Durham University

Electrical Engineering II

ENGI2191

Circuit Analysis in Phasor Domain

Dr Hamed Bahmani
Hamed.h.bahmani@Durham.ac.uk
E108

Lecture Objectives:

Aim of this lecture:

The aim of this lecture is to analyse electric circuits in phasor domain.

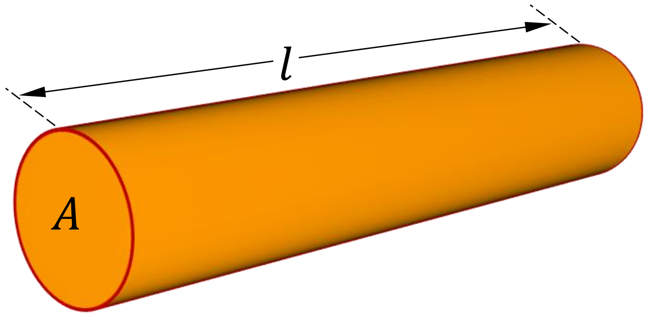
Intended Learning Outcomes:

At the completion of the lecture and associated problems you should be able to:

- Understand the characteristics of sinusoidal functions.
- Identify phasor representation of sinusoids.
- Converting between time and frequency domains.
- Identify the performance of the basic ideal circuit elements in phasor domain.
- Analyse the electric circuits in phasor domain.

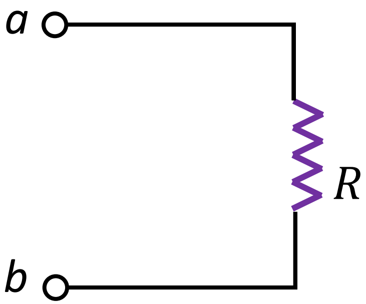
Physics of Basic Electric Elements

Ideal Electric Elements:

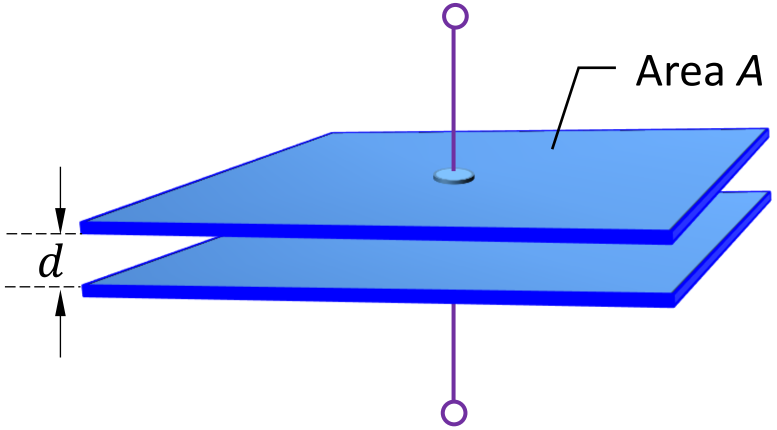


$$R = \rho \frac{l}{A}$$

$$V = R\, I$$

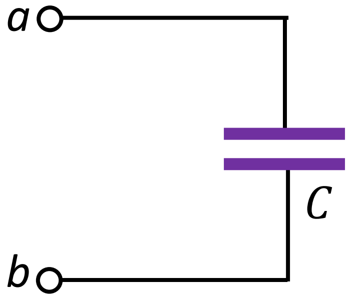


Ideal resistor

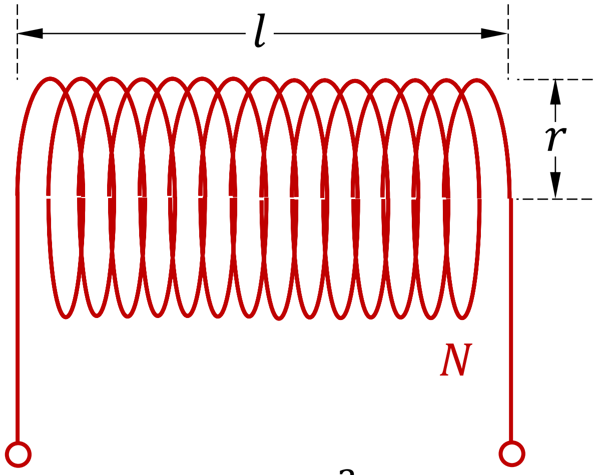


$$C = \varepsilon_0 \frac{A}{d}$$

$$q = C\, V$$

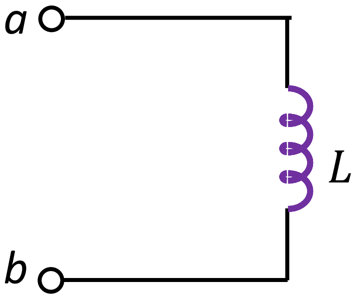


Ideal capacitor



$$L = \mu_0 \frac{N^2 A}{l}$$

$$\Phi_B = LI$$



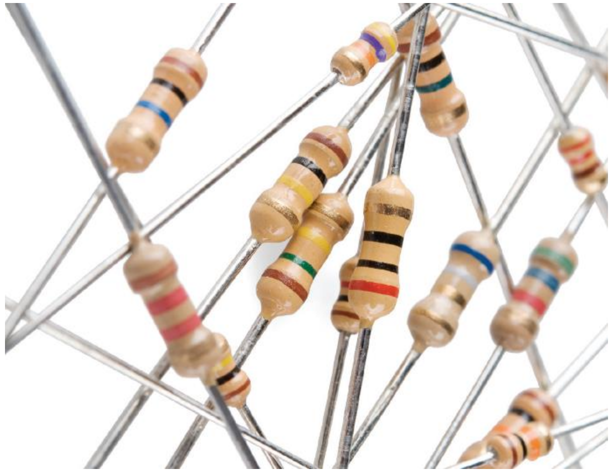
Ideal inductor

Resistance R, capacitance C and self-inductance L are purely geometric.

Ideal Electric Elements:

Different types of Resistors:

Colour Code Resistor



Power Resistor

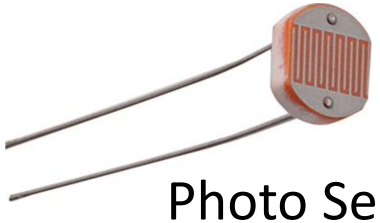
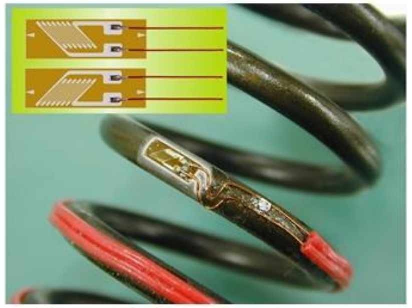
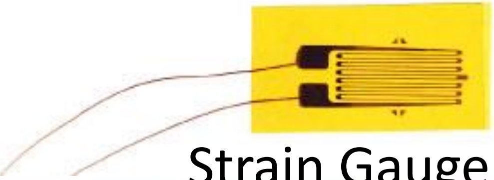
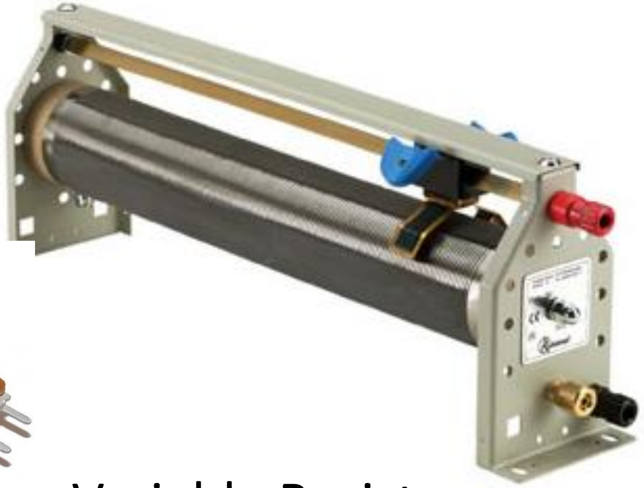


Photo Sensor





Strain Gauge



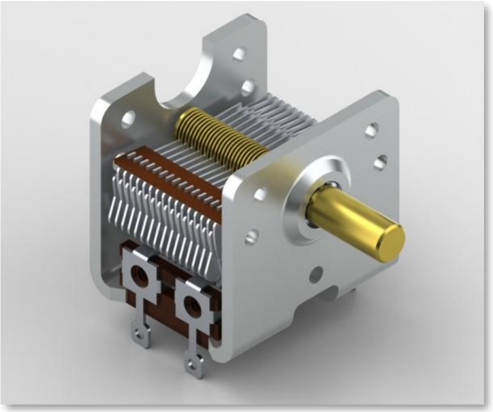
Variable Resistors

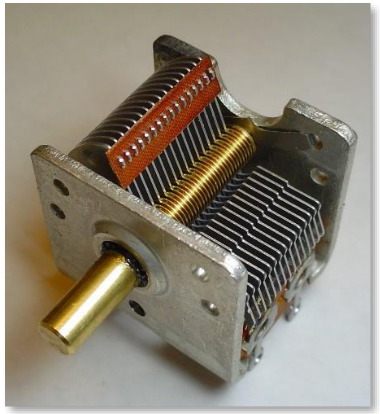
www.electronics-tutorials.ws

Ideal Electric Elements:

Different types of Capacitors:

Fixed capacitors



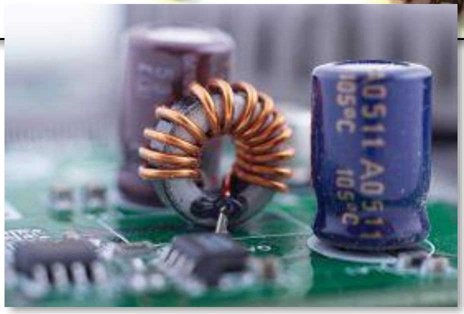


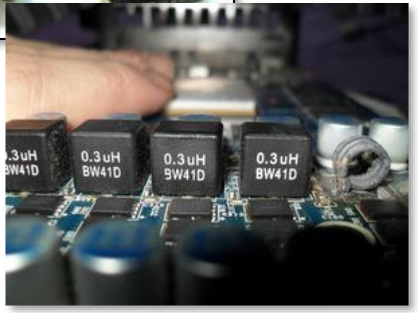
Variable capacitors

400 kV capacitor bank in power system

Ideal Electric Elements:

Different types of Inductors:





Inductors used in electronic circuits

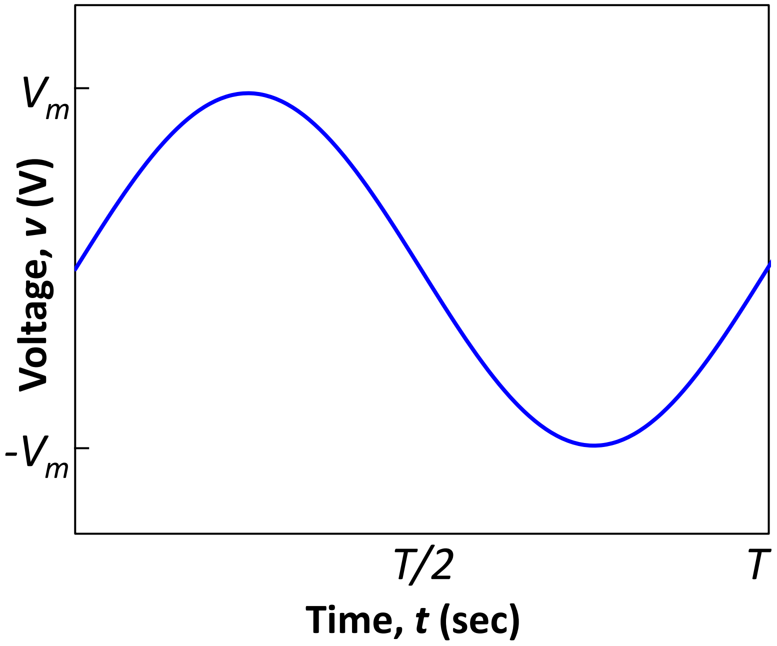
Current-limiting inductor installed at a 345 kV power plant in South Korea

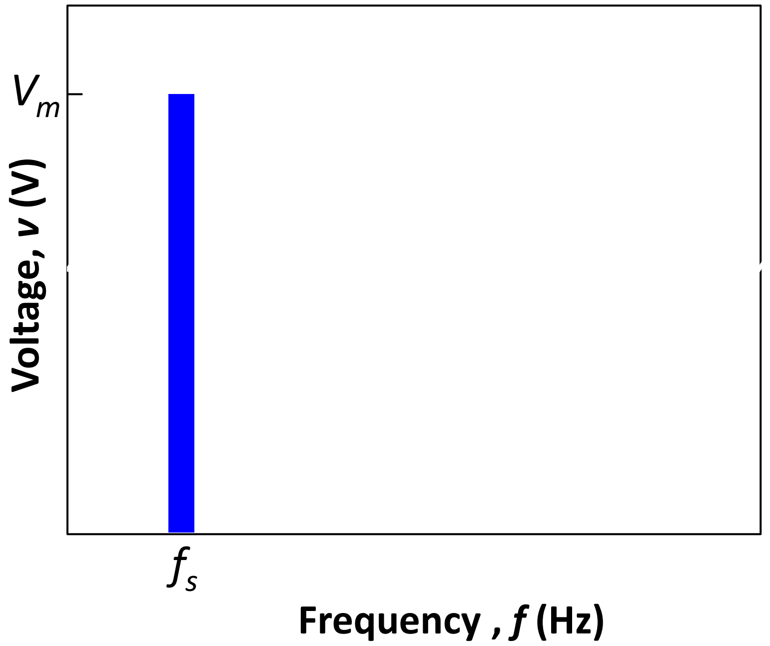
Performance of ideal circuit elements in ac circuits

Alternating Currents and Phasors:

$$v(t) = V_m \sin\,(\omega t \pm \varphi)$$

$$V(\omega) = V_m \angle{\pm\ \varphi}$$

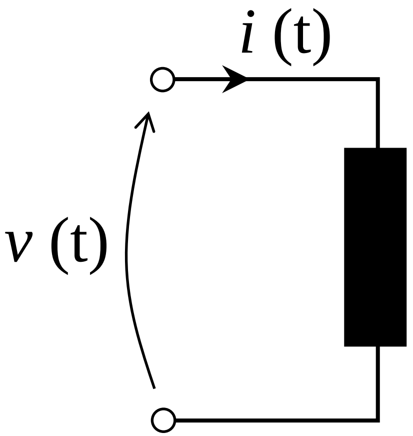




In engineering and physics, phasor is a complex number representing a sinusoidal function whose amplitude, angular frequency, and phase angle are time-invariant.

Alternating Currents and Phasors:

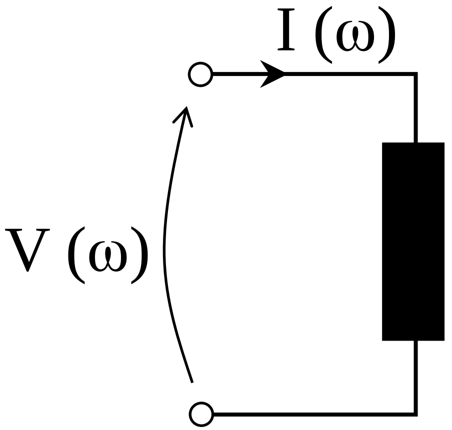
Voltage and current representation in "time domain"



$$v(t) = V_m \, sin \, (\omega t + \theta_v)$$

$$i(t) = I_m \, sin \, (\omega t + \theta_i)$$

Voltage and current representation in "frequency domain"

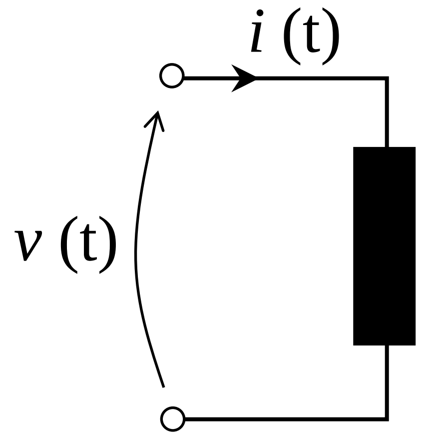


$$\boldsymbol{V}(\omega) = V_{rms} \angle \theta_v$$

$$\boldsymbol{I}(\omega) = I_{rms} \angle \theta_i$$

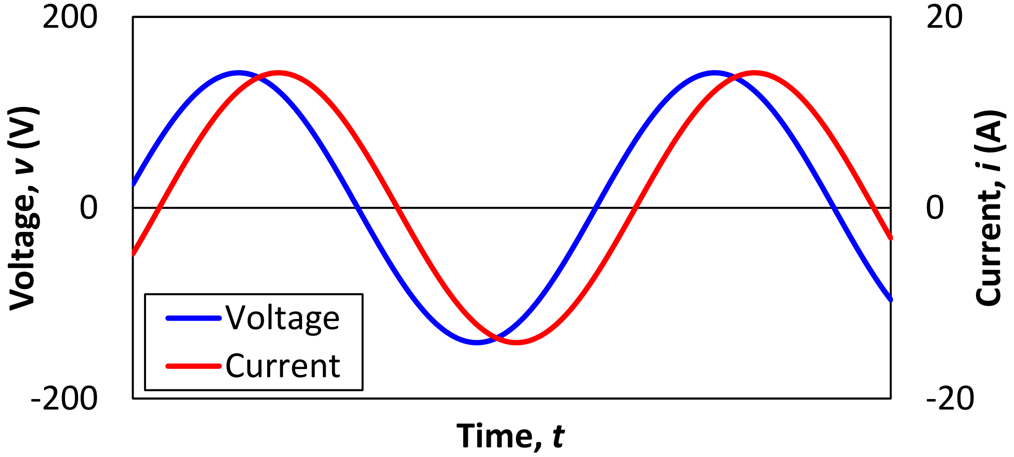
In network analysis, phasor of voltage and current are given as the *rms* values.

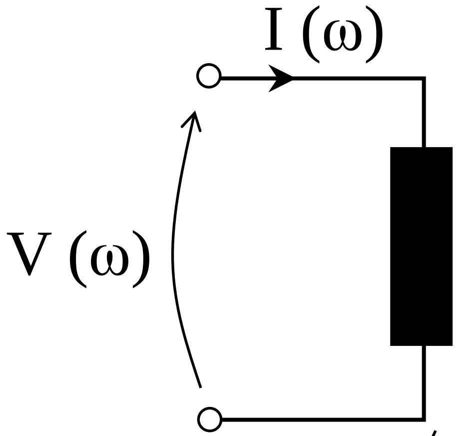
Alternating Currents and Phasors:



$$v(t) = 100 \times \sqrt{2}\, Sin\,(\omega t + 10)$$

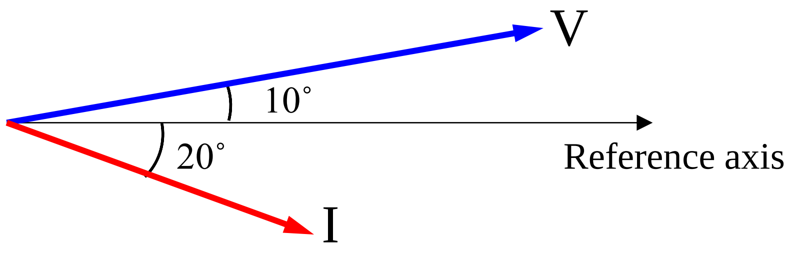
$$i(t) = 10\ \times \sqrt{2}\, Sin\,(\omega t - 20)$$



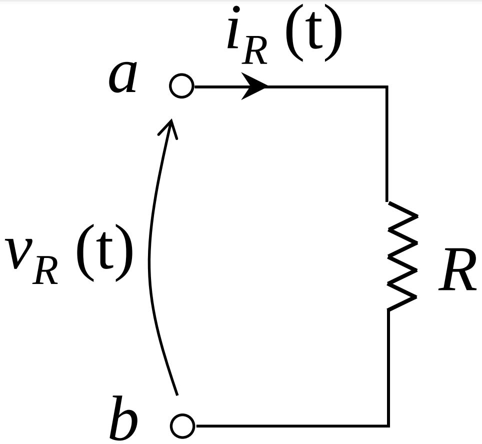


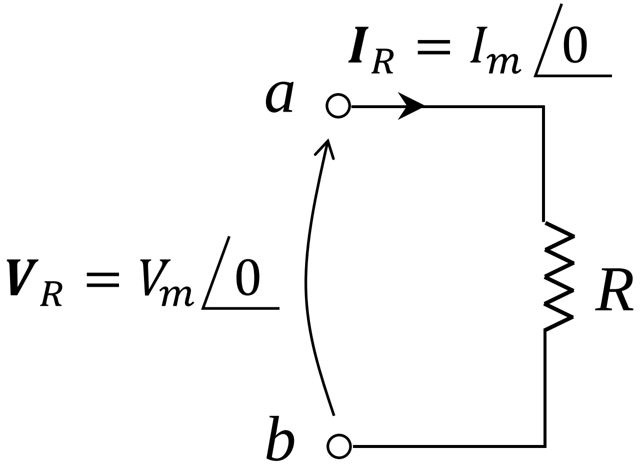
$$\boldsymbol{V}(\omega) = 100\ \angle{+10^\circ} \quad \text{V}$$

$$\boldsymbol{I}(\omega) = 10\ \angle{-20^\circ} \quad \text{A}$$



Performance of a resistor in ac circuit:



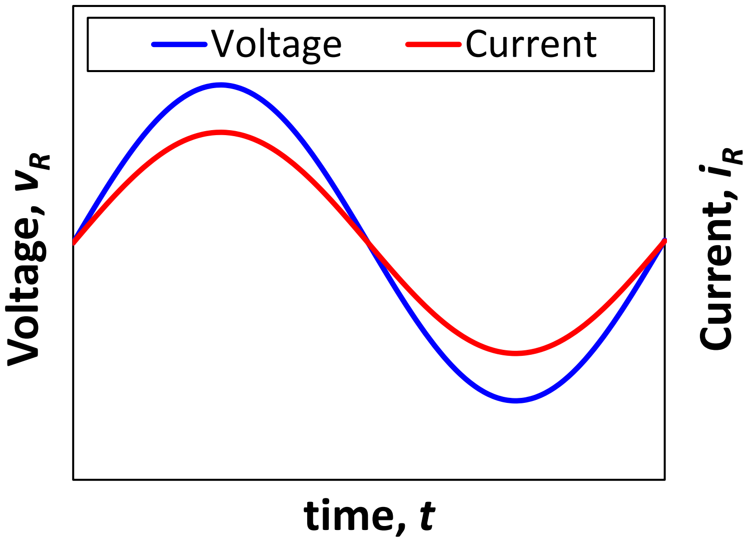


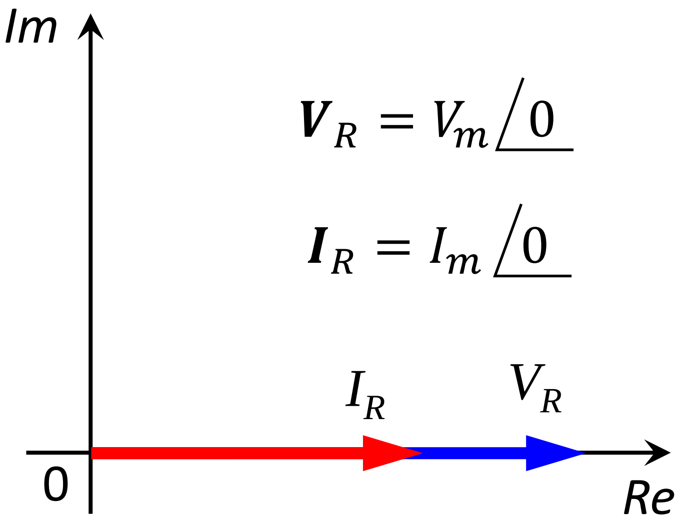
$$i_R(t) = I_m \sin(\omega t)$$

We know from Ohm's law:

$$v_R(t) = R\, i_R(t)$$

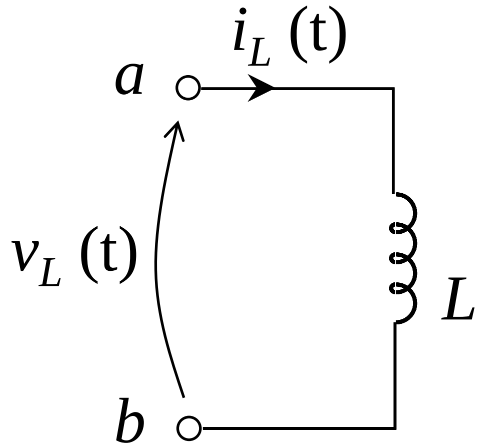
$$v_R(t) = R\, I_m \sin(\omega t)$$





In a pure resistor, the current and voltage are both "in-phase" as there is no phase difference between them.

Performance of an inductor in ac circuit:



$$i_L(t) = I_m\,sin\,(\omega t)$$

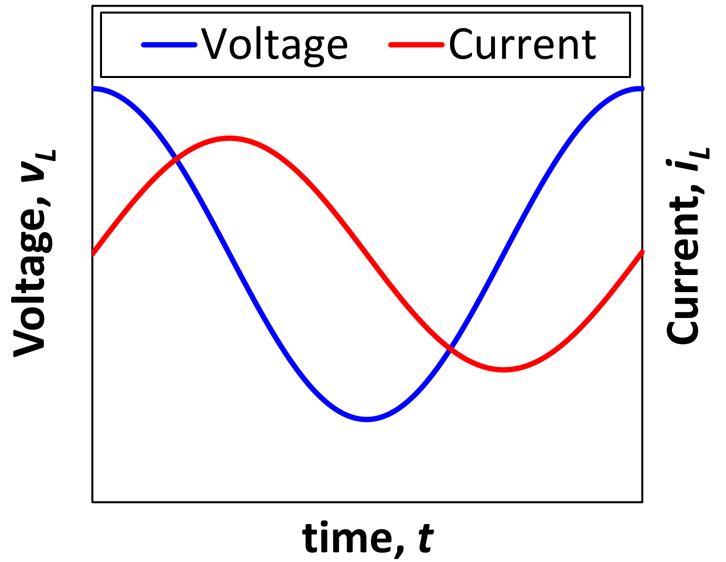
$$v_L(t) = L\frac{di_L(t)}{dt} = L\frac{d(I_m\,sin(\omega t))}{dt}$$

$$v_L(t) = L\omega\,I_m\,cos(\omega t)$$

$$v_L(t) = L\omega\,I_m\,sin(\omega t + 90°)$$

$$i_L(t) = I_m\,sin\,(\omega t)$$

$$v_L(t) = L\omega\,I_m\,sin(\omega t + 90°)$$



In pure inductive circuit:

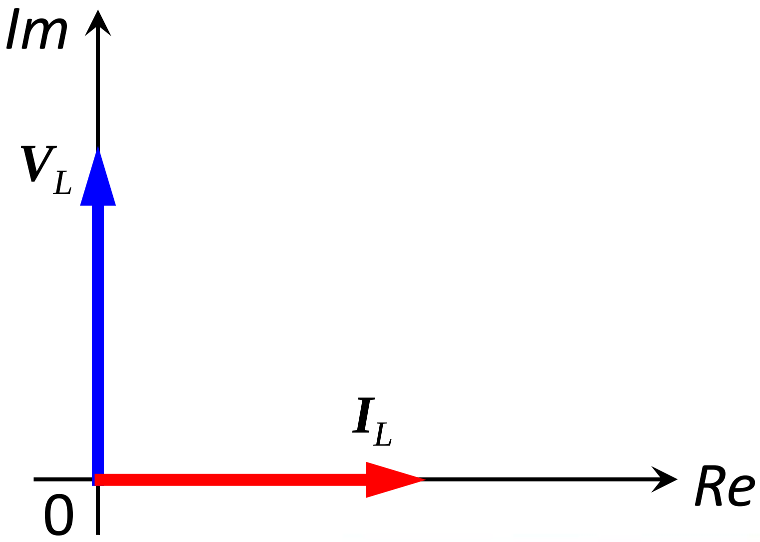
Current lags the voltage by 90°.

Voltage leads the current by 90°.

Performance of an inductor in ac circuit:

$$\begin{cases} i_L(t) = I_m \sin(\omega t) \\ v_L(t) = L\omega\, I_m \sin(\omega t + 90°) \end{cases}$$

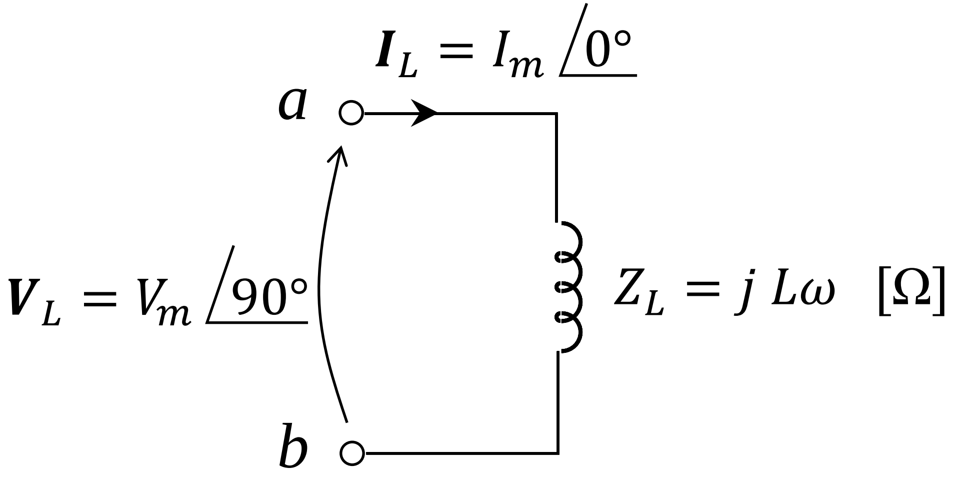
$$\begin{cases} \boldsymbol{I}_L = I_m \angle 0° & \text{[A]} \\ \boldsymbol{V}_L = L\omega\, I_m \angle 90° & \text{[V]} \end{cases}$$



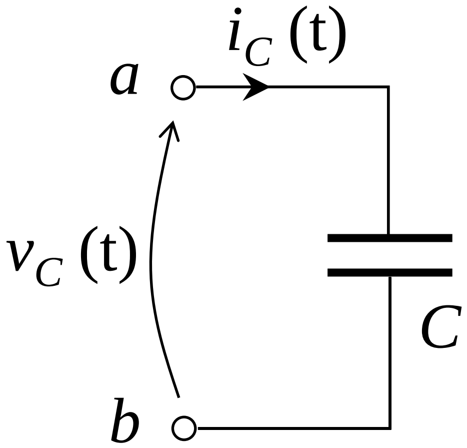
From Ohm's law:

$$Z_L = \frac{\boldsymbol{V}_L}{\boldsymbol{I}_L} = \frac{L\omega I_m \angle 90°}{I_m \angle 0°} = L\omega \angle 90° \qquad [\Omega]$$

$Z_L = j\,L\omega$ $\quad[\Omega]$ Inductive impedance



Performance of an capacitor in ac circuit:



$$v_C(t) = V_m \, sin\,(\omega t)$$

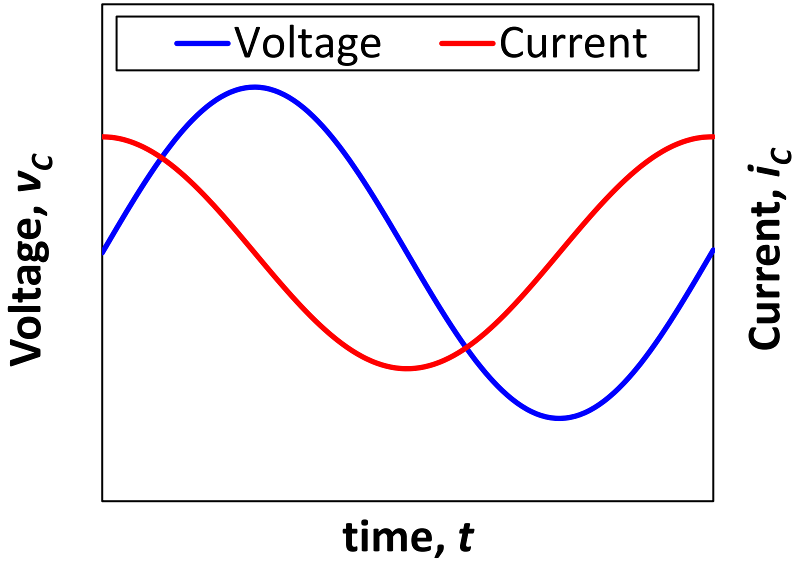
$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{d(V\,sin(\omega t))}{dt}$$

$$i_C(t) = C\omega\, V_m \, cos(\omega t)$$

$$i_C(t) = C\omega\, V_m \, sin(\omega t + 90°)$$

$$v_C(t) = V_m \, sin\,(\omega t)$$

$$i_C(t) = C\omega\, V_m \, sin(\omega t + 90°)$$



In a pure capacitive circuit:

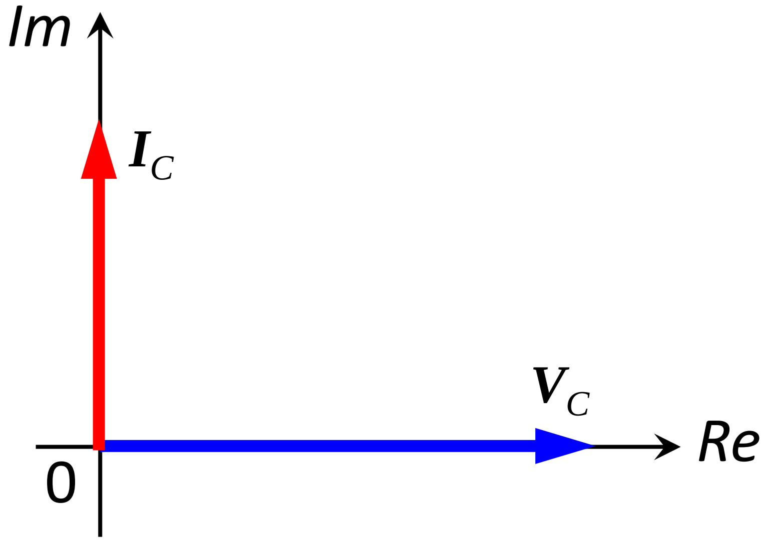
Voltage lags the current by 90°.

Current leads the voltage by 90°.

Performance of an capacitor in ac circuit:

$$\begin{cases} v_C(t) = V_m \sin(\omega t) \\ i_C(t) = C\omega\, V_m \sin(\omega t + 90°) \end{cases}$$

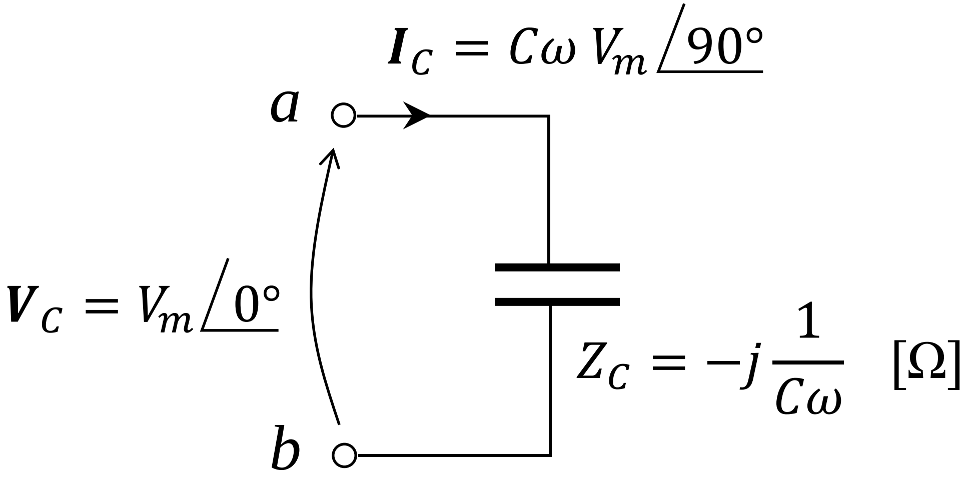
$$\begin{cases} \boldsymbol{V}_C = V_m \underline{/0°} \qquad \text{[A]} \\ \boldsymbol{I}_C = C\omega\, V_m \underline{/90°} \qquad \text{[V]} \end{cases}$$



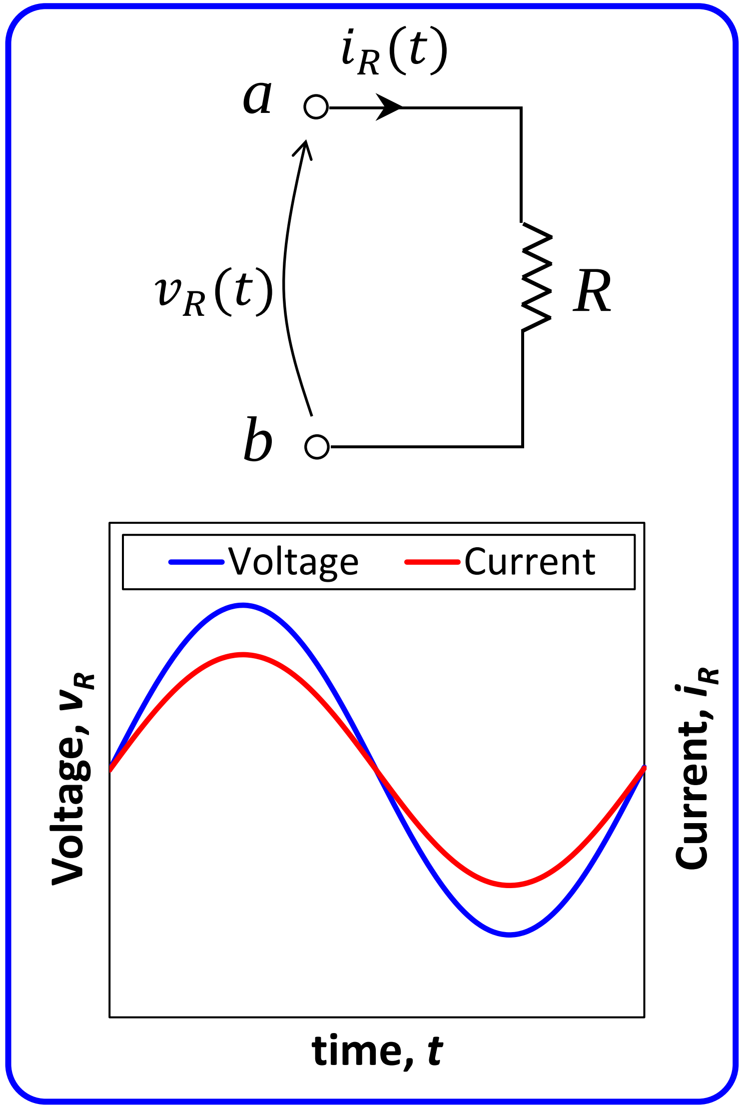
From Ohm's law:

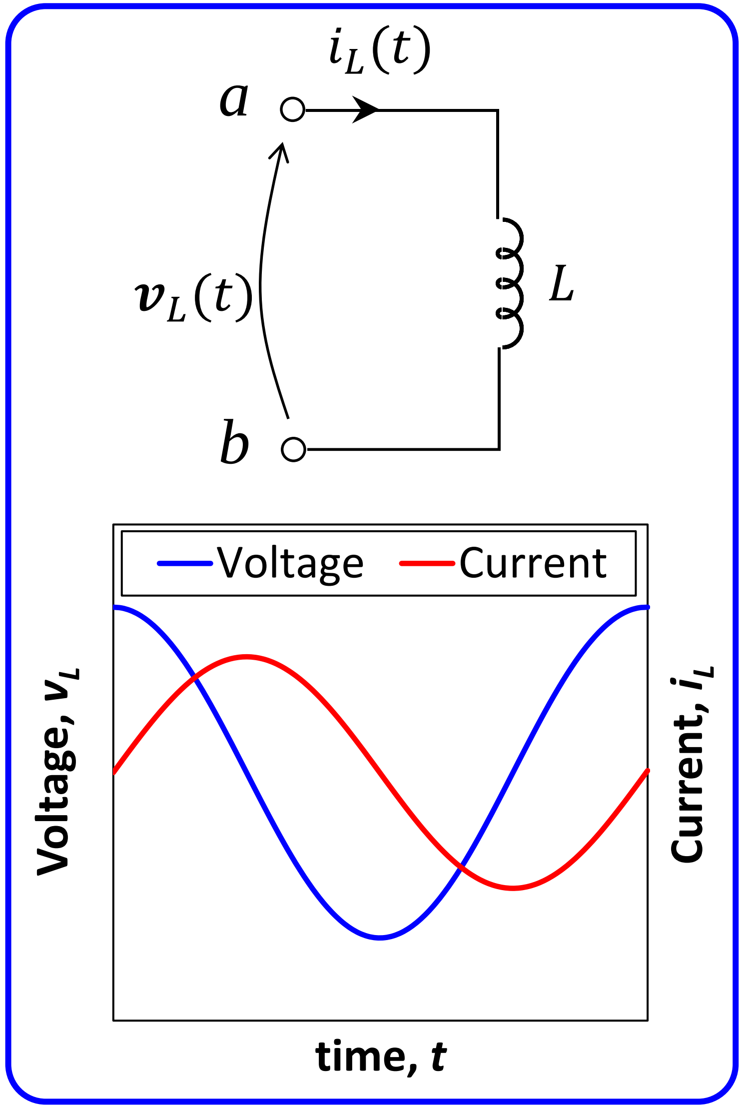
$$Z_C = \frac{\boldsymbol{V}_C}{\boldsymbol{I}_C} = \frac{V_m \underline{/0°}}{C\omega V_m \underline{/90°}} = \frac{1}{C\omega} \underline{/-90°} \quad [\Omega]$$

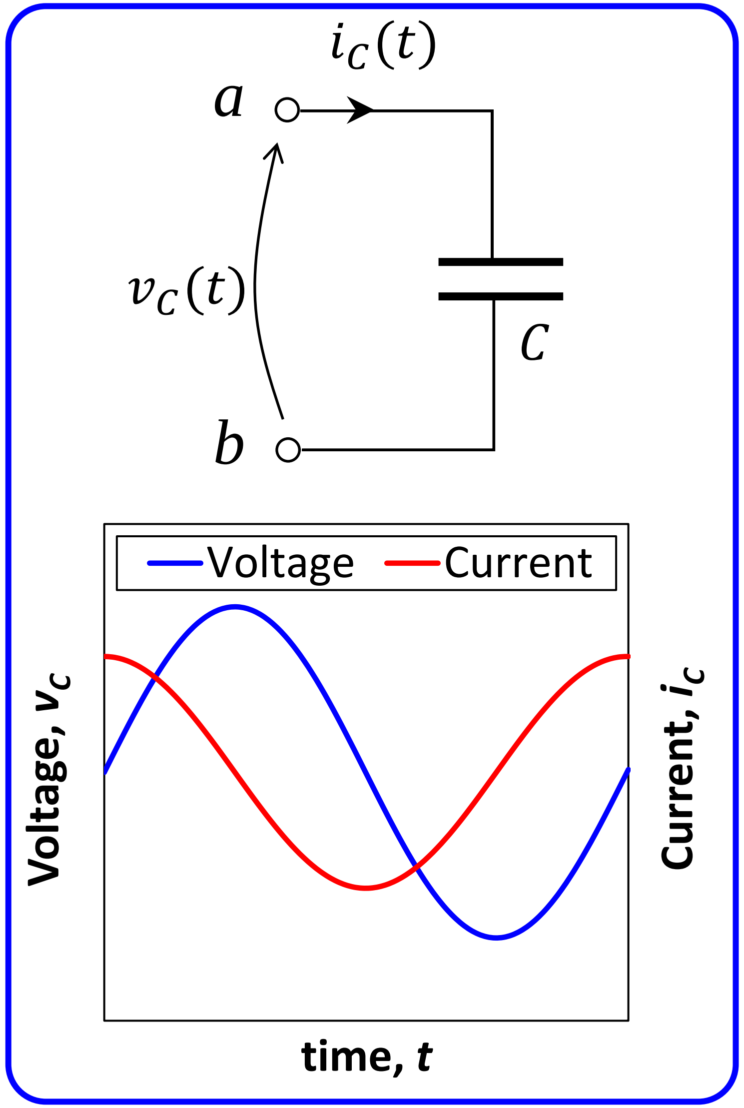
$$Z_C = -j\frac{1}{C\omega} \quad [\Omega]$$ Capacitive impedance



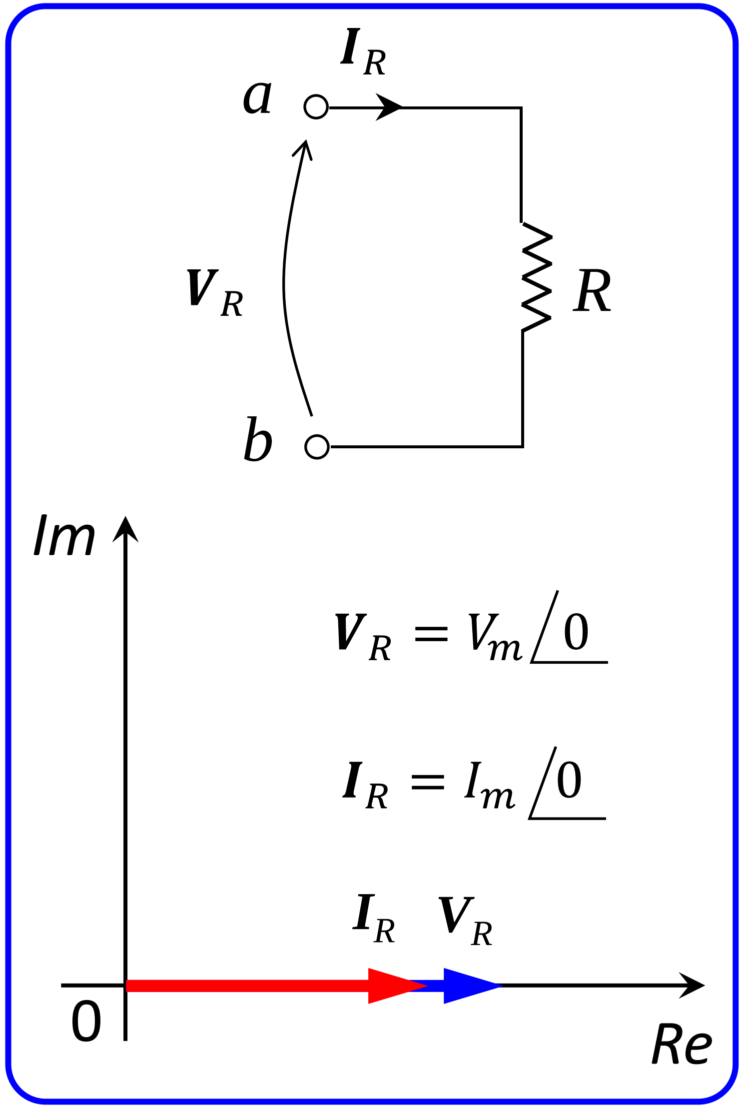
Performance of ideal circuit elements in ac circuit:

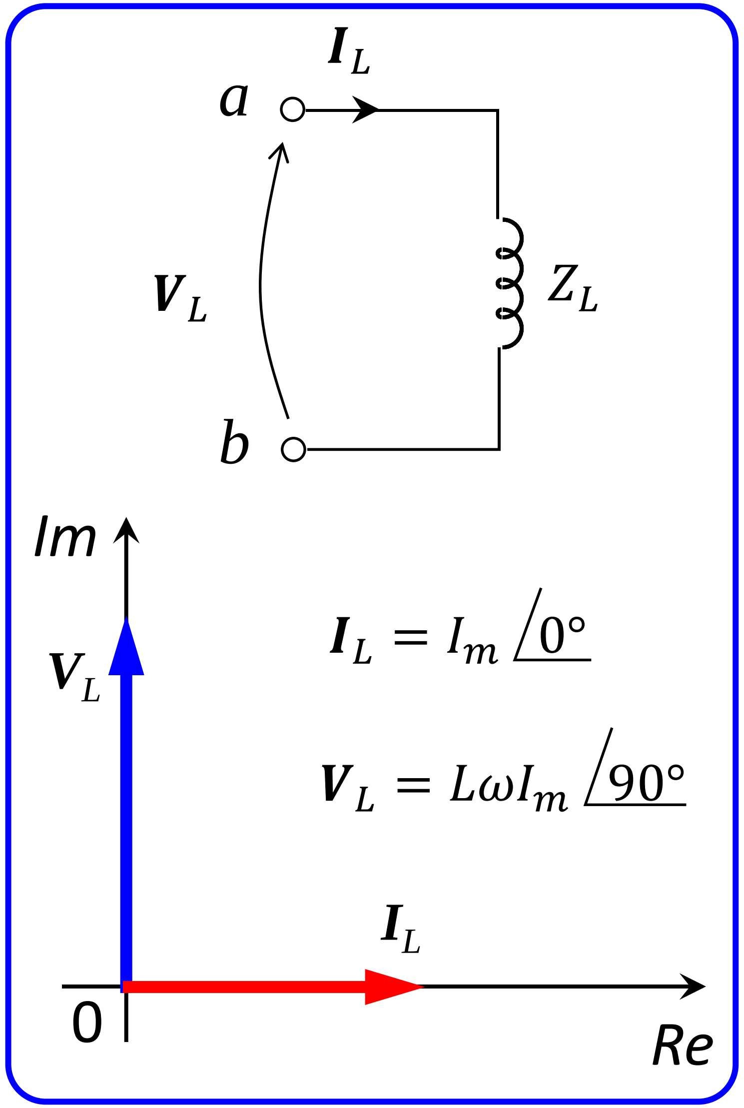


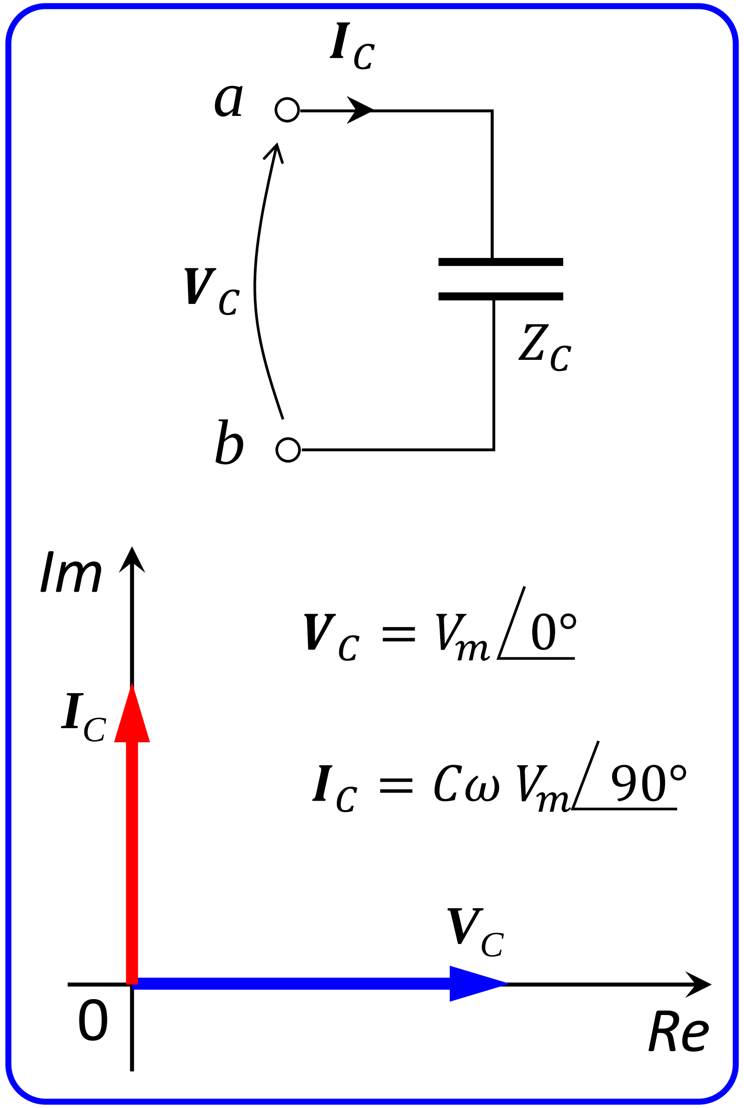




Performance of ideal circuit elements in ac circuit:

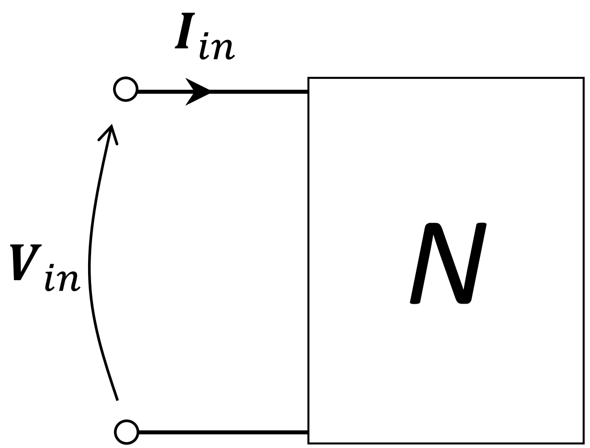






Performance of ideal circuit elements in ac circuit:

Phasor impedance (Reactance):



$$\boldsymbol{Z}_{in} = \frac{\boldsymbol{V}_{in}}{\boldsymbol{I}_{in}} = \frac{V_{rms}\angle\theta_v}{I_{rms}\angle\theta_i}$$

$$\boldsymbol{V}_{in} = V_{rms}\angle\theta_v$$

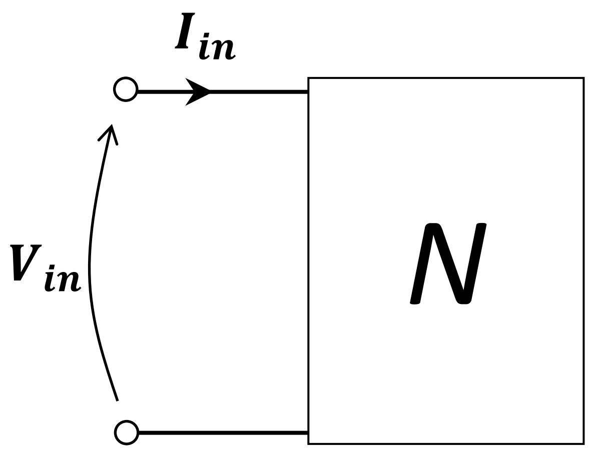
$$\boldsymbol{I}_{in} = I_{rms}\angle\theta_i$$

$$\boldsymbol{Z}_{in} = |Z|\angle\theta_v - \theta_i = |Z|\angle\varphi \qquad [\Omega]$$

$$\boldsymbol{Z}_{in} = \underbrace{|Z|\cos\varphi}_{\text{Resistive component}} + j\underbrace{|Z|\sin\varphi}_{\text{Reactive component (Inductive or Capacitive)}} \qquad [\Omega]$$

Performance of ideal circuit elements in ac circuit:

Example:



$$\boldsymbol{V_{in}} = 75\angle 60^\circ \quad \text{V}$$

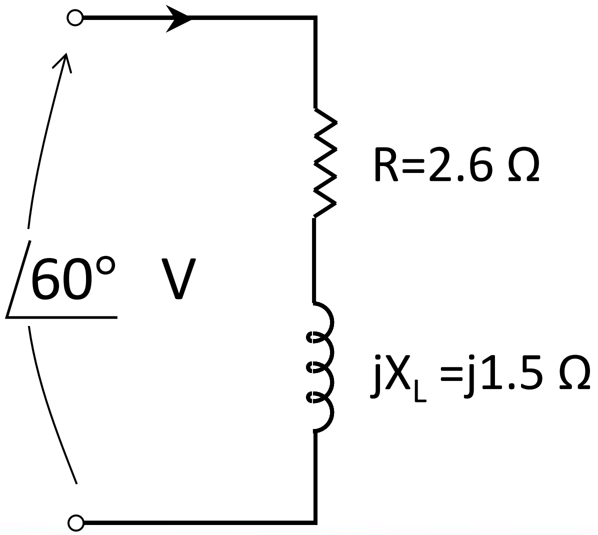
$$\boldsymbol{I_{in}} = 25\angle 30^\circ \quad \text{A}$$

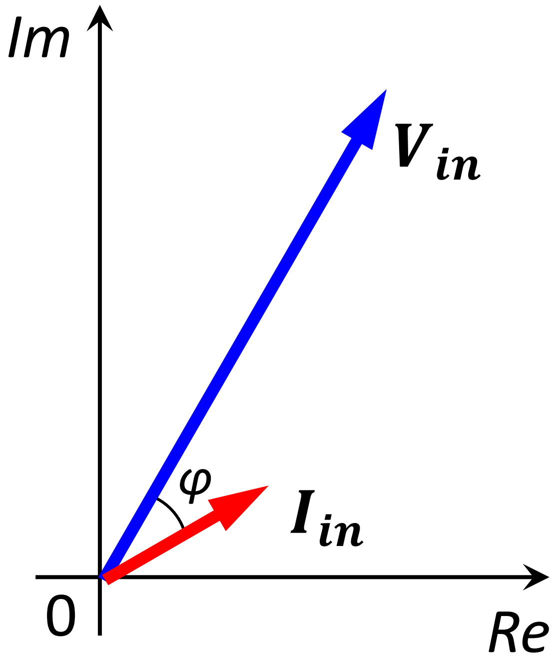
$$\boldsymbol{Z_{in}} = \frac{\boldsymbol{V_{in}}}{\boldsymbol{I_{in}}} = \frac{75\angle 60^\circ}{25\angle 30^\circ} = 3\angle 30 \quad [\Omega]$$

$$\boldsymbol{Z_{in}} = 2.6 + j1.5 \quad [\Omega]$$

$$\boldsymbol{I_{in}} = 25\angle 30^\circ \quad \text{A}$$

$$\boldsymbol{V_{in}} = 75\angle 60^\circ \quad \text{V}$$



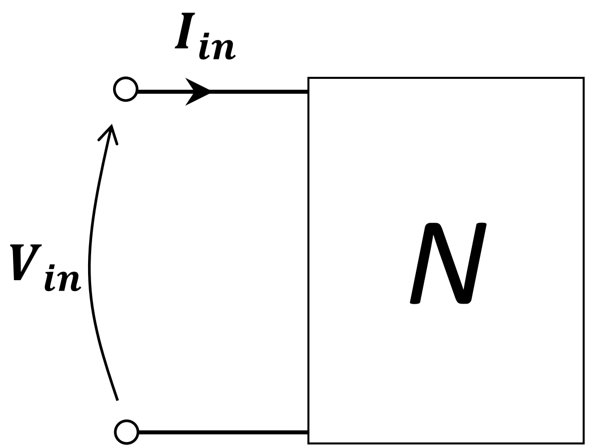


Performance of ideal circuit elements in ac circuit:

Example:

$$\boldsymbol{Z_{in}} = \frac{\boldsymbol{V_{in}}}{\boldsymbol{I_{in}}} = \frac{75\ \angle 30^\circ}{25\ \angle 60^\circ} = 3\angle -30 \quad [\Omega]$$

$$\boldsymbol{Z_{in}} = 2.6 - j1.5 \quad [\Omega]$$

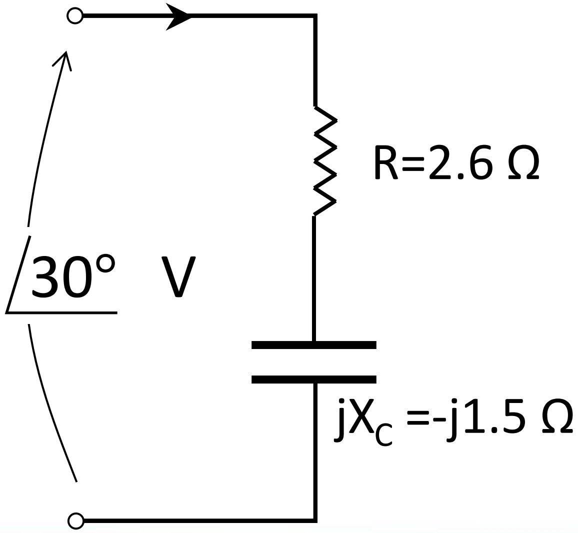


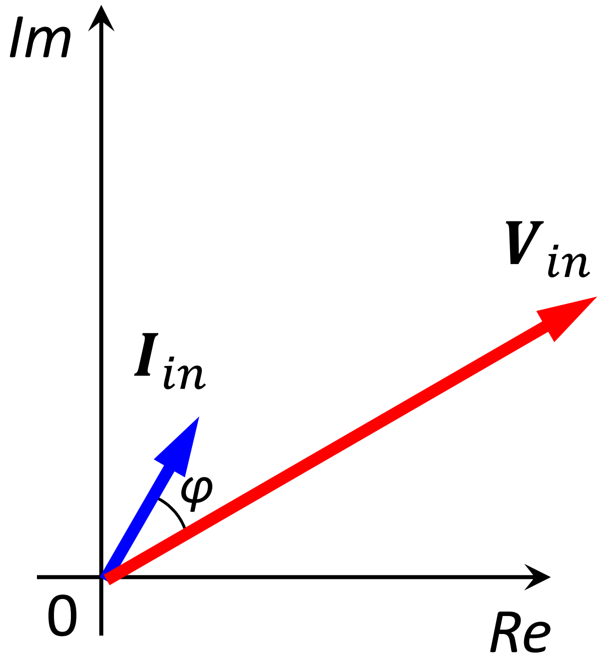
$$\boldsymbol{V_{in}} = 75\ \angle 30^\circ \quad \text{V}$$

$$\boldsymbol{I_{in}} = 25\ \angle 60^\circ \quad \text{A}$$

$$\boldsymbol{I_{in}} = 25\ \angle 60^\circ \quad \text{A}$$

$$\boldsymbol{V_{in}} = 75\angle 30^\circ \quad \text{V}$$





Performance of ideal circuit elements in ac circuit:

Example:

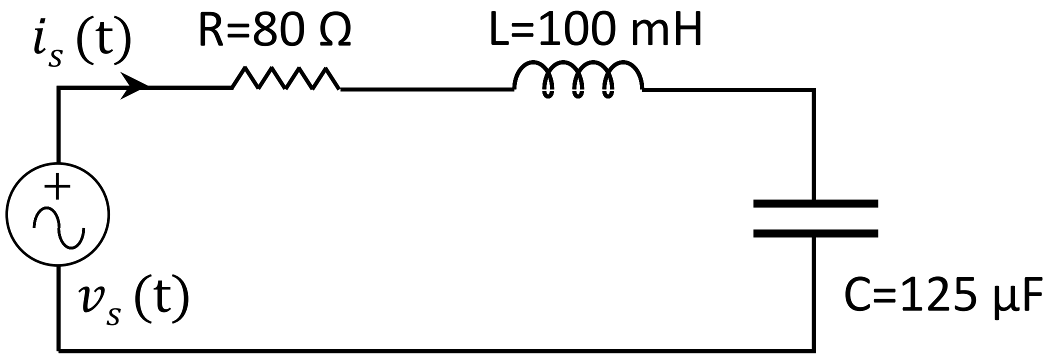
Consider the following circuit:

a) Convert the circuit to phasor

b) Find the current

c) Find the voltage drop on each element

d) Draw the phasor diagram of the circuit

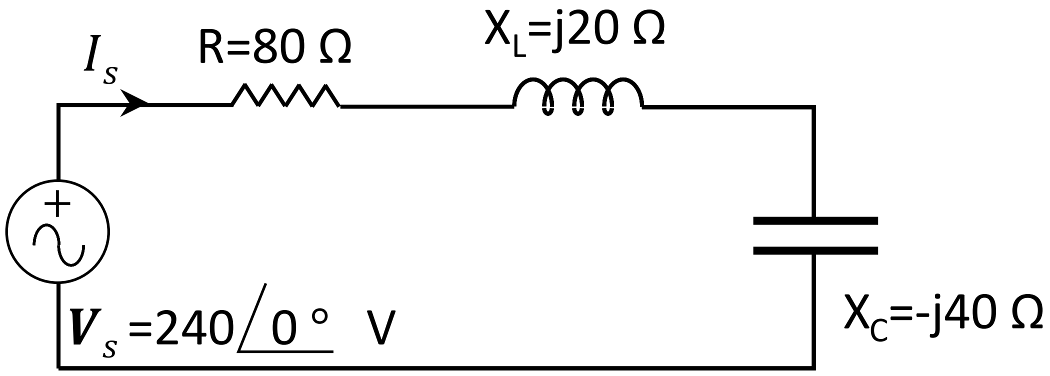


$$v_s(t) = 240\sqrt{2}\, sin\,(200t)$$

$$X_L = \mathrm{j}\,\omega L = j\,200 \times 100 \times 10^{-3} = j\,20\,\Omega$$

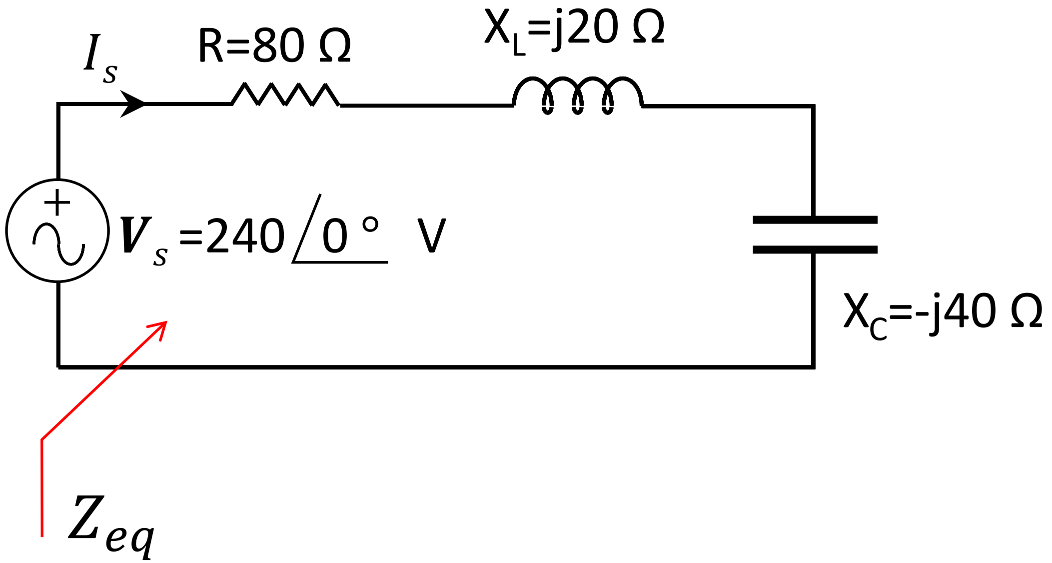
$$X_C = -\mathrm{j}\frac{1}{\omega\,C} = -j\frac{1}{200 \times 125 \times 10^{-6}} = -j\,40\,\Omega$$

$$\boldsymbol{V_s} = 240\angle 0 \quad \mathrm{V}$$



Performance of ideal circuit elements in ac circuit:

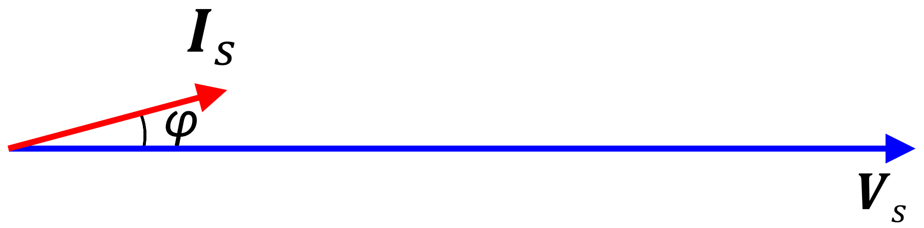
Example:



$$\boldsymbol{Z}_{eq} = 80 + \mathrm{j}20 - \mathrm{j}\,40 = 80 - \mathrm{j}20\ \ \Omega$$

$$\boldsymbol{Z}_{eq} = 82.46\angle{-14.03°}\quad \Omega$$

$$\boldsymbol{I}_s = \frac{240\angle 0°}{82.46\angle{-14.03°}} = 2.91\angle 14.03°\quad \mathrm{A}$$



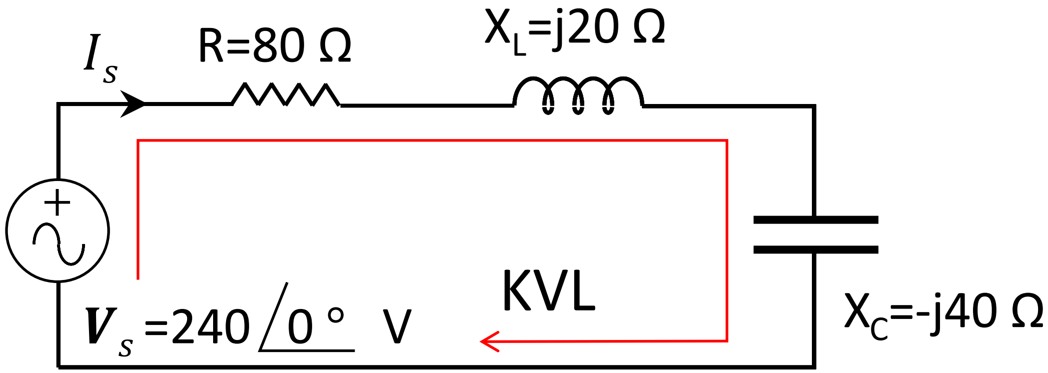
$$\boldsymbol{V}_R = \mathrm{R}\,\boldsymbol{I}_s = 233.8\ \angle 14.03°\qquad \mathrm{V}$$

$$\boldsymbol{V}_{X_L} = \mathrm{j}\,X_L\boldsymbol{I}_s = 58.2\ \angle 104.03°\qquad \mathrm{V}$$

$$\boldsymbol{V}_{X_C} = -\mathrm{j}\,X_C\boldsymbol{I}_s = 116.4\ \angle{-75.97°}\quad \mathrm{V}$$

Performance of ideal circuit elements in ac circuit:

Example:



KVL:

$$-\boldsymbol{V}_s + R\,\boldsymbol{I}_S + \mathrm{j}\,X_L\,\boldsymbol{I}_S - \mathrm{j}\,X_C\,\boldsymbol{I}_S = 0$$

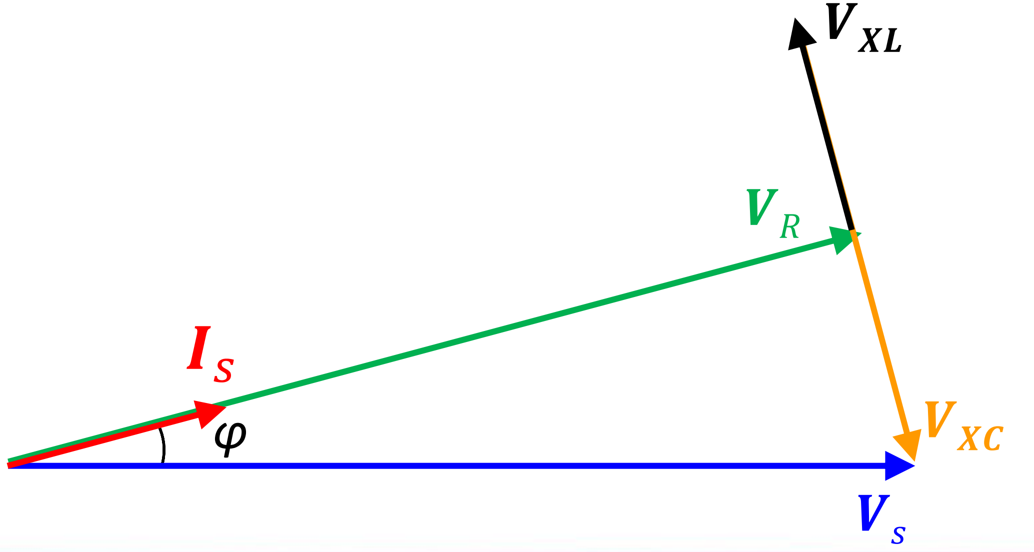
$$-240\,\underline{/0} + 80\,\boldsymbol{I}_S + \mathrm{j}\,20\,\boldsymbol{I}_S - \mathrm{j}\,40\,\boldsymbol{I}_S = 0$$

$$\boldsymbol{I}_s = 2.91\,\underline{/14.03°} \quad \mathrm{A}$$

$$\boldsymbol{V}_R = \mathrm{R}\,\boldsymbol{I}_s = 233.8\,\underline{/14.03°} \quad \mathrm{V}$$

$$\boldsymbol{V}_{X_L} = \mathrm{j}\,X_L\boldsymbol{I}_s = 58.2\,\underline{/104.03°} \quad \mathrm{V}$$

$$\boldsymbol{V}_{X_c} = -\mathrm{j}\,X_C\boldsymbol{I}_s = 116.4\,\underline{/-75.97°} \quad \mathrm{V}$$



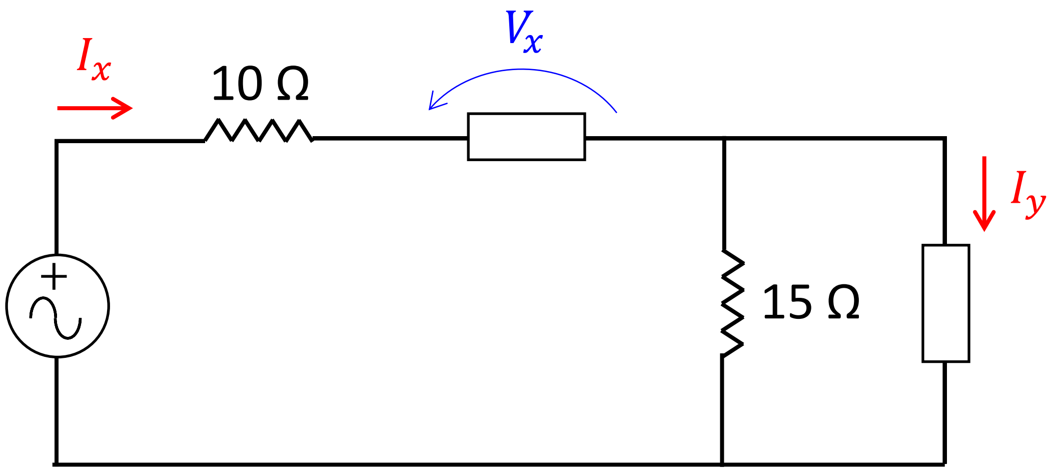
Example:

Drill:

In the following circuit $I_x = 5 < 35°\ A$, $V_x = 25 < 15°\ \ V$, and $I_y = 3 < -25°\ A$.

a) Find current of the 15 Ω resistor

b) Find phasor of the voltage source V_s.



Answer:

a) $I_{15} = 4.35 < 71.52°\ \ \ A$

b) $V_s = 129.52 < 48.3°\ \ \ V$

Module description:

Recommended text books:

- DeCarlo Lin, "*Linear Circuit Analysis*", Oxford University Press, Second Edition, 2003
- W H Hayt, J E Kemmerly, S M Durbin, "*Engineering Circuit Analysis*", McGraw-Hill, 9th Edition, 2019