Electrical Engineering 2

ENGI2191

Complex Power and Components

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E108

Lecture Objectives:

Aim of this lecture:

The aim of this lecture is to understand the concepts of complex power and its components in electrical networks.

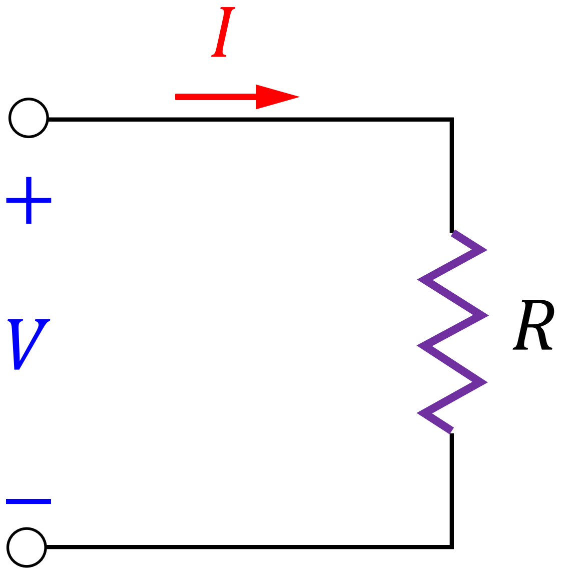
Intended Learning Outcomes:

At the completion of the lecture and associated problems you should be able to:

- Understand the fundamental concepts of complex power
- Identify the theoretical basis of complex power and its importance in electrical networks.
- Identify the concept of power factor in electrical system.

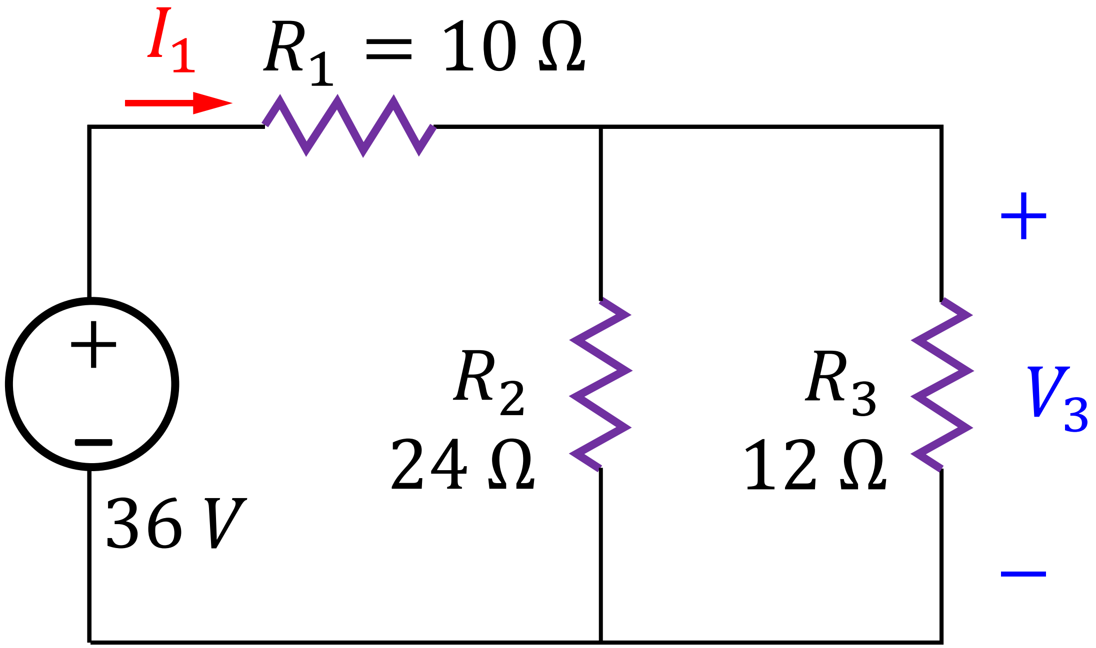
Complex Power and Components:

Power of a resistor:



$$P = V\,I \qquad [W]$$

$$V = R\,I \quad \ggg \quad \begin{cases} P = R\,I^2 & [W] \\ P = \dfrac{V^2}{R} & [W] \end{cases}$$

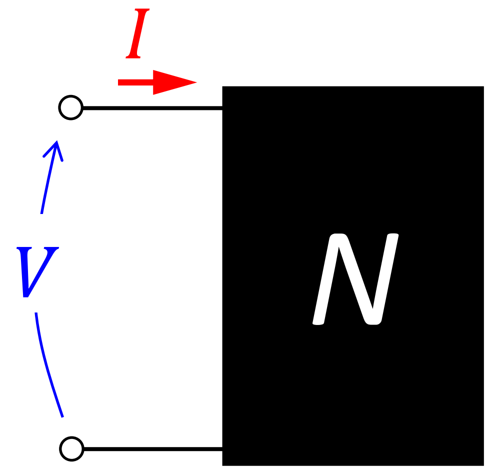


$$I_1 = 2\ A \qquad V_3 = 16\ V$$

$$\begin{cases} P_1 = R_1 (I_1)^2 = 40 & [W] \\ P_3 = \dfrac{(V_3)^2}{R_3} = 21.3 & [W] \end{cases}$$

Complex Power and Components:

In AC networks we deal with phasors, so what is the mechanism of power analysis?



"*Complex power*" of an element or a network is a complex number defined by:

$\boldsymbol{S} = \boldsymbol{V}\,\boldsymbol{I}^{*}$ $\Rightarrow$ $\boldsymbol{S} = V_{rms} < \theta_v \,.\, I_{rms} < -\theta_i$

$\boldsymbol{S} = V_{rms} I_{rms} < (\theta_v - \theta_i)$

$\boldsymbol{V} = V_{rms} < \theta_v$

$\boldsymbol{I} = I_{rms} < \theta_i$

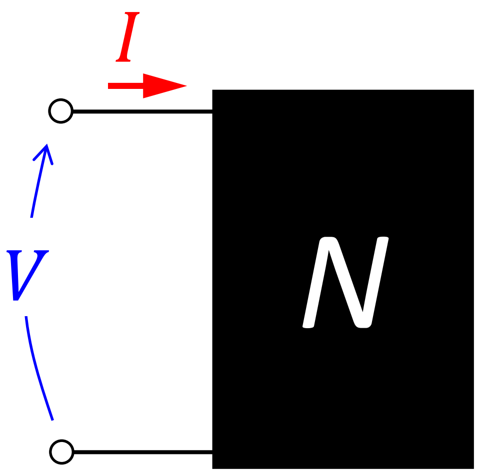
And in form of a complex number:

$$\boldsymbol{S} = V_{rms} I_{rms} \cos(\theta_v - \theta_i) + j\, V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

It is more convenience to define $\varphi = \theta_v - \theta_i$

$$\boldsymbol{S} = V_{rms} I_{rms} \cos(\varphi) + j\, V_{rms} I_{rms} \sin(\varphi)$$

Complex Power and Components:



$$\boldsymbol{V} = V_{rms} < \theta_v$$

$$\boldsymbol{I} = I_{rms} < \theta_i$$

$$\boldsymbol{S} = \underbrace{V_{rms} I_{rms} \cos(\varphi)}_{\text{Active power, P}} + j\, \underbrace{V_{rms} I_{rms} \sin(\varphi)}_{\text{Reactive power, Q}}$$

$$P = \mathrm{Re}\,[\mathbf{S}] = V_{rms} I_{rms} \cos(\varphi)$$ **W, kW, MW**

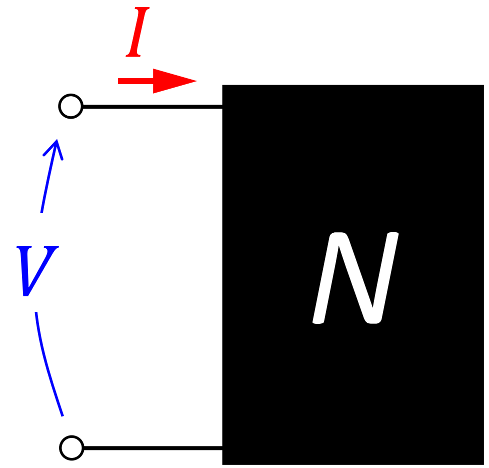
$$Q = \mathrm{Im}\,[\mathbf{S}] = V_{rms} I_{rms} \sin(\varphi)$$ **VAR, kVAR, MVAR**

$$\boldsymbol{S} = P + jQ$$

The magnitude of the complex power *S* is defined as apparent power:

$$S = |\boldsymbol{S}| = \sqrt{P^2 + Q^2} = V_{rms}\, I_{rms}$$ **VA, kVA, MVA**

Complex Power and Components:



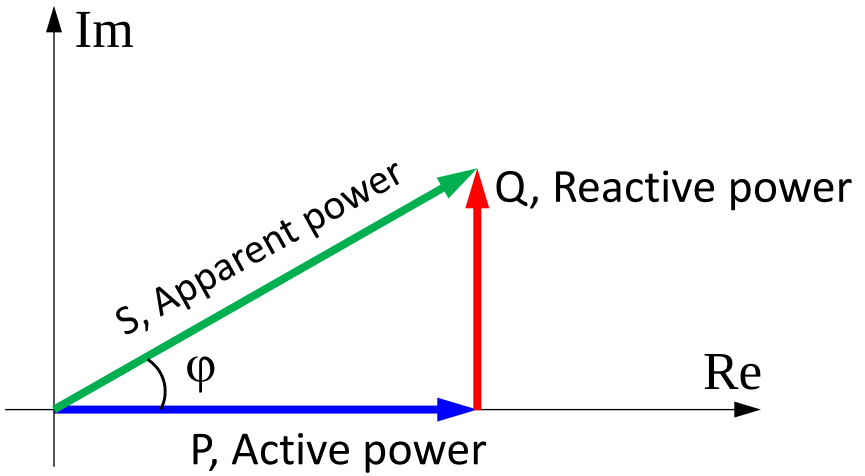
$$\boldsymbol{V} = V_{rms} < \theta_v$$

$$\boldsymbol{I} = I_{rms} < \theta_i$$

$$\boldsymbol{S} = V_{rms}I_{rms}\ \cos(\varphi) + j\ V_{rms}I_{rms}\ sin\ (\varphi)$$

$$\boldsymbol{S} = P + jQ$$

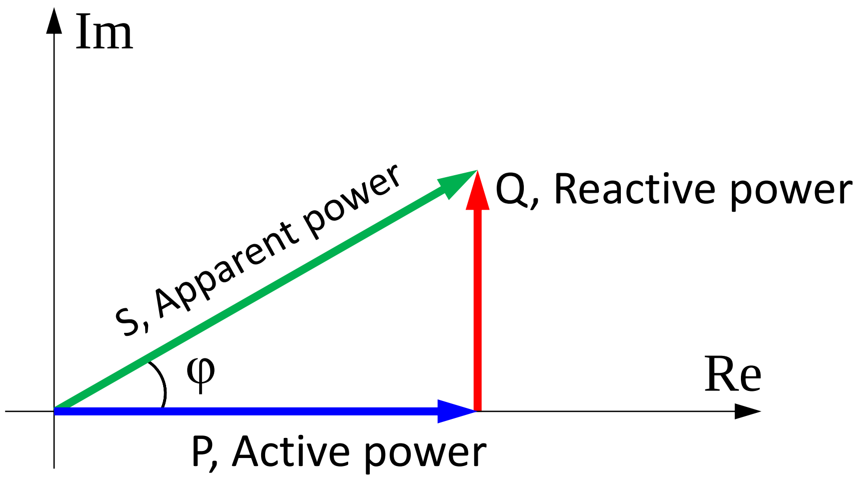
The relationship between complex, active and reactive power is shown by a phasor diagram, known as “power triangle”:

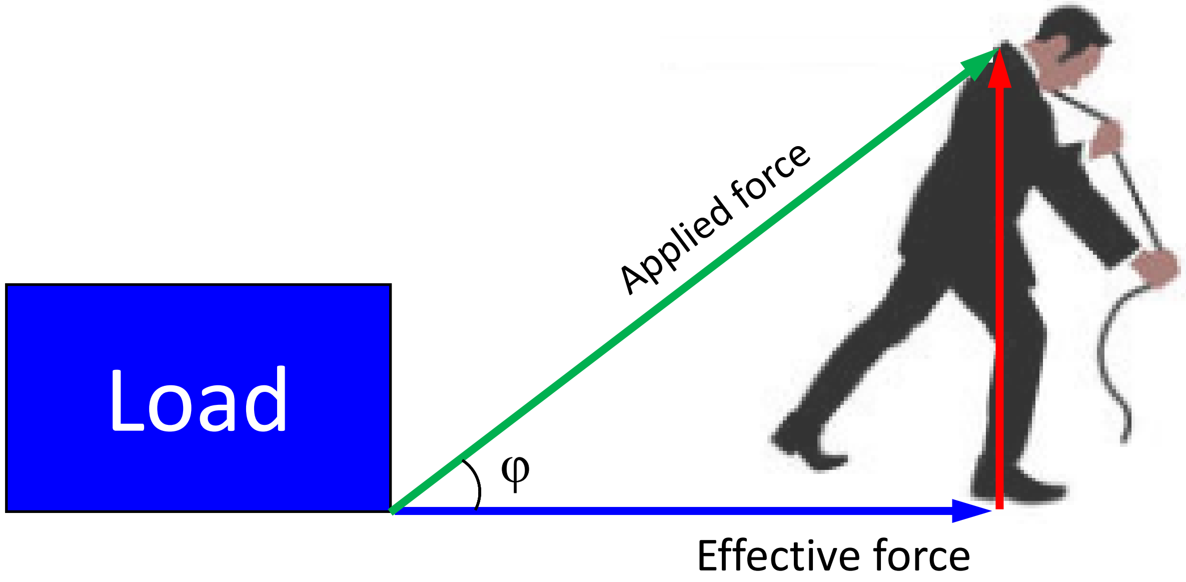


$$\begin{cases} P = S\ \cos(\varphi) \\ Q = S\ \sin(\varphi) \end{cases}$$

Complex Power and Components:

Physical concept of complex power:



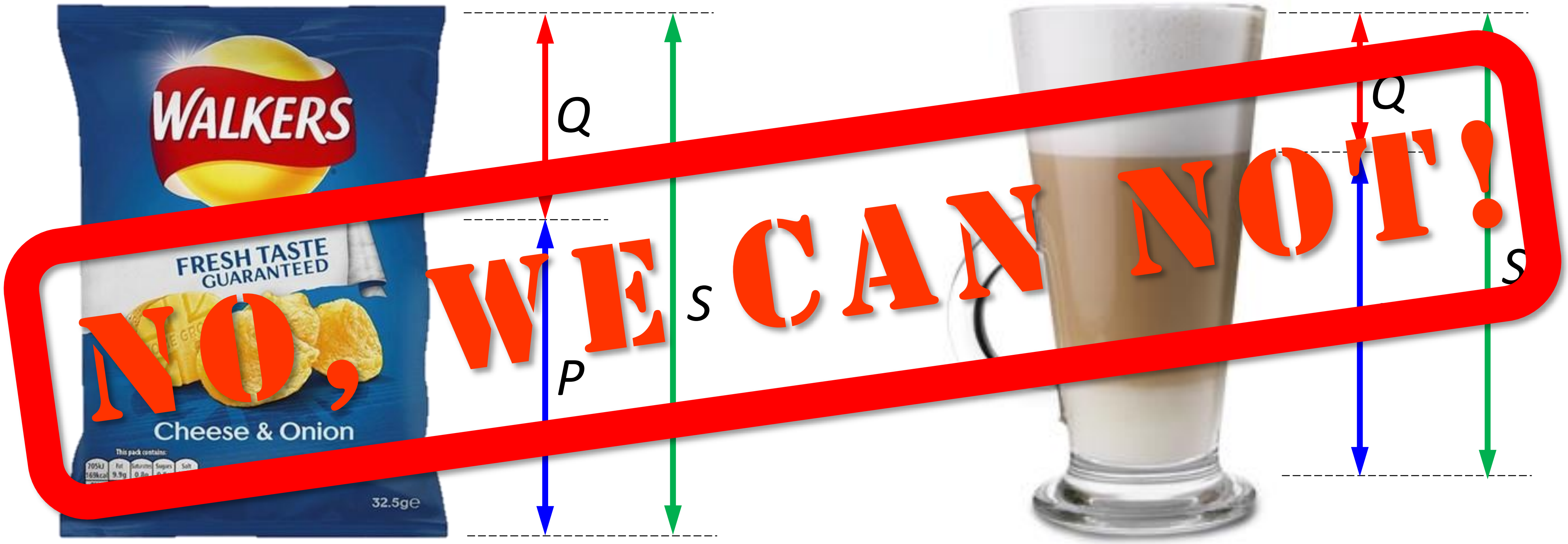


$$\text{Active power} = S \times \cos(\varphi)$$

$$\text{Effective Force} = F \times \cos(\varphi)$$

Complex Power and Components:

Can we say this?



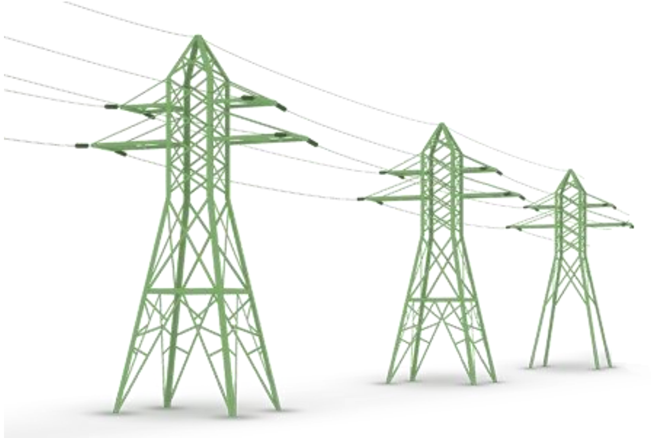
How much crisp you can get from the bag?

How much coffee you can get from the mug?

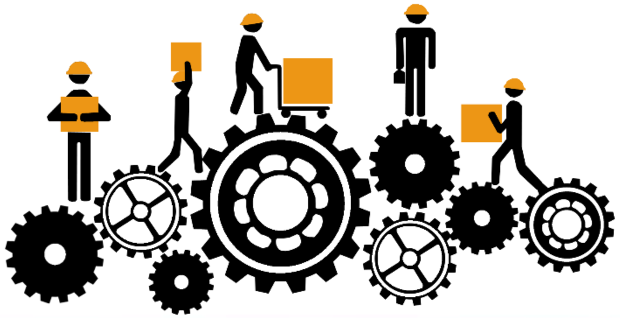
Complex Power and Components:

We need reactive power to generate electricity at the power stations.

We need reactive power to transfer electricity to the national grid.



We need reactive power to run industry.



Complex Power and Components:

Power Factor (*p.f.*)

The ratio of active power *p* to apparent power *S* is known as power factor (*p.f.*) and calculated by:

$$p.f. = \frac{P}{S} = \frac{V_{rms} I_{rms}\ \cos(\theta_v - \theta_i)}{V_{rms} I_{rms}} = \cos(\theta_v - \theta_i) = \cos\varphi$$

Power factor is a measure of how efficiently electrical power is converted into useful work.

The phase angle $(\theta_v - \theta_i)$ is angle of the load impedance and is known as the *power factor angle*.

$$\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{V_{rms} < \theta_v}{I_{rms} < \theta_i} = Z < (\theta_v - \theta_i) = Z < \varphi$$

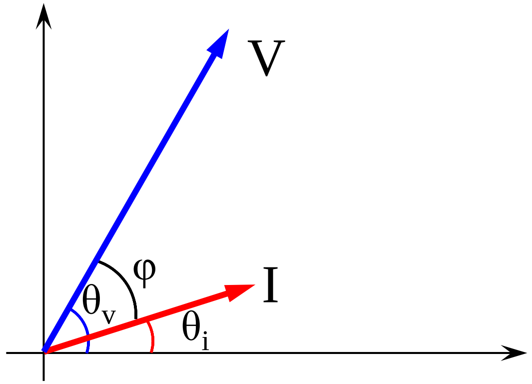
The ideal power factor is unity, or one, that can be achieved with pure resistive loads.

Unity power factor in a dream that never came true!

Complex Power and Components:

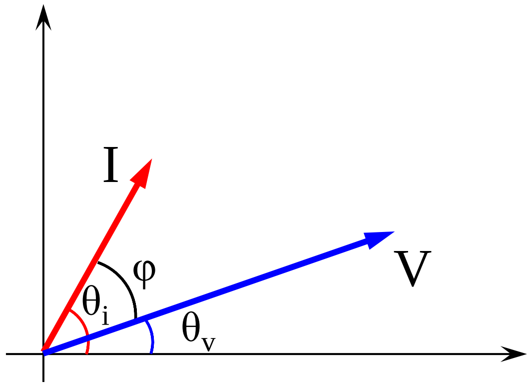
p.f. is lagging in inductive loads

$$(\theta_v - \theta_i) > 0$$



p.f. is leading in capacitive loads

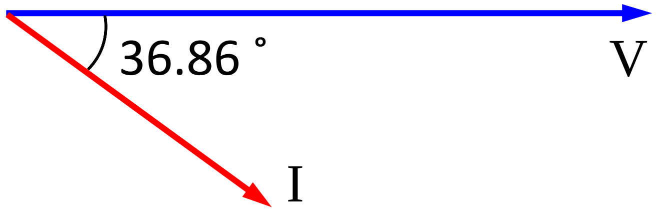
$$(\theta_v - \theta_i) < 0$$



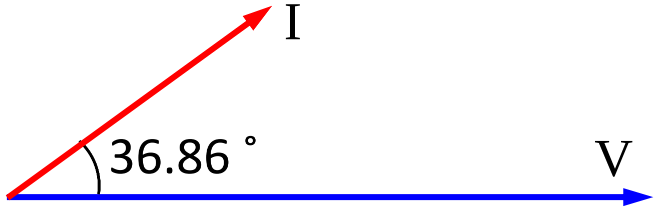
Understanding power factor is significantly important, because it conveys the "*load behaviour*".

$$\begin{cases} p.f. = 0.8 \\ \varphi = \cos^{-1}(0.8) = 36.86^\circ \end{cases}$$

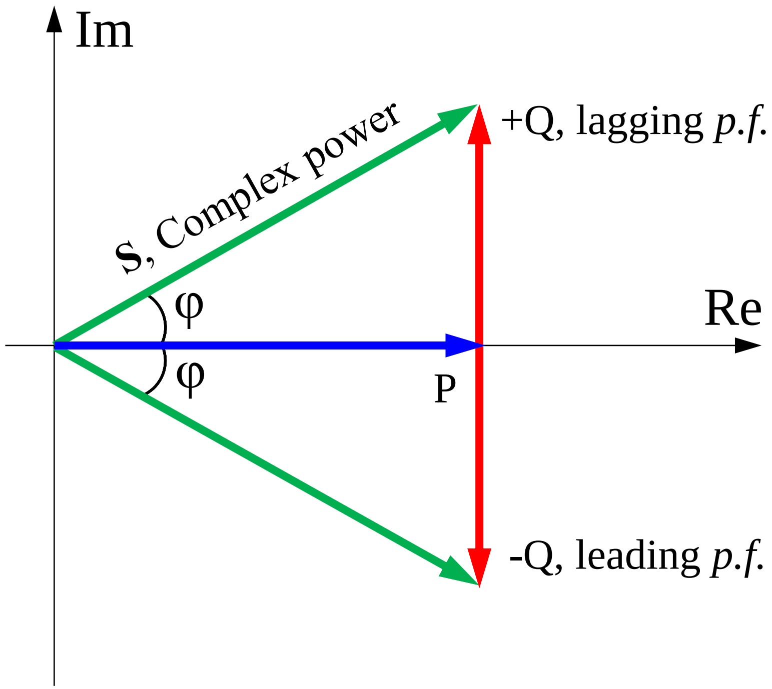
Lagging

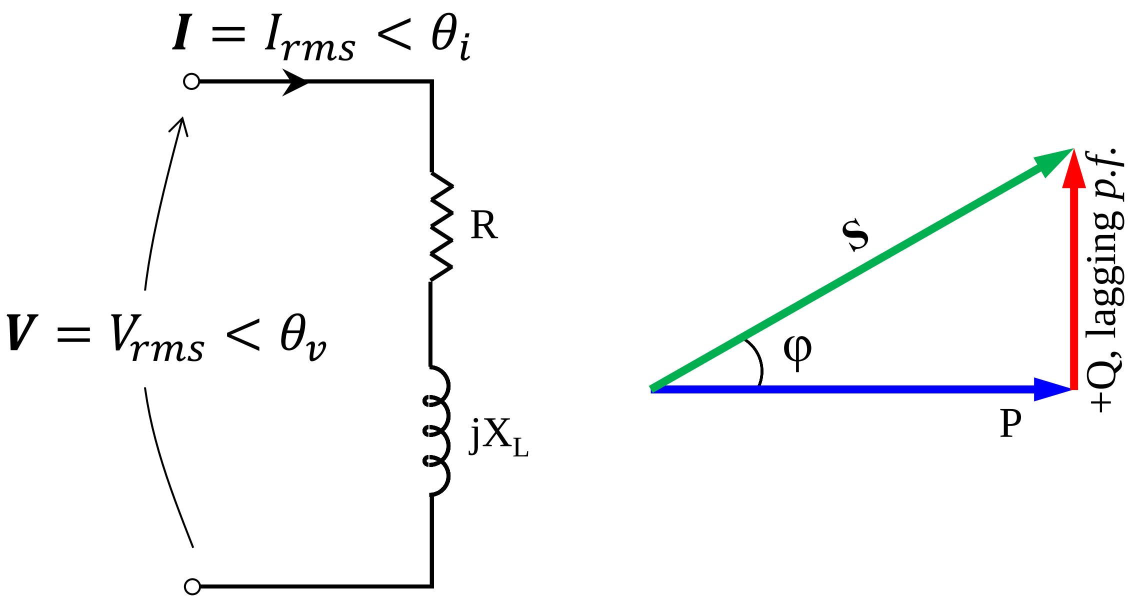


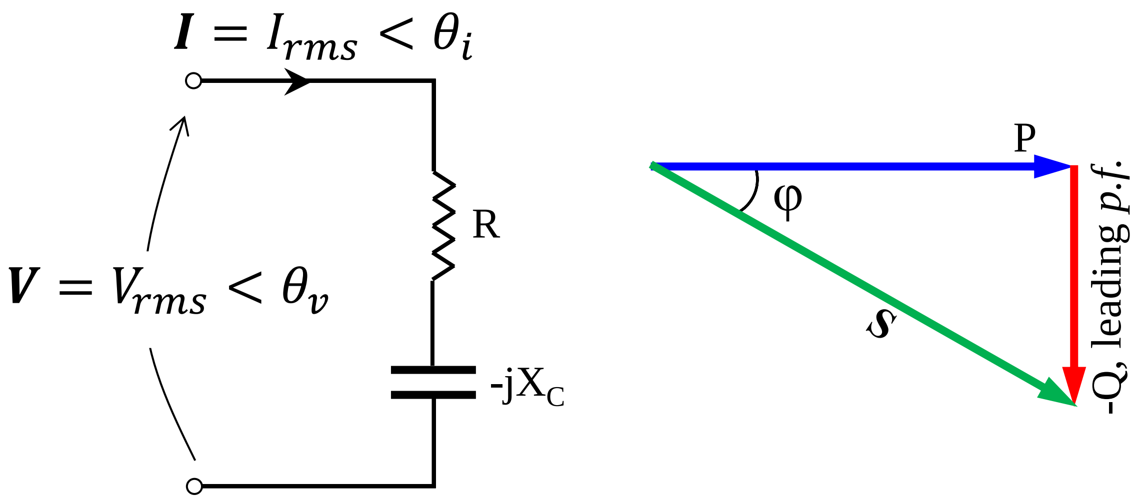
Leading



Complex Power and Components:



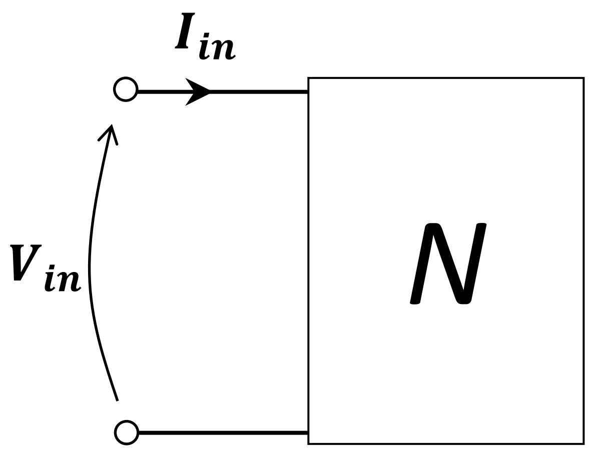




Complex Power and Components:

Example:

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{75\angle 60^\circ}{25\angle 30^\circ} = 3\angle 30 \quad [\Omega]$$

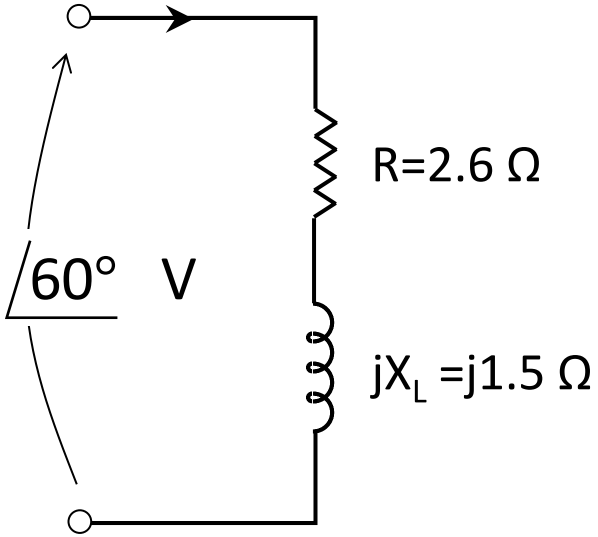


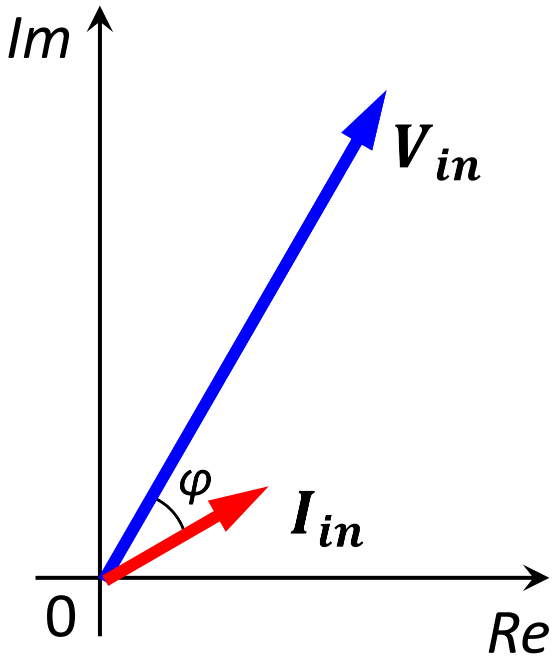
$$Z_{in} = 2.6 + j1.5 \quad [\Omega]$$

$$V_{in} = 75\angle 60^\circ \quad \text{V}$$

$$I_{in} = 25\angle 30^\circ \quad \text{A}$$

$$I_{in} = 25\angle 30^\circ \quad \text{A}$$

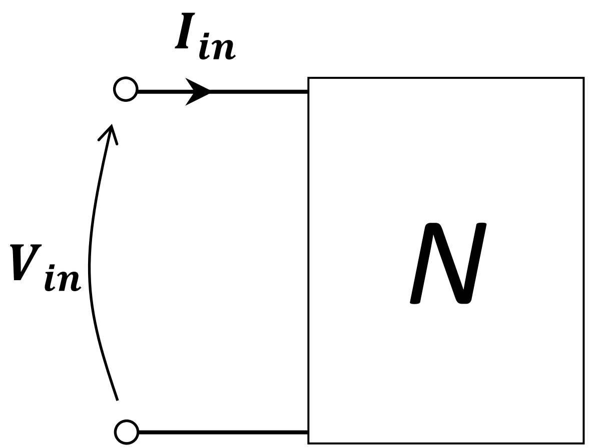




$$p.f. = 0.866 \quad lagging$$

Complex Power and Components:

Example:



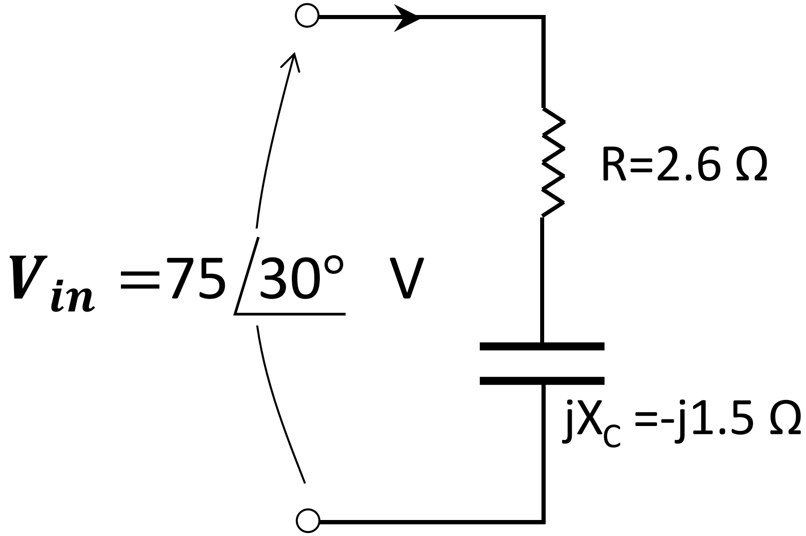
$$V_{in} = 75\ \angle 30° \quad \text{V}$$

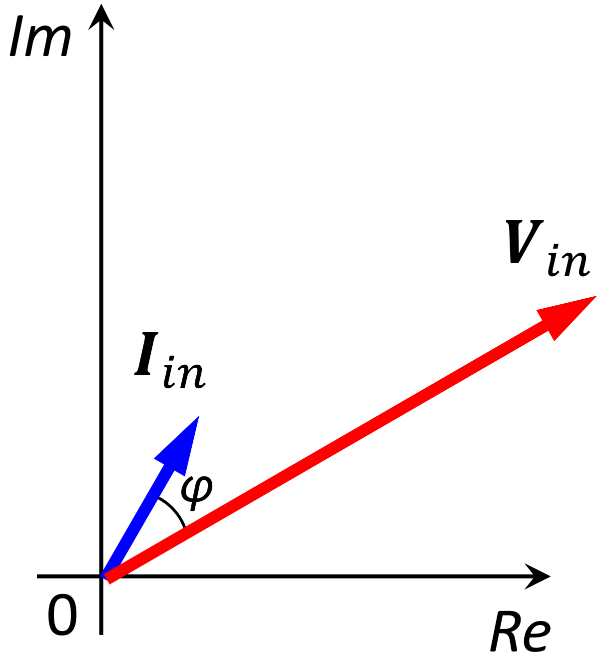
$$I_{in} = 25\ \angle 60° \quad \text{A}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{75\ \angle 30°}{25\ \angle 60°} = 3\angle -30 \qquad [\Omega]$$

$$Z_{in} = 2.6 - j1.5 \qquad [\Omega]$$

$$I_{in} = 25\angle 60° \quad \text{A}$$



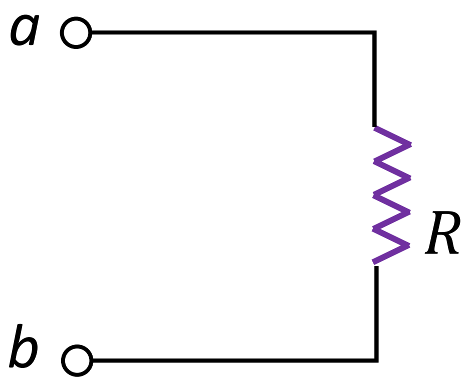


$$p.f. = 0.866 \quad leading$$

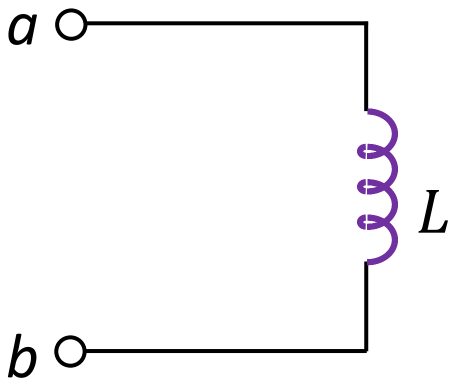
Complex Power and Components:

Quiz:

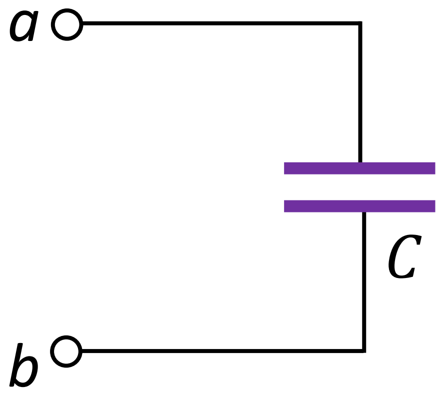
- How much is the reactive power in a pure resistor?
- How much is active power in a pure inductor and a pure capacitor?



Ideal resistor



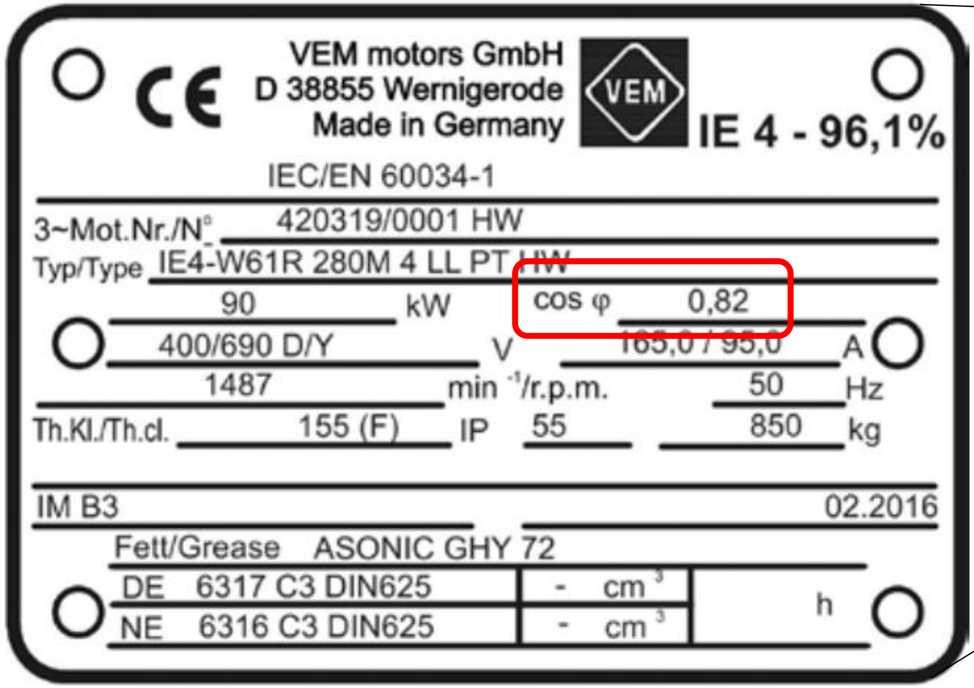
Ideal inductor



Ideal capacitor

Complex Power and Components:

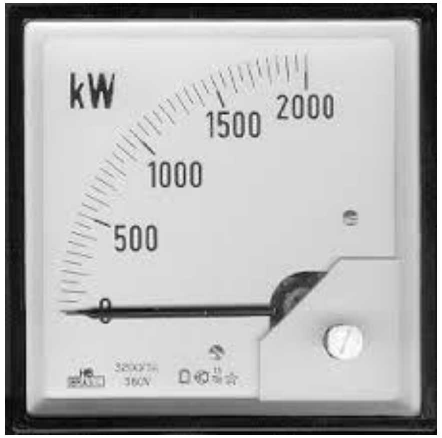
Apart from rated voltage, power, current and frequency, **power factor** is also an important parameter to characterise inductive loads, e.g. an electric motor.



Durham University

Electrical Measuring Instruments

Complex Power and Components:



Complex Power and Components:

Electric meter is a measuring device to measure <u>ELECTRIC ENERGY</u> in kWh.

Electromagnetic meter

Digital meter

Smart meter

Complex Power and Components:

KVARh

Reactive Energy, KVARh

KWh

Active Energy, KWh

Electromechanical meters

Digital meter

Example:

An industrial load with apparent power of 20 kVA and power factor of 0.65 (lagging) is supplied by a voltage source with *rms* voltage of 240 V and frequency of 50 Hz. Find the following values:

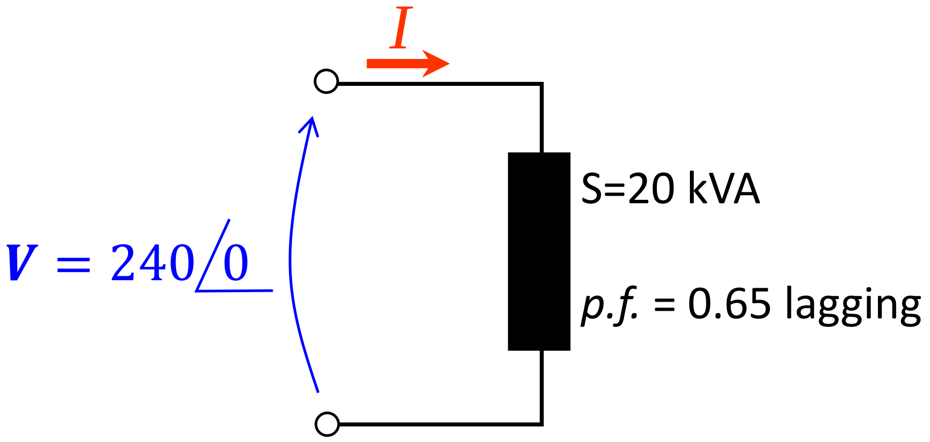
a) Load current

b) Load impedance

c) Complex, active and reactive power

d) Draw the power triangle of the load

Solution:

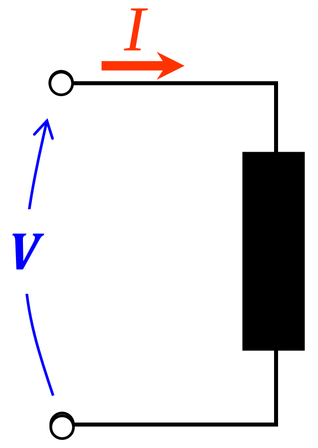


$$S = V_{rms}\, I_{rms} \ggg I_{rms} = \frac{S}{V_{rms}} = \frac{20\times 10^3}{240} = 83.33\ A$$

$$\text{p.f.} = \cos(\varphi) = 0.65 \ggg \begin{cases} \varphi = \cos^{-1}(0.65) \\ \varphi = 49.45\,^{\circ} \end{cases}$$

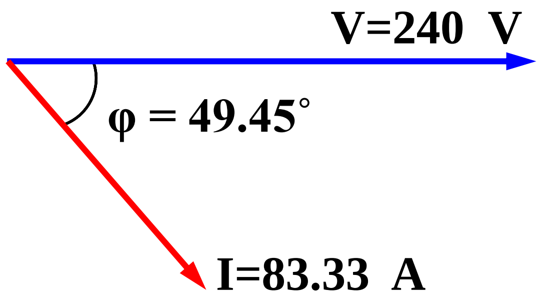
$$\boldsymbol{I} = 83.33\ \angle -49.45\ \ A$$

Example:



$$\boldsymbol{V} = 240\angle 0 \quad \text{V}$$

$$\boldsymbol{I} = 83.33\ \angle -49.45\ \ A$$

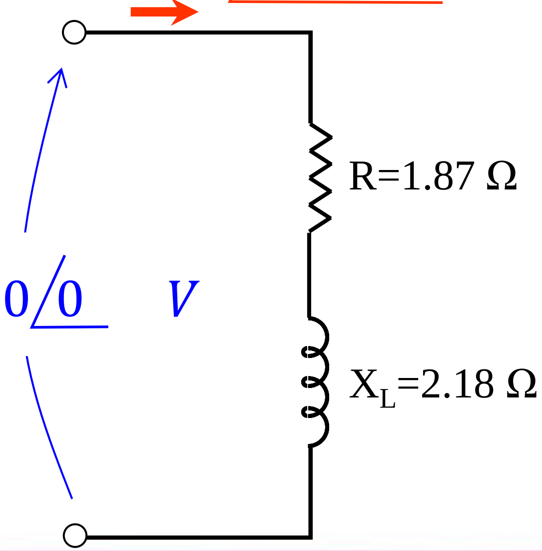


$$\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{240\angle 0}{83.33\ \angle -49.45} = 2.88\angle 49.45\ \ \Omega$$

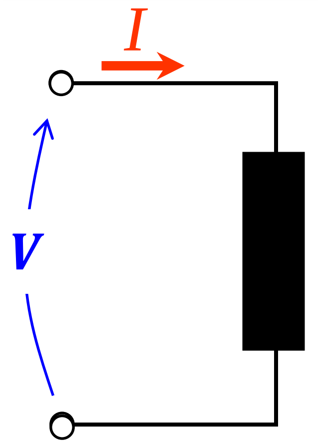
$$\boldsymbol{Z} = 2.88\angle 49.45 = 1.87 + j2.18\ \ \Omega \quad \begin{cases} R = 1.87\ \ \Omega \\ X_L = 2.18\ \ \Omega \end{cases}$$

$$\boldsymbol{I} = 83.33\ \angle -49.45\ \ A$$

$$\boldsymbol{V} = 240\angle 0 \quad V$$



Example:



$$\boldsymbol{V} = 240\angle 0 \quad \text{V}$$

$$\boldsymbol{I} = 83.33\angle -49.45 \quad A$$

$$\boldsymbol{S} = \boldsymbol{V}\,\boldsymbol{I}^*$$

$$\boldsymbol{S} = V_{rms}\angle\theta_v\ I_{rms}\angle -\theta_i$$

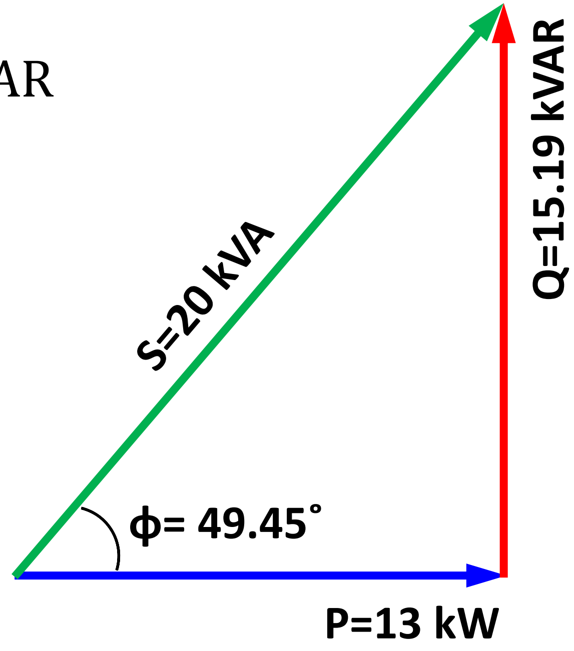
$$\boldsymbol{S} = (240 < 0)(83.33 < 49.45)$$

$$\boldsymbol{S} = 20 < 49.45° \qquad kVA$$

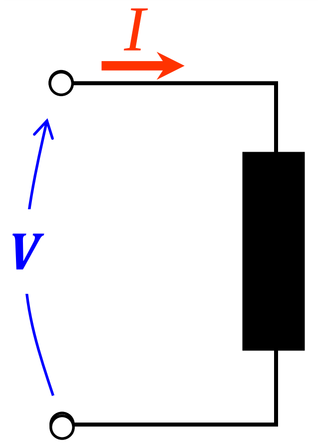
$$\boldsymbol{S} = 20 < 49.45° \qquad kVA$$

$$\boldsymbol{S} = 13 + j15.19 \qquad kVA$$

$$\begin{cases} P = 13\ \text{kW} \\ Q = 15.19\ \ \text{kVAR} \end{cases}$$



Example:

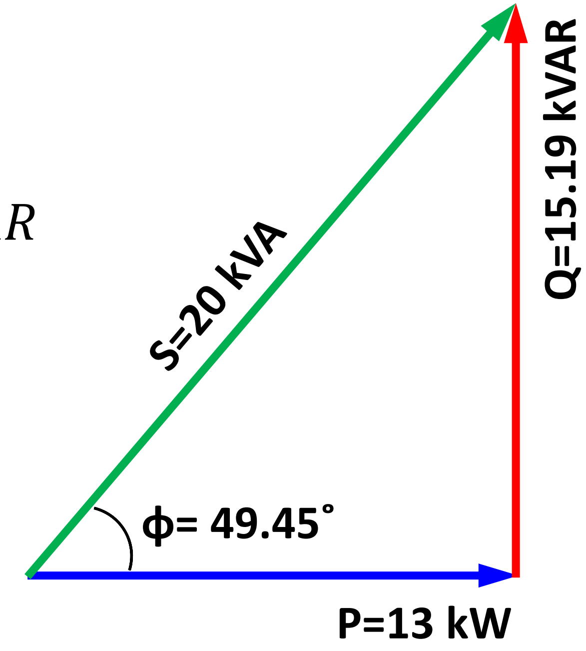


$$\boldsymbol{V} = 240\angle 0 \quad \text{V}$$

$$\boldsymbol{I} = 83.33\ \angle -49.45\ \ A$$

$$\begin{cases} P = V_{rms} I_{rms}\ \cos(\varphi) = 240 \times 83.33 \times \cos(49.45) = 13\ kW \\ Q = V_{rms} I_{rms}\ \sin(\varphi) = 240 \times 83.33 \times \sin(49.45) = 15.19\ kVAR \end{cases}$$

$$\begin{cases} P = S\ \cos(\varphi) = 20 \times \cos(49.45) = 13\ kW \\ Q = S \sin(\varphi) = 20 \times \sin(49.45) = 15.19\ kVAR \end{cases}$$

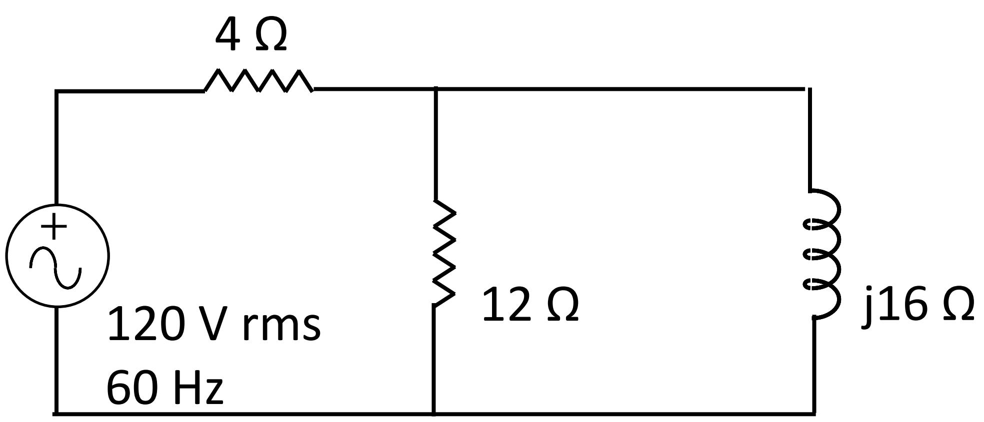


Example:

Drill:

Consider the following circuit:

a) Find the operating power factor of the voltage source
b) Find the current of the voltage source
c) Find the active and reactive power of the voltage source
d) Draw the power triangle



Answer:

$p.f. = 0.89$

$I_s = 9.21\ \angle -26.25^{o}\ \text{A}$

Reading list:

Recommended text books:

- DeCarlo Lin, “*Linear Circuit Analysis*”, Oxford University Press, Second Edition, 2003
- W H Hayt, J E Kemmerly, S M Durbin, “*Engineering Circuit Analysis*”, McGraw-Hill, 9th Edition, 2019