

Worst case

$$\begin{aligned}
 T(n) &= T(n-1) + cn \\
 &= T(n-2) + c(n-1) + cn \\
 &= T(n-3) + c(n-2) + c(n-1) + cn \\
 &\quad \dots \\
 &= (c + 2c + 3c + \dots + c(n-1) + cn) \\
 &= c(1+2+3+\dots+n) \\
 &= c \frac{n(n+1)}{2} = \Theta(n^2)
 \end{aligned}$$

Best case

$$\begin{aligned}
 T(n) &= T(n/2) + T(n-1-\frac{n}{2}) + cn \\
 &= T(n/2) + T(n/2-1) + cn \\
 &= T(n/2) + T(n/2) + cn
 \end{aligned}$$

According to master theorem, $a=2$ $b=2$ $d=1 \Rightarrow \Theta(n \lg n)$

Average case

$$\begin{aligned}
 T(n) &= T(n-k) + T(k) \\
 &= \frac{1}{n} \left(\sum_{i=1}^{n-1} T(i) + \sum_{i=1}^{n-1} T(n-i) \right) \\
 &= \frac{2}{n} \sum_{i=1}^{n-1} T(i)
 \end{aligned}$$

$$nT(n) = 2 \sum_{i=1}^{n-1} T(i) \quad (1)$$

$$(n-1)T(n-1) = 2 \sum_{i=1}^{n-2} T(i) \quad (2)$$

$$(1) - (2) \quad nT(n) - (n-1)T(n-1) = 2T(n-1) + n^2 - c(n-1)^2$$

$$nT(n) = (n+1)T(n-1) + 2cn$$

$$\frac{T(n)}{n+1} = \frac{T(n-1)}{n} + \frac{2c}{n+1}$$

$$\frac{T(n-1)}{n} = \frac{T(n-2)}{n-1} + \frac{2c}{n}$$

$$\frac{T(n)}{n+1} = \frac{T(n-2)}{n-1} + \frac{2c}{n+1} + \frac{2c}{n}$$

$$\frac{T(n-2)}{n-1} = \frac{T(n-3)}{n-2} + \frac{2c}{n-1}$$

⋮

$$\frac{T(n)}{n+1} = \frac{T(1)}{2} + 2c \left[\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \frac{1}{n+1} \right]$$

$$T(n) = 2c \log_2 n(n+1) \quad \Theta(n \log_2 n)$$