Durham University

Electrical Engineering 2

ENGI2191

Three phase systems

Dr Hamed Bahmani
Hamed.h.bahmani@Durham.ac.uk
E108

Lecture Objectives:

Aim of this lecture:

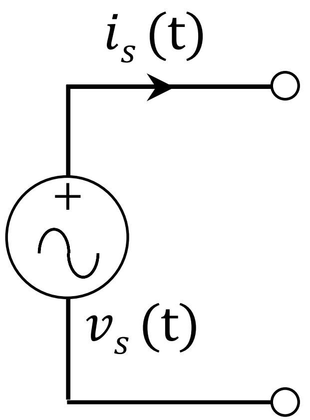
The aim of this lecture is to understand the basic concepts of three-phase systems in electrical networks and power systems.

Intended Learning Outcomes:

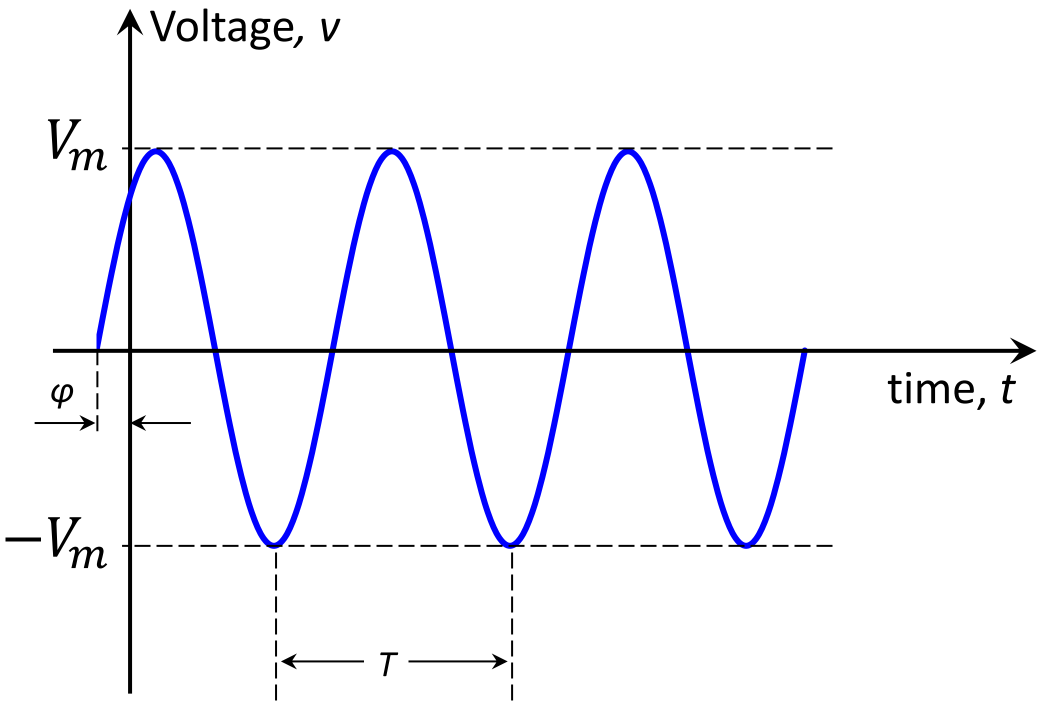
At the completion of the lecture and associated problems you should be able to:

- Discuss the differences between single-phase and three-phase systems.
- Discuss the characteristics of Y and Δ connections.
- Calculate voltage and current values for both Y and Δ connections.
- Analyse the balanced three phase circuits.
- Calculate complex power in three phase systems.

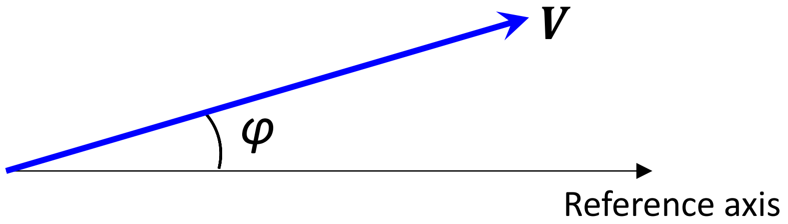
Single phase voltage source:



$$v(t) = V_m \, Sin \, (\omega t + \varphi)$$

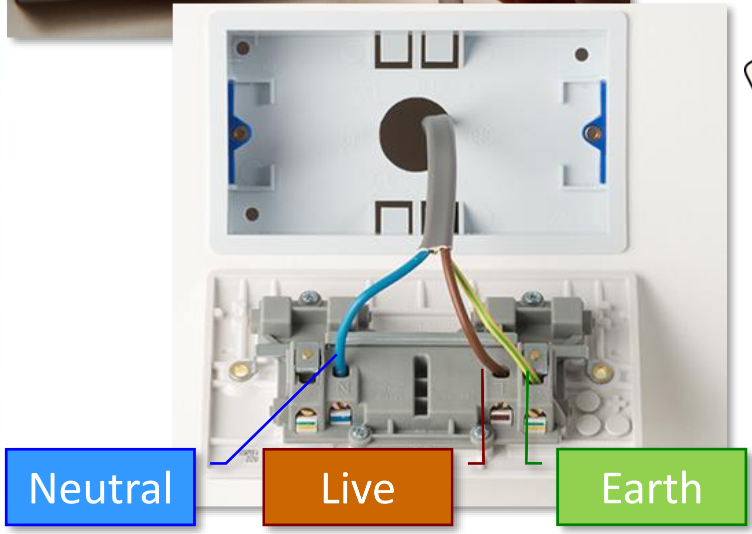


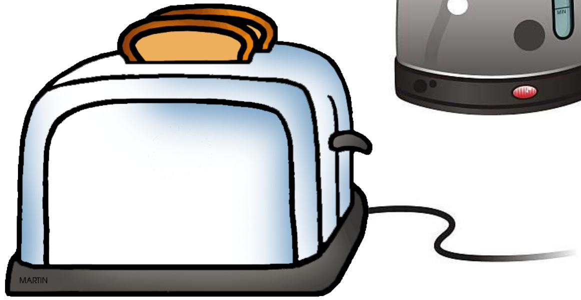
$$\boldsymbol{V} = V_m < \varphi°$$



Single phase voltage source:

Households and non-industrial small businesses use single phase power.

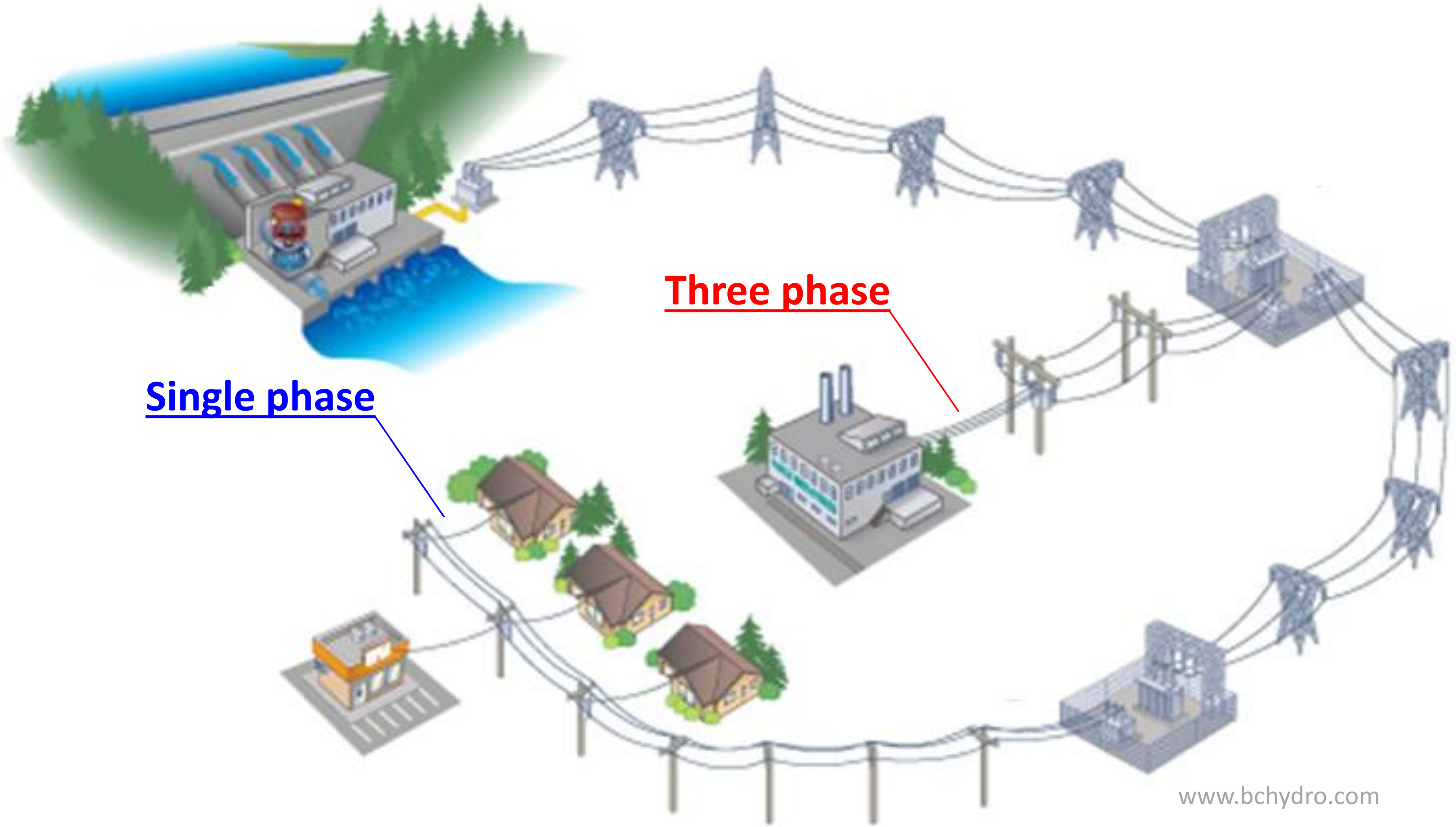




Three Phase Systems

Three phase voltage source:

Electrical energy is generated, transferred and distributed in three phase system.



Three phase generators:

Three phase generator
in hydro power station

turbosquid.com

Three phase generator
in wind turbine

Three phase transmission lines:

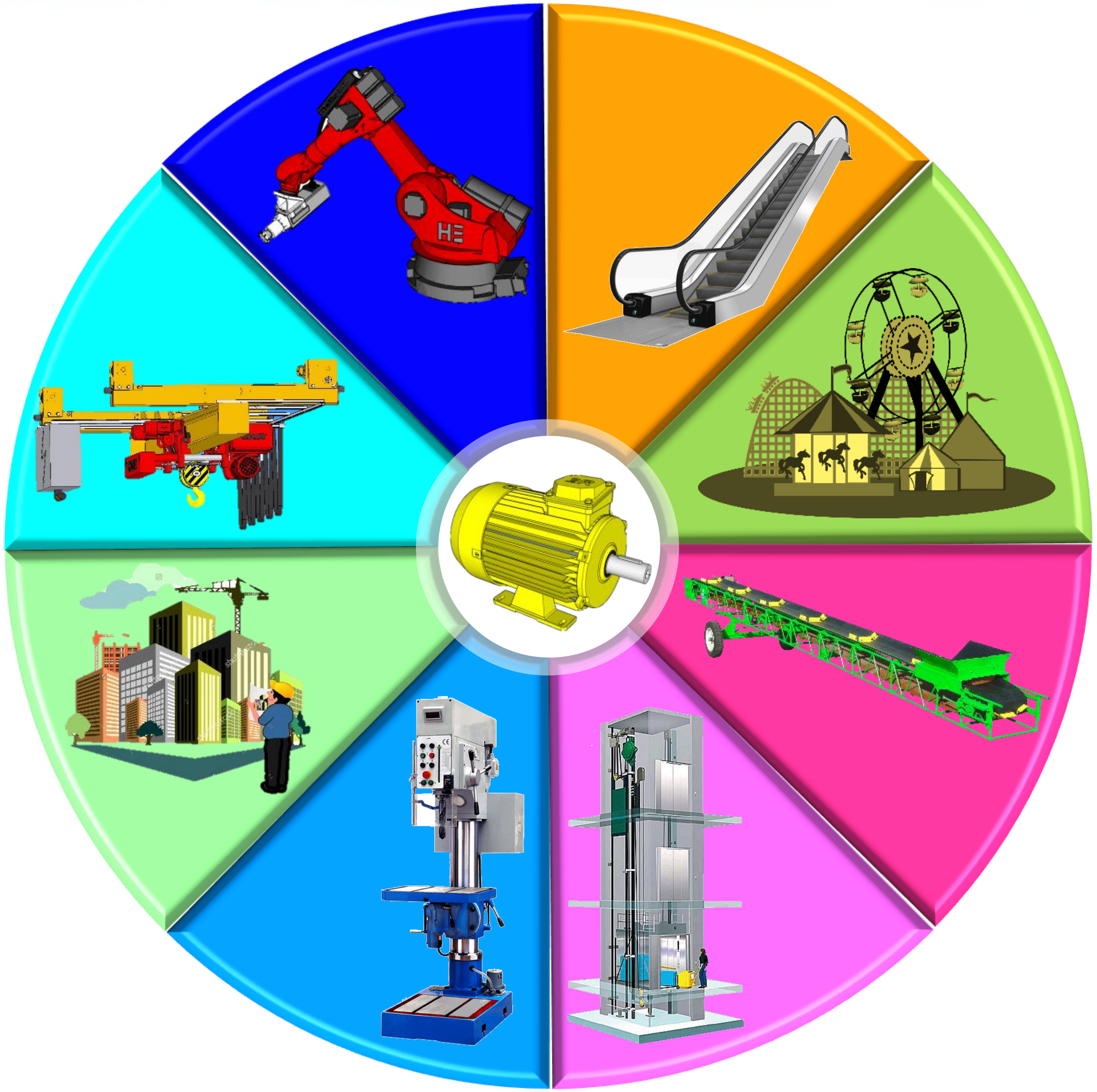
400 kV power line
Cheshire

132 kV power line
Western Power Distribution

11 kV distribution system

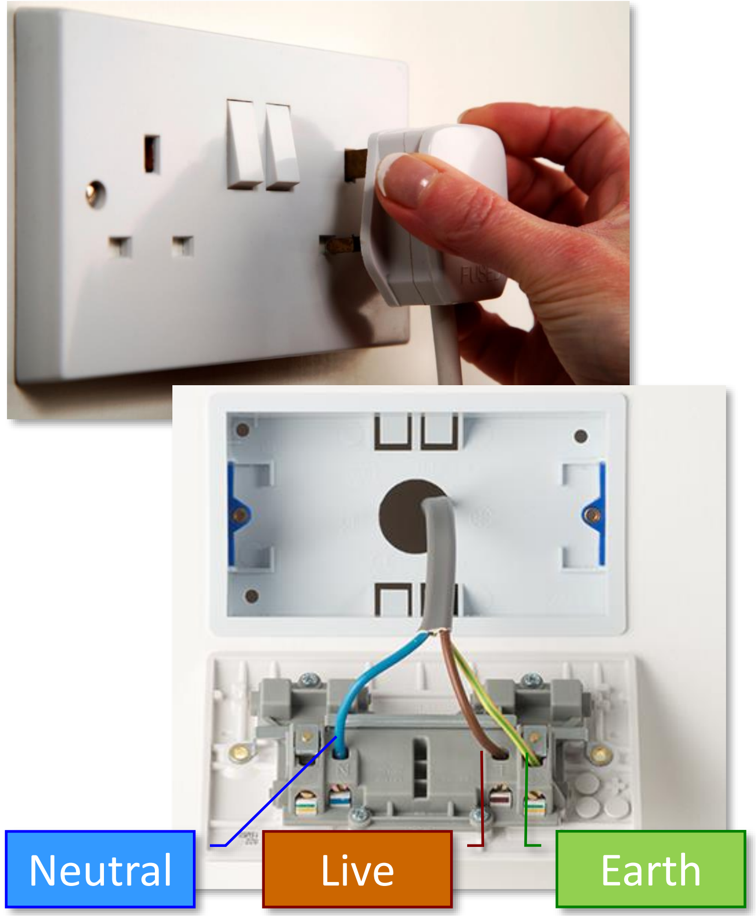
Three phase loads:

Three phase induction motors are one of the most popular electrical machines in industrial drives.



Three phase loads:

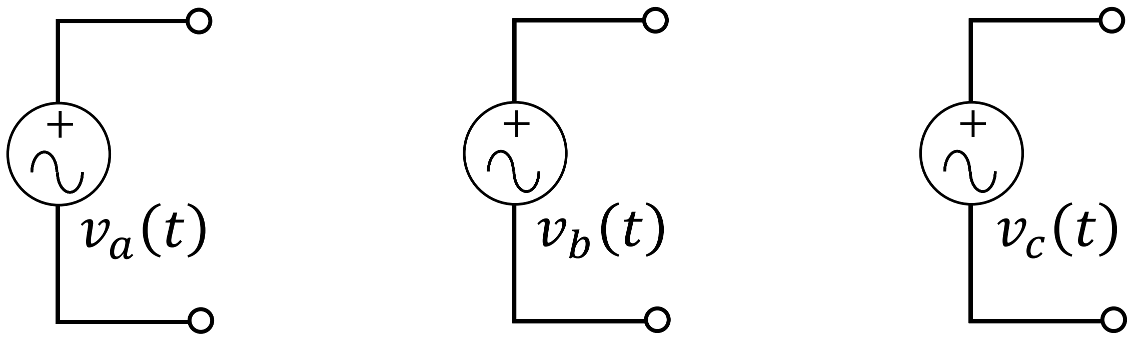
Single-phase plug and socket



Three-phase plug and socket

Three phase voltage source:

A three-phase voltage source consists of three independent single phase sinusoidal voltage sources with the same frequency and amplitude, but with phase angles separated by 120°.



$$v_a(t) = V_m\ Sin\ (\omega t)$$

$$v_b(t) = V_m\ Sin\ (\omega t - 120°)$$

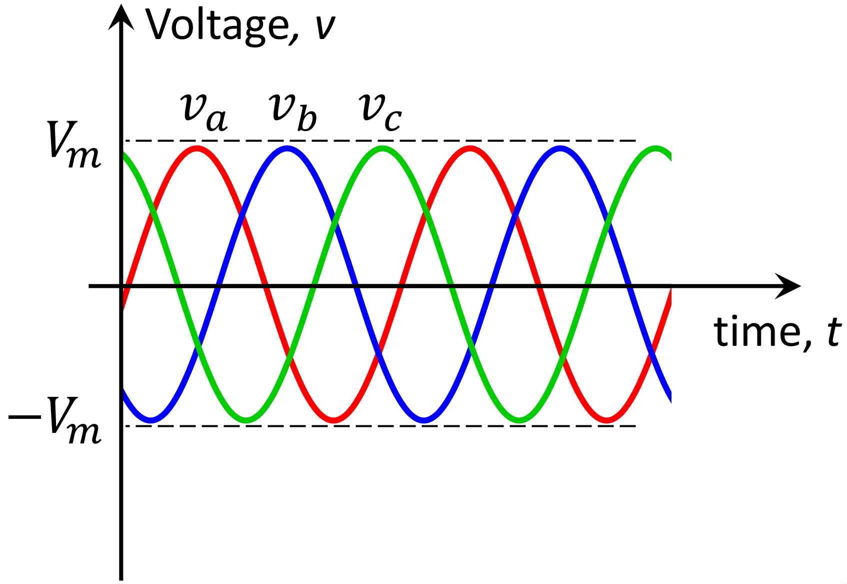
$$v_c(t) = V_m\ Sin\ (\omega t + 120°)$$

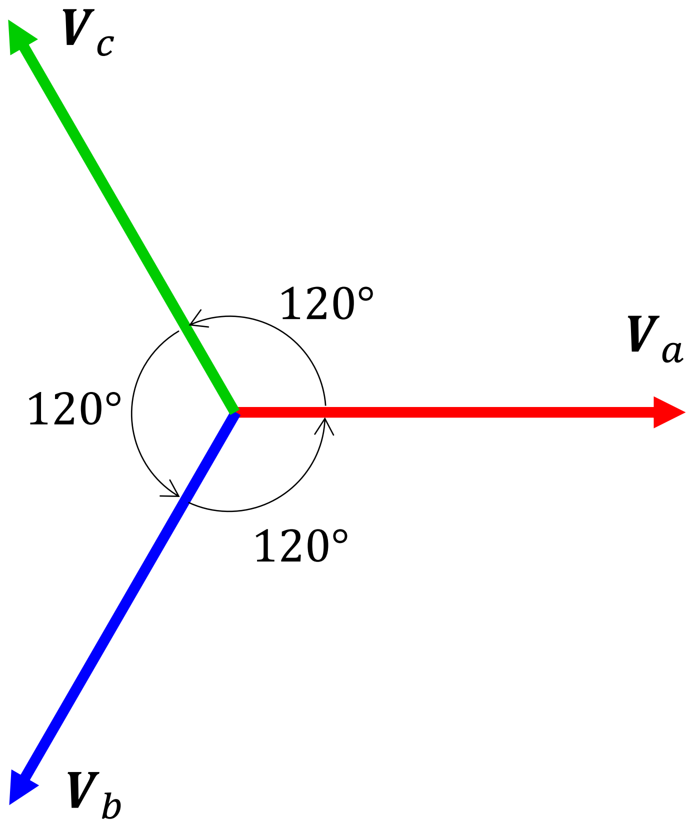
$$\boldsymbol{V}_a = V_{rms} < 0°$$

$$\boldsymbol{V}_b = V_{rms} < -120°$$

$$\boldsymbol{V}_c = V_{rms} < 120°$$

Three phase systems:





$$v_a(t) = V_m \, Sin\,(\omega t)$$

$$v_b(t) = V_m \, Sin\,(\omega t - 120°)$$

$$v_c(t) = V_m \, Sin\,(\omega t + 120°)$$

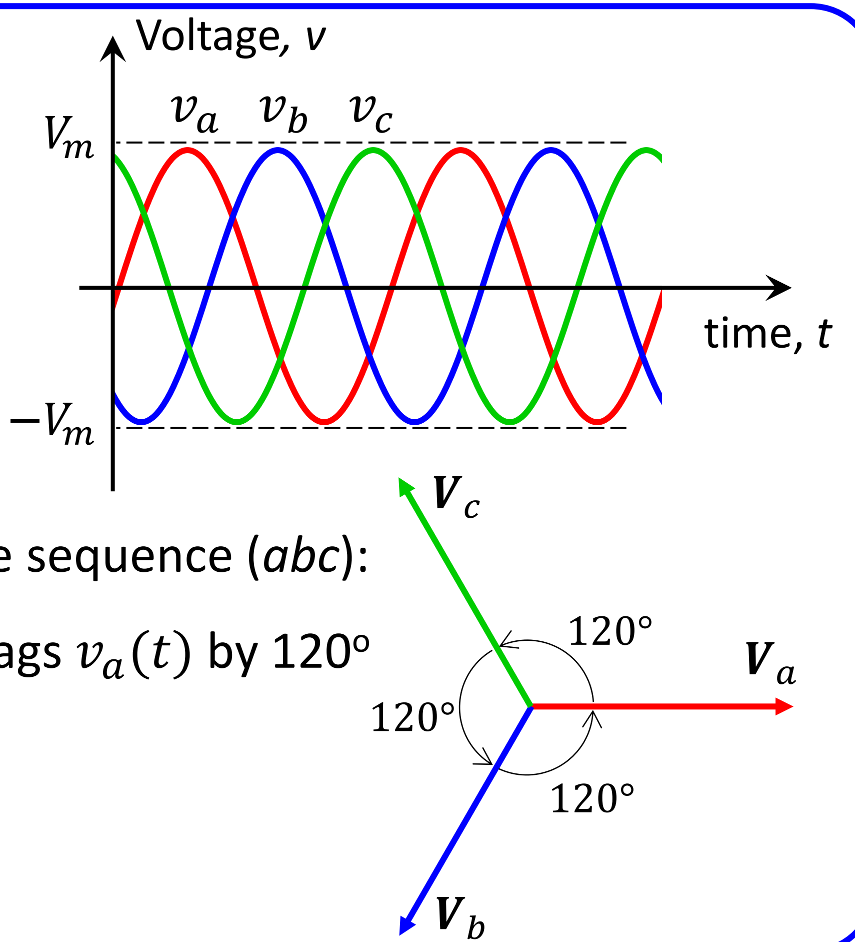
$$\boldsymbol{V}_a = V_{rms} < 0°$$

$$\boldsymbol{V}_b = V_{rms} < -120°$$

$$\boldsymbol{V}_c = V_{rms} < 120°$$

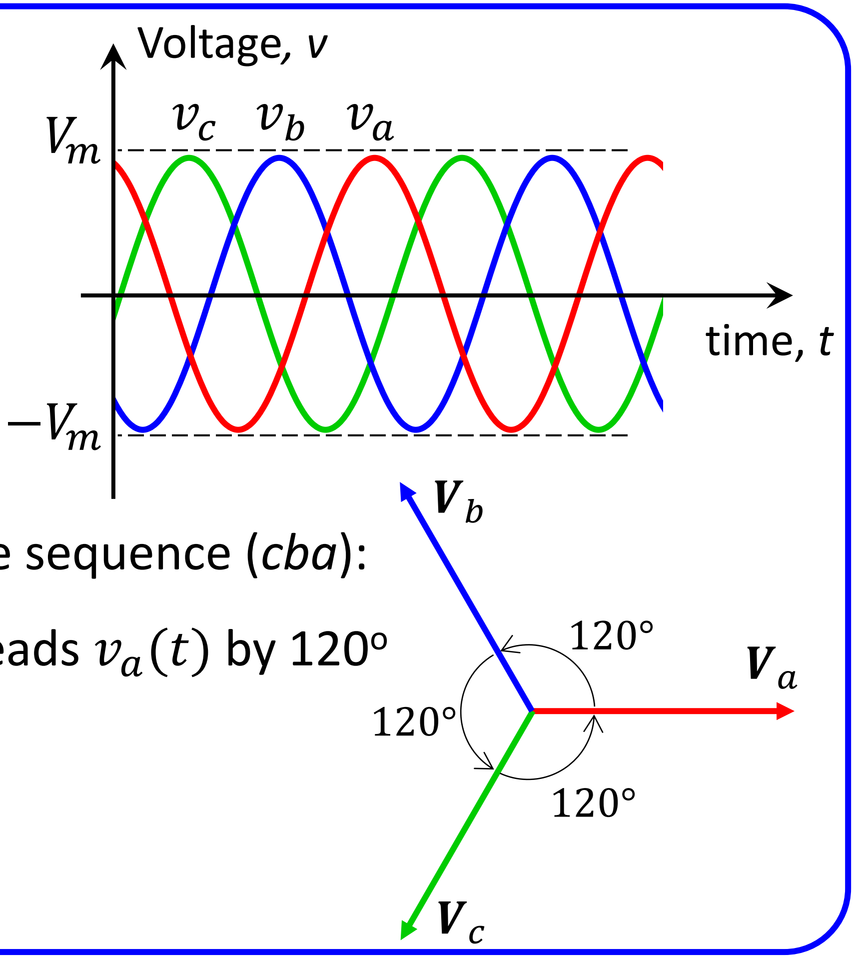
Three phase systems:

Based on the rotating direction of the three-phase generator, two phase sequences are defined:



Positive sequence (*abc*):

$v_b(t)$ lags $v_a(t)$ by 120°



Negative sequence (*cba*):

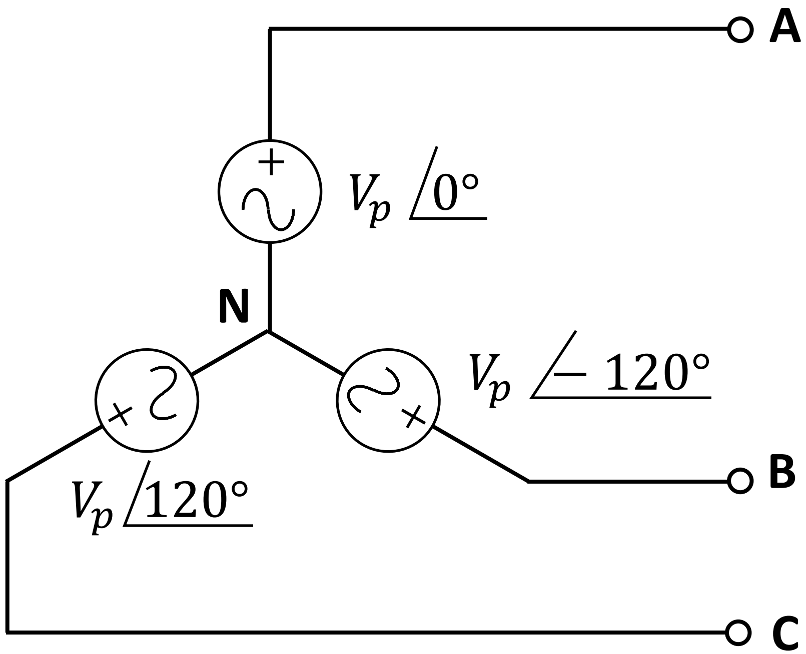
$v_b(t)$ leads $v_a(t)$ by 120°

In both systems, V_a is considered as the reference voltage.

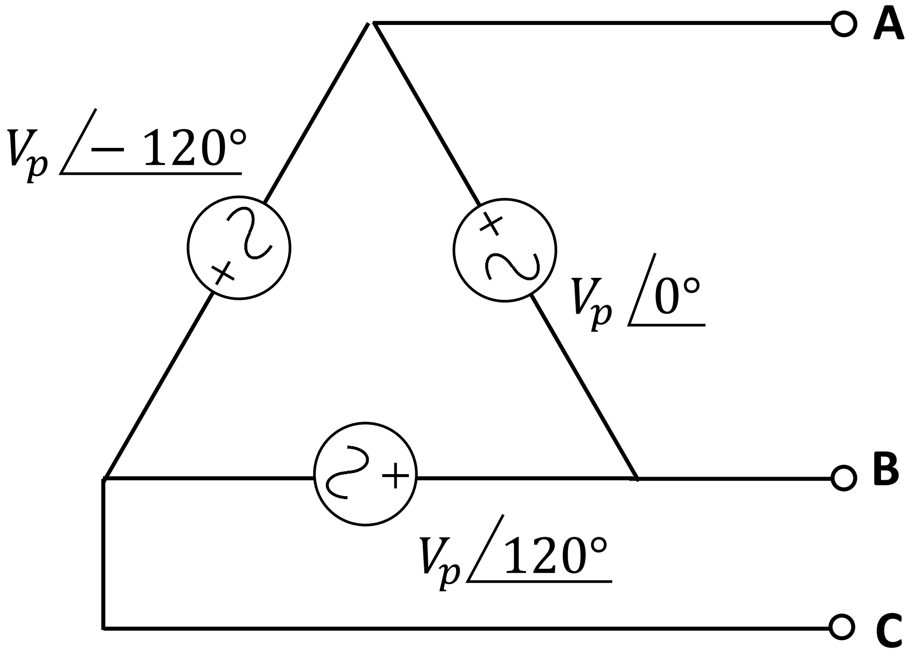
Three phase systems:

Three phase voltage source:

There are two common connections in three phase systems



Y, Wye or star connected three-phase voltage source



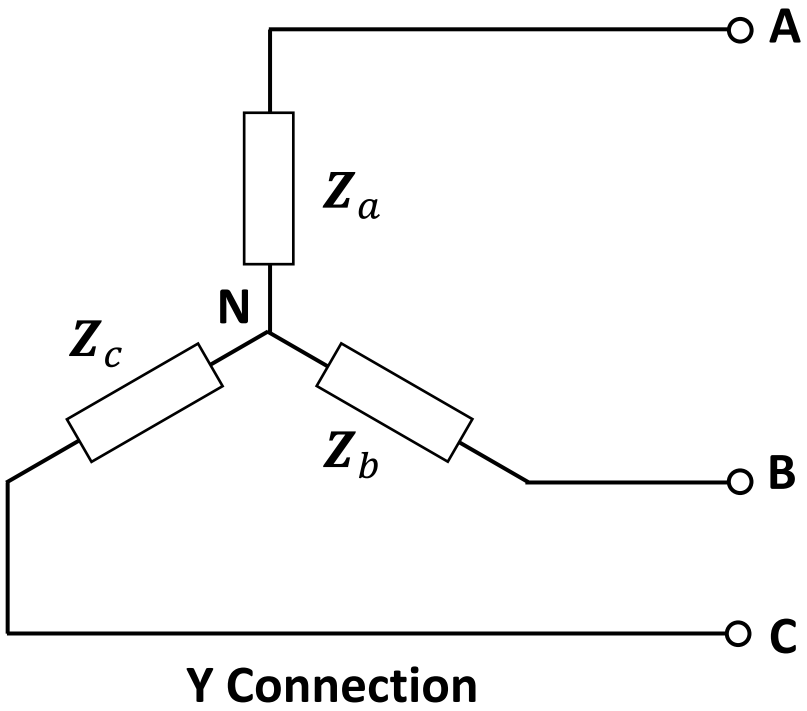
Δ connected three-phase voltage source

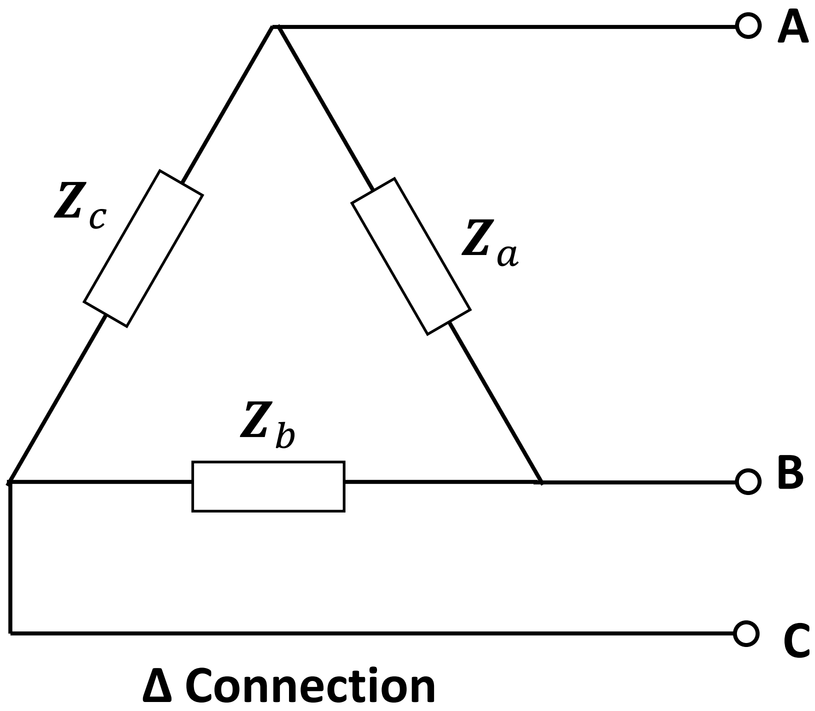
V_p is called phase voltage, is voltage across each single-phase voltage source.

Three phase systems:

Three phase loads:

Three phase load impedance are also connected either Y or Δ connection.

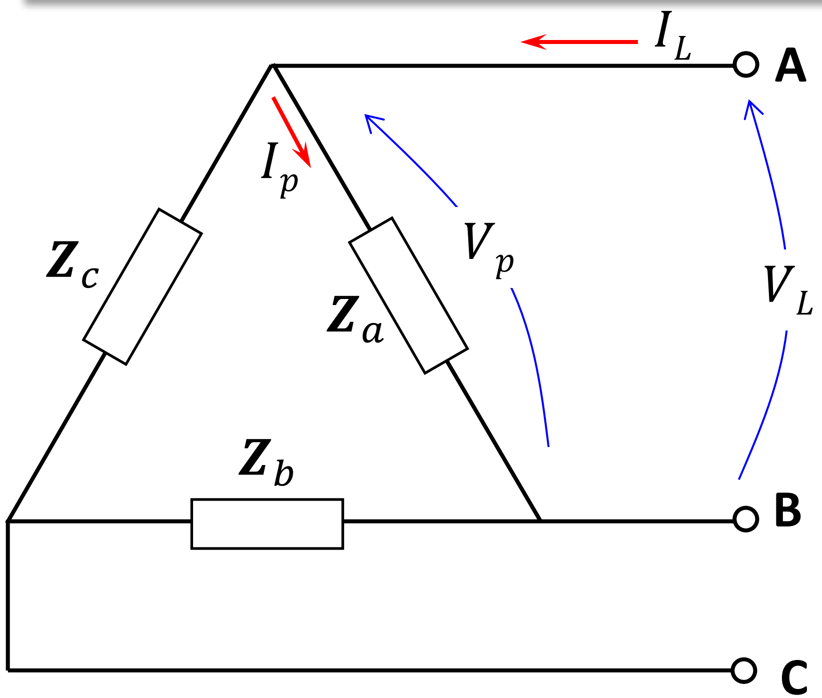


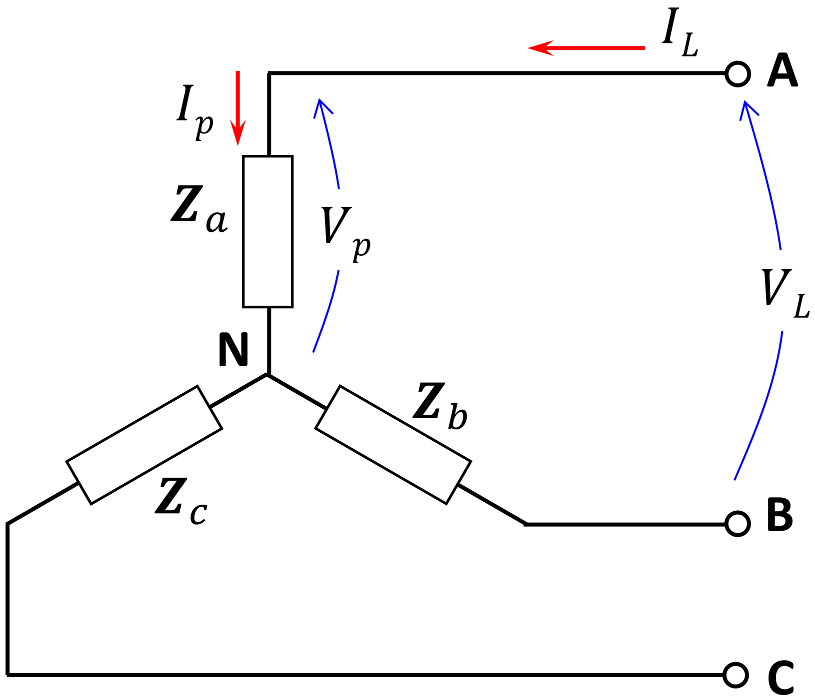


In a balance three phase load: $Z_a = Z_b = Z_c$

Any circuit that does not have all loads with the same impedance in all three phases, is unbalanced load.

Three phase systems:





In three phase loads:

Phase voltage, V_p:

Voltage across each impedance.

Phase current, I_p:

Current flowing through each impedance.

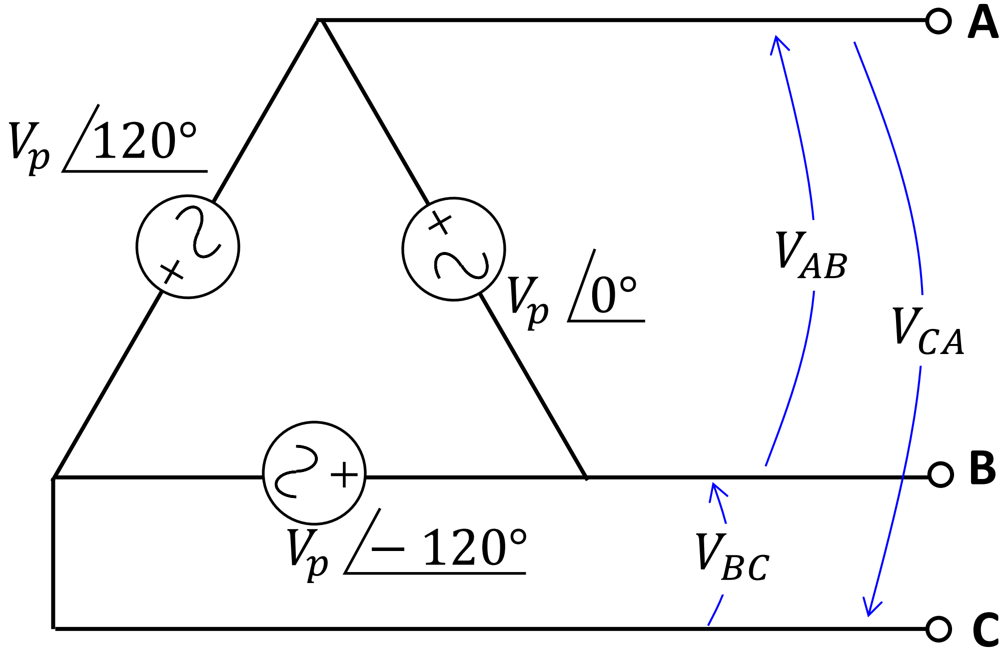
Line voltage, V_L:

Voltage between any pair of lines or terminals.

Line current, I_L:

Current flowing through each line.

Three phase systems:



$$\begin{cases} \boldsymbol{V}_{AB} = V_p \angle 0° \\ \boldsymbol{V}_{BC} = V_p \angle -120° \\ \boldsymbol{V}_{CA} = V_p \angle 120° \end{cases}$$

In three phase voltage sources:

Phase voltage, V_p:

Voltage across each source.

Line voltage, V_L:

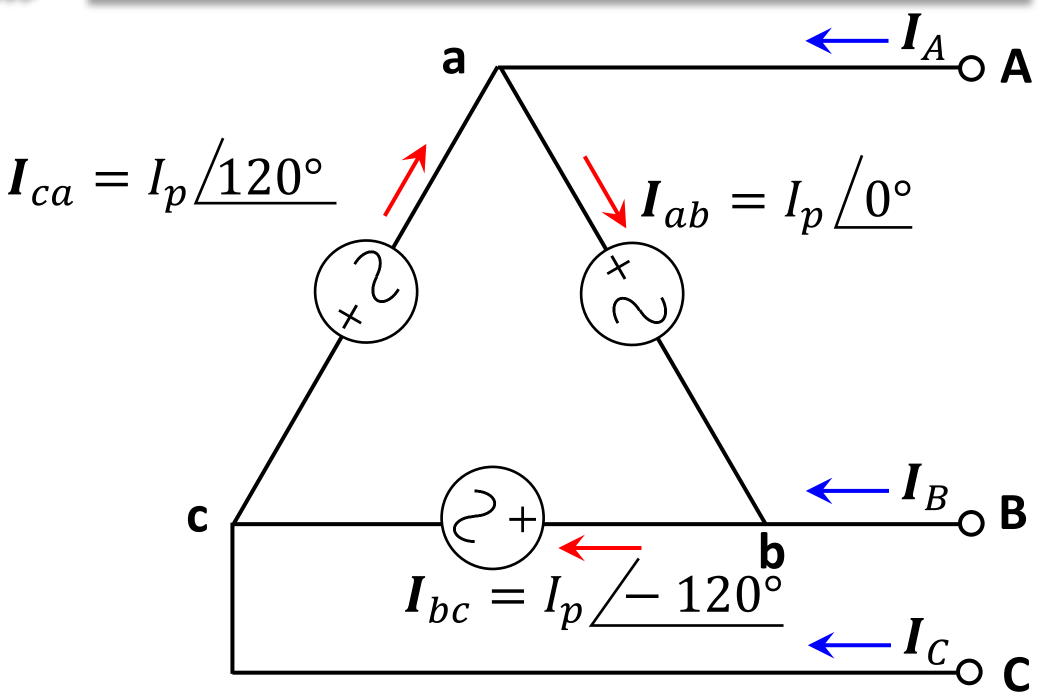
Voltage between any pair of lines or terminals.

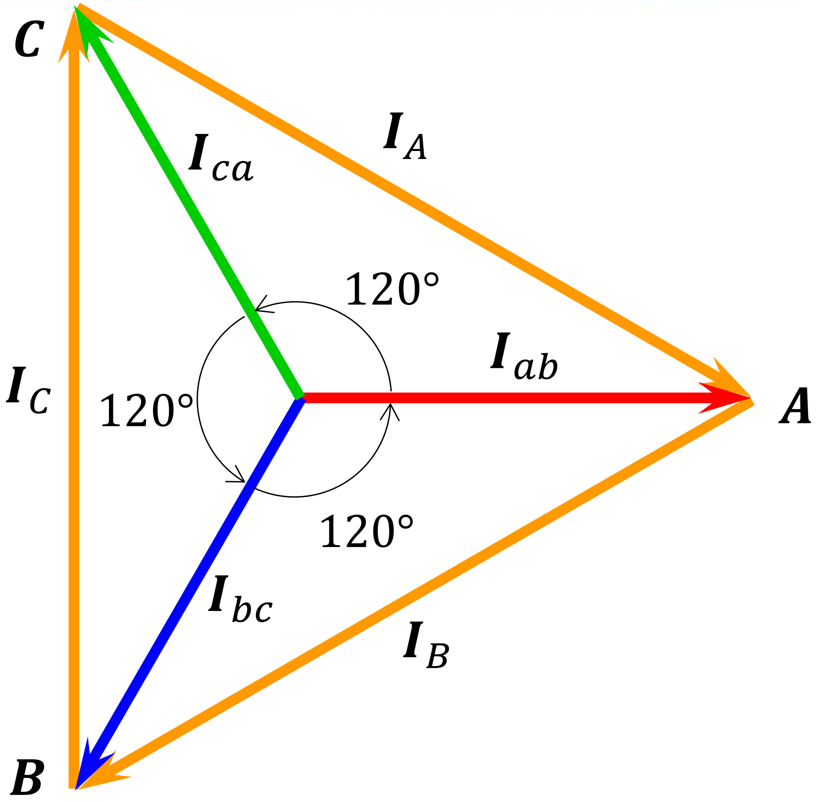
In three phase Δ-connected systems:

In Δ connection:

$$V_L = V_p$$

Three phase systems:





From KCL the relation between the phase and line currents can be obtained:

$$I_A = I_{ab} - I_{ca} = I_p\angle 0° - I_p\angle 120° = \sqrt{3}\ I_p\angle -30°$$

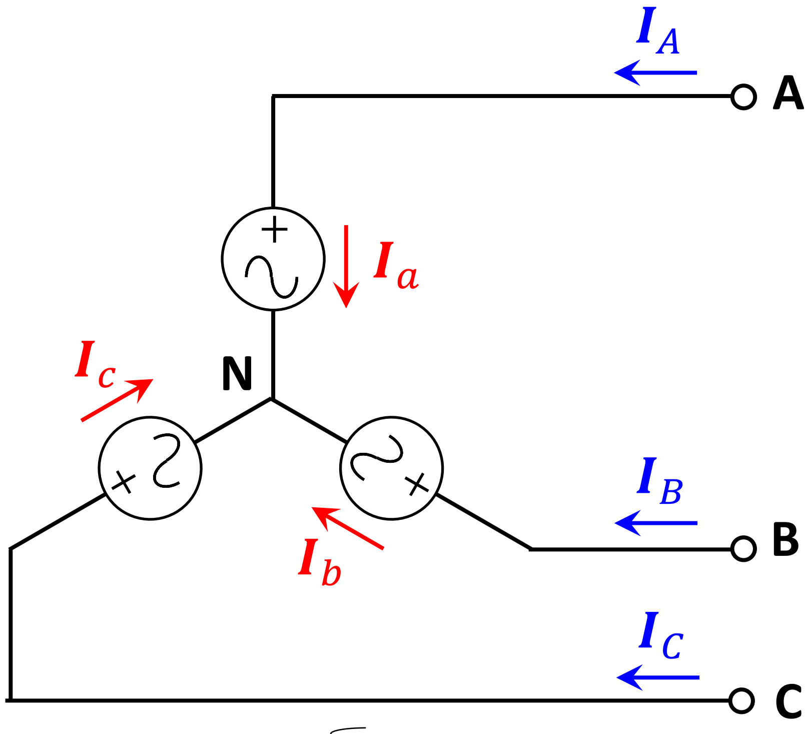
$$I_B = I_{bc} - I_{ab} = I_p\angle -120° - I_p\angle 0° = \sqrt{3}\ I_p\angle 210°$$

$$I_C = I_{ca} - I_{bc} = I_p\angle 120° - I_p\angle -120° = \sqrt{3}\ I_p\angle 90°$$

In Δ connection:

$$I_L = \sqrt{3}\ I_p$$

Three phase systems:



$$\begin{cases} \boldsymbol{I}_A = \boldsymbol{I}_a \\ \boldsymbol{I}_B = \boldsymbol{I}_b \\ \boldsymbol{I}_C = \boldsymbol{I}_c \end{cases}$$

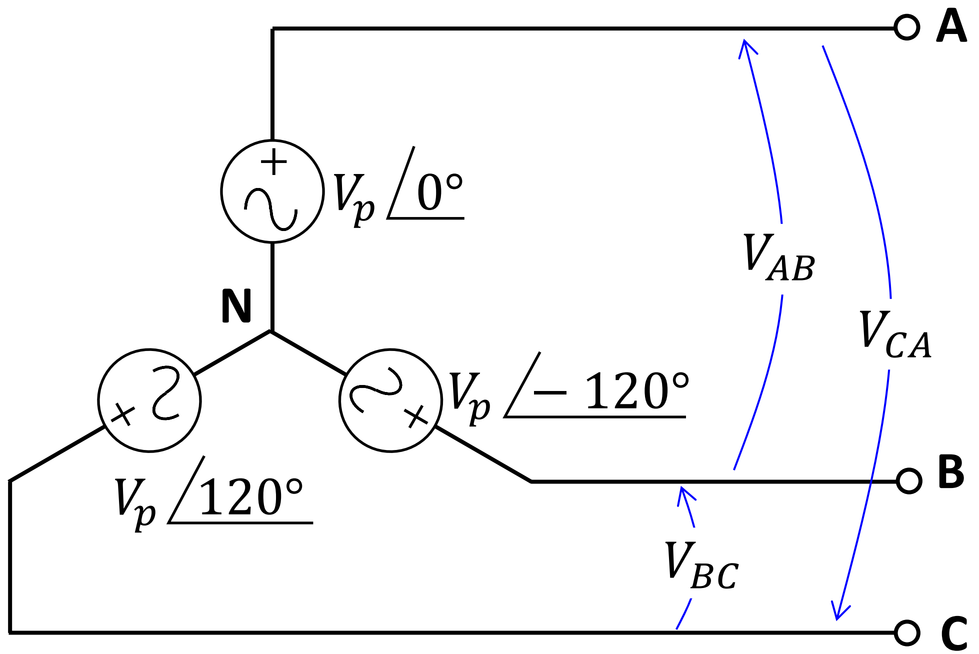
$\boldsymbol{I}_a$, $\boldsymbol{I}_b$ and $\boldsymbol{I}_c$ are phase currents.

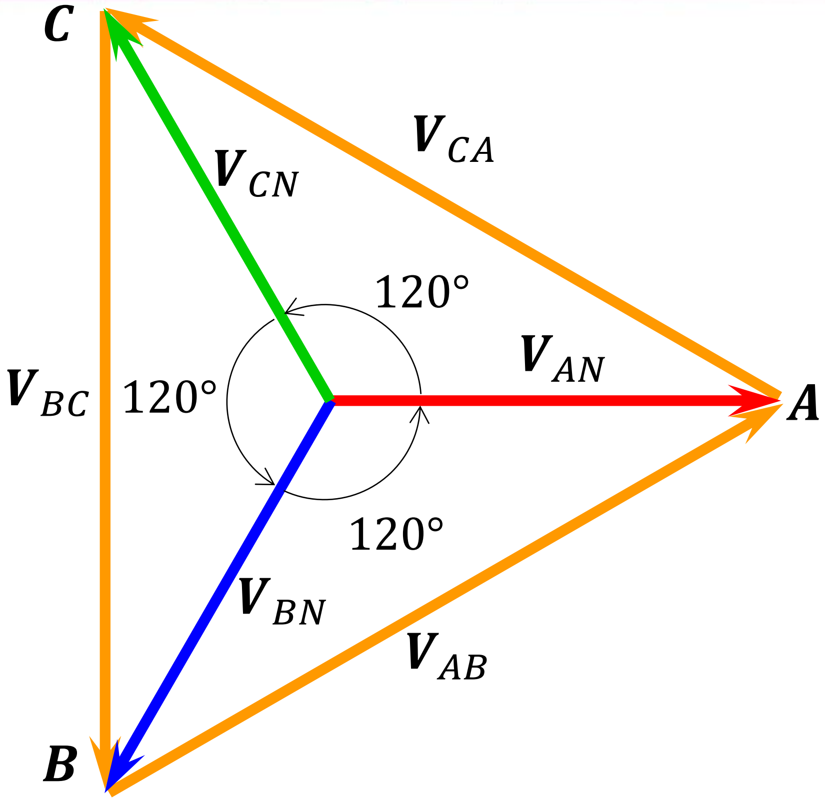
$\boldsymbol{I}_A$, $\boldsymbol{I}_B$ and $\boldsymbol{I}_C$ are line currents.

In Y connection:

$$I_L = I_p$$

Three phase systems:





From KVL the relation between the phase and line voltages can be obtained:

$$\boldsymbol{V}_{AB} = \boldsymbol{V}_{AN} - \boldsymbol{V}_{BN} = V_p \angle 0° - V_p \angle -120° = \sqrt{3}\, V_p \angle 30°$$

$$\boldsymbol{V}_{BC} = \boldsymbol{V}_{BN} - \boldsymbol{V}_{CN} = V_p \angle -120° - V_p \angle 120° = \sqrt{3}\, V_p \angle -90°$$

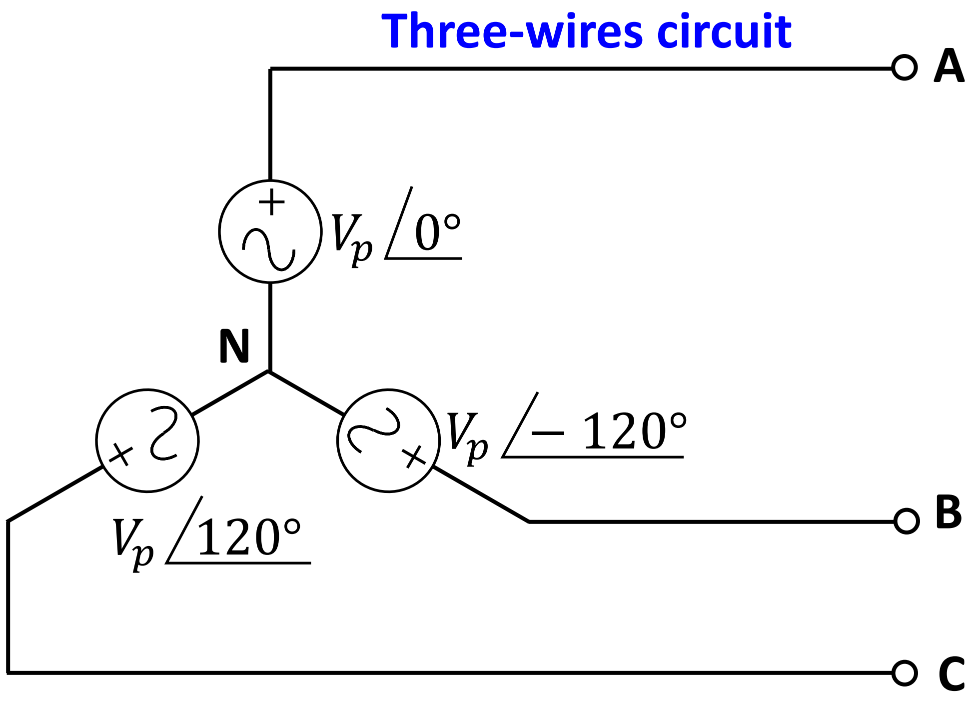
$$\boldsymbol{V}_{CA} = \boldsymbol{V}_{CN} - \boldsymbol{V}_{AN} = V_p \angle 120° - V_p \angle 0° = \sqrt{3}\, V_p \angle 150°$$

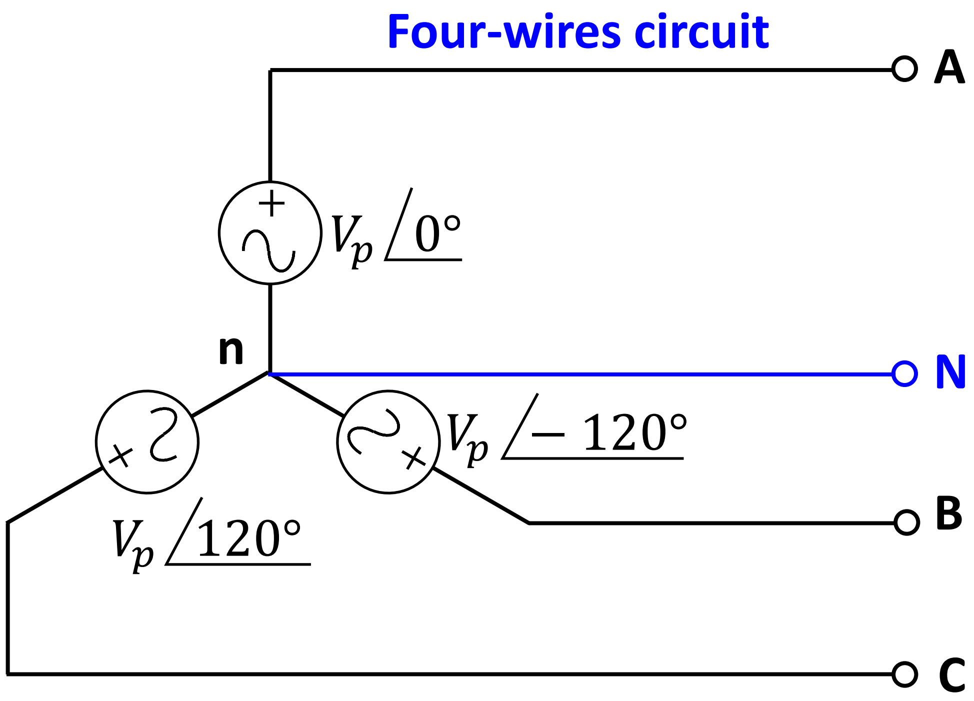
In Y connection:

$$V_L = \sqrt{3}\, V_p$$

Three phase systems:

Three phase Y-connected voltage sources are available in two configurations:



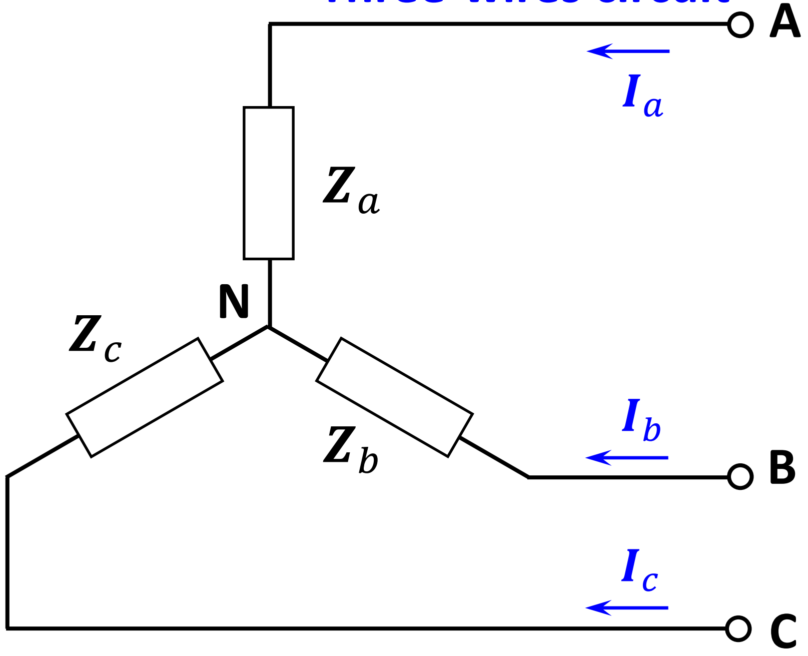


A four-wire system with symmetrical voltages between phase and neutral is obtained when the neutral is connected to the "common star point" of all supply windings. The neutral point is normally grounded for protection purposes.

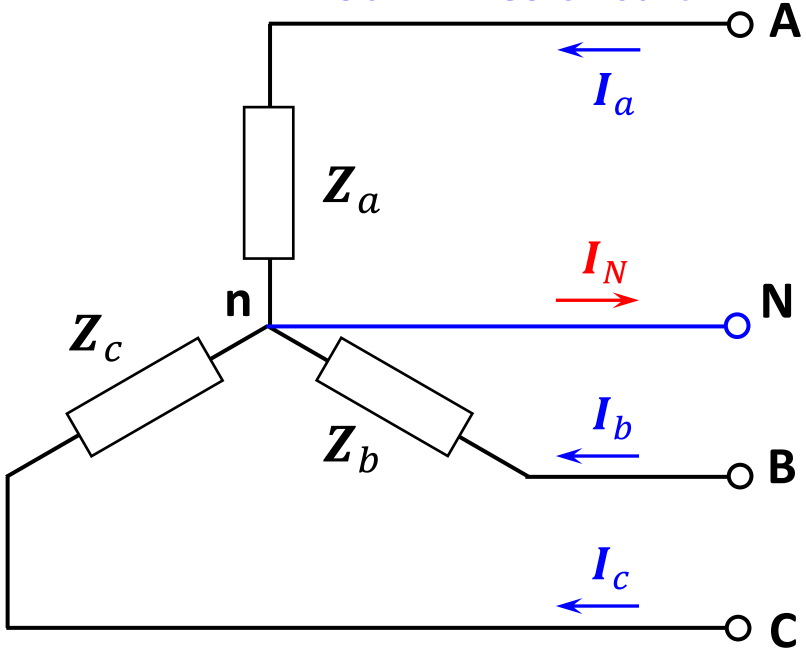
Three phase systems:

Similarly, three phase Y-connected loads are available in two configurations:

Three-wires circuit



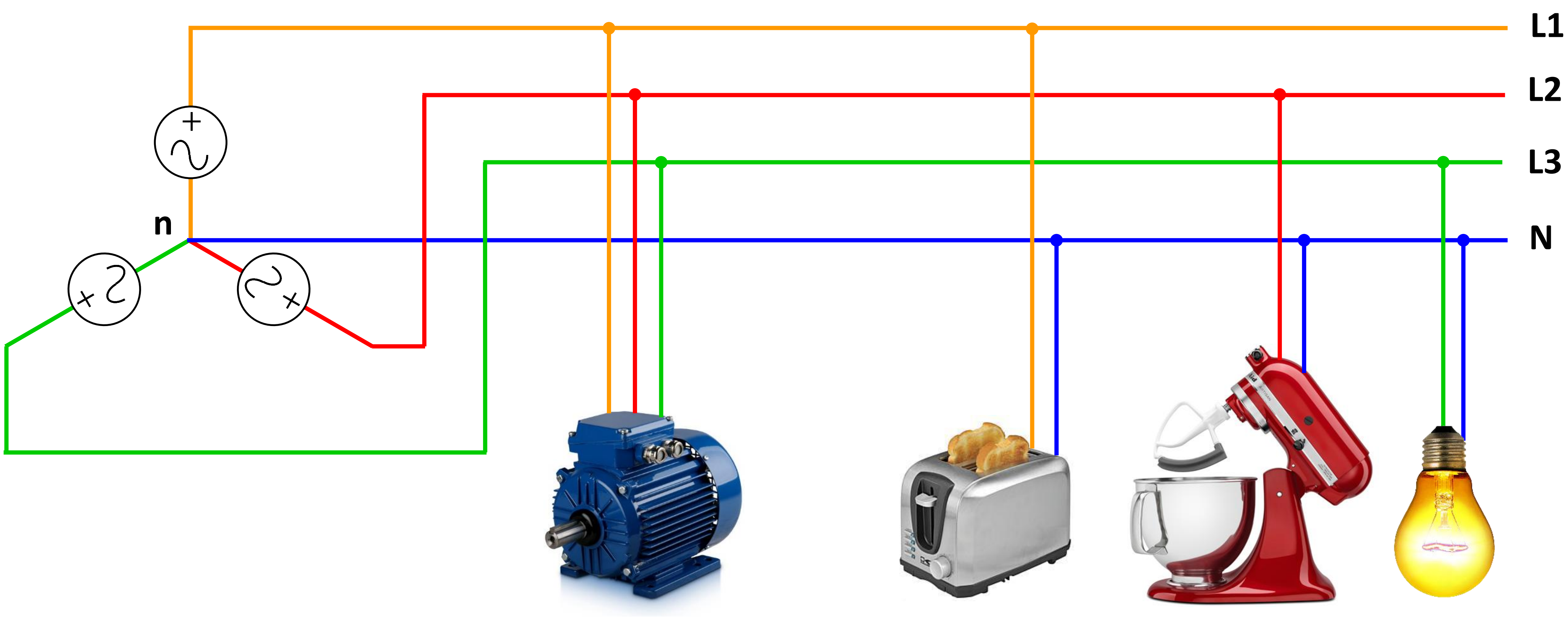
Four-wires circuit



In a balance three phase load: $I_N = 0$

In unbalanced loads, it is necessary to use a four-wire system to maintain a balance voltage across the load impedance.

Three phase systems:



3-phase Y-connected

4-wires power supply

Three phase motor

Industrial load

Single phase facilities

Household consumption

Three phase systems:

Since the load distribution in the distribution networks is practically unbalanced, all of the distribution networks employ four wire system.

4 wires distribution system

5 wires distribution system

In distribution network of the UK, phase voltage is $V_p = 230\ V$, line voltage is $V_L = 400\ V$ and $f = 50\ Hz$

Durham University

Image: National Grid

Power in Three-Phase Systems

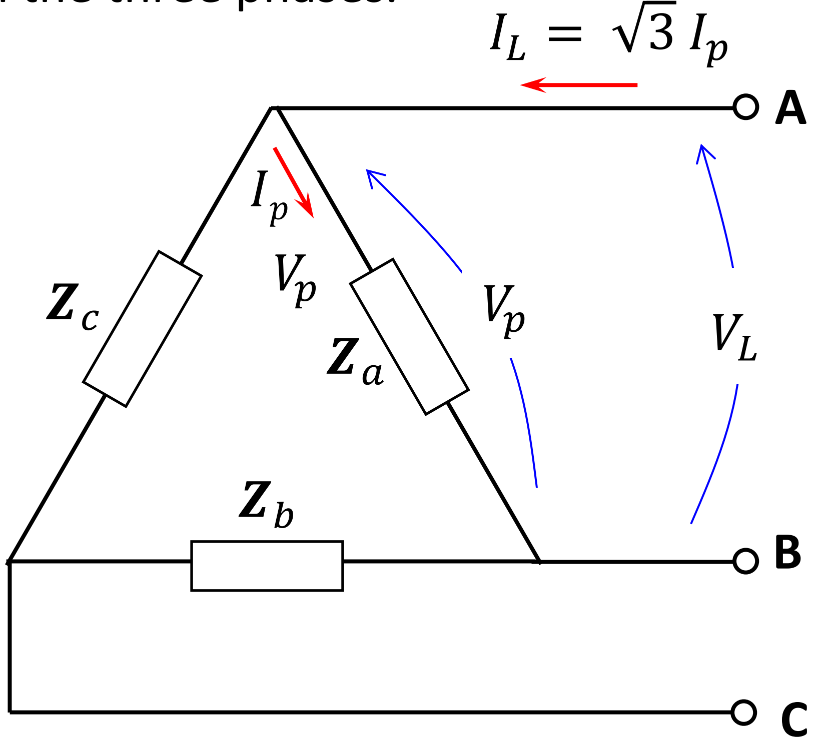
Power in three phase systems:

Total power in three phase system is the sum of powers in the three phases.

$$P_t = 3\, V_p\, I_p \cos\varphi$$

In 3-phase systems it is more convenient to use line voltage and line current. For Δ-connection:

$$V_L = V_p \qquad \& \qquad I_L = \sqrt{3}\, I_p$$



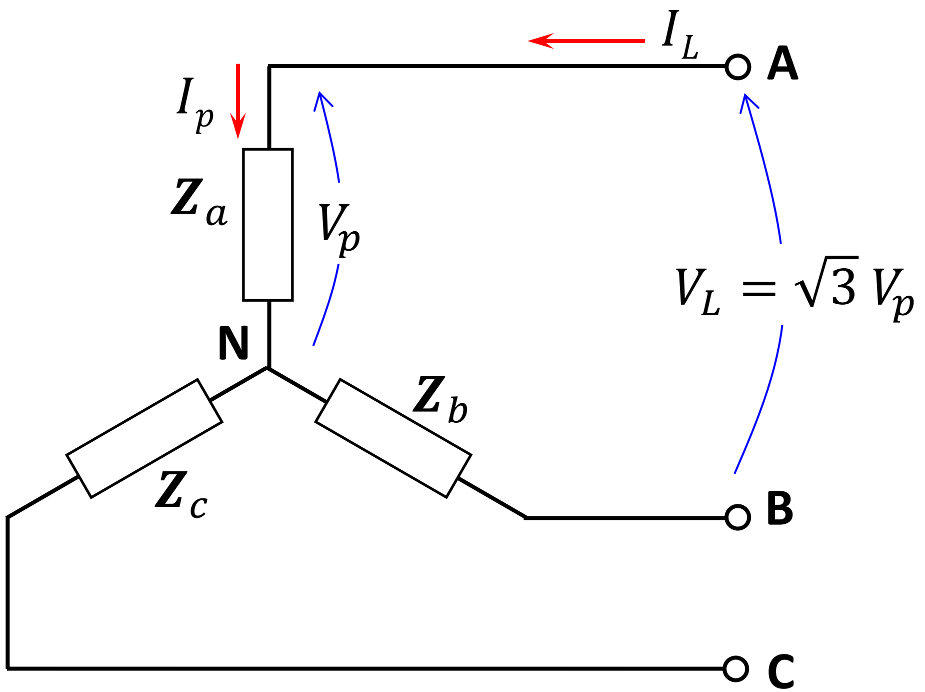
Power in Δ connected three-phase balance system

$$P_t = 3\, V_L \left(\frac{I_L}{\sqrt{3}}\right) \cos\varphi \quad \ggg \quad P_t = \sqrt{3}\; V_L\, I_L \cos\varphi$$

Power in three phase systems:

Similarly, in a Y-connected three phase system total power is given as:

$$P_t = 3\, V_p\, I_p \cos\varphi$$



In Y connection:

$$V_L = \sqrt{3}\, V_p \qquad \& \qquad I_L = I_p$$

Power in Y-connected three-phase balance system

$$P_t = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\varphi \quad \ggg \quad P_t = \sqrt{3}\; V_L\, I_L \cos\varphi$$

Power in three phase systems:

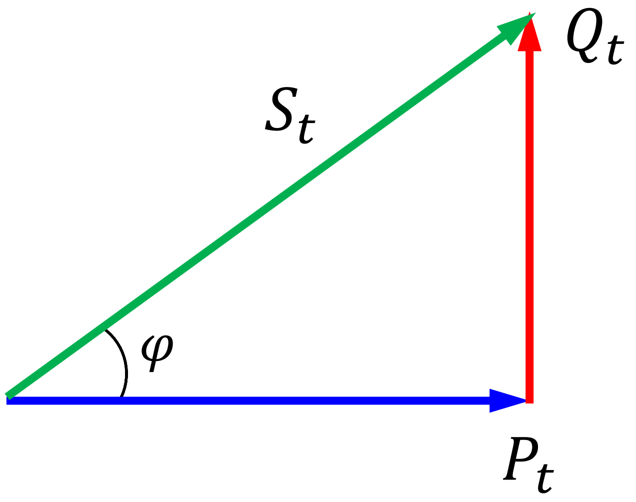
Similar principle is applied to other components of power:

$$P_t = \sqrt{3}\ V_L\ I_L \cos\varphi \qquad W, kW \text{ or } MW$$

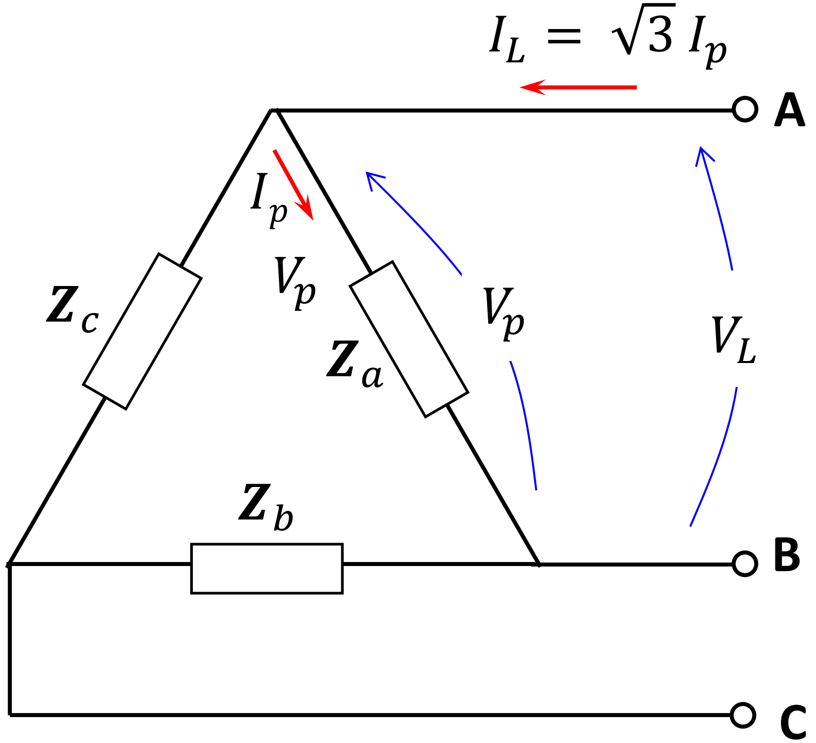
$$Q_t = \sqrt{3}\ V_L\ I_L \sin\varphi \qquad VAR, kVAR \text{ or } MVAR$$

$$S_t = \sqrt{3}\ V_L\ I_L \qquad VA, kVA \text{ or } MVA$$

$$\boldsymbol{S_t} = P_t + j\,Q_t \qquad VA, kVA \text{ or } MVA$$



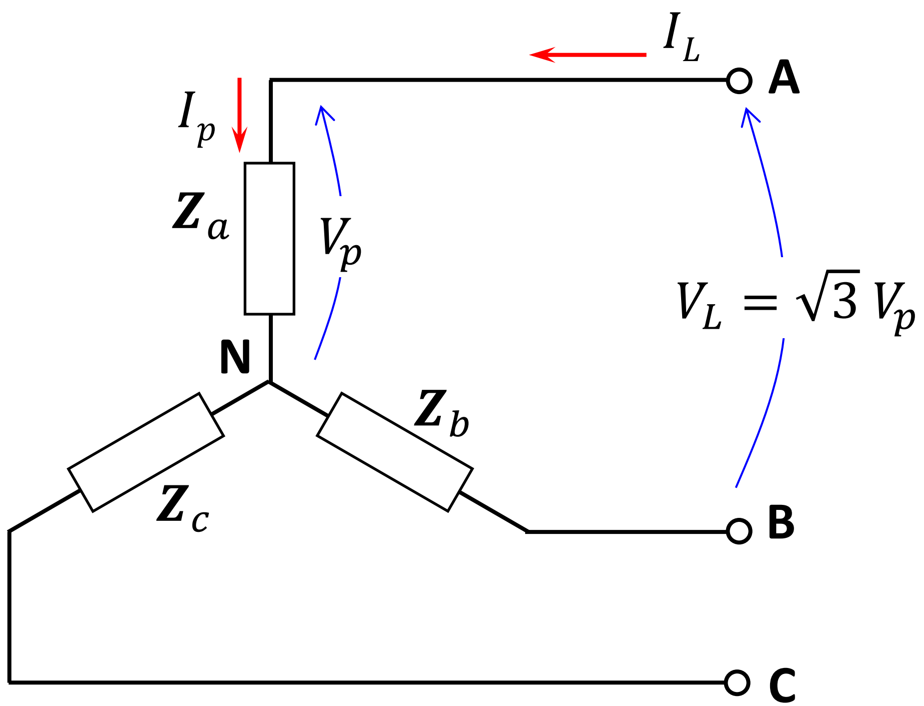
Three phase systems:



$V_L = V_p$ $\qquad$ $I_L = \sqrt{3}\, I_p$

$P_t = \sqrt{3}\ V_L\, I_L \cos\varphi$

Only 3-wires system is possible



$V_L = \sqrt{3}\, V_p$ $\qquad$ $I_L = I_p$

$P_t = \sqrt{3}\ V_L\, I_L \cos\varphi$

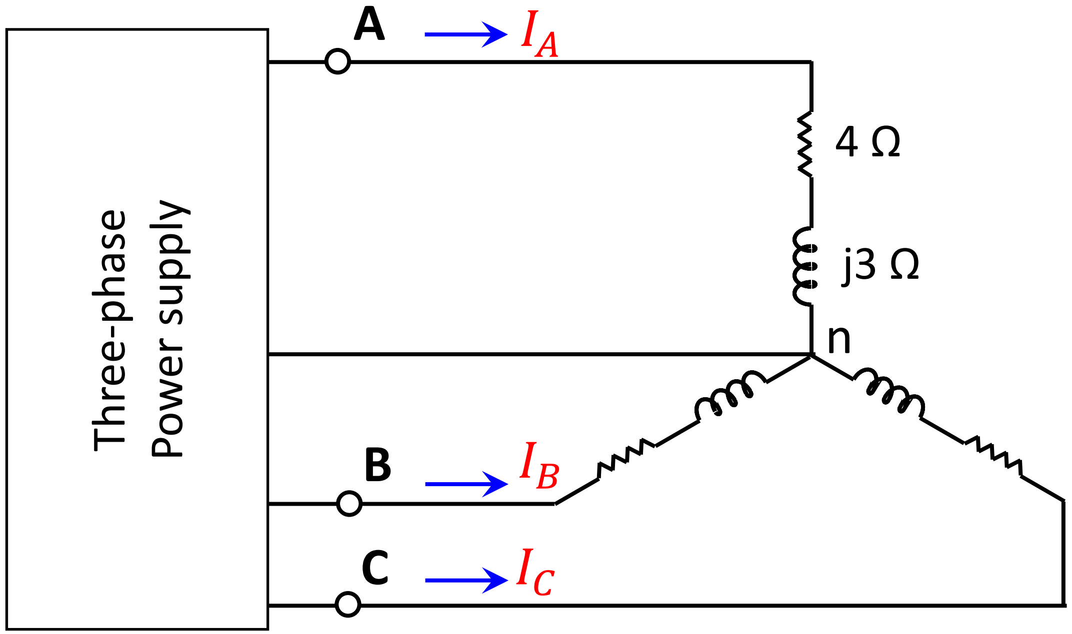
3-wires and 4-wires systems are possible

Three phase systems:

Example:

Consider a Y-connected three-phase balance load with phase impedance of $Z_p = (4 + j3)\ \Omega$. This load is supplied through a 3-phase 4-wires power supply with line voltage of $V_L = 208\ V$.

a) Find the line current I_L b) Find the neutral current I_N c) Find the complex power of the load.



$$V_L = 208 \quad \text{V} \qquad V_p = \frac{208}{\sqrt{3}} = 120 \quad \text{V}$$

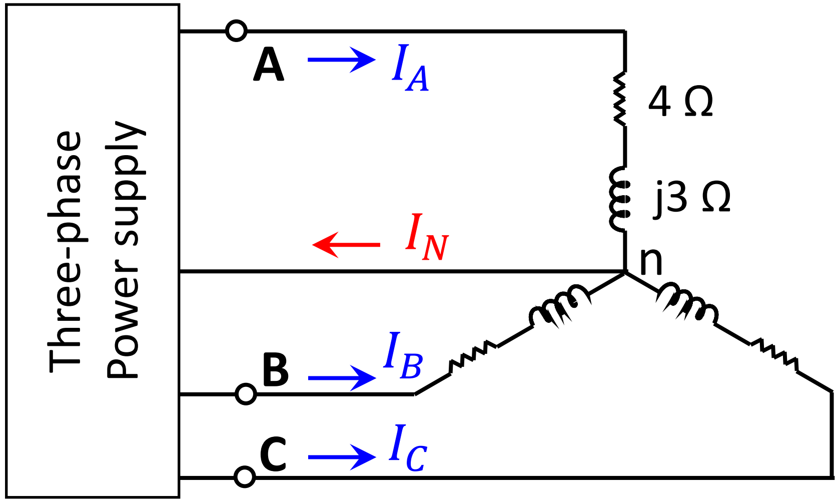
Phase voltages across the load impedance:

$$V_{An} = 120\ \angle 0^\circ \quad V$$

$$V_{Bn} = 120\ \angle 120^\circ \quad V$$

$$V_{Cn} = 120\ \angle -120^\circ \quad V$$

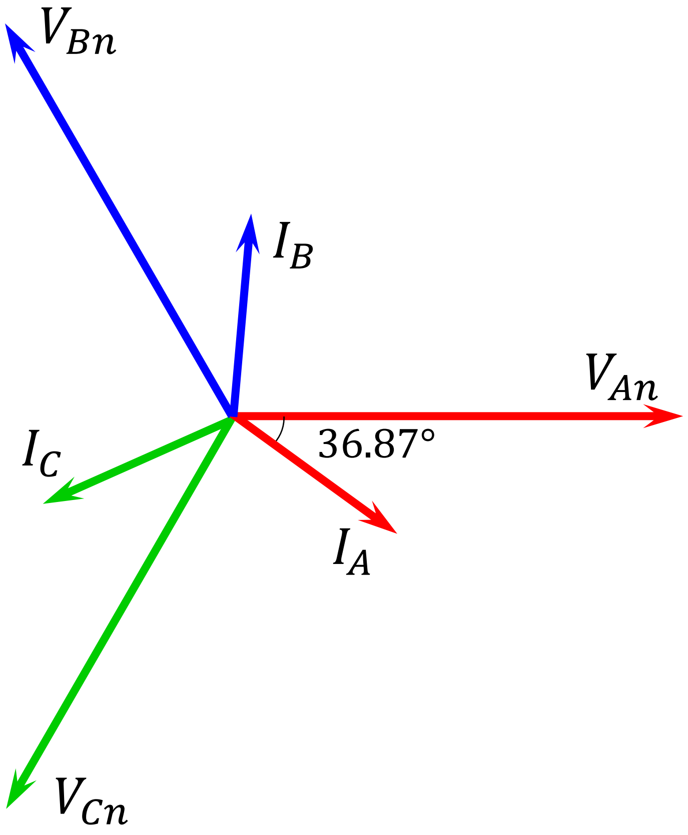
Three phase systems:



$$I_N = I_A + I_B + I_C$$

$$I_N = 24\underline{/-36.87°} + 24\underline{/83.13°} + 24\underline{/-156.87°}$$

$$I_N = 0$$

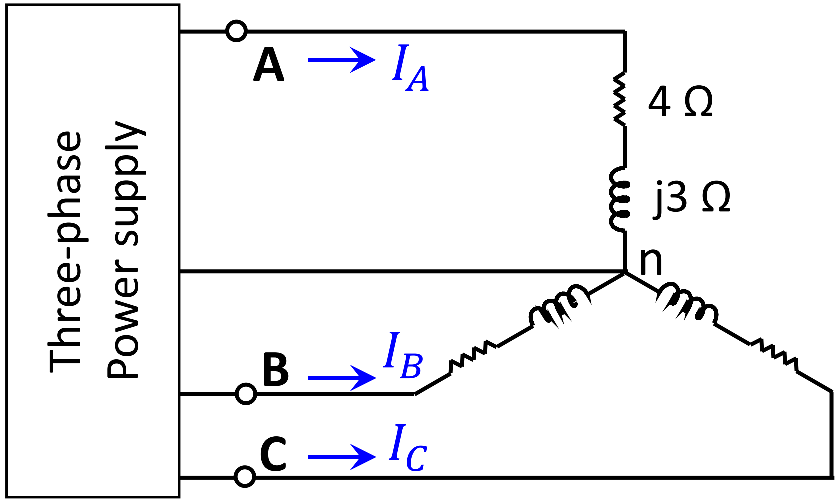


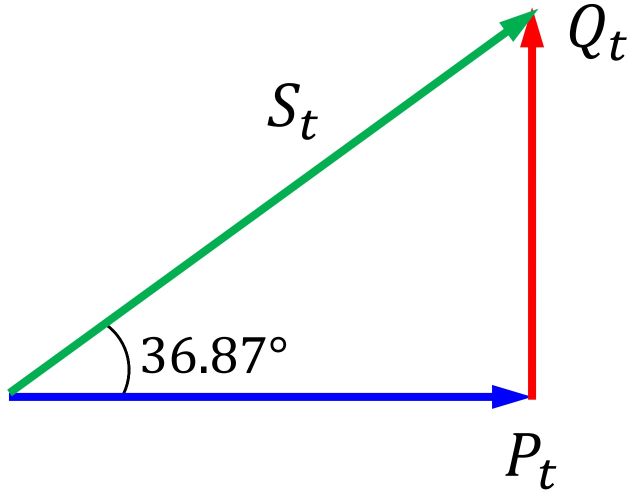
$$I_A = \frac{V_{An}}{Z_A} = \frac{120\underline{/0°}}{(4+j3)} = \frac{120\underline{/0°}}{5\underline{/36.87°}} = 24\underline{/-36.87°}\ \text{A}$$

$$I_B = \frac{V_{Bn}}{Z_B} = \frac{120\underline{/120°}}{(4+j3)} = \frac{120\underline{/120°}}{5\underline{/36.87°}} = 24\underline{/83.13°}\ \text{A}$$

$$I_C = \frac{V_{Cn}}{Z_C} = \frac{120\underline{/-120°}}{(4+j3)} = \frac{120\underline{/-120°}}{5\underline{/36.87°}} = 24\underline{/-156.87°}\ \text{A}$$

Three phase systems:





$$P_t = \sqrt{3}\ V_L\ I_L \cos\varphi = \sqrt{3}\ \times 208\ \times 24 \times \cos(36.87°) = 6.92\ \text{kW}$$

$$Q_t = \sqrt{3}\ V_L\ I_L \sin\varphi = \sqrt{3}\ \times 208\ \times 24 \times sin(36.87°) = 5.19\ \text{kVAR}$$

$$\boldsymbol{S}_t = P_t + j\,Q_t = 6.92 + j\,5.19 \qquad \text{kVA}$$

$$S_t = \sqrt{(P_t^2 + \ Q_t^2)} = \sqrt{(6.92)^2 + (5.19)^2} = 8.65\ \text{kVA}$$

Drill:

Repeat the last example for a Δ connected load, with everything else unchanged.

a) Find the line current I_L b) Plot the phasor diagram of the voltages and currents c) Find the complex power of the load d) How do you compare the current s and voltages with the case of star connected load (results of the last example)?

Hint! In this case the load is supplied through a three phase three wires voltage source.

Answer:

$I_{ph} = 41.7\ A$ $\qquad I_L = 72.2\ A$

$\mathrm{P} = 20.87\ kW$ $\qquad \mathrm{Q} = 15.65\ kVAR$


Imagine a Blue Planet!


Imagine a Green Planet!

Together we can fight the climate change.
Together we can save our planet.


Stay Green,
Have a Green Day.

Reading list:

Recommended text books:

- DeCarlo Lin, “*Linear Circuit Analysis*”, Oxford University Press, Second Edition, 2003
- W H Hayt, J E Kemmerly, S M Durbin, “*Engineering Circuit Analysis*”, McGraw-Hill, 9th Edition, 2019