The large-scale structure of the universe

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CUSO Doctoral Program in Physics
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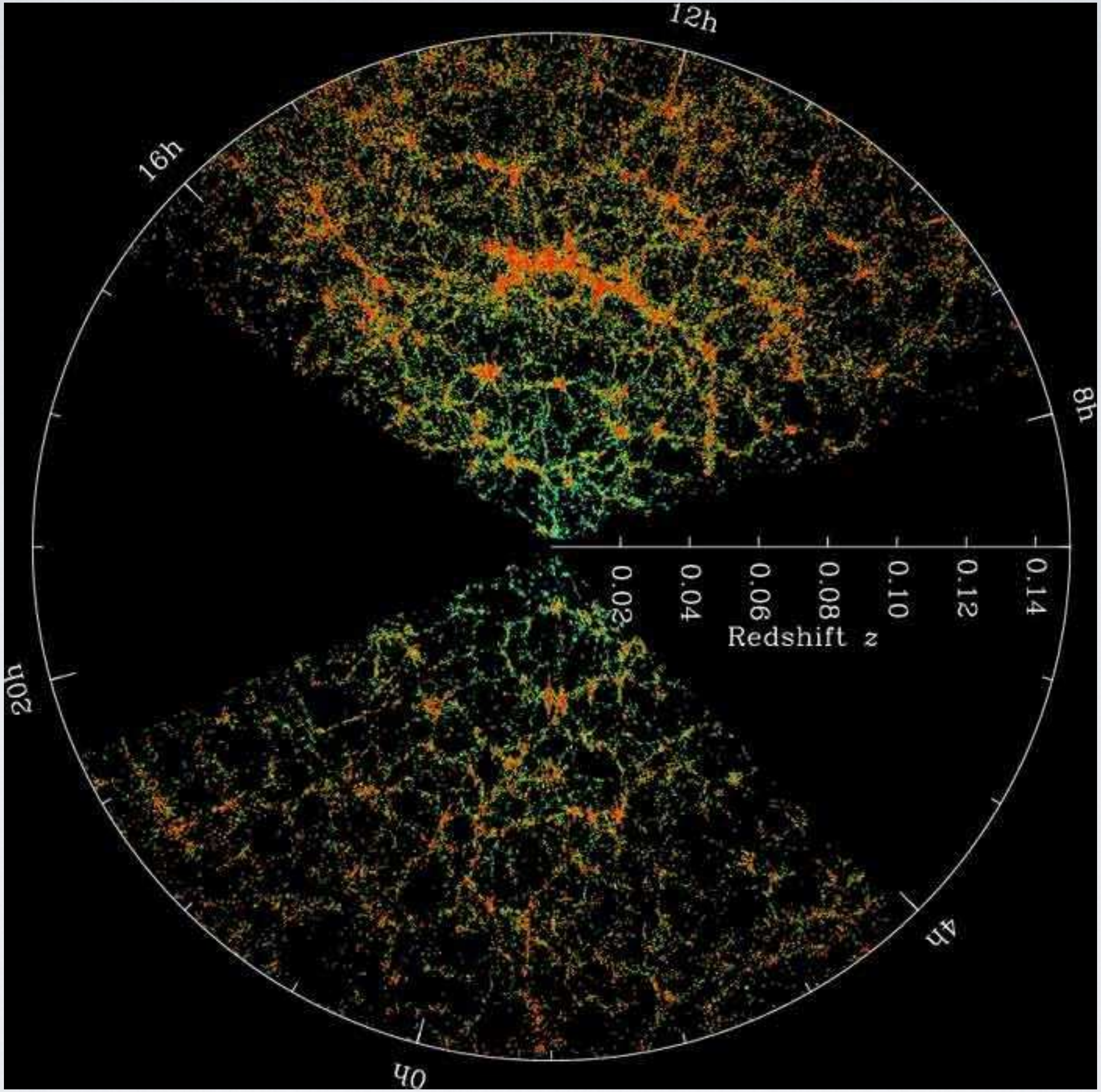
Galaxy survey

Credit: M. Blanton, SDSS



Outline

Aim

- Follow the evolution of the large-scale structure from inflation until today.

- Determine what the large-scale structure can tell us about our universe: dark matter, baryons, dark energy, gravity?

Message

The large-scale structure is a very powerful cosmological probe. It is complementary to the CMB and it allows us to follow the evolution of perturbations in the late universe.

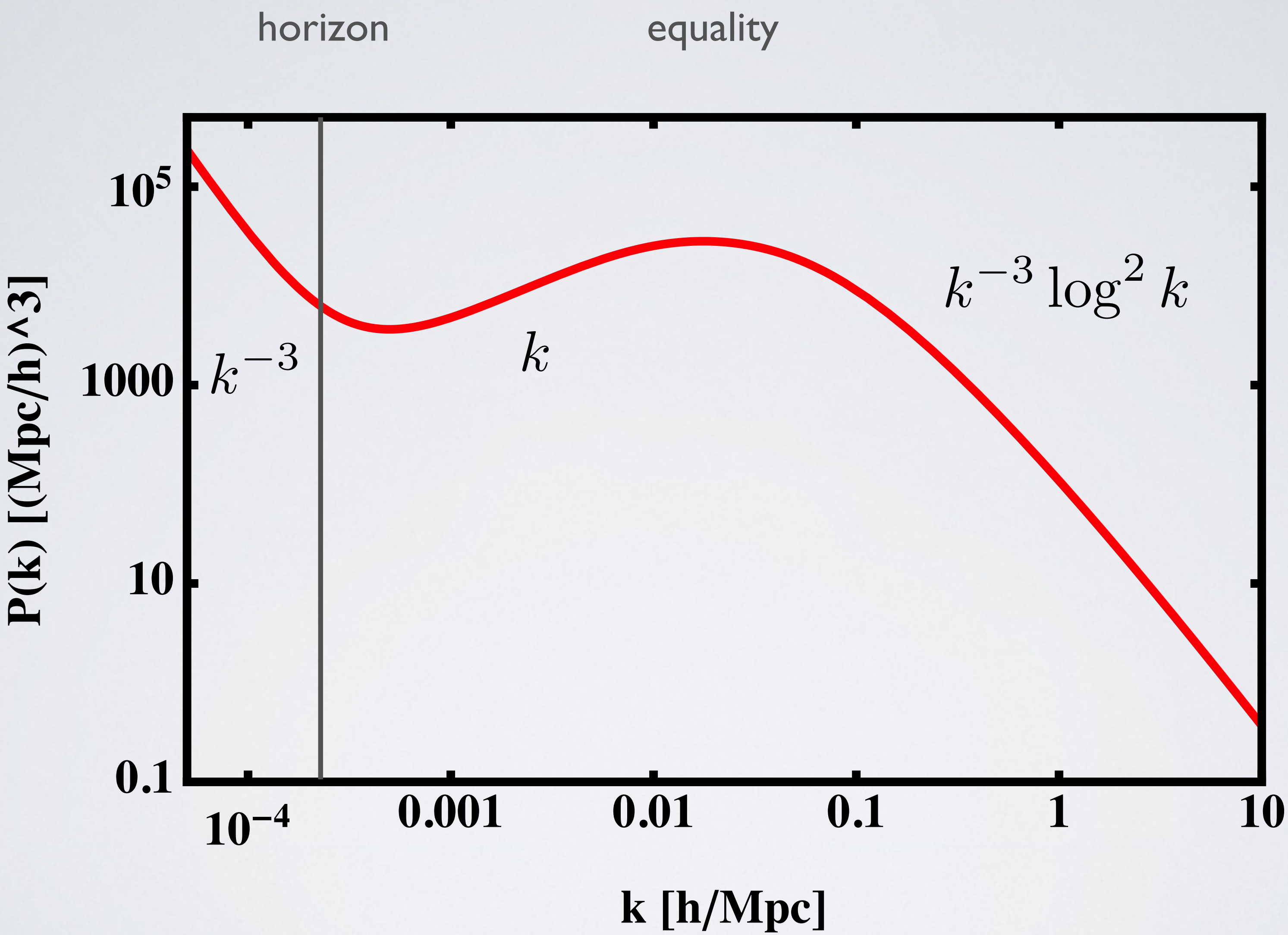
Outline

Evolution of perturbations: from inflation until today.

- Dark matter only (no baryons)
- No dark energy
- Linear calculation
- Real-space calculation
- Sub-horizon calculation: keep only dominant terms in $k/\mathcal{H}$

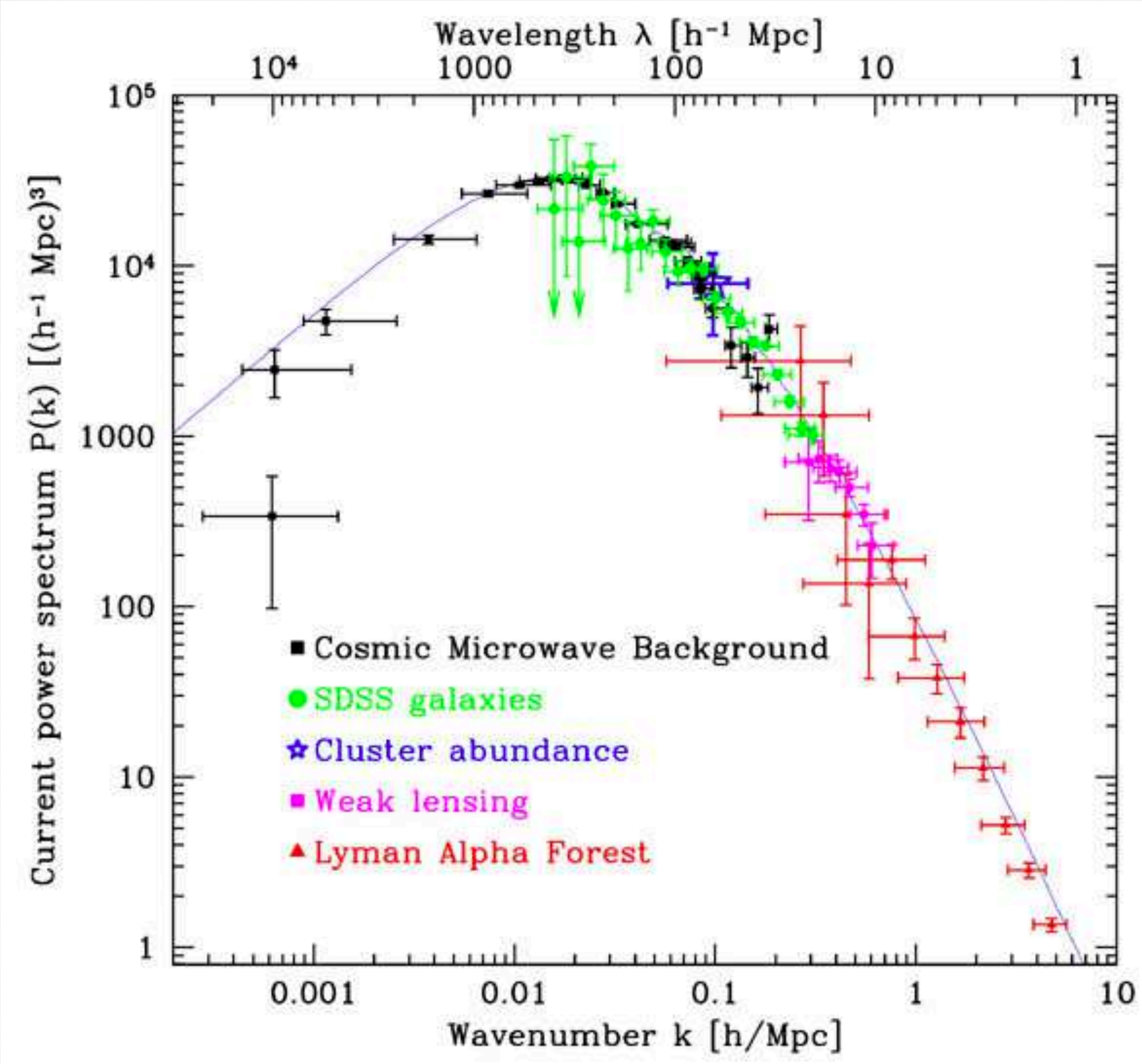
→ Dark matter **power spectrum**

Power spectrum



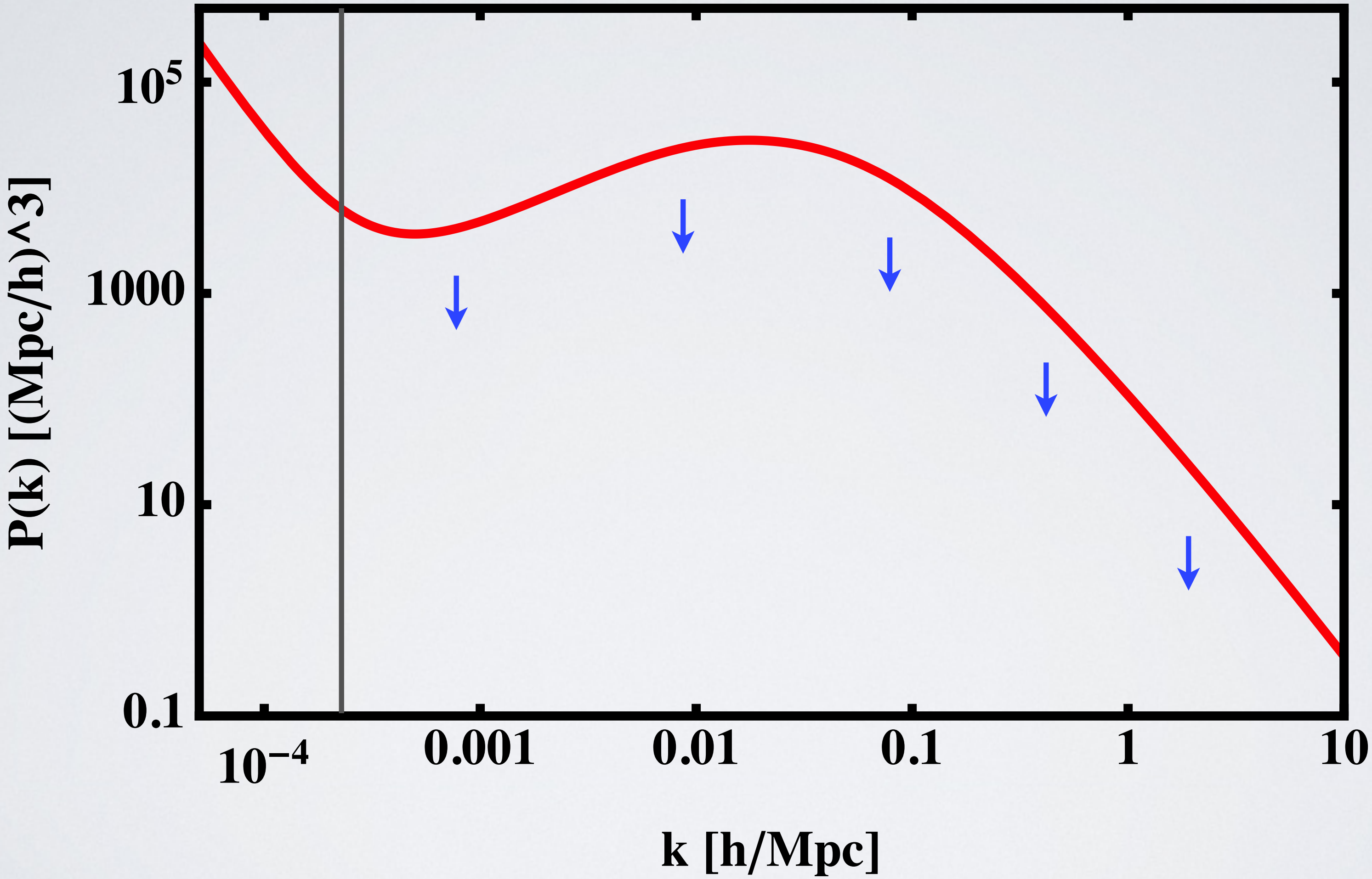
Power spectrum

Credit: Tegmark et al. (2002)



Beyond our approximations

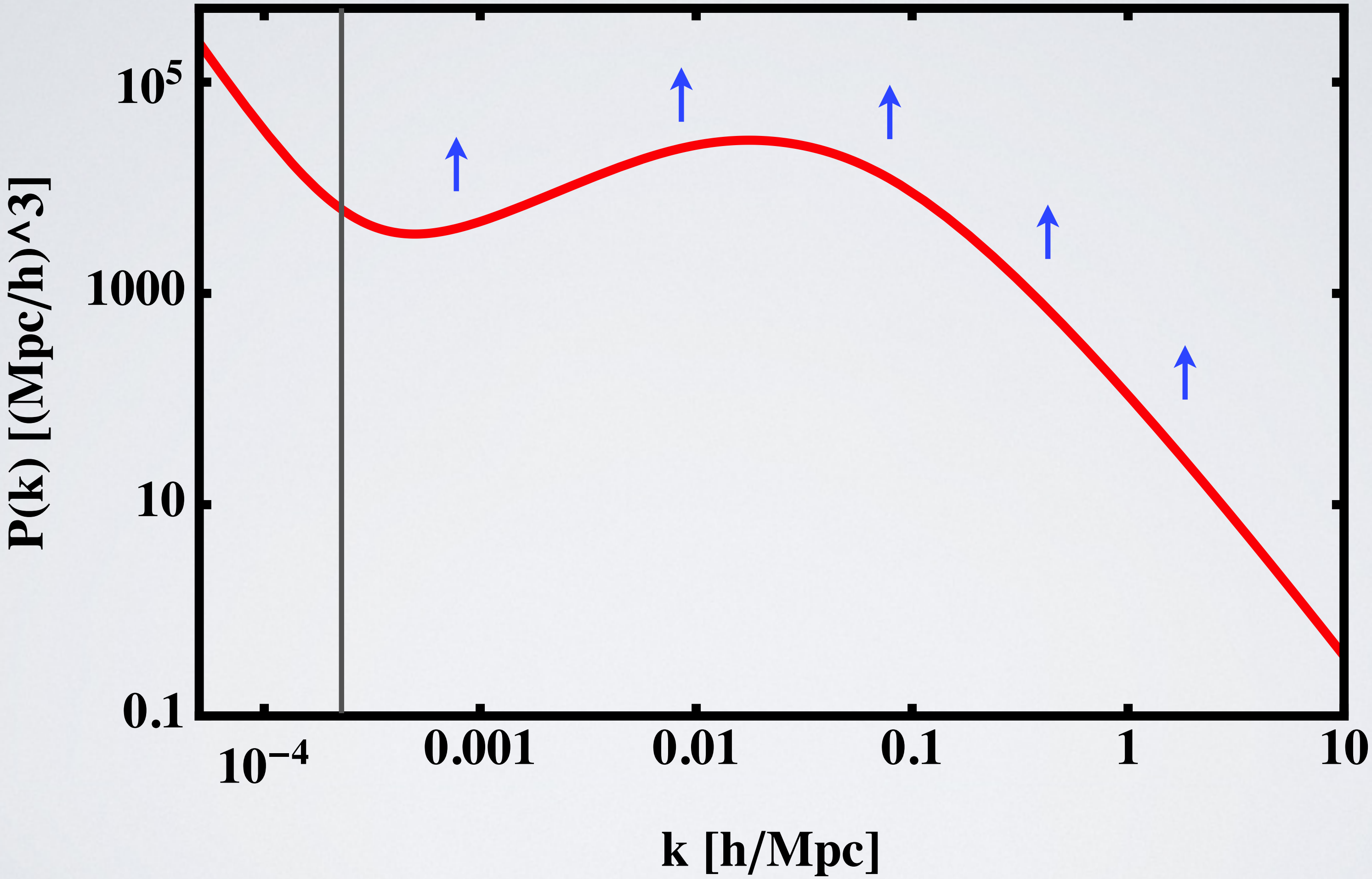
Dark energy



Beyond our approximations

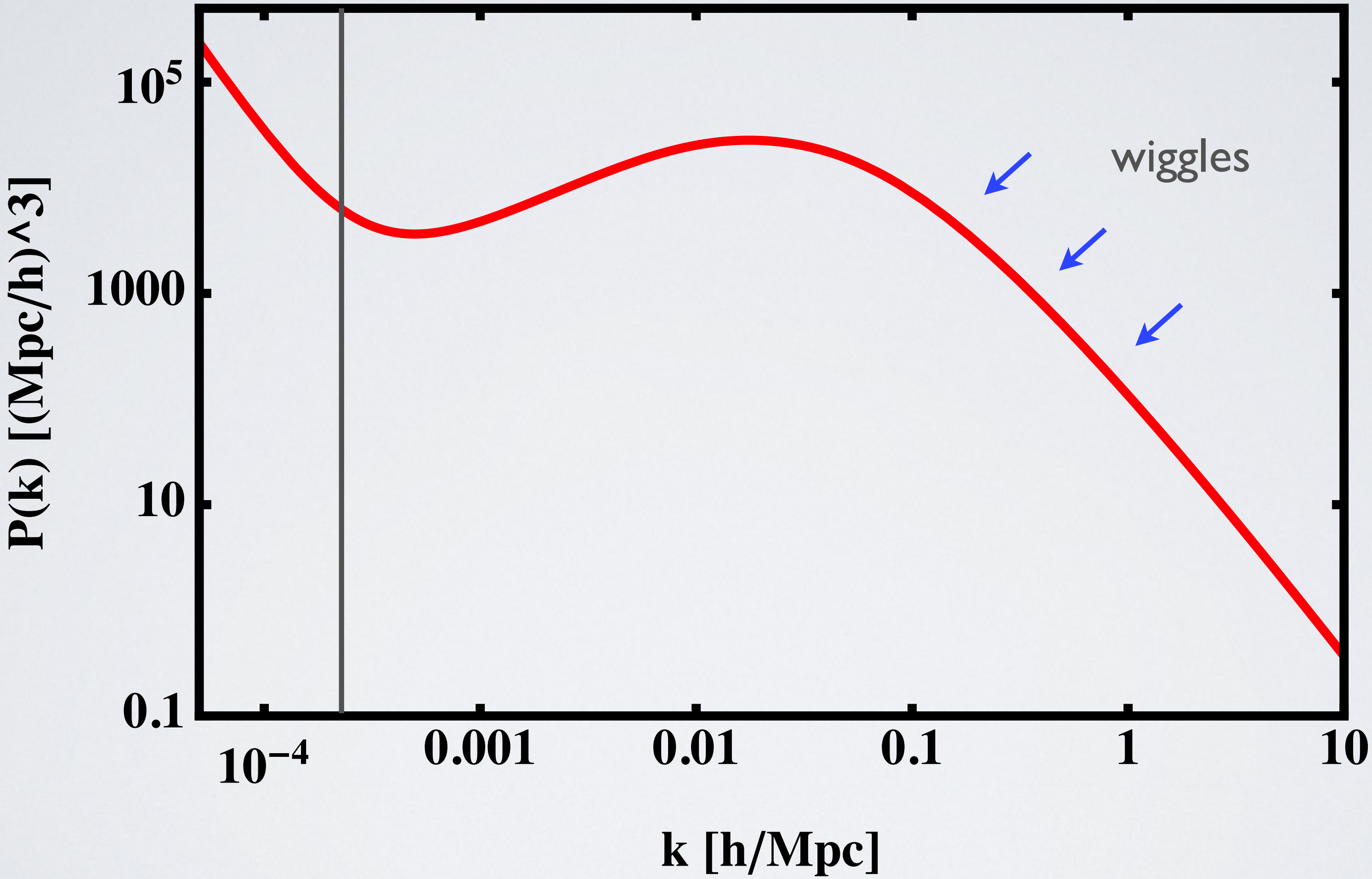
Redshift distortions

and breaking of isotropy



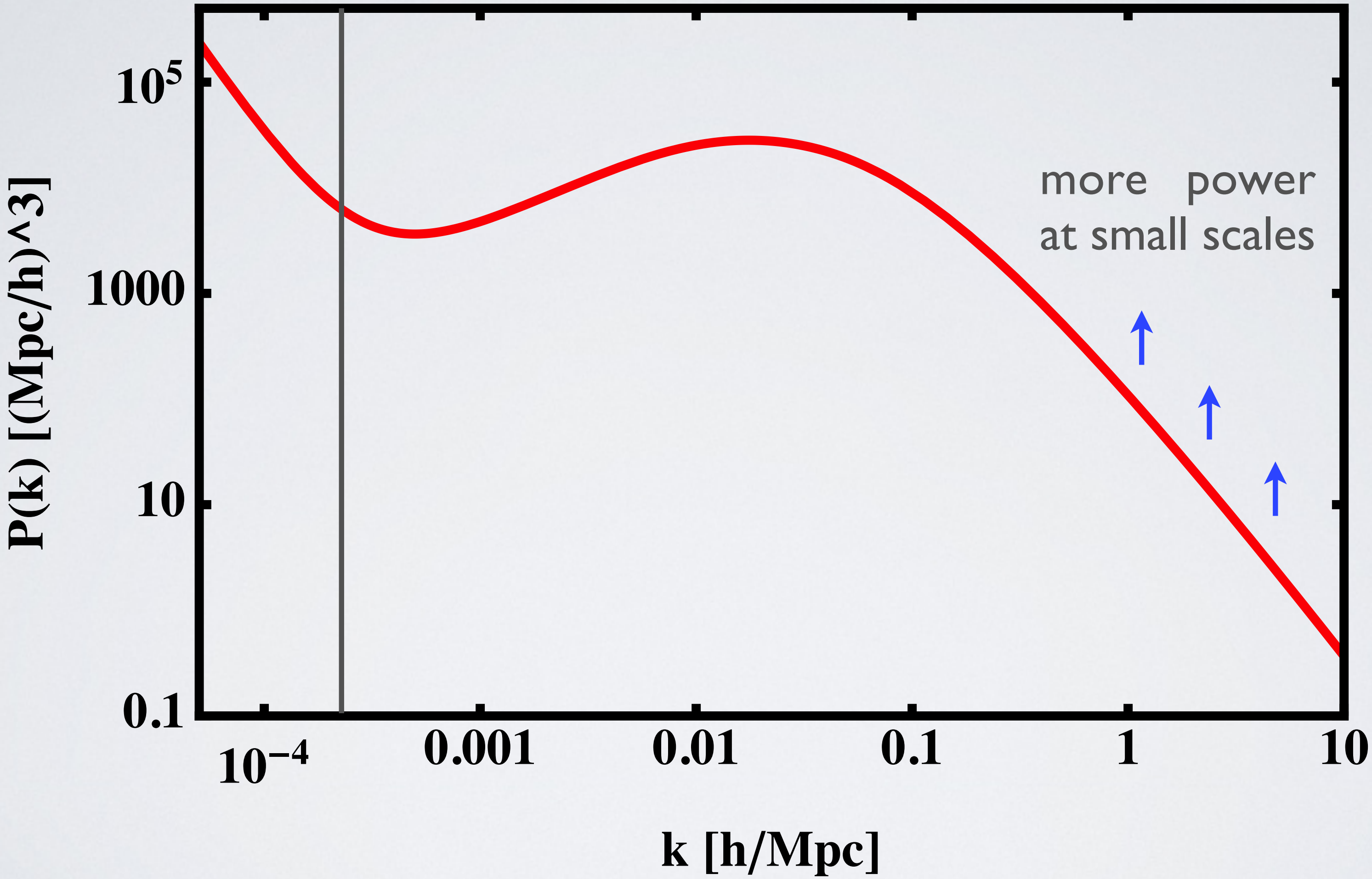
Beyond our approximations

Baryons



Beyond our approximations

Non-linearities



Relativistic effects

- The standard calculation of the power spectrum keeps only track of **dominant terms** in $k/\mathcal{H}$.
- Surveys become more **precise** and observe **larger area**

  → subdominant terms may become important.

- Relativistic derivation of the **galaxy number counts**.
- Impact on angular power spectrum and correlation function.
- New techniques to **isolate** relativistic effects.
- Impact of relativistic effects on **21cm intensity mapping** and **lensing**.

Evolution of perturbations

Initial conditions

At the **end of inflation**: we have fluctuations in the energy density of matter and radiation.

We want to understand how these initial fluctuations **evolve** to give rise to the large-scale structure.

The properties of the large-scale structure depend on the properties of the initial fluctuations.

We need to know the characteristics of the initial fluctuations. The details depend on the model of inflation but the **general characteristics** are common.

Properties of initial fluctuations

Properties expressed in terms of the primordial **gravitational potential** Φ_p , related to the density via Poisson equation.

In Fourier space: $$\Phi_p(\mathbf{k}) = \int d^3\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}}\, \Phi_p(\mathbf{x})$$

$$\langle \Phi_p(\mathbf{k})\Phi_p(\mathbf{k}')\rangle = (2\pi)^3 P(k)\delta_D(\mathbf{k}+\mathbf{k}')$$

statistical homogeneity and isotropy

Nearly **scale-invariant** spectrum: $P(k) = \frac{A}{k^3}\left(\frac{k}{k_*}\right)^{n_s-1}$

Gaussian field:

- Completely determined by the two-point function.
- Three-point function vanishes.

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Nearly **scale-invariant** spectrum:

Power per logarithmic k-bins

$$k^3 P(k) \simeq \text{const}$$

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Properties of initial fluctuations

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$$\langle \Phi_p(\mathbf{k})\Phi_p(\mathbf{k}')\rangle = (2\pi)^3 P(k)\delta_D(\mathbf{k}+\mathbf{k}')$$

slightly smaller than 1

Nearly **scale-invariant** spectrum: $$P(k) = \frac{A}{k^3}\left(\frac{k}{k_*}\right)^{n_s-1}$$

Gaussian field:

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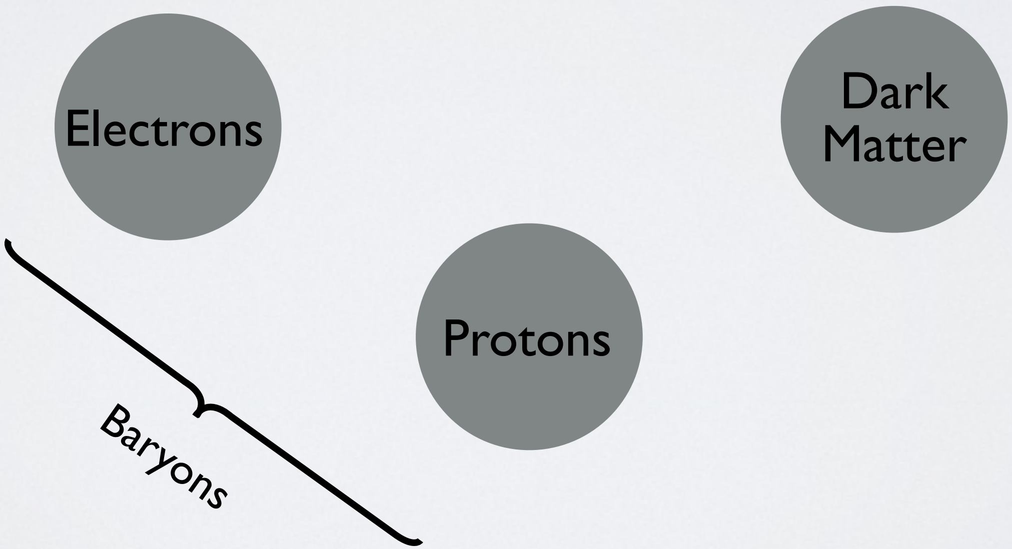
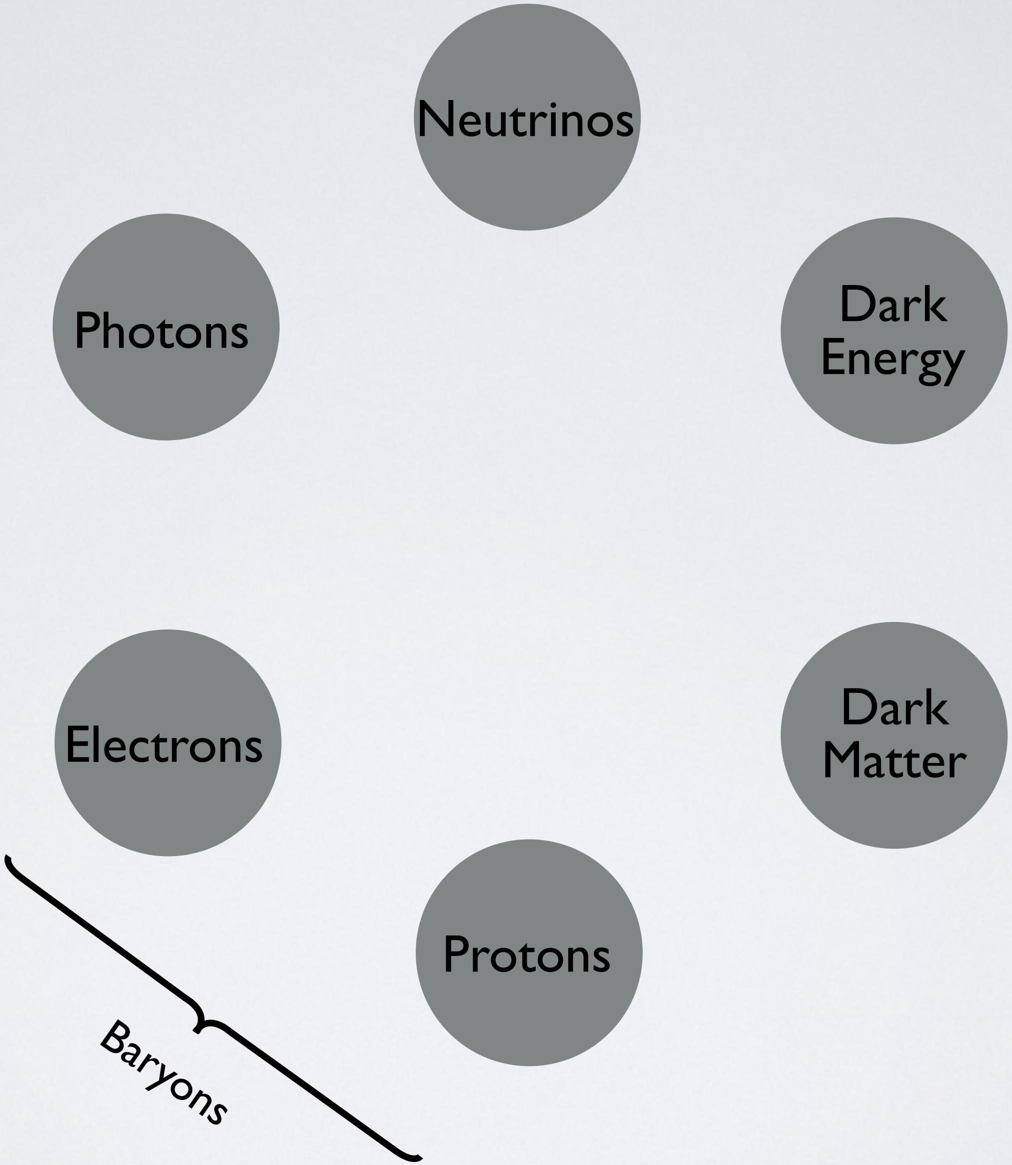
General interactions

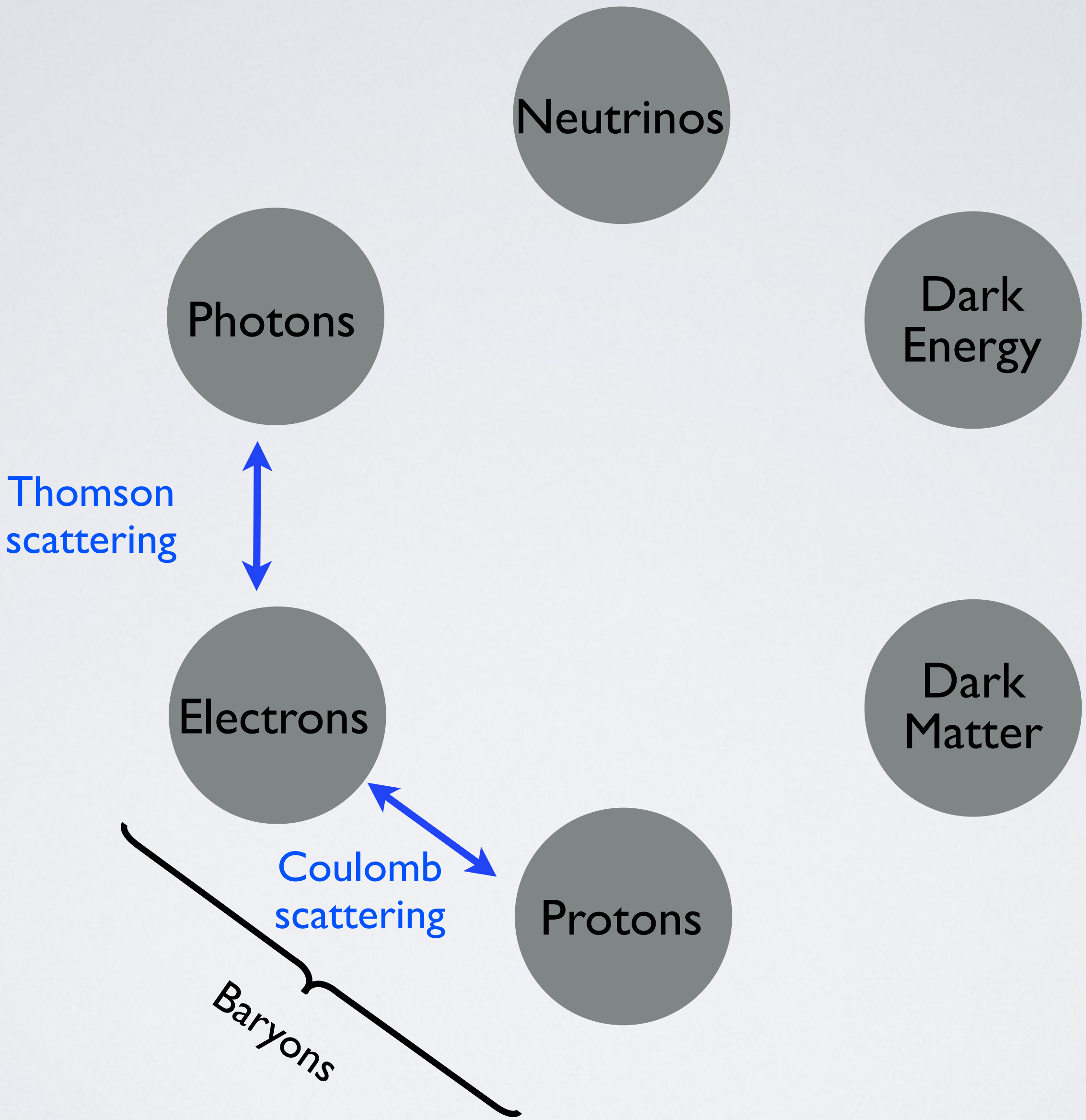
$$N(\mathbf{x}, t) \leftrightarrow \rho_m(\mathbf{x}, t)$$



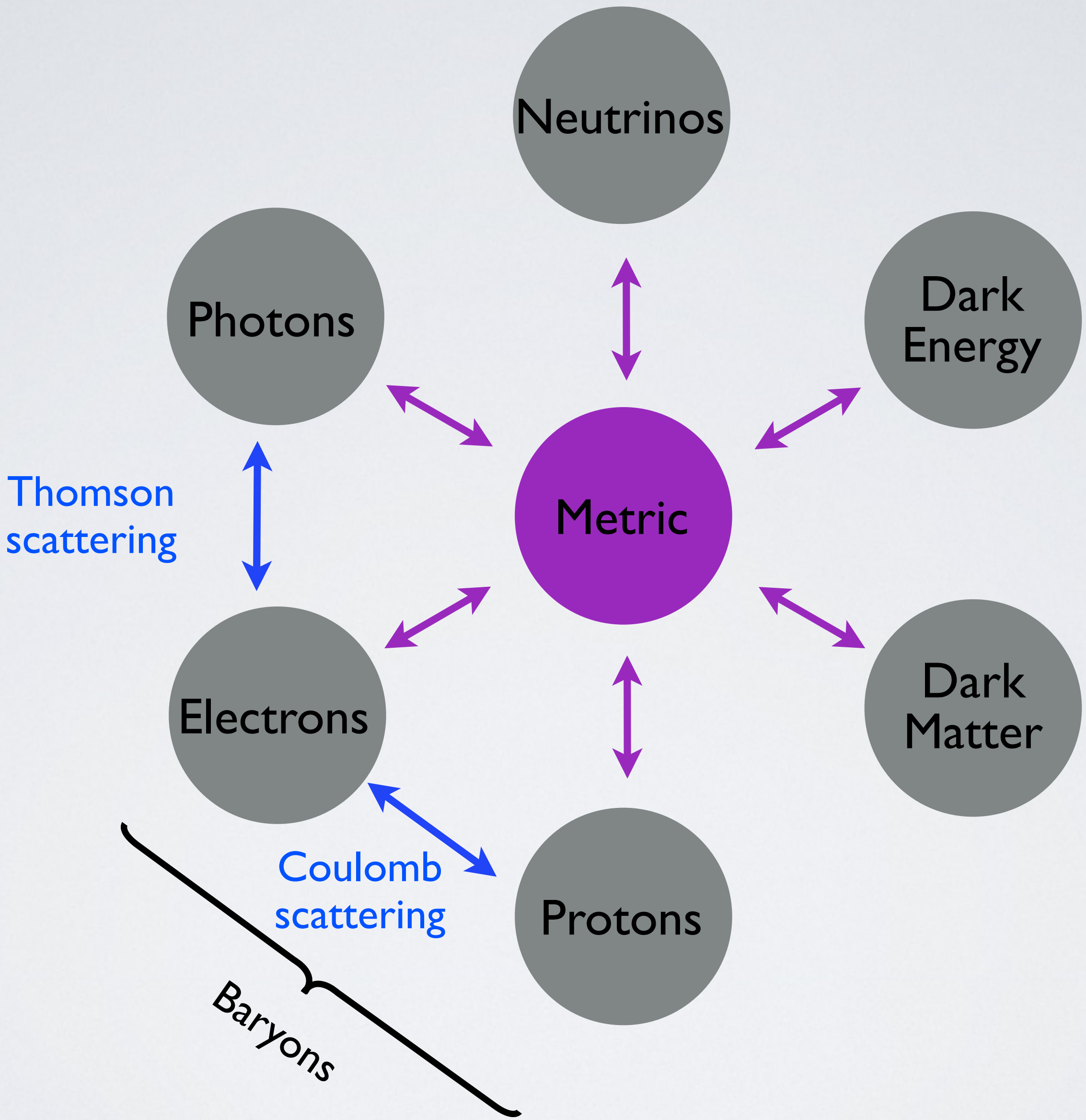
General interactions



General interactions



General interactions



Evolution

Evolution of various components, from inflation up to today.

We **split** the variables into **background** plus **perturbations**.

Example: photon **energy density**

$$\rho_\gamma(\mathbf{x},t) = \underbrace{\bar\rho_\gamma(t)}_{\text{average density}} + \underbrace{\delta\rho_\gamma(\mathbf{x},t)}_{\text{inhomogeneities}}$$

Photon distribution described by:

- **Pressure** $P_\gamma(\mathbf{x},t) = \bar P_\gamma(t) + \delta P_\gamma(\mathbf{x},t)$
- **Velocity**: background velocity encoded in expansion $a(t)$ → Hubble flow

  peculiar velocity, due to inhomogeneities $v^i_\gamma(\mathbf{x},t)$

Evolution

Similar split for baryons, dark matter, neutrinos and dark energy.

The **metric** can also be split.

- Friedmann universe: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$
- Perturbations (Newtonian gauge):

$$ds^2 = -\Big(1 + 2\Psi(\mathbf{x}, t)\Big)dt^2 + a^2(t)\Big(1 - 2\Phi(\mathbf{x}, t)\Big)\delta_{ij}dx^i dx^j$$

two gravitational potentials

We want a set of **equations** describing the **evolution** and **interaction** between matter, radiation and metric:

- Boltzmann equations
- Einstein's equations

Background

$$\bar{\rho}_i(t), \quad \bar{P}_i(t), \quad a(t), \qquad i = b,\, dm,\, \gamma,\, \nu,\, de$$

Einstein's equations relate the geometry to the content.

- **Friedmann equations**:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\bar{\rho} \qquad \text{and} \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\bar{\rho} + 3\bar{P})$$

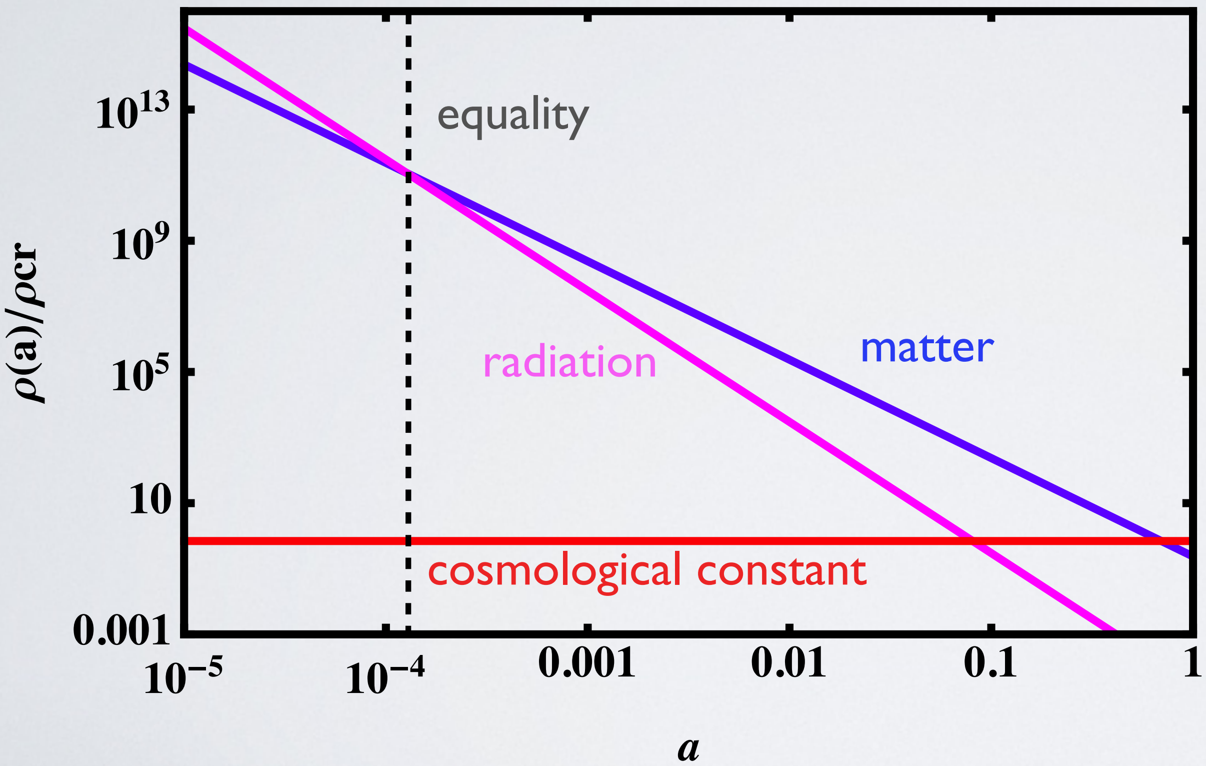
The expansion $a(t)$ is determined by the total energy density $\bar{\rho}(t) = \sum_i \bar{\rho}_i(t)$ and pressure $\bar{P}(t) = \sum_i \bar{P}_i(t)$

- **Conservation equations** for each components:

$$\dot{\bar{\rho}}_i = -3H(\bar{\rho}_i + \bar{P}_i)$$

Density evolution

- Radiation: photons and neutrinos $\bar{\rho}_r \sim a^{-4}$
- Matter: baryons and dark matter $\bar{\rho}_m \sim a^{-3}$
- Dark energy $\bar{\rho}_\Lambda \sim \text{const}$



Three distinct **stages** for the evolution of universe:

- radiation era
- matter era
- dark energy era

During each stage we know $a(t)$ and $H(t)$.

Perturbations

Boltzmann equations describe interactions between particles and evolution under gravitation.

Coupled system: Boltzmann equations plus Einstein's equation.

To describe the evolution of **large-scale structure**, we can follow the evolution of **dark matter** only. We assume that baryons fall into the potential well generated by dark matter.

♦ Dark matter only interacts **gravitationally**.

→ Energy-momentum conserved.

♦ **Cold dark matter**: $\delta P_{dm} = 0$

→ Two quantities describe DM: $\delta_{dm} = \dfrac{\delta\rho_{dm}}{\bar{\rho}_{dm}}$ and v^i_{dm}

Dark matter perturbations

$$ds^2 = -a^2(\eta)\Big(1 + 2\Phi(\mathbf{x},\eta)\Big)d\eta^2 + a^2(\eta)\Big(1 - 2\Phi(\mathbf{x},\eta)\Big)\delta_{ij}dx^i dx^j$$

Conformal time $d\eta = \frac{dt}{a}$ and Fourier space

- **Continuity equation**: conservation of energy

$$\delta'_{dm} = kv_{dm} + 3\Phi'$$

- **Euler equation**: conservation of momentum

$$v'_{dm} + \mathcal{H}v_{dm} = -k\Phi$$

$$\boxed{\delta''_{dm} + \mathcal{H}\delta'_{dm} = -k^2\Phi + 3\mathcal{H}\Phi' + 3\Phi''}$$

dilution from expansion

gravitational term

Gravitational potential

Evolution of the gravitational potential

$$\boxed{\Phi'' + 3(1 + c_S^2)\mathcal{H}\Phi' + c_S^2 k^2 \Phi = 0}$$

valid for constant $c_S^2 = w$

Depends on:

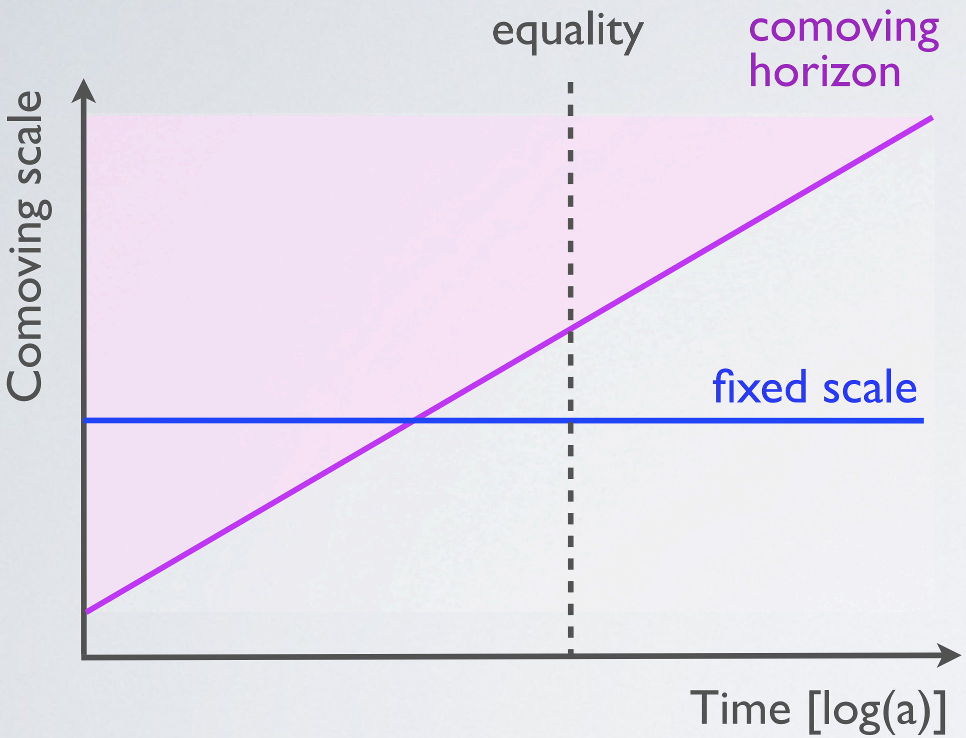
- **Hubble parameter**: how the universe expands. This differs in a radiation-dominated universe and a matter-dominated universe.

- **Sound speed** c_S^2 : how the fluids cluster $\quad c_S^2 = \dfrac{\delta P}{\delta \rho}$

→ We solve the equation in different steps.

Super-horizon scales

Just after inflation, all the modes of interests are outside the horizon $\lambda \sim k^{-1} \gg d_H \sim \mathcal{H}^{-1} \quad \rightarrow \quad k \ll \mathcal{H}$



negligible

$$\Phi'' + 3(1+c_S^2)\mathcal{H}\Phi' + \cancel{c_S^2 k^2}\Phi = 0$$

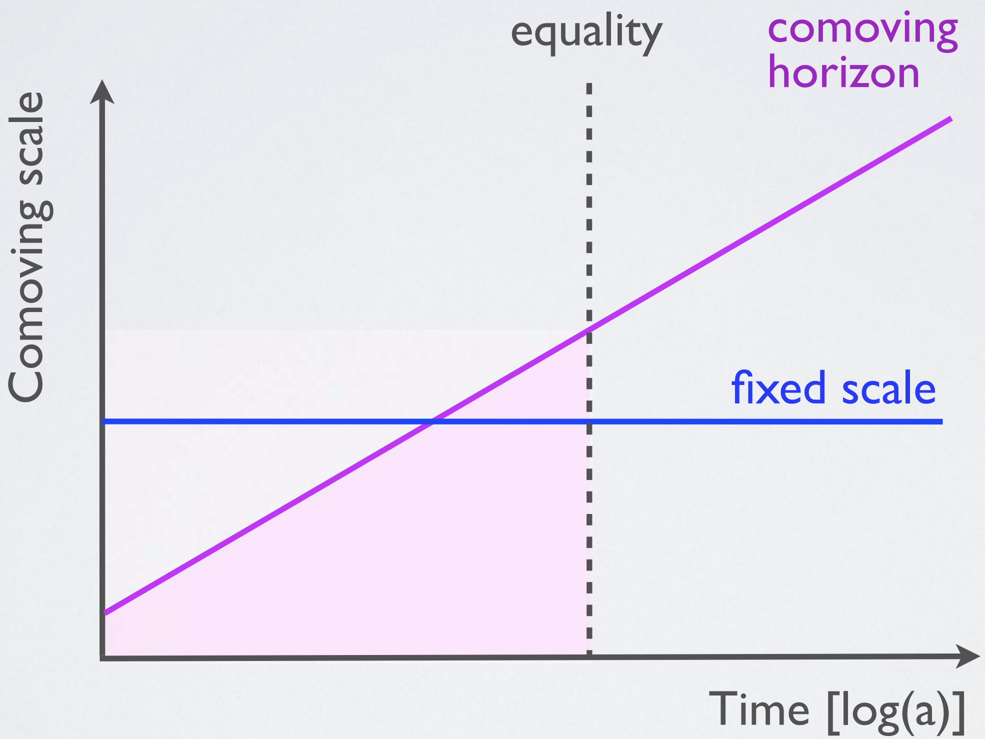
$$\boxed{\Phi(\mathbf{k},\eta) = \text{const}}$$ frozen

Initial conditions: $\Phi_p(\mathbf{k}) \sim k^{-3/2}$

At the transition from radiation to matter era,
$\Phi(\mathbf{k},\eta)$ decreases by $9/10$

Radiation era

As the universe expands, the **horizon grows**, modes start to enter inside the horizon $k > \mathcal{H}$ and the perturbations evolve.



Radiation era

As the universe expands, the **horizon grows**, modes start to enter inside the horizon $k > \mathcal{H}$ and the perturbations evolve.

$$\Phi'' + 3(1+c_S^2)\mathcal{H}\Phi' + c_S^2 k^2 \Phi = 0$$

$\mathcal{H} = 1/\eta$ and $c_S^2 = 1/3$

Solution well **inside** the **horizon**:

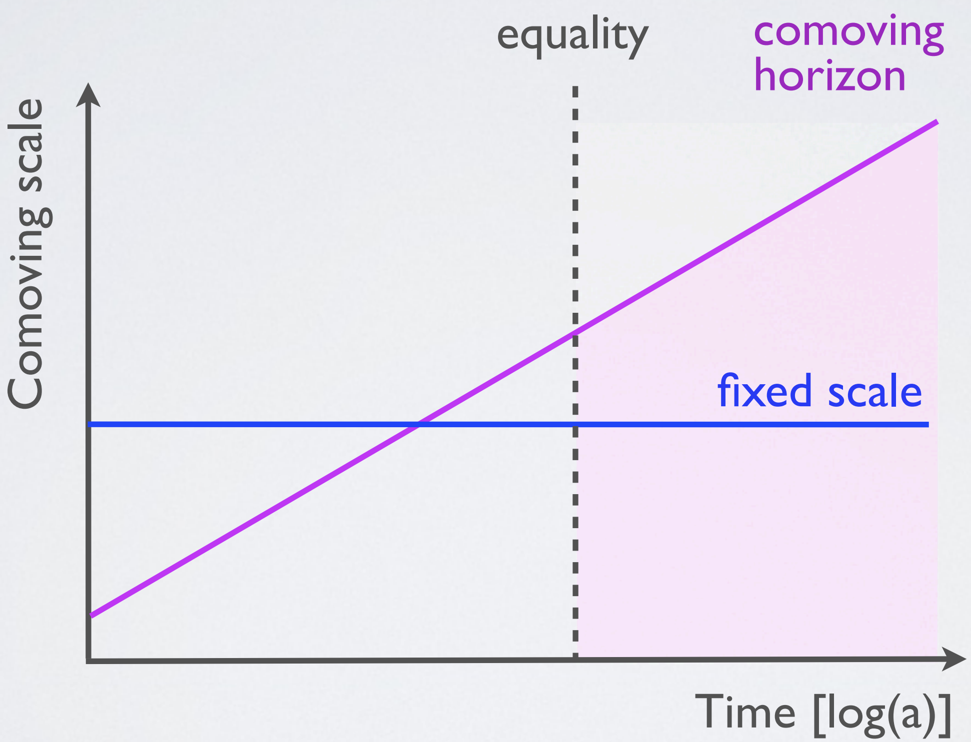
$$\Phi(\mathbf{k},\eta) = -\Phi_p(\mathbf{k})\left(\frac{a_{eq}}{\eta_{eq}}\right)^2 \cos\left(\frac{k\eta}{\sqrt{3}}\right)\frac{1}{(ka)^2}$$

oscillates due to radiation pressure

decreases due to expansion

Matter era

Finally, at late time the universe is dominated by matter.



Matter era

$$\Phi'' + 3(1 + c_S^2)\mathcal{H}\Phi' + c_S^2 k^2 \Phi = 0$$

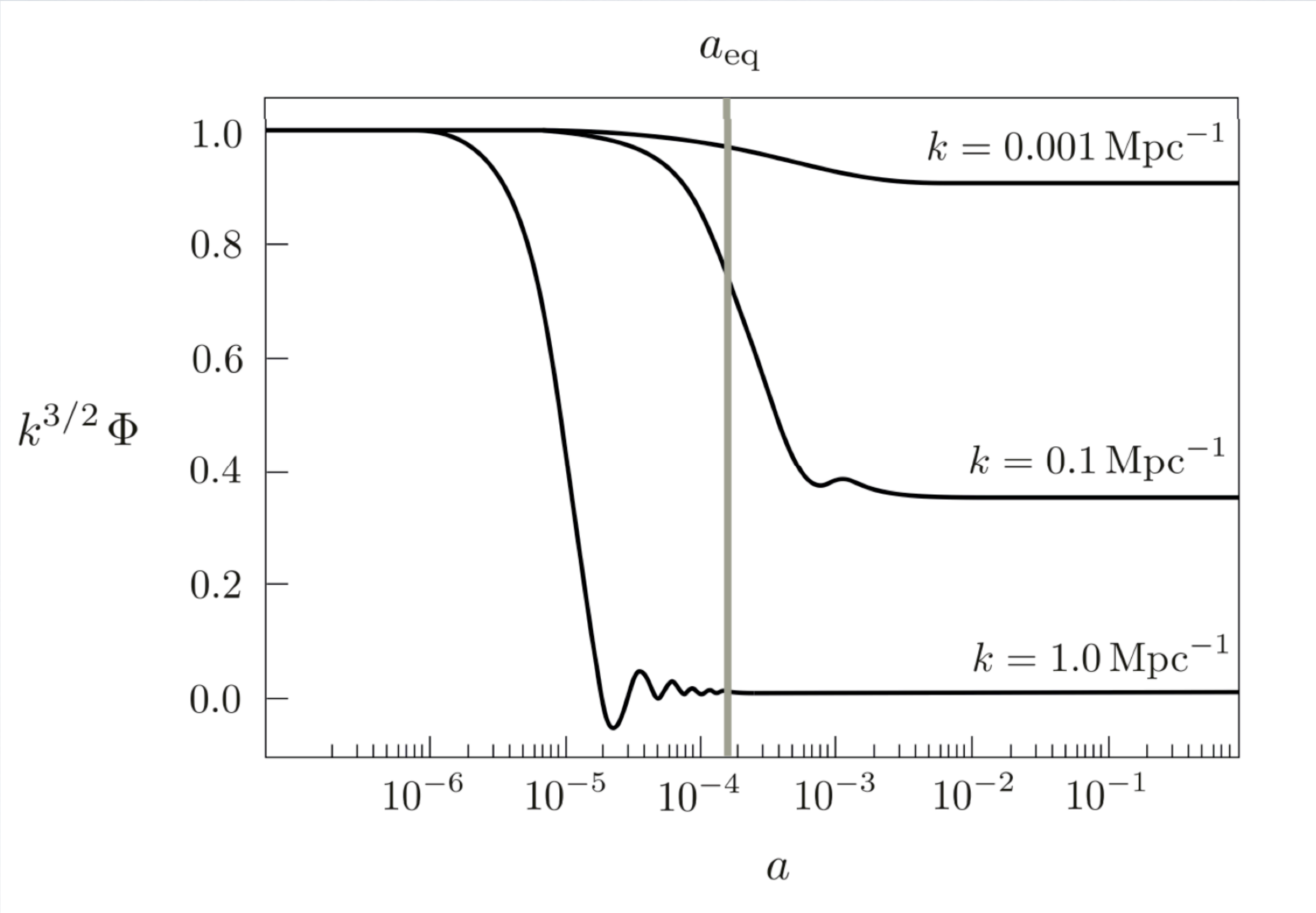
$\mathcal{H} = 2/\eta$ and $c_S^2 = 0$

$$\Phi'' + \frac{6}{\eta}\Phi' = 0$$

Solution (at all scales) $\Phi = \mathrm{const}$

No pressure to counteract gravity, but the expansion prevents the potential to grow.

Summary: potential evolution



Credit: D. Baumann, cosmology lecture

Modes which enter earlier have more time to decay before the transition to the matter era.

Dark matter perturbations

We use the solution $\Phi(\mathbf{k},\eta)$ as a **source** for the density perturbations.

$$\delta''_{dm} + \mathcal{H}\delta'_{dm} = -k^2\Phi + 3\mathcal{H}\Phi' + 3\Phi''$$

Outside horizon $\quad k \ll \mathcal{H} \qquad \Phi = \text{const}$

$$\delta''_{dm} + \mathcal{H}\delta'_{dm} = 0 \qquad \delta_{dm} = \text{const}$$

No growth of structure **outside** the **horizon**.

Initial conditions through Poisson equation:

$$\delta = -\frac{2}{3}\cancel{\left(\frac{k}{\mathcal{H}}\right)}^2\Phi - 2\Phi - \cancel{\frac{2}{\mathcal{H}}}\Phi'$$

Adiabatic initial conditions: $\quad \delta_{dm} = \dfrac{3\delta}{4} = -\dfrac{3\Phi}{2}$

Radiation era

The potential oscillates. The solution for the dark matter density fluctuation is:

$$\delta_{dm} = 9\,\Phi_p(\mathbf{k})\Big[\log(k\eta) - 1/2\Big]$$

Dark matter perturbations grow **logarithmically** with a.

Not very efficient growth, due to the radiation pressure.

Modes which enter **earlier** inside the horizon have **more time to grow** before the transition to the matter era.

Matter era

The potential is constant. The equation for the dark matter density fluctuation is:

$$\delta''_{dm} + \frac{2}{\eta}\delta'_{dm} = -k^2\Phi$$

Combine with Poisson $-k^2\Phi = \frac{3}{2}\mathcal{H}^2\delta$

And use that, during matter domination: $\delta \simeq \delta_{dm}$

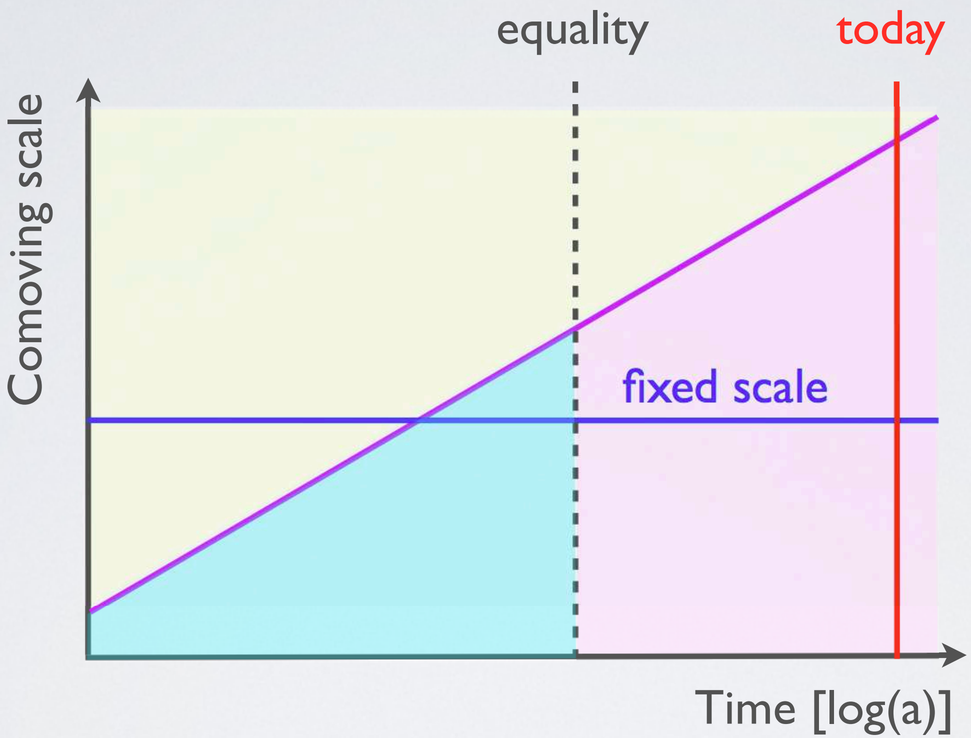
$$\delta''_{dm} + \frac{2}{\eta}\delta'_{dm} - \frac{6}{\eta^2}\delta_{dm} = 0$$

Solution: **linear growth** $\delta_{dm} \sim \eta^2 \sim a$

Growth of perturbation much **more efficient** than during the radiation era.

The power spectrum

We are interested in δ at a fixed time (today) but at different scales (different k).



$$\langle \delta(\mathbf{k}, \eta_0)\delta(\mathbf{k}', \eta_0)\rangle = (2\pi)^3 P_\delta(k, \eta_0)\delta_D(\mathbf{k} + \mathbf{k}')$$

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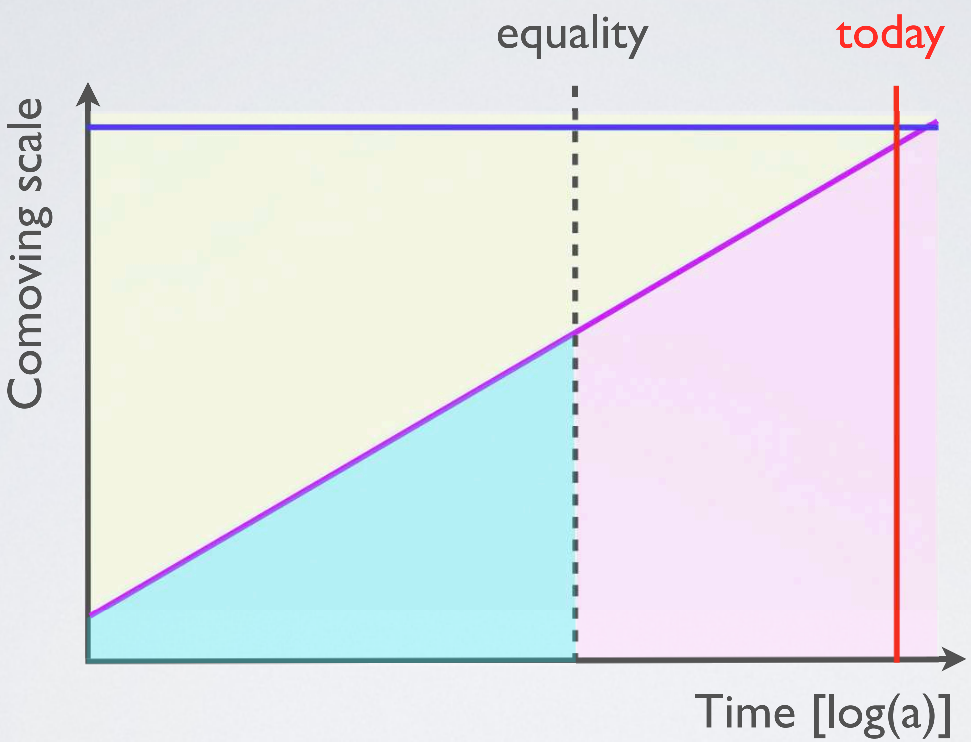
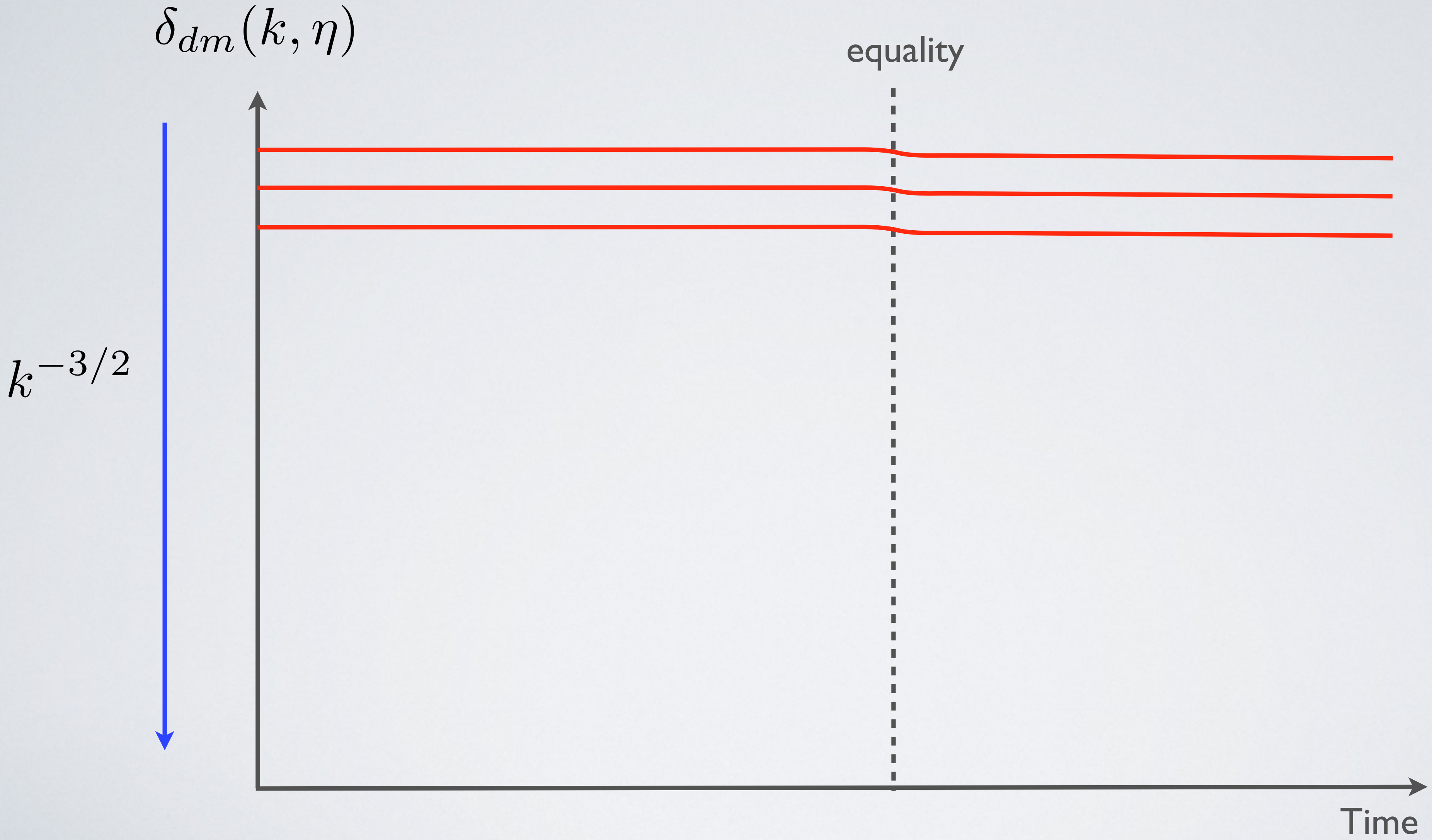


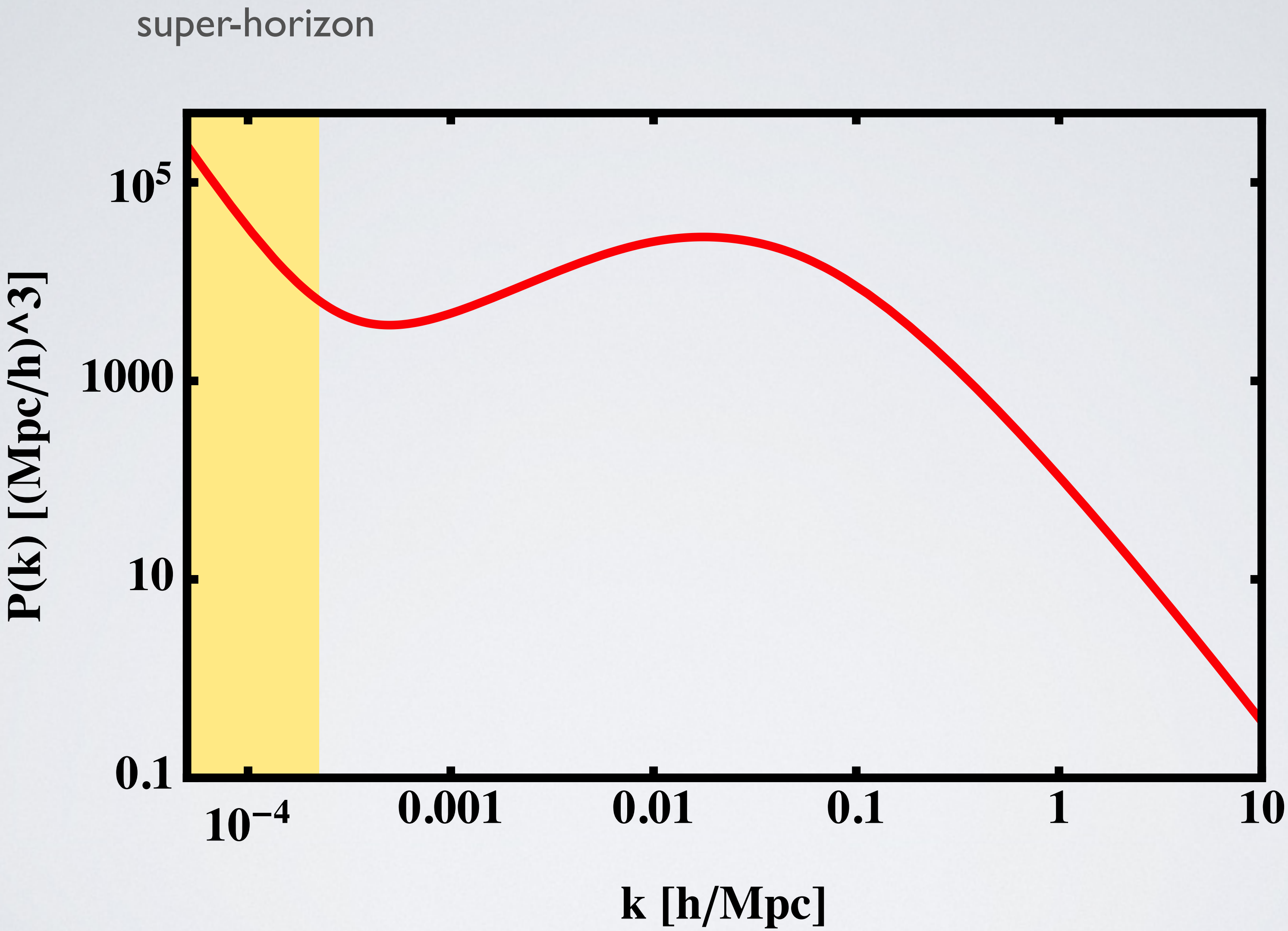
$$\langle\delta(\mathbf{k},\eta_0)\delta(\mathbf{k}',\eta_0)\rangle=(2\pi)^3P_\delta(k,\eta_0)\delta_D(\mathbf{k}+\mathbf{k}')$$

Dark Matter Power Spectrum

Outside the horizon



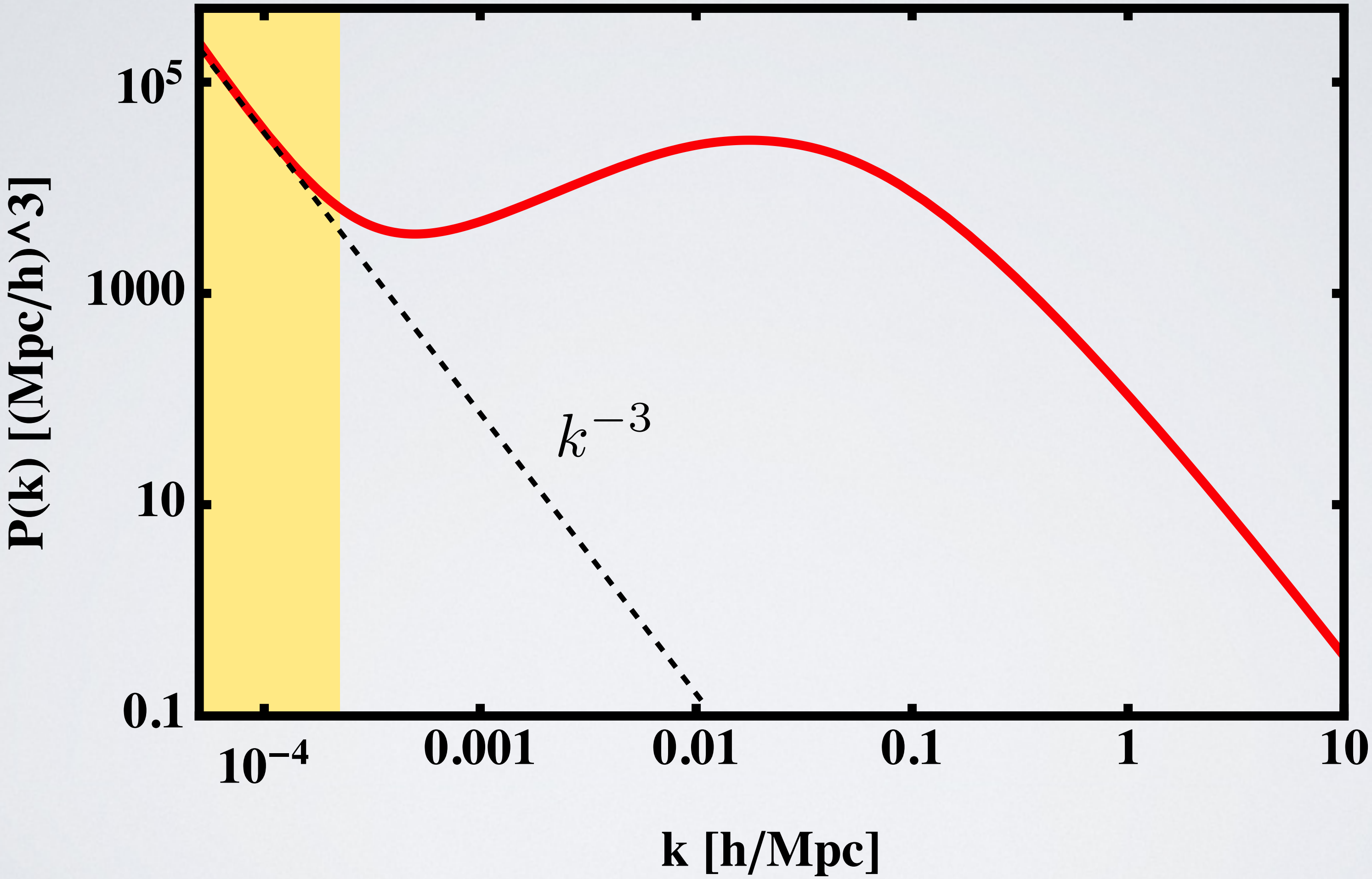
Dark Matter Power Spectrum



Dark Matter Power Spectrum

$$P_\delta \sim P \sim \frac{1}{k^3}$$

super-horizon



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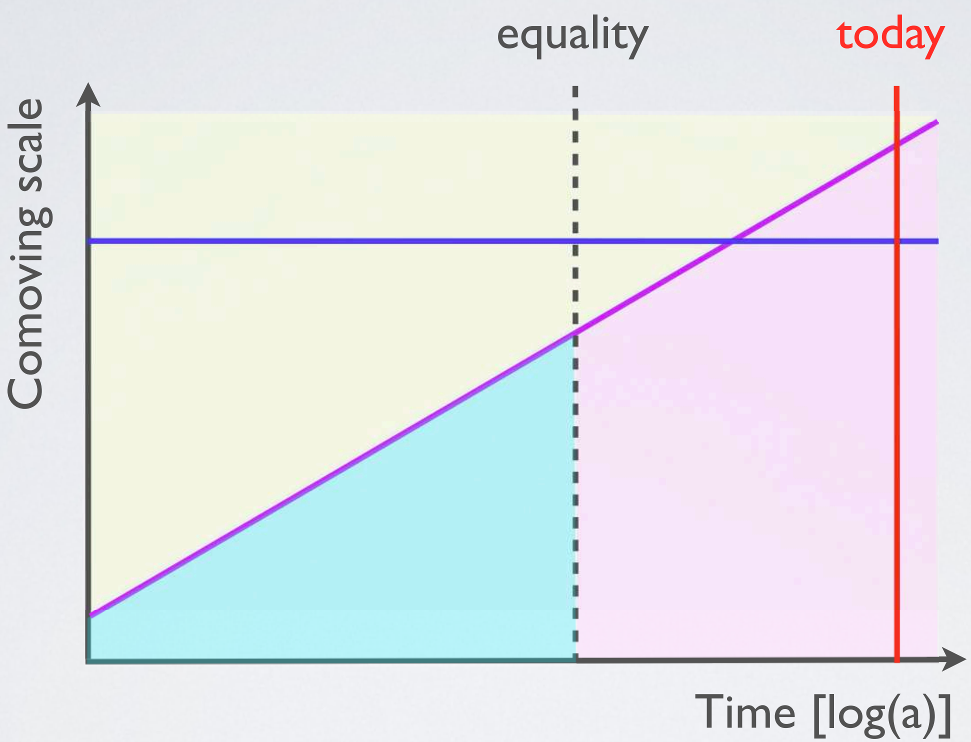
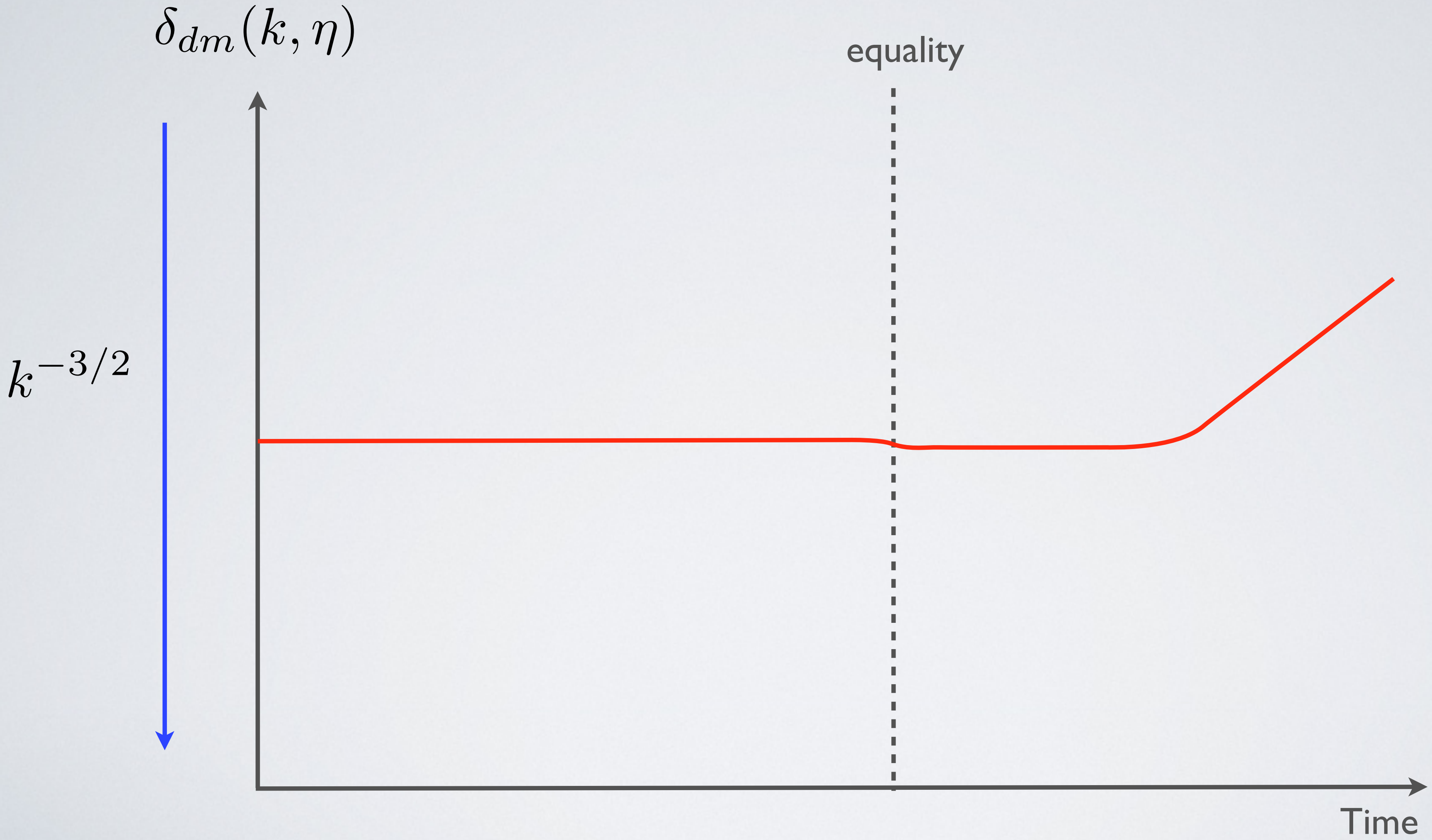


$$\langle \delta(\mathbf{k}, \eta_0)\delta(\mathbf{k}', \eta_0)\rangle = (2\pi)^3 P_\delta(k, \eta_0)\delta_D(\mathbf{k}+\mathbf{k}')$$

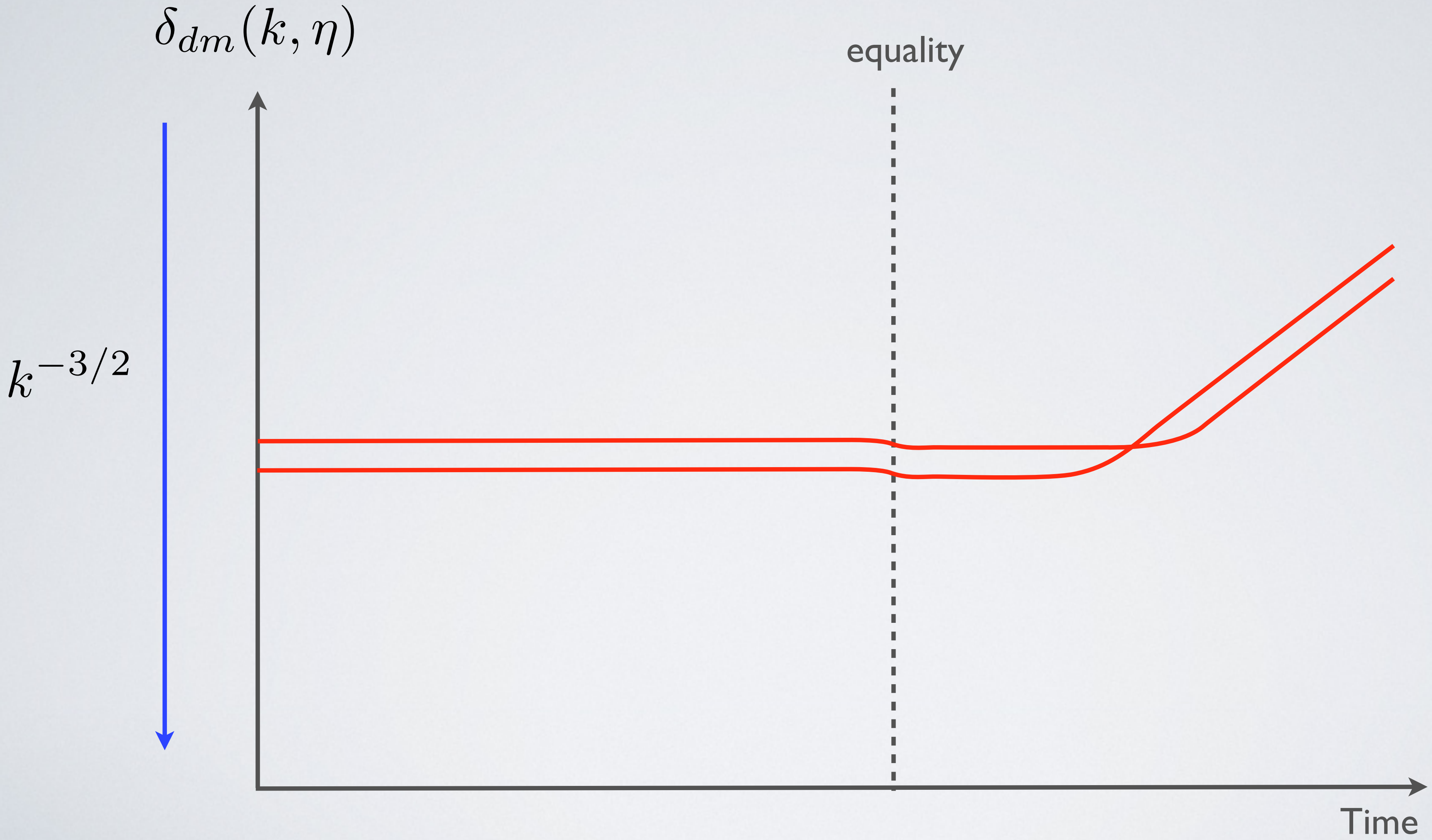
Dark Matter Power Spectrum

Modes entering the horizon after equality.



Dark Matter Power Spectrum

Modes entering the horizon after equality.



Dark Matter Power Spectrum

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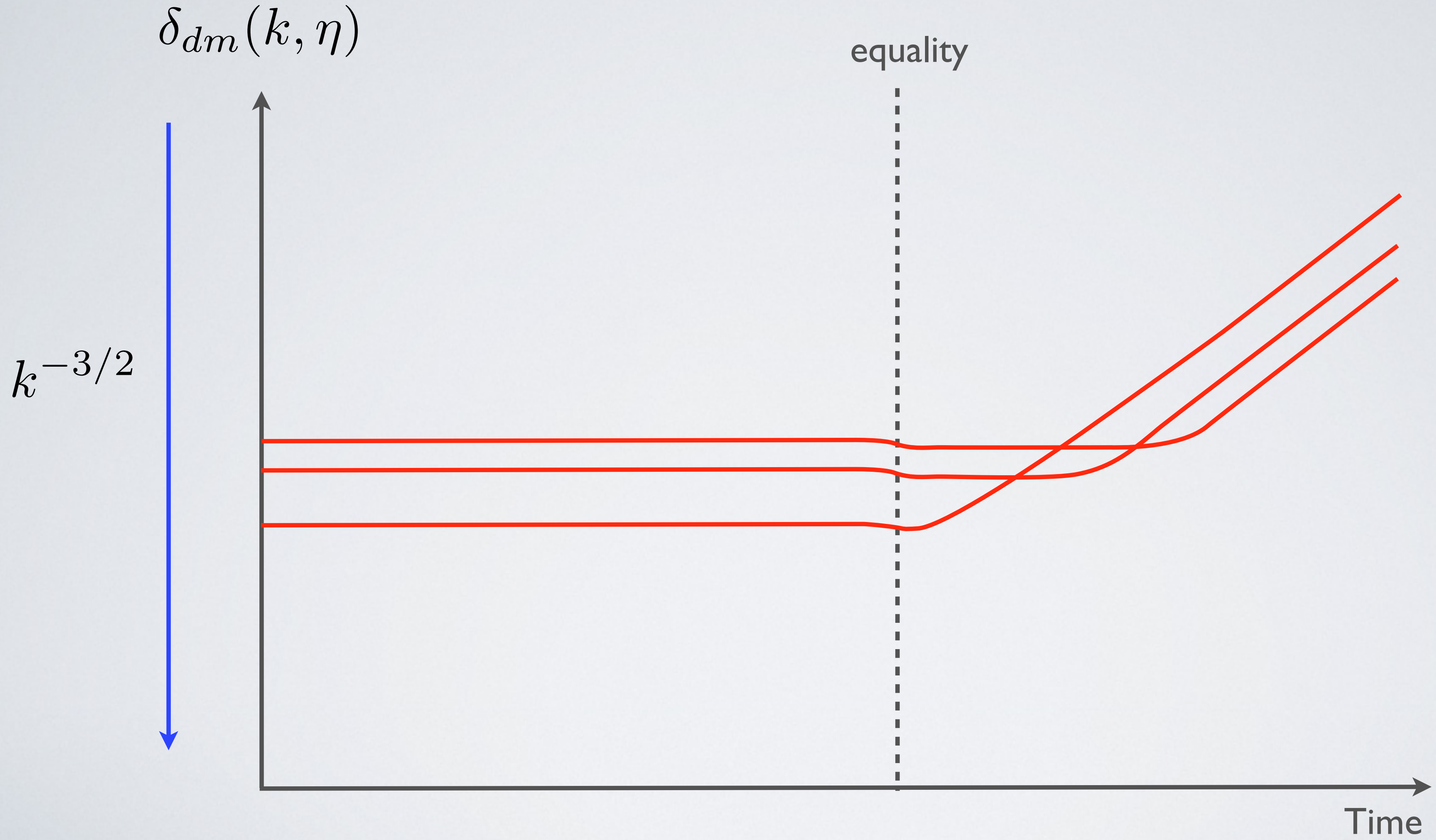
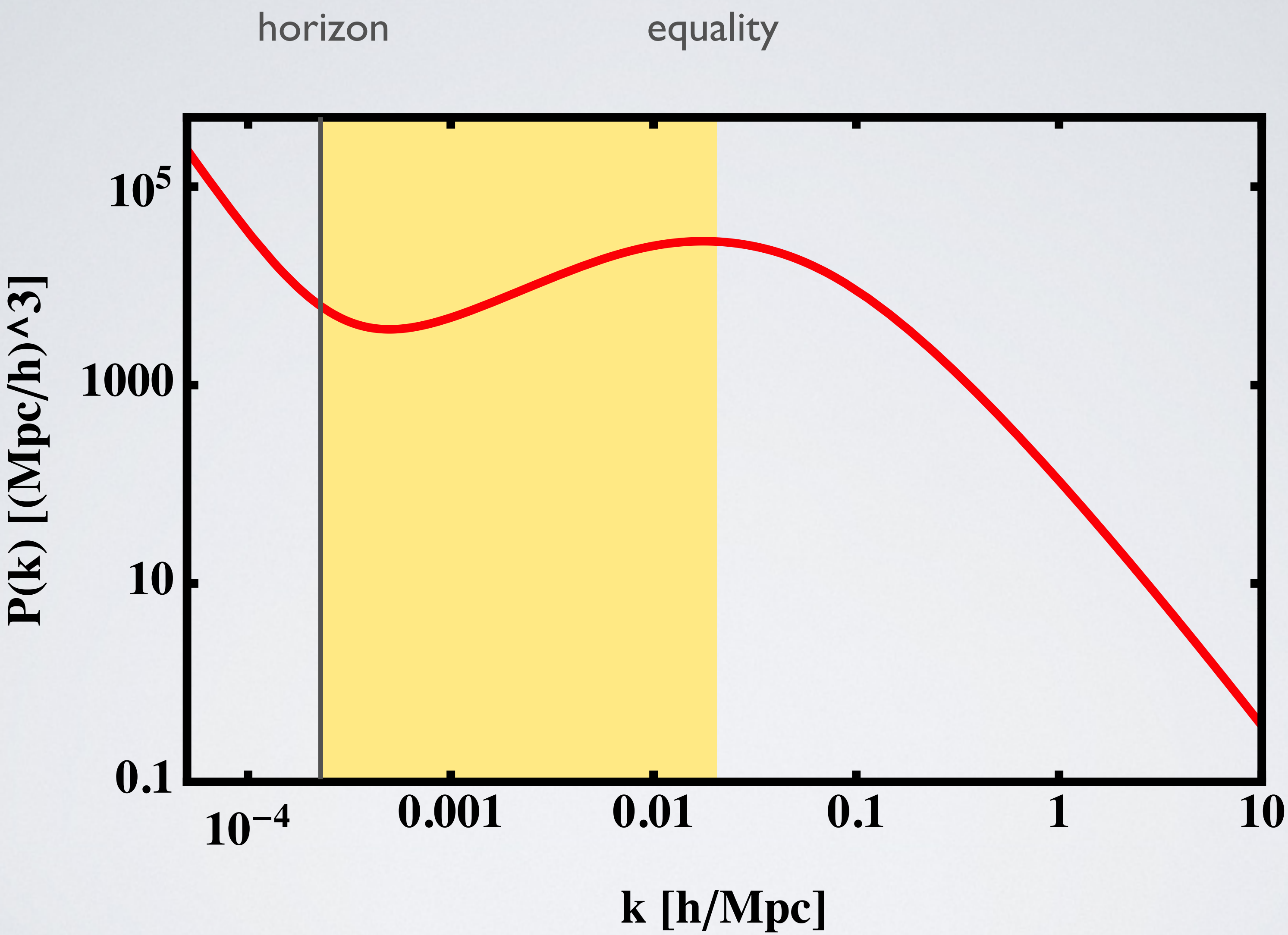


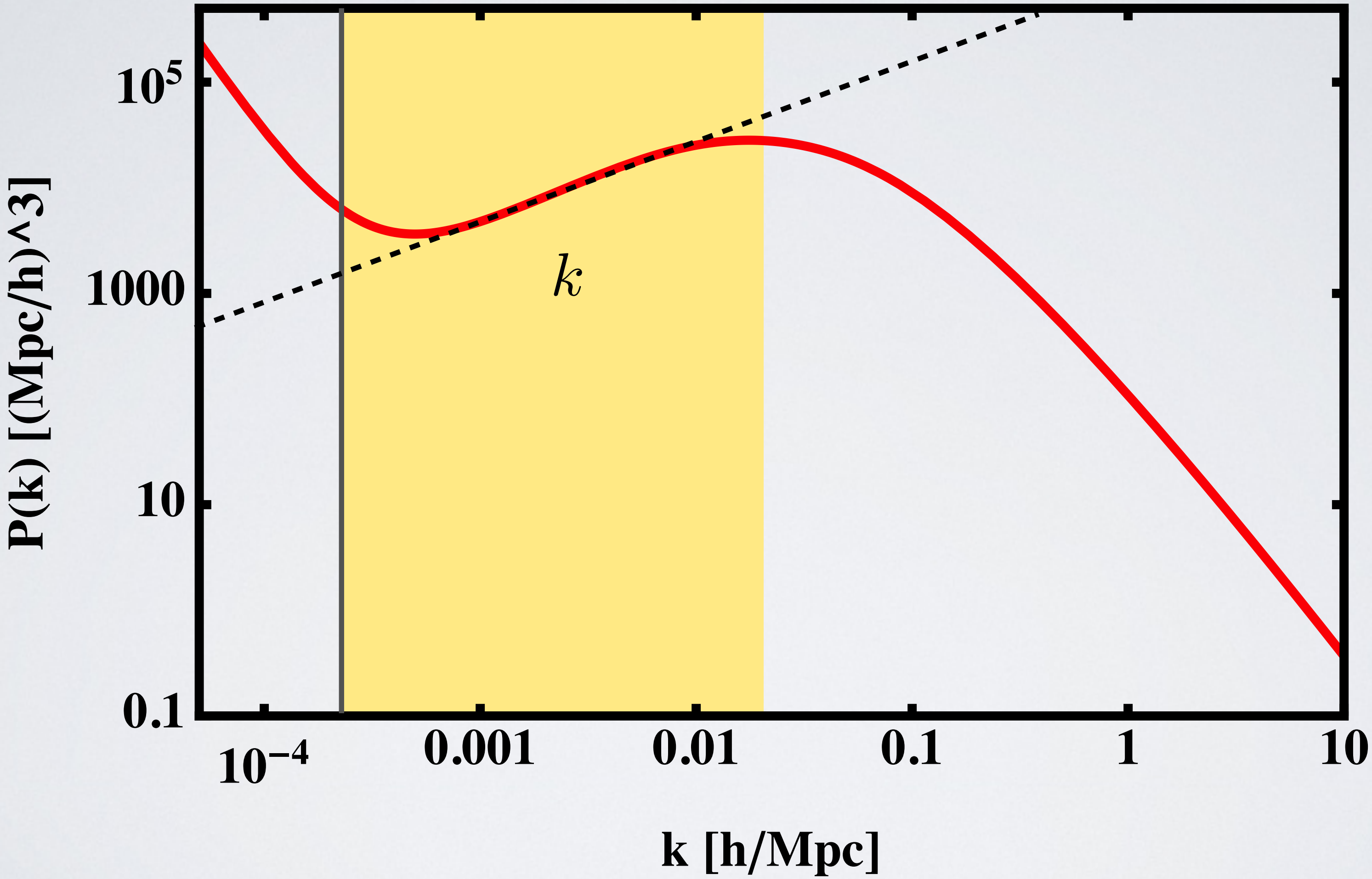
Dark Matter Power Spectrum



Dark Matter Power Spectrum

$$\delta \sim k^2\,\Phi \qquad P_\delta \sim k^4 P \sim k$$

horizon equality



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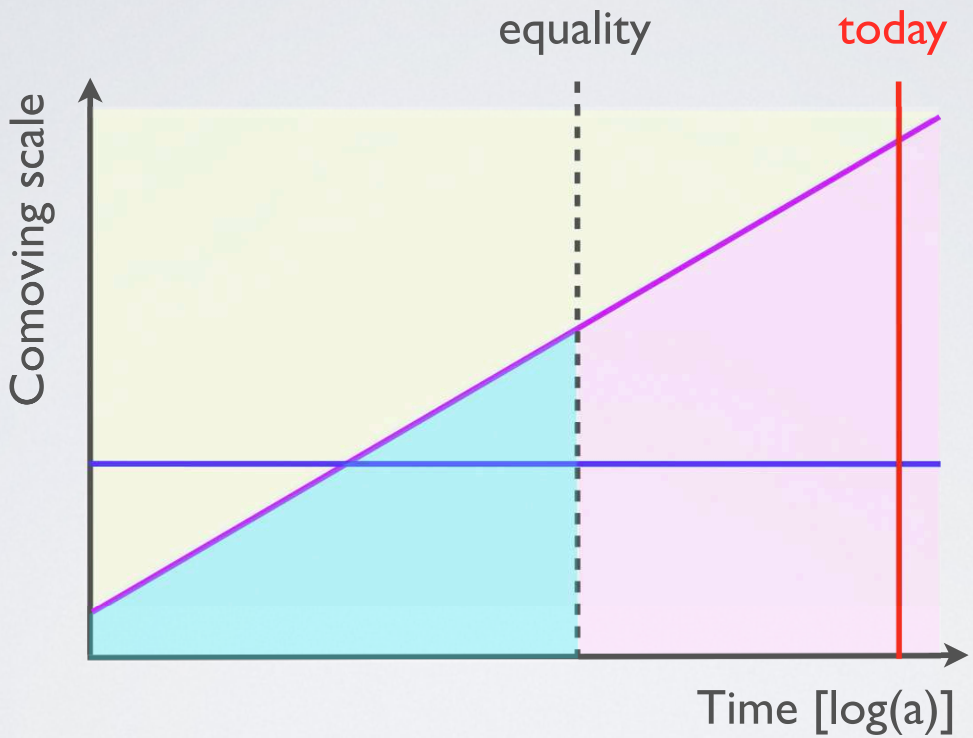
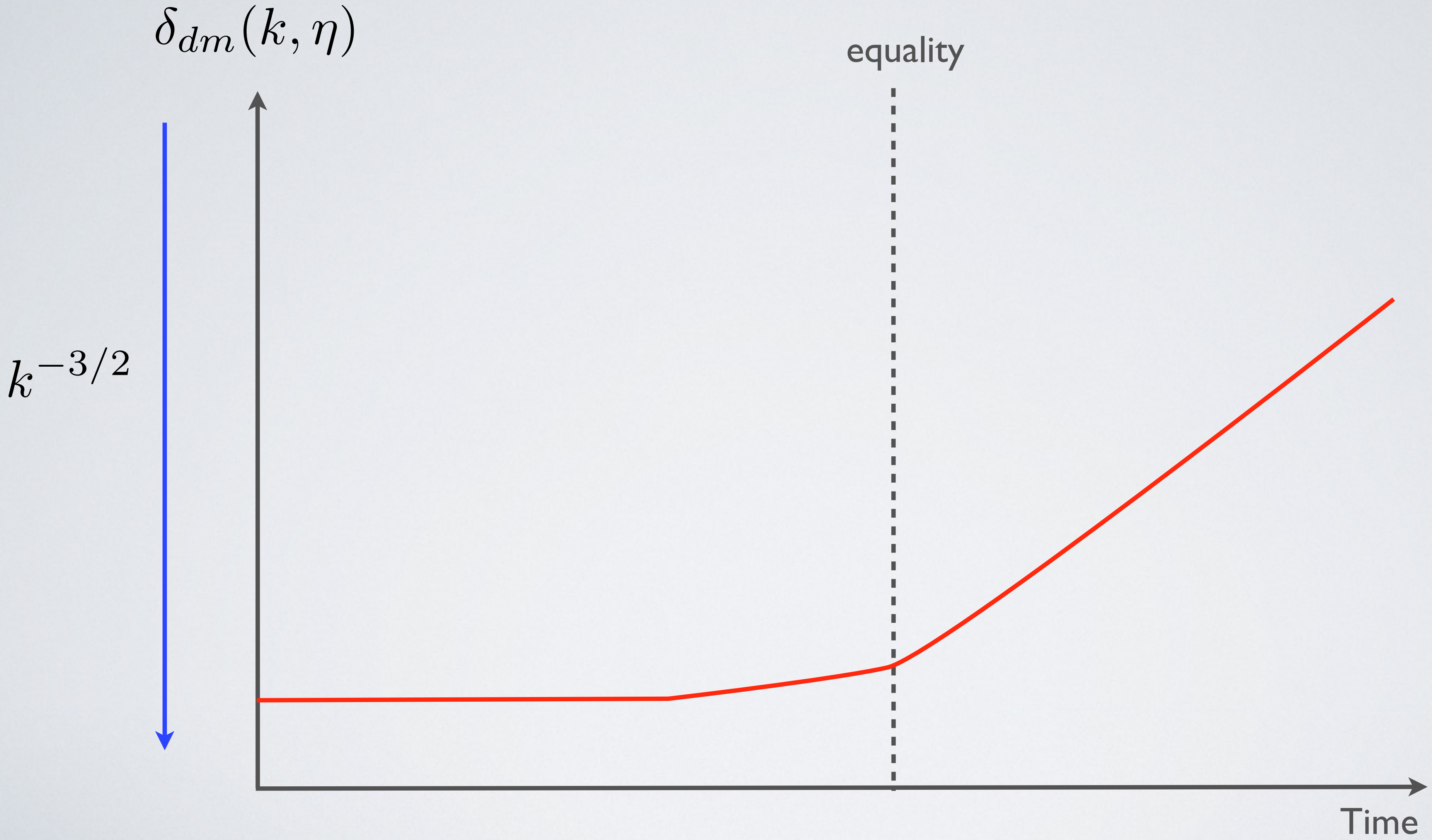


$$\langle \delta(\mathbf{k},\eta_0)\delta(\mathbf{k}',\eta_0)\rangle = (2\pi)^3 P_\delta(k,\eta_0)\delta_D(\mathbf{k}+\mathbf{k}')$$

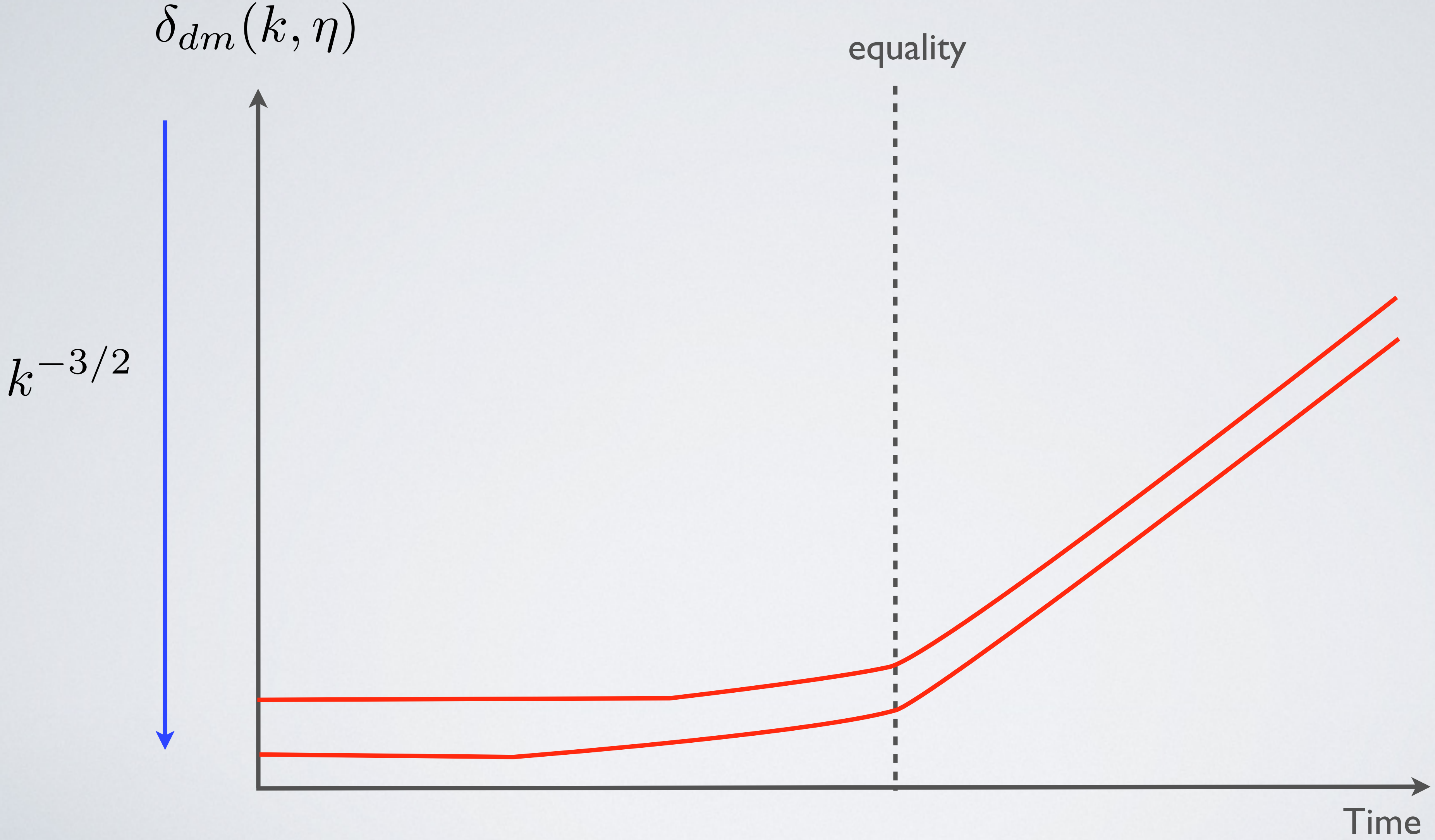
Dark Matter Power Spectrum

Modes entering the horizon before equality.

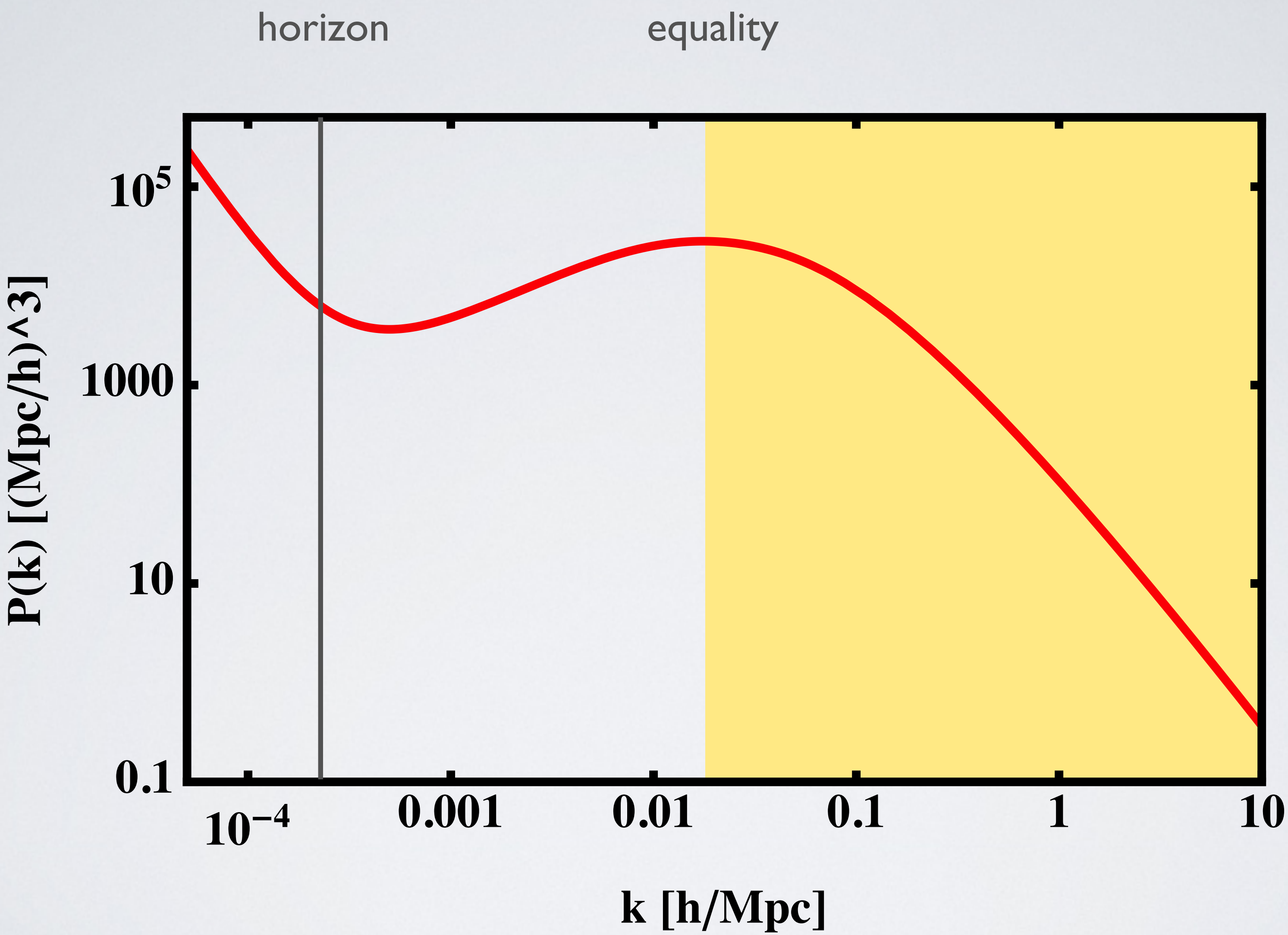


Dark Matter Power Spectrum

Modes entering the horizon before equality.



Dark Matter Power Spectrum



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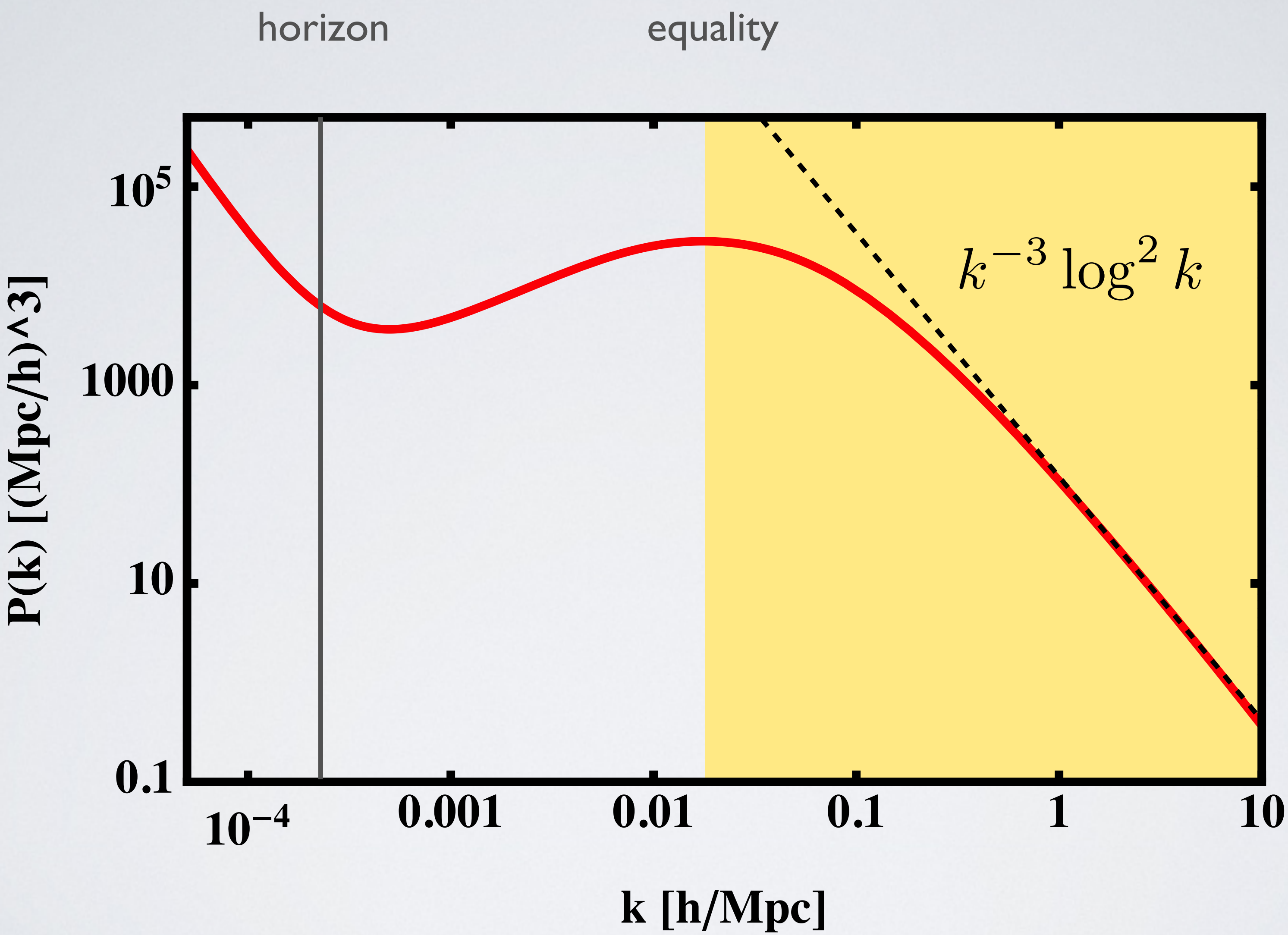
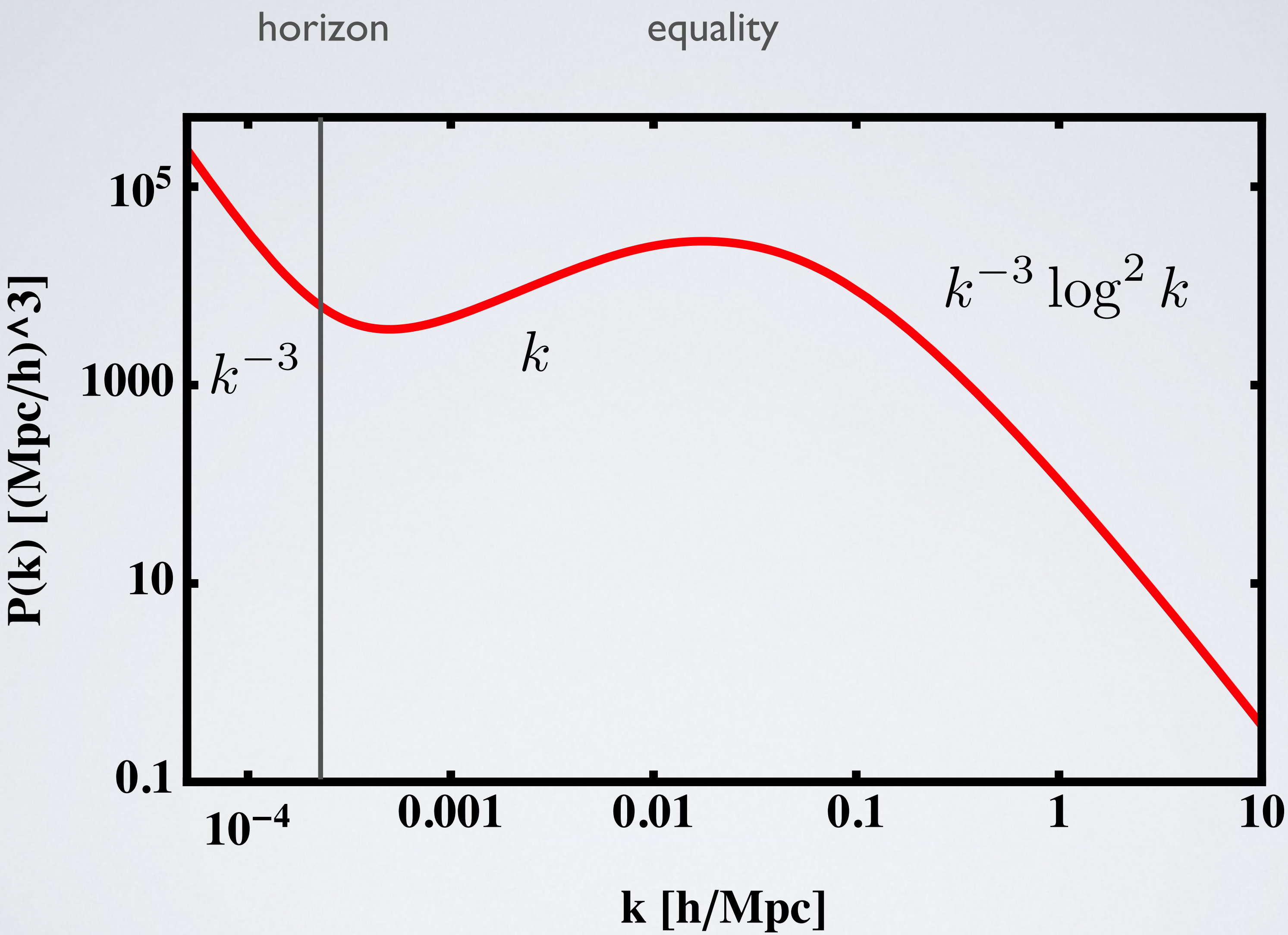
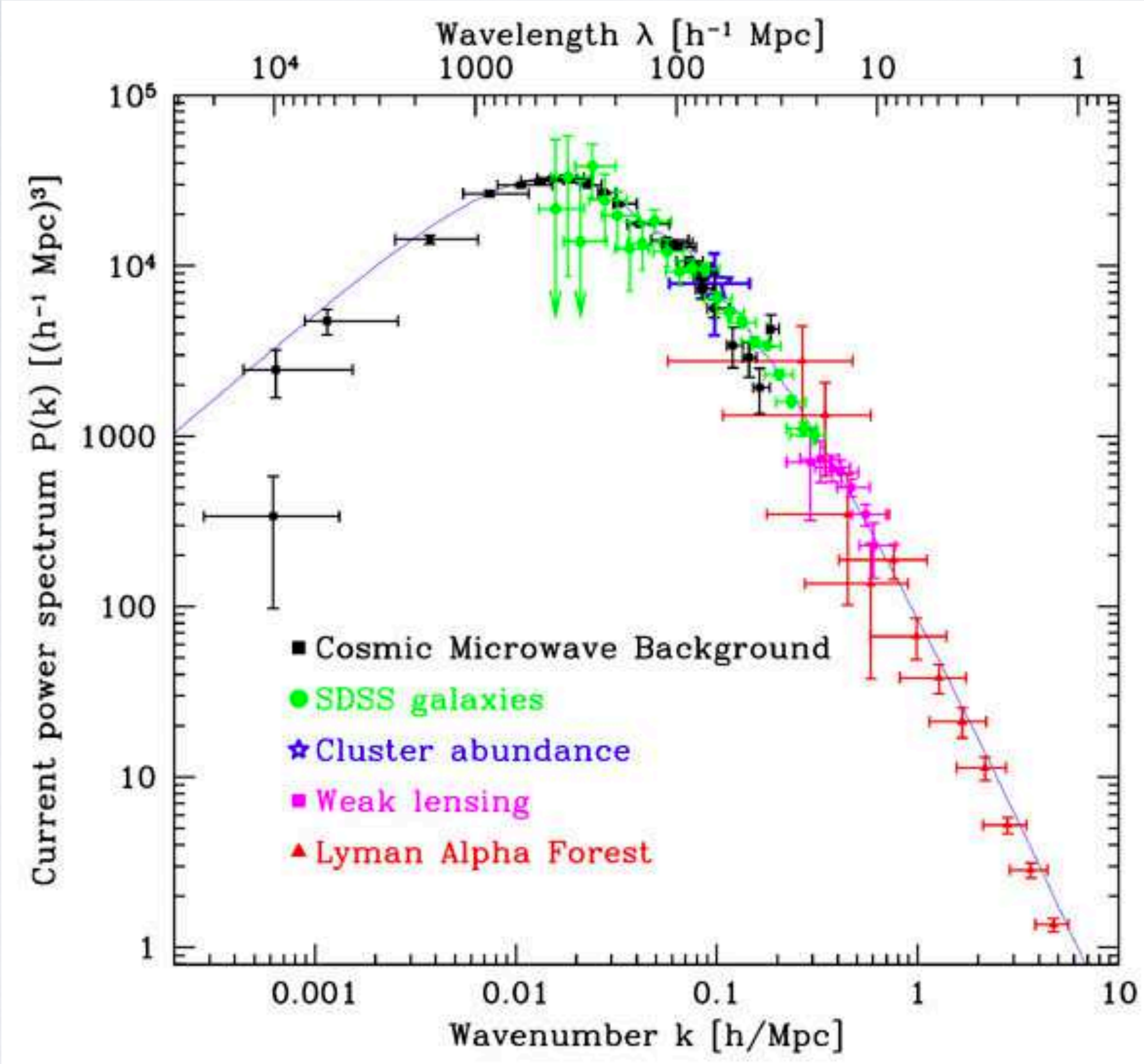


Dark Matter Power Spectrum



Observation

Credit: Tegmark et al. (2002)

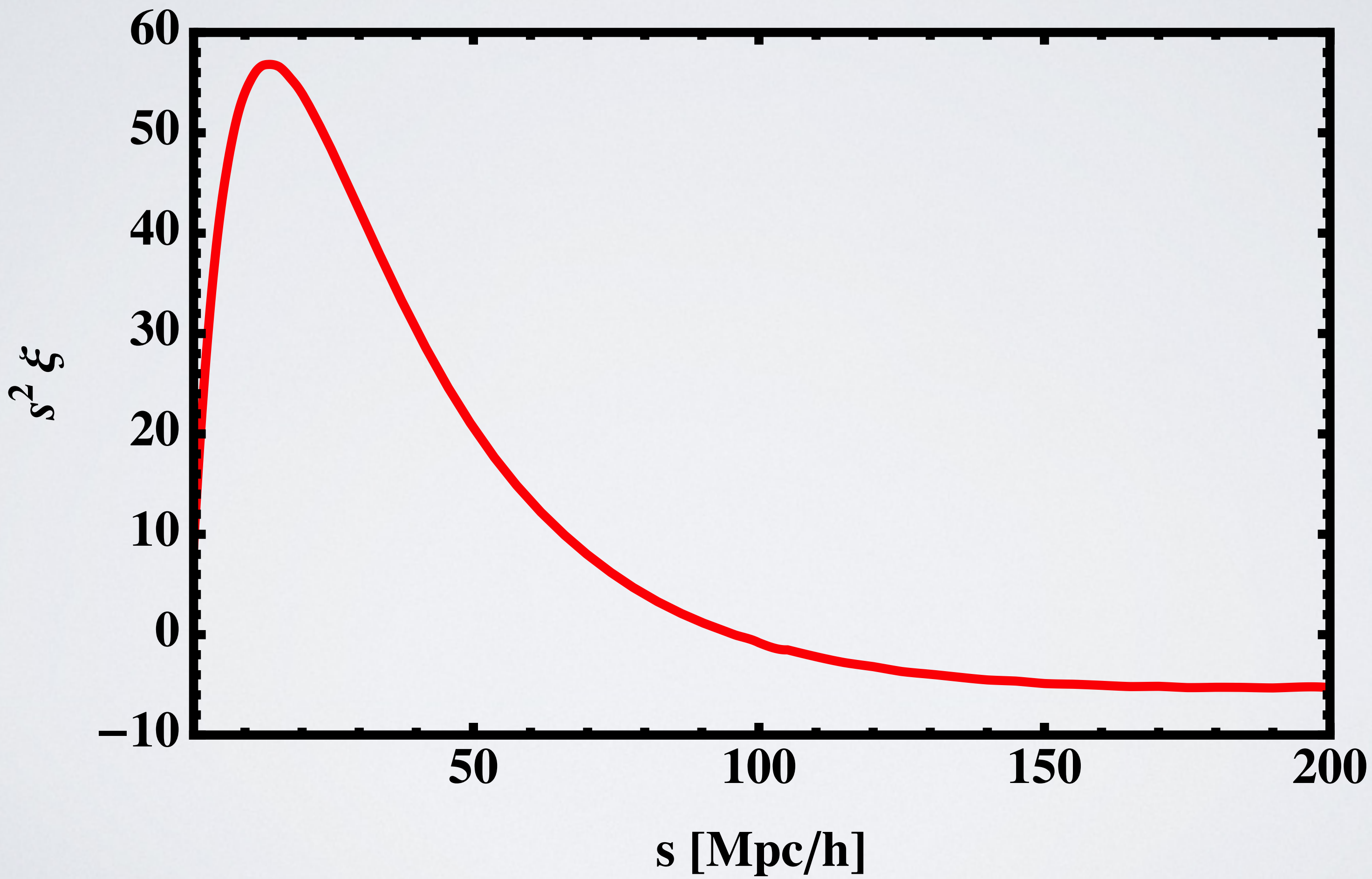


Correlation function

statistical homogeneity and isotropy

$$\xi(|\mathbf{x}-\mathbf{x}'|,\eta_0)=\langle\delta_{dm}(\mathbf{x},\eta_0)\delta_{dm}(\mathbf{x}',\eta_0)\rangle$$

$$=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\,e^{-i\mathbf{k}(\mathbf{x}-\mathbf{x}')}P_\delta(k,\eta_0)=\int\frac{dk\,k^2}{2\pi^2}P_\delta(k,\eta_0)j_0\big(k|\mathbf{x}-\mathbf{x}'|\big)$$



Missing elements

- Dark energy
- Redshift-space distortions
- Baryon acoustic oscillations
- Non-linearities
- Relativistic effects

Impact of Dark Energy

through the background

through additional clustering

Dark Energy

- From supernovae measurement we know that the Universe started **accelerating recently** at $z \sim 0.5$.

- For most of the dark matter evolution, dark energy was **negligible**.

- Dark energy affects all the observable **scales** in the **same** way, because they were all inside the horizon when dark energy started dominating.

- Dark energy will modify the **amplitude** of the power spectrum, but not its shape → the density can be expressed as:

At late time: $$\delta_{dm}(\mathbf{k},\eta) = D_1(a)T_\delta(k)\Phi_p(\mathbf{k})$$

initial conditions

growth rate: independent of scale

transfer function: independent of time

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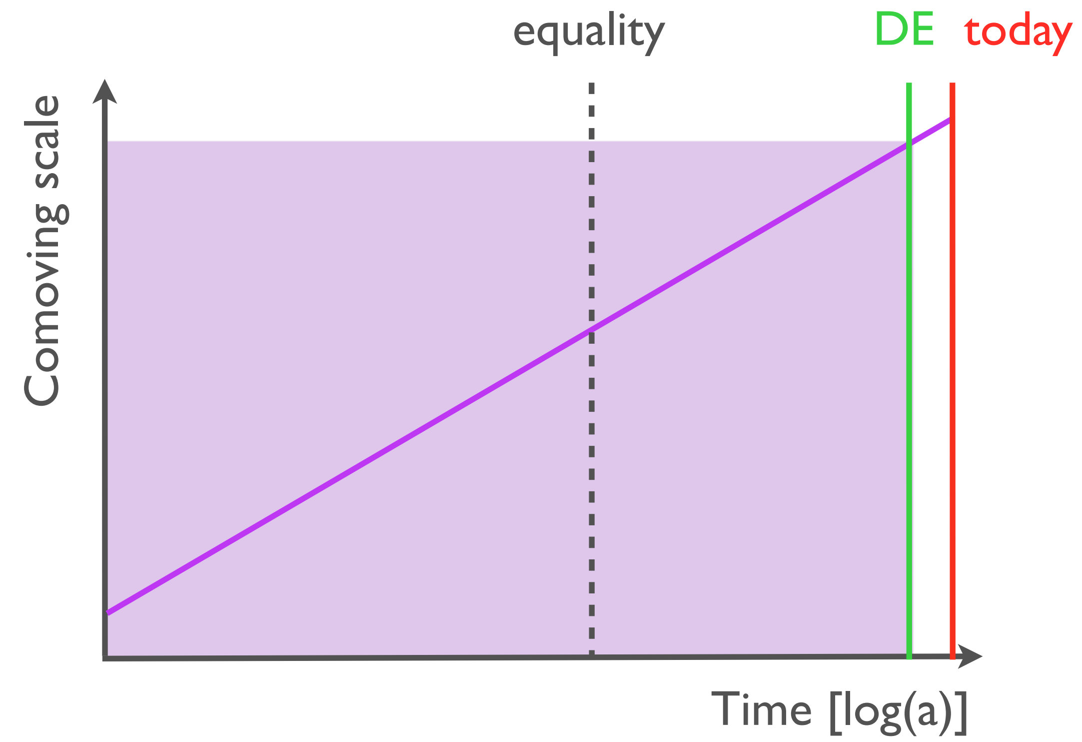

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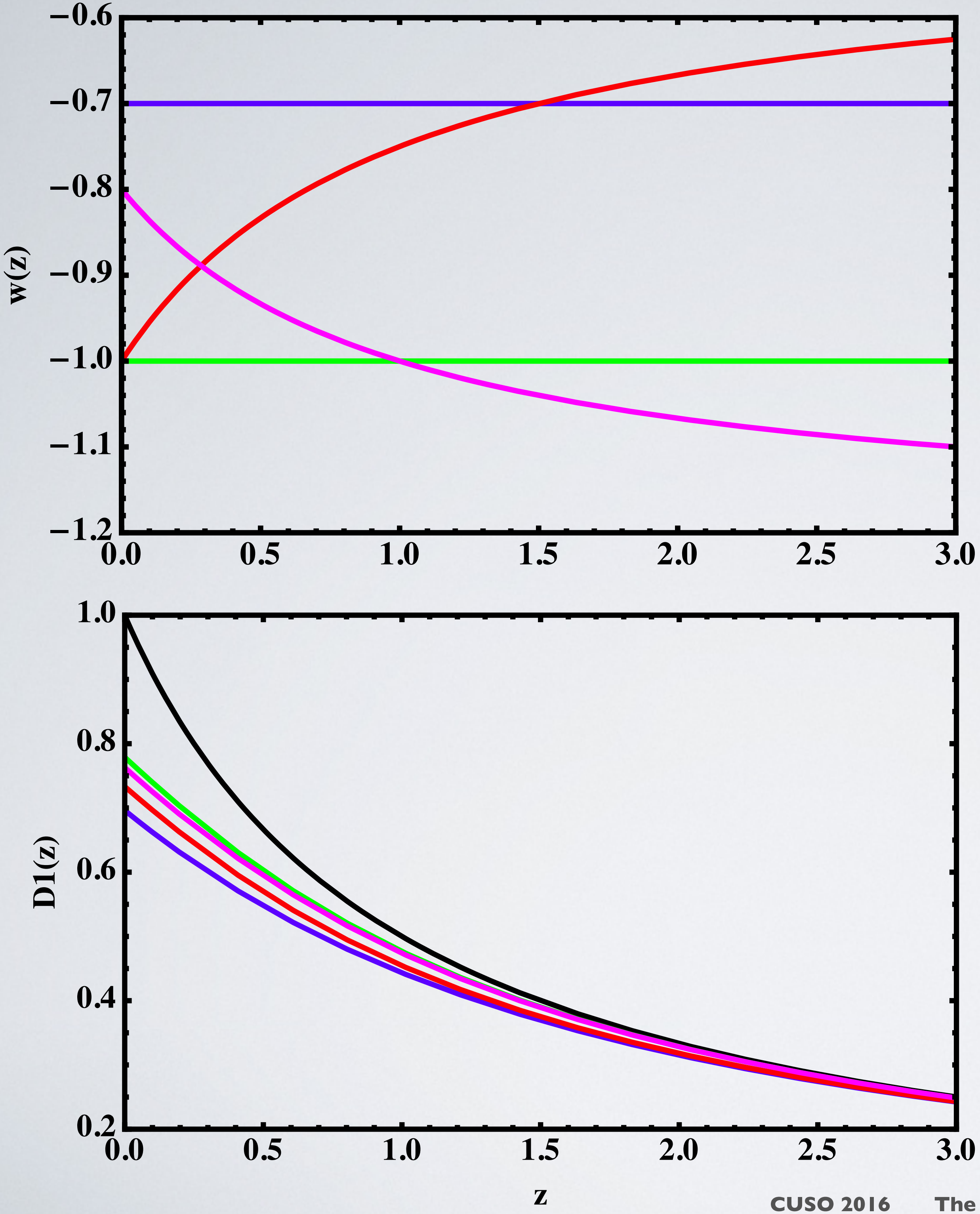
Growth function

- In a **matter dominated** universe: $D_1(a) = a$

- How does this change with **dark energy**?

- Intuitively we expect a **slower** growth (acceleration).

- Calculation: $\delta''_{dm} + \mathcal{H}\delta'_{dm} = -k^2\Phi + 3\mathcal{H}\Phi' + 3\Phi''$



no dark energy

$w = -1$

$w = -0.7$

$w_0 = -1 \quad w_a = 0.5$

$w_0 = -0.8 \quad w_a = -0.4$

$$w = w_0 + w_a \frac{z}{1+z}$$

Clustering dark energy

- We can split the dark energy into a homogeneous component plus **perturbations**.

- Evolution equations for $\delta_{\rm DE}$ and $v_{\rm DE}$

  If $w=-1$ then $\delta_{\rm DE}=0$ and $v_{\rm DE}=0$ are solutions.

  If $w\neq-1$ we **automatically** have dark energy **perturbations**.

- The evolution of these perturbations depend on the **speed of sound** of dark energy.

- Above the **sound horizon**, perturbations can grow → the growth occurs only for scales between the (causal) horizon and the sound horizon $\mathcal{H}\leq k\leq\mathcal{H}/c_S$.