Durham University

Electrical Engineering 2

ENGI2191

Three Phase Systems

A review on phasor diagrams

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E108

Lecture Objectives:

Aim of this lecture:

The aim of this lecture is to understand the basic concepts of three-phase systems in electrical networks and power systems.

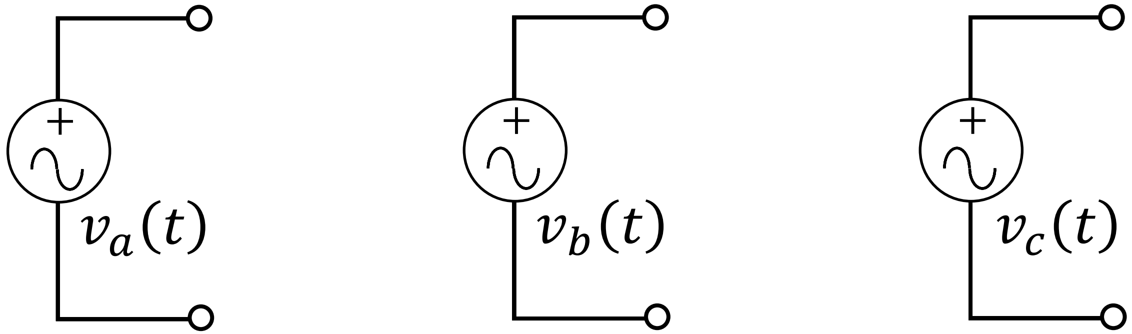
Intended Learning Outcomes:

At the completion of the lecture and associated problems you should be able to:

- Discuss the differences between single-phase and three-phase systems.
- Discuss the characteristics of Y and Δ connections.
- Calculate voltage and current values for both Y and Δ connections.
- Analyse the balanced three phase circuits.
- Calculate complex power in three phase systems.

Three phase voltage source:

A three-phase voltage source consists of three independent single phase sinusoidal voltage sources with the same frequency and amplitude, but with phase angles separated by 120°.



$$v_a(t) = V_m \, Sin\,(\omega t)$$

$$v_b(t) = V_m \, Sin\,(\omega t - 120°)$$

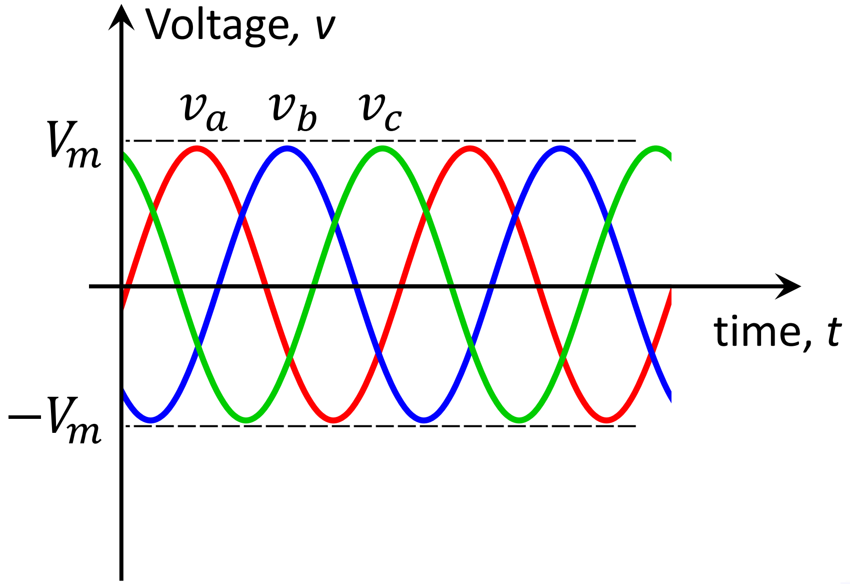
$$v_c(t) = V_m \, Sin\,(\omega t + 120°)$$

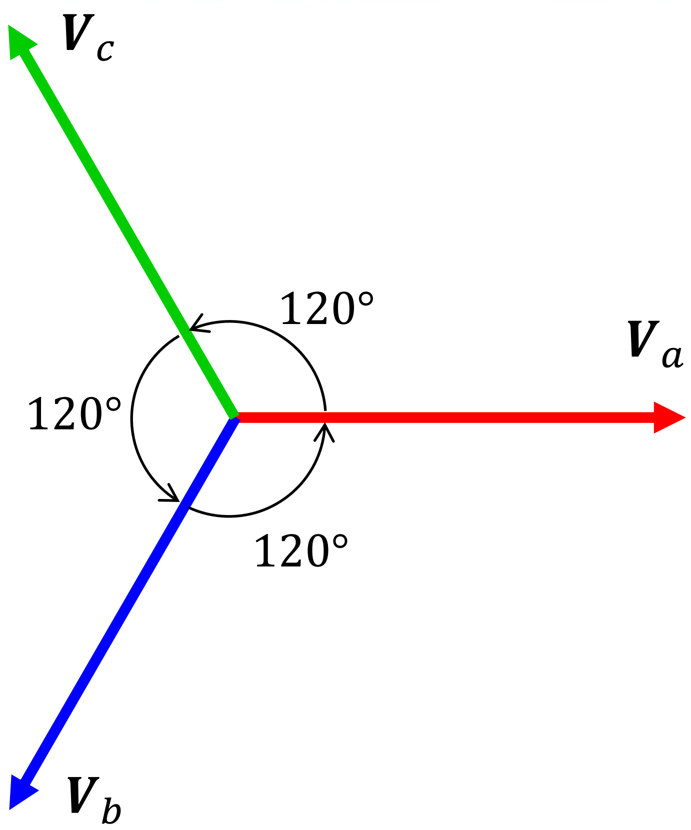
$$\boldsymbol{V}_a = V_m \angle 0°$$

$$\boldsymbol{V}_b = V_m \angle -120°$$

$$\boldsymbol{V}_c = V_m \angle 120°$$

Three phase systems:





$$v_a(t) = V_m\, Sin\,(\omega t)$$

$$v_b(t) = V_m\, Sin\,(\omega t - 120°)$$

$$v_c(t) = V_m\, Sin\,(\omega t + 120°)$$

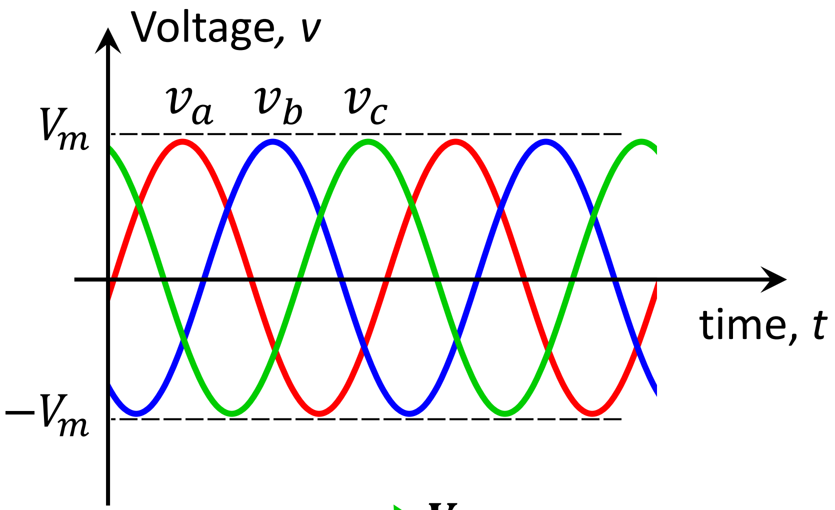
$$\boldsymbol{V}_a = V_m \angle 0°$$

$$\boldsymbol{V}_b = V_m \angle -120°$$

$$\boldsymbol{V}_c = V_m \angle 120°$$

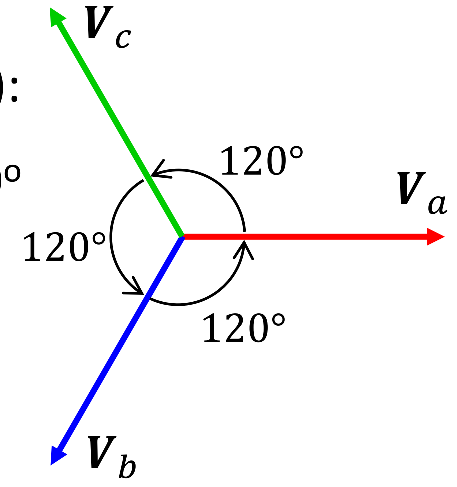
Three phase systems:

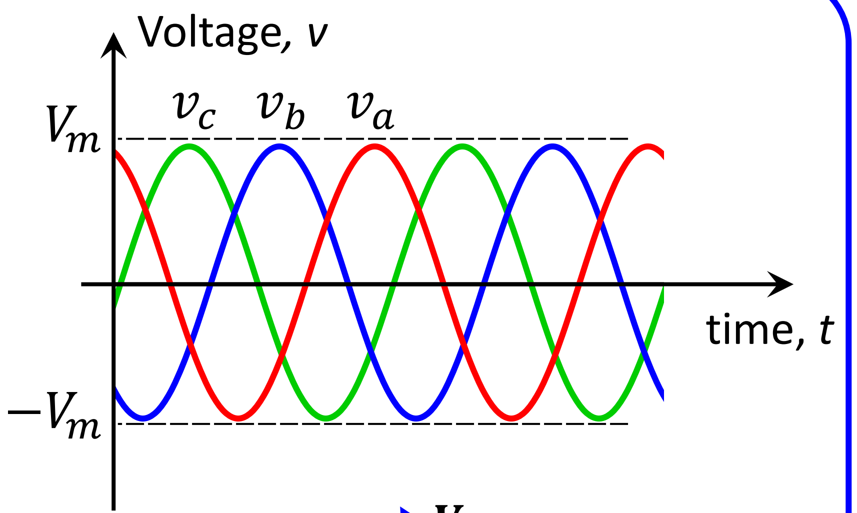
Based on the rotating direction of the three phase generator, two phase sequences are defined:



Positive sequence (abc):

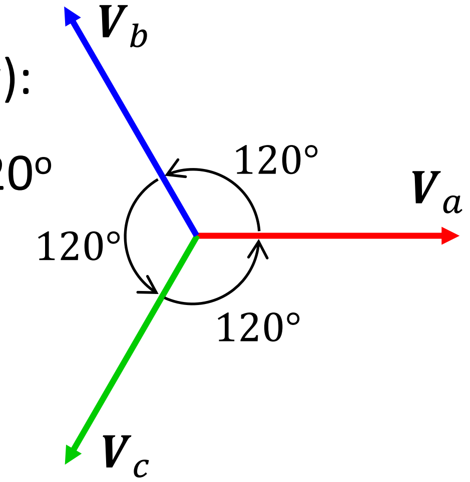
$v_b(t)$ lags $v_a(t)$ by 120°





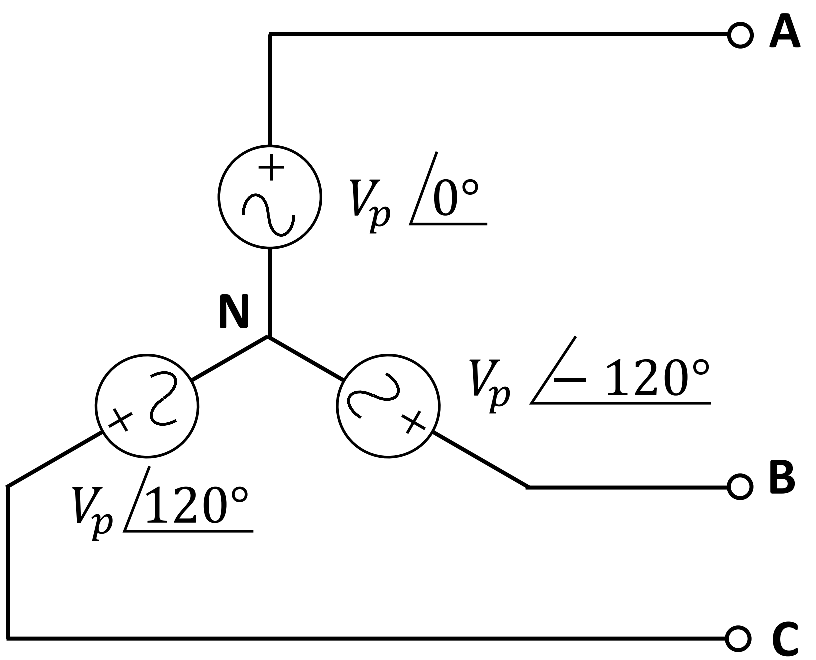
Negative sequence (cba):

$v_b(t)$ leads $v_a(t)$ by 120°

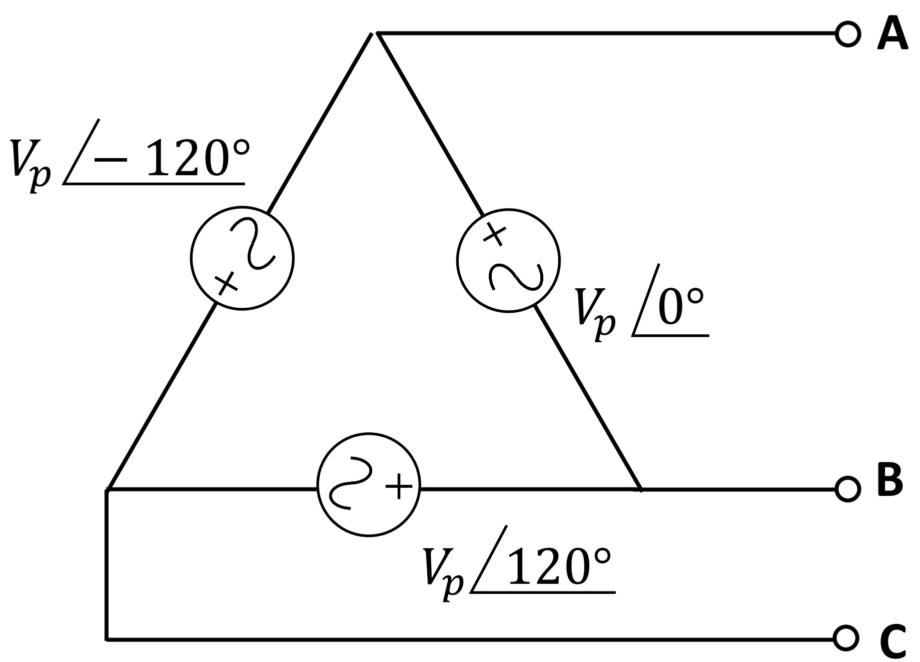


Three phase systems:

Three phase voltage source:



Y, Wye or star connected three-phase voltage source



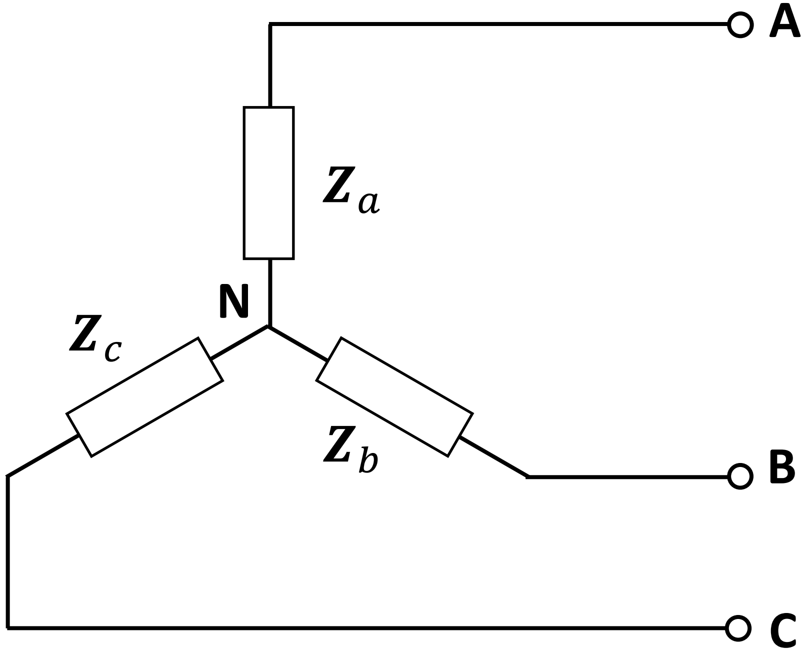
Δ connected three-phase voltage source

V_p is called phase voltage, is voltage across each single-phase voltage source.

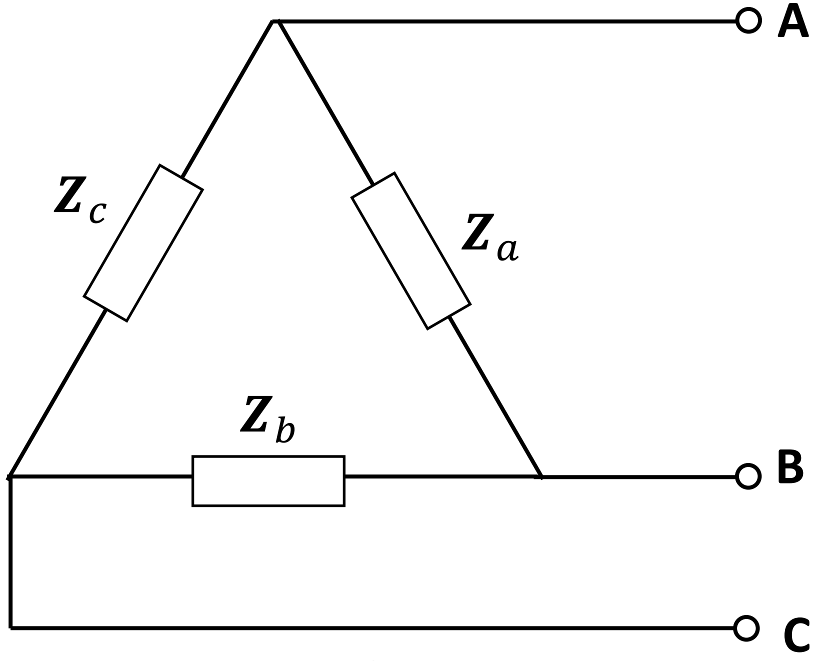
Three phase systems:

Three phase loads:

Three phase load impedance are also connected either Y or Δ connection.



Y Connection

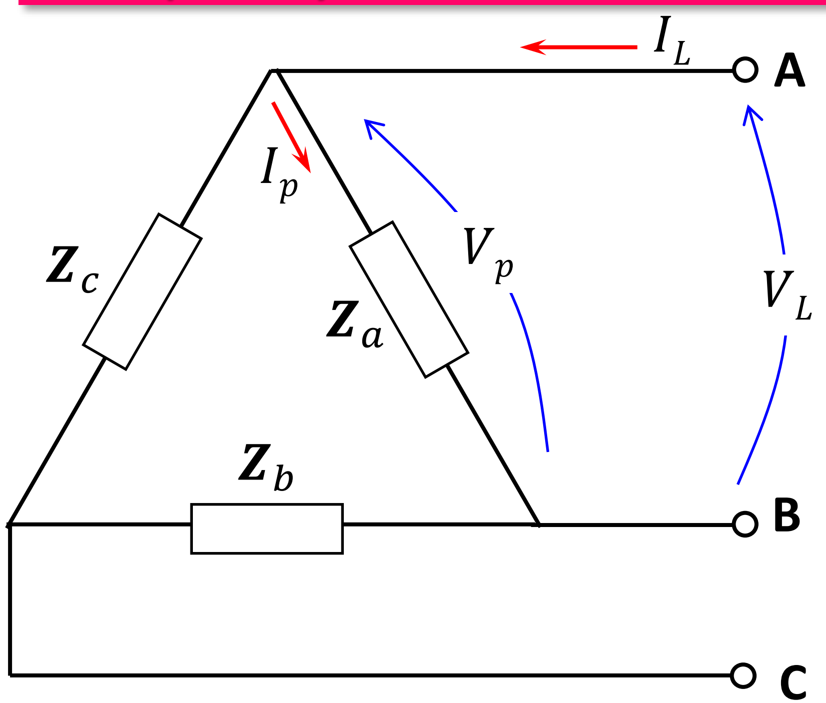


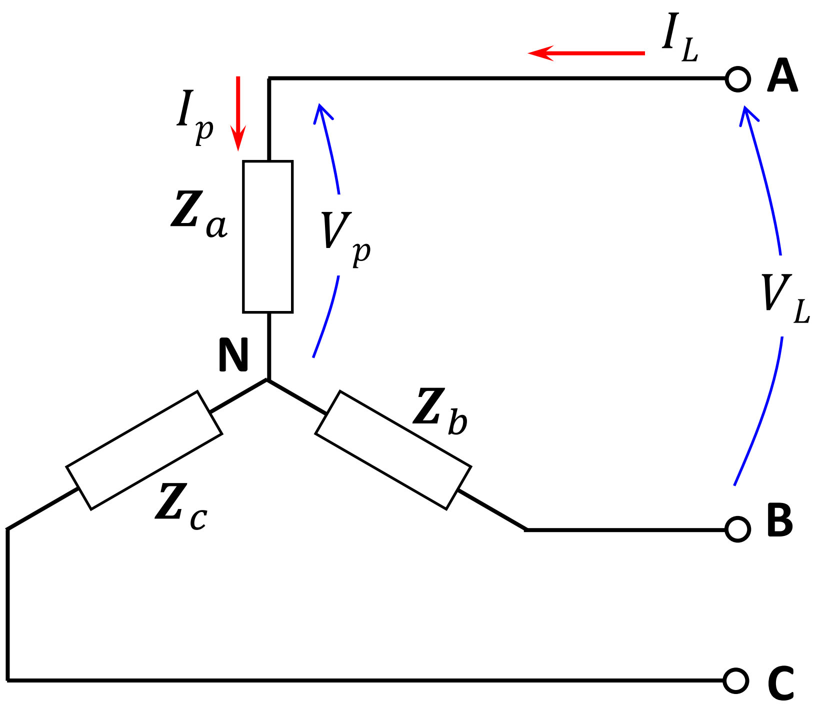
Δ Connection

In a balance three phase load: $Z_a = Z_b = Z_c$

Any circuit that does not have all loads with the same impedance in all three phases, is unbalanced load.

Three phase systems:





In three phase loads:

Phase voltage, V_p:

Voltage across each impedance.

Line voltage, V_L:

Voltage between any pair of lines or terminals.

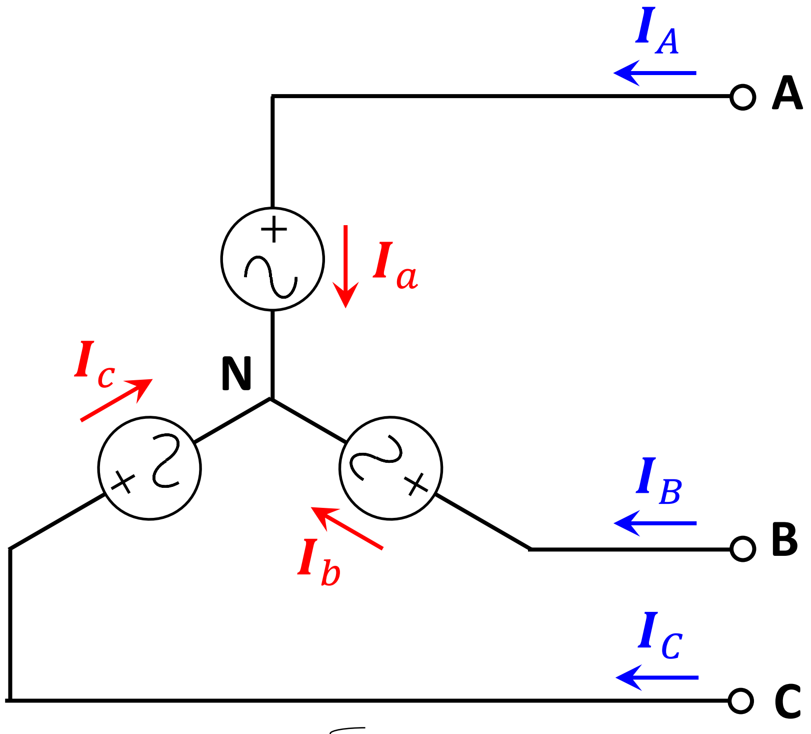
Phase current, I_p:

Current flowing through each impedance.

Line current, I_L:

Current flowing through each line.

Three phase systems:



$$\begin{cases} \boldsymbol{I}_A = \boldsymbol{I}_a \\ \boldsymbol{I}_B = \boldsymbol{I}_b \\ \boldsymbol{I}_C = \boldsymbol{I}_c \end{cases}$$

In three phase systems:

Phase current, I_p:

Current passing through each source (or impedance).

Line current, I_L:

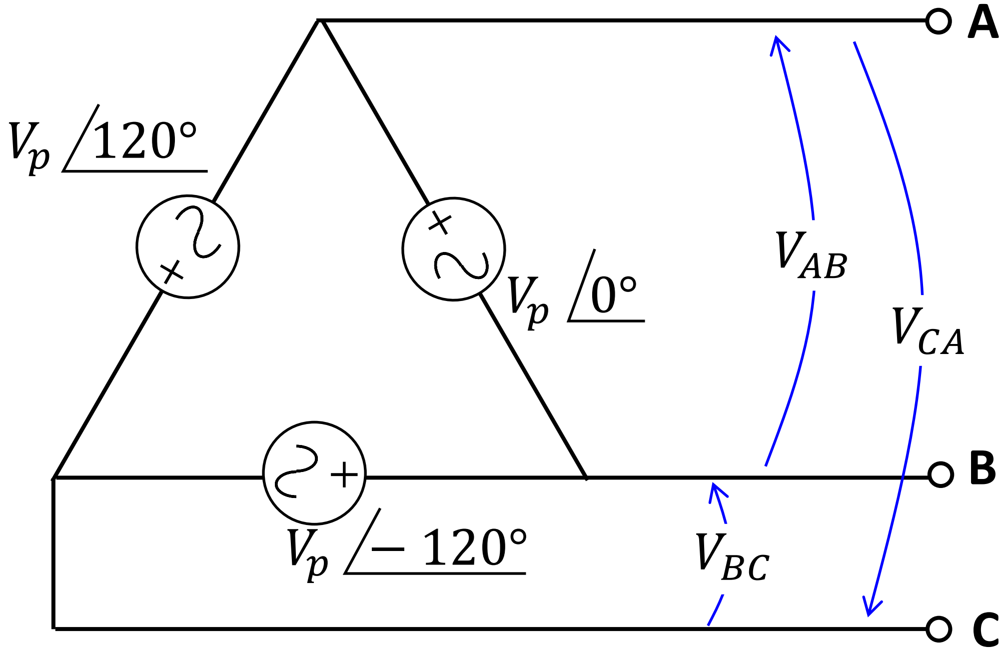
Current passing through a each line.

In three phase Y-connected systems:

In Y connection:

$$I_L = I_p$$

Three phase systems:



$$\begin{cases} \boldsymbol{V}_{AB} = V_p \angle 0° \\ \boldsymbol{V}_{BC} = V_p \angle -120° \\ \boldsymbol{V}_{CA} = V_p \angle 120° \end{cases}$$

In three phase voltage sources:

Phase voltage, V_p:

Voltage across each source.

Line voltage, V_L:

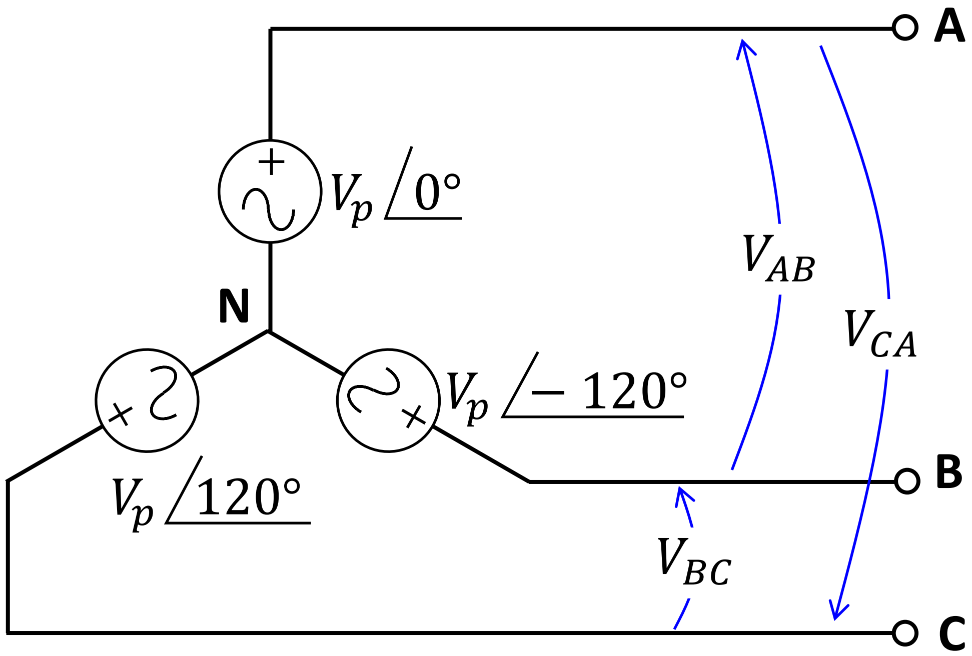
Voltage between any pair of lines or terminals.

In three phase Δ-connected systems:

In Δ connection:

$$V_L = V_p$$

Three phase systems:

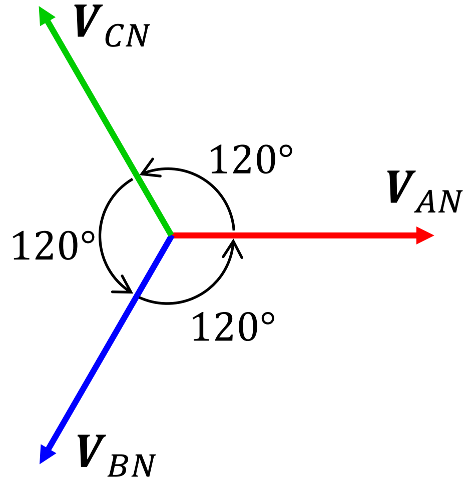


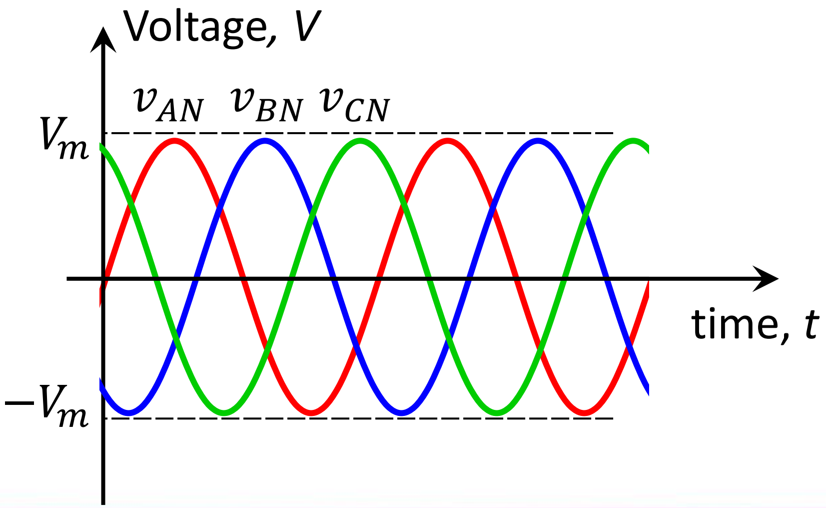
Phase voltages:

$$\begin{cases} v_{AN}(t) = V_m\,Sin\,(\omega t) \\ v_{BN}(t) = V_m\,Sin\,(\omega t - 120°) \\ v_{CN}(t) = V_m\,Sin\,(\omega t + 120°) \end{cases}$$

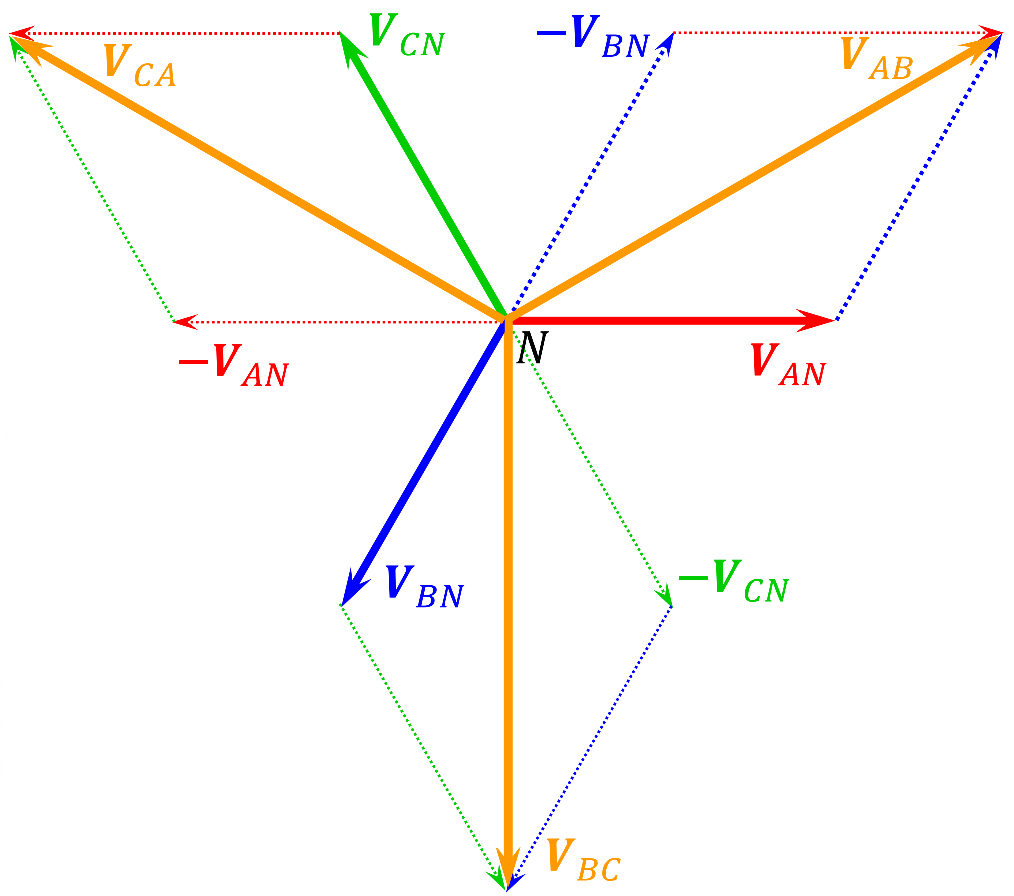
Phase voltages:

$$\begin{cases} \boldsymbol{V}_{AN} = V_{ph} \angle 0° \\ \boldsymbol{V}_{BN} = V_{ph} \angle -120° \\ \boldsymbol{V}_{CN} = V_{ph} \angle 120° \end{cases}$$





Three phase systems:

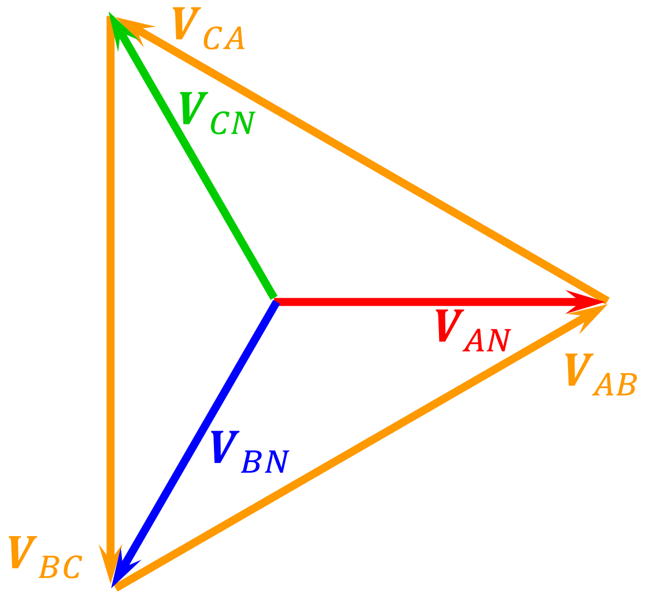


Phase voltages:

$$\begin{cases} V_{AN} = V_{ph}\angle 0° \\ V_{BN} = V_{ph}\angle -120° \\ V_{CN} = V_{ph}\angle 120° \end{cases}$$

Line voltages:

$$\begin{cases} V_{AB} = V_{AN} - V_{BN} \\ V_{BC} = V_{BN} - V_{CN} \\ V_{CA} = V_{CN} - V_{AN} \end{cases}$$



Line voltages:

$$\begin{cases} V_{AB} = V_L\angle 30° \\ V_{BC} = V_L\angle -90° \\ V_{CN} = V_L\angle 150° \end{cases}$$

$$V_L = \sqrt{3}\,V_p$$

Three phase systems:

From KVL the relation between the phase and line voltages can be obtained:

$$\boldsymbol{V}_{AB} = \boldsymbol{V}_{AN} - \boldsymbol{V}_{BN} = V_p \angle 0° - V_p \angle -120°$$

$$= V_p - \left(-\frac{1}{2}V_p - j\frac{\sqrt{3}}{2}V_p\right) = \frac{3}{2}V_p + j\frac{\sqrt{3}}{2}V_p$$

$$\boldsymbol{V}_{AB} = \sqrt{3}\,V_p \angle 30°$$

Do the same calculations for $\boldsymbol{V}_{BC}$ and $\boldsymbol{V}_{CA}$:

$$\boldsymbol{V}_{BC} = \boldsymbol{V}_{BN} - \boldsymbol{V}_{CN} = V_p \angle -120° - V_p \angle 120° = \sqrt{3}\,V_p \angle -90°$$

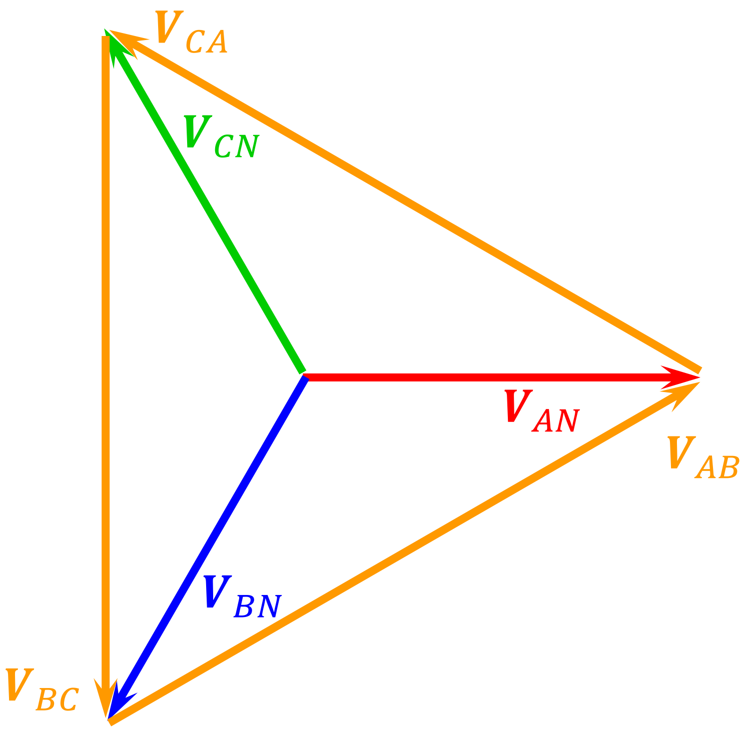
$$\boldsymbol{V}_{CA} = \boldsymbol{V}_{CN} - \boldsymbol{V}_{AN} = V_p \angle 120° - V_p \angle 0° = \sqrt{3}\,V_p \angle 150°$$

In Y connection:

$$V_L = \sqrt{3}\,V_p$$

Three phase systems:

Phasor diagram of a three phase system with positive phase sequence (*abc*):



Phase voltages:

$$\begin{cases} \boldsymbol{V}_{AN} = V_{ph}\angle 0° \\ \boldsymbol{V}_{BN} = V_{ph}\angle -120° \\ \boldsymbol{V}_{CN} = V_{ph}\angle 120° \end{cases}$$

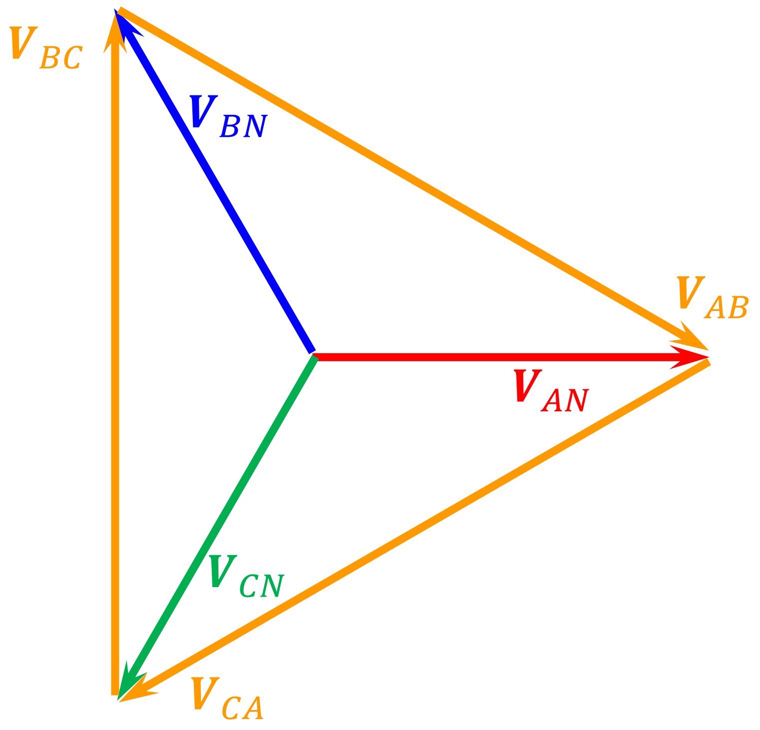
Line voltages:

$$\begin{cases} \boldsymbol{V}_{AB} = V_L\angle 30° \\ \boldsymbol{V}_{BC} = V_L\angle -90° \\ \boldsymbol{V}_{CA} = V_L\angle 150° \end{cases}$$

$$V_L = \sqrt{3}\, V_p$$

Three phase systems:

Phasor diagram of a three phase system with <u>negative phase sequence</u> (*cba*):



Phase voltages:

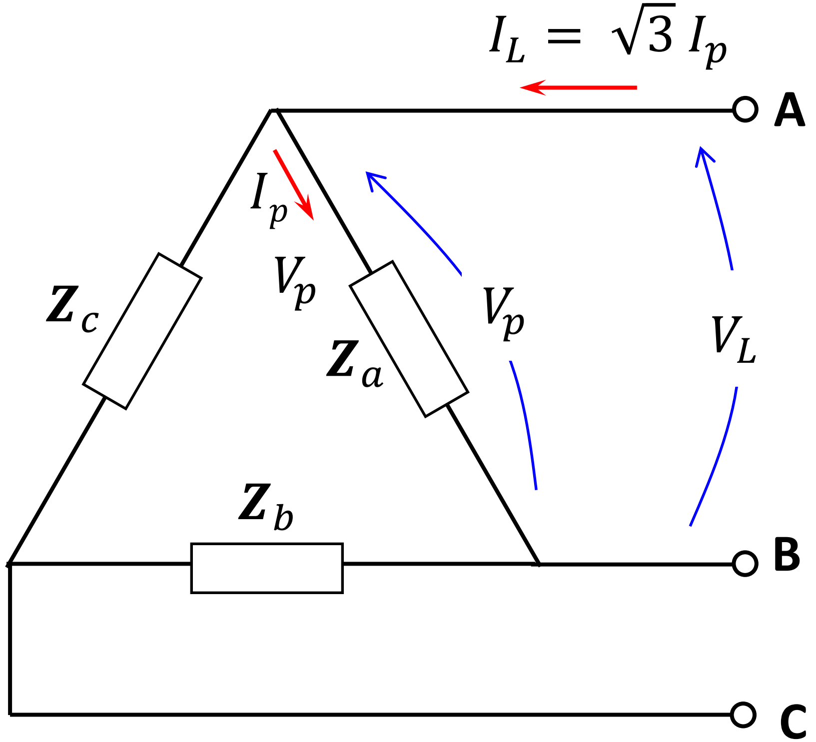
$$\begin{cases} \boldsymbol{V}_{AN} = V_{ph}\angle 0° \\ \boldsymbol{V}_{BN} = V_{ph}\angle 120° \\ \boldsymbol{V}_{CN} = V_{ph}\angle -120° \end{cases}$$

Line voltages:

$$\begin{cases} \boldsymbol{V}_{AB} = V_L\angle -30° \\ \boldsymbol{V}_{BC} = V_L\angle 90° \\ \boldsymbol{V}_{CA} = V_L\angle -150° \end{cases}$$

$$V_L = \sqrt{3}\, V_p$$

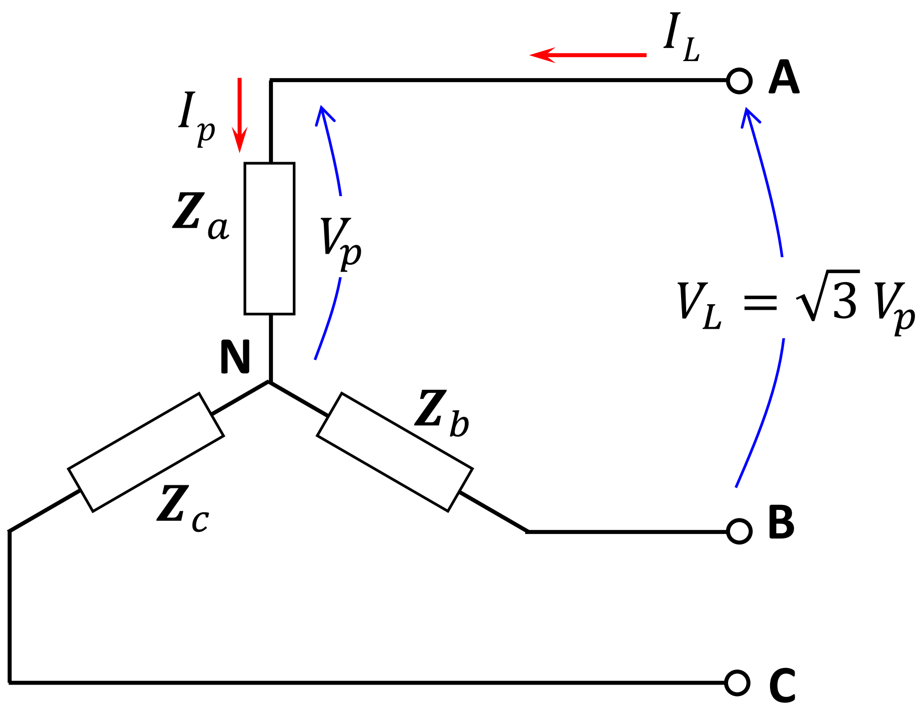
Three phase systems:



$V_L = V_p$ $\quad$ $I_L = \sqrt{3}\, I_p$

$P_t = \sqrt{3}\; V_L\, I_L \cos\varphi$

Only 3-wires system is possible



$V_L = \sqrt{3}\, V_p$ $\quad$ $I_L = I_p$

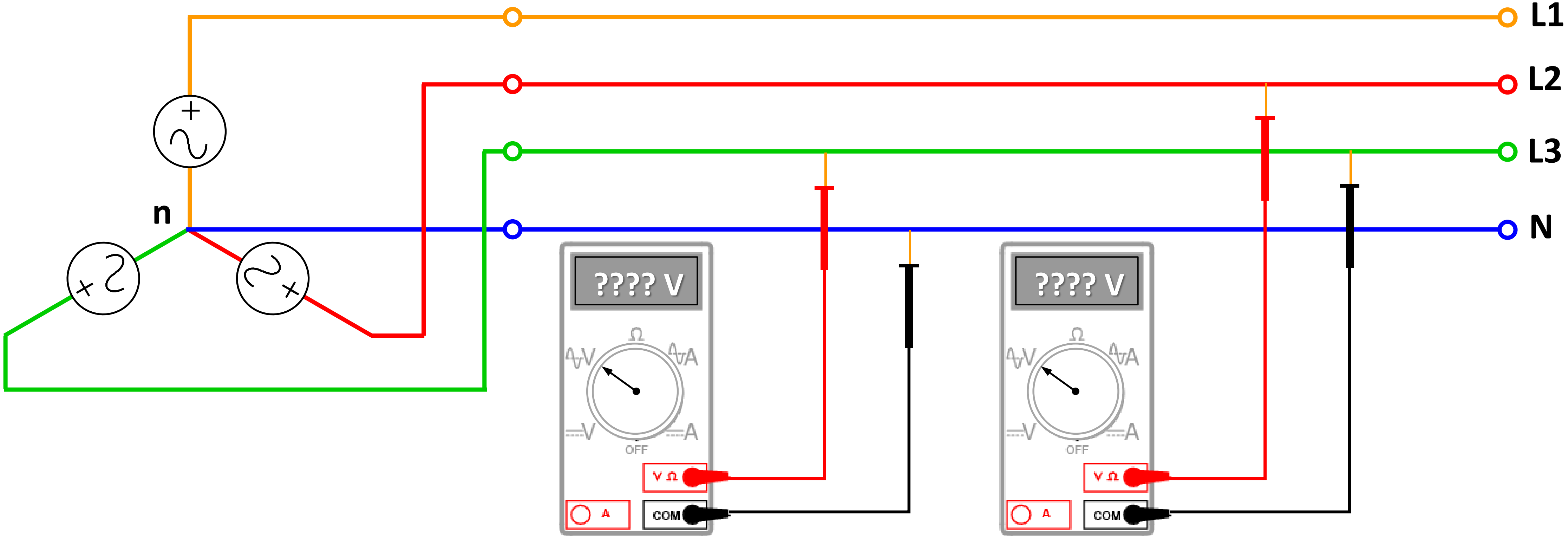
$P_t = \sqrt{3}\; V_L\, I_L \cos\varphi$

3-wires and 4-wires systems are possible

Three phase systems:

Practice:

Measure the phase and line voltages in the Lab (ask one demonstrator to help), and find the ration of line voltage to phase voltage.



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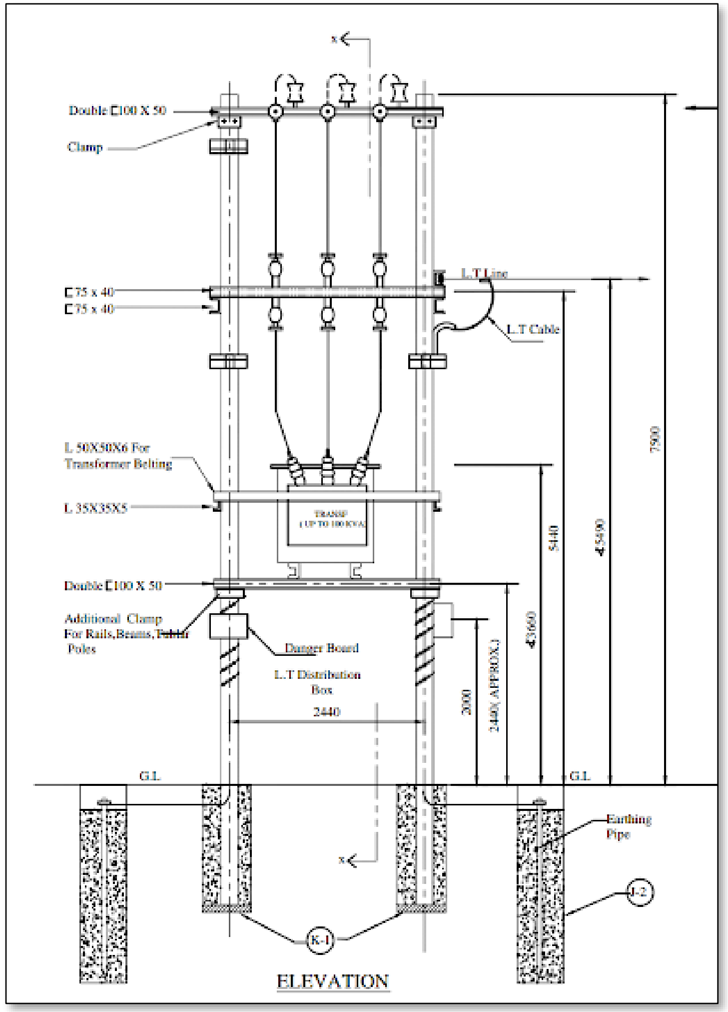
Transmission lines, motorways of the power systems

Image: National Grid

Three phase transmission lines:

11 kV overhead distribution line

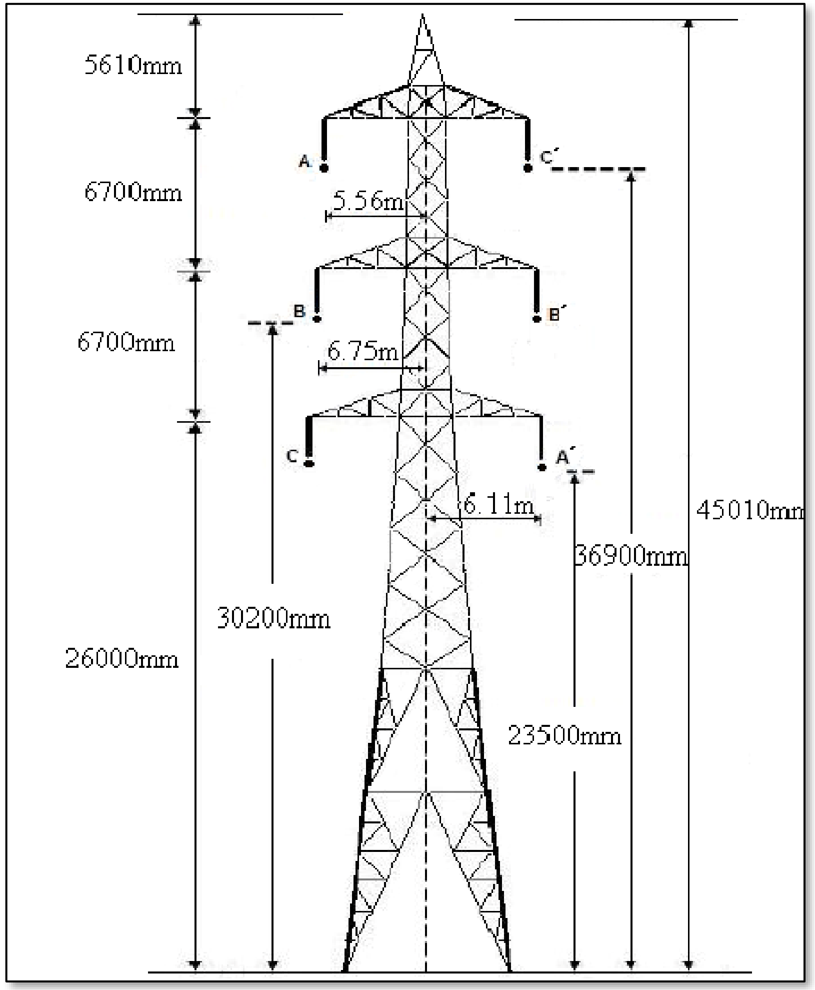
Post-mounted distribution transformer 11kV/400 V



Schematic diagram

Three phase transmission lines:

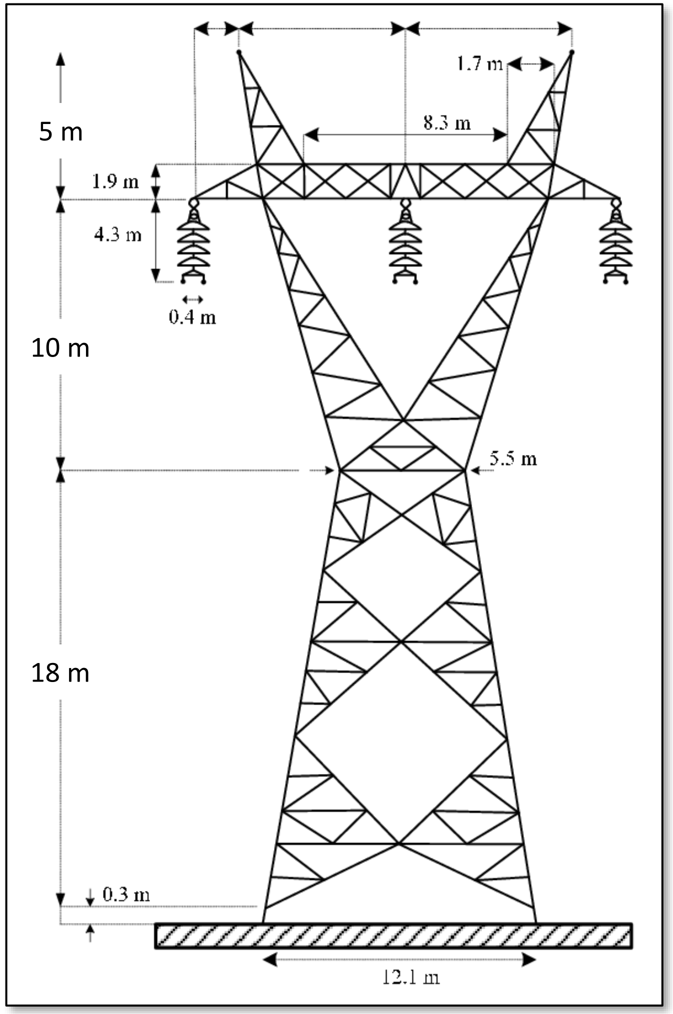
132 kV Double Circuit Transmission Line



132 KV Double Circuit Pylon

Three phase transmission lines:

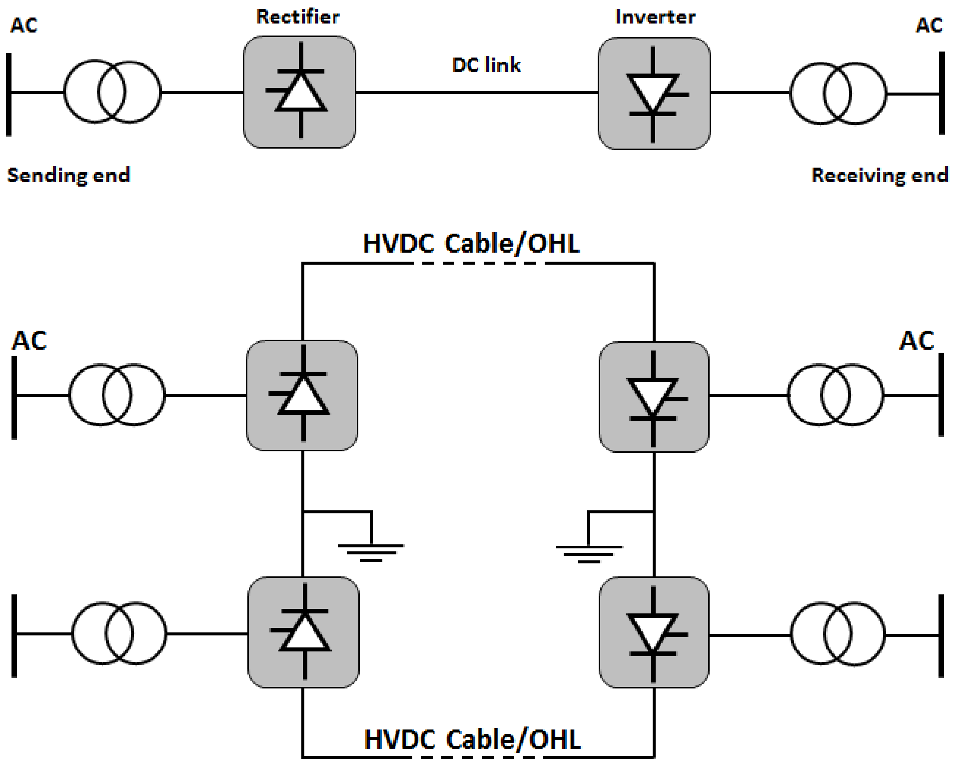
400 kV Single Circuit Transmission Line



400 KV Single Circuit Pylon

Three phase transmission lines:

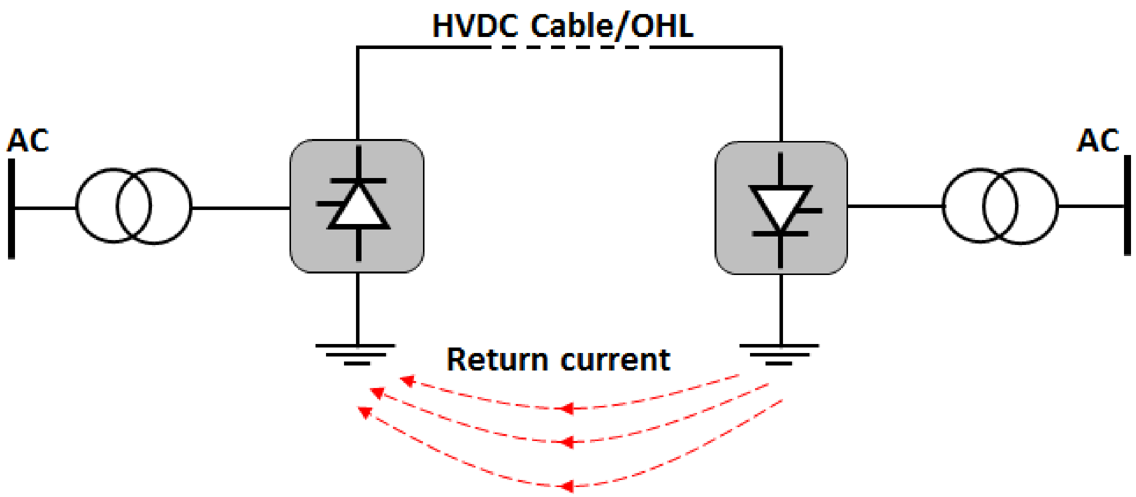
±800 kV HVDC transmission line, Nidhura to Agra, India



- HVDC systems are more advantageous for bulk power transmission over long distances than their counterpart HVAC systems.
- HVDC systems are found around the world, in the USA, Canada, Brazil, China and India.

Three phase transmission lines:

±533 KV Cahora-Bassa HVDC transmission line
Apollo HVDC converter station, in south Africa



Monopole HVDC Configuration:

In some applications, one HV conductor is installed as an OHL and the ground or sea is implemented as a conductive path for the current return path

Three phase systems:

Example:

The measuring instruments at a distribution substation shows the following for one of the transmission lines:

Line voltage: $V_L = 11\ kV$

Active power: $P = 450\ kW$

Power factor: $\cos\varphi = 0.87\ lagging$

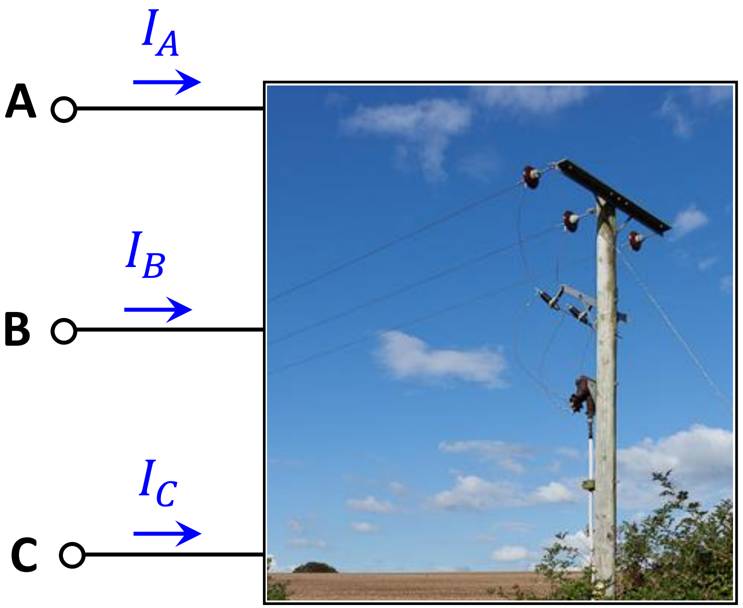
(a) Find the line currents I_A, I_B and I_C

(b) Find the reactive power of the transmission line

(c) Considering a λ connected load, find the phase impedance of the load.

Three phase systems:

Solution:



$V_L = 11\ kV$

$P = 450\ kW$

$\cos\varphi = 0.87\ lag$

(a)

$$P = \sqrt{3}\ V_L\ I_L \cos\varphi$$

$$I_L = \frac{P}{\sqrt{3}\ V_L\ \cos\varphi} = \frac{450 \times 10^3}{\sqrt{3}\ \times 11 \times 10^3 \times 0.87}$$

$$I_L = 27.15\ A$$

$\cos(\varphi) = 0.87$ ⟹ $\varphi = 29.54°$

Considering a positive (*abc*) phase sequence:

$$\begin{cases} I_{LA} = 27.15 < -29.54° \quad A \\ I_{LB} = 27.15 < -149.54° \quad A \\ I_{LC} = 27.15 < 90.46° \quad A \end{cases}$$

Three phase systems:

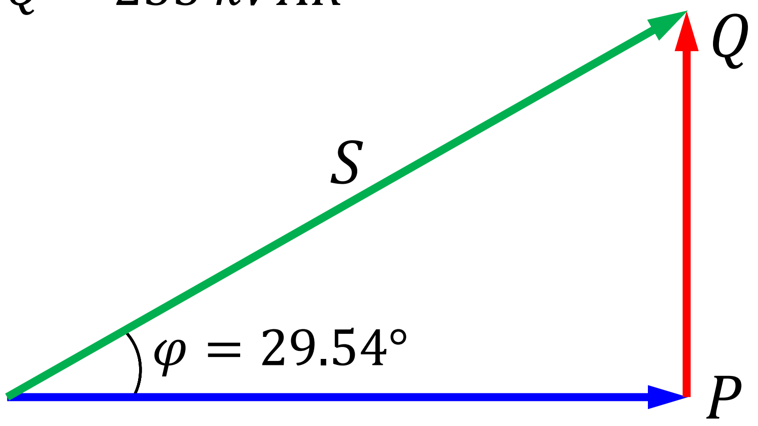
Solution:

(b)

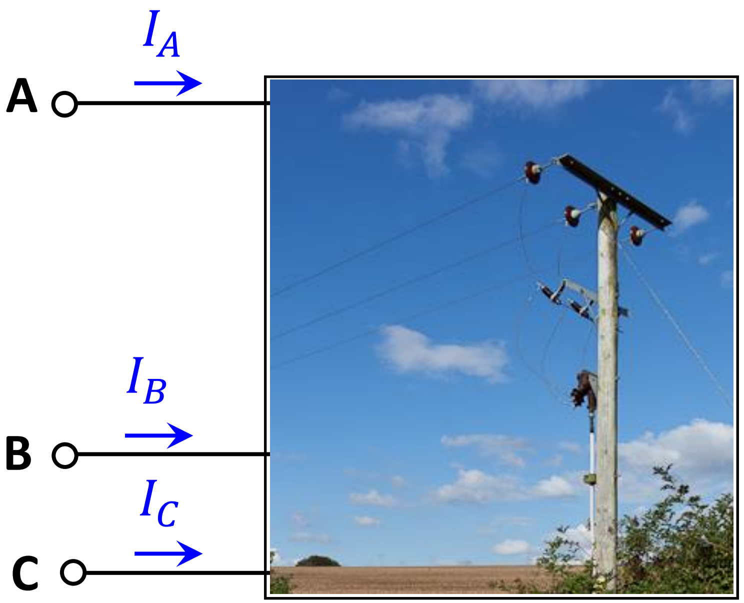
$$Q = \sqrt{3}\ V_L\ I_L\ sin\,\varphi$$

$$Q = \sqrt{3}\ \times 11 \times 27.15\, sin(29.54)$$

$$Q = 255\ kVAR$$



(c)



We know in λ connection and *abc* phase sequence :

$$V_p = \frac{V_L}{\sqrt{3}} = 6.35\ kV \quad V_{AN} = 6.35 < 0\ \ kV$$

$$Z_A = \frac{V_{AN}}{I_A} = \frac{6.35 < 0}{27.15 < -29.54°} = 233 < 29.54°\ \Omega$$

Three phase systems:

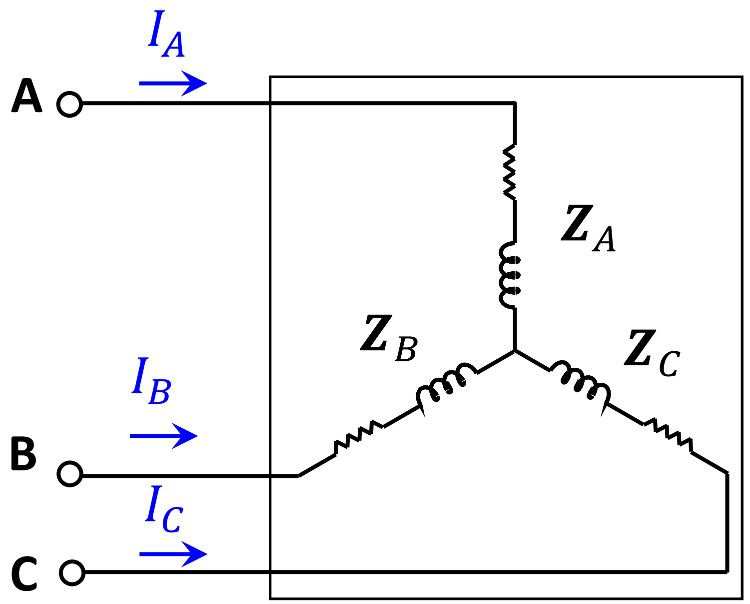
Solution:

$$Z_A = 233 < 29.54° \, \Omega$$

$$Z_A = 202.71 + j114.8 \, \Omega$$

We know for balanced loads:

$$Z_A = Z_B = Z_C$$



$V_L = 11 \, kV$

$V_p = 6.35 \, kV$

$I_L = 27.15 \, A$

$P = 450 \, kW$

$Q = 255 \, kVAR$

$\cos\varphi = 0.87 \, lag$

$Z_A = 202.71 + j114.8 \, \Omega$

Drill:

Repeat the last example for a Δ connected load (with everything else unchanged).

(a) Find the line currents I_A, I_B and I_C

(b) Find the phase currents I_{AB}, I_{BC} and I_{CA}

(c) Find the phase impedance of the load.

Answer:

$I_L = 27.15\ A$ $\qquad$ $I_{ph} = 15.67\ A$

$Z_A = 608.2 + j344.5\ \Omega$

Plastic's Carbon Footprint:

Plastic Has a Big Carbon Footprint:

Plastic was invented around 70 years ago! At the time people were happy to use it in an any form and shape, because it is flexible and water proof.

But no one thought about its destructive impacts on the environment!

Plastic's Carbon Footprint:

PLEASE, Reduce your plastic waste.

PLEASE, Recycle, as much as you can.

A small step in the right direction can always make an impact!

Module description:

Recommended text books:

- DeCarlo Lin, “*Linear Circuit Analysis*”, Oxford University Press, Second Edition, 2003
- W H Hayt, J E Kemmerly, S M Durbin, “*Engineering Circuit Analysis*”, McGraw-Hill, 9th Edition, 2019