Solution to Exercise Session 1

Problem 1   Multiple Choice Questions

A) *A geostationary satellite is orbiting the Earth at an altitude of 36'000 km. Assuming the satellite is stopped instantaneously and starts to fall, at what speed will it reach the top of the Earth atmosphere, 100 km above the Earth surface?*

(1) 89.2 km/s
(2) 10.2 km/s
(3) 14.8 km/s
(4) 7.4 km/s

This result is computed using the conservation of mechanical energy in the Earth-centered inertial frame:

$$\begin{aligned} E_{\text{pot1}} + E_{\text{kin1}} &= E_{\text{pot2}} + E_{\text{kin2}} \\ -\mu \frac{m}{R_\oplus + h_1} + 0 &= -\mu \frac{m}{R_\oplus + h_2} + \frac{1}{2} m v^2 \\ \Rightarrow \qquad v &= \sqrt{2\mu \left(\frac{1}{R_\oplus + h_2} - \frac{1}{R_\oplus + h_1} \right)} = \mathbf{10.21\ km/s} \end{aligned}$$

with Earth gravitational parameter $\mu = G \cdot M_\oplus = 3.986 \cdot 10^{14}$ m^{3}s^{-2}, Earth radius $R_\oplus = 6378$ km, $h_1 = 36'000$ km and $h_2 = 100$ km.

B) *Assuming that a spacecraft moves towards the Sun along the line joining their centers, at which distance from the Earth center does the spacecraft feels no net gravitational force ?*

(1) $\mathbf{2.58 \cdot 10^5}$ km
(2) $1.48 \cdot 10^6$ km
(3) $2.59 \cdot 10^8$ km
(4) $1.49 \cdot 10^8$ km

$$F_{G\oplus} = F_{G\odot} \quad \Longrightarrow \quad m_{\text{S/C}} \frac{\mu_\odot}{\ell_\odot^2} = m_{\text{S/C}} \frac{\mu_\oplus}{\ell_\oplus^2} \underbrace{\Longrightarrow}_{\ell_\odot + \ell_\oplus = d_{\oplus\odot}} \sqrt{\frac{\mu_\odot}{\mu_\oplus}} \ell_\oplus = d_{\oplus\odot} - \ell_\oplus$$

$$\Longrightarrow \quad \ell_\oplus = \frac{d_{\oplus\odot}}{1 + \sqrt{\frac{\mu_\odot}{\mu_\oplus}}} \approx \mathbf{2.58 \cdot 10^5\ km}$$

where $\odot$ is the symbol for the Sun, $\oplus$ the symbol for the Earth, $d_{\oplus\odot}$ is the distance Sun–Earth and ℓ the distance to the spacecraft (S/C) from the Sun or the Earth.

C) *Estimate the equilibrium temperature of the Earth given the radiation power from the Sun, and the self-radiation power from the Earth into space, and solving for temperature. Use the blackbody assumption $\alpha/\epsilon = 1$.*

(1) $21°C$
(2) $\boldsymbol{6°C}$
(3) $-21°C$
(4) $0°C$

Solution. The power of the Sun absorbed by Earth is given by :

$$P_a = \alpha S A_n$$

Where $A_n = \pi R_\oplus^2$ is the cross-section (disc) intercepting the sun's radiation. Earth emitted power is approximated using the black-body radiation :

$$P_e = \epsilon \sigma T^4 A_{tot}$$

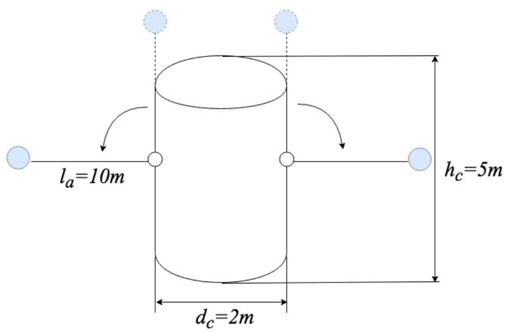
Where A_{tot} is the emitting surface (sphere) : $A_{tot} = 4\pi R_\oplus^2$. At equilibrium, we have $P_e = P_a$; therefore we can isolate T :

$$T = \left(\frac{\alpha}{\epsilon}\right)^{1/4} \times \left(\frac{S A_n}{\sigma A_{tot}}\right)^{1/4}$$

$$\begin{aligned} T = \left(\frac{S}{4\sigma}\right)^{1/4} &= \left(\frac{1.367 \times 10^3}{4 \cdot 5.67 \times 10^{-8}}\right)^{1/4} \\ &= 278.63°K \approx 6°C \end{aligned}$$

Note: If we consider that only the fraction of the solar radiation reaching the Earth contributes to heating (total solar radiation minus the albedo), then we find $T = 251.8°K = -21°C$.

D) *The Cassini-Huygens spacecraft was launched in 1997 towards Saturn. It was roughly of cylindrical shape with two straight arms along the cylinder whose purpose was to deploy antennas far from the main body some time after launch.*



After deployment from the launcher, the spacecraft was turning along its main axis at a rate of 1 rpm or 6 degrees per second. After deployment of the antennas 90 degrees off the axis of the *spacecraft, what was the rotation rate of the spacecraft? (consider the antennas as point masses and the arms are supposed massless)*

Data : Cassini total mass $M_c = 5600\ kg$ without the antennas, height $h_c = 5\ m$, diameter $d_c = 2\ m$. Mass of each antenna $m_a = 50\ kg$, length of each arm $l_a = 10\ m$

(1) 7.61 deg/s
(2) 0.57 deg/s
(3) 1.16 deg /s
(4) 3.84 deg/s

Solution. The angular momentum before the deployment of the arm is equal to the moment to the angular momentum after the deployment : $L_1 = L_2$ where $L_1 = I_c \cdot \omega_1$ and $L_2 = I_{c+a} \cdot \omega_2$.

$$I_c = \frac{1}{2} M_c \left(\frac{d_c}{2}\right)^2 + 2m_a \left(\frac{d_c}{2}\right)^2$$
$$I_{c+a} = \frac{1}{2} M_c \left(\frac{d_c}{2}\right)^2 + 2m_a \left(l_a + \frac{d_c}{2}\right)^2$$
$$\Rightarrow \omega_2 = \omega_1 \frac{I_c}{I_{c+a}} \approx 1.16 \deg /s$$

Problem 2 Escape velocity

The Rosetta spacecraft launched by the European Space Agency successfully entered the orbit of the comet 67P/Churyumov Gerasimenko in August 2014. November 12 2014, the Philae lander was released and touched down 7 hours later at a speed of $1 m/s$.
The harpoon mechanism which was supposed to secure the lander failed and it bounced off the comet. Assuming purely elastic impact, will the lander leave the comet or return at some point?

Data: Mass of the lander : $m_l = 100\ kg$, Mass of the comet : $M_c = 3.14 \cdot 10^{12} kg$, Radius : $R_c = 2\ km$ (assume a spherical shape).

Solution. Due to the elastic impact and important mass difference between the lander and the comet, we can assume that the post-impact (f) velocity is the same (in amplitude) as the velocity before (i) the impact : $|\vec{v}_f| = |\vec{v}_i|$

The escape velocity is the minimum velocity that an object or spacecraft has to be given to escape a celestial body forever (reaching infinity at zero velocity).

Conservation of Energy:

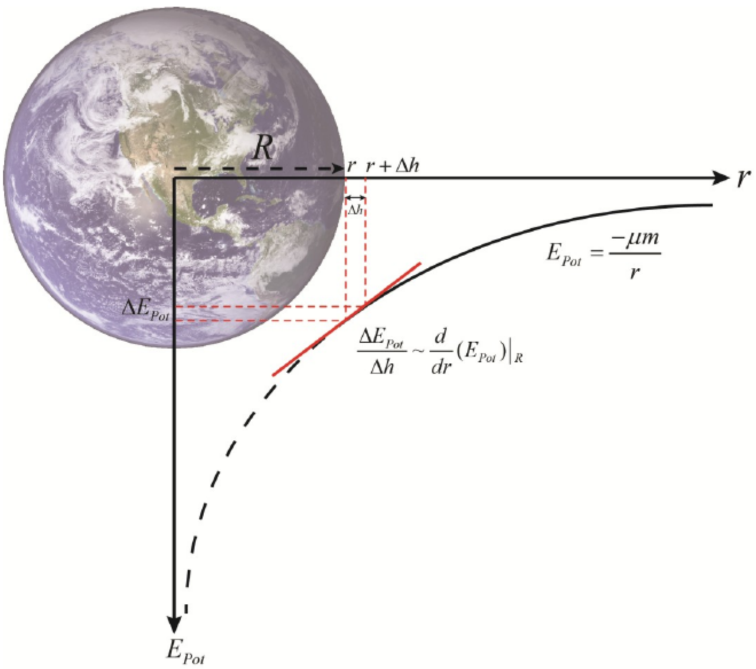
$$\underbrace{-\frac{\mu}{R_c} + \frac{1}{2} v_e(R_c)^2}_{On\ the\ surface} = \underbrace{0 + 0}_{At\ infinity} \implies$$

$$v_e(R_c) = \sqrt{\frac{2\mu_c}{R_c}} = 0.458 m/s,$$

where R_c is the radius of the comet.
The velocity of the lander after impact is more than twice as high as the escape velocity which shows that if the impact is perfectly elastic, the lander will escape the comet and never come back.

Problem 3 Potential energy close to the surface of the Earth

The general expression for the potential energy of a mass m in the Earths gravitational field is $E_{pot} = -\frac{m\mu}{r}$, r being the distance to the center of the Earth. In the vicinity of the surface of the Earth, the difference in potential energy for a mass m when the height above the ground is changed by Δh is equal to $mg\Delta h$, where g is the gravitational acceleration at the surface of the Earth. Derive this approximate expression from the general expression.



Intuitive solution. The difference in potential energy can be explained as the derivative of general expression of potential energy with respect to r :

$$\frac{\mathrm{d}}{\mathrm{d}r}E_{\text{pot}} = \frac{\mathrm{d}}{\mathrm{d}r}\left(-\frac{m\mu}{r}\right) = \frac{m\mu}{r^2} = mg(r)$$

The gravitational acceleration of the Earth is $g(r) \equiv \frac{\mu}{r^2}$. At Earth's surface, $g(R) = 9.81$ m/s^2. At the vicinity of the Earth's surface, when r varies as Δh ($\Delta h \ll R$), we can approximate the above derivative:

$$\left.\frac{\mathrm{d}}{\mathrm{d}r}E_{\text{pot}}\right|_R = \frac{m\mu}{R^2} \sim \frac{\Delta E_{\text{pot}}}{\Delta h} \implies \Delta E_{\text{pot}} \sim mg(R)\Delta h$$

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Problem 4 Radiation balance

A) *Consider two spherical satellites with a radius r, respectively $2r$. Determine the radiation balance of these objects if they are exposed to solar radiation only and compare their temperatures.*

B) *Consider a cylindrical satellite (radius = 1 m, height = 2 m) that is spin-stabilised, hence turning on its longitudinal axis. It is supposed to be on an orbit where eclipses are negligible, and the satellite's longitudinal axis of rotation stays perpendicular to the sun rays. The external structure of the satellite is made of steel (AM 350) with a (α/ε) ratio of 1.79. We only consider the Sun's radiation on the satellite and neglect the Earth's albedo and self IR radiation.*
During a space shuttle mission, the science instrument of this satellite has to be replaced, and a spacewalk of two crewmembers is planned. Will it be safe for the astronauts performing this task to touch the surface of the satellite with their gloves if the "touch" – "no touch" limit is at 80 °C ?

Solution. The satellite's temperature can be determined with the radiation balance and is given by the following formula

$$T = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{SA_n}{\sigma A_{\text{tot}}}\right)^{1/4}$$

A) As for a spherical satellite $A_n = \pi r^2$ and $A_{\text{tot}} = 4\pi r^2$, this corresponds to

$$T = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{S\pi r^2}{\sigma 4\pi r^2}\right)^{1/4} = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{S}{4\sigma}\right)^{1/4}$$

The temperature of a spherical satellite is therefore not influenced by its radius.

B) As the Sun rays remain perpendicular to the rotational axis of the satellite, the surface perpendicular to Sun's direction given by

$$A_n = 2rh$$

where r is the radius of the cylinder and h its height.

The satellite emits over the whole surface of the cylinder, the total surface is therefore

$$A_{\text{tot}} = 2\pi rh + 2\pi r^2$$

Hence the temperature of the satellite is given by :

$$T = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{SA_n}{\sigma A_{\text{tot}}}\right)^{1/4} = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{S \cdot 2rh}{\sigma \cdot 2\pi r(h+r)}\right)^{1/4} = \left(\frac{\alpha}{\varepsilon}\right)^{1/4} \left(\frac{S \cdot h}{\sigma \cdot \pi(h+r)}\right)^{1/4}$$

This yields a temperature of the surface of

$$T = (1.79)^{1/4} \left(\frac{1.4 \cdot 10^3 \cdot 2}{5.67 \cdot 10^{-8} \cdot \pi(2+1)}\right)^{1/4} = 311.2 \text{ K}$$

The surface temperature of the satellite is 311.2 K or 38 °C and it is below the critical limit of "touch" – "no touch" and the astronauts can touch its surface without any risk.