Solution to Exercise Session 3

Problem 1 Multiple Choice Questions

A) *We perform a rendezvous with the ISS which orbits the Earth in a circular LEO at 400 km altitude. During the approach, the chaser (Space Shuttle) is in an elliptical orbit in the same plane as the ISS with an apogee at 400 km and a perigee at 370 km, behind the ISS. On each successive apogee crossing, will the Shuttle get closer to the ISS or further away ? By how many kilometers ? (Give your answer as measured on the orbit of the ISS).*

(1) Closer	(1) -282 km	(4) -4.3 km
(2) Further away	(2) 282 km	**(5) 141 km**
	(3) 7.7 km	(6) -152 km

Catch up rate (per orbit) for the chaser in an elliptical orbit of semi-major axis **a** and the TGT in a circular orbit of radius **r**: $\Delta x = 3\pi(r - a)$.

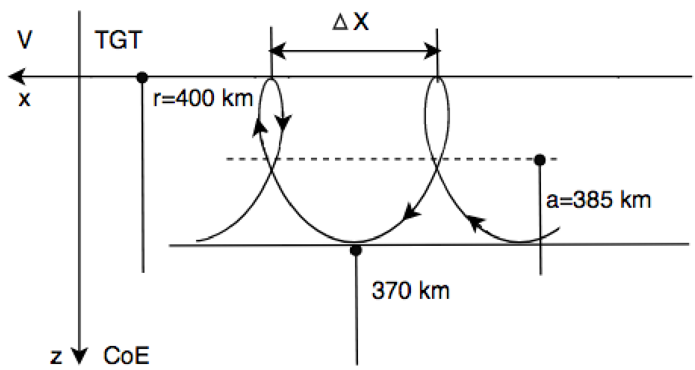

Figure 1: *Chaser relative trajectory*

Thus r - a =15km, and $\Delta x = 3\pi 15\text{km} = 141\text{km}$

The chaser (Shuttle) will get closer to the TGT (ISS) on each apogee crossing.

B) *In order to circularize its orbit to reach the ISS, the Space Shuttle will have to do different maneuvers. During rendezvous, a maneuver is performed at apogee which raises the perigee by 10 km. What is the Δv needed for this maneuver ?*

(1) 0.22 m/s
(2) 1.43 m/s
(3) 2.86 m/s

(4) 0.32 km/s
(5) 7.11 km/s

We use the LEO approximation $\Delta r \simeq 3.5\Delta v$ with $\Delta r = 10$ km for the perigee altitude change:

$$\Delta v \simeq \frac{\Delta r}{3.5} = \frac{10}{3.5} = \mathbf{2.86 \quad m/s}.$$

C) *CHEOPS was launched in 2019. This exoplanet observation satellite, which is partly designed at EPFL, is in a Sun-Synchronous orbit with an inclination of 98.6° to avoid long eclipses and therefore avoid carrying many heavy batteries. To achieve this minimization of eclipses strategy, what are the local mean solar times when the satellite crosses the equator ?*

(1) Noon/midnight
(2) 3 pm / 9 pm
(3) 6 am / 6 pm (Sunrise/Sunset)
(4) 10 am / 4 pm
(5) it does not matter

A satellite on a sun-synchronous orbit crosses the equator always at about the same mean solar time. To minimize eclipses, the orbit of CHEOPS is close to the terminator, the day/night boundary on the surface of the Earth, so that the satellite is nearly always illuminated by the Sun. This results in equatorial crossing at about 6AM/6 PM mean solar time (Sunrise/Sunset).

D) *We want to inject a GPS satellite from a circular parking orbit at 230 km altitude to a final circular orbit at 20'000 km altitude, using a Hohmann transfer without orbital plane change. What are the amounts of the two maneuvers Δv_1 and Δv_2 ?*

(1) 2.1, 3.7 km/s
(2) 1.4, 1.1 km/s
(3) 1.4, 2.1 km/s
(4) 2.1, 1.4 km/s
(5) 4.7, 4.4 km/s

The circular orbital velocities are given by $v = \sqrt{\frac{\mu_\oplus}{R_\oplus + h}}$. Therefore :

$$\begin{aligned} v_{\text{LEO}} &= \sqrt{\frac{\mu_\oplus}{R_\oplus + 230 \text{ km}}} = 7.76 \text{ km/s} \\ v_{\text{GPS}} &= \sqrt{\frac{\mu_\oplus}{R_\oplus + 20\,000 \text{ km}}} = 3.89 \text{ km/s} \end{aligned}$$

Using Fig. 2, one can compute the Δv required:

$$\begin{aligned} \Delta v_1 &= 2.05 \text{ km/s} \\ \Delta v_2 &= 1.43 \text{ km/s} \\ \Delta v_{\text{tot}} &= \Delta v_1 + \Delta v_2 = 3.48 \text{ km/s} \end{aligned}$$

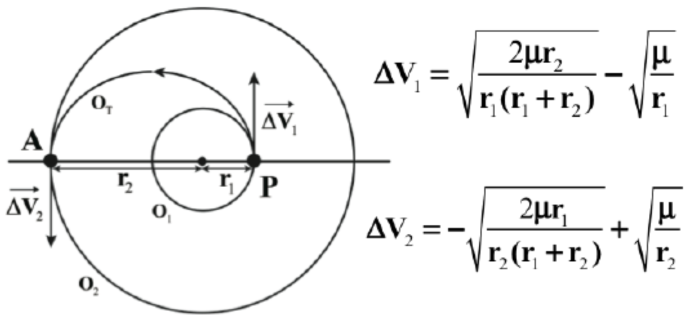

Figure 2: *Hohmann Transfer*

Problem 2 Chaser and Target

For each of the configurations and initial conditions (i) listed below, draw the trajectory of the chaser (thick line orbit) vs. target (thin line orbit). In all cases the direction of motion is counter clockwise, for the chaser as well as for the target.

Solution.