Electrical Engineering II

ENGI2191

Modelling in three phase systems, Part I

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E108

Lecture Objectives:

Aim of this lecture:

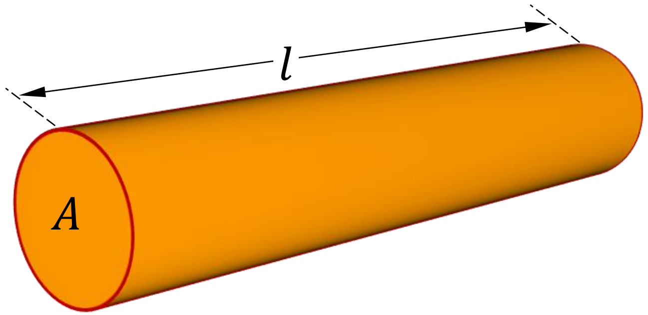
The aim of this lecture is to understand the concepts of modelling in three-phase systems and per-phase equivalent circuit in network analysis.

Intended Learning Outcomes:

At the completion of the lecture and associated problems you should be able to:

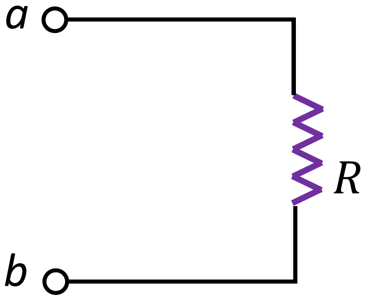
- Identify basic equivalent circuit of different parts of a three-phase system.
- Analyse three-phase balanced systems based on the per-phase equivalent circuit.
- Calculate power loss of three-phase transmission lines.

Ideal Electric Elements:

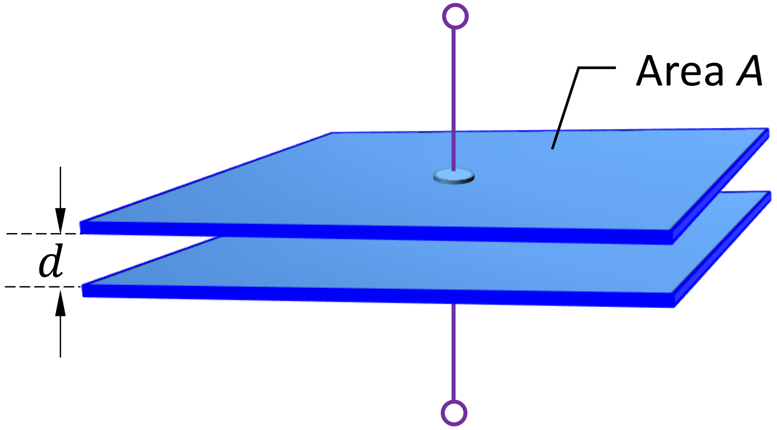


$$R = \rho \frac{l}{A}$$

$$V = R\,I$$

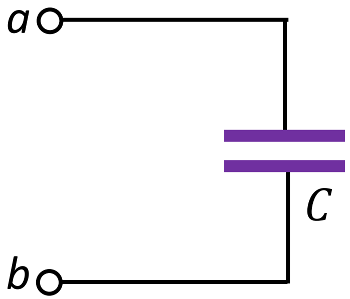


Ideal resistor

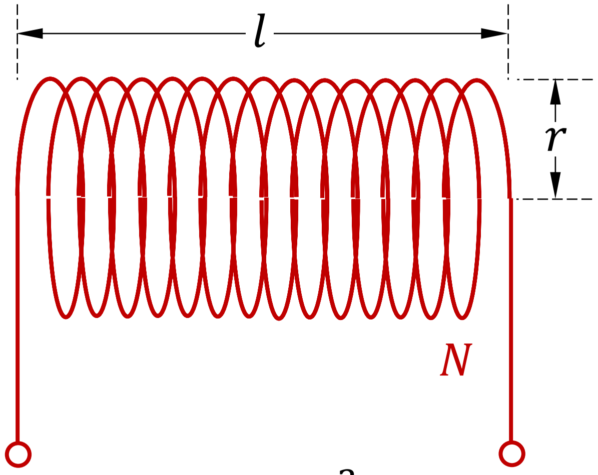


$$C = \varepsilon_0 \frac{A}{d}$$

$$q = C\,V$$

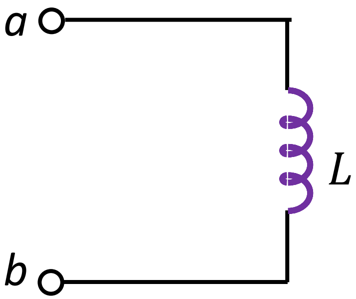


Ideal capacitor



$$L = \mu_0 \frac{N^2 A}{l}$$

$$\Phi_B = LI$$

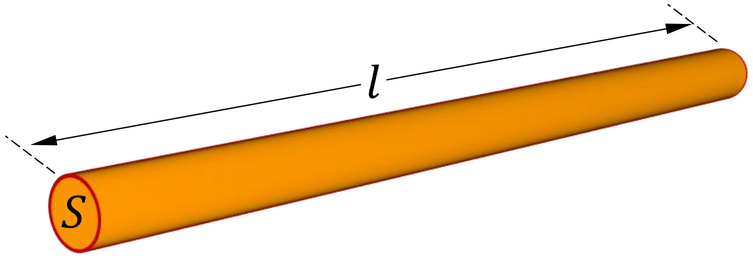


Ideal inductor

Resistance R, capacitance C and self-inductance L are purely geometric.

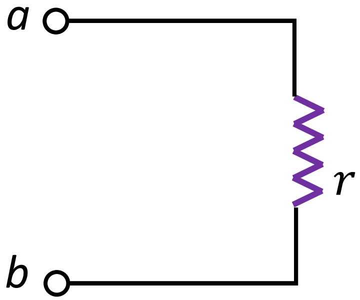
Three phase systems (modelling):

Principle of modelling in electrical engineering:

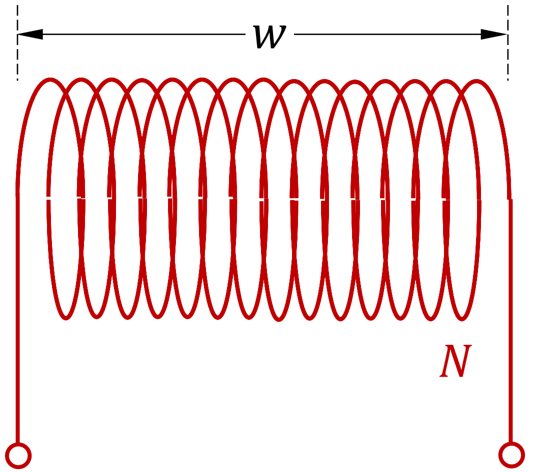


$$r = \rho \frac{l}{S}$$

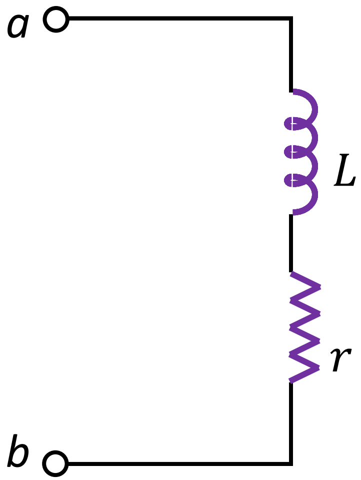
$$\rho = 1.77 \times 10^{-8}\ \Omega.m$$



$r \ll$



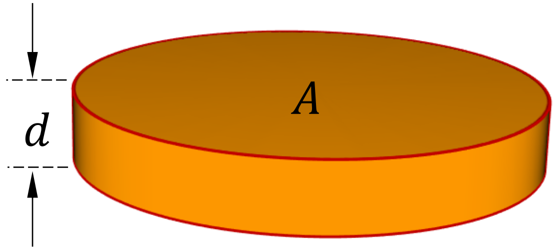
$$\begin{cases} L = \mu_0 \dfrac{N^2 A}{w} \\ r = \rho \dfrac{l}{S} \end{cases}$$



A *practical* inductor can be modelled as a series combination of an *ideal* inductor and an *ideal* resistor.

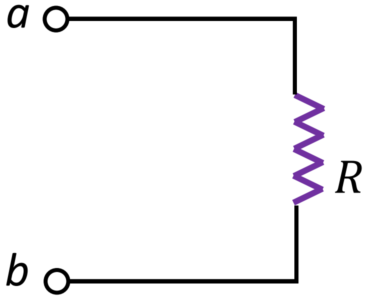
Three phase systems (modelling):

Principle of modelling in electrical engineering:

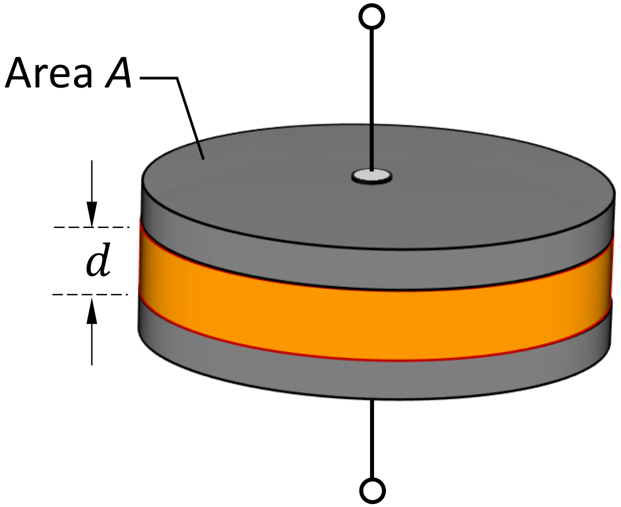


$$R = \rho \frac{d}{A}$$

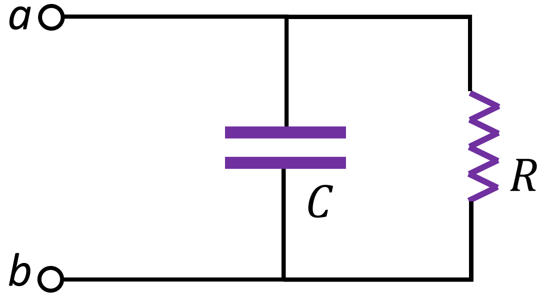
$$\rho = 10^{10} - 10^{19} \Omega.m$$



$R \gg$

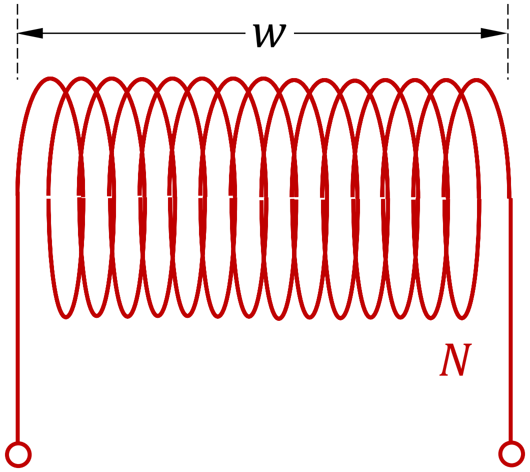


$$\begin{cases} C = \varepsilon_0 \varepsilon_r \dfrac{A}{d} \\ R = \rho \dfrac{d}{A} \end{cases}$$

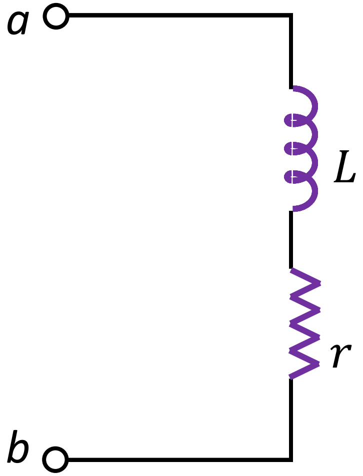


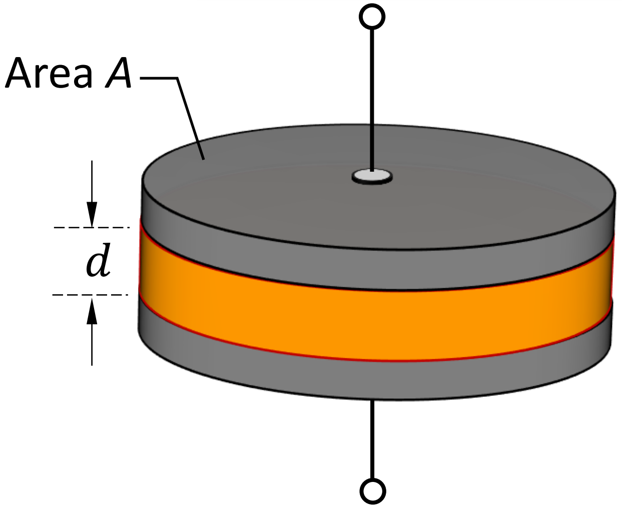
A *practical* capacitor can be modelled as a parallel combination of an *ideal* capacitor and an *ideal* resistor.

Three phase systems (modelling):



$$\begin{cases} L = \mu_0 \dfrac{N^2 A}{w} \\ r = \rho \dfrac{l}{S} \end{cases}$$





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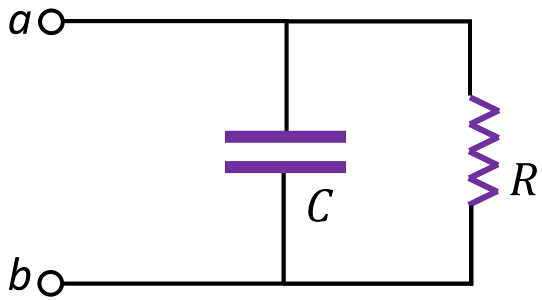
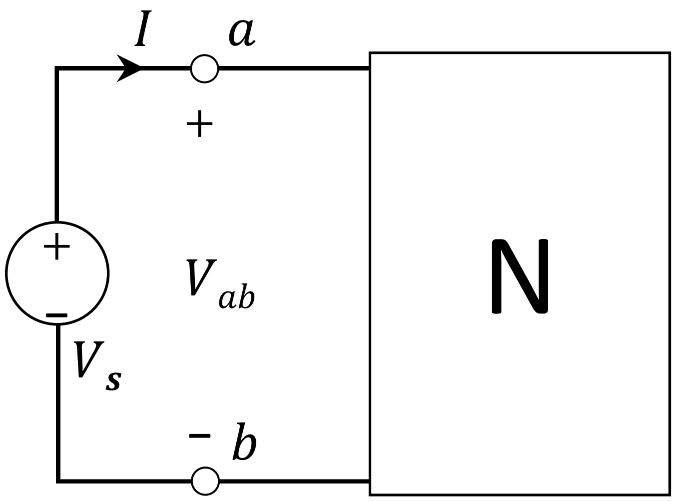
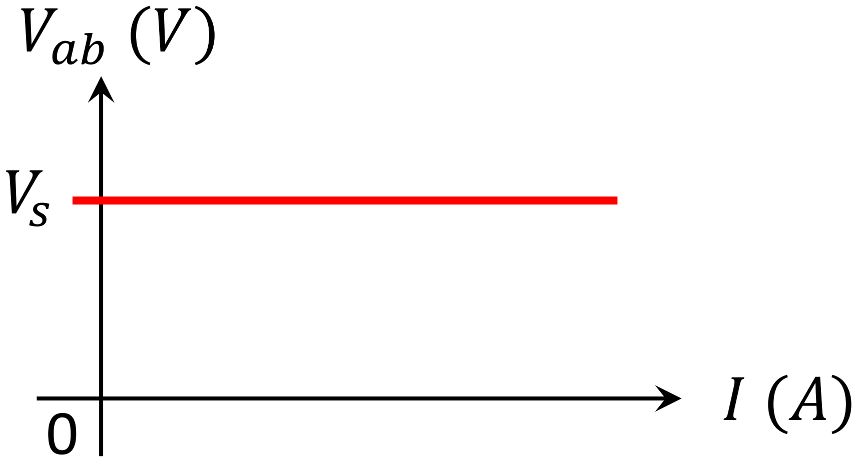


This is the first step of modelling in electrical engineering systems.

Three phase systems (modelling):

Ideal voltage source:

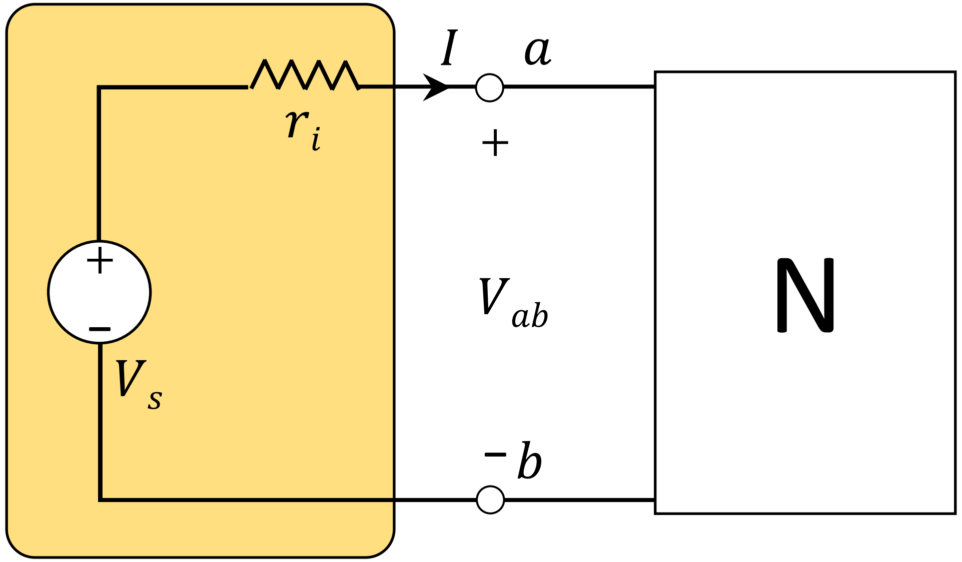


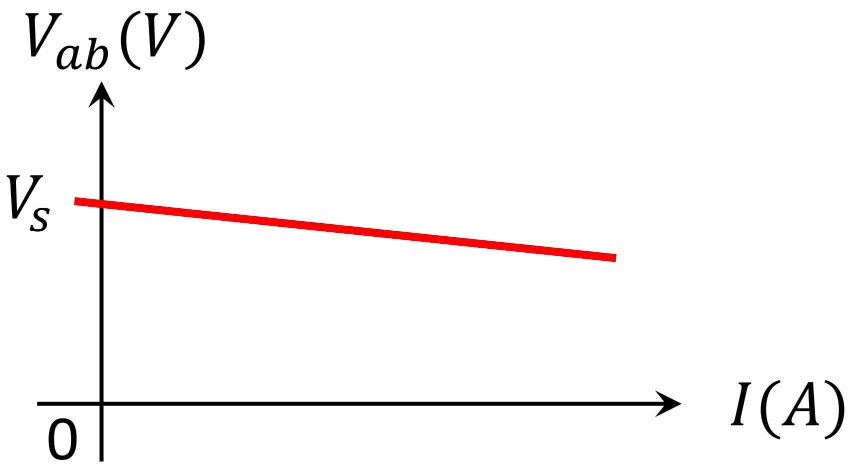


V_{ab} = constant, no matter what the load current is.

Three phase systems (modelling):

Real voltage source:



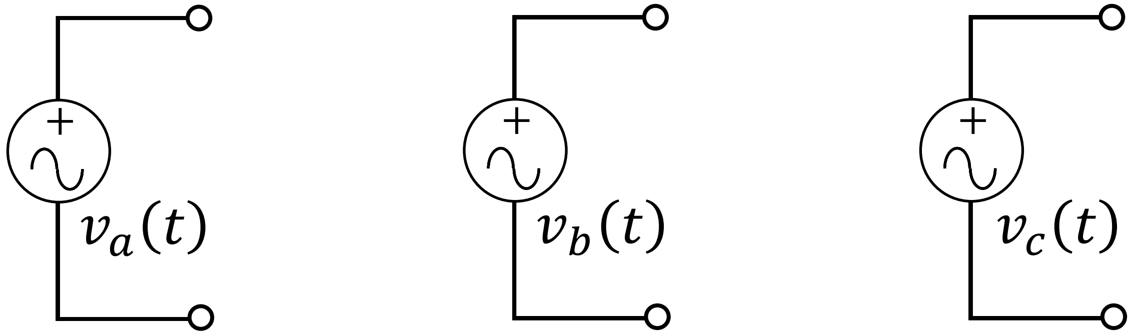


$$V_{ab} = V_s - r_i I$$

- r_i represents the internal resistance of the voltage source.
- A practical voltage source has a very low internal resistance; often much less than 1 Ω.

Three phase voltage source:

A three-phase voltage source consists of three independent single phase sinusoidal voltage sources with the <u>same frequency</u> and <u>amplitude</u>, but with <u>phase angles separated by 120°</u>.



$$v_a(t) = V_m\, Sin\,(\omega t)$$

$$v_b(t) = V_m\, Sin\,(\omega t - 120°)$$

$$v_c(t) = V_m\, Sin\,(\omega t + 120°)$$

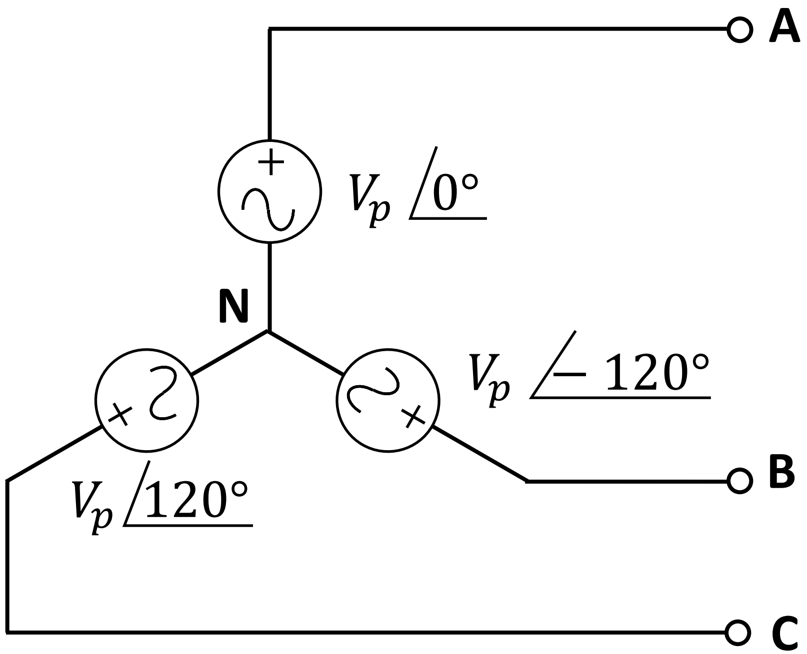
$$\boldsymbol{V}_a = V_m < 0°$$

$$\boldsymbol{V}_b = V_m < -120°$$

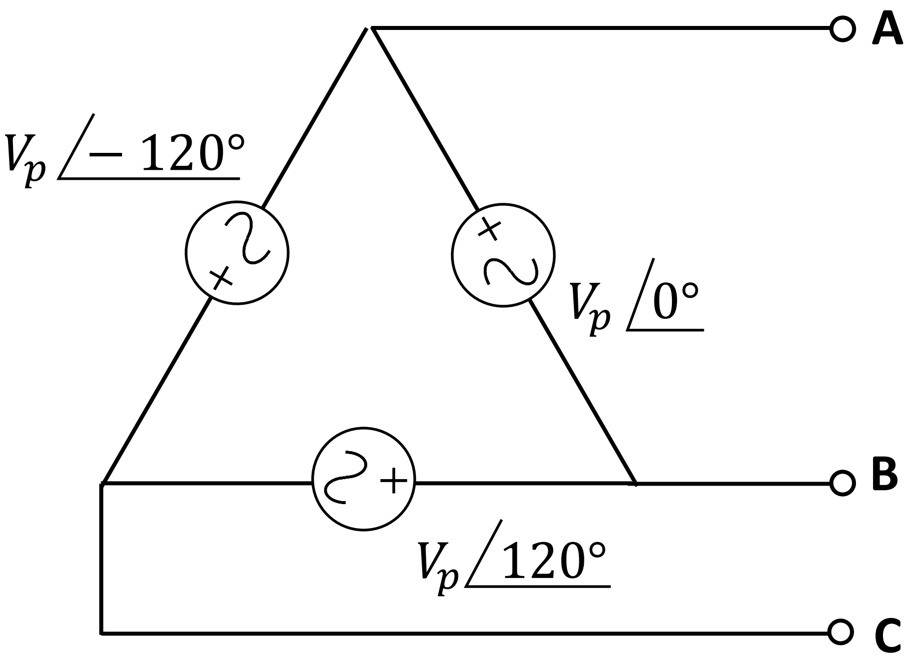
$$\boldsymbol{V}_c = V_m < 120°$$

Three phase systems (modelling):

Three phase voltage source:



Y, Wye or star connected three-phase voltage source



Δ connected three-phase voltage source

These are ideal three phase voltage sources (generators).

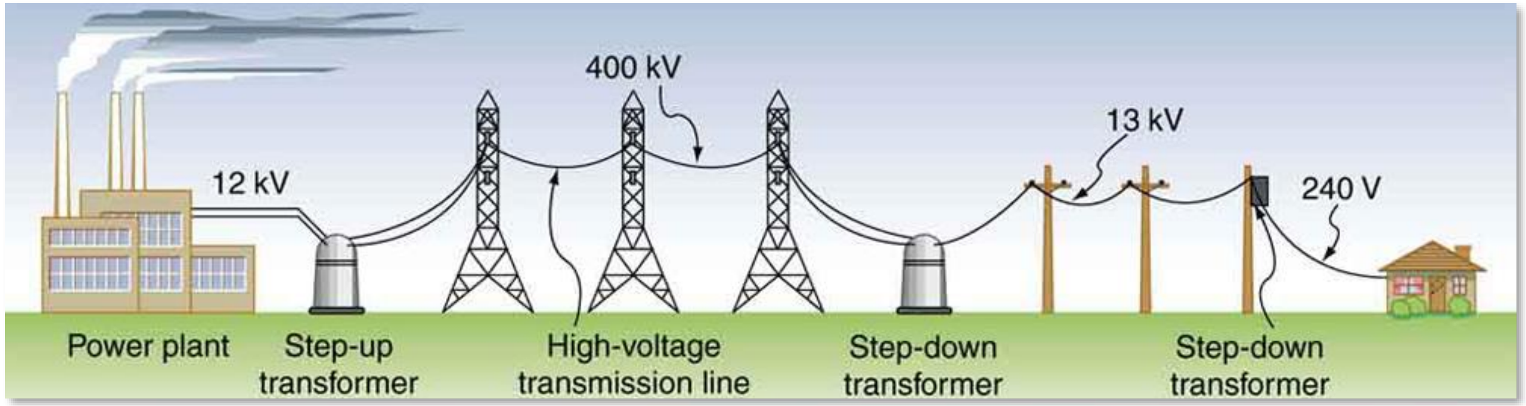
Modelling of three phase system

Three phase systems (modelling):

www.bchydro.com

A power system consists of:

- Power station
- HV substation
- Transmission line
- Major distribution network
- Distribution network



Three phase systems (modelling):

Electrical energy is generated in power stations, or power plant.

Thermal power station

Hydro power station

Three phase systems (modelling):

Electrical energy is produced in power stations, or power plant.


Wind energy

Solar energy


Three phase systems (modelling):

The generated electricity needs to be transferred to the load centres. e.g cities, town, etc.

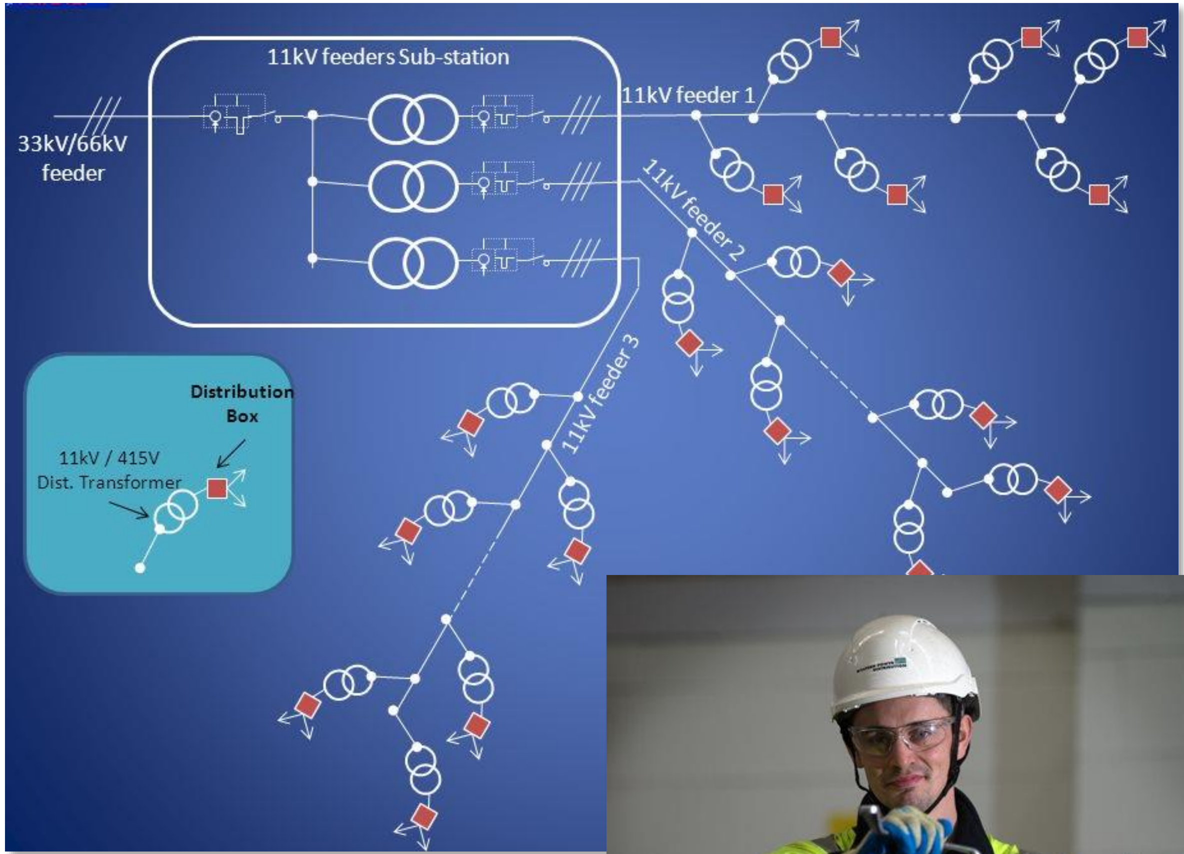
This is done by HV substations, and transmission lines.

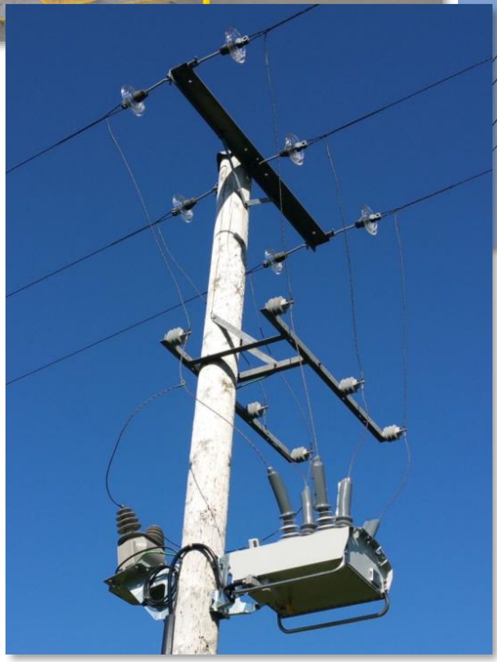





Three phase systems (modelling):

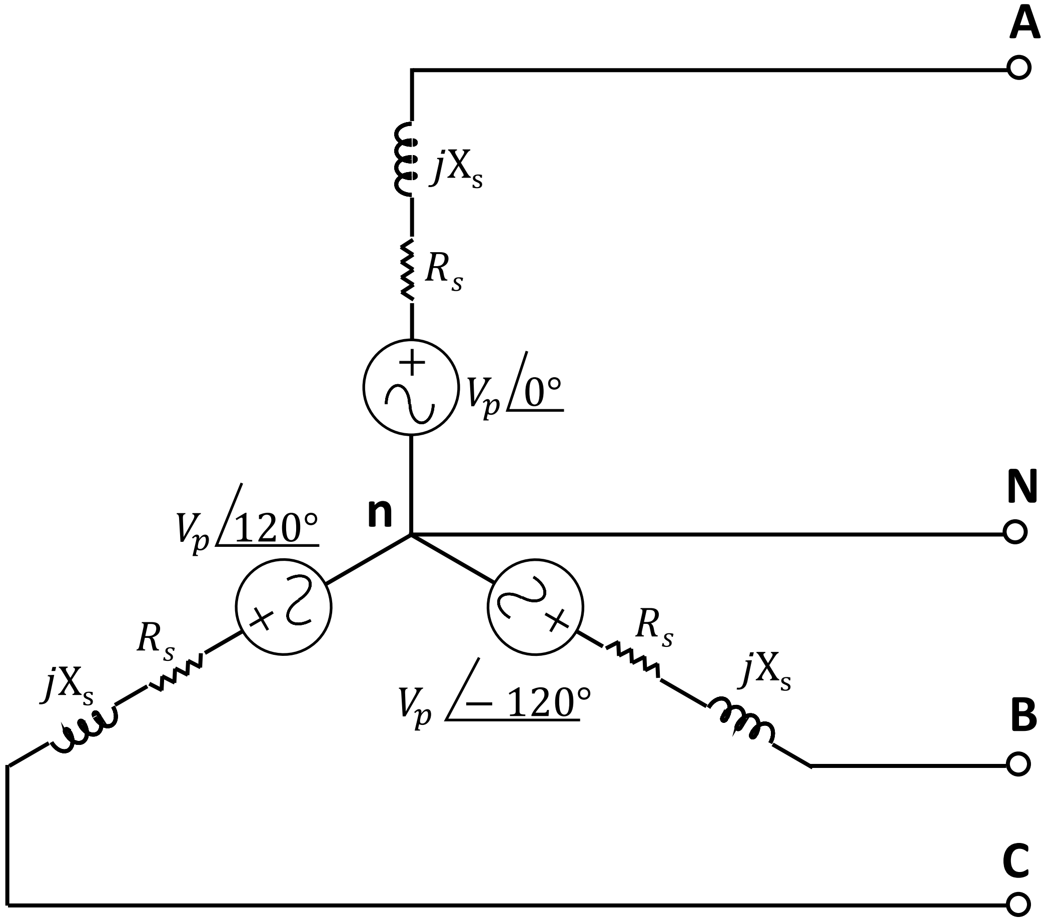
At the last stage, power is distributed to different parts of the city, town, etc.

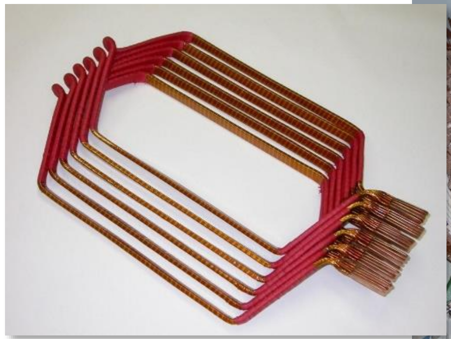




Three phase systems (modelling):

Equivalent circuit of practical three-phase systems (internal impedance of the power supply):

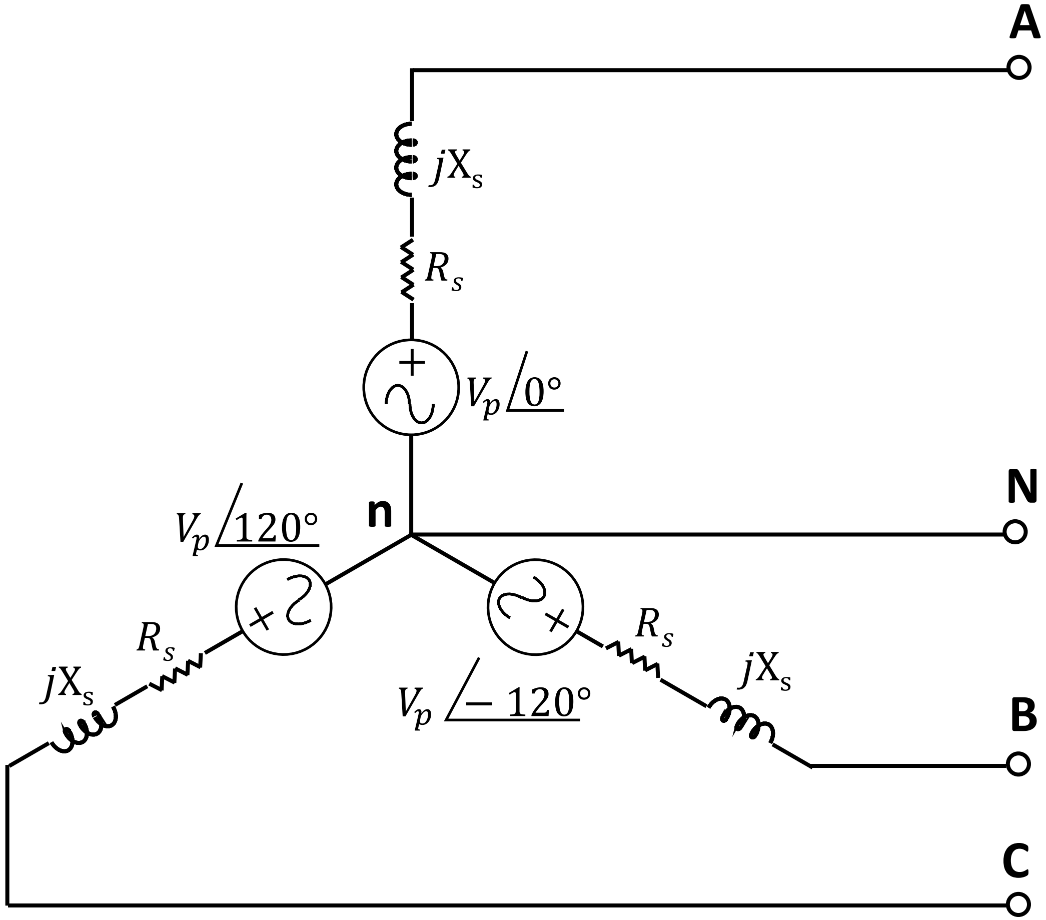


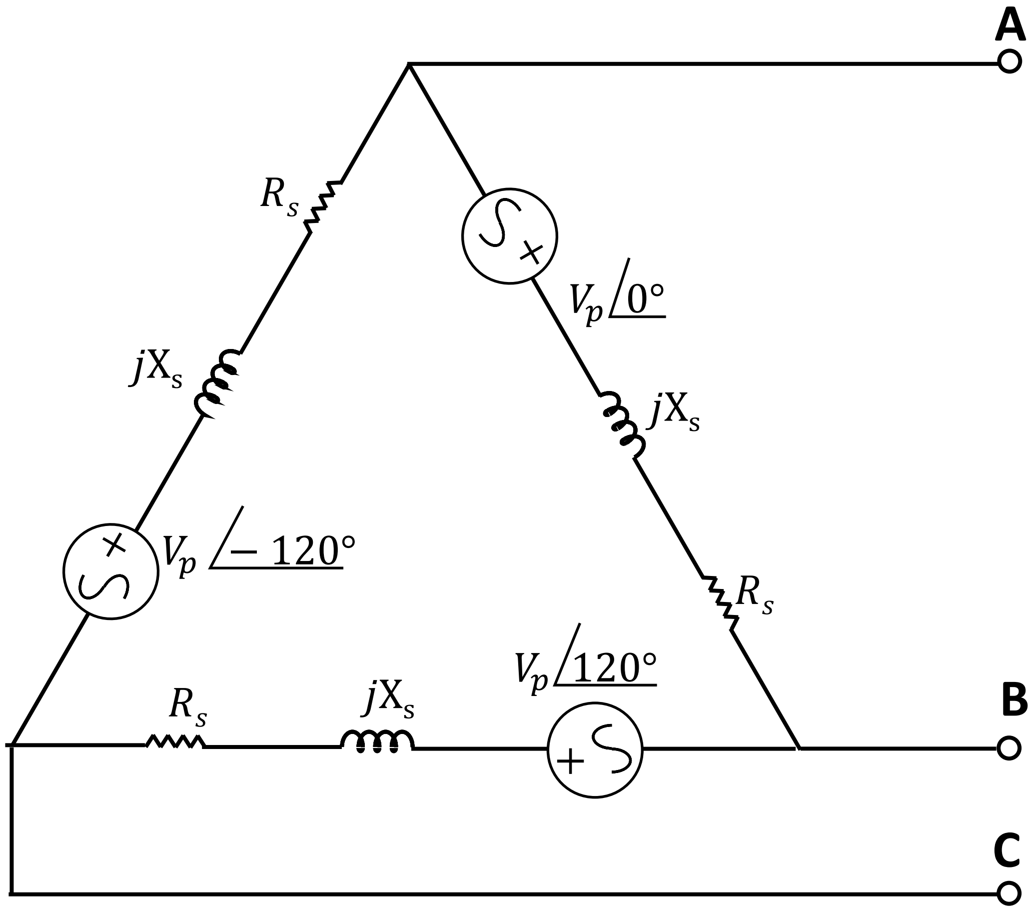


$R_s + jX_s$ is internal impedance of the voltage source (generator).

Three phase systems (modelling):

Equivalent circuit of practical three-phase systems (internal impedance of the power supply):





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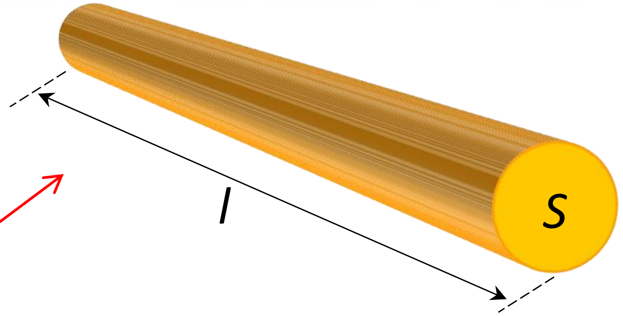
Three phase systems (modelling):



Electric resistance of any material is given by:

$$R = \rho \frac{l}{s}$$

R: Resistance of the material in Ω

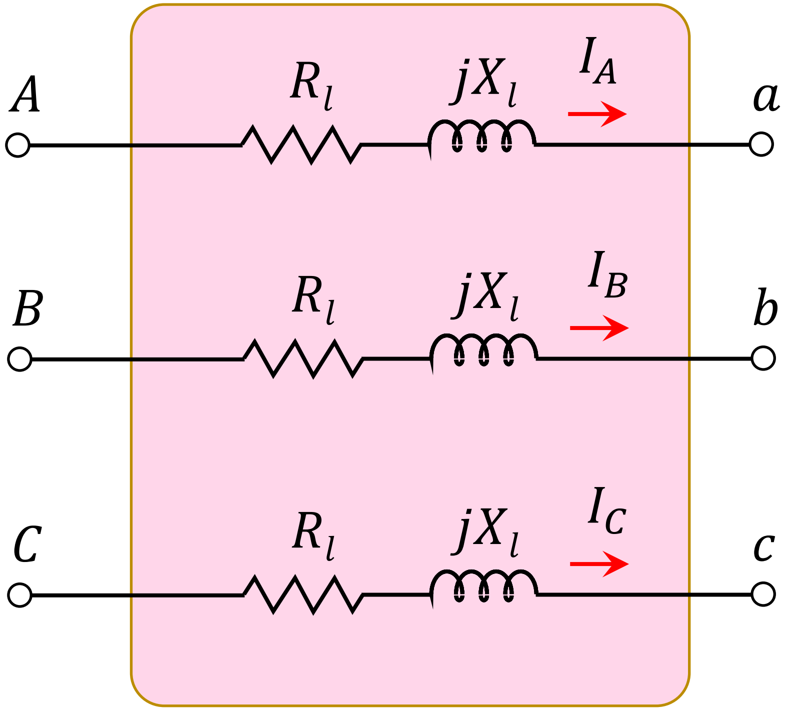
l: Length in m

s: cross sectional area in m^2

ρ: Resistivity of the material in Ω.m

- Transmission lines are usually characterised based on the resistance in Ω/km.
- Transmission lines have also inductive behaviour, which can be modelled by equivalent inductance in H/km.

Three phase systems (modelling):

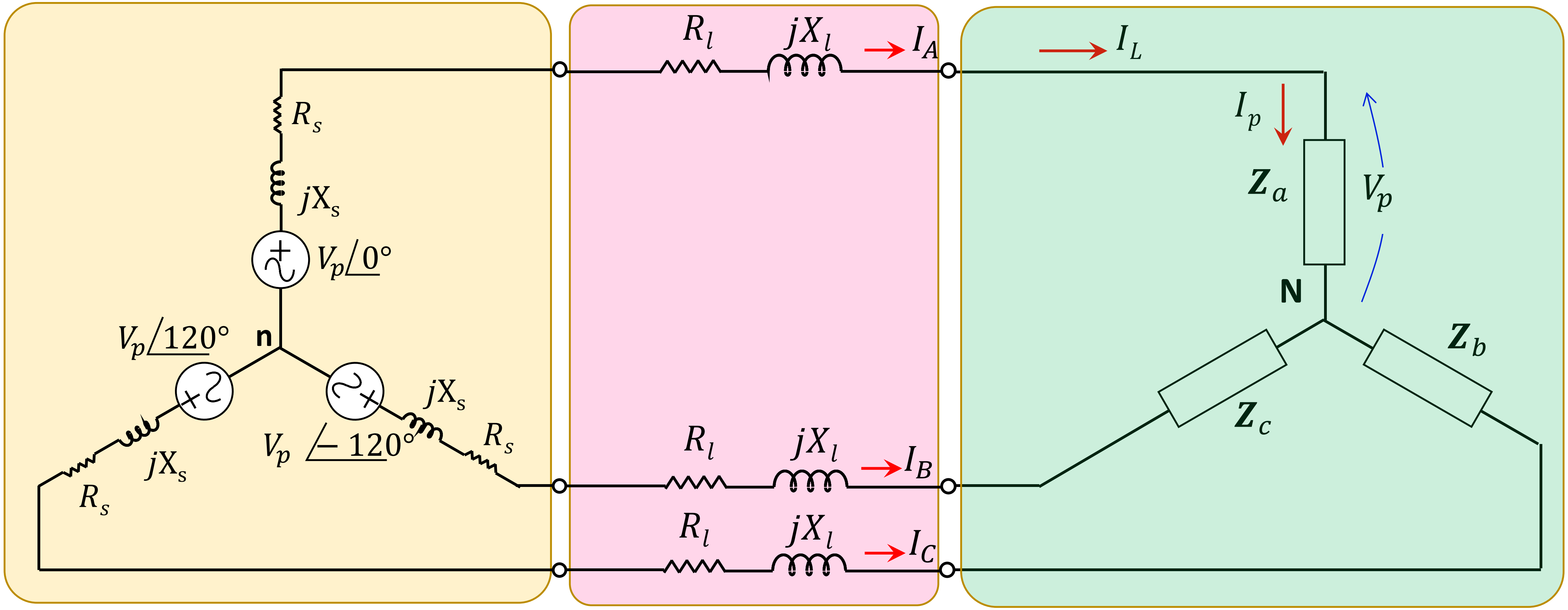


Model of the transmission line

$R_l + jX_l$ is the transmission line impedance.

Three phase systems (modelling):

Equivalent circuit of practical three-phase systems (impedance of the transmission line):



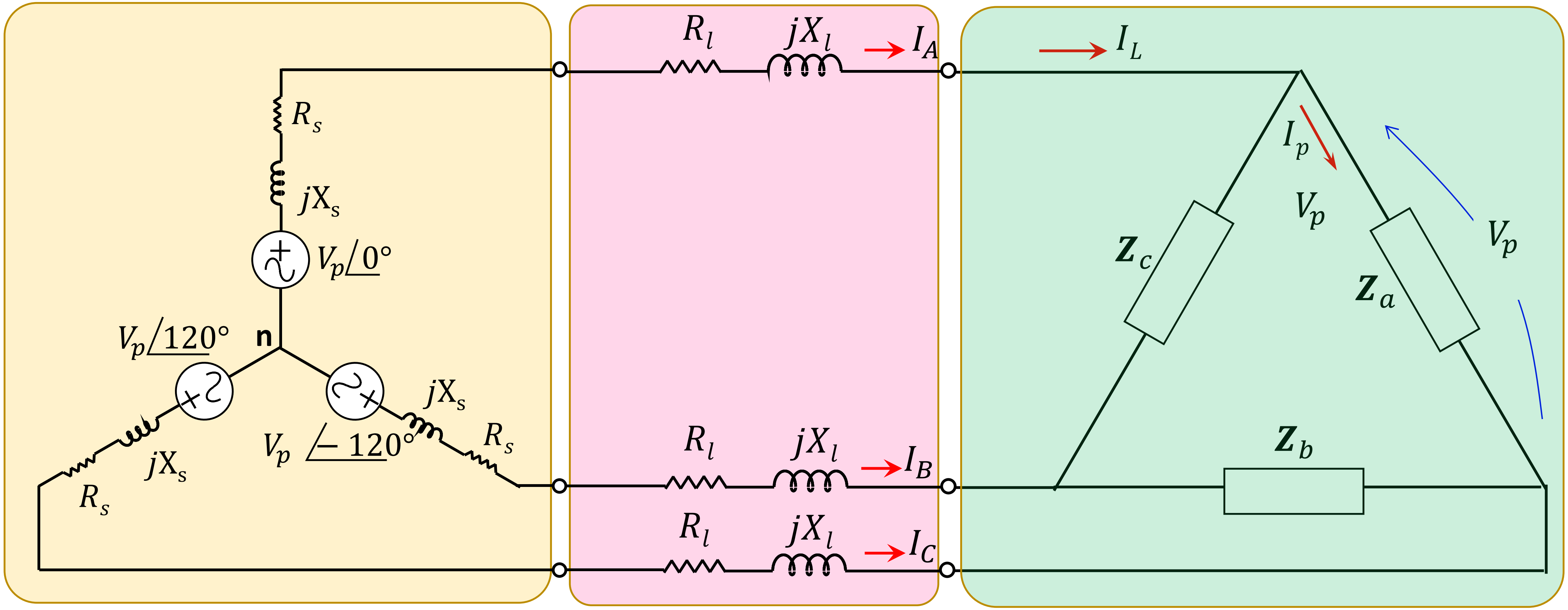
Model of the 3-phase power supply

Model of the transmission line

Model of the 3-phase load

Three phase systems (modelling):

Equivalent circuit of practical three-phase systems (impedance of the transmission line):



Model of the 3-phase power supply

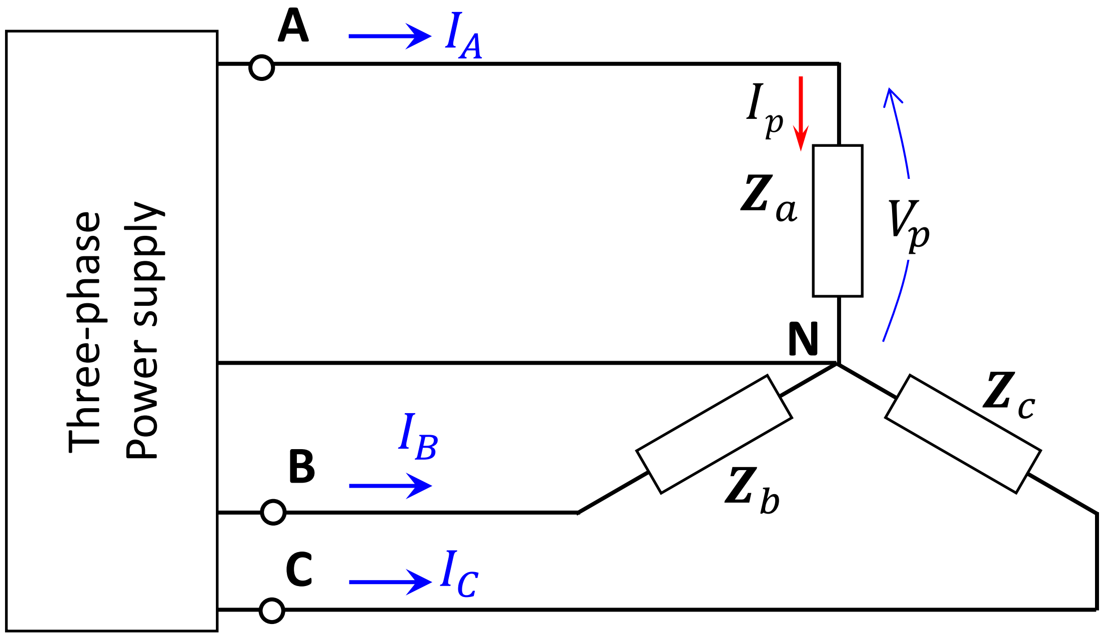
Model of the transmission line

Model of the 3-phase load

Three phase systems (modelling):

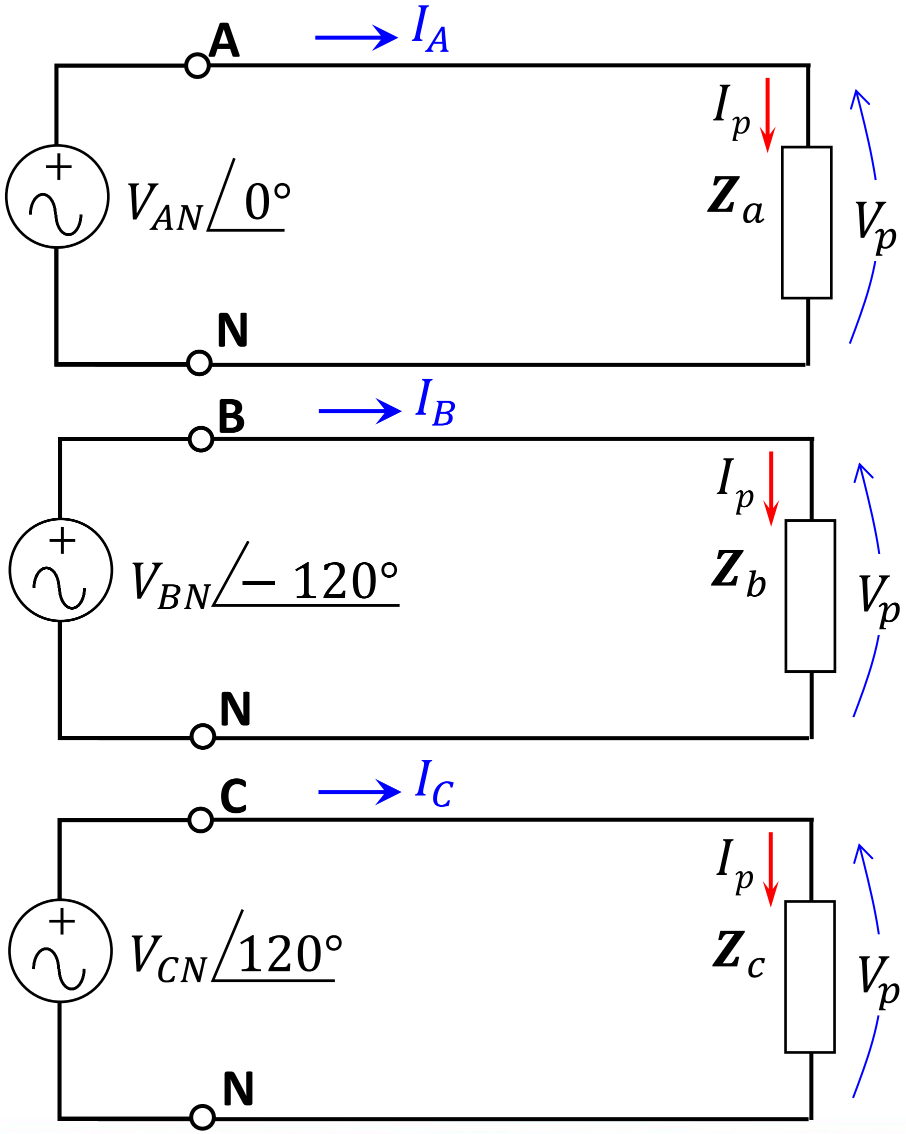
Per-phase equivalent circuit for three phase,

Y-connected **balanced load**



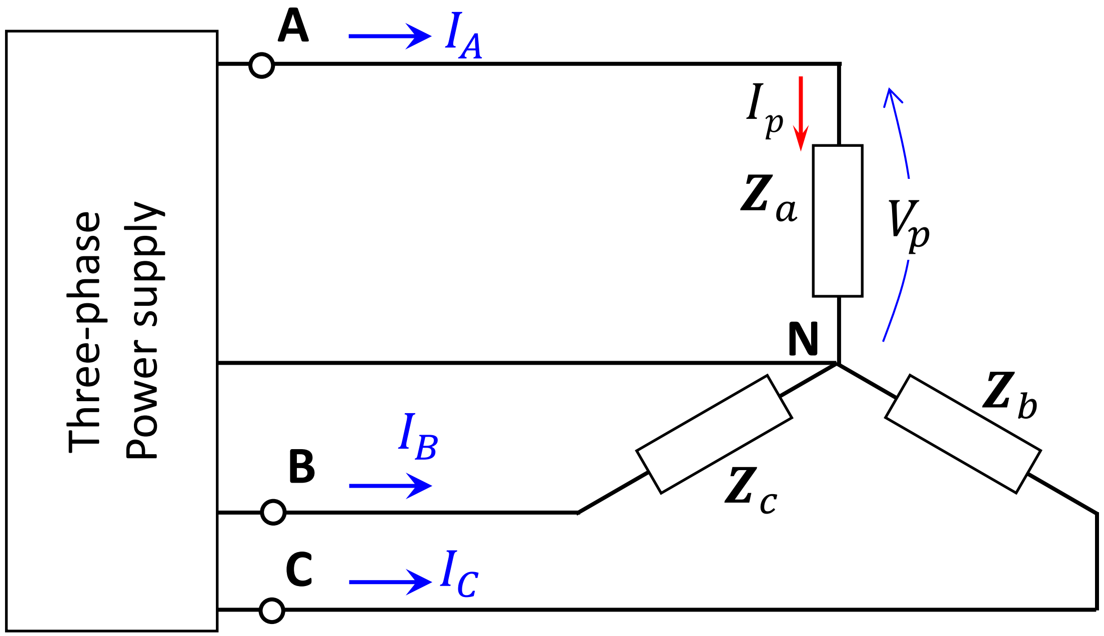
This system can be replaced by three identical independent single phase systems, joined at node N.

Per-phase equivalent circuits

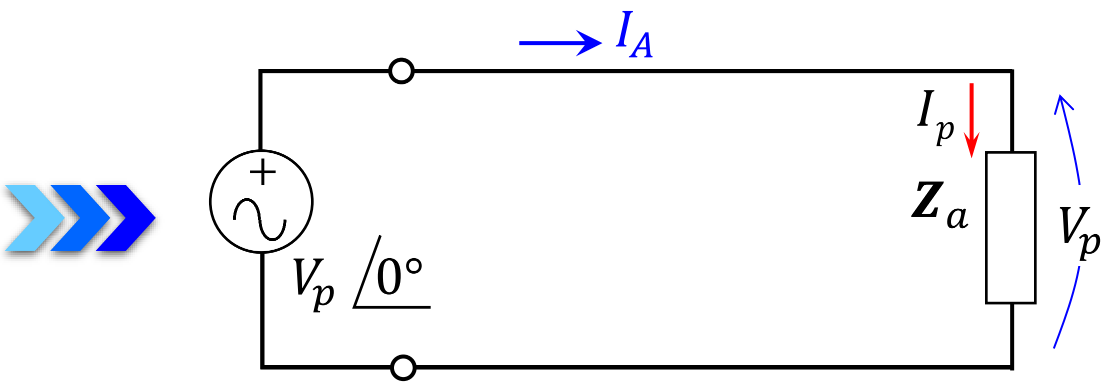


Three phase systems (modelling):

Therefore, analysing phase A leads to knowledge of phases B and C.



3-phase, Y-connected balanced load



Per-phase equivalent circuit

Three phase systems (modelling):

Example:

Consider a Y-connected three phase voltage source (generator) with phase voltage of $120\ V$, *abc* sequence and internal impedance of $Z_g = (0.05 + j0.15)\ \Omega$. A three phase Y-connected balanced load with phase impedance of $Z_L = (4 + j3)\ \Omega$ is connected to this power supply, through a transmission line with per-phase impedance of $Z_l = (0.1 + j0.2)\ \Omega$.

a) Draw the equivalent circuit of this system.

b) Find the line currents.

c) Find voltage across each load impedance.

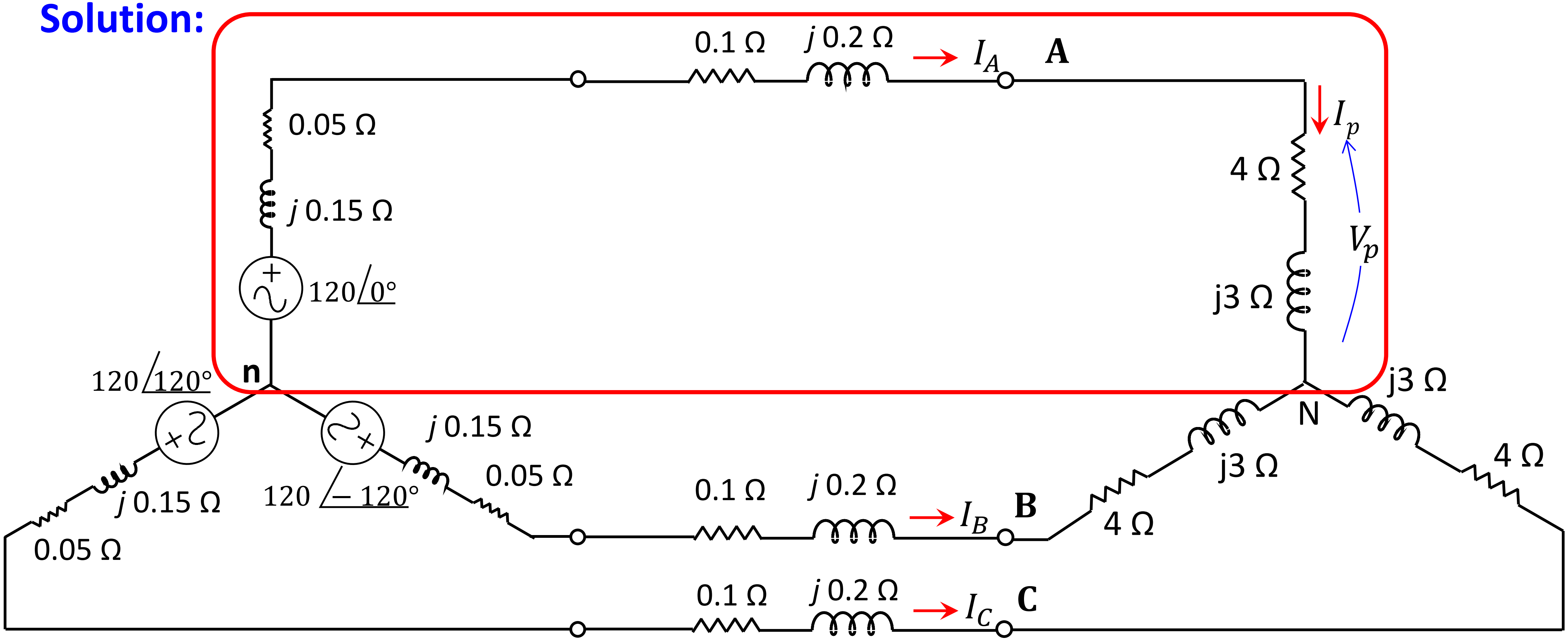
d) Find the power delivered to the load.

e) Find power supplied by the generator.

f) Find the total power loss of the system.

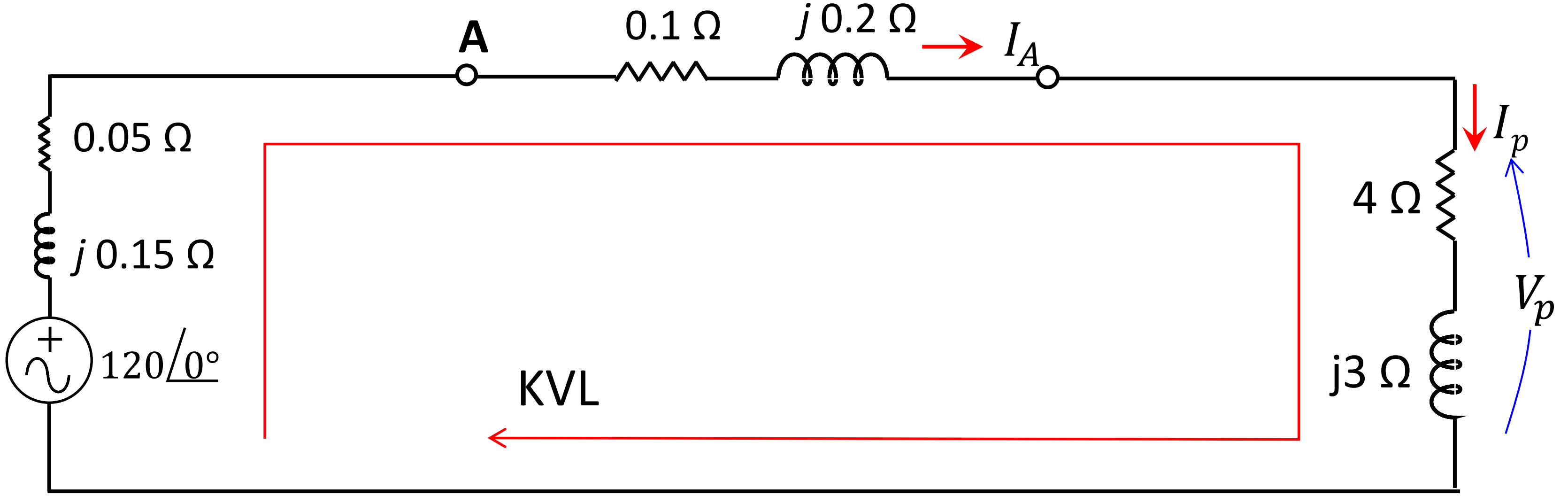
Three phase systems (modelling):

Solution:



Three phase systems (modelling):

Solution:



KVL:

$$-120\angle 0 + (0.05 + j0.15)\boldsymbol{I}_A + (0.1 + j\,0.2)\boldsymbol{I}_A + (4 + j3)\boldsymbol{I}_A = 0$$

$$\boldsymbol{I}_A = \frac{120\angle 0^\circ}{4.15 + j\,3.35} = \frac{120\angle 0^\circ}{5.33\angle 38.9^\circ} = 22.5\angle -38.9^\circ \quad \text{A}$$

Three phase systems (modelling):

Solution:

For a positive phase sequence:

$$\boldsymbol{I}_A = 22.5\angle{-38.9°} \quad \text{A} \qquad \boldsymbol{I}_B = 22.5\angle{-158.9°} \quad \text{A} \qquad \boldsymbol{I}_C = 22.5\angle{81.8°} \quad \text{A}$$

Find voltage across each load impedance.

$$\boldsymbol{V}_{AN} = Z_A \boldsymbol{I}_A = (4 + j\,3)\,22.5\angle{-38.9°} = 112.5\angle{-2.03°} \quad \text{V}$$

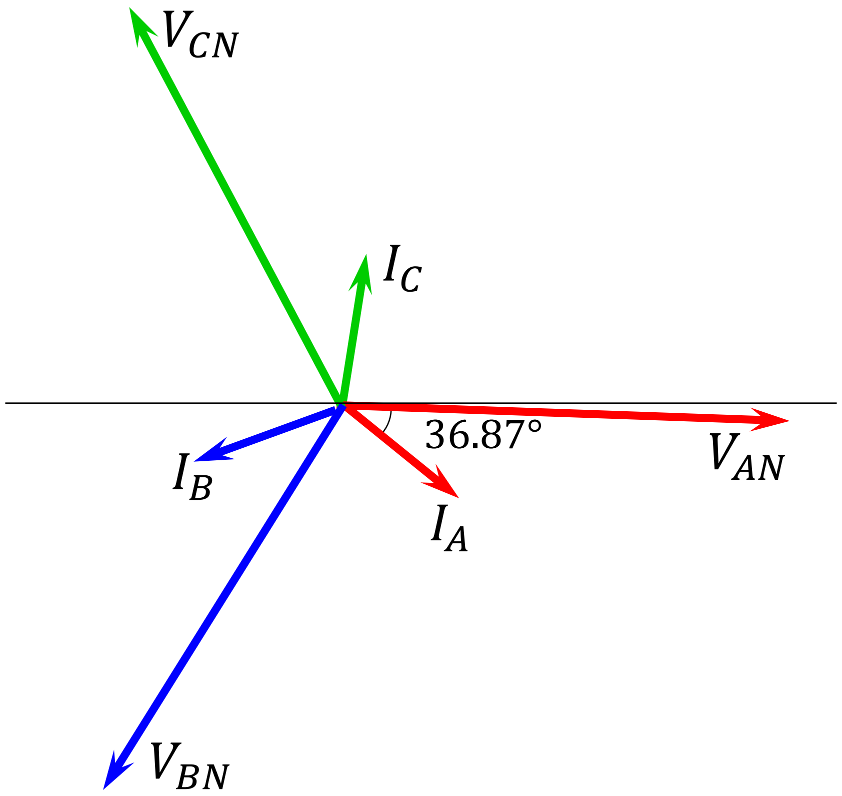
$$\boldsymbol{V}_{BN} = Z_B \boldsymbol{I}_B = (4 + j\,3)\,22.5\angle{-158.9°} = 112.5\angle{-122.03°} \quad \text{V}$$

$$\boldsymbol{V}_{CN} = Z_C \boldsymbol{I}_C = (4 + j\,3)\,22.5\angle{81.8°} = 112.5\angle{117.97°} \quad \text{V}$$

Note: Voltage across the load impedance is less than the source voltage, **WHY**?

Three phase systems (modelling):

Solution:



Total power delivered to the load:

$$P_L = \sqrt{3}\ V_L\, I_L \cos\varphi$$

$$V_L = \sqrt{3} V_{AN} = \sqrt{3}\ 112.5 = 194.85 \quad \mathrm{V}$$

$$\varphi = \varphi_L = \tan^{-1}\frac{3}{4} = 36.87°$$

$$P_L = \sqrt{3}\ 194.85 \times 22.5\ \cos(36.87°) = 6.075\ \mathrm{kW}$$

$$P_L = 3\, R_L\, (I_L)^2 = 3 \times 4 \times (22.5)^2 = \ 6.075\ \mathrm{kW}$$

Three phase systems (modelling):

Solution:

Total power supplied by the generator:

$$P_g = \sqrt{3}\ V_L\, I_L \cos\varphi$$

$$V_L = \sqrt{3}\, V_p\ = \sqrt{3}\ \times 120 = 208\ \ \mathrm{V}$$

In this case, φ is the phase angle of the total impedance seen by the generator:

$$\varphi = 38.9°$$

$$P_g = \sqrt{3}\ \ 208 \times 22.5\ \cos(38.9°)$$

$$P_g = 6.308\ \mathrm{kW}$$

The generator supplies $6.308\ \mathrm{kW}$ active power, but the power delivered to the load is $6.075\ \ \mathrm{kW}$.

Where is the rest of the power?

It is power loss of the system!

Three phase systems (modelling):

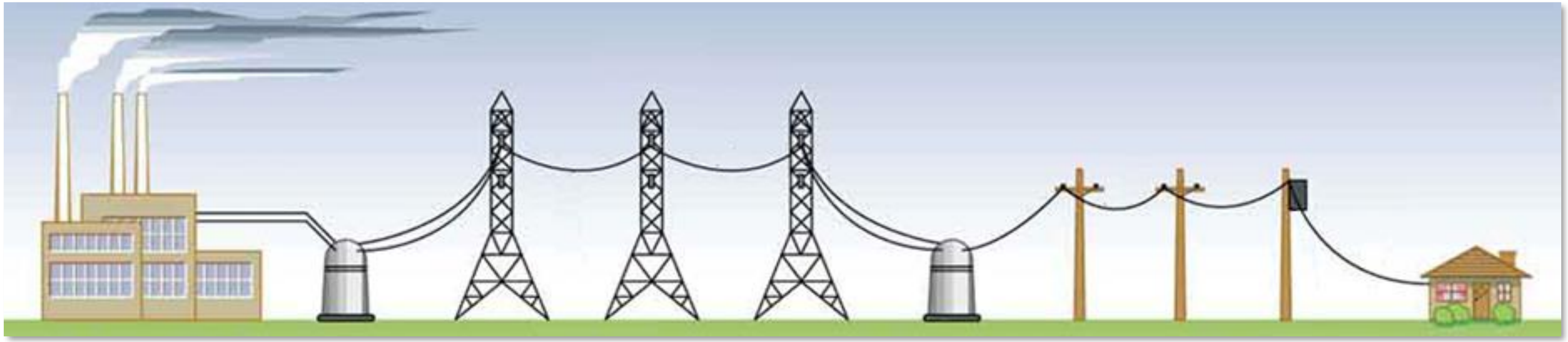
Power loss classification:

It is fact that the amount of electric energy generated by the power station, does not match with the distributed energy to the loads or consumers. Some percentage of the units is lost in the power system.

This difference in the generated and distributed units is known as **Transmission and Distribution loss**.

Transmission and Distribution loss are the amounts that are not paid for by users.

T&D Losses = (Energy Input to feeder (Kwh) – Billed Energy to Consumer (Kwh))



Three phase systems (modelling):

Solution:

For this example, the total power loss is:

$$P_{loss} = P_g - P_L = 6.308 - 6.075 \quad \Rightarrow \quad P_{loss} = 0.233\ kW$$

This power is dissipated in the resistive components of the generator and transmission line:

$$P_{loss} = 3(R_g + R_l)(I_L)^2$$

$$P_{loss} = 3(0.05 + 0.1)(22.5)^2 = 0.233\ kW$$

In power systems, power loss (or energy loss) is calculated in percentage. In this example power loss of the system is 3.69 %

Three phase systems (modelling):

Drill I:

A balanced *abc*-sequence of voltages feeds a balanced three phase $Y - Y$ connection. The line and load impedance are $Z_l = (0.6 + j0.9)\ \Omega$ and $Z_L = (8 + j12)\ \Omega$, respectively. Internal impedance of the source is ignored. Phase voltage at the load terminal on phase *a* is $V_{AN} = 116 < 10°\ V$.

a) Find the line currents I_A, I_B and I_C

b) Find voltage of the source

c) Find the total power supplied by the source

Answer:

$I_A = 8.07 < -46.31°\ A$

$V_{SA} = 125 < 10°\ V$

$P_S = 1.68\ \text{kW}$

Drill II:

A balanced *abc*-sequence of voltages feeds a balanced three phase $Y - Y$ connection. The line and load impedance are $Z_l = (0.6 + j0.2)\ \Omega$ and $Z_L = (16 + j10)\ \Omega$, respectively. Internal impedance of the source is $Z_g = (0.4 + j0.8)\ \Omega$. Total power observed by the load is $30\ kW$.

a) Find the line currents I_A, I_B and I_C

b) Find voltage of the source

c) Find the total power supplied by the source

Answer:

$I_A = 25 < 0°\ \ A$

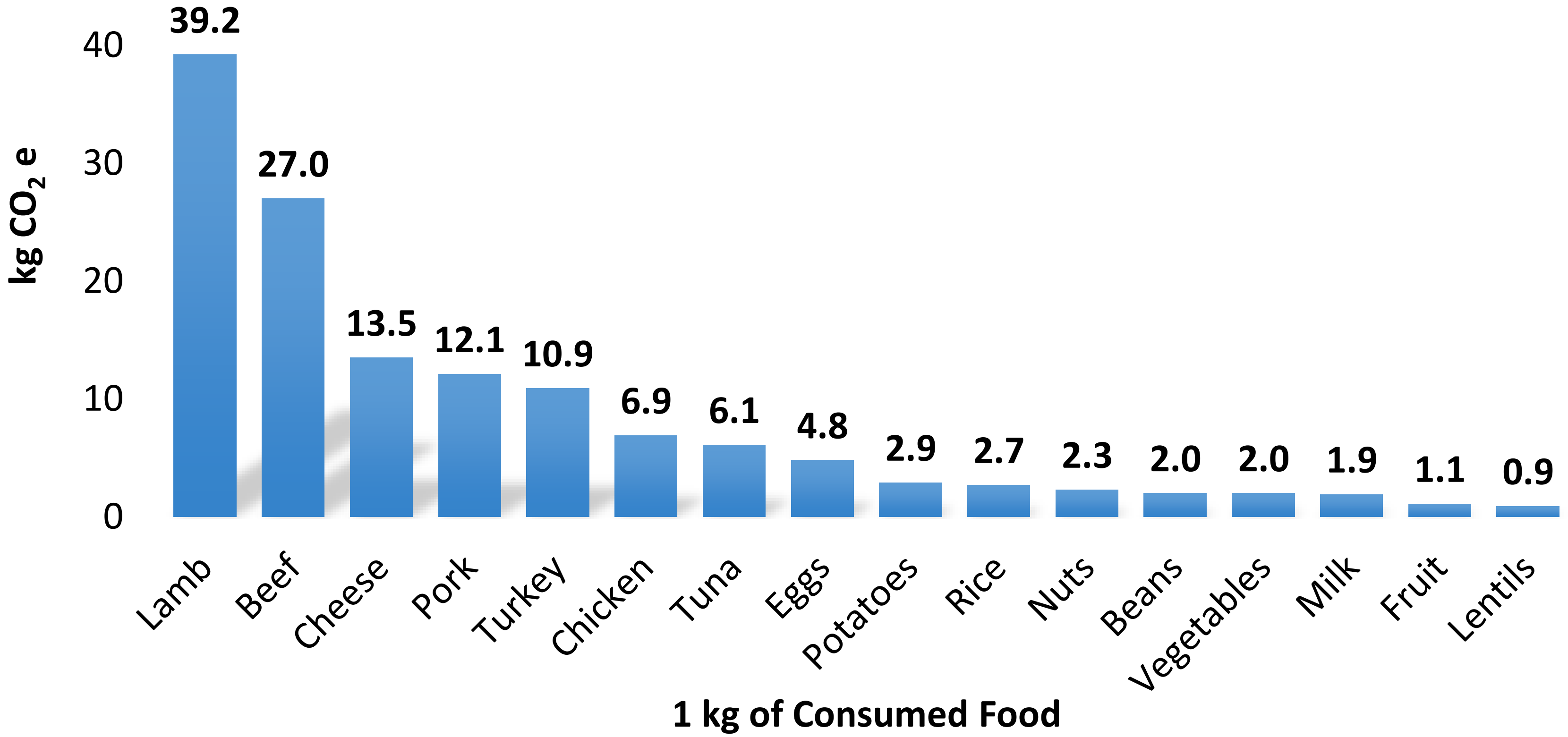
$V_{SA} = 506.25 < 32.91°\ \ V$

$P_S = 31.875\ \mathrm{kW}$

Food's Carbon Footprint:

CO_2 e produced by one kilo of each food

This includes all the emissions produced on the farm, in the factory, on the road, in the shop and in your home.

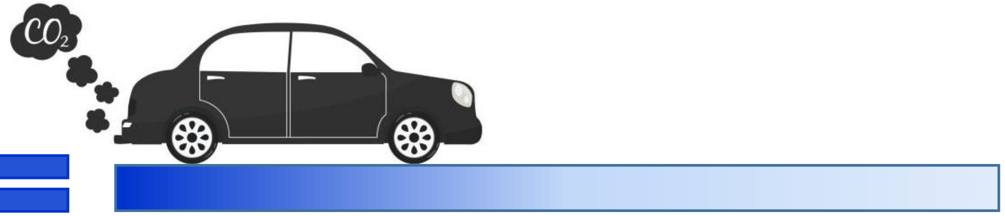


Food's Carbon Footprint:

A research team at the National Institute of Livestock and Grassland Science in Japan, showed that the CO_2 e of 1 kg beef are equivalent to the amount of CO_2 released by an average car every 160 miles, and the energy consumption is equal to 20, 100 W bulb being left on for the whole day.

1 kg beef

=



0 100 160
Miles

=

20 × 100 W

Can we change our diet to reduce our carbon footprint and protect the environment?

Three phase systems (modelling):

Reading list:

- DeCarlo Lin, "*Linear Circuit Analysis*", Oxford University Press, Second Edition, 2003
- W H Hayt, J E Kemmerly, S M Durbin, "*Engineering Circuit Analysis*", McGraw-Hill, 9th edition, 2019
- S J Chapman, "*Electric Machinery Fundamentals*", New York, McGraw-Hill, 2012