Space Mission Design and Operations

Space Mission Design and Operations

Chapters 2.2 to 3.3

Prof. Claude Nicollier

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Outline

2.2 Concept of gravitational well; escape and transfer velocities

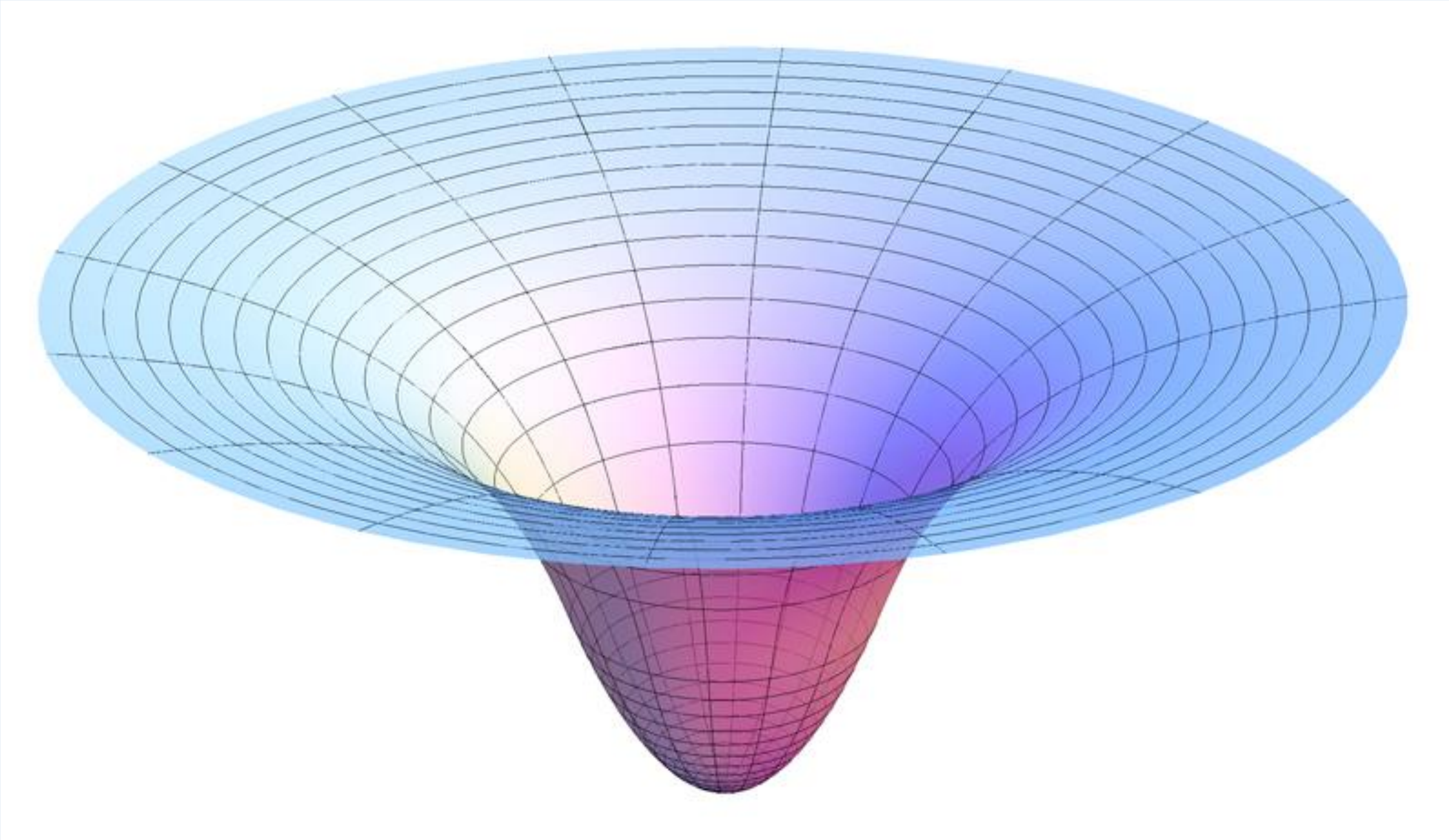
2.3 Orbital motion and Kepler's laws

2.4 Circular and elliptical orbits

2.5 Reference frames; orbital parameters and calendars

3.2 Orbital maneuvers and Hohmann transfer

3.3 Geosynchronous and geostationary orbits; nodal regression and Sun-synchronous orbits; Lagrange points



2.2.1 Concept of gravitational well

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Credits: Wikipedia, AllenMcC

Gravitational field

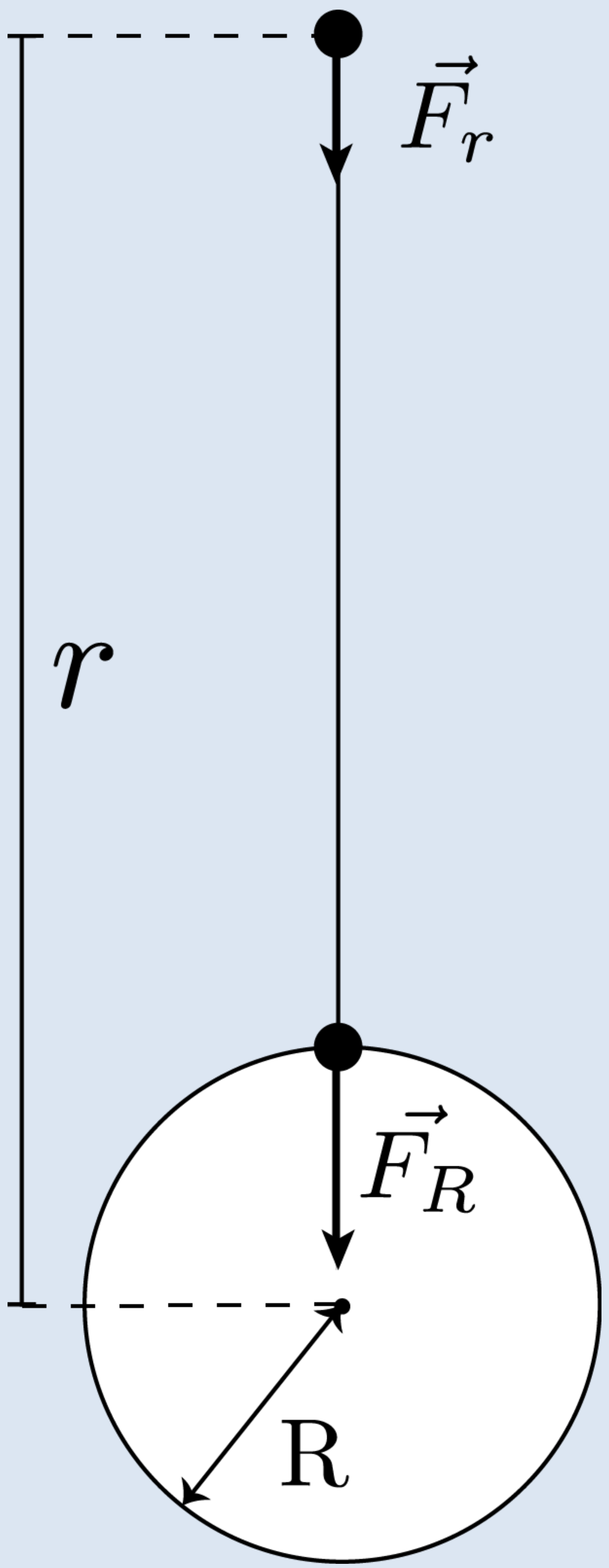
At distance ***r*** :

$$F_r = G\frac{Mm}{r^2} = g_r m$$ Gravitational force

$$g_r = G\frac{M}{r^2} = \frac{\mu}{r^2}$$ Gravitational acceleration

$$G = 6.67259 \times 10^{-11}\ \mathrm{m^3 kg^{-1} s^{-2}}$$

$$\mu = GM$$



On the surface of Earth (distance ***R***):

$$F_R = G\frac{Mm}{R^2} = g_0 m$$

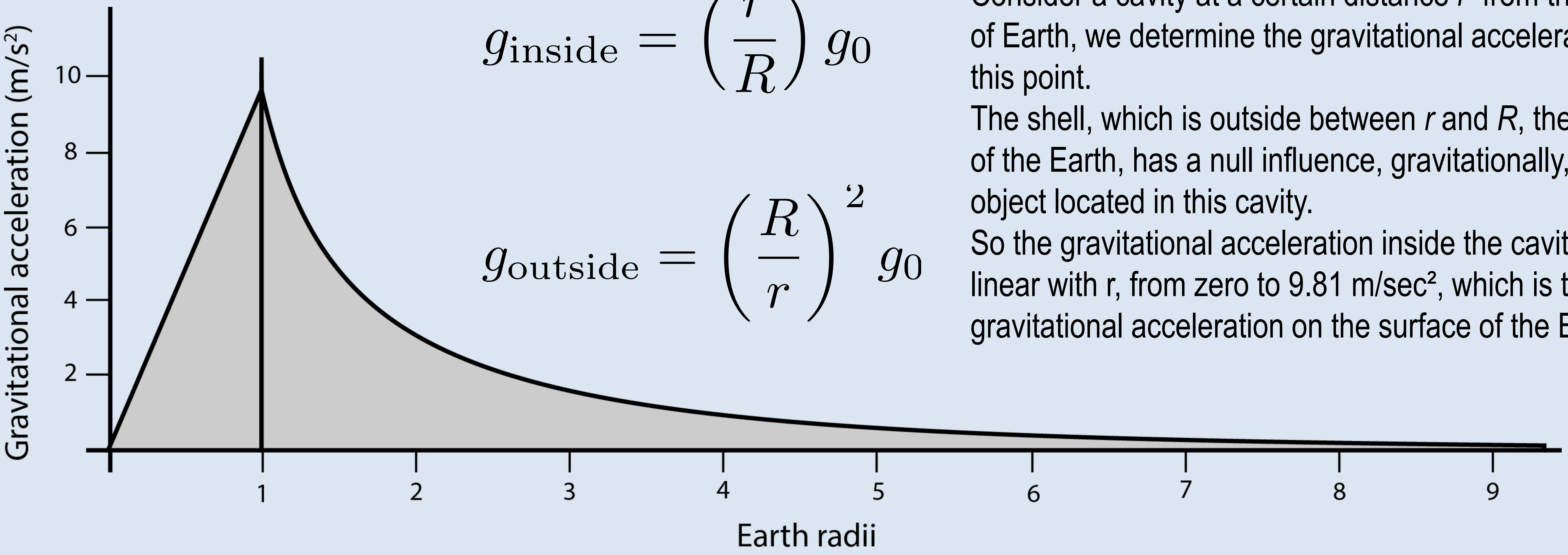
$$g_0 = G\frac{M}{R^2} = \frac{\mu}{R^2} = g$$

$$g_0 = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2} = 32.2\ \frac{\mathrm{ft}}{\mathrm{s}^2}$$

$$R = 6.378 \times 10^6\ \mathrm{m}$$

$$M = 5.9742 \times 10^{24}\ \mathrm{kg}$$

Gravitational acceleration profile inside and outside Earth



$$g_{\text{inside}} = \left(\frac{r}{R}\right) g_0$$

$$g_{\text{outside}} = \left(\frac{R}{r}\right)^2 g_0$$

Consider a cavity at a certain distance *r* from the center of Earth, we determine the gravitational acceleration at this point.
The shell, which is outside between *r* and *R*, the radius of the Earth, has a null influence, gravitationally, on any object located in this cavity.
So the gravitational acceleration inside the cavity is linear with r, from zero to 9.81 m/sec², which is the gravitational acceleration on the surface of the Earth.

The linear gravitational acceleration profile « Inside Earth » is valid only for an homogeneous Earth.

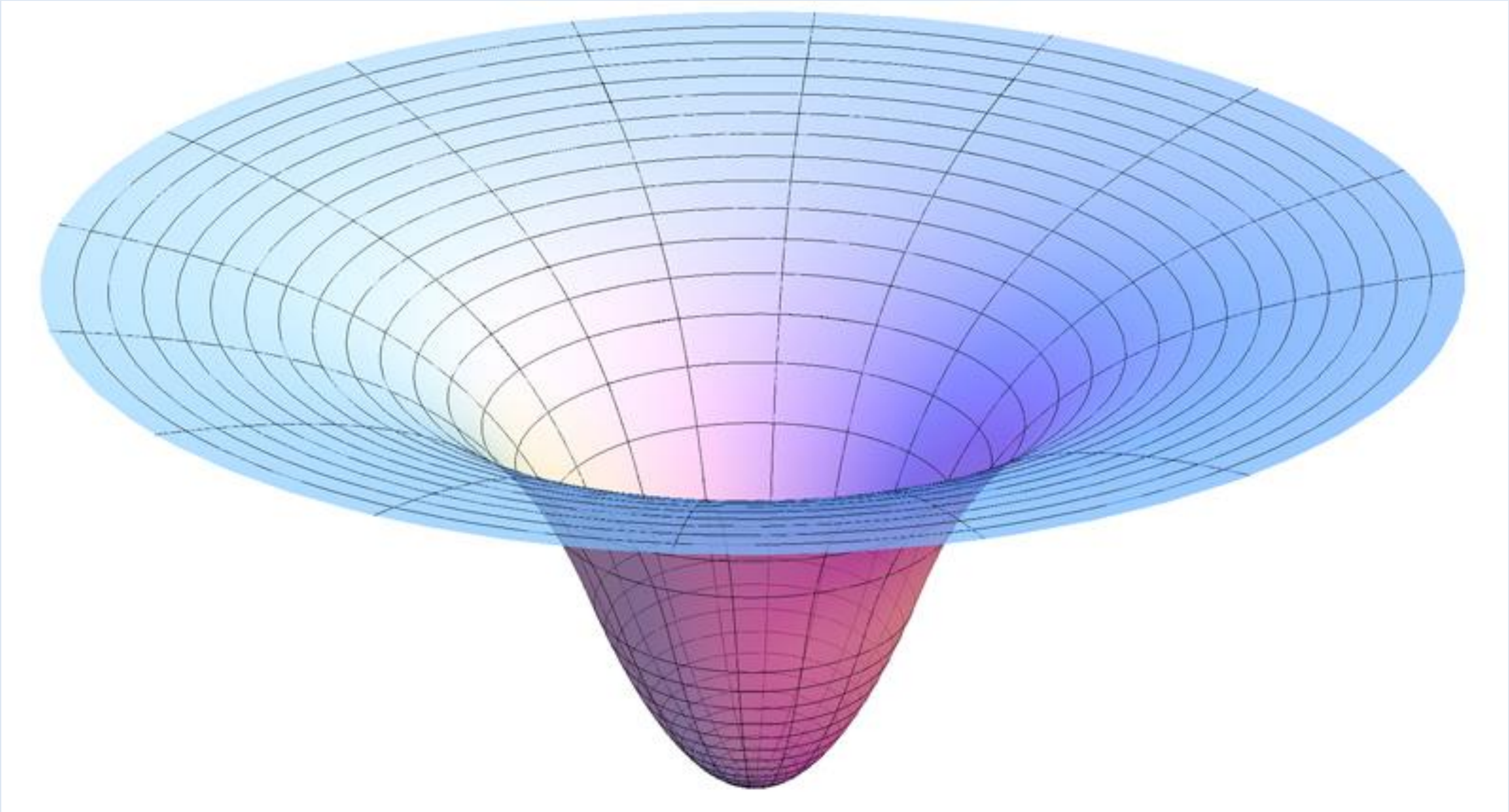
Gravitational figures for bodies in the solar system

Body	Gravity E=1	Time to fall 4.9 m (sec)	Escape velocity $\left(\frac{km}{s}\right)$	Circ. velocity on surface $\left(\frac{km}{s}\right)$
Sun	28	0.2	618	437
Mercury	0.26	2.0	3.5	2.5
Venus	0.90	1.1	10.4	7.3
Earth	1	1	11.2	7.9
Moon	0.16	2.5	2.3	1.6
Mars	0.38	1.6	5.0	3.6
Phobos	0.001∗	30.0∗	0.01∗	0.01∗
Jupiter	2.65	0.6	60	42.5
Ganymede	0.2∗	2.0∗	3.0∗	2.0∗
Saturn	1.14	0.9	36	25
Titan	0.2∗	2.0∗	3.0∗	2.0∗
Uranus	0.96	1.0	22	15.5
Neptune	1.0	1.0	23	16

∗Approximative figures

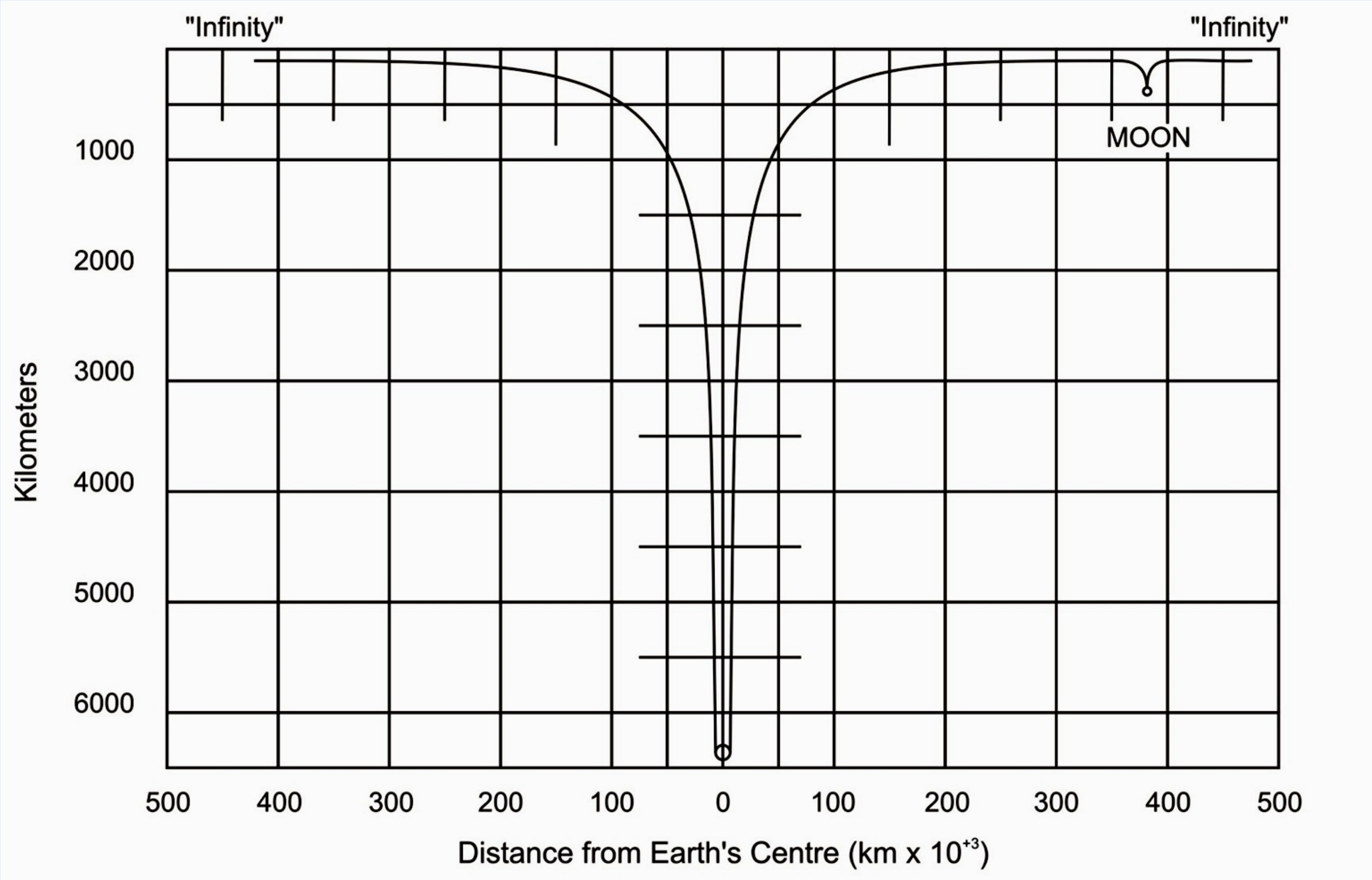
The escape velocity is equal to the square root of two times the circular velocity for a given distance to the center of the attracting object.

The concept of gravitational well



- A gravitational well is a conceptual model of the gravitational field surrounding a body in space.
- Entering space from the surface of a planet means climbing out of the gravitational well.
- The deeper a planet‘s gravitational well is, the more energy it takes to escape from it.

Credits: Wikipedia, AllenMcC



2.2.2 Concept of gravitational well: development

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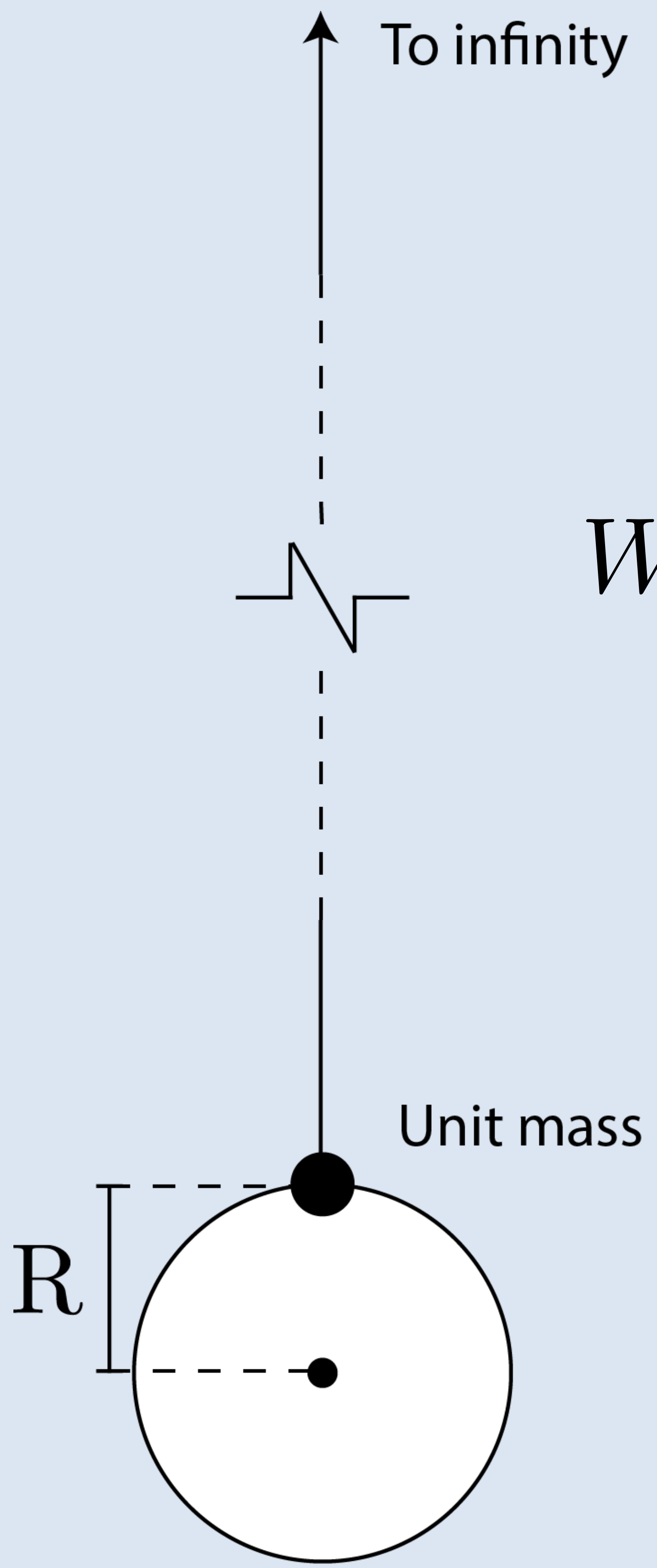
Credits: Adapted from « Ascent to Orbit », Arthur C. Clarke

The concept of gravitational well: work



- Work performed to bring a unit mass to infinity, from the Earth's surface

$$W_R = \int_R^\infty \frac{\mu}{r^2} dr = \frac{\mu}{R^2} R = g_0 R$$

$$\boxed{W_R = g_0 R}$$

- ***R***: Earth's radius
- ***g_0***: Gravitational acceleration on Earth's surface.

The work necessary to lift a unit mass from the surface of Earth to infinity is the constant gravitational acceleration *g_0* times the Earth's radius.

We say: The depth of the Earth's gravitational well is equal to the radius of the Earth R

The concept of gravitational well: work

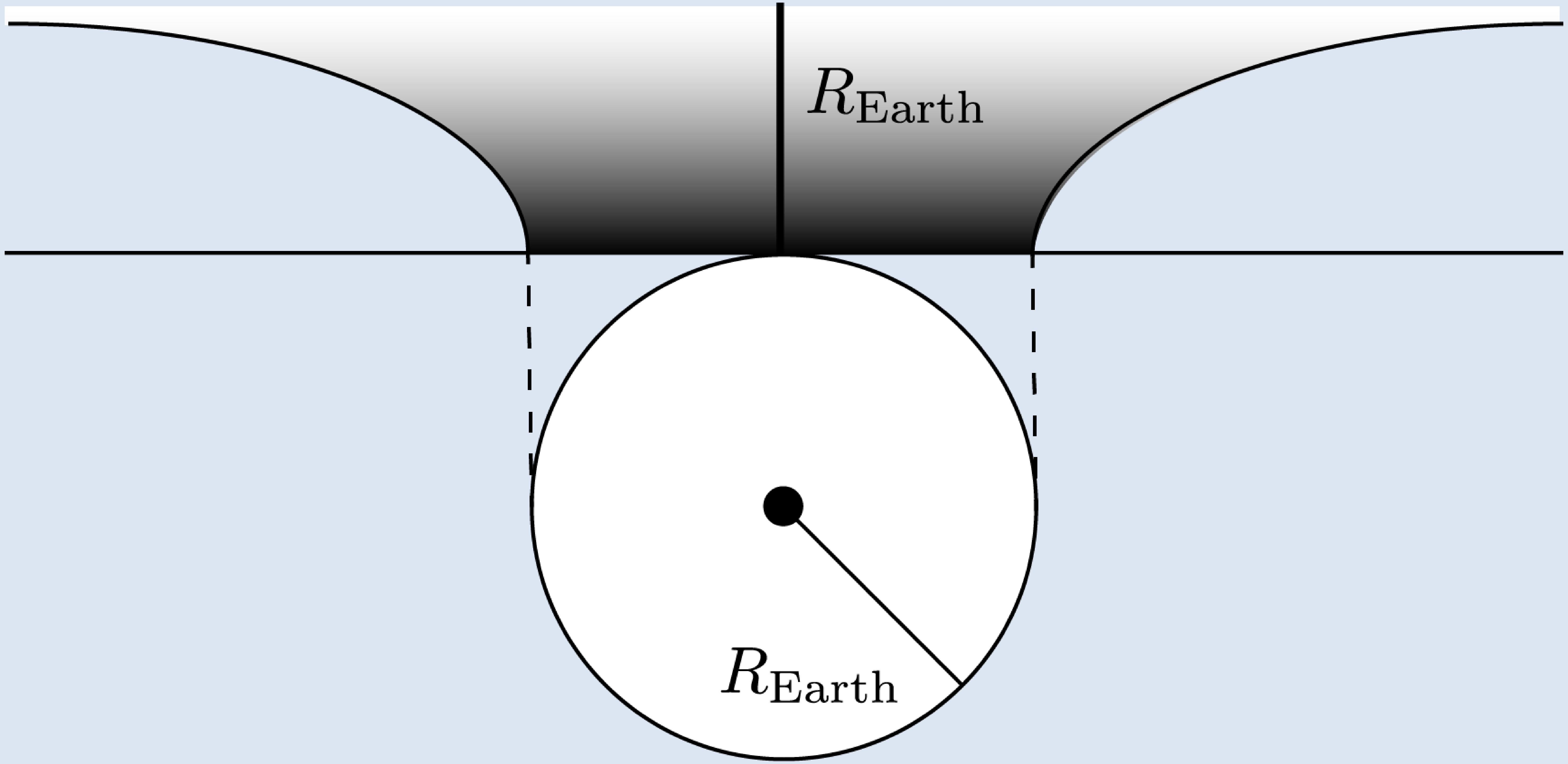
- Starting at a distance ***r*** from the Earth's center, the work performed is:

$$W_r = g_r r = \frac{g_r}{g_0}\frac{r}{R}W_R = \frac{R}{r}W_R$$

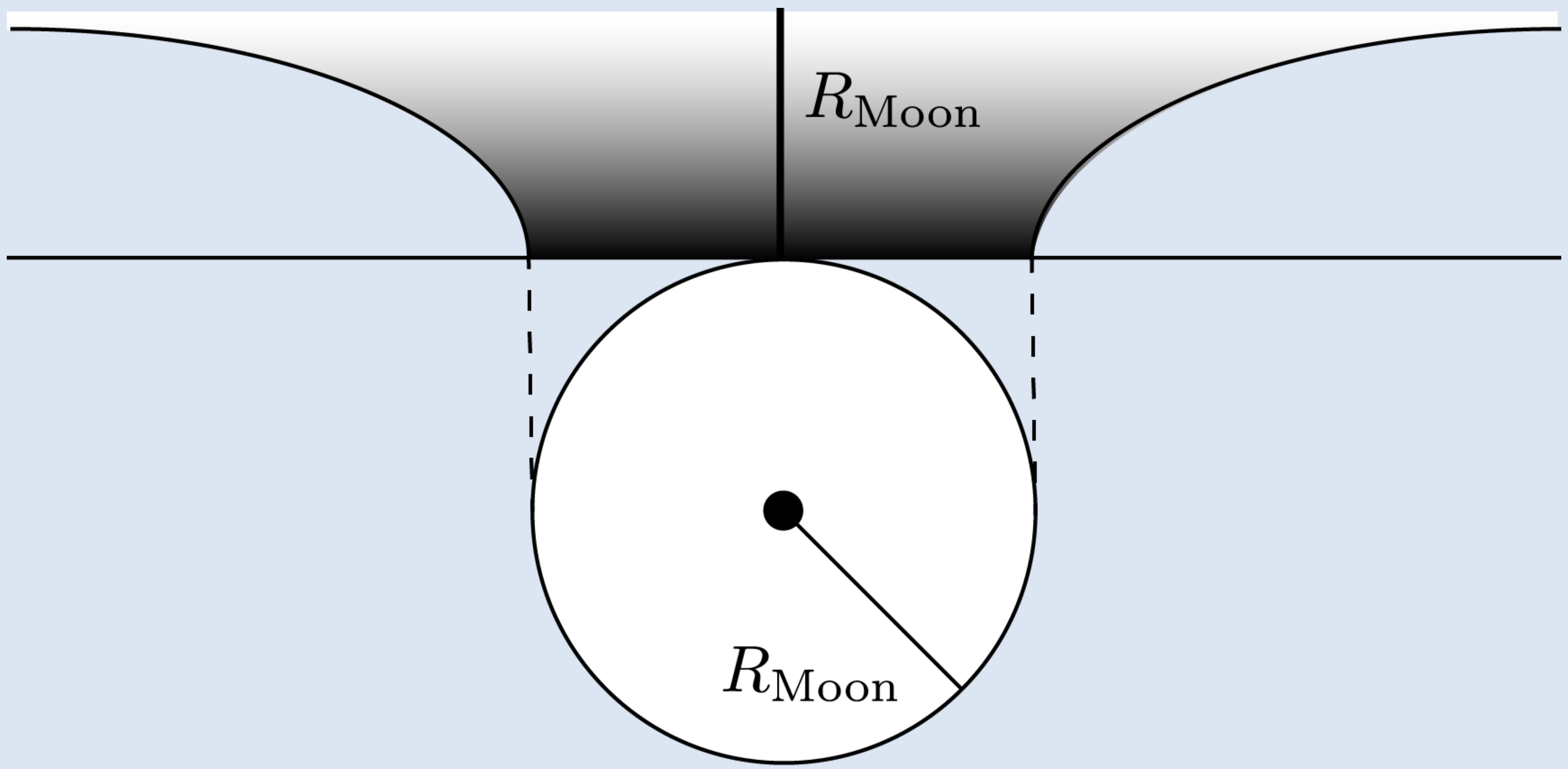
The work necessary to lift a unit mass from the distance *r*, larger than the radius of the Earth,to infinity, is equal to the work from the surface of the Earth multiplied by the factor *R/r*

The profile of the gravitational well is in *1/r*

Earth's gravitational well: depth = R_{Earth}

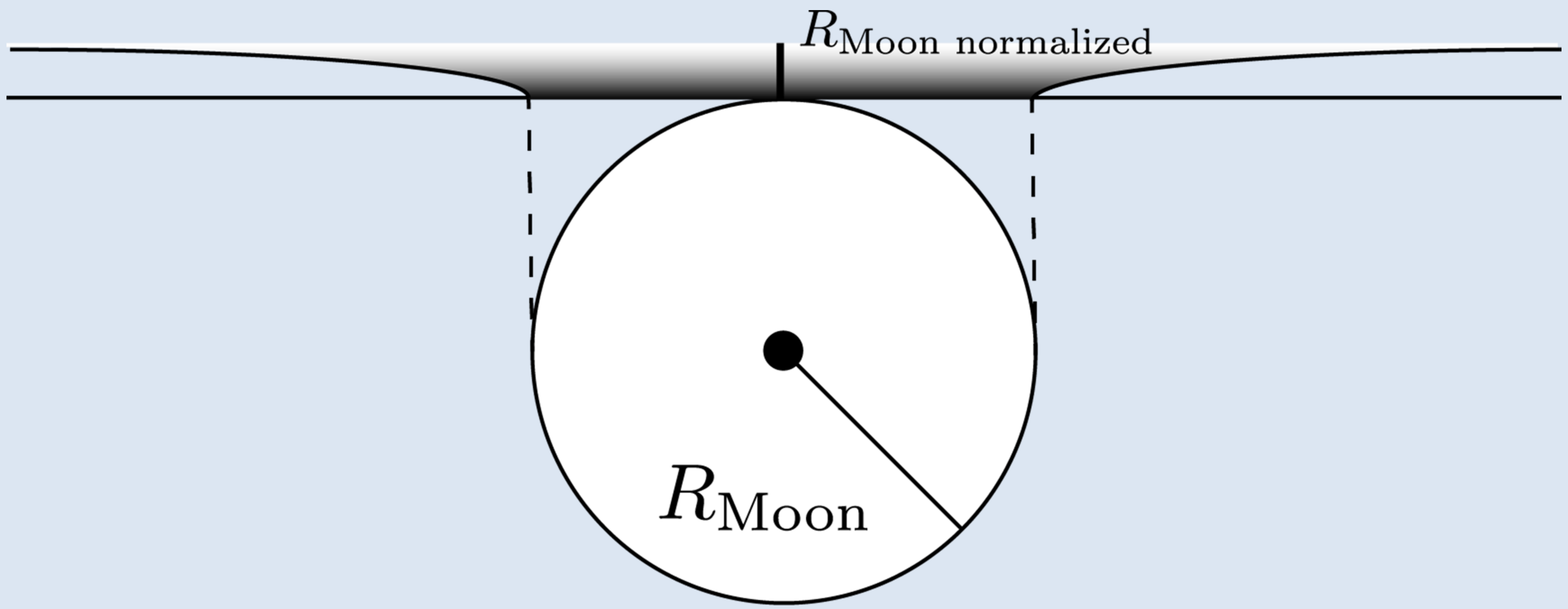


Moon’s gravitational well: depth = R_{Moon}



The work needed to bring a unit mass from the surface of the Moon to infinity is equal to the work done to take that unit mass from the surface of the Moon to the radius of the Moon away from the Moon's surface, with a constant force equal to the gravitational force on the surface.

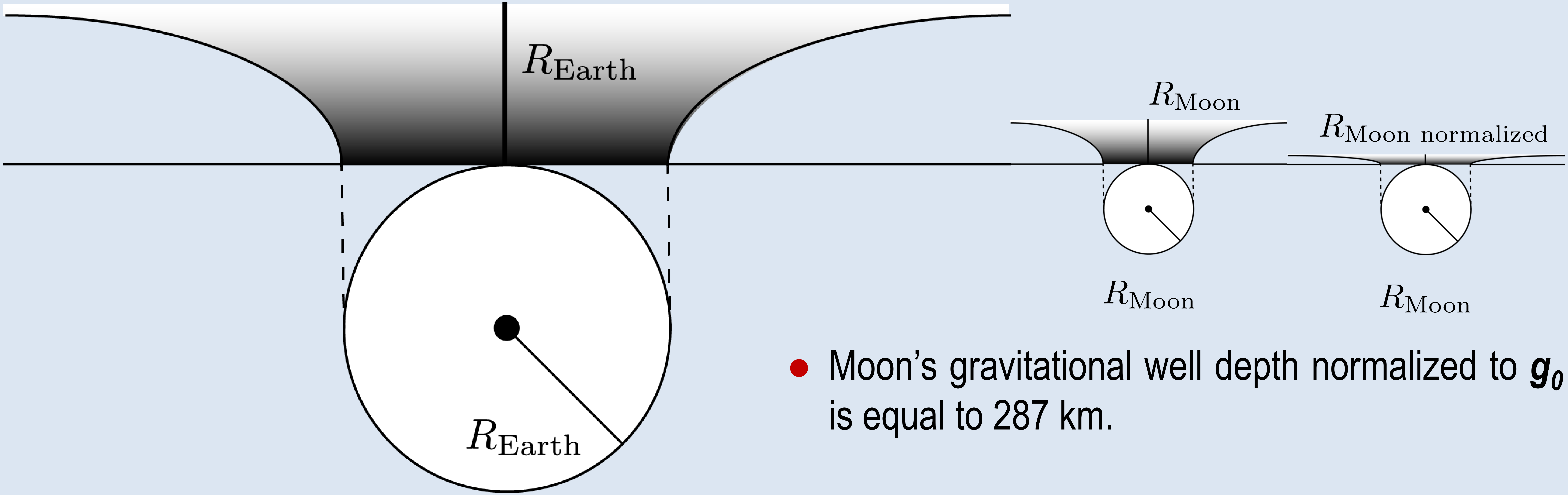
Moon's gravitational well normalized



Considering that the gravitational acceleration on the Moon is only 1/6 of the value on Earth, the normalized depth of the gravitational well of the Moon is the radius of the Moon divided by six.

The depth of gravitational well of any spherical objects in the solar system or elsewhere, is always normalized to the gravitational acceleration of the Earth for comparison purposes. It is the radius of that object, multiplied by the ratio between the gravitational acceleration on the surface of that object and the one on the surface of the Earth

Comparison of gravitational wells

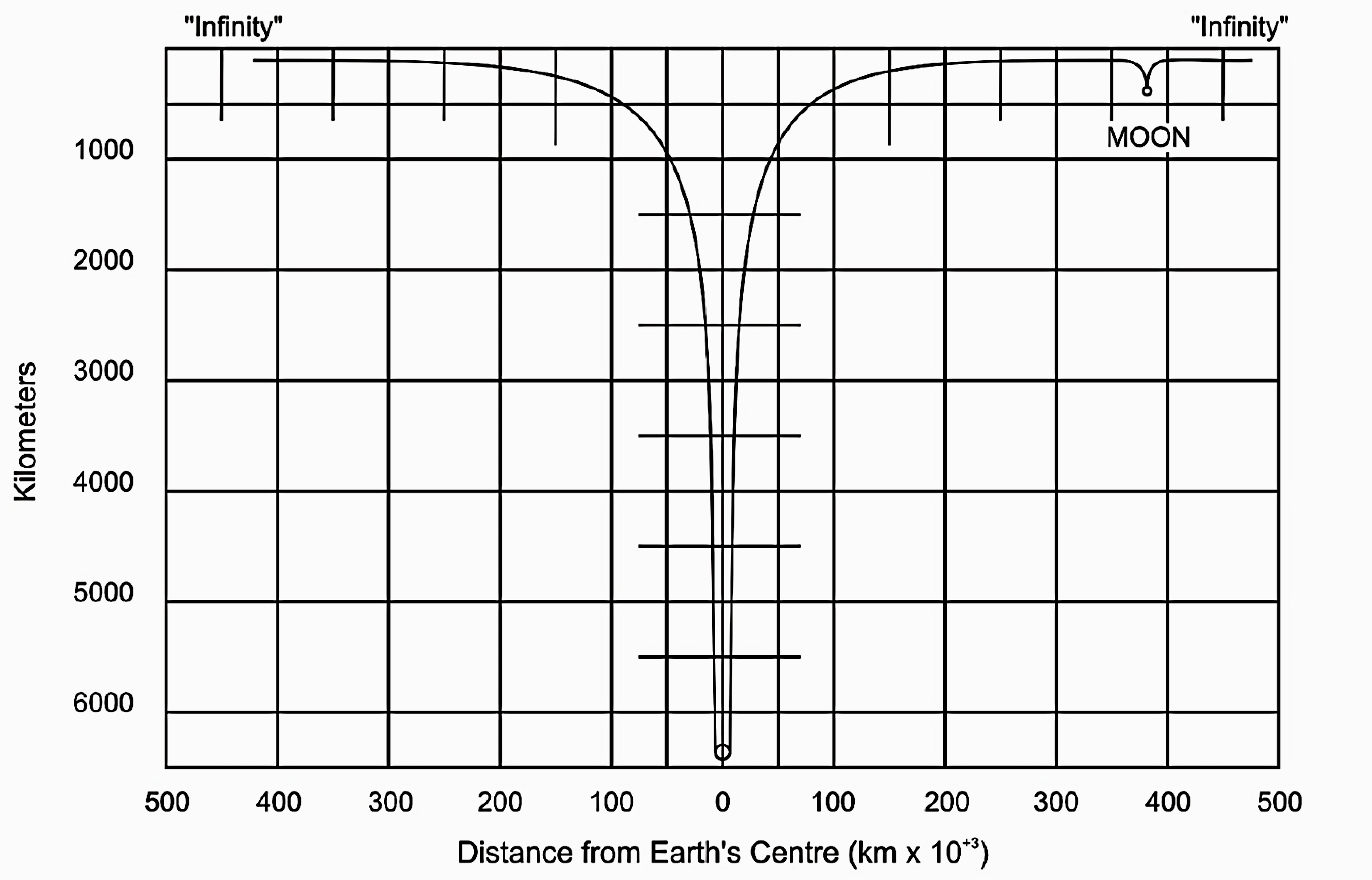


- Moon's gravitational well depth normalized to g_0 is equal to 287 km.
- It is small in comparison with the 6378 km and 1738 km for Earth and Moon radius respectively.

From left to right: at the same scale, gravitational well of the Earth, non-normalized gravitational well of the Moon (equal to the radius of the Moon) and normalized gravitational well of the Moon.

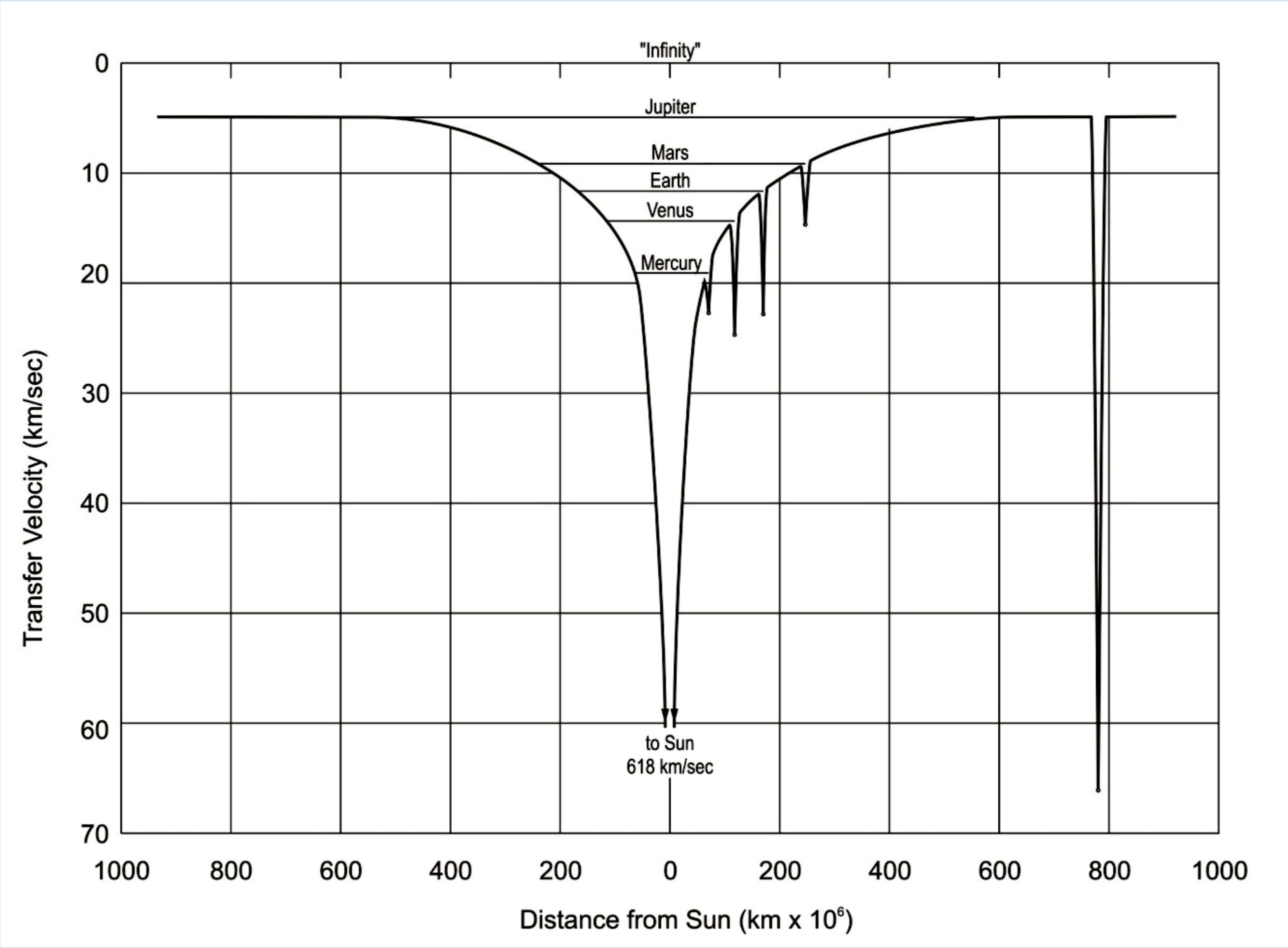
For a very small object like an asteroid or the nucleus of a comet, the normalized depth of the gravitational well could be equal to less than one meter or even a few centimeters.

Profile of Earth's and Moon's gravitational wells



Normalized profile of gravitational well of the Earth with a profile of *1/R* and of the Moon

Credits: Adapted from « Ascent to Orbit », Arthur C. Clarke



2.2.3 Escape and transfer velocities

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Credits: Adapted from « Ascent to Orbit », Arthur C. Clarke

Concept of escape velocity

- How to escape from the Earth's gravitational influence?
- Slow method: inefficient.
- Rapid, with a single impulse: **escape velocity concept.**
- Work performed to bring a unit mass from the Earth's surface to infinity = $\boldsymbol{g_0 R}$.

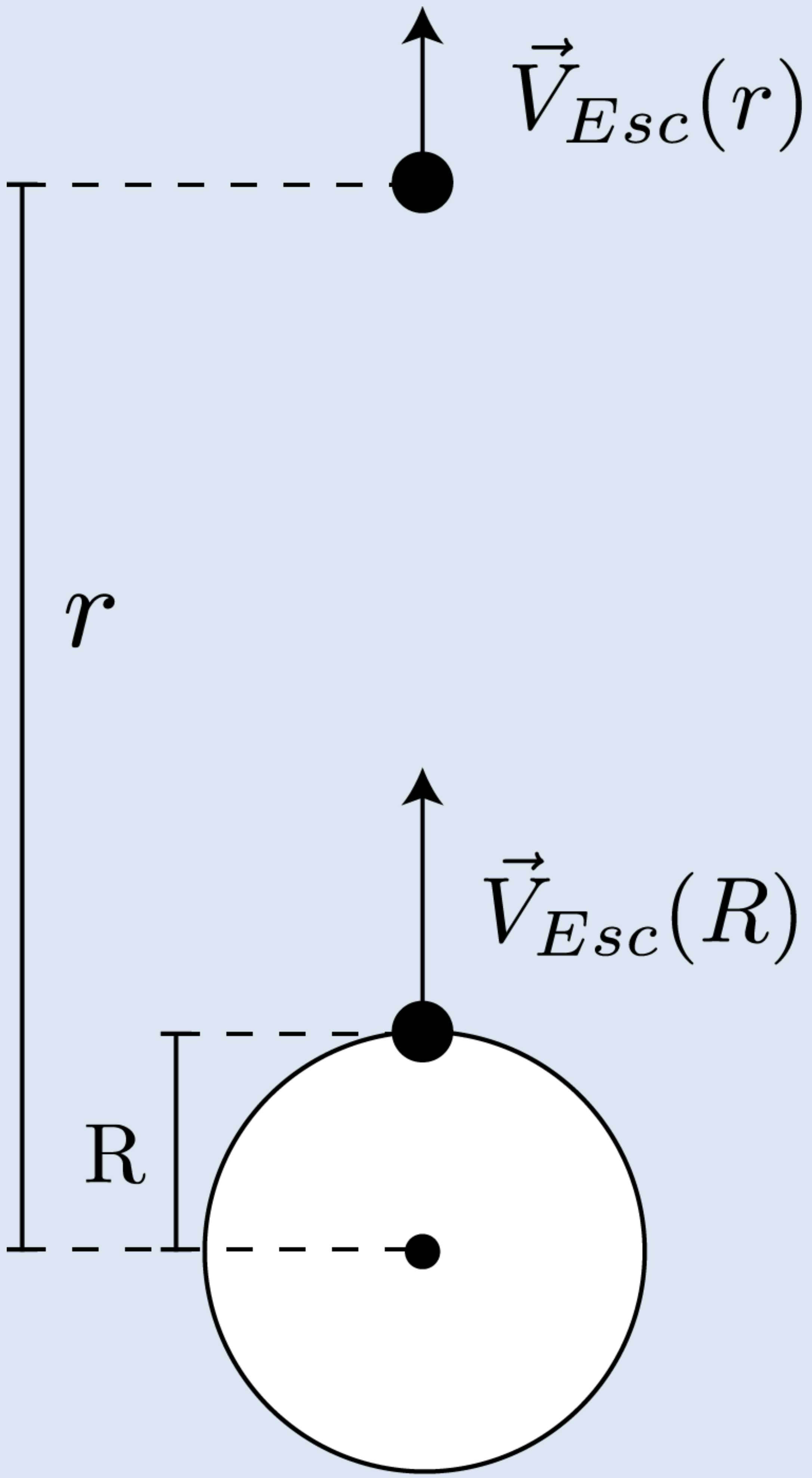
$$\frac{1}{2}V_{Esc}^2 = g_0 R \Rightarrow V_{Esc} = \sqrt{2 g_0 R} = \sqrt{\frac{2\mu}{R}}$$

Escape velocity is the velocity at which a spacecraft has to leave the surface of the Earth in order to reach infinity with a zero velocity (in the absence of an atmosphere).

If velocity at infinity is not zero, you have done more than what is needed to just escape the gravitational influence of the Earth.

The work needed to bring a unit mass (the spacecraft) from the surface of the Earth to infinity is equal to the initial kinetic energy.

Escape velocity

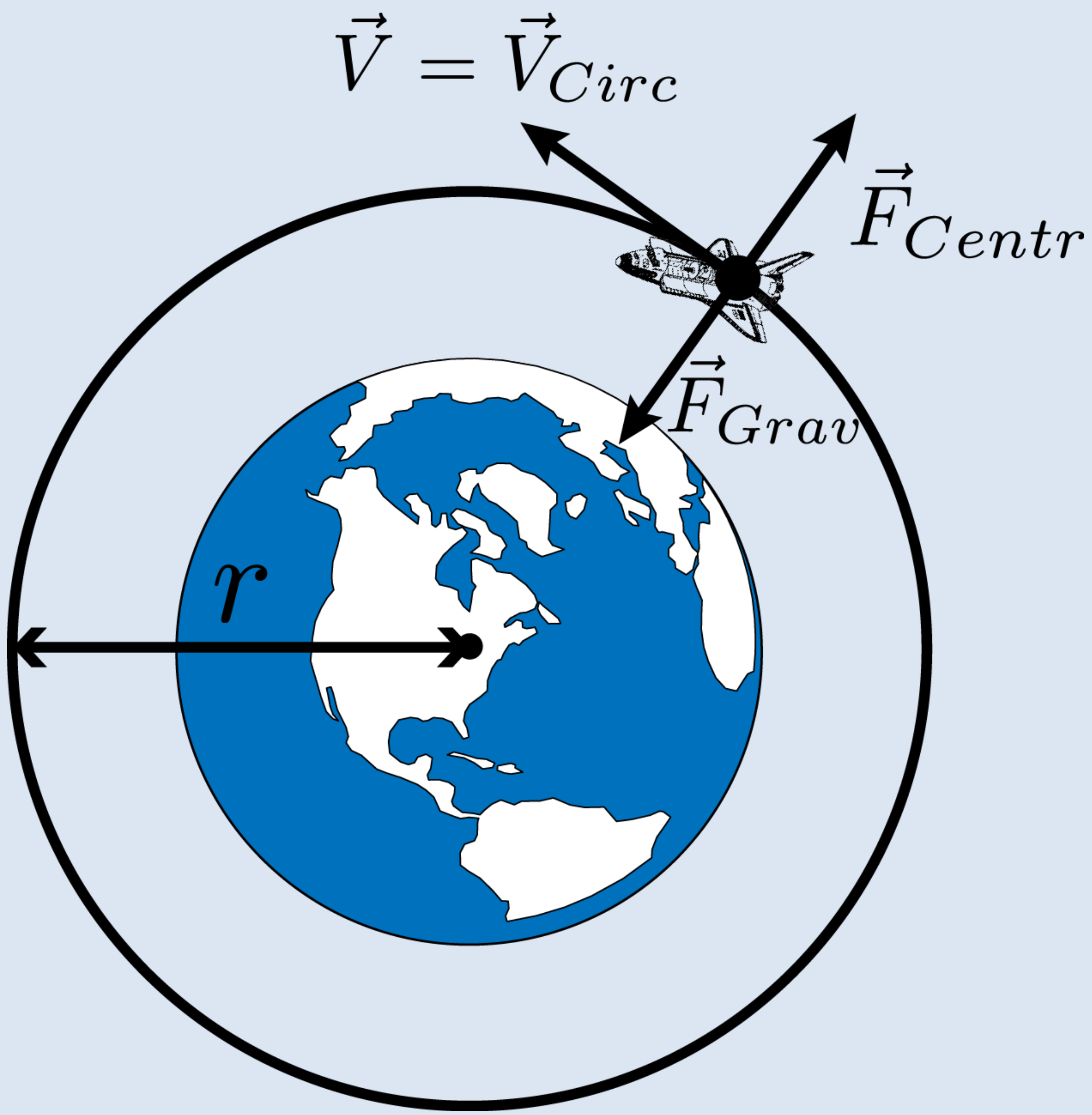


Generally, for a distance ***r*** from the center of the Earth:

$$\frac{1}{2}V_{Esc}^2 = g_r r$$

$$\Rightarrow V_{Esc} = \sqrt{2g_r r} = \sqrt{\frac{2\mu}{r}}$$

Circular velocity



The Space Shuttle on a circular Low Earth Orbit has a velocity of the order of 7.7—7.8 km/s, or going around the Earth in about an hour and a half.

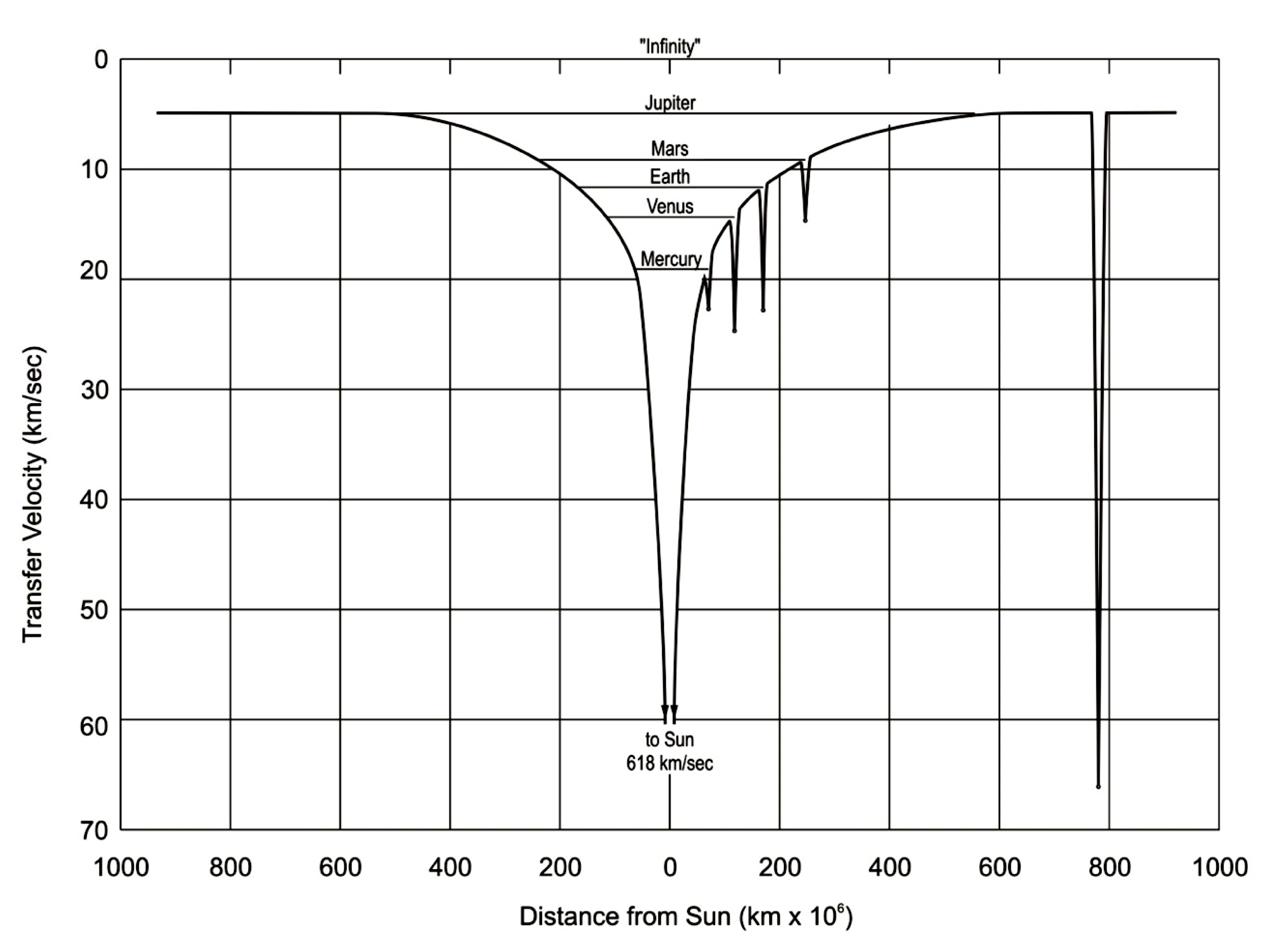
The centrifugal force resulting from the curved orbital trajectory is equal in magnitude to the gravitational force on the orbiting Shuttle:

$$F_{\mathrm{Centr}} = F_{\mathrm{Grav}}$$

$$\Rightarrow \frac{V_{\mathrm{Circ}}^2}{r} = \frac{\mu}{r^2}$$

$$\Rightarrow V_{\mathrm{Circ}} = \sqrt{\frac{\mu}{r}}$$

Gravitational well in term of transfer velocity



- The transfer velocity, for a given planet, is the velocity that has to be added to the planet's circular velocity for a transfer to infinity from this location in the Sun's gravitational well, i.e. as if to leave the solar system.

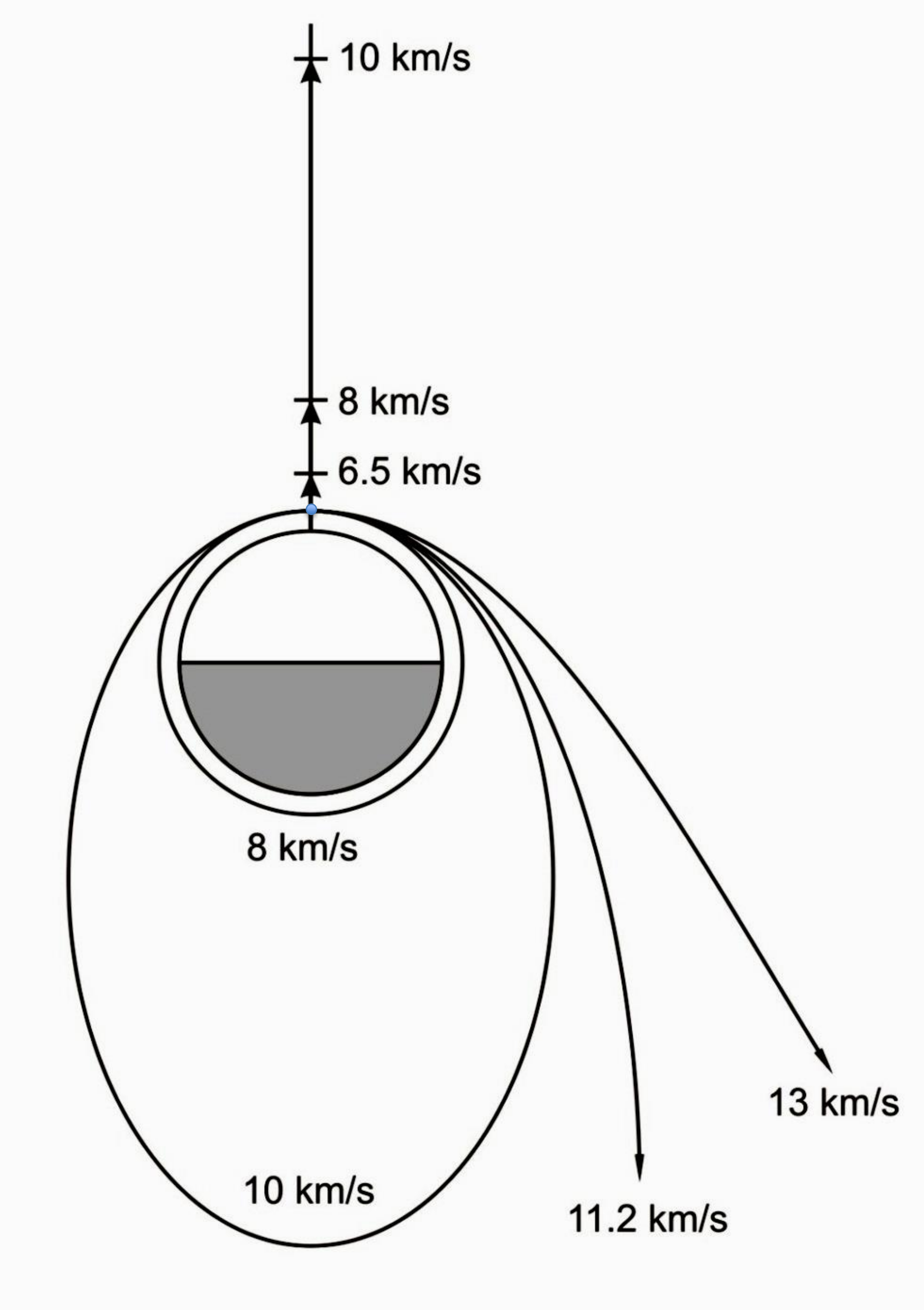
- To determine the «two steps» escape velocity out of the solar system from the surface of a planet, the escape velocity from the planet itself should be added and is illustrated by the depth of the « icicles » around each planet. Typically, for the Earth the icicle has an amplitude of 11.2 km/s.

Credits: Adapted from « Ascent to Orbit », Arthur C. Clarke

Example of escape velocity

- Escape velocity out of the solar system from Mercury's orbit: **68 km/s**.
- Average orbital velocity of Mercury: **48 km/s**.
- Transfer velocity out of the solar system from Mercury's orbit: **68 - 48 = 20 km/s**.
- Escape velocity from the surface of Mercury: **3.5 km/s**.
- «Two steps» escape velocity out of the solar system from the surface of Mercury: **20 + 3.5 = 23.5 km/s**.

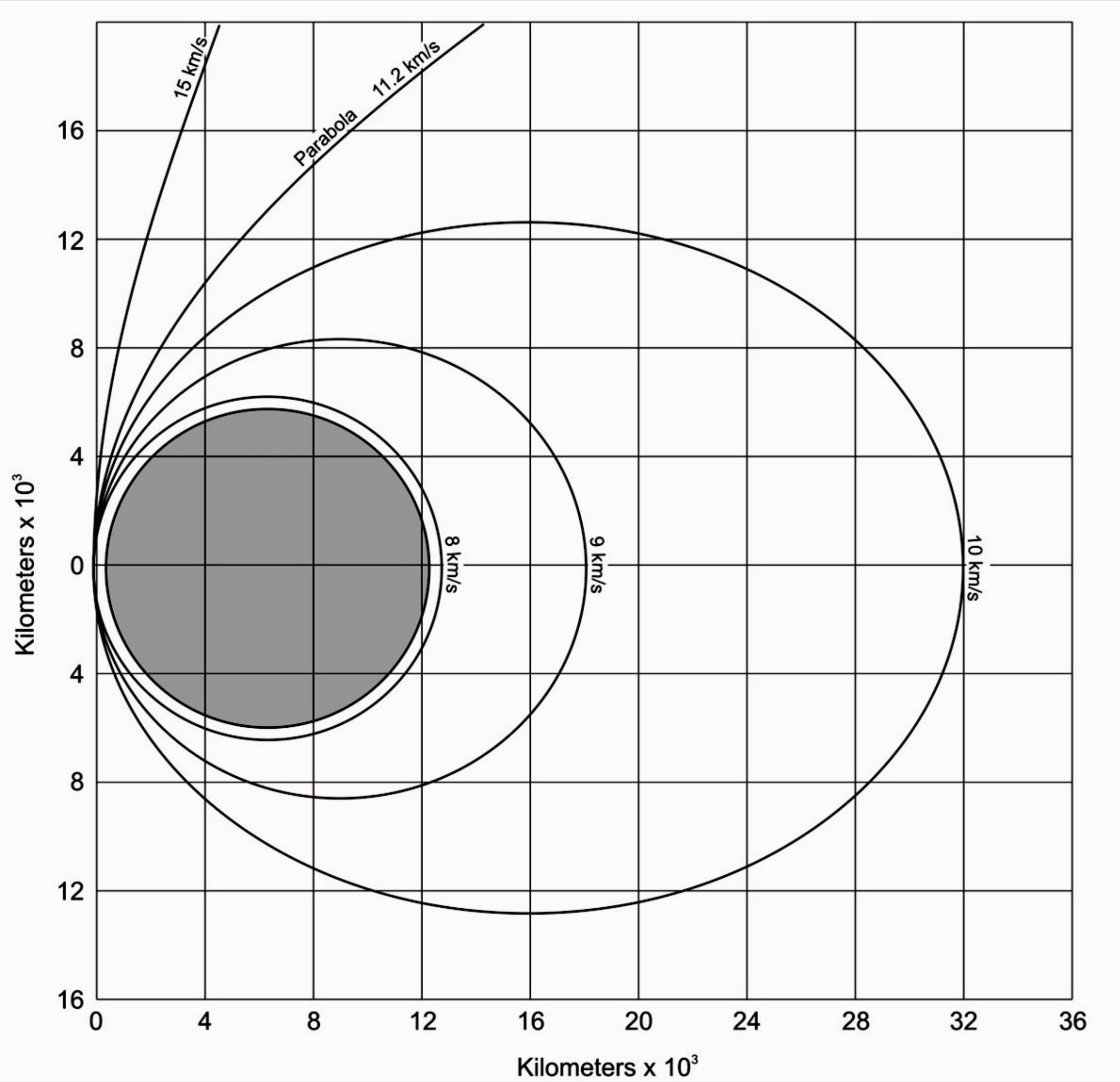
Escape velocity vs. direction of escape



The escape velocity is independent of the direction of the initial impulse (as long as escape really takes place)

If the initial impulse is horizontal (from a point just outside of the Earth's atmosphere, or a little above 100 km altitude):

- At 11.2 km/s the trajectory will be parabolic;
- At a larger velocity than 11.2 km/s the trajectory will be hyperbolic;
- If less than 11.2 km/s, it will be an elliptical orbit, degrading because of low perigee;
- At 8 km/sec the trajectory will be initially a circular orbit, but rapidly degrading



2.3.1 Orbital motion and Kepler’s laws

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Credits: Adapted from « Ascent to Orbit », Arthur C. Clarke

Two-body problem

- The two-body problem is to determine the motion of the two bodies that interact only with each other.

- Common examples include a satellite orbiting a planet, a planet orbiting a star, two stars orbiting each other (binary star).

Credits: NASA

Newton's law and hypotheses for the rest of the course

$$F_{Grav} = G\frac{Mm}{r^2} = m\frac{\mu}{r^2}$$

- **Hypotheses:** Unless otherwise noted, we will consider:
  - Central body of mass ***M*** + spacecraft only.
  - Mass of spacecraft ***m*** << mass of the central body ***M***.
  - Bodies spherical and homogeneous.
  - No perturbations.

Gravitational perturbations for LEO (less than 2000 km altitude)

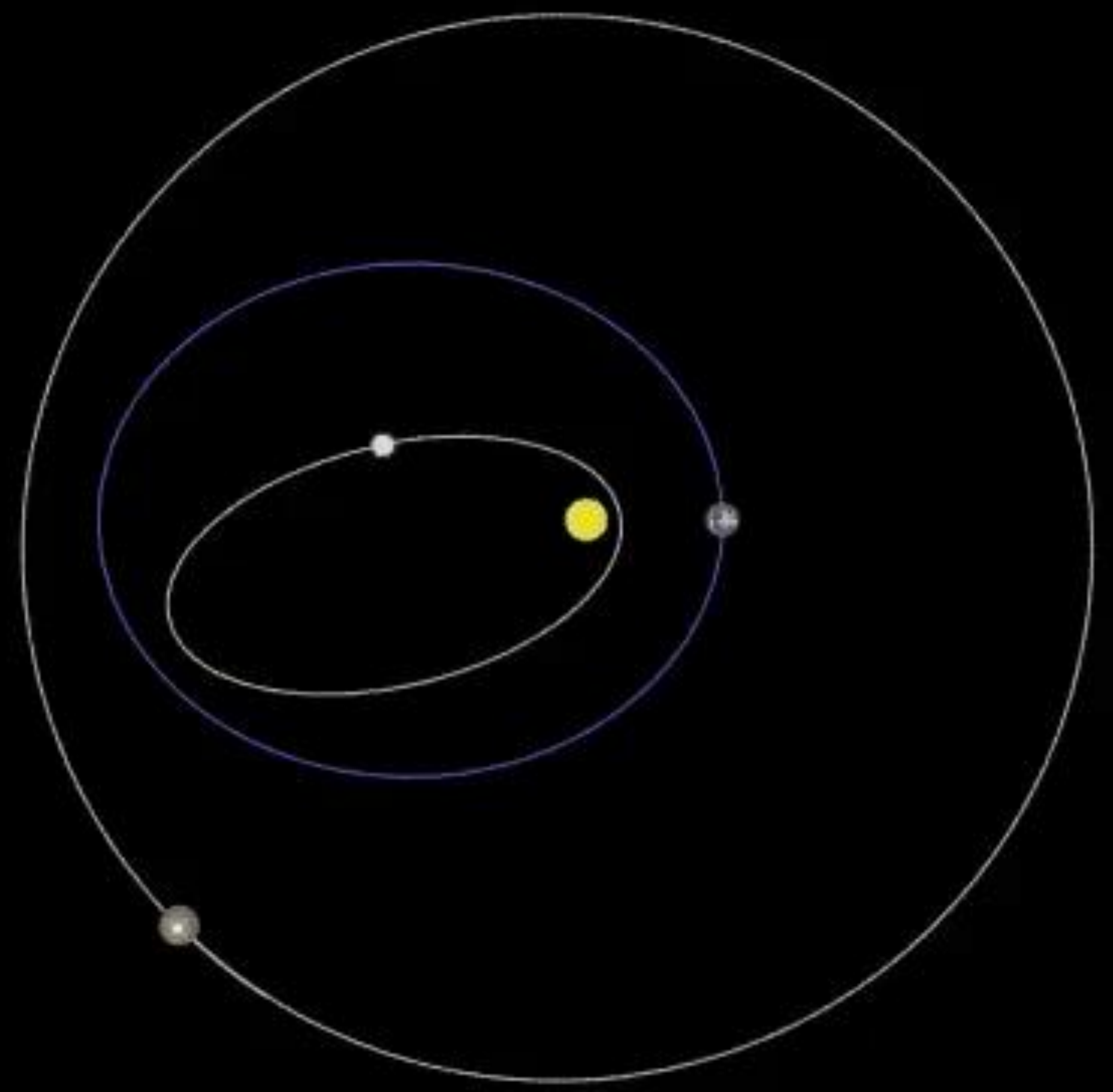
Body	Acceleration, g
Earth	~ 0.9
Sun	6×10^{-4}
Moon	3×10^{-6}
Jupiter	3×10^{-8}
Venus	2×10^{-8}

The major perturbations in LEO are coming from the drag due to the residual atmosphere, and the Sun.

Credits: Dover Publications Inc

Kepler’s laws (1609-1619) – First law

The orbit of every planet is an ellipse with the Sun at one of the two foci.



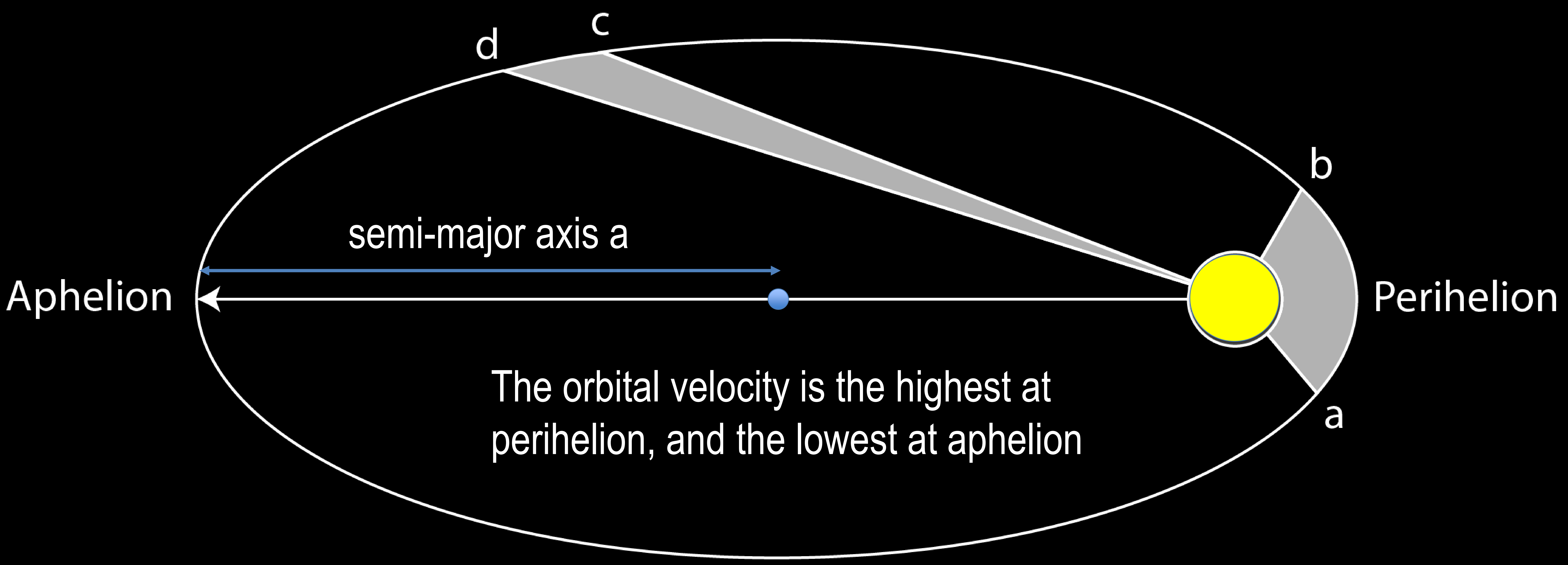
Kepler's laws were established at the beginning of the 17th century from observations of the motion of Mars in the sky made by Tycho Brahe.

The first Kepler’s law can be generalized in the case of a two-body problem: the orbit of the small body versus the large body is, generally speaking, a conic, i.e. an ellipse, a parabola, or hyperbola.

Credits: Animations for Physics and Astronomy Education

Kepler's laws (1609-1619) – Second law

A line joining a planet and the Sun sweeps equal areas during equal intervals of time.



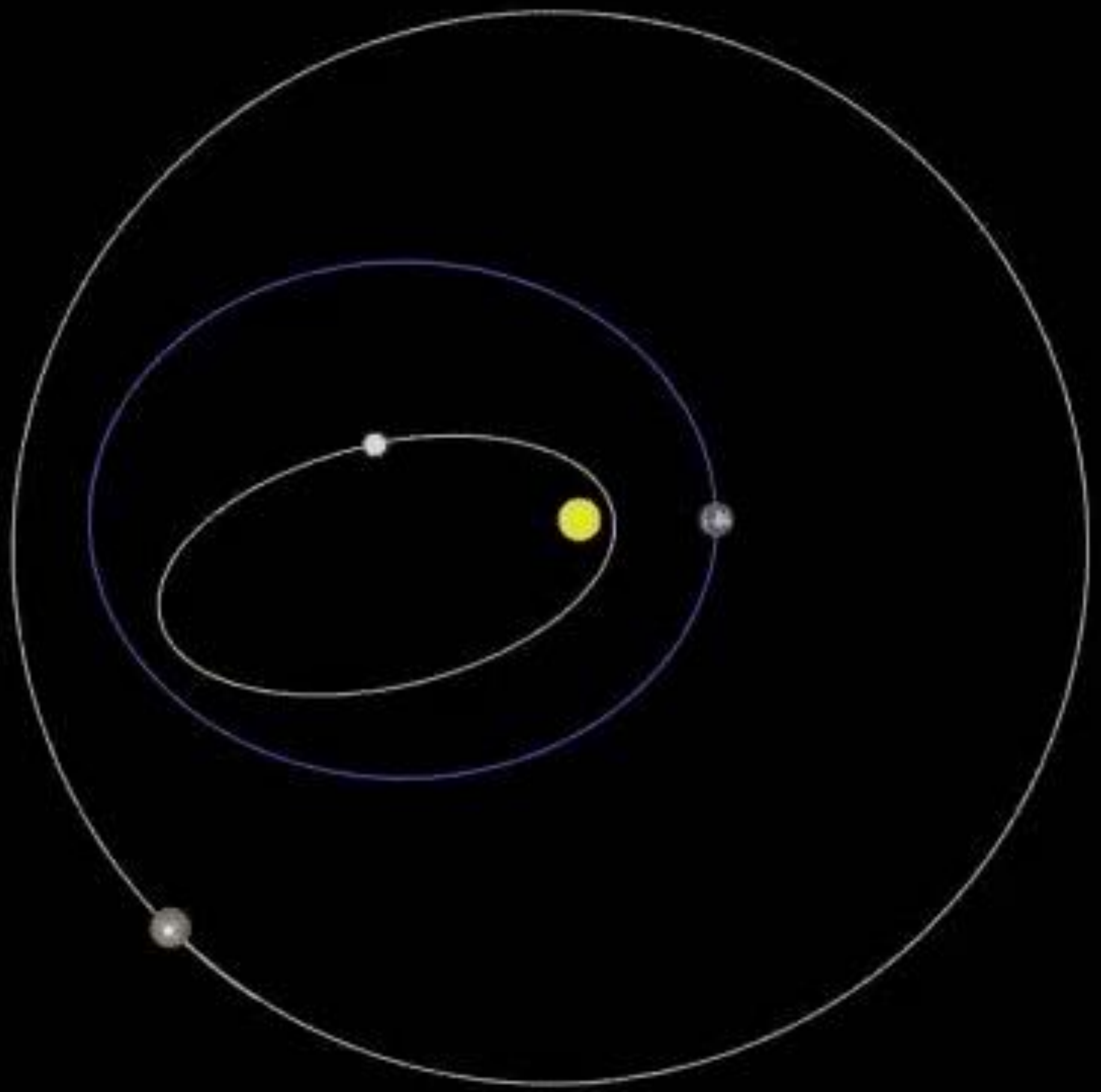
The orbital velocity is the highest at perihelion, and the lowest at aphelion

Kepler's laws (1609-1619) – Third law

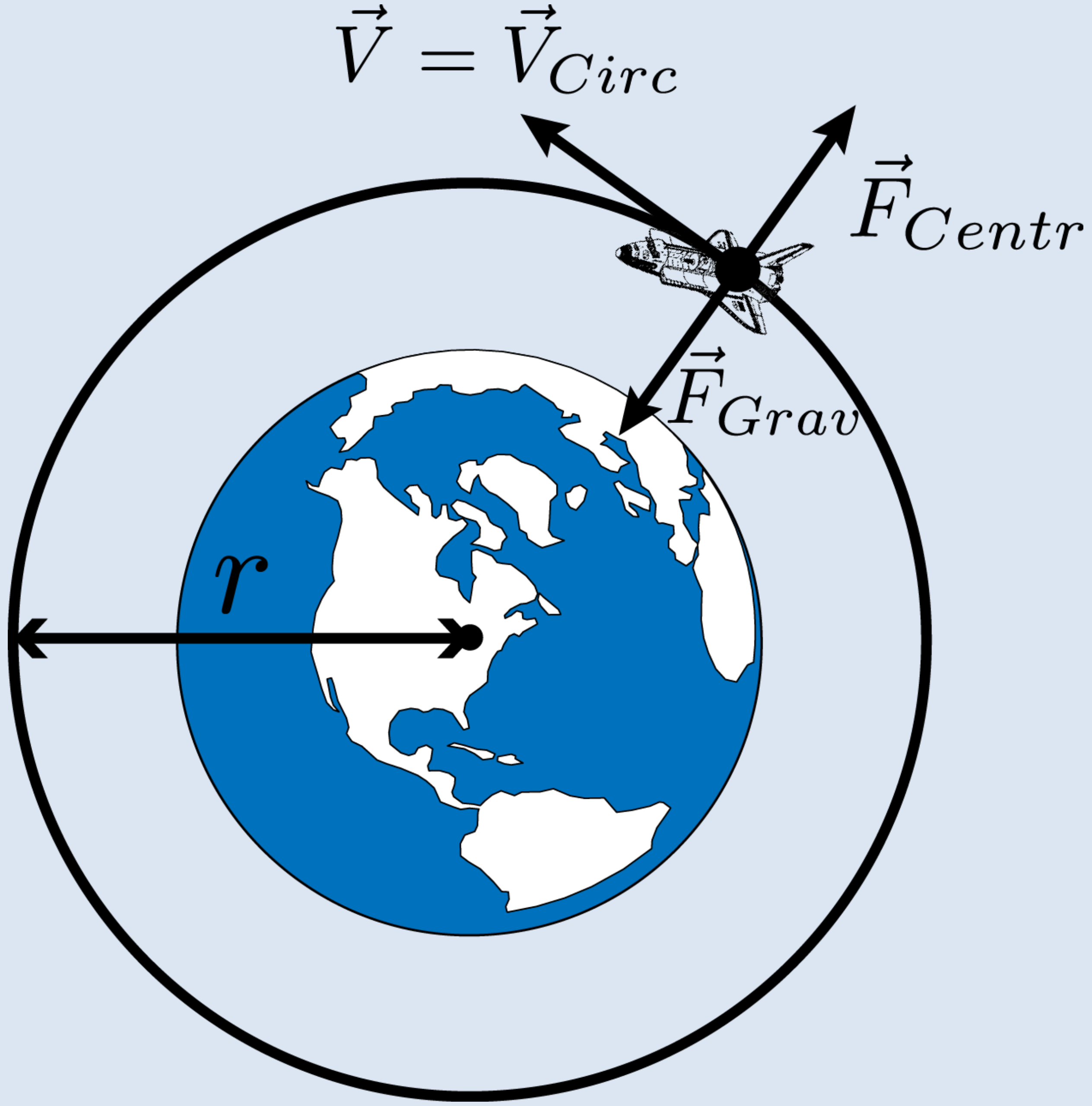
The square of the orbital period of a planet is proportional
to the cube of the semi-major axis of the orbit.

$$T^2 \sim a^3 \longrightarrow T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Kepler's laws (1609-1619) – Summary



Credits: Animations for Physics and Astronomy Education



2.4.1 Circular and elliptical orbits

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Orbital period

Velocity on a circular orbit:

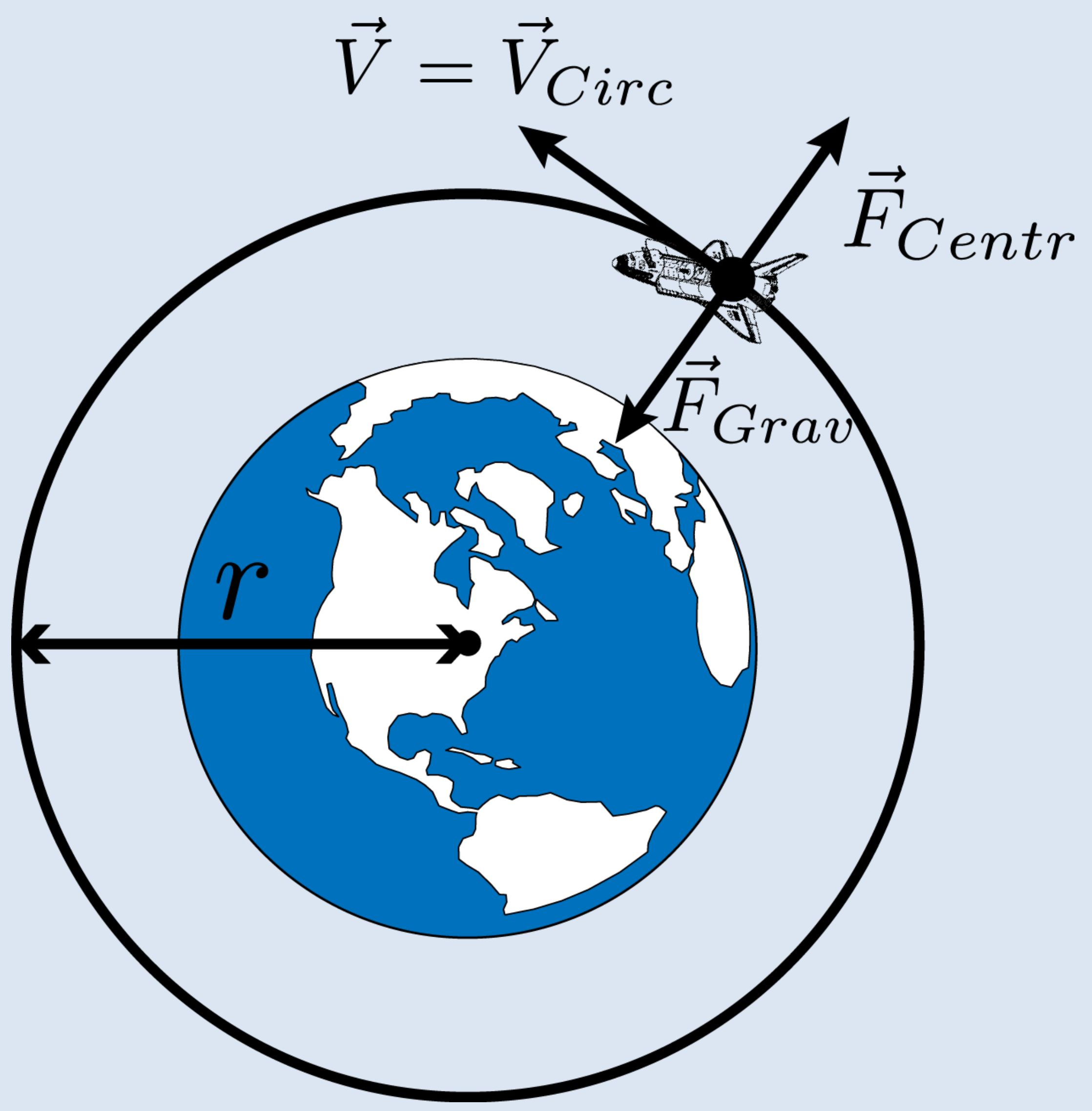
$$F_C = F_{\text{Grav}} \Rightarrow \frac{V^2}{r} = \frac{\mu}{r^2} \Rightarrow V = \sqrt{\frac{\mu}{r}}$$

Period (circular orbit):

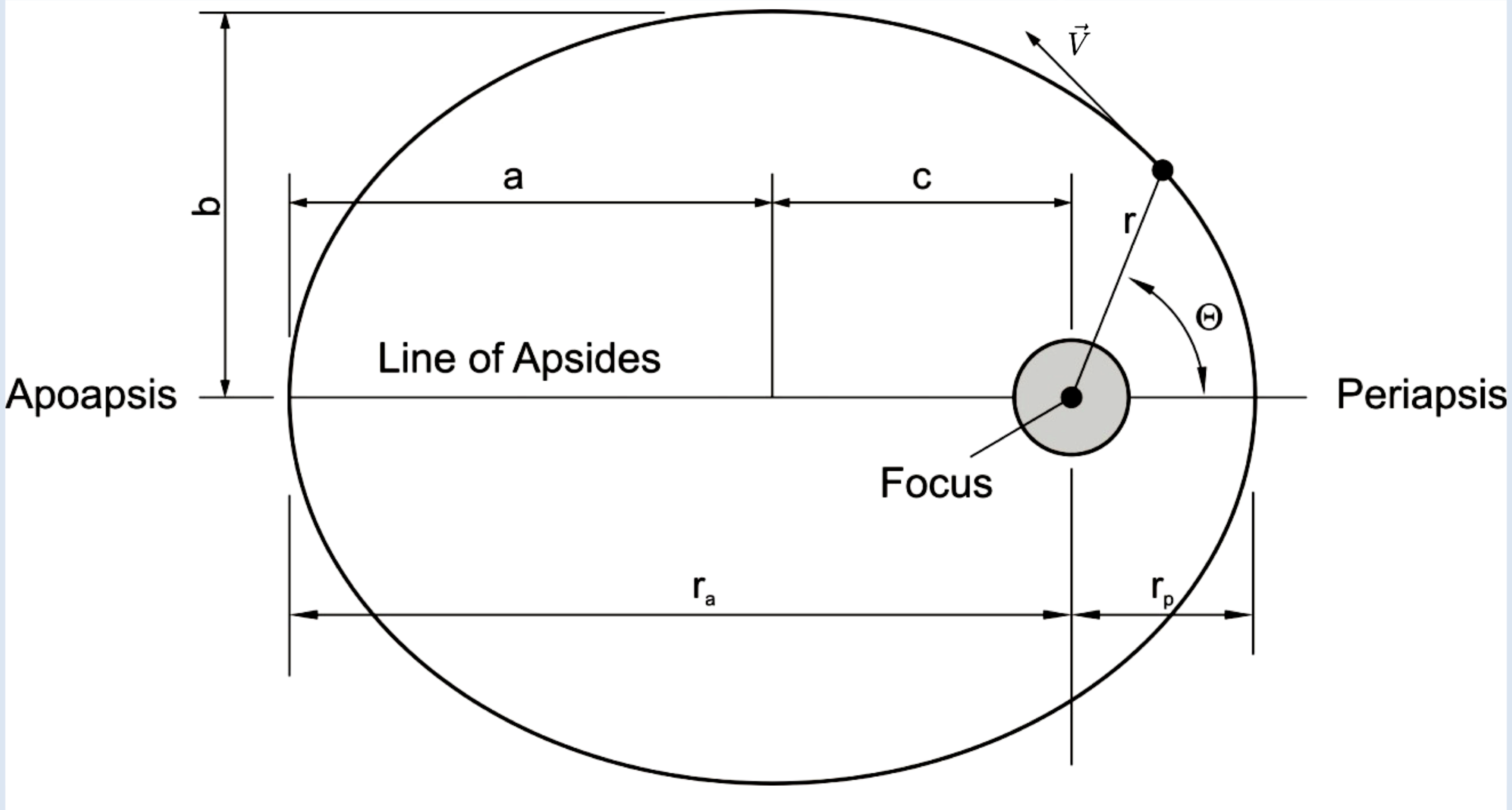
$$T = \frac{2\pi r}{V} = 2\pi\sqrt{\frac{r^3}{\mu}}$$

and for an elliptical orbit:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$



Elliptical orbits



- ***a***: Semi-major axis
- ***b***: Semi-minor axis
- ***c*** = ***ae***: Eccentricity ***e*** < 1

- r_a: Distance to the apoastris
- r_p: Distance to the periastris

- $\vec{V}$: Velocity
- Θ: True anomaly

Periapsis and apoapsis are general terms. Periastris and apoastris sometimes used for a star as central body. If the Earth is the central body, we talk about perigee and apogee; if it is the Sun, perihelion and aphelion.
The *True anomaly* is the angle between the direction of the periapsis from the central body and the radius vector to the spacecraft or the planet.
if *e* = 0 the orbit is circular. If e = 1, the orbit is parabolic (a to infinity)

Energy of the orbital motion and orbital velocity

- Energy of the orbital motion, per unit mass:

$$\boxed{\epsilon = \frac{V^2}{2} - \frac{\mu}{r}}$$

$$\boxed{\epsilon = -\frac{\mu}{2a}}$$

depends on a only

- Orbital velocity at any location on an elliptical or circular orbit:

elliptical (Vis Viva equation):

$$\boxed{V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}}$$

circular:

$$\boxed{V = \sqrt{\frac{\mu}{r}}}$$

The total energy in a gravitational field is the sum of the kinetic energy and the potential energy.

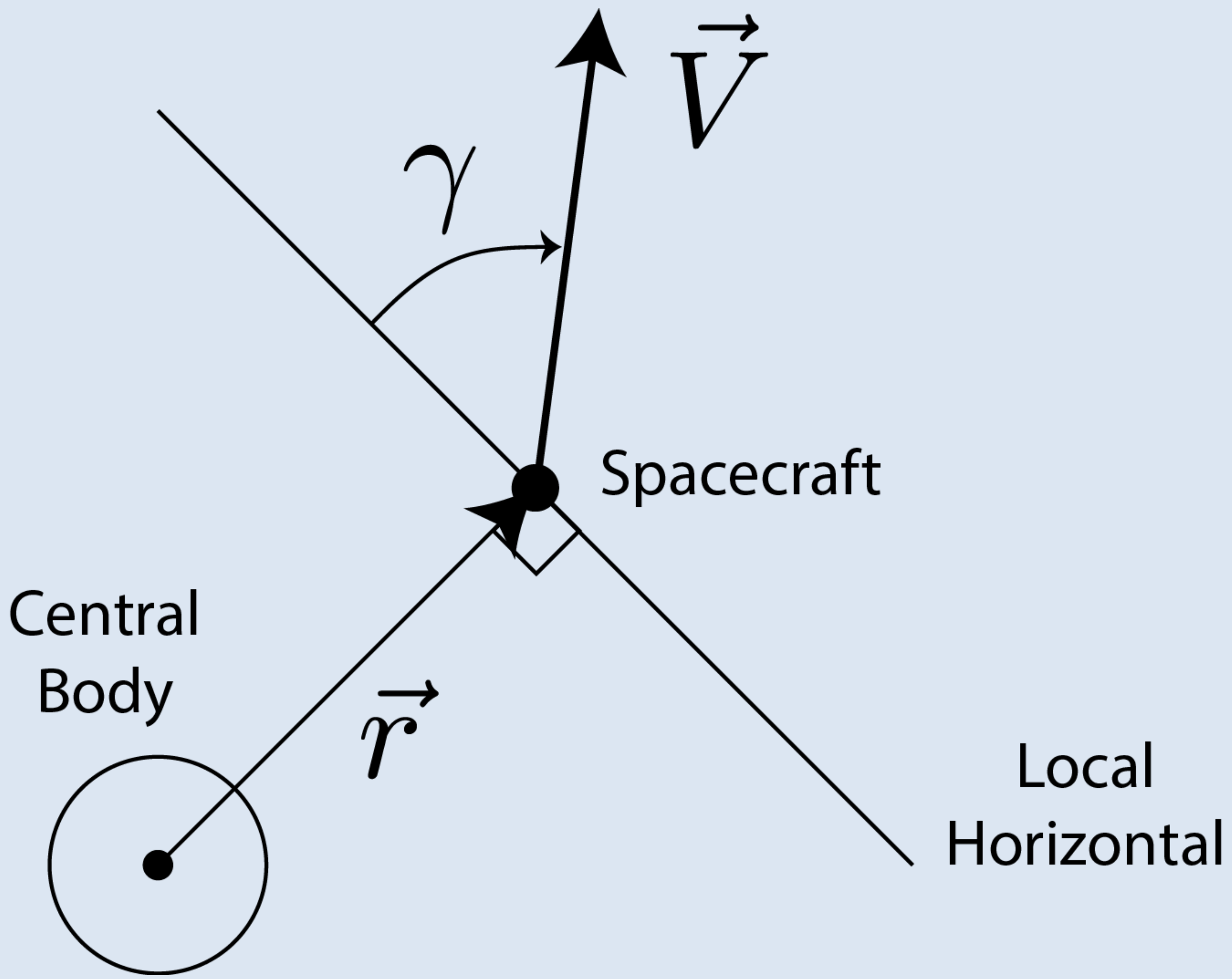
In case of a very elongated ellipse, the total energy is close to zero.

In the limit case of a parabolic orbit, the total energy is equal to zero.

If V < escape velocity, which is the case for a closed orbit, elliptical or circular, the total energy is negative.

If the orbit is hyperbolic, the total energy is positive

Angular momentum



- Angular momentum of a spacecraft $\vec{j}$, per unit mass:

$$\vec{j} = \vec{r} \times \vec{V}$$

$$|\vec{j}| = r \cdot V \cos\gamma$$

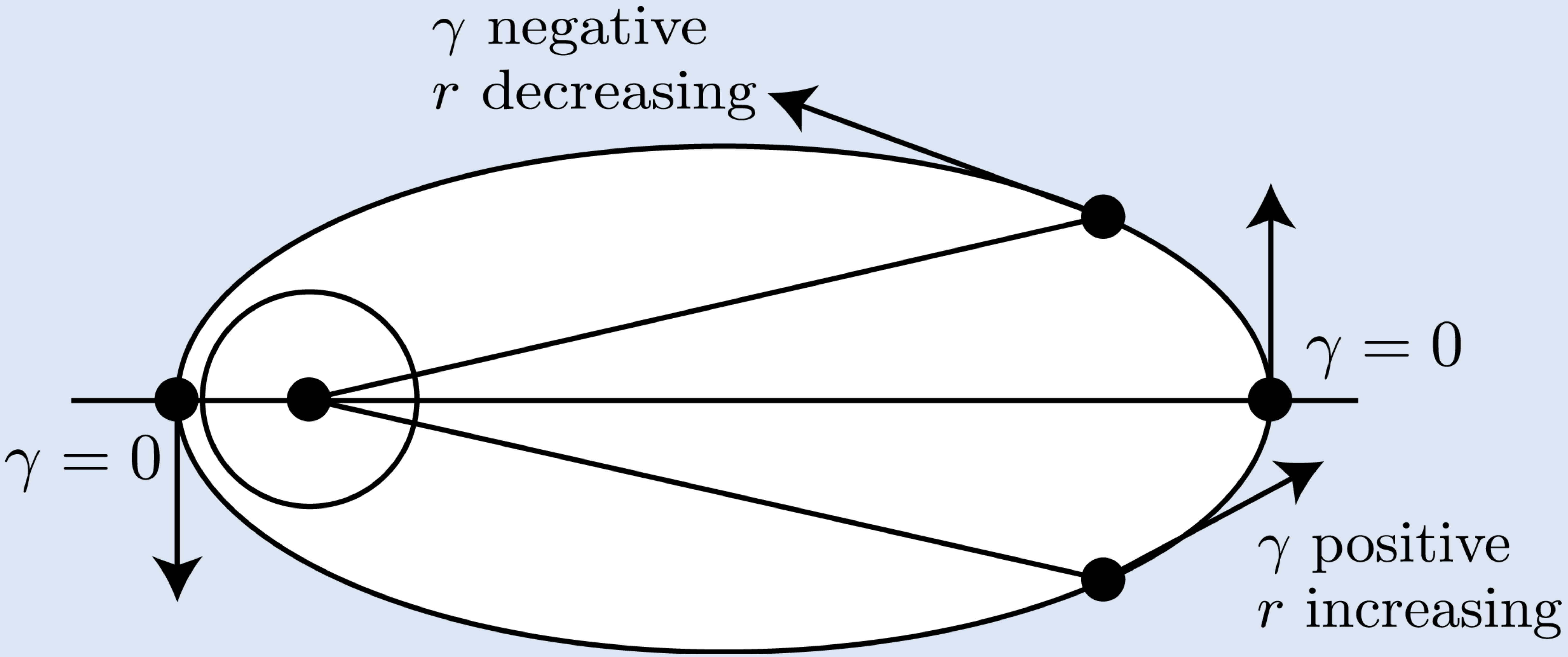
- γ is the flight path angle.

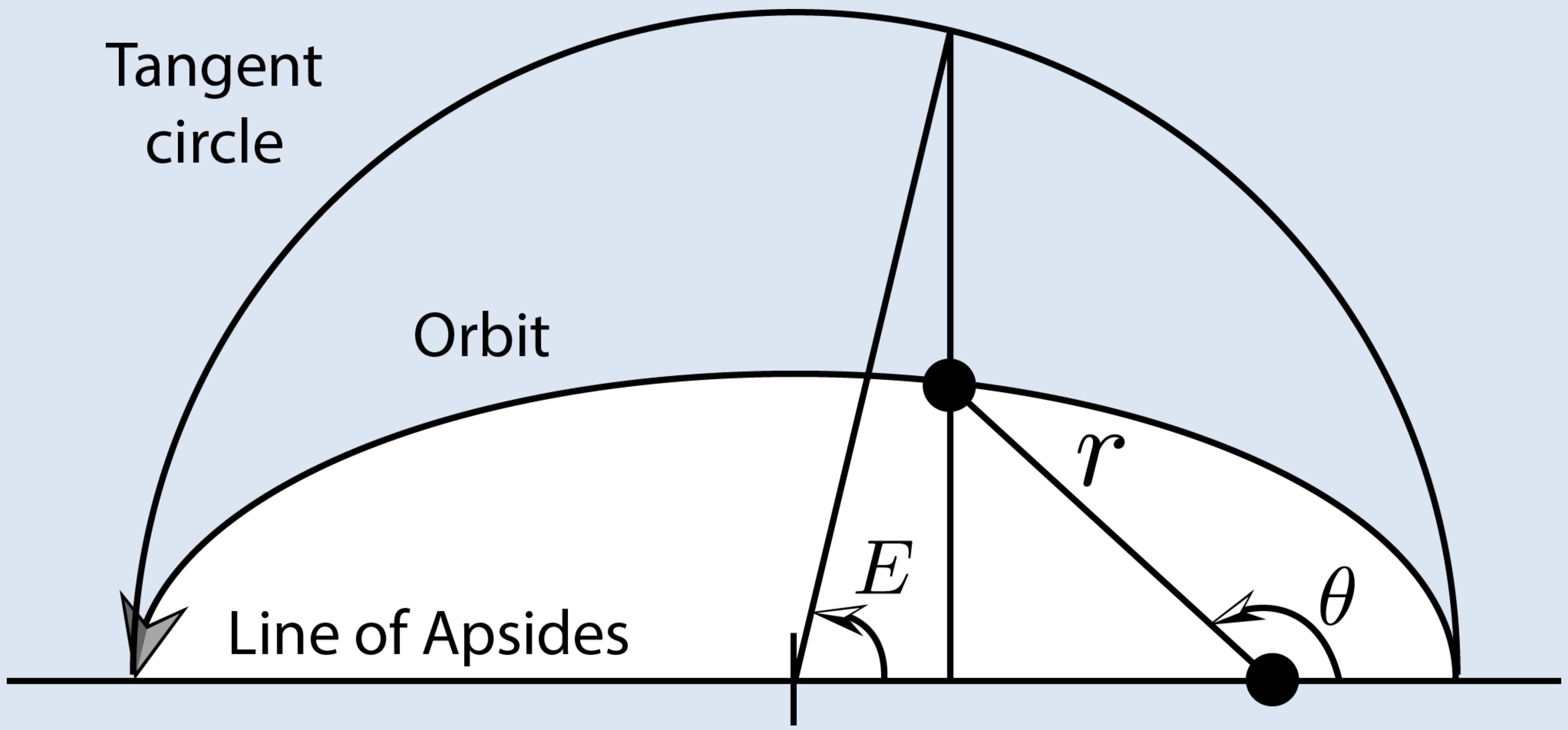
The flight path angle is the angle between the direction of the velocity vector and the perpendicular to the radius vector at the point where the spacecraft is.

Variation of the flight path angle (γ)

The flight path angle is equal to zero at the apogee and perigee (or apoapsis and periapsis). It is positive from the perigee to the apogee and negative from the apogee to the perigee.

Variation of the flight path angle γ on an elliptical orbit as a function of position:





2.4.2 Elliptical orbits: examples and Kepler's equation

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Elliptical orbits – Useful formulas

- Eccentricity $$e = \frac{c}{a} = \frac{(r_a - r_p)}{(r_a + r_p)} = \frac{r_a}{a} - 1 = 1 - \frac{r_p}{a} = \frac{r_2 - r_1}{r_1 \cos\theta_1 - r_2 \cos\theta_2}$$
- Flight path angle $$\tan\gamma = \frac{e \sin\theta}{1 + e\cos\theta}$$
- Mean motion (rad/sec) $$n = \sqrt{\frac{\mu}{a^3}}$$
- Period $$T = \frac{2\pi}{n} = 2\pi\sqrt{\frac{a^3}{\mu}}$$
- Radius $$r = \frac{a(1-e^2)}{(1+e\cos\theta)} = \frac{r_p(1+e)}{(1+e\cos\theta)}$$
- Apoapsis radius $$r_a = a(1+e) = 2a - r_p = r_p\frac{(1+e)}{(1-e)}$$

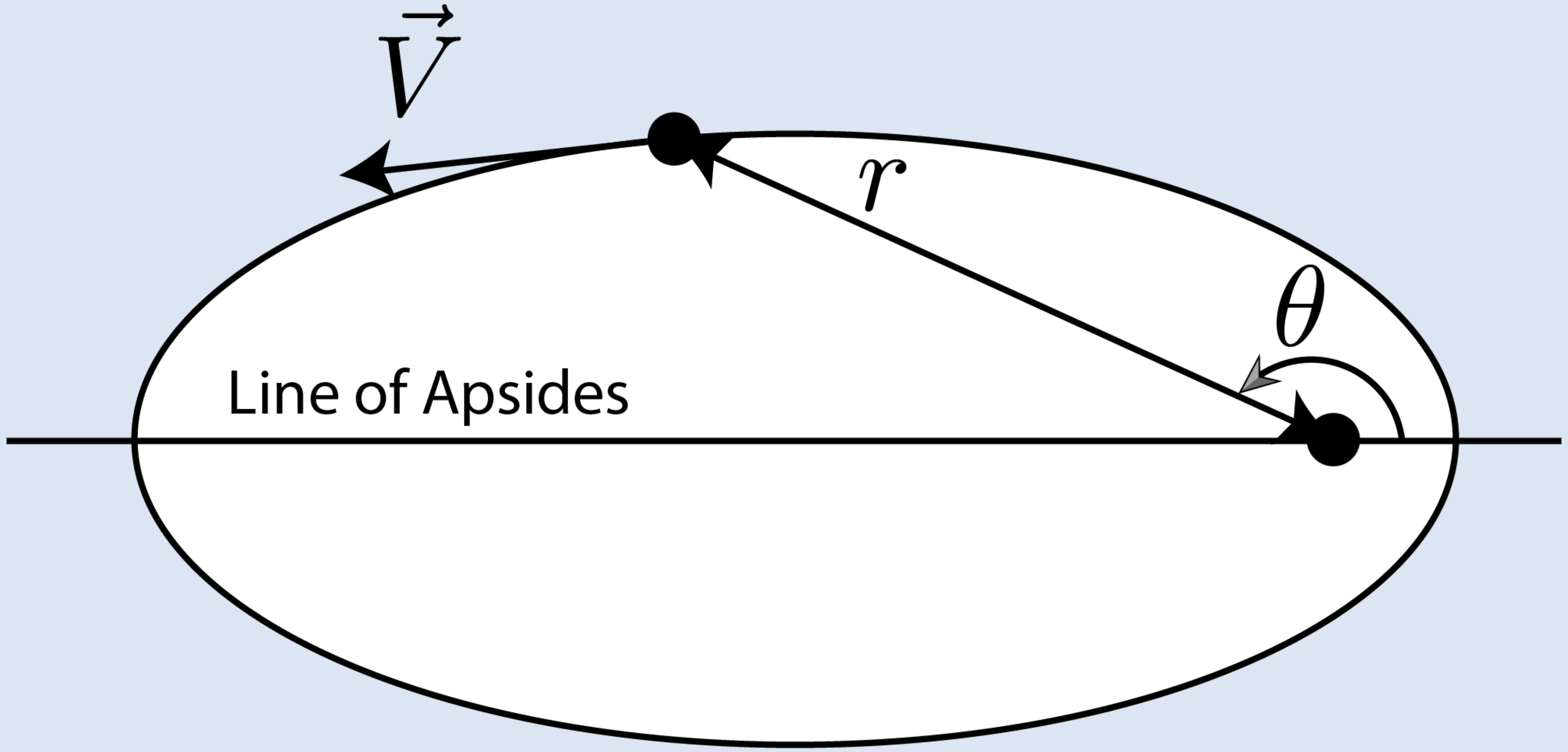
Elliptical orbits – Useful formulas

- Periapsis radius $$r_p = a(1-e) = r_a\frac{(1-e)}{(1+e)} = 2a - r_a = \frac{r_1(1+e\cos\theta_1)}{1+e}$$
- True anomaly $$\cos\theta = \frac{r_p(1+e)}{re} - \frac{1}{e} = \frac{a(1-e^2)}{re} - \frac{1}{e}$$
- Semi-major axis $$a = \frac{(r_a+r_p)}{2} = \frac{r_p}{(1-e)} = \frac{r_a}{(1+e)}$$
- Time since periapsis $$t = \frac{(E - e\sin E)}{n}$$
- Eccentric anomaly $$cos E = \frac{e+\cos\theta}{1+e\cos\theta}$$
- Velocity $$V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} \qquad r_pV_p = r_aV_a$$

Elliptical orbits – Example 1

Determination of ***r*** knowing ***a***, ***e***, ***θ***, or ***r_p***, ***e*** and ***θ***.

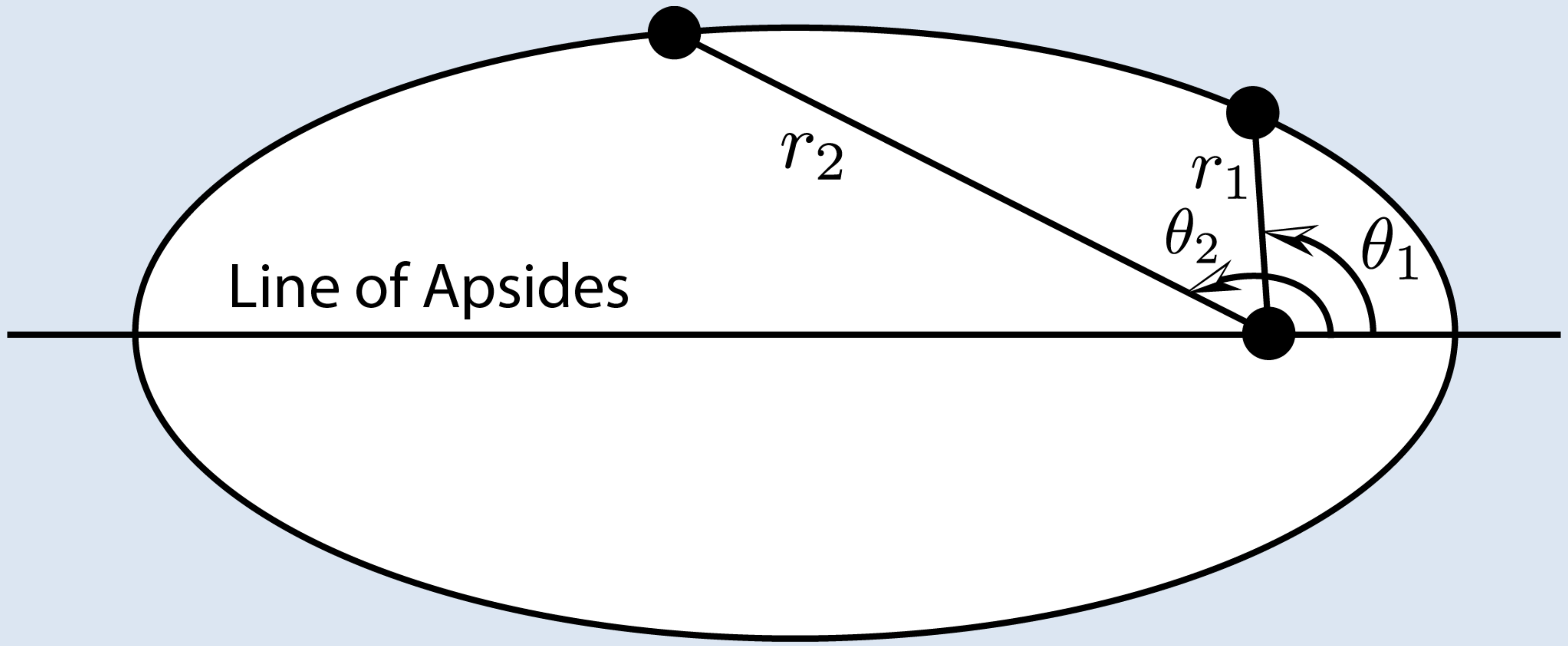
$$r = \frac{a(1-e^2)}{(1+e\cos\theta)} \qquad r = \frac{r_p(1+e)}{(1+e\cos\theta)}$$



Elliptical orbits – Example 2

Determination of parameters of an elliptical orbit knowing two points on the orbit.

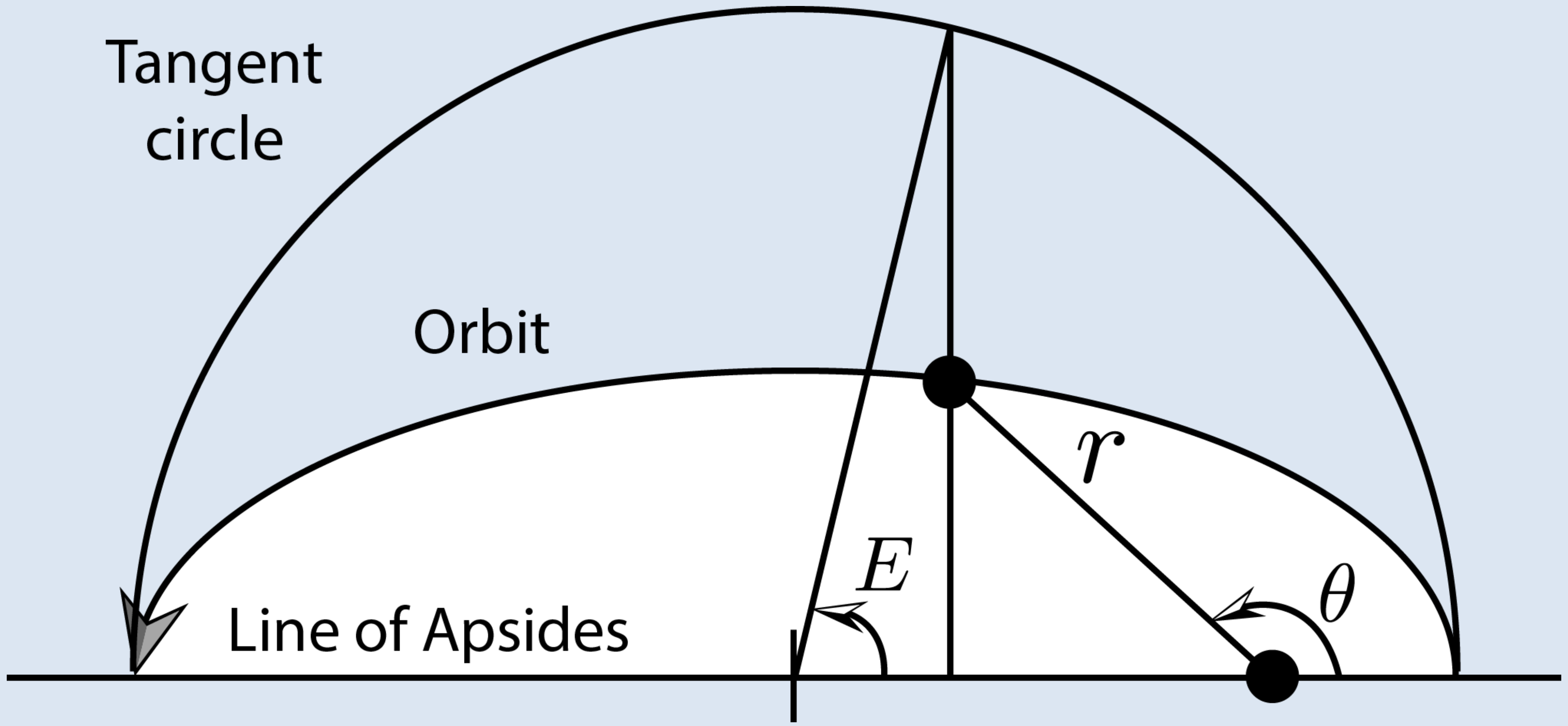
$$r_1 = \frac{r_p(1+e)}{1+e\cos\theta_1} \qquad r_2 = \frac{r_p(1+e)}{1+e\cos\theta_2}$$



$$e = \frac{r_2 - r_1}{r_1\cos\theta_1 - r_2\cos\theta_2}$$

$$r_p = r_1\frac{(1+e\cos\theta_1)}{1+e} \qquad r_a = r_p\frac{(1+e)}{(1-e)}$$

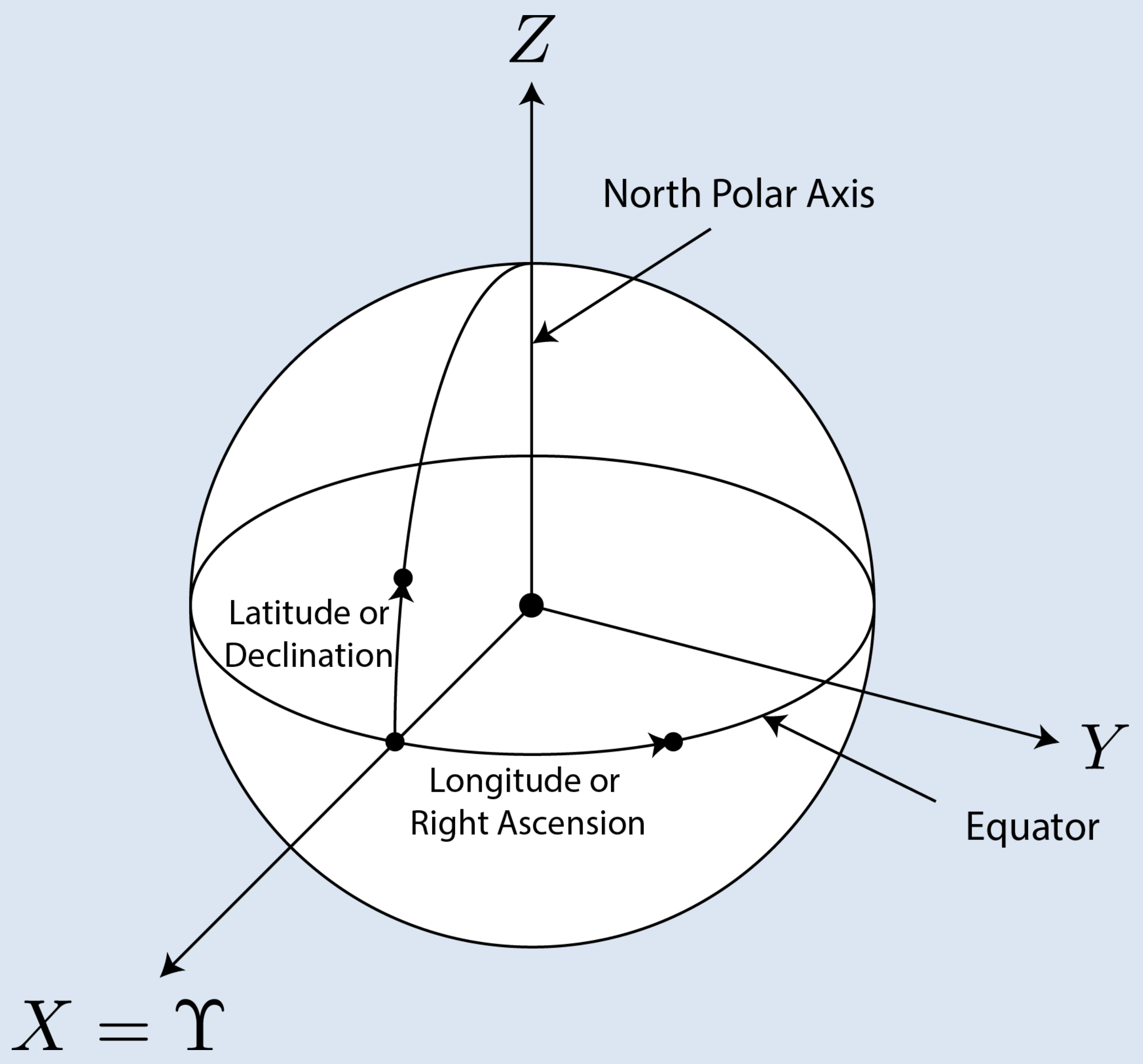
Elliptical orbits – Kepler's equation



E, the eccentric anomaly, is an angular parameter that defines the position of a body that is moving along an elliptical Keplerian orbit.

The Kepler's equation is a transcendental equation that cannot be solved for *E* but expresses the time evolution of *E*, the eccentric anomaly since passing the periapsis.

$$\boxed{t = \frac{(E - e\sin E)}{n}}$$

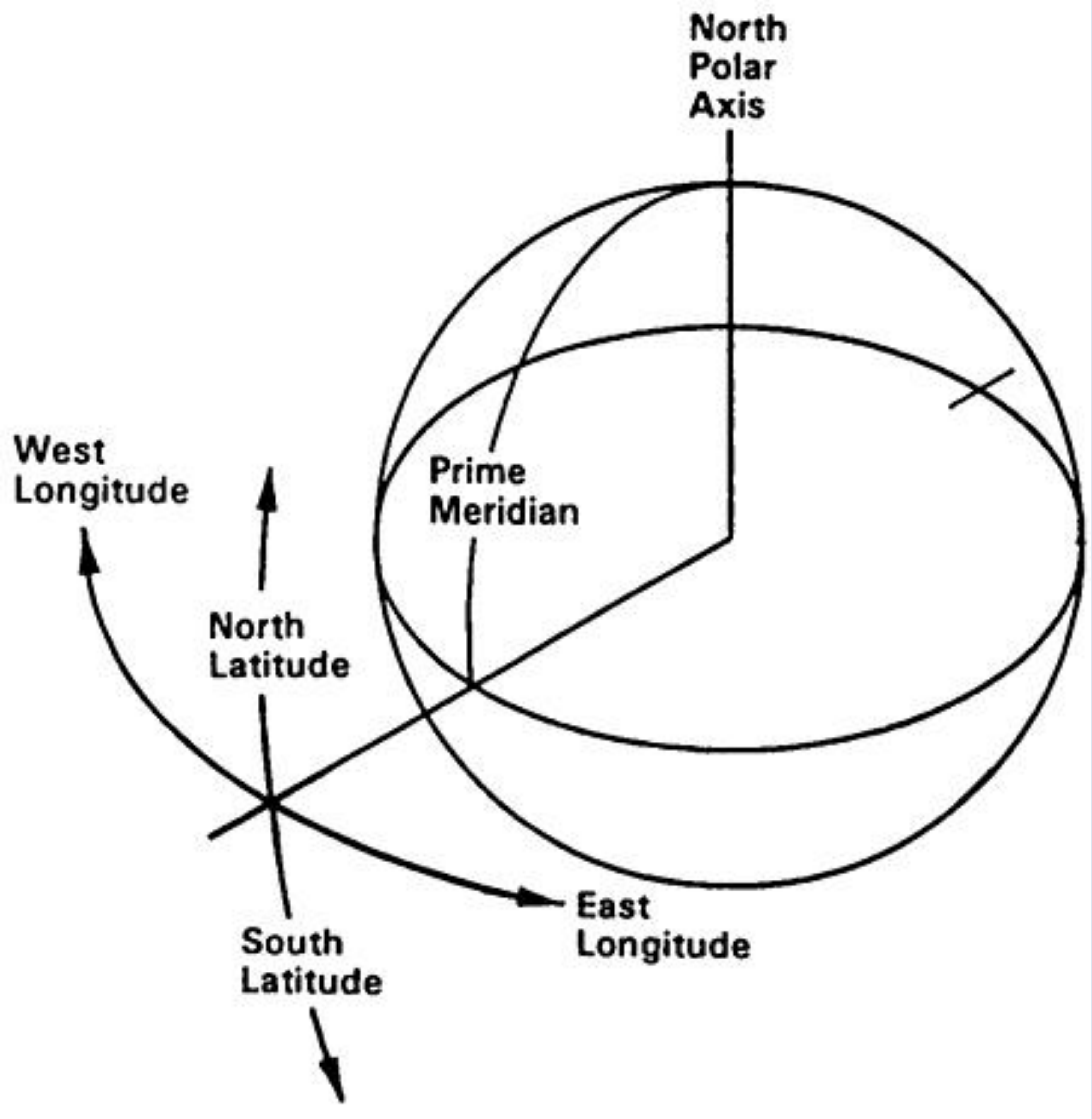


2.5.1 Reference frames

Space Mission Design and Operations

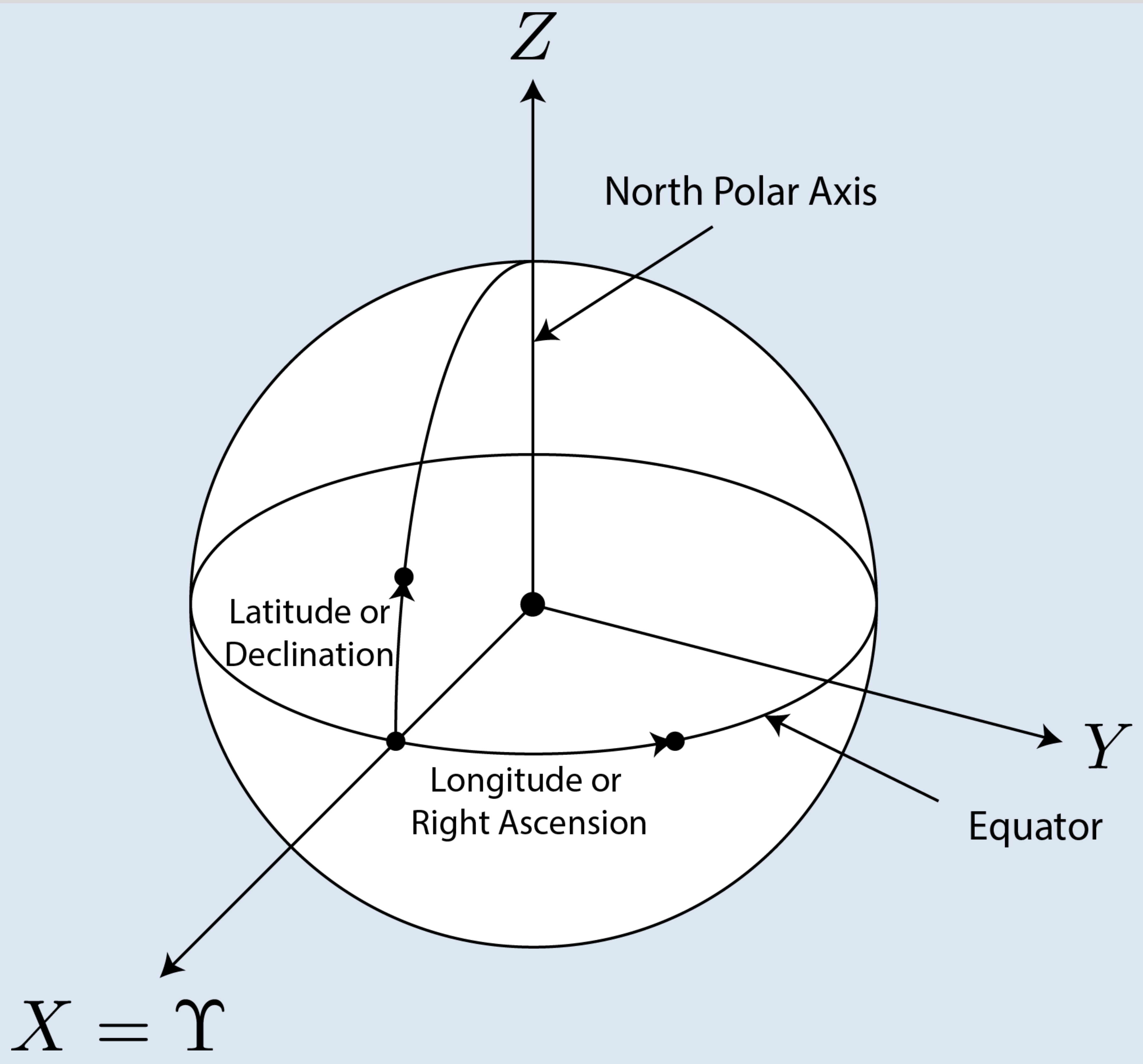
Prof. Claude Nicollier

Geographic coordinate system



The geographic coordinate system (longitude, latitude) is used to specify a location on the surface of the Earth.

Geocentric-inertial coordinate system



An inertial frame is an orthogonal frame of reference XYZ, with respect to which the laws of motion, the laws of Newton, are valid.

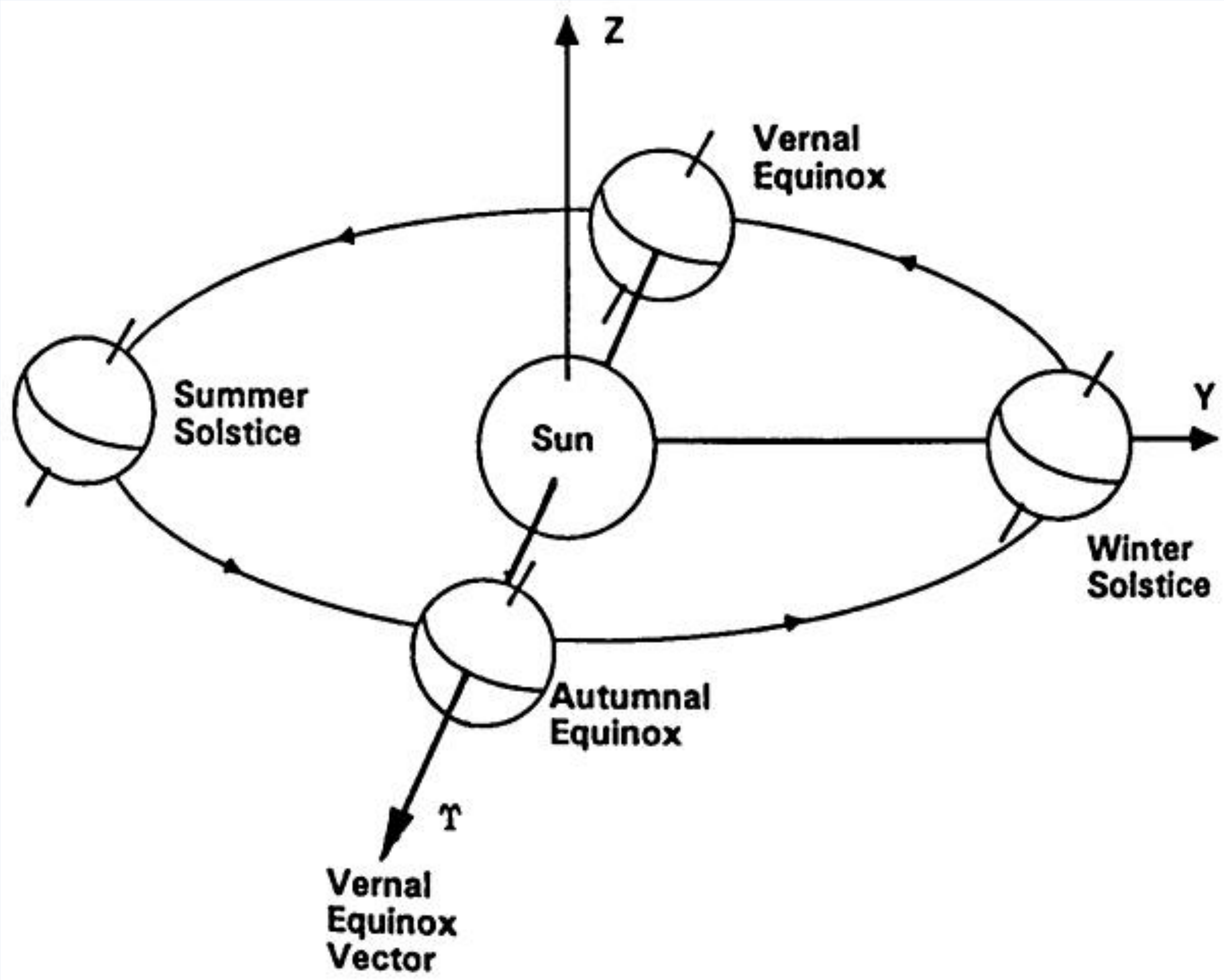
The center of the geocentric inertial coordinate system is the center of the Earth.

The plane of reference is the plane of the Equator, where the direction of X is the direction of the vernal equinox.

The vernal equinox ♈ is a point on the Equator that the Sun crosses when it goes from the southern celestial hemisphere to the northern celestial hemisphere around the 21st of March.

This point is very slowly migrating to the west (precession of equinoxes, about 0.014 degrees per year), so, when using the geocentric-inertial coordinate system, the year shall be specified. Currently the year 2000 is used.

Heliocentric-inertial coordinate system

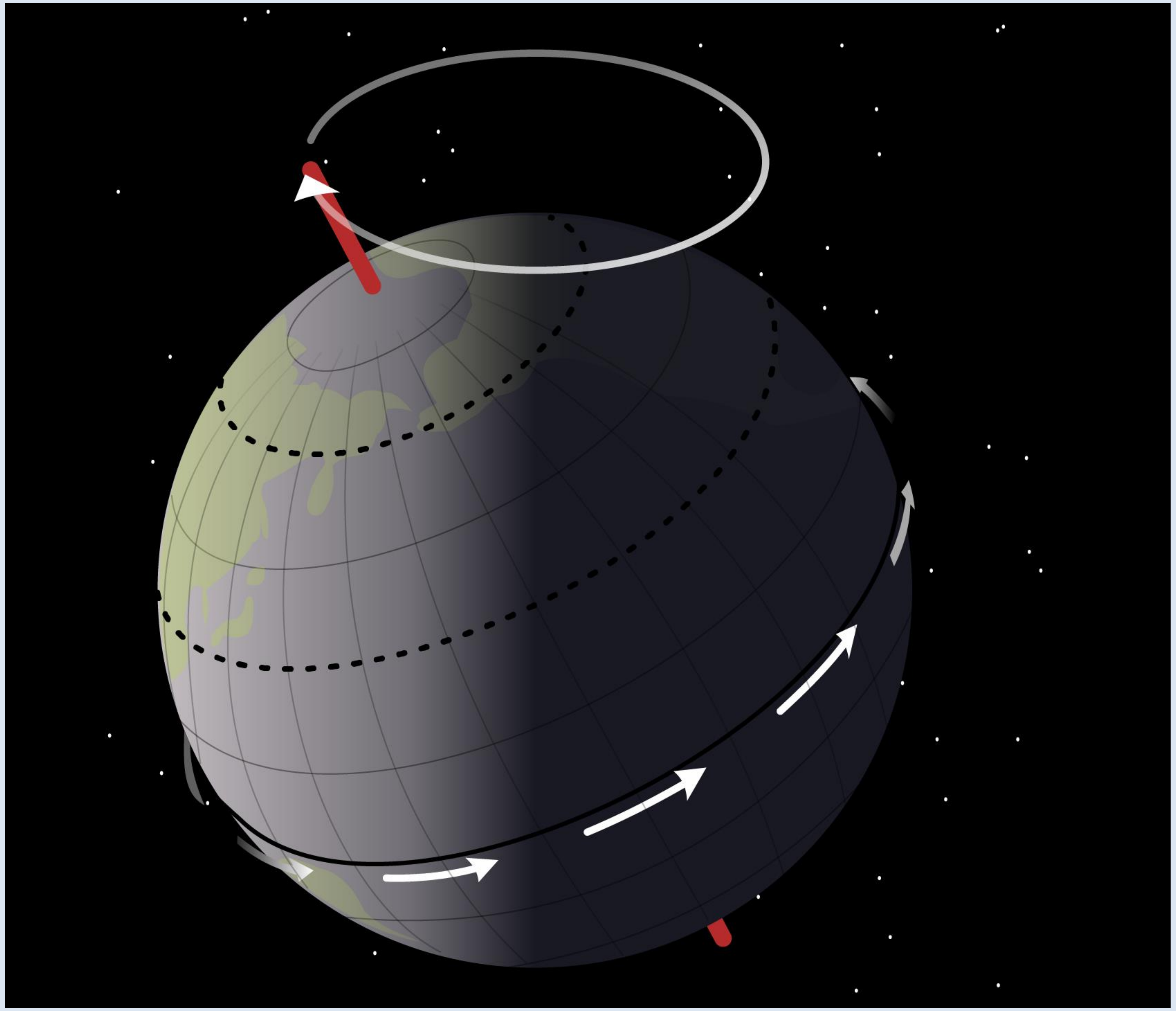


The heliocentric-inertial coordinate system has the same directions of axes as the geocentric-inertial coordinate system.

The center of this coordinate system is in the center of the Sun.

The plane of reference is the plane of the ecliptic, or plane of the Earth's orbit around the Sun

Precession of the equinoxes



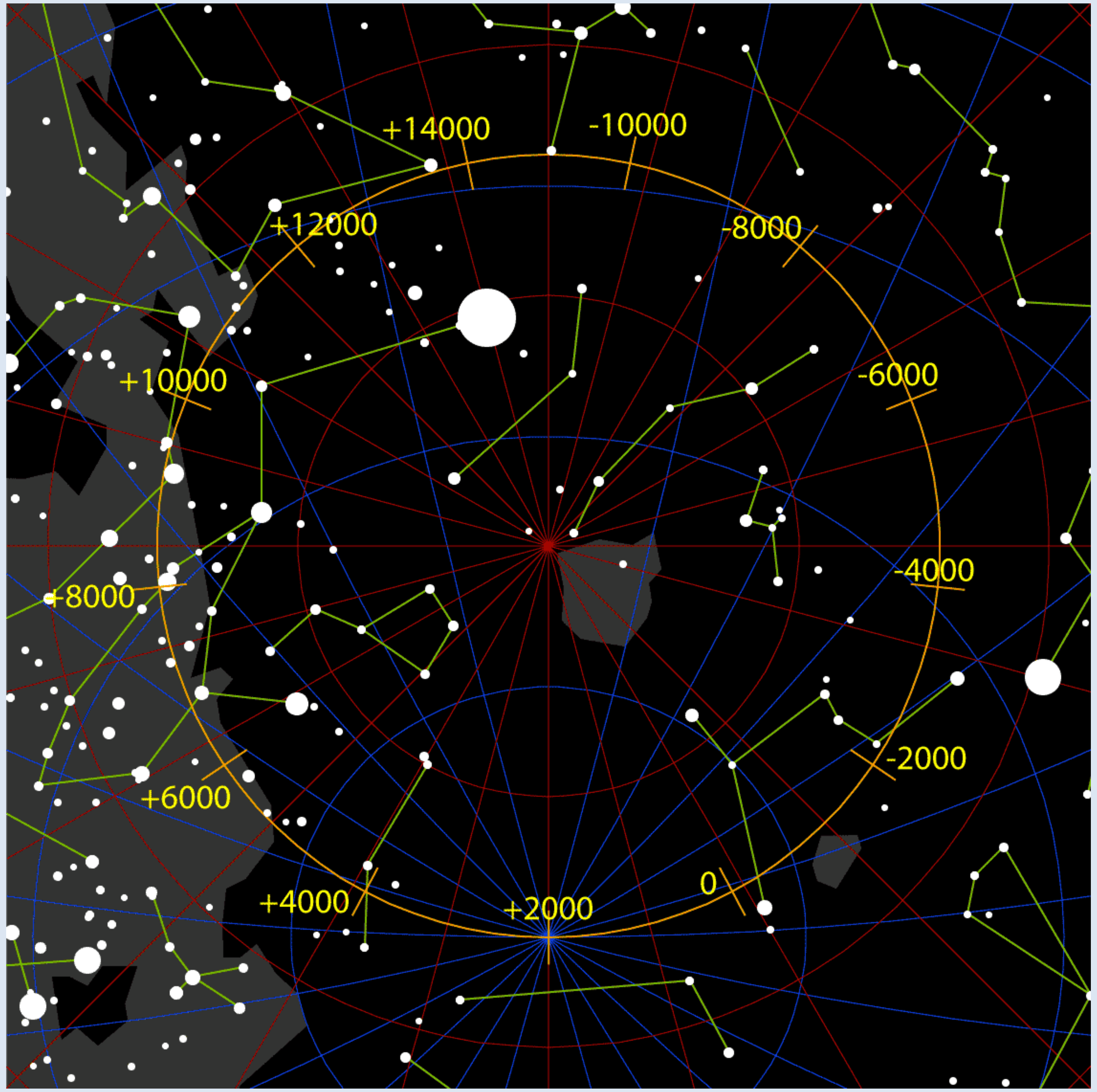
Credits: NASA GSFC, Robert Simmon

- Earth's rotational axis has a tilt of 23.5 degrees vs. a perpendicular to the ecliptic plane
- Axial precession is the displacement of the rotational axis of an astronomical body.
- Earth goes through one such complete precessional cycle in about 26'000 years.

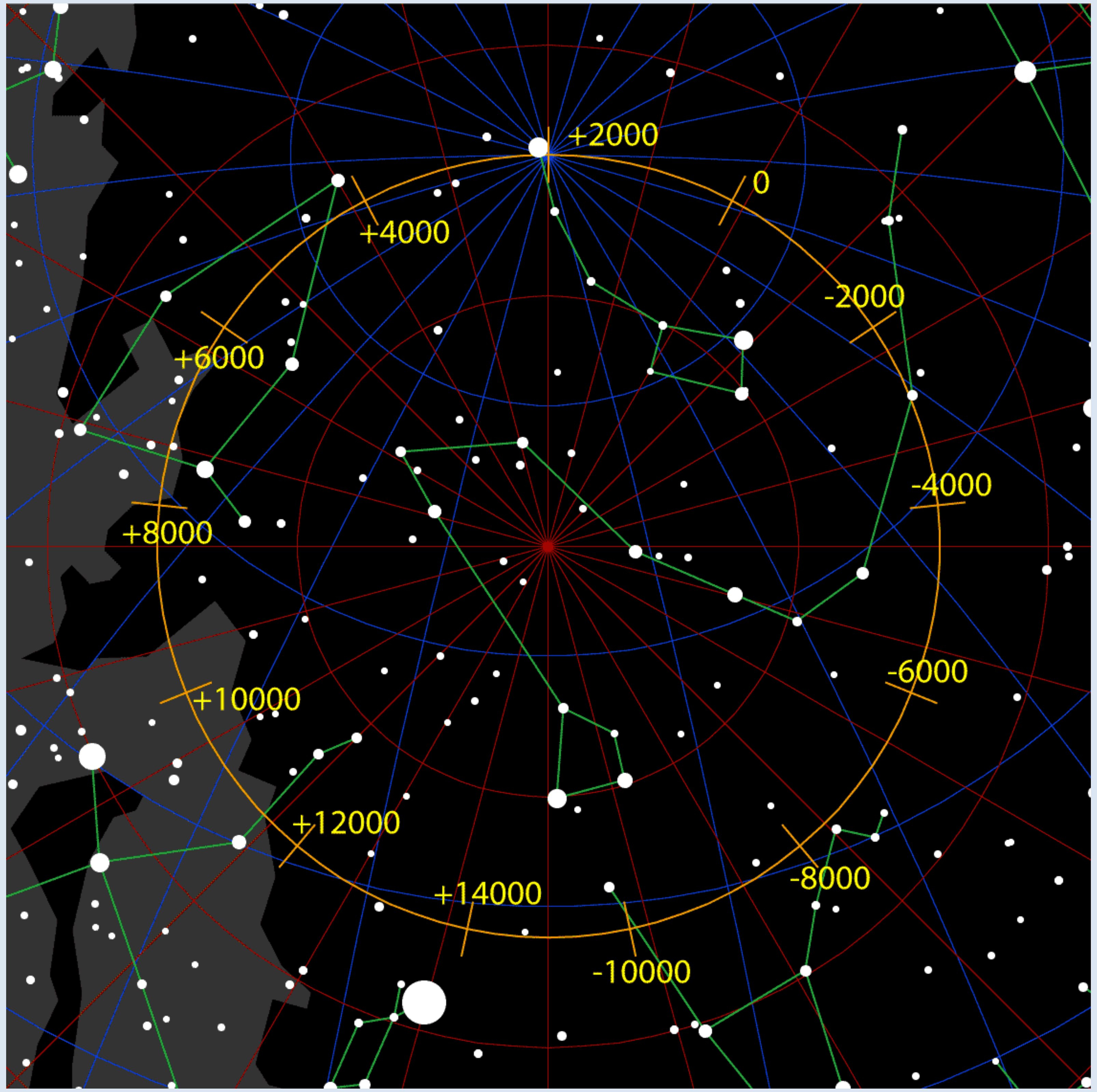
The Earth is not a perfect sphere, but has an equatorial bulge, and the gravitational force, from the Sun and the Moon, on a non-spherical body, causes the precession.

Precession of Earth’s axis

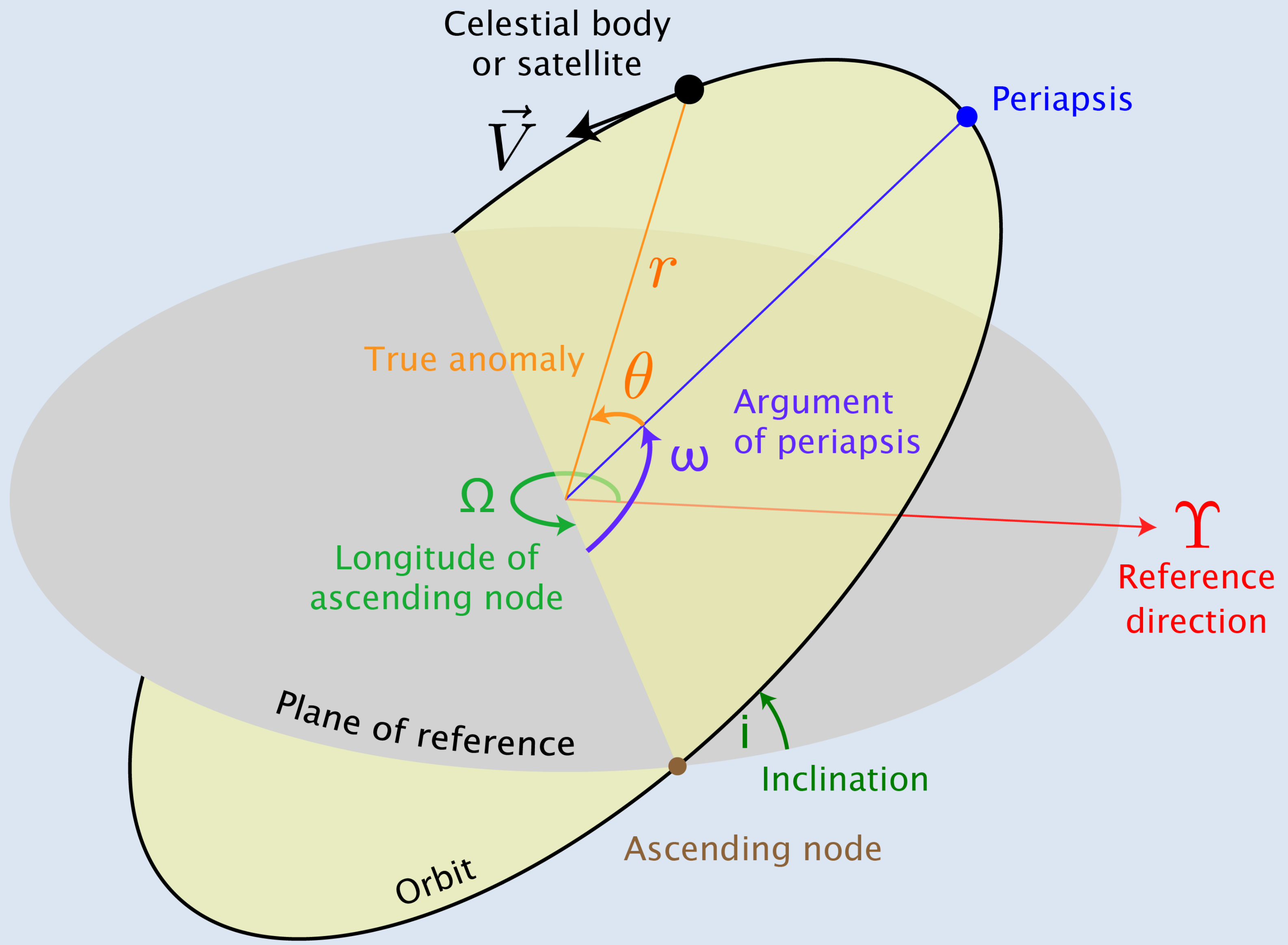
Precession of Earth’s axis
around the south ecliptic pole



Precession of Earth’s axis
around the north ecliptic pole



Credits: Wikipedia,



2.5.2 Orbital parameters and calendars

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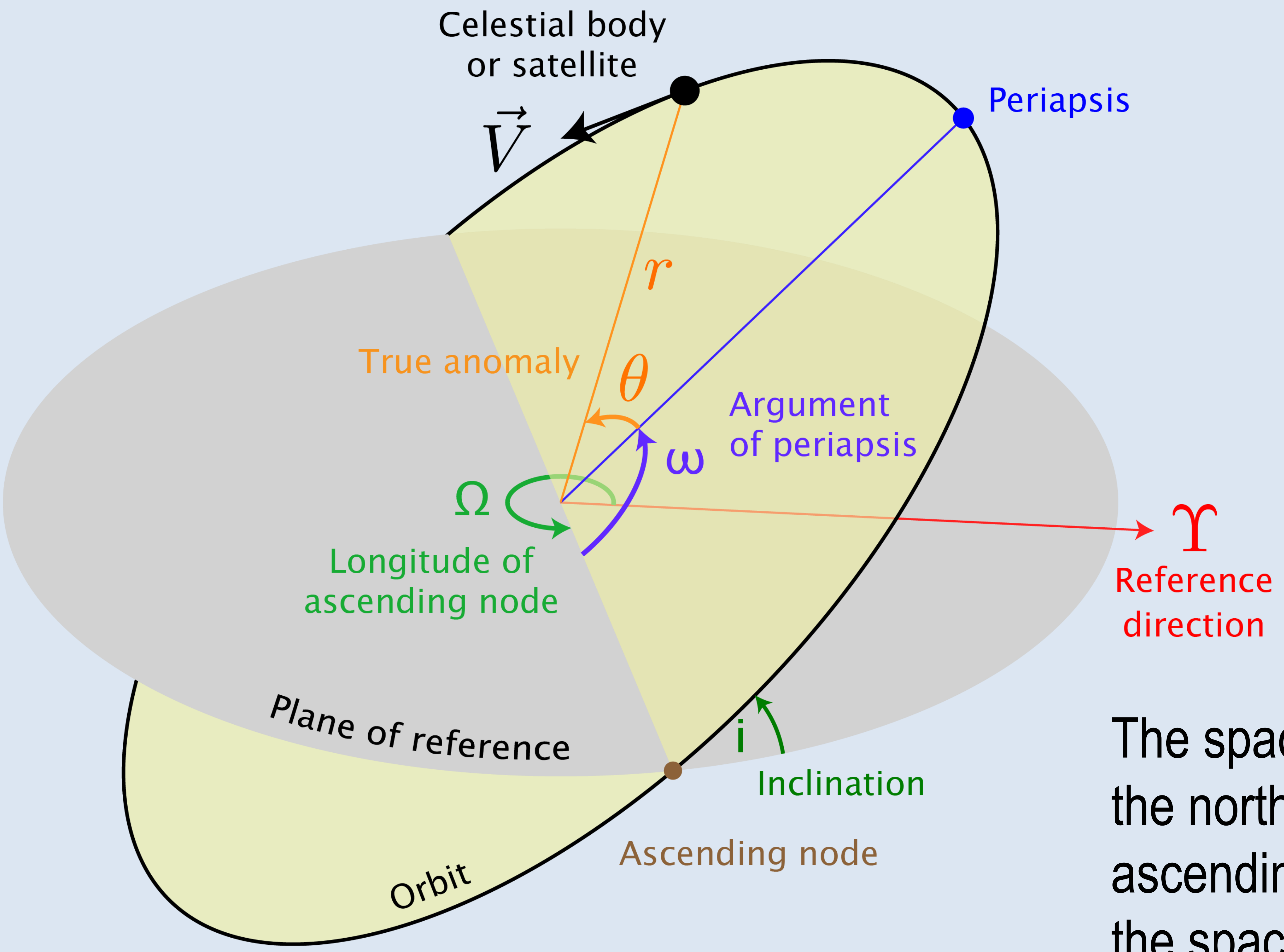
Prof. Claude Nicollier

Credits: Adapted from
Wikipedia, Lasunnkty

Classical orbital parameters



- ***e*, *a***, then:
- ***i*** = inclination of the orbital plane.
- ***Ω*** = longitude or Right Ascension of the Ascending Node (RAAN) in the plane of reference).
- ***ω*** = argument of periapsis (in the orbital plane).
- ***T_p*** = time of periapsis transit.
- Current time ***t*** allowing a determination of the exact position of the celestial body or satellite.

The spacecraft is passing from the southern celestial hemisphere to the northern on a point on the plane of reference which is called the ascending node. The descending node is on the other side, where the spacecraft goes from the north to the south.

Credits: Adapted from Wikipedia, Lasunnkty

Spacecraft's state vector

$$(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}, t)$$

$$\Updownarrow$$

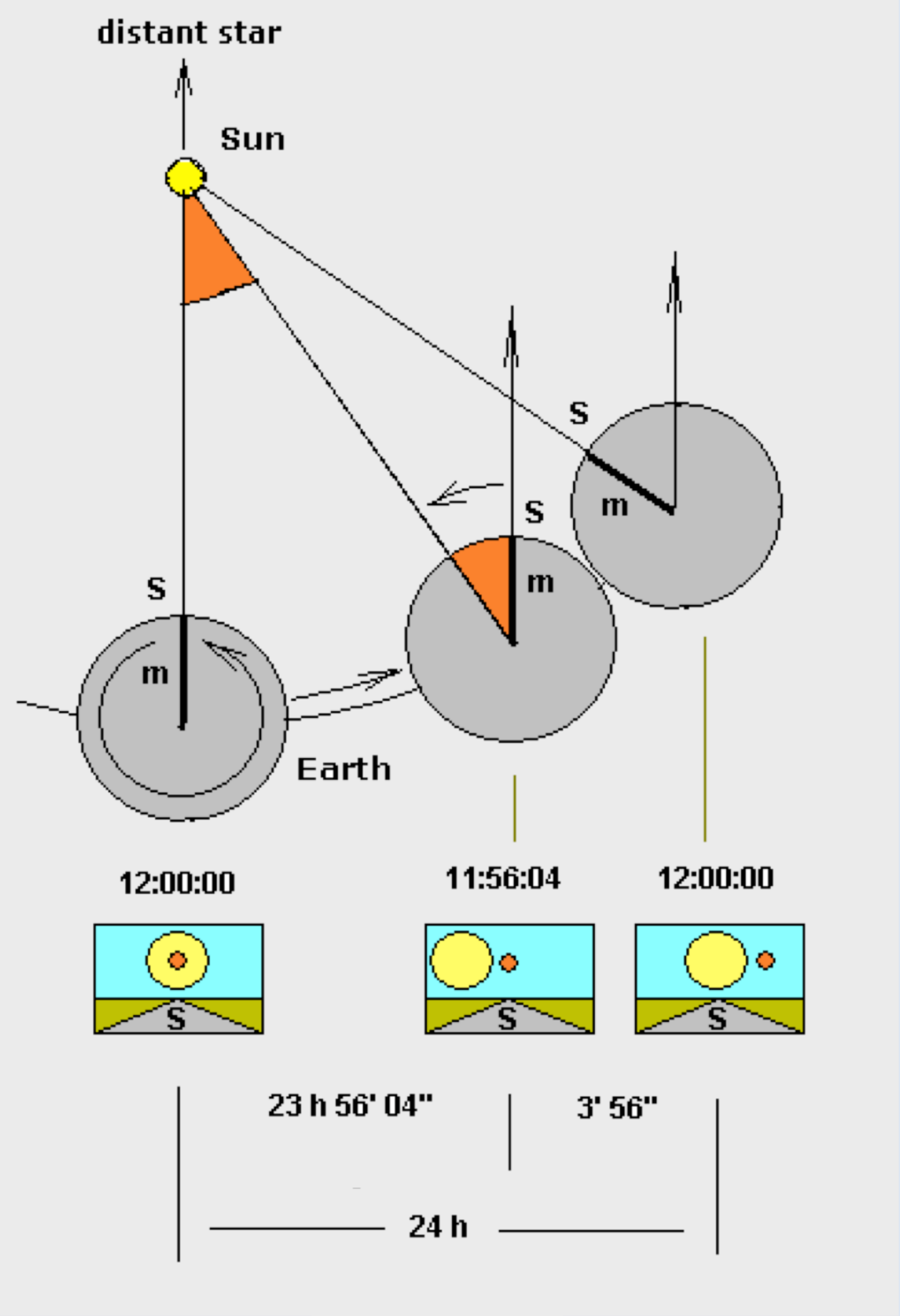
$$(a, e, i, \Omega, \omega, T_p, t)$$

The spacecraft's state vector is functionally equivalent to the six orbital parameters plus the time ***t***.

X, Y, Z, $\dot{X}$, $\dot{Y}$, $\dot{Z}$ are in the geocentric-inertial (or heliocentric-inertial) coordinate system.

On board the Space Shuttle, the current state vector was constantly propagated for a knowledge of the estimated current position and velocity vector. When GPS became available, there was a constant updating of this current position and velocity estimation, with a substantially increased accuracy!

Mean solar day – sidereal day



Credits: Wikipedia, Francisco Javier Blanco González

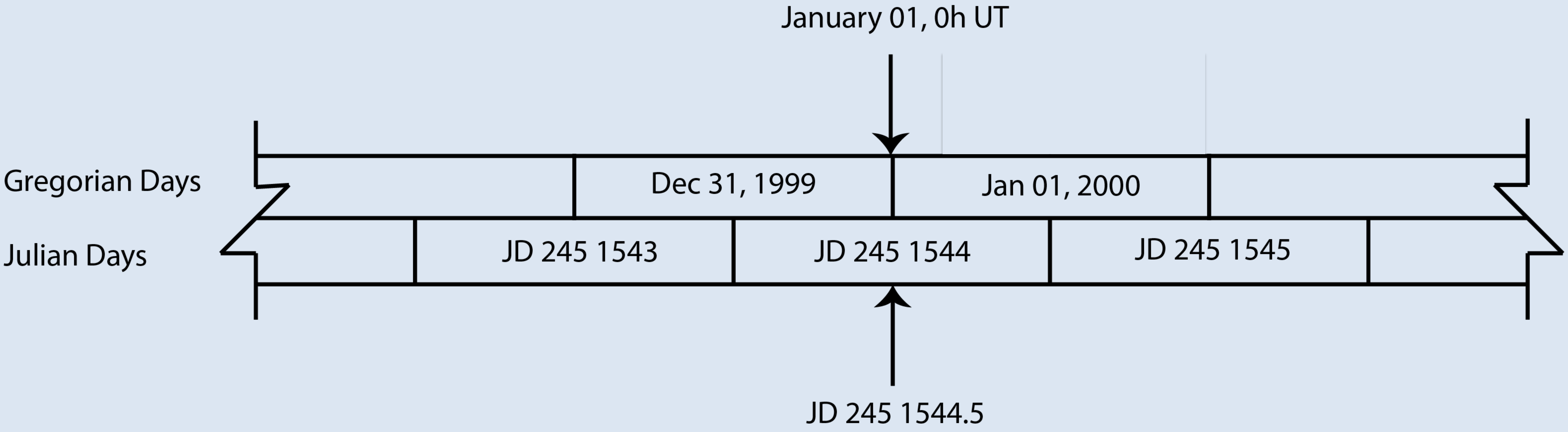
- Picture left: a distant star (the small red circle) and the Sun are at culmination, on the local astronomical meridian.
- Picture center: the distant star is again at culmination, after one sidereal day.
- Picture right: few minutes later the Sun is on the local astronomical meridian again at culmination. A solar day is complete.

The sidereal day is the time it takes for the Earth to make one full rotation with respect to the stars.

The mean solar day is the time it takes for the Earth to make one full rotation with respect to the mean Sun.

The duration of the mean solar day is 24 hours, but the duration of the sidereal day is about four minutes less.

Gregorian days vs. Julian days



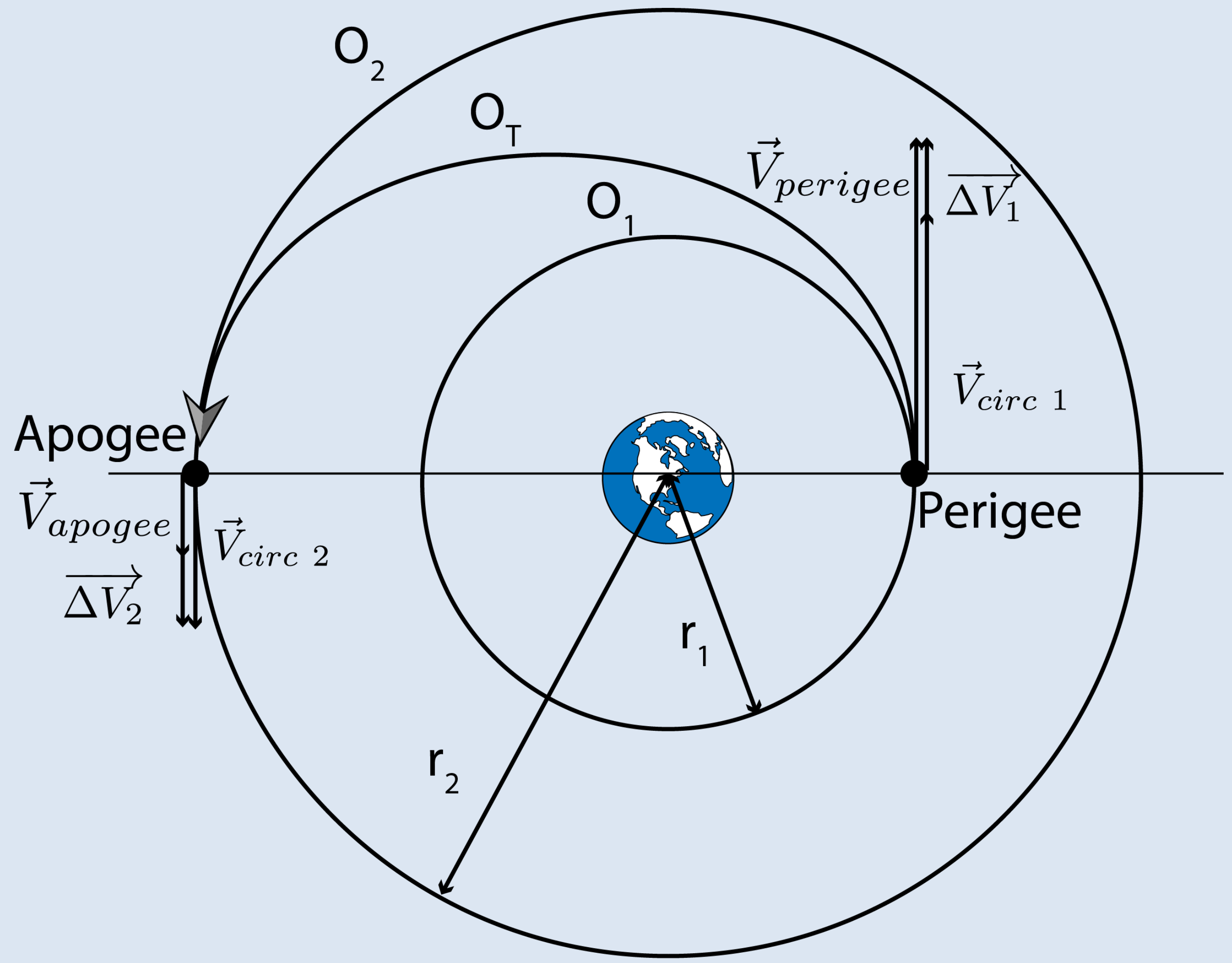
Julian day is used in the Julian date (JD) system of time measurement for scientific use by the astronomy community, presenting the interval of time in days and fractions of a day since January 1st, 4713 BC Greenwich noon. Julian date is recomended for astronomical use by the International Astronomical Union.

Conversion of Gregorian days to Julian days

$$J = 367Y - 7\left[\frac{Y + \frac{M+9}{12}}{4}\right] + \frac{275M}{9} + D + 1'721'013.5$$

- ***J*** = Julian day number
- ***Y*** = calendar year
- ***M*** = calendar month number (e.g., July = 7)
- ***D*** = calendar day and fraction

- All divisions must be integer divisions. Only the integer is kept; the fraction is discarded.
- Good from 1901 to 2099.

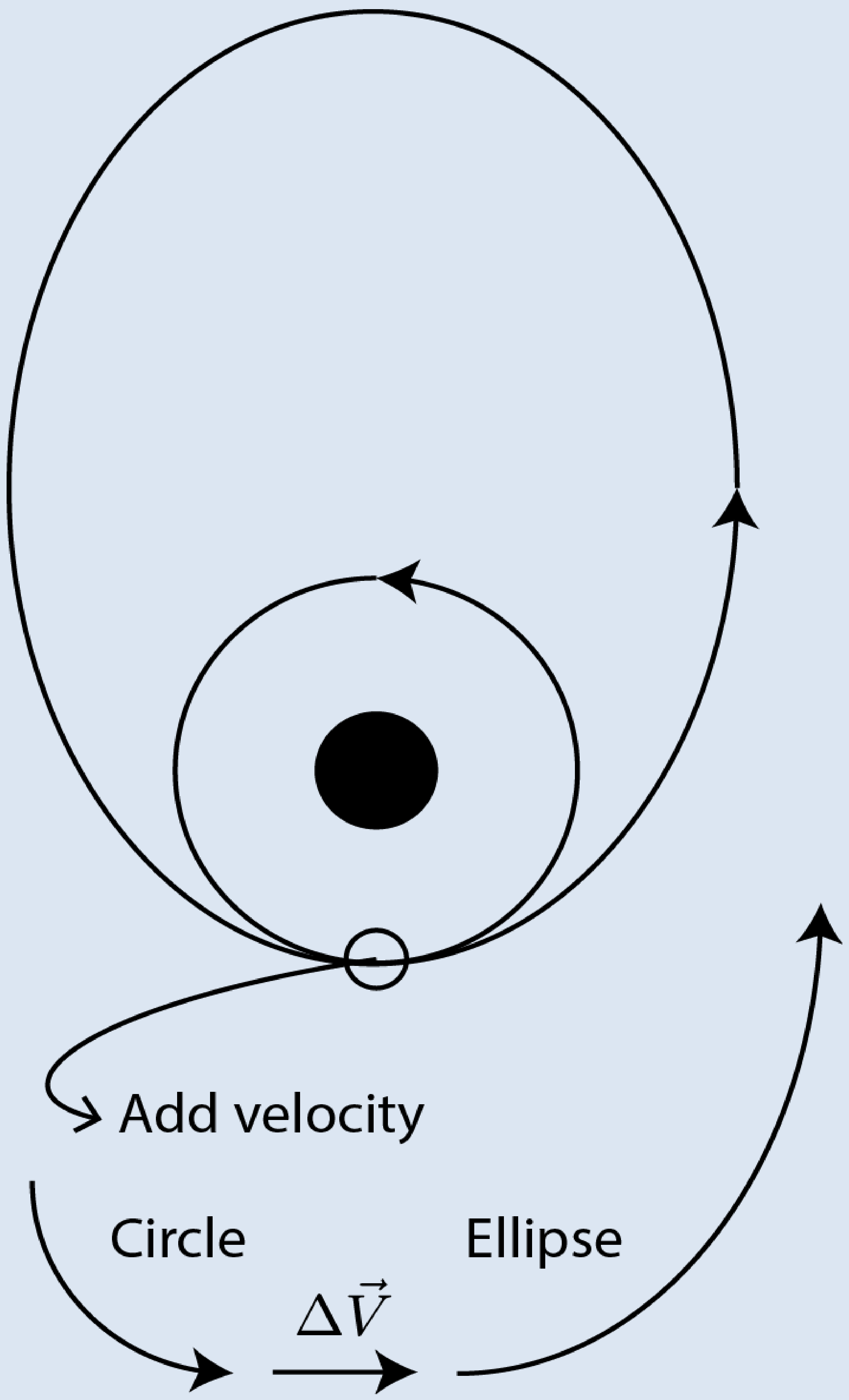


3.2.1 Orbital maneuvers and Hohmann transfer

Space Mission Design and Operations

Prof. Claude Nicollier

Maneuvers in-orbit

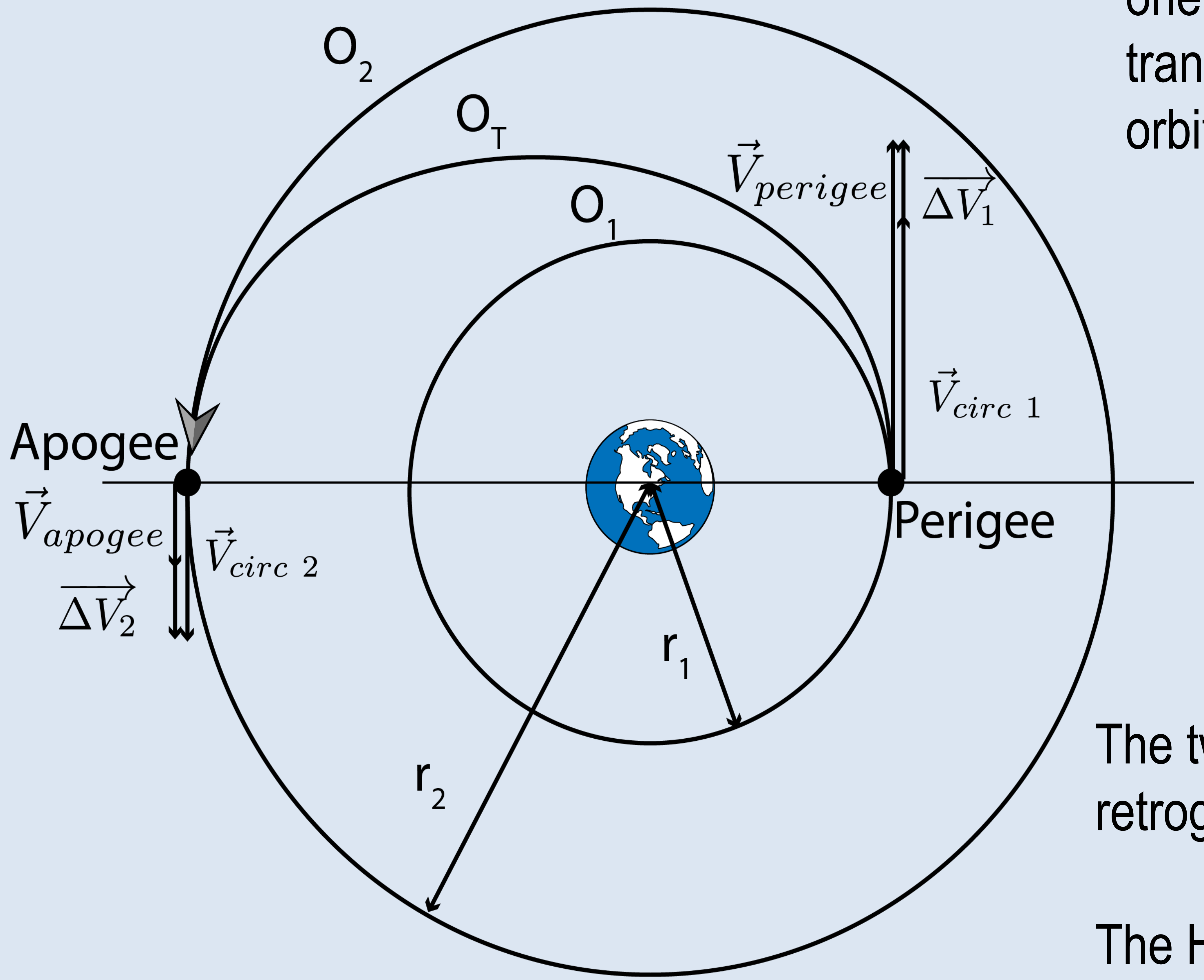


A maneuver in orbit is a vectorial ΔV causing a change of the orbital elements of an orbiting spacecraft.

We will use indifferently the terms maneuver, Delta-V or “Burn”, because, in nearly all cases, the maneuver is performed using thrusters that burn a propellant in presence of an oxidizer!

In this course, we will only consider instantaneous maneuvers.

Hohmann Transfer



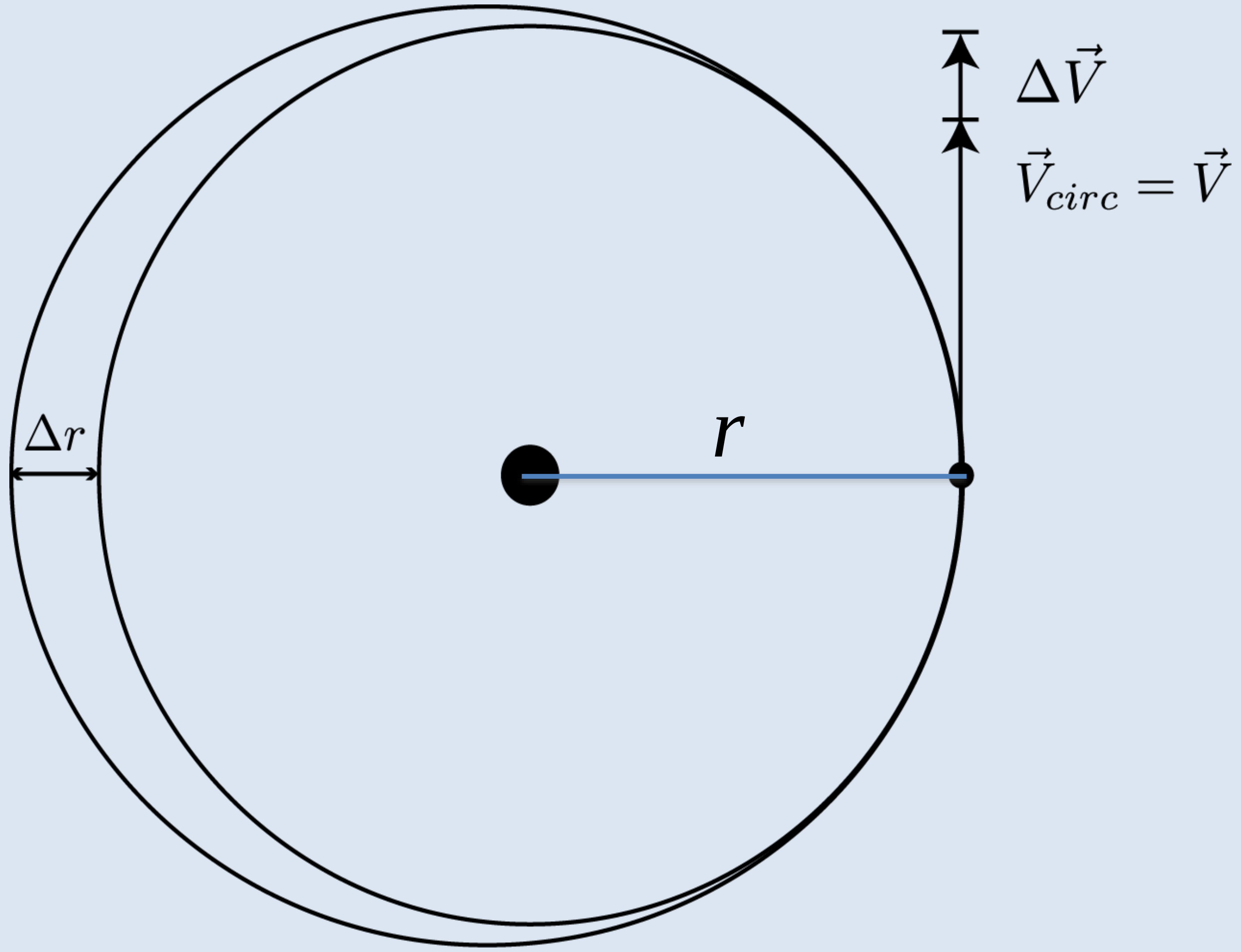
The Hohmann transfer is a very common method of transfer from one circular orbit to another, around the same central body. The transfer orbit is tangent to both the initial orbit and the destination orbit.

$$\Delta V_1 = \sqrt{\frac{2\mu r_2}{r_1(r_1+r_2)}} - \sqrt{\frac{\mu}{r_1}}$$

$$\Delta V_2 = -\sqrt{\frac{2\mu r_1}{r_2(r_1+r_2)}} + \sqrt{\frac{\mu}{r_2}}$$

The two ΔVs are posigrade for a transfer to a higher orbit, and retrograde for a transfer to a smaller orbit.

The Hohmann transfer is the most efficient transfer because the changes in velocity are used entirely for changes in kinetic energy.



3.2.2 Hohmann transfer – Case of small ΔV

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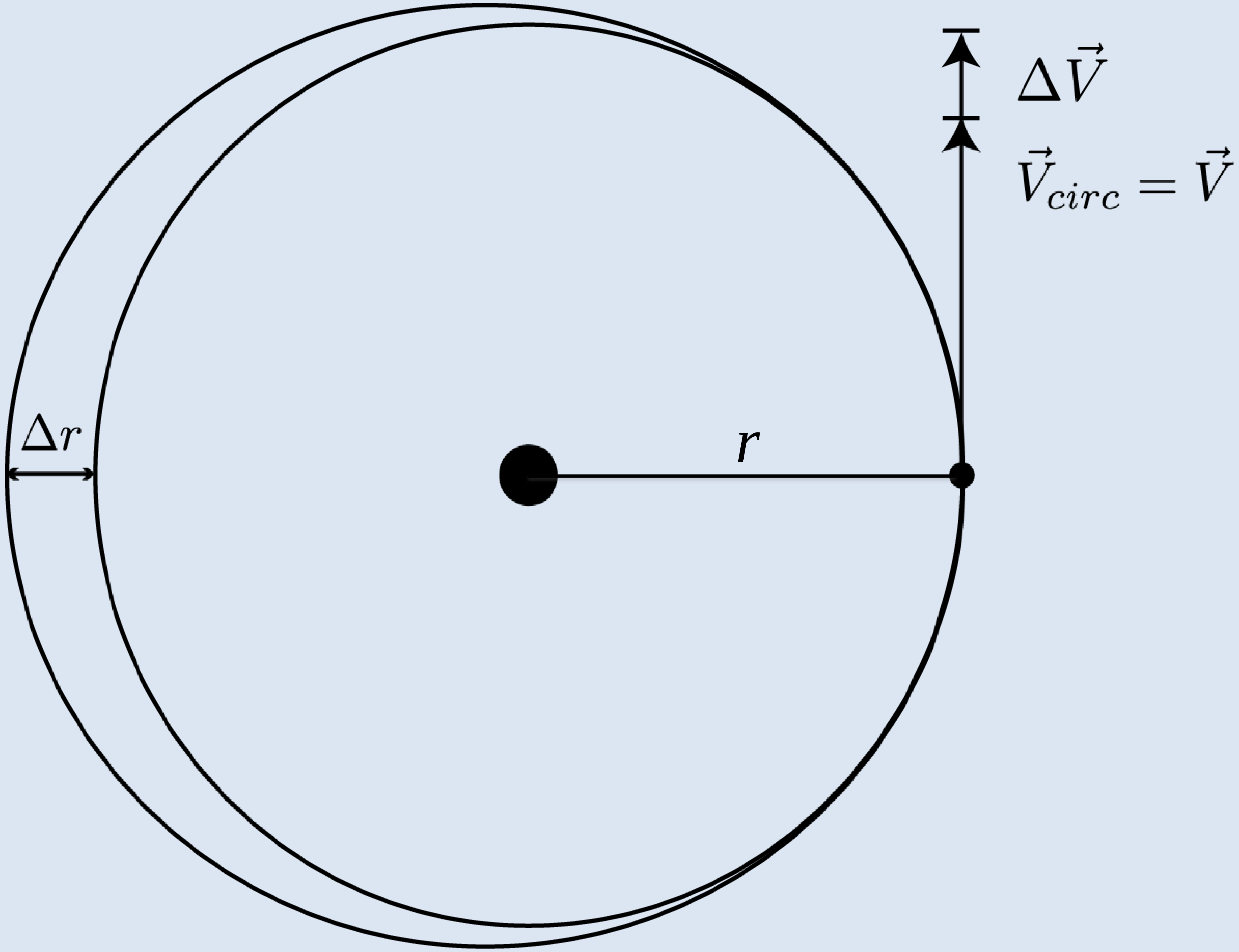
Hohmann transfer – Case of small ΔV for the first burn



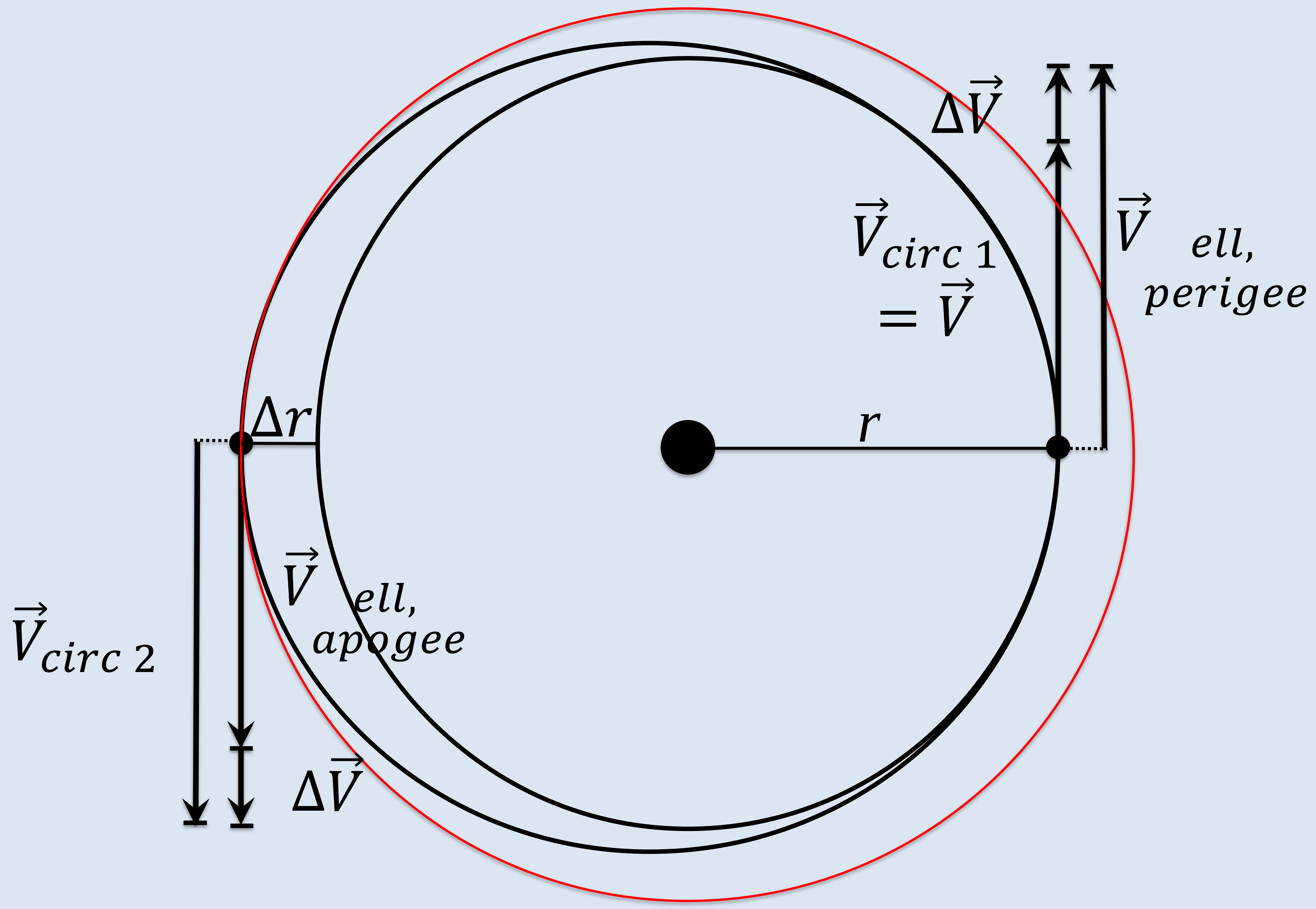
$$\boxed{\frac{\Delta r}{r} \cong 4\frac{\Delta V}{V}}$$

For LEO $\boxed{\Delta r \cong 3.5\Delta V}$

Δr in km and ΔV in $\frac{m}{s}$

Hohmann transfer: $\Delta V_{total} = 2\Delta V$ for the full transfer (small ΔV)

EPFL

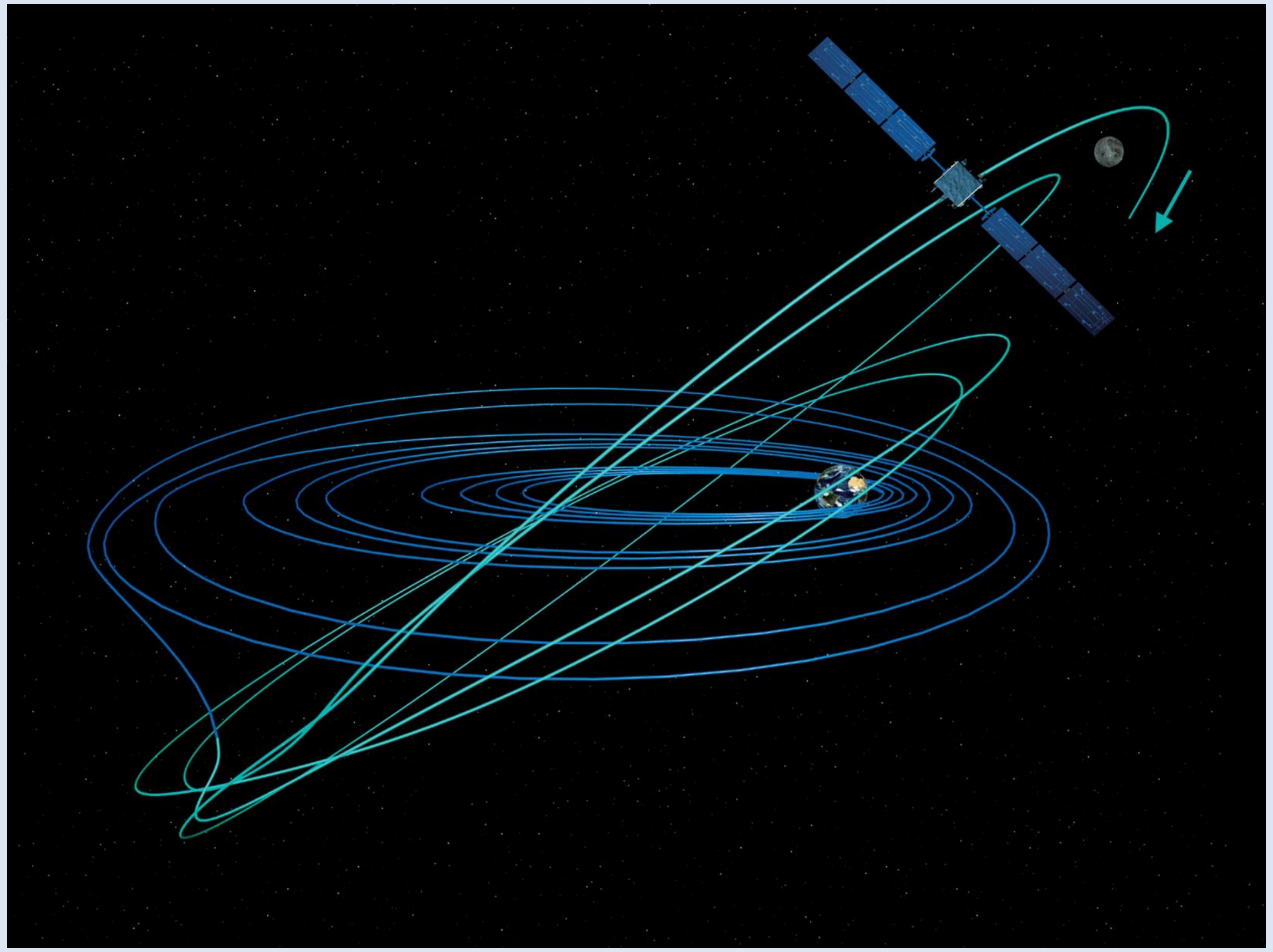


Resulting orbit in red following the 2 ΔVs

$$\boxed{\Delta V_{total} = 2\Delta V = \frac{1}{2} V \frac{\Delta r}{r}}$$

For LEO $\boxed{\Delta V_{total} \cong 0.57\Delta r}$

Δr in km and ΔV in $\frac{m}{s}$



Smart 1 orbits the Earth and Moon in ever-increasing ellipses

3.2.3 Change of orbital plane

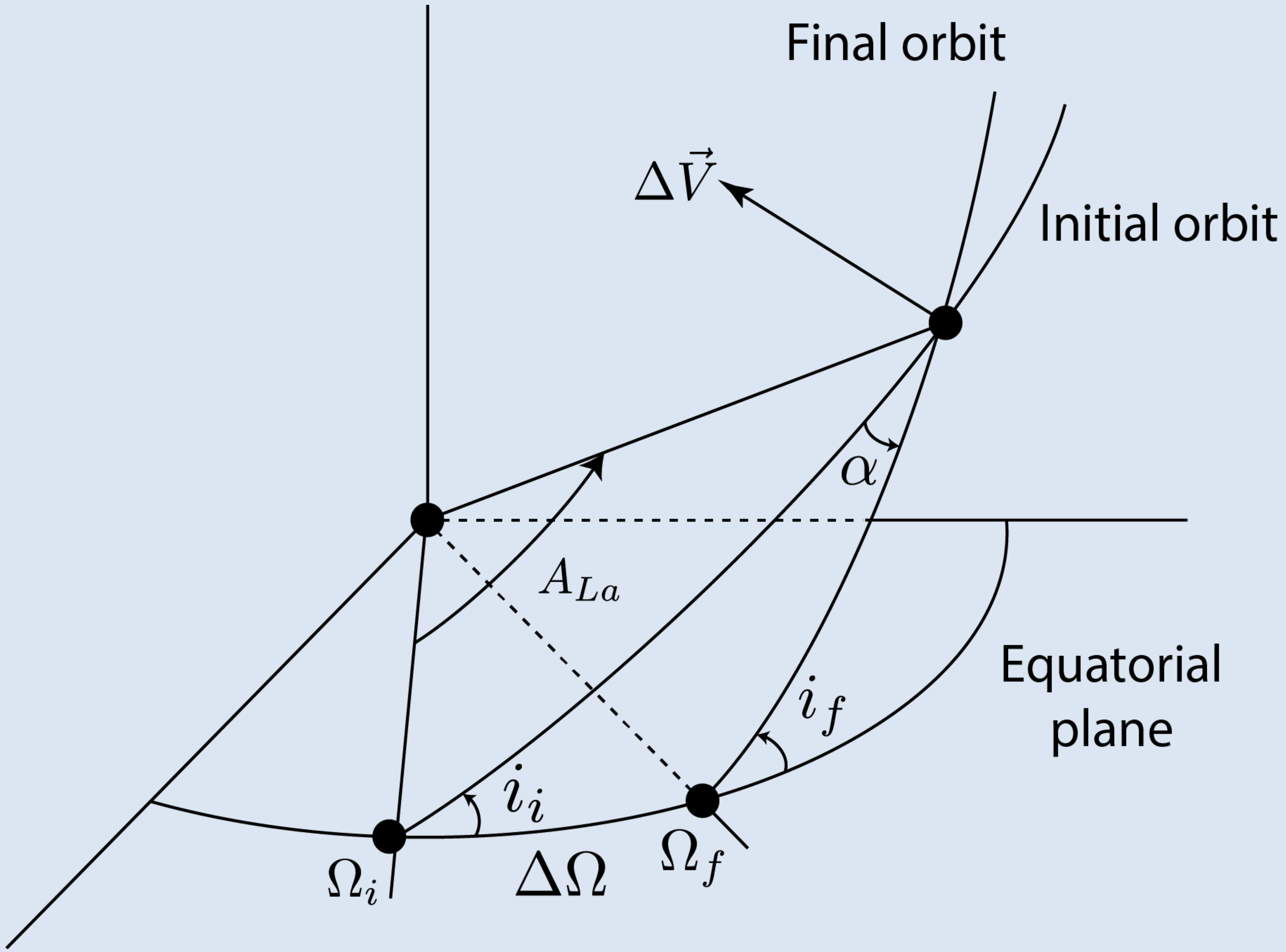
Space Mission Design and Operations

Prof. Claude Nicollier

Credits: AOES Medialab, ESA 2002

Change of orbital plane

$$\Delta\vec{V} \rightarrow \alpha \rightarrow \Delta\Omega \leftrightarrow \Delta i$$



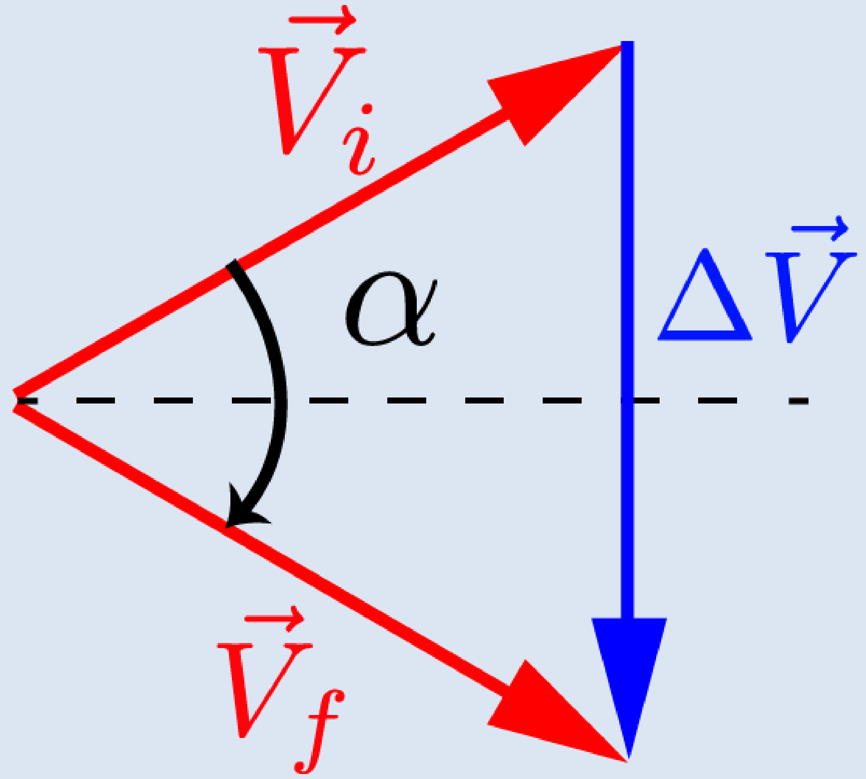
A maneuver with an out-of-plane component, typically a vectorial *ΔV* perpendicular to the orbital plane, will cause a change in a multiplicity of orbital parameters, in particular Ω and i.

As illustrated here, the vectorial *ΔV* perpendicular to *V* causes a change of direction α in the instantaneous velocity vector. Changes in Ω and i involve spherical trigonometry calculations which will not be detailed here.

An orbital plane change is best performed at equator crossing (simplicity and efficiency)

ΔV needed for a change of orbital plane

EPFL



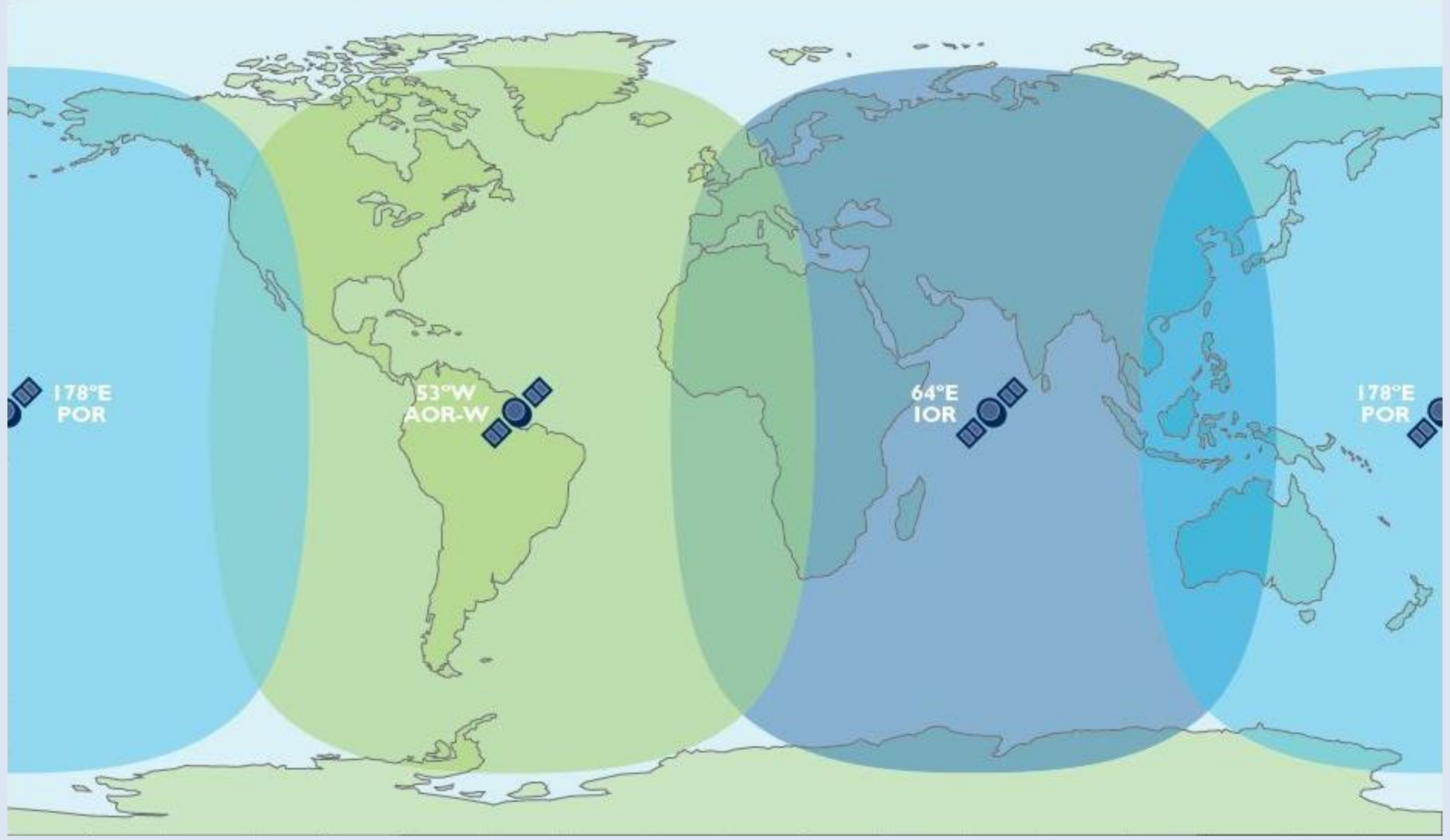
$$|\vec{V}_i| = |\vec{V}_f|$$

Change of orbital plane best done at equator crossing.

$$\boxed{\Delta V = 2V_i \sin\frac{\alpha}{2}}$$

Because orbital velocities in LEO are large, of the order of 7.7 km/sec, it is clear that any orbital plane change with an out-of-plane ΔV will be expensive in propellant!

EPFL



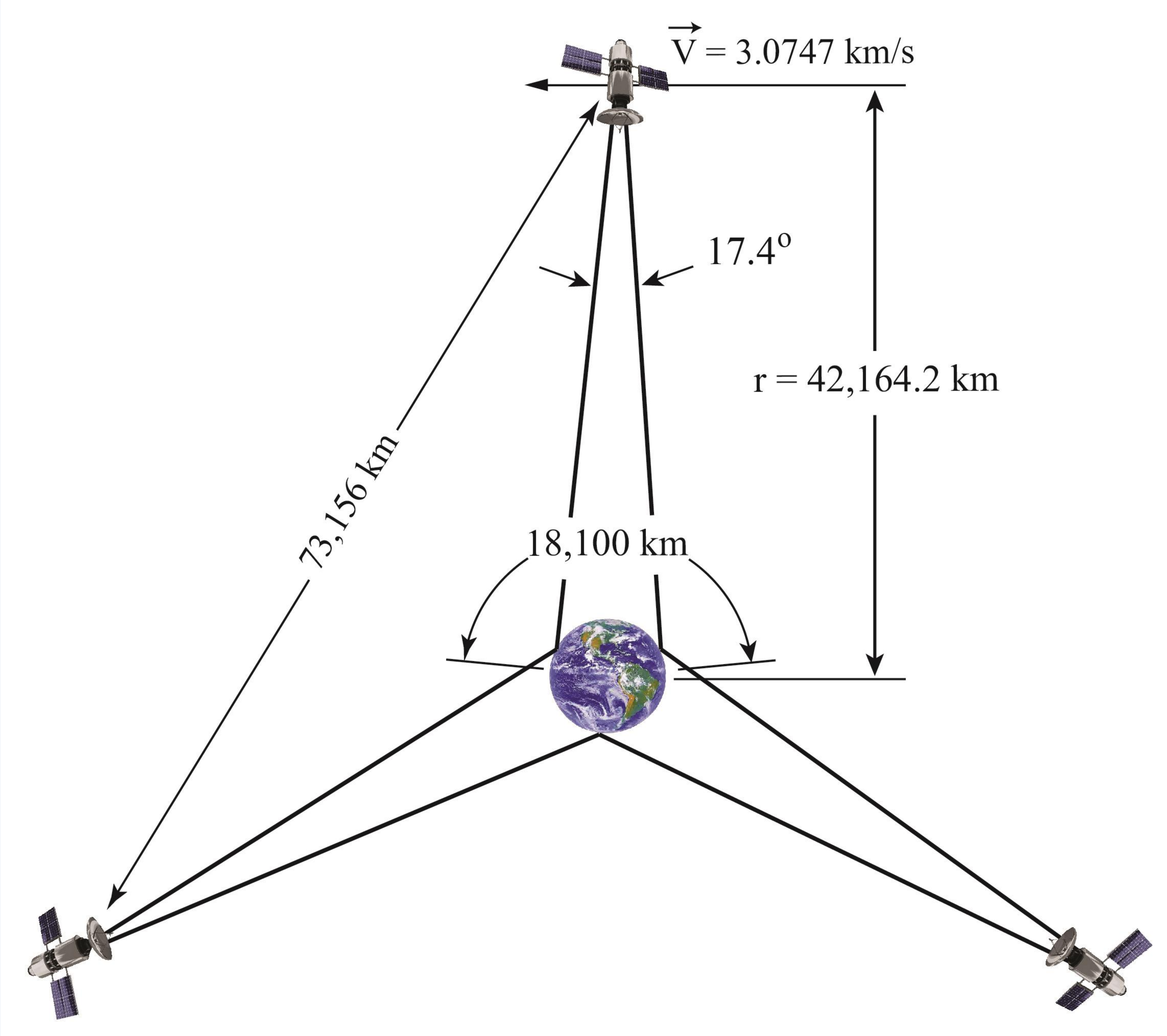
3.3.1 Geosynchronous and geostationary orbits

Space Mission Design and Operations

Prof. Claude Nicollier

Credits: Inmarsat

Geosynchronous and geostationary orbits

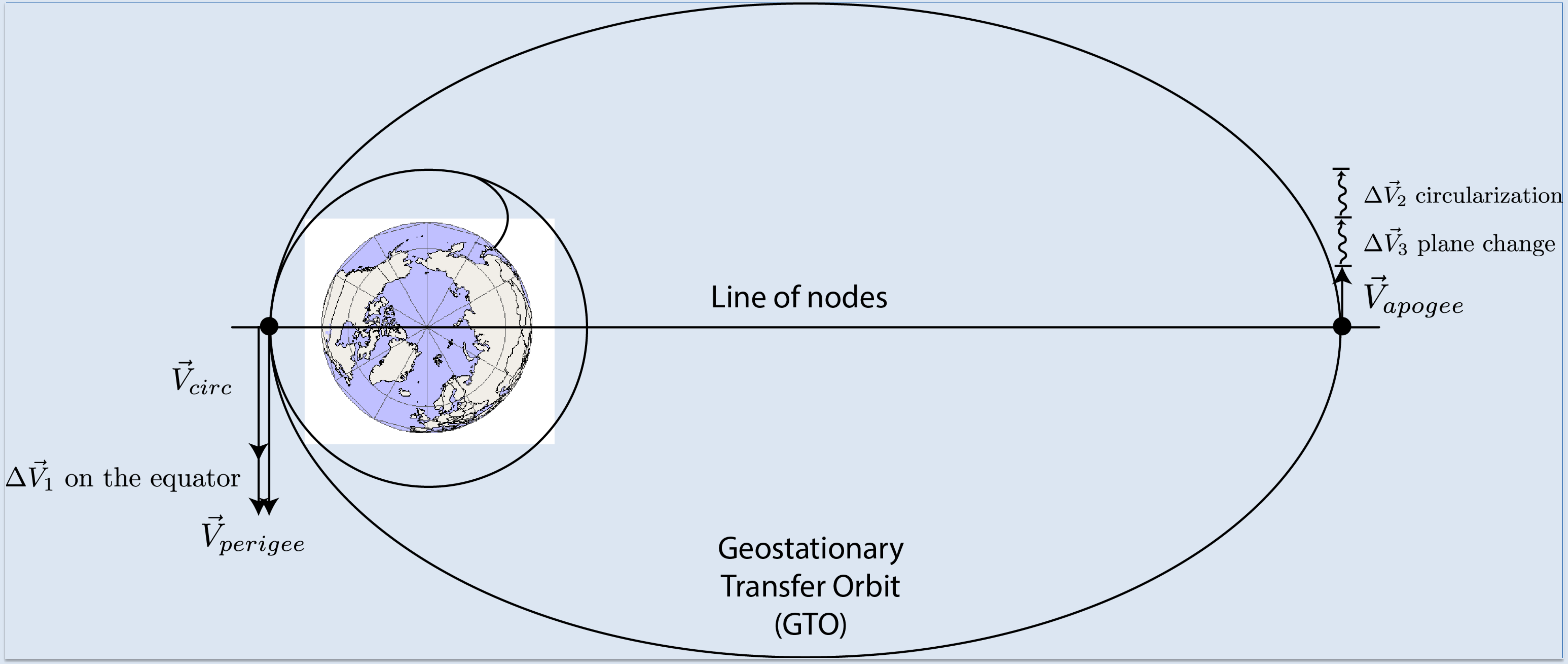


$$T = \frac{2\pi r}{V} = 2\pi\sqrt{\frac{r^3}{\mu}}$$

- Look for **r** satisfying ***T*** = 23h 56min 4.09s. this is the condition for a geosynchronous orbit.
- Geostationary = geosynchronous circular on the equatorial plane (**e** = 0 and ***i*** = 0).

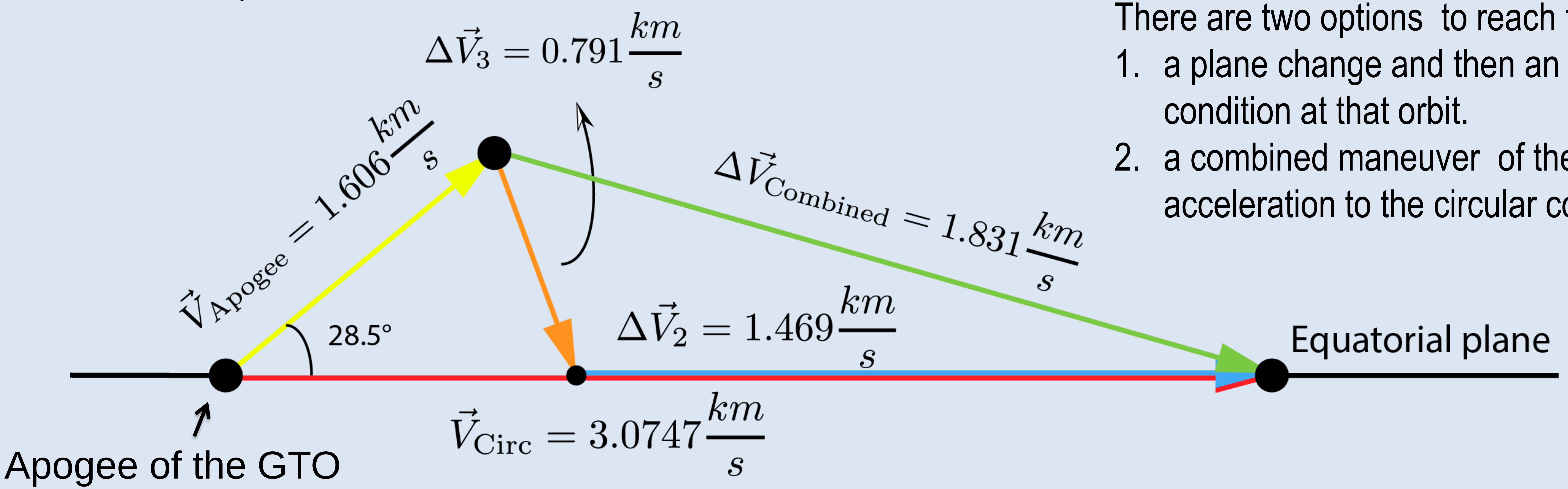
Strategy to reach the geostationary orbit

The line of nodes is the intersection between the plane of the satellite orbit and the equatorial plane.
To reach the geostationary orbit, a Hohmann transfer is necessary followed by a plane change of the orbit from the initial plane to the equatorial condition. The apogee shall be at the geostationary altitude, about 36,000 km above the Earth's surface.
The geostationary transfer orbit is not on the Equator, it has a certain inclination versus the Equator which is usually equal to the latitude of the launch site. Typically 7 degrees for a launch from French Guiana, in Kourou, and 28.5 degrees if the launch takes place from Kennedy Space Center, Florida.



Combined maneuver

Exemple of a combined maneuver at the apogee of the transfer orbit for insertion into a geostationary equatorial orbit for a launch from Kennedy Space Center, Florida (Lat. 28.5°).



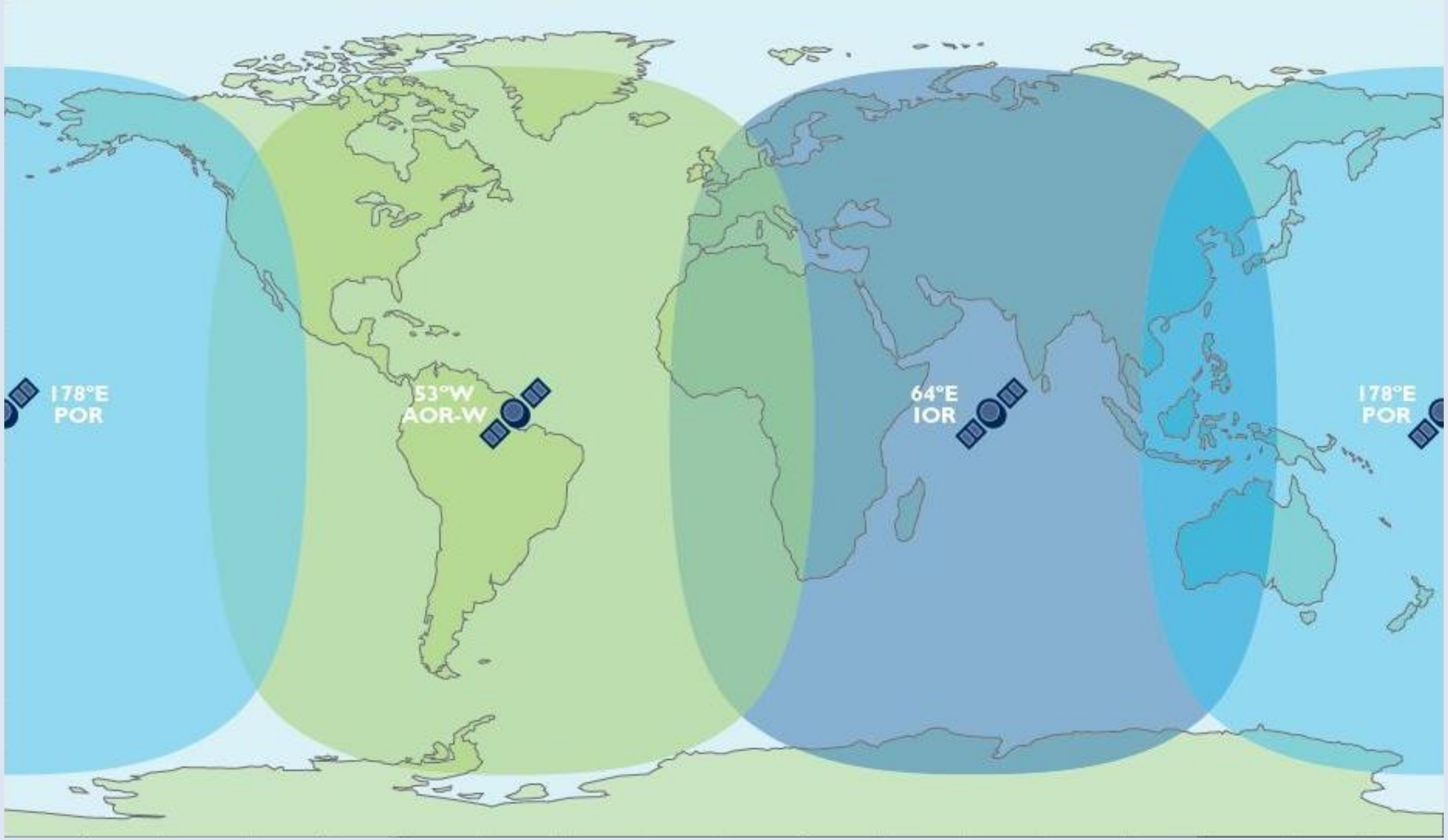
There are two options to reach the geostationary orbit:

1. a plane change and then an acceleration to the circular condition at that orbit.
2. a combined maneuver of the plane change and acceleration to the circular conditions

$$\text{Separate maneuvers: } |\Delta\vec{V}_{\text{Total}}| = |\Delta\vec{V}_2| + |\Delta\vec{V}_3| = 2.260\frac{km}{s}$$

$$\text{Combined maneuvers: } |\Delta\vec{V}_{\text{Total}}| = |\Delta\vec{V}_{\text{Combined}}| = 1.831\frac{km}{s}$$

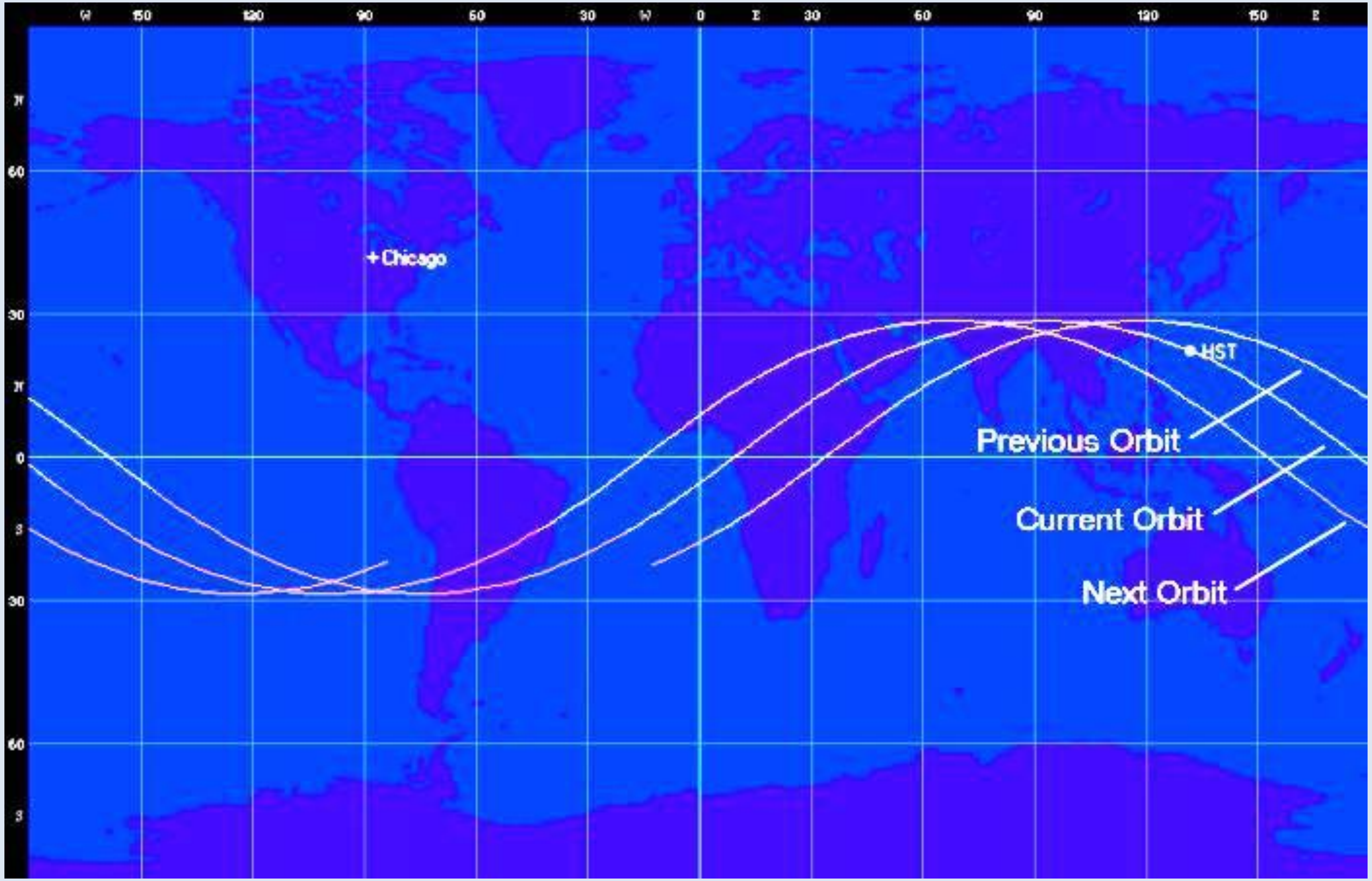
Visibility of Earth's surface from GEO



The Inmarsat system blankets the Earth, connecting geostationary satellites for near total coverage.

Credits: Inmarsat

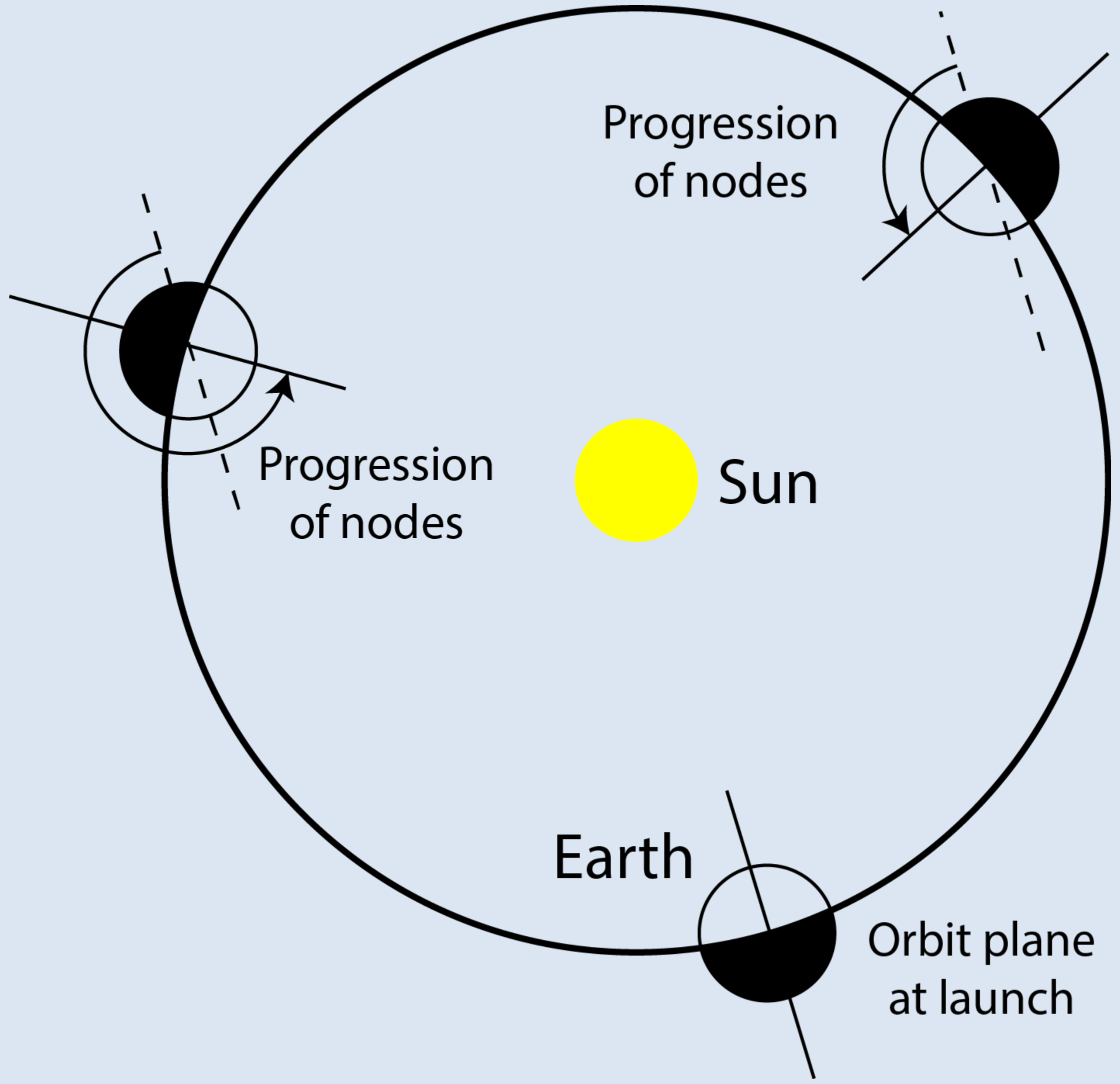
Ground track



The ground track for a geostationary orbiting satellite is a point on the equator.

For a satellite in LEO, it is a set of curves with succcessive equatorial crossings (at the ascending nodes) 22.5 degrees apart, and spreading to the west (22.5 degrees = orbital period of 1.5 hour, multiplied by 15 degrees per hour Earth rotation rate)

Credits: Aerospaceweb.org

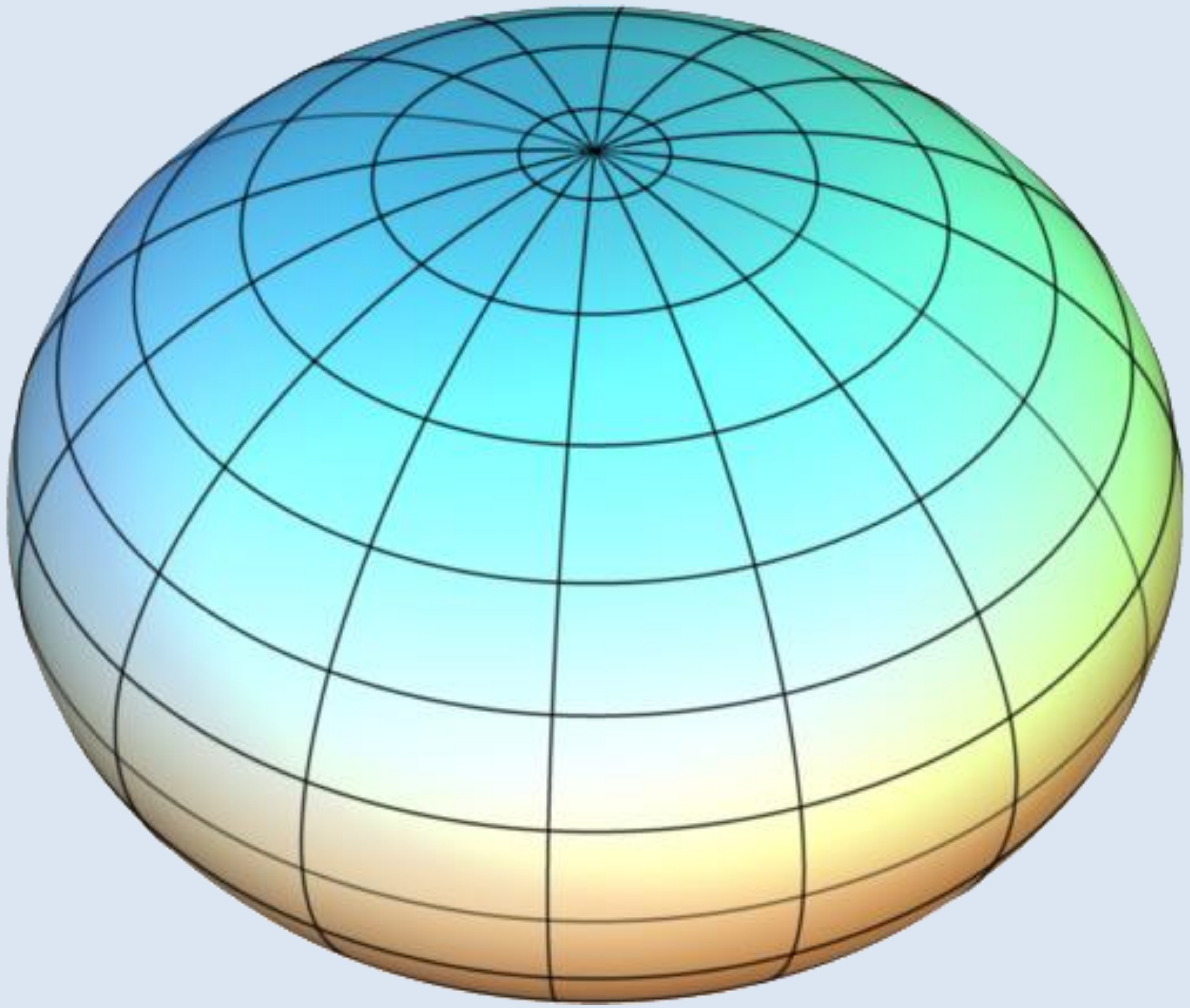


3.3.2 Nodal regression and Sun synchronous orbits

Space Mission Design and Operations

Prof. Claude Nicollier

Equatorial bulge



- The flattening of the Earth's surface is 1:298 (which corresponds to a radius difference of 21.38 km of the Earth radius 6378.13 – 6356.75 km).

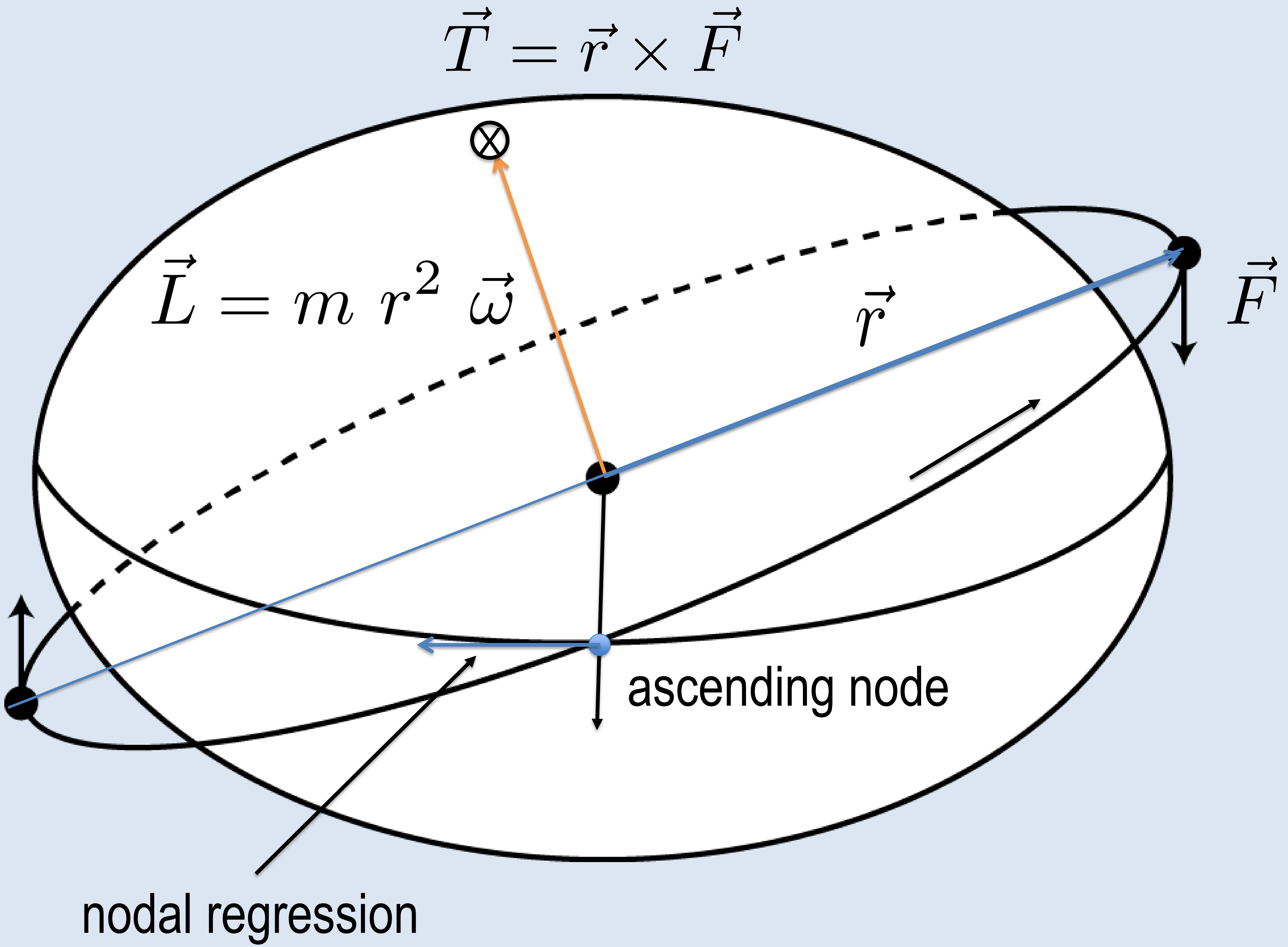
- The Earth's equatorial bulge has two consequences:

1. Nodal regression

2. Rotation of the line of apsides
 (not covered in this course)

Credits: Wikipediai

Nodal regression for LEO – Forces and torques



The equatorial bulge causes the gravitational force on a satellite to deviate from a purely central force, except at equator crossings, and for purely equatorial or polar orbits.

$$\vec{T} = \vec{r} \times \vec{F}$$

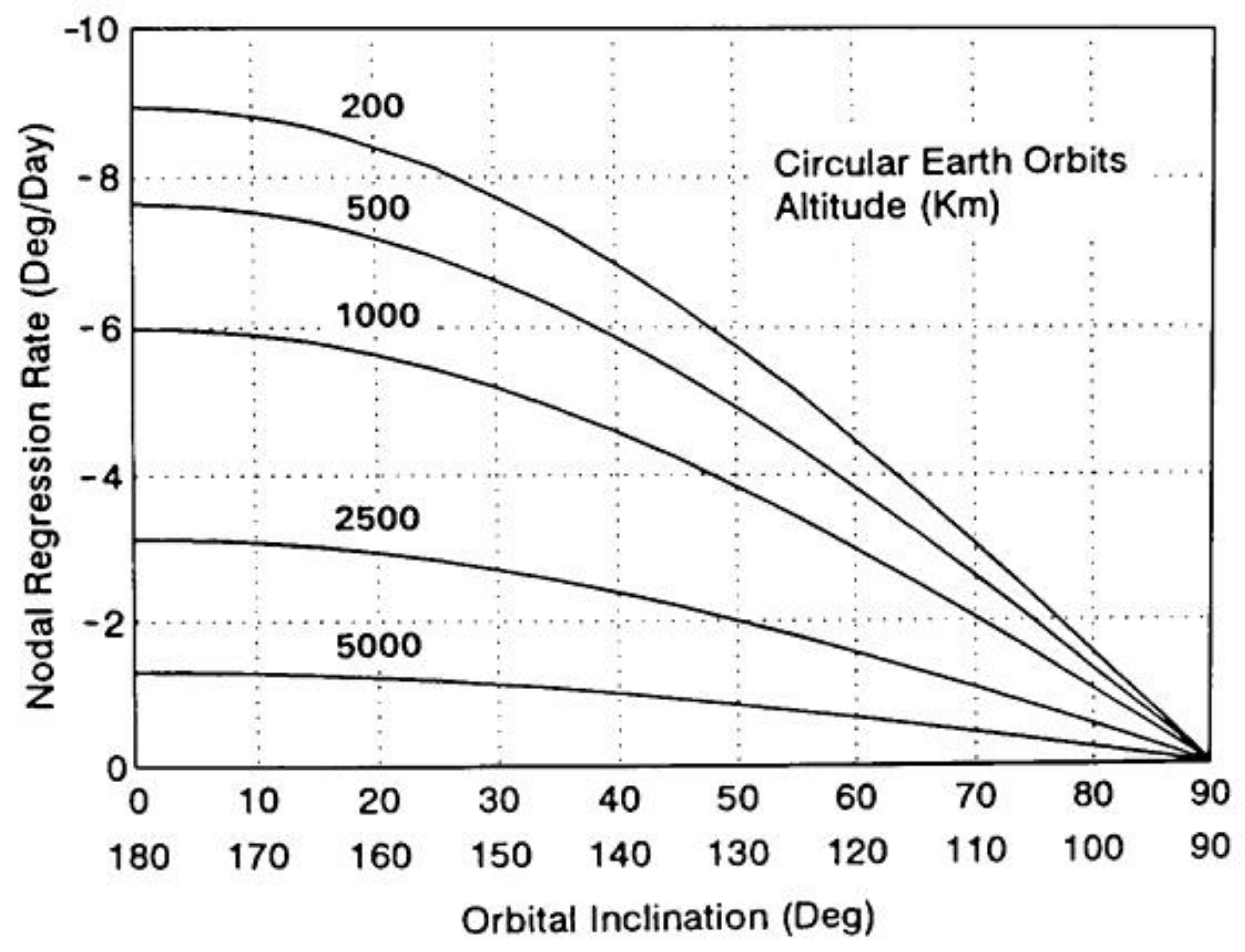
$$\boxed{\vec{T} = \frac{d\vec{L}}{dt}}$$

Nodal regression rate for LEO

$$\boxed{\frac{d\Omega}{dt} = -2.06474 \times 10^{14} \frac{\cos i}{a^{3.5}(1-e^2)^2}}$$

in degrees per mean solar day with the semi-major axis ***a*** in kilometers.

Nodal regression rate vs. orbital inclination and altitude

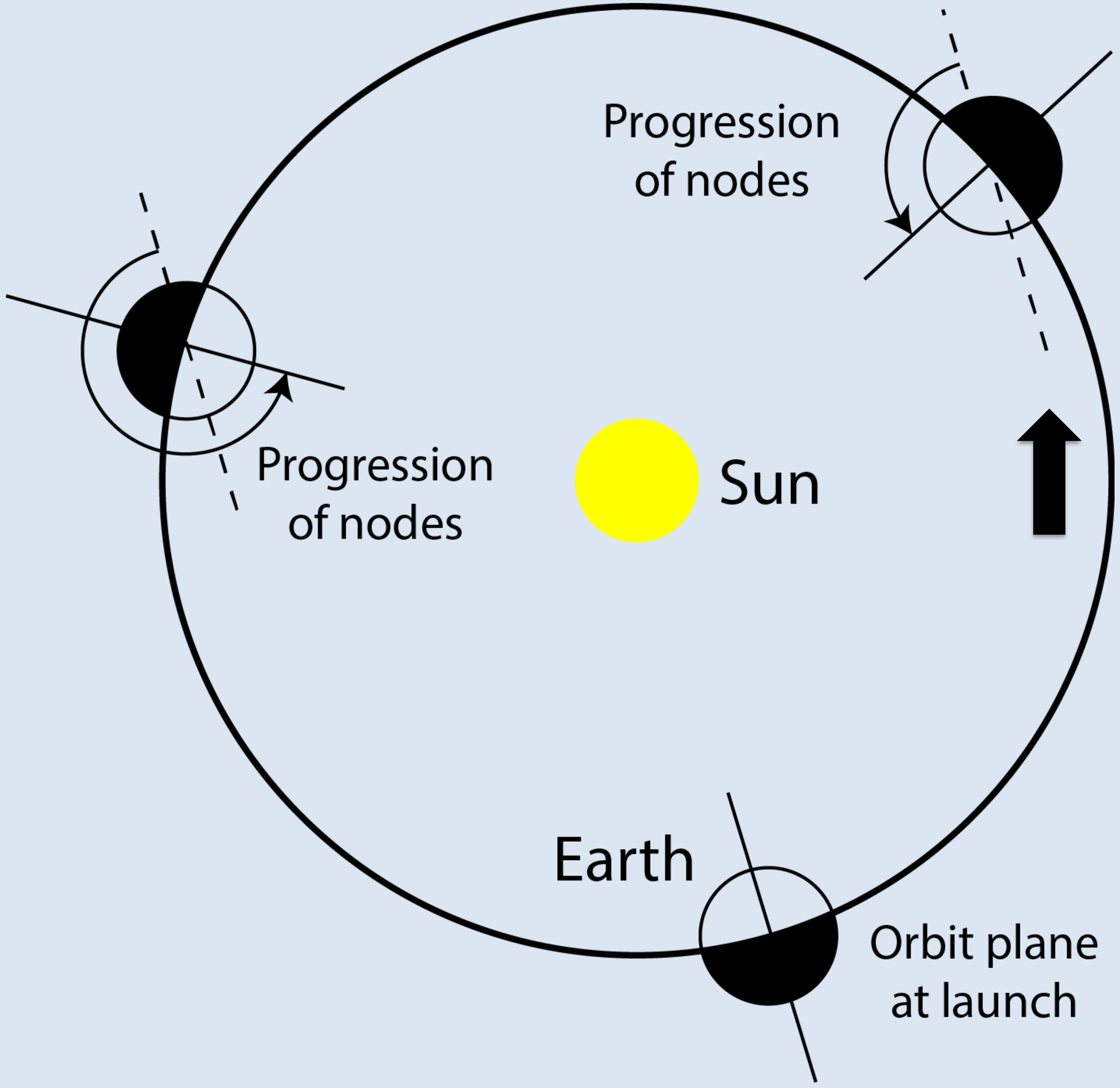


The line of nodes drifts:

- To the west for i = 0 – 90°
  (posigrade or direct orbit)
  Nodal regression
- To the east for i = 90 – 180°
  (retrograde orbit)
  Nodal progression

Sun-synchronous orbit

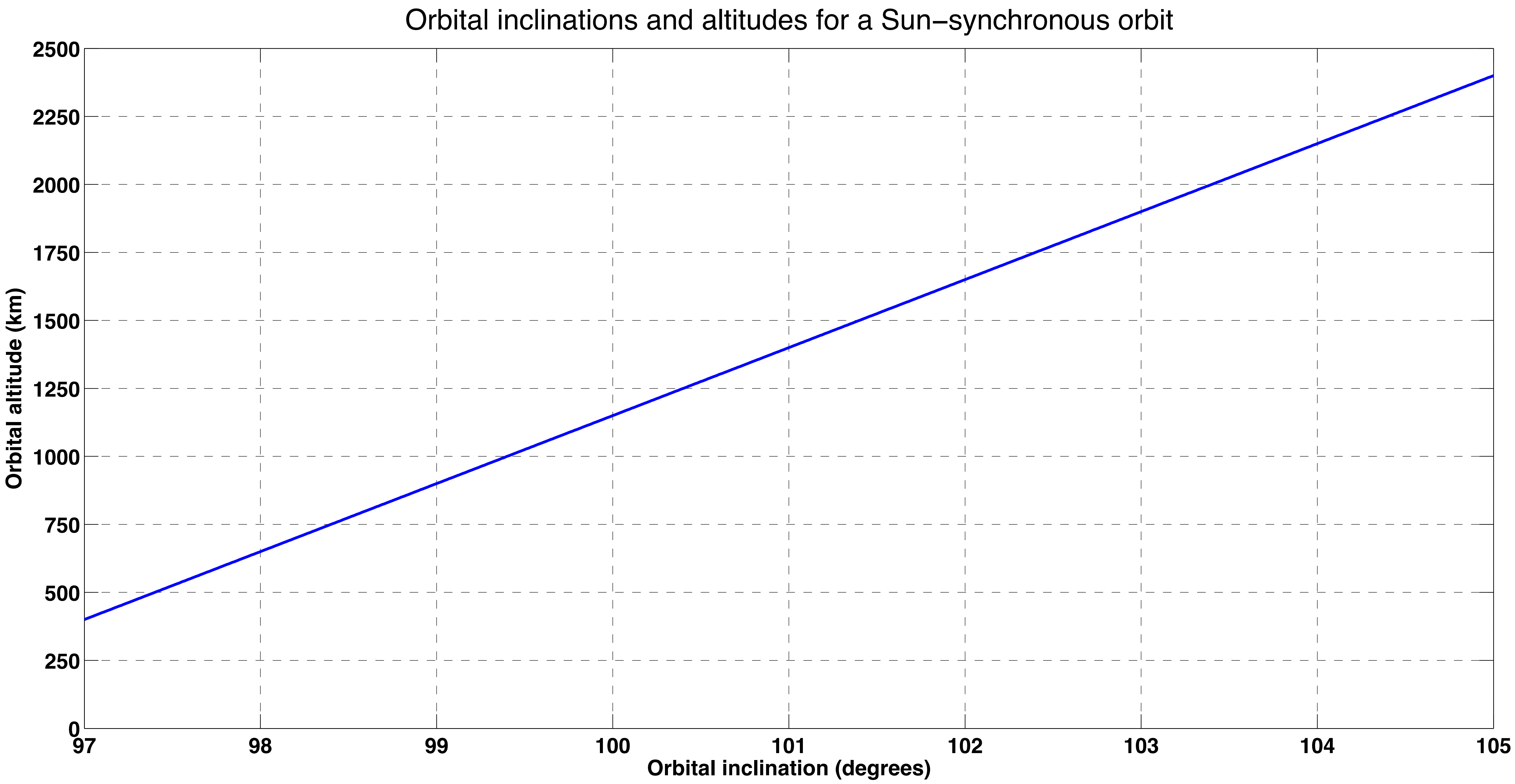
A Sun-synchronous orbit is an Earth centered orbit which maintains a constant orientation with respect to the Sun during the whole year. It is easy to understand that it means a drift of the line of nodes to the east, of about one degree per day.



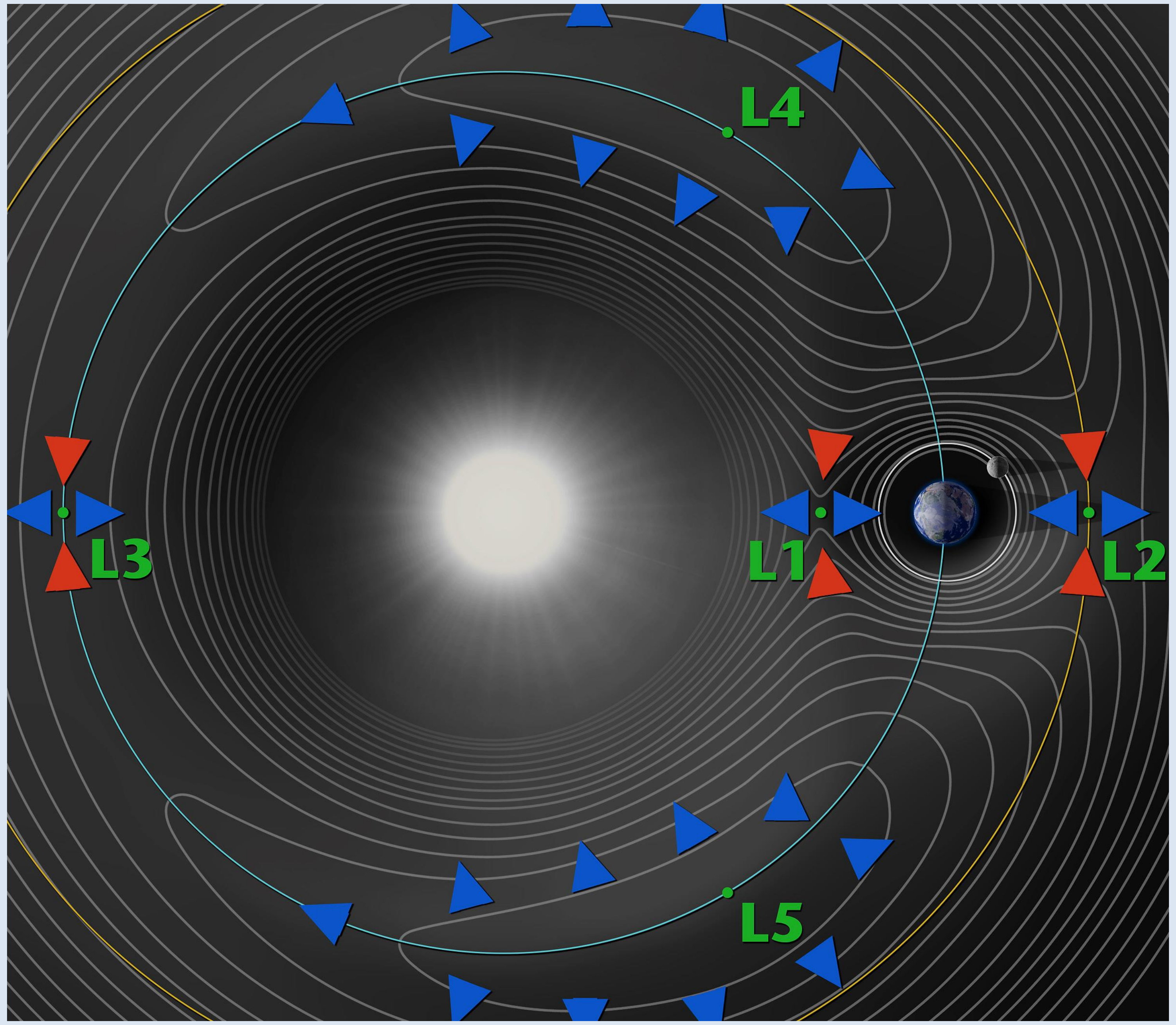
Requirement on $\frac{d\Omega}{dt}$ for a Sun-synchronous orbit:

$$\frac{360 \text{ deg}}{365.242 \text{ days}} = 0.9856 \ \frac{\text{deg}}{\text{day}}$$

Orbital inclinations and altitudes for a Sun-synchronous orbit



In order to have a drift of the line of nodes to the east, the orbit should have a little more than 90 degrees inclination, typically between 97 and 105 degrees inclination depending on the altitude.



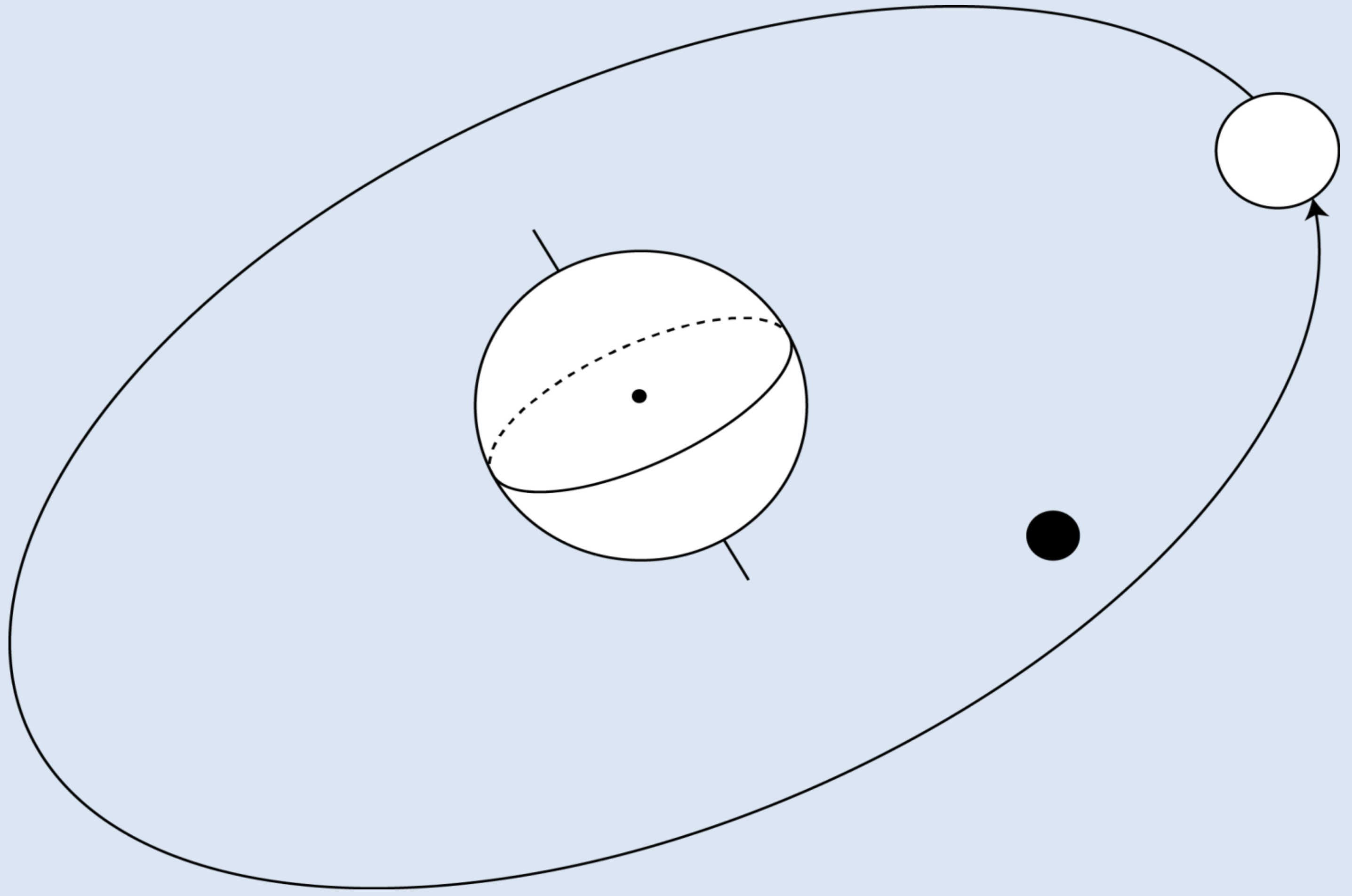
3.3.3 Lagrange points

Space Mission Design and Operations

Prof. Claude Nicollier

Credits: NASA, WMAP Science Team

Restricted three-body problem

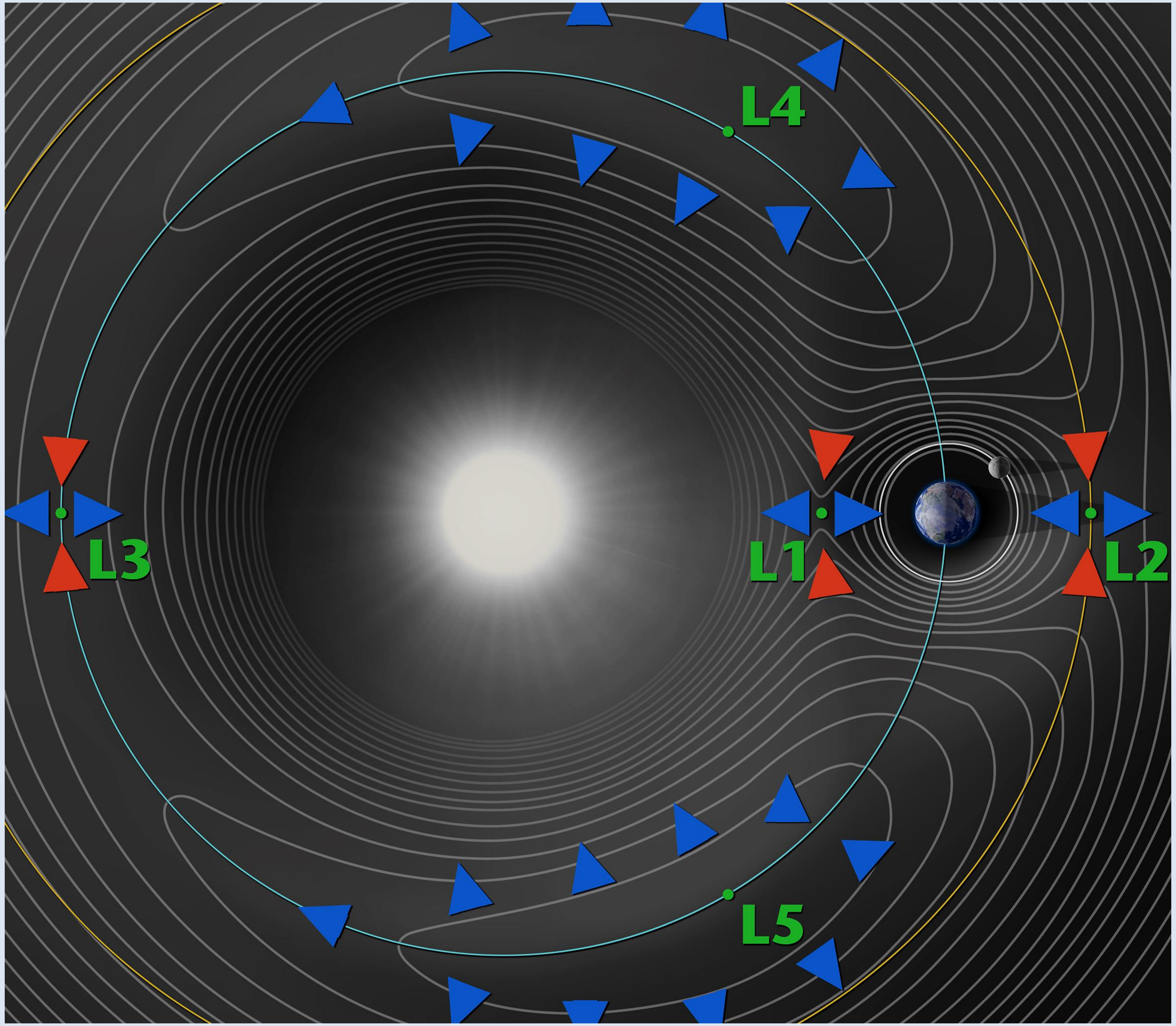


The general three-body problem is complex and not covered in this course. We will consider only the restricted three-body problem: two relatively large bodies and a smaller body : the spacecraft.

Assumptions:

- The two main bodies are on circular orbits around the center of mass of the system.
- The mass of the third body (satellite) is very small compared to the mass of the two main bodies.
- The third body is in an orbit contained in the plane of the orbits of the two main bodies.

Lagrange points



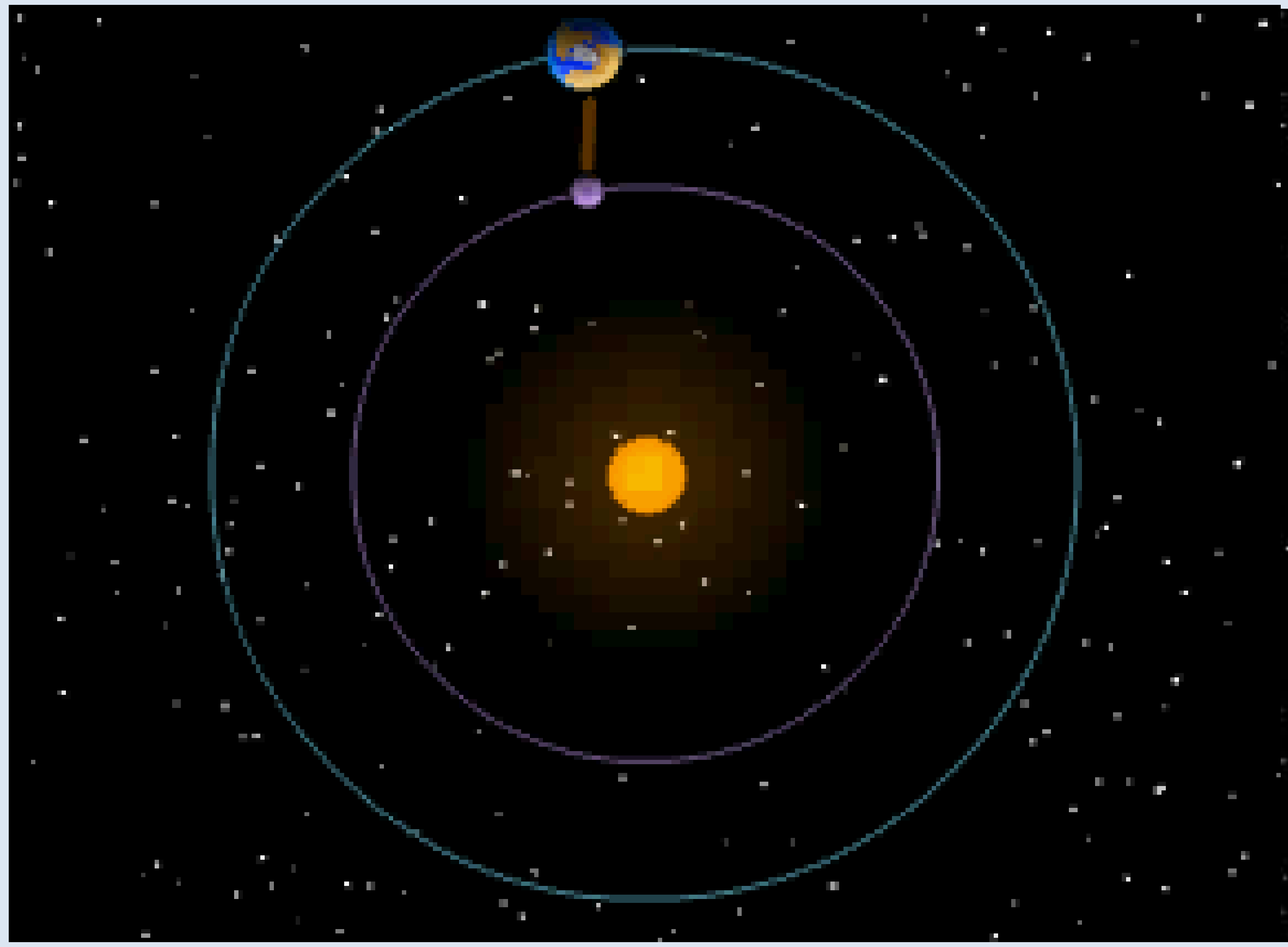
Credits: NASA, WMAP Science Team

The gravitational fields of two massive bodies with the addition of the inertial force due to the satellite's circular motion are in balance at the Lagrange points, allowing the third body or satellite to be stationary with respect to the first two bodies.

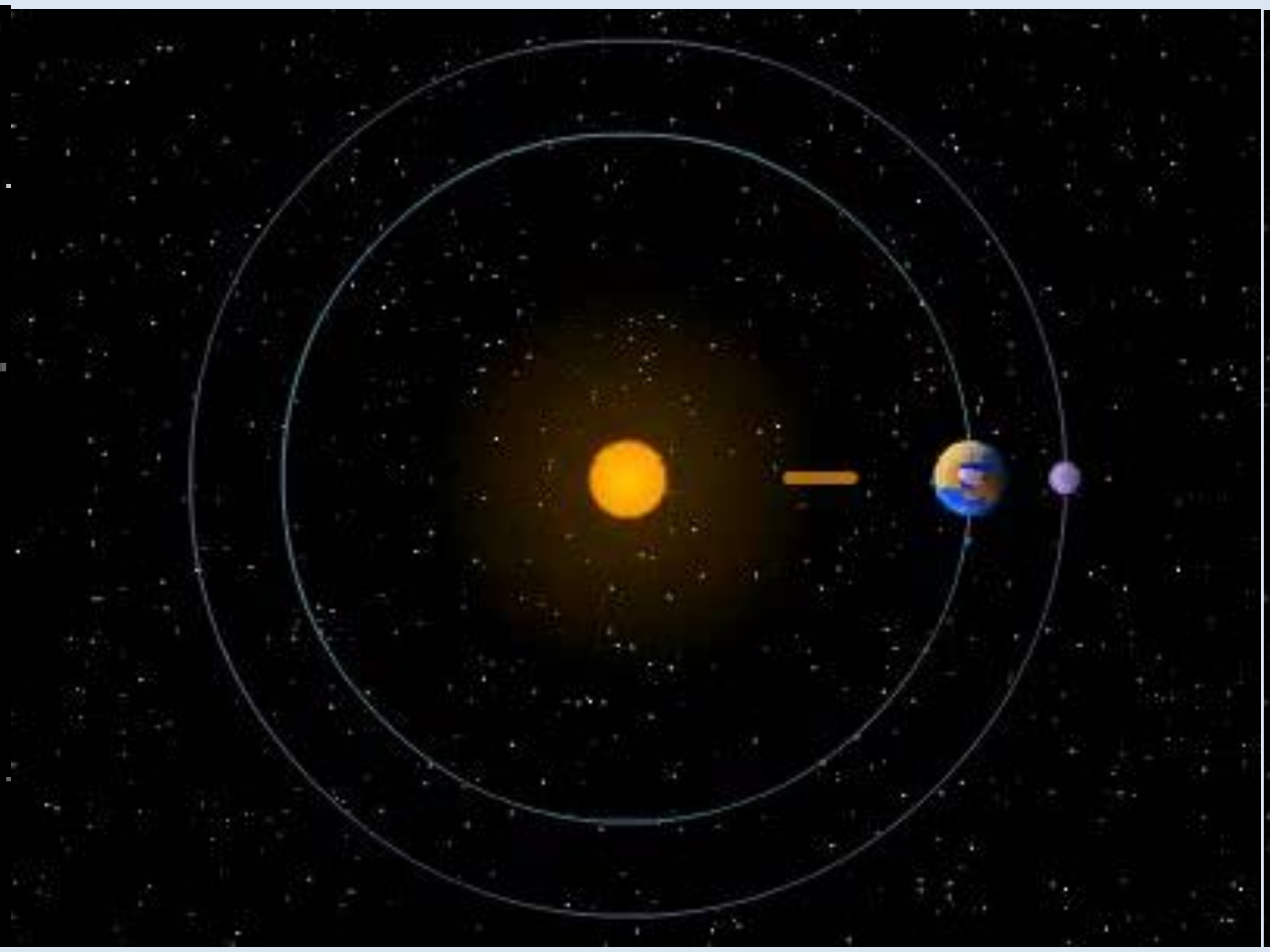
- In the Sun-Earth system, the distance to the Earth of the L1 Lagrange point is about 1.5 million km. The Lagrange point L2 is 1.5 million km away from the Earth in the anti-Sun direction.
- L4 and L5 are locally stable points.
- For L1, L2, and L3 there is stability across the L2 to L3 line
- Red arrows indicate stability, blue arrows instability

Lagrange points of the Sun-Earth system (not to scale)

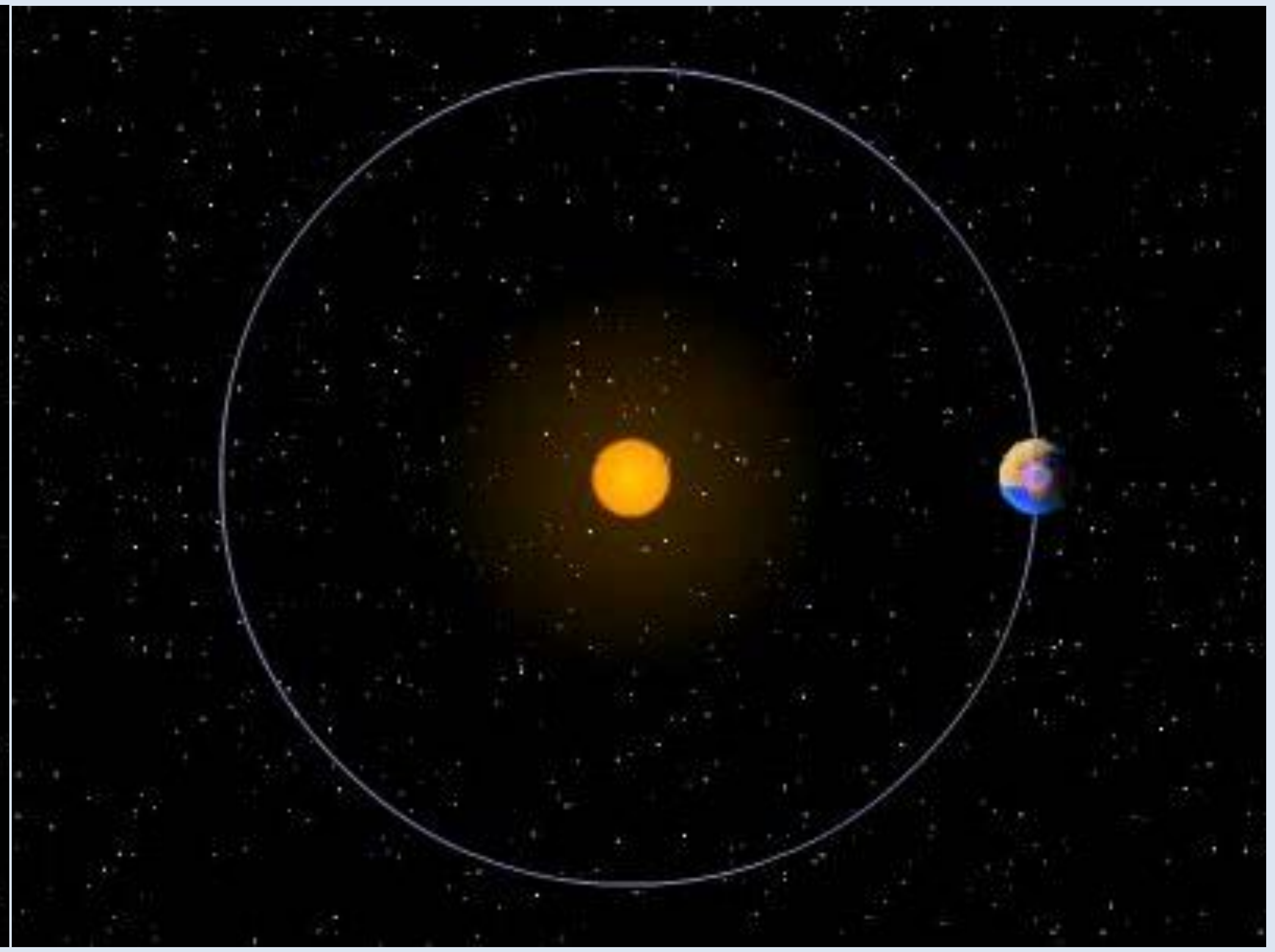
- In the case of L3, the spacecraft is not exactly on the orbit of the Earth but slightly further away.
- L3 is never used. L4 and L5 (not represented here) are not used for the Sun-Earth system.
- L1 is used for spacecraft looking at the Sun: SOHO for instance. The James Web Space Telescope will be located at L2.



Spacecraft on L1



Spacecraft on L2



Spacecraft on L3

Credits: ESA