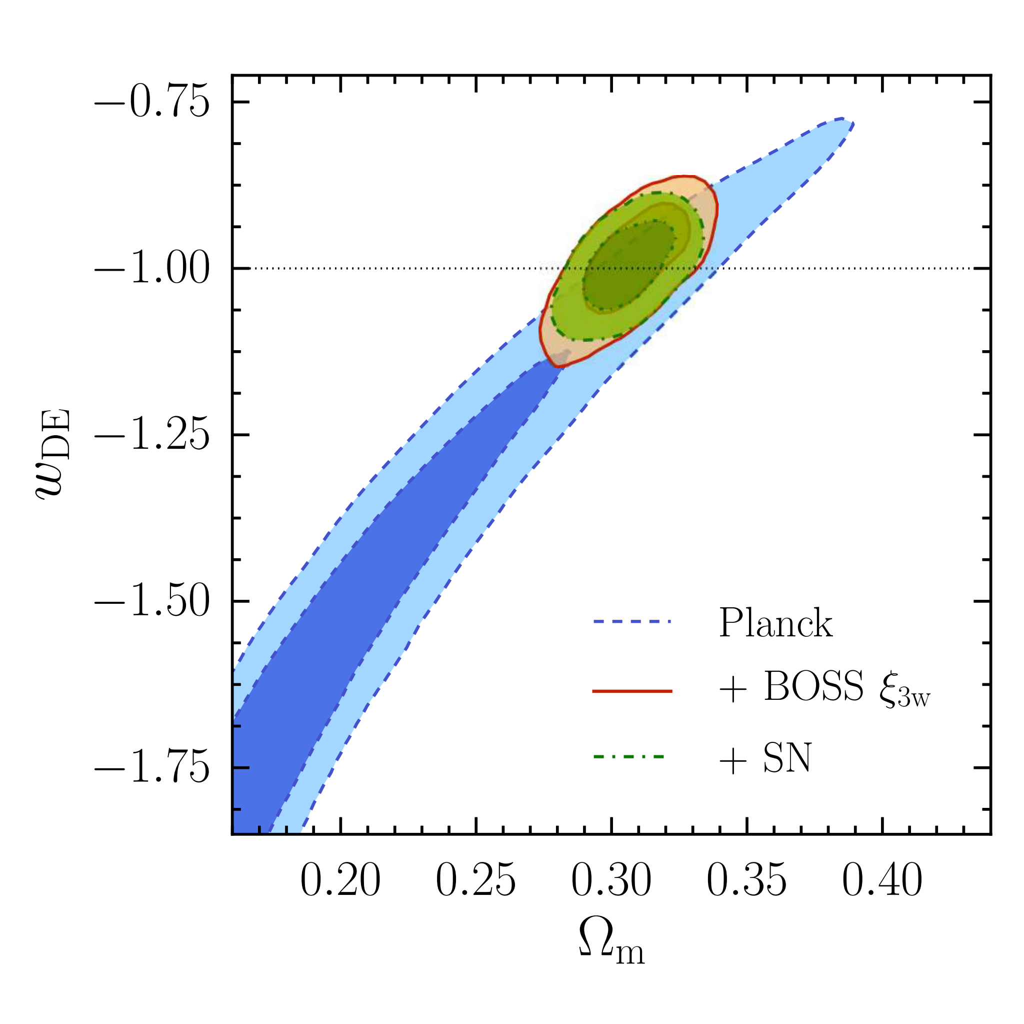


Growth rate evolution

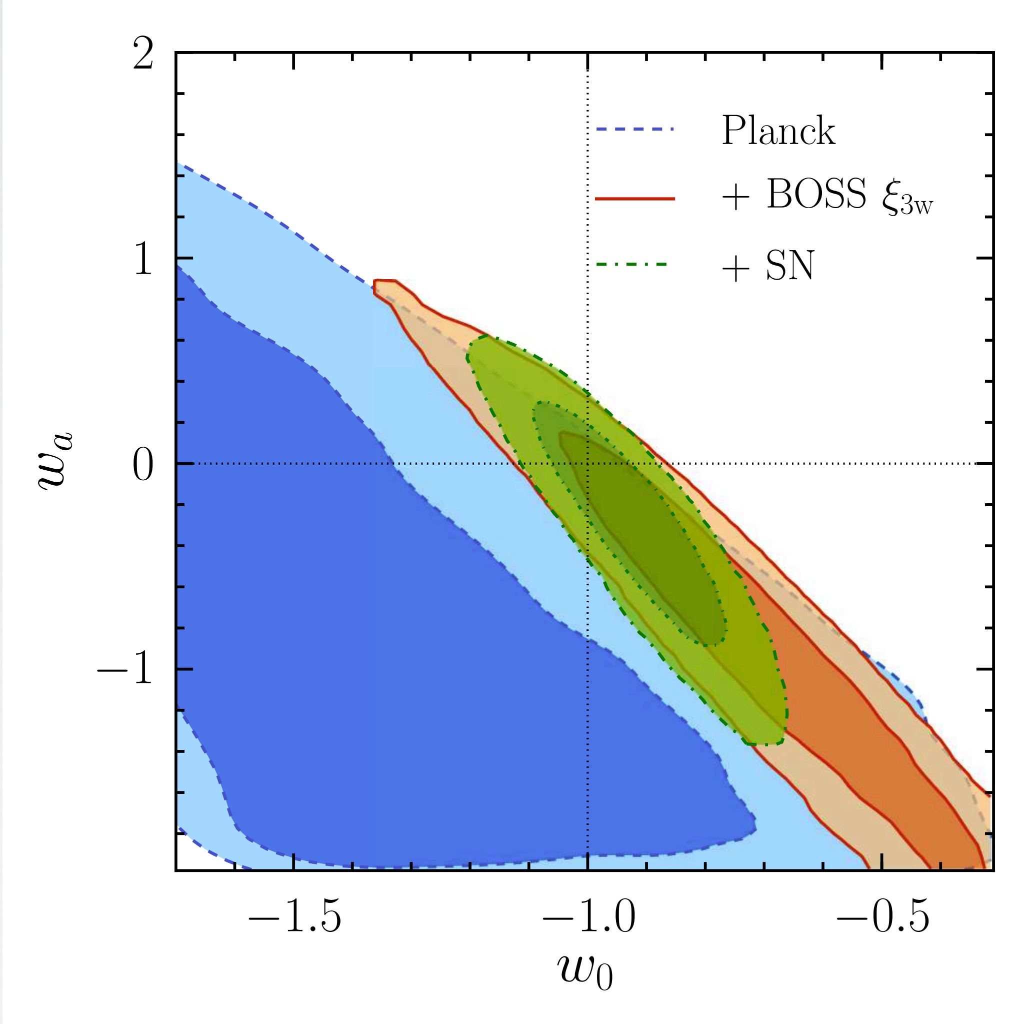
Satpathy et. al., arXiv:1607.03148



Cosmological constraints



Sanchez et. al., arXiv:1607.03147



$$w = w_0 + w_a \frac{z}{1+z}$$

Consistency of General Relativity

How can we quantify **deviations** from general relativity?

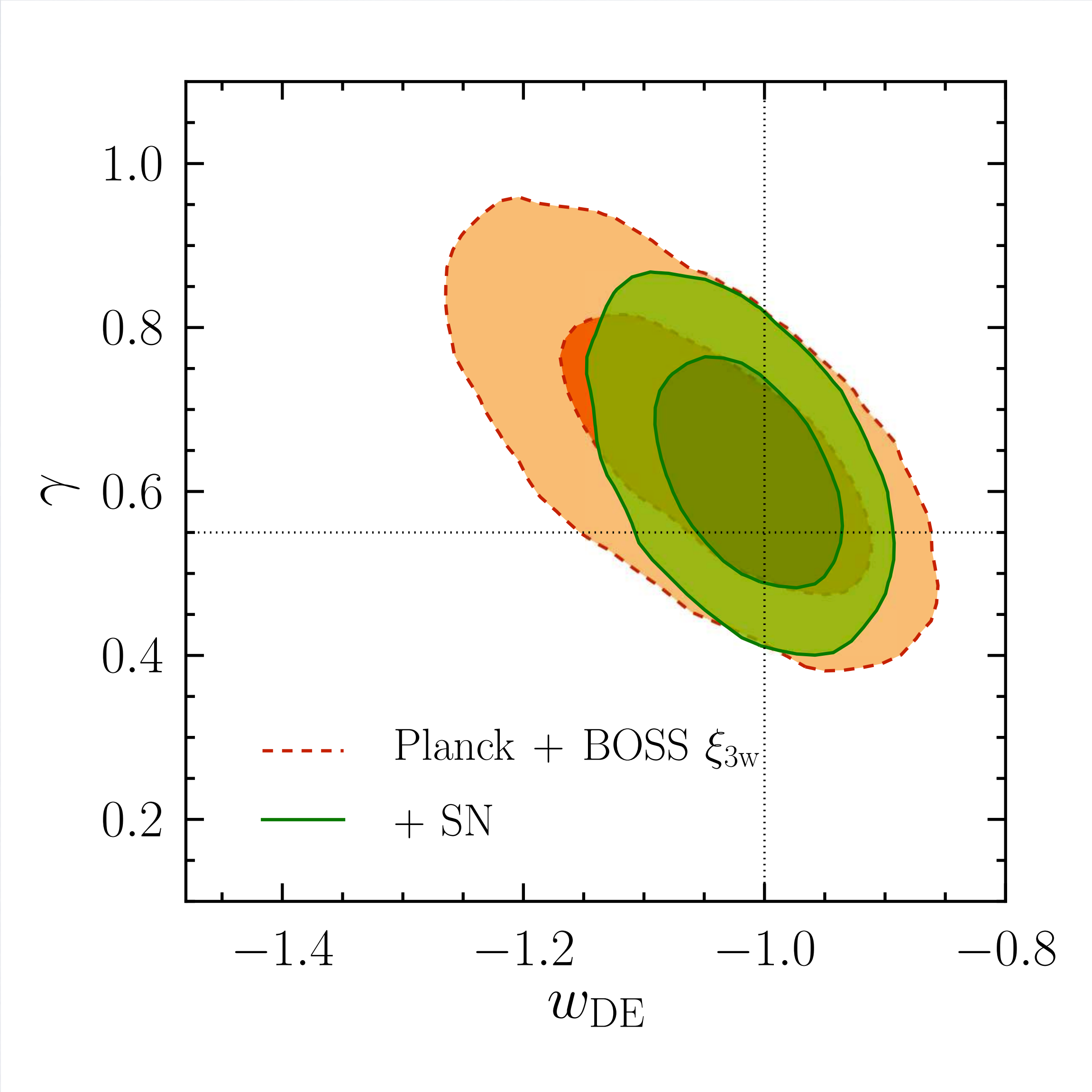
Useful **parameterisation**: $f(a) = \Omega_m(a)^\gamma$ Peebles (1980) Wang and Steinhardt (1998)

In general relativity with a cosmological constant: $\gamma = 0.55$

Observing a different values would mean a **deviation** from ΛCDM.

This is not a general parametrisation but it allows to test the **consistency** of general relativity.

Consistency of General Relativity



Sanchez et al, arXiv:1607.03147