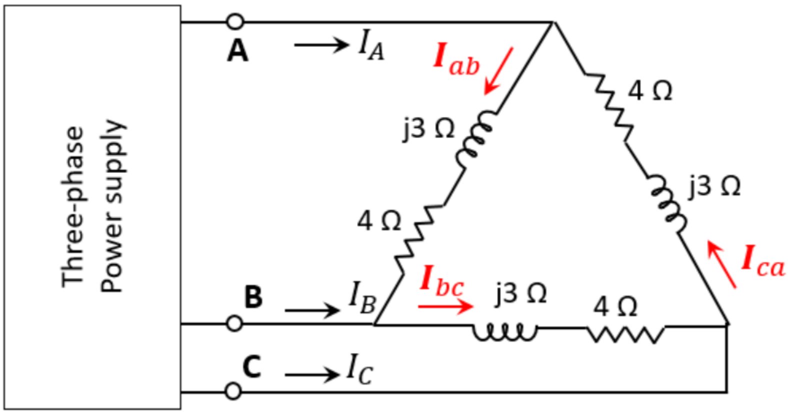
1 Consider a Δ-connected three phase balance load with phase impedance of Z_p = (4 + j3) Ω. This load is supplied through a 3-phase power supply with line voltage of V_L= 208 V.

a) Find the line current I_L b) Find the complex power of the load

c) How do you compare the currents and voltages with the case of star connected load (results of the Week 5 class example)?

Solution:

a) Schematic diagram of this three-phase system is given here:



Line voltage of this system is 208 V, so considering *cba* phase sequence a set of phase and line voltages for the power supply are:

Phase voltages:	Line voltages:
$V_{AN} = 120 < 0° \quad V$ $V_{BN} = 120 < 120° \quad V$ $V_{CN} = 120 < -120° \quad V$	$V_{AB} = 208 < -30° \quad V$ $V_{BC} = 208 < 90° \quad V$ $V_{CA} = 208 < -150° \quad V$

For Δ-connected loads, line voltages are applied across the phase impedance (on other word $V_{ph} = V_L$), therefore in this analysis line voltages are used.

To find the line currents, first we need to find the phase currents:

$$I_{ab} = \frac{V_{AB}}{Z_{AB}} = \frac{208 < -30°}{4 + j3} = \frac{208 < -30°}{5 < 36.87°} = 41.7 < -66.87° \quad A$$

$$I_{bc} = \frac{V_{BC}}{Z_{BC}} = \frac{208 < 90°}{4 + j3} = 41.7 < 53.13° \quad A$$

$$I_{ca} = \frac{V_{CA}}{Z_{CA}} = \frac{208 < -150°}{4 + j3} = 41.7 < -186.87° \quad A$$

Line currents can be calculated by applying KCL to each node of the circuit:

$$I_A = I_{ab} - I_{ca} = 41.7 < -66.87° - 41.7 < -186.87° = 72 < -36.9° \quad A$$

$$I_B = I_{bc} - I_{ab} = 72 < 83.1° \quad A$$

$$I_C = I_{ca} - I_{bc} = 72 < 203.1° \quad A$$

b) To find the complex power of the load, we can calculate active and reactive powers individually:

$$P_L = \sqrt{3} V_L I_L \cos\varphi = \sqrt{3} \times 208 \times 72 \times \cos(36.87°) = 20.87 \quad kW$$

$$Q_L = \sqrt{3} V_L I_L \sin\varphi = \sqrt{3} \times 208 \times 72 \times \sin(36.87°) = 15.65 \quad kVAR$$

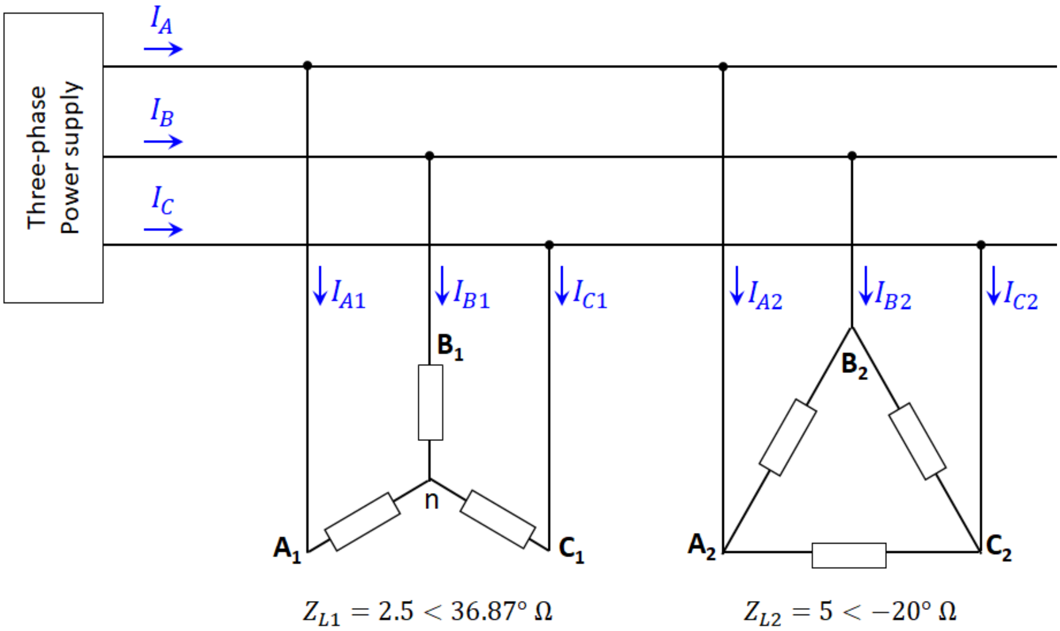
$$\boldsymbol{S}_L = 20.87 + j15.65 \quad KVA$$

c) Ratio of the line current of Δ connection to Y connection is:

$$\frac{I_{L\Delta}}{I_{L\lambda}} = \frac{72}{24} = 3$$

This is the case for any three phase balanced load.

2 A three phase power supply with line voltage of V_L=400 V supplies to balanced loads as shown in the schematic diagram. Load # 1 has phase impedance of $Z_{L1} = 2.5 < 36.87°\ \Omega$ with λ connection, and load # 2 has phase impedance of $Z_{L2} = 5 < -20°\ \Omega$ with Δ connection.



a) Find the line current of the source

b) Find the complex power of the source

Solution:

a) In this network first we need to find current of each load. Considering a positive phase sequence, line and phase voltages are:

Phase voltages	Line voltages
$V_A = 230 < 0°\ \ V$	$V_{AB} = 400 < 30°\ \ V$
$V_B = 230 < -120°\ \ V$	$V_{BC} = 400 < -90°\ \ V$
$V_C = 230 < 120°\ \ V$	$V_{CA} = 400 < 150°\ \ V$

For the Y connected load:

$$I_{A1} = \frac{V_A}{Z_{L1}} = \frac{230 < 0°}{2.5 < 36.86°} = 92 < -36.86° \quad A$$

$$I_{B1} = 92 < -156.86° \quad A \qquad I_{C1} = 92 < 83.14° \quad A$$

Therefore, power analysis of this load is:

$$P_{L1} = \sqrt{3}V_L I_L \cos\varphi = \sqrt{3} \times 400 \times 92 \times \cos(36.86°) = 51.2 \quad kW$$

$$Q_{L1} = \sqrt{3}V_L I_L \sin\varphi = \sqrt{3} \times 400 \times 92 \times \sin(36.86°) = 38.4 \quad kVAR$$

$$\boldsymbol{S}_{L1} = 51.2 + j38.4 \quad KVA$$

Similar analysis for the Δ connected load:
Phase currents:

$$I_{AB2} = \frac{V_{AB}}{Z_{p2}} = \frac{400 < 30°}{5 < -20°} = 80 < 50° \quad A$$

$$I_{BC2} = 80 < -70° \quad A \qquad I_{CA2} = 80 < 170° \quad A$$

Line currents:

$$I_{A2} = I_{AB2} - I_{CA2} = 80 < 50° - 80 < 170° = 138.55 < 20° \quad A$$

$$I_{B2} = I_{BC2} - I_{AB2} = 138.55 < -100° \quad A$$

$$I_{C2} = I_{CA2} - I_{BC2} = 138.55 < 140° \quad A$$

And power analysis of this load is:

$$P_{L2} = \sqrt{3}V_L I_L \cos\varphi = \sqrt{3} \times 400 \times 138.55 \times \cos(-20°) = 90.2 \quad kW$$

$$Q_{L2} = \sqrt{3}V_L I_L \sin\varphi = \sqrt{3} \times 400 \times 138.55 \times \sin(-20°) = -32.83 \quad kVAR$$

$$\boldsymbol{S}_{L2} = 90.2 - j32.83 \quad KVA$$

b) Total current of the source:

$$I_A = I_{A1} + I_{A2} = 92 < -36.86° + 138.55 < 20° = 203.95 < -2.2° \quad A$$

$$I_C = I_{B1} + I_{B2} = 203.95 < -122.2° \quad A$$

$$I_C = I_{C1} + I_{C2} = 203.95 < 117.8° \quad A$$

And finally power analysis of the source is:

$$P_{Lt} = \sqrt{3} V_L I_L \cos\varphi = \sqrt{3} \times 400 \times 203.95 \times \cos(2.2°) = 141.2 \quad kW$$

$$Q_{Lt} = \sqrt{3} V_L I_L \sin\varphi = \sqrt{3} \times 400 \times 203.95 \times \sin(2.2°) = 5.45 \quad kVAR$$

$$\boldsymbol{S}_{Lt} = 141.2 + j5.45 \quad KVA$$

Total power of the source can be also calculated by adding power of the individual load:

$$P_{Lt} = P_{L1} + P_{L2} \qquad\qquad Q_{Lt} = Q_{L1} + Q_{L2}$$

Negative sign of the total current I_A shows that the total impedance seen by the source has an inductive nature with lagging power factor.