Fast and Accurate Modeling of Parametric Array Loudspeakers in the Frequency Domain

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PennState
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Outline

- Introduction
- Properties of sound fields generated by a PAL
- Computational models
- Conclusions and future work

(Evaluate my presentation for the Early Career Presenter Competition)

- PAL: radiates only ultrasound!
- Mechanism: **nonlinear interactions** of **intense** ultrasonic waves (e.g., 130 dB)

$$f_1, f_2 \xrightarrow{\text{second order}} \boxed{f_2 - f_1}, f_1 + f_2, 2f_1, 2f_2$$

- $f_1 = 60\,\text{kHz}, f_2 = 61\,\text{kHz}$, $\boxed{f_2 - f_1 = 1\,\text{kHz}}$
- Sharp directivity [1]

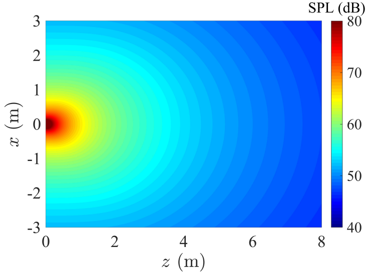
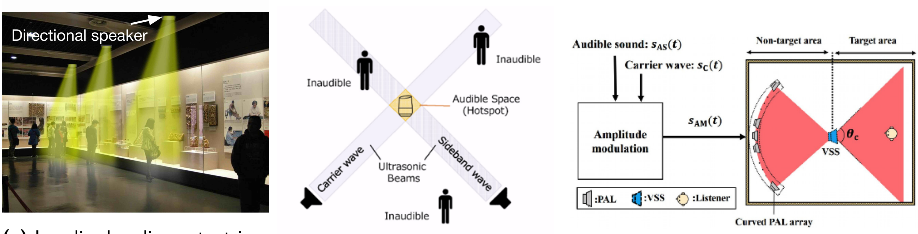


Fig 1. SPL distribution at 1 kHz: (left) a conventional loudspeaker; (right) a PAL with an aperture size of 0.1 m ($\underline{0.3\lambda_\text{a}}$)

[1] W.-S. Gan et al. In: **Appl. Acoust.** 73.12 (2012), pp. 1211–1219

Applications of PALs (1/2)

- Communications with a high level privacy [1]
- Remotely attack on voice assistant systems (Siri, Google Assistant) [2]
- Wave field synthesis [3]



(a) Localized audio content in museums and art galleries [1]

(b) Audio hotspot attack [2]

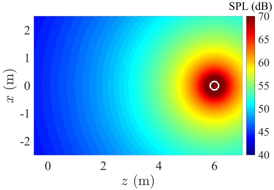
(c) Virtual sound source [3]

Fig 2. Selected examples of applications of PALs

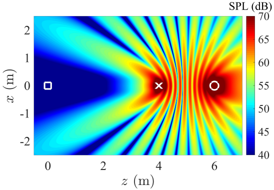
1. D. Ortega et al. In: EuroNoise 2021. Madeira, Portugal, 2021
2. R. Iijima et al. In: **IEEE Trans. Emerg.** 9.4 (2019), pp. 2004–2018
3. Y. Ogami et al. In: **J. Acoust. Soc. Am.** 146.2 (2019), pp. 1314–1325

Using PALs in active noise control (ANC) systems

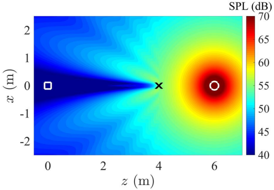
- **Spillover effect**: the noise around the target (error) point is reduced (quiet zone), but **the noise in the other areas is amplified**!
- Reason: the **omni-directivity** of conventional loudspeakers
- Solution: using **directional** loudspeakers
- Single [1] and multi [2] channel ANC systems



(a) Primary (noise) sound generated by a point source



(b) Controlled by a **conventional loudspeaker**



(c) Controlled by a **parametric array loudspeaker**

Fig 3. Sound pressure level (SPL) distributions at 1 kHz. ○: noise source; □: target point; ×: secondary source.

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1. N. Tanaka et al. In: **J. Acoust. Soc. Am.** 127.6 (2010), pp. 3526–3537
2. J. Zhong et al. In: **J. Acoust. Soc. Am.** 151.2 (2022), pp. 1235–1245

Fast and accurate modeling of PALs in the frequency domain

- Complexity due to the nonlinear process
- Many terms
    - Near field, Westervelt far field, inverse-law far field
    - Local effects, cumulative effects, Lagrangian density
    - KZK equation, Westervelt equation, Kuznetsov equation
    - Paraxial approximation, quasilinear approximation
    - ......
- **No versatile modeling methods for all applications**
- Many papers published recently yet no review

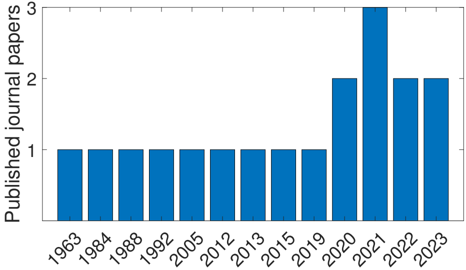


Fig 4. Important published papers on the modeling methods for PALs.

Sound fields generated by a PAL

Sound fields on **front side** [1]

- **Near field**: second-order nonlinear or Kuznetsov equation (local effects are dominant)
- **Westervelt far field**: Westervelt equation (local effects are negligible)
- **Inverse-law far field**: $p_a \propto 1/r$
- Select appropriate methods in the region of interest

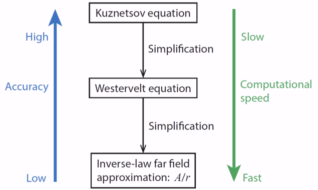


Fig 5. Accuracy and computational speed

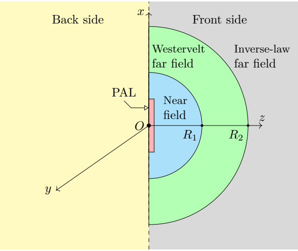


Fig 6. Regions of sound fields

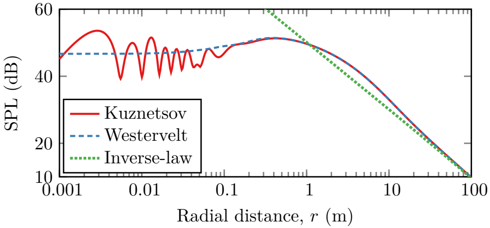


Fig 7. Axial audio SPL at 1 kHz

1 J. Zhong et al. In: J. Acoust. Soc. Am. 149.3 (2021), pp. 1524–1535

Governing equations and quasilinear approximation

- Second-order nonlinear equation [1]

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)p + \frac{\delta}{c_0^4}\frac{\partial^3 p}{\partial t^3} = -\frac{\beta}{\rho_0 c_0^4}\frac{\partial^2 p^2}{\partial t^2} - \left(\nabla^2 + \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\mathcal{L} \qquad (1)$$

- **Lagrangian density**: $\mathcal{L} = \rho_0 \mathbf{v}\cdot\mathbf{v}/2 - p^2/(2\rho_0 c_0^2)$
- **Westervelt equation**: $\mathcal{L} = 0$
- **Quasilinear approximation**: nonlinear equation $\Longrightarrow$ two coupled linear equations
  - Condition: ultrasound < 130 dB
  - When $\mathcal{L} = 0$ in the frequency domain

$$\begin{cases} \left(\nabla^2 + k_\mathrm{u}^2\right)p_\mathrm{u} = 0, \quad \mathrm{u} = 1, 2, & \text{(Ultrasound)} \\ \left(\nabla^2 + k_\mathrm{a}^2\right)p_\mathrm{a} = \beta k_\mathrm{a}^2 \dfrac{p_1^* p_2}{\rho_0 c_0^2}, & \text{(Audio sound)} \end{cases} \qquad (2)$$

  - When $\mathcal{L} \neq 0$ [2]

$$\tilde{p}_\mathrm{a} = p_\mathrm{a} - \left[\frac{\rho_0}{2}\mathbf{v}_1^*(\mathbf{r})\cdot\mathbf{v}_2(\mathbf{r}) - \left(\frac{\omega_1}{\omega_2} + \frac{\omega_2}{\omega_1} - 1\right)\frac{p_1^*(\mathbf{r})p_2(\mathbf{r})}{2\rho_0 c_0^2}\right] \qquad (3)$$

1. S. I. Aanonsen et al. In: **J. Acoust. Soc. Am.** 75.3 (1984), pp. 749–768
2. M. Červenka et al. In: **J. Acoust. Soc. Am.** 151.6 (2022), pp. 4046–4052

2D model:

- $u_\mathrm{u}(y)$ — Source profile; $H_0(k_\mathrm{u}\rho)$ — Hankel function; $*$ — Convolution
- Complexity: 3-fold integral (convolution)

$$\text{Ultrasound:} \quad p_\mathrm{u}(\boldsymbol{\rho}) \propto u_\mathrm{u}(y) * H_0(k_\mathrm{u}\sqrt{x^2+y^2}) \tag{4}$$

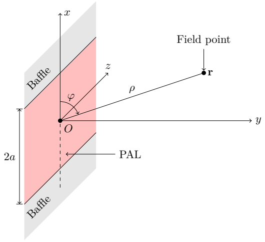
$$\text{Audio:} \quad p_\mathrm{a}(\boldsymbol{\rho}) \propto (p_1^* p_2)(\boldsymbol{\rho}) ** H_0(k_\mathrm{a}\sqrt{x^2+y^2}) \tag{5}$$

3D model:

- $\mathrm{h}_0(k_\mathrm{u}r) = \mathrm{e}^{\mathrm{i}k_\mathrm{u}r}/(\mathrm{i}k_\mathrm{u}r)$ — Spherical Hankel function
- Complexity: 5-fold integral (convolution) (**Time-consuming!**)

$$\text{Ultrasound:} \quad p_\mathrm{u}(\mathbf{r}) \propto u_i(x,y) ** \mathrm{h}_0(k_\mathrm{u}\sqrt{x^2+y^2+z^2}) \tag{6}$$

$$\text{Audio:} \quad p_\mathrm{a}(\mathbf{r}) \propto (p_1^* p_2) *** \mathrm{h}_0(k_\mathrm{a}\sqrt{x^2+y^2+z^2}) \tag{7}$$



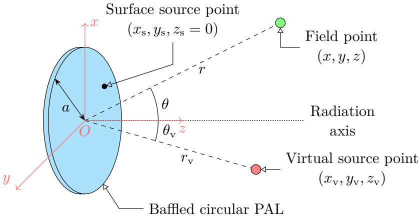


Fig 8. Sketch of (left) 2D and (right) 3D models

Finite Element Method (FEM)

Development

- First FEM model for modeling PALs [1]
- COMSOL Multiphysics [2–3]

Pros

- Valid in the near field and Westervelt far field
- Accurate and versatile

Cons

- Require large memory
    - Wavelength of the ultrasound frequency: 5.7 mm to 8.58 mm (40 kHz to 60 kHz)
    - Dimensions of region of interests: **100 to 1000 wavelengths** (1 m to 5 m)
- Very time-consuming
- **Almost impossible** for 3D problems

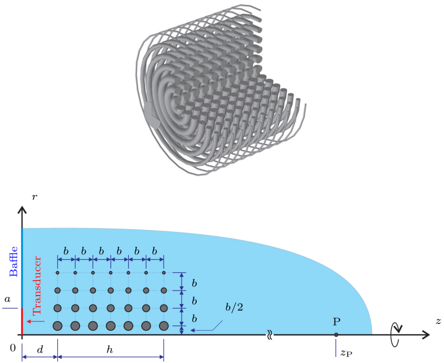


Fig 9. Above, a gradient-index phononic crystal (PC); bottom, sketch of a PAL lensed by a PC [3]

1. Y. Kagawa et al. In: **J. Sound Vib.** 154.1 (1992), pp. 125–145
2. M. Červenka et al. In: **J. Acoust. Soc. Am.** 146.4 (2019), pp. 2163–2169
3. M. Červenka et al. In: **J. Acoust. Soc. Am.** 149.6 (2021), pp. 4534–4542

Spherical Wave Expansion (SWE) Method

Decomposing 3D Green's functions into **spherical harmonics**

$$\mathrm{h}_0(k_\mathrm{u}R) = \frac{\mathrm{e}^{\mathrm{i}k_i R}}{\mathrm{i}k_\mathrm{u}R} = 4\pi \sum_{\ell=0}^{\infty} \mathrm{j}_\ell(k_\mathrm{u} r_{\mathrm{s},<}) \mathrm{h}_\ell(k_\mathrm{u} r_{\mathrm{s},>}) \sum_{m=-\ell}^{\ell} Y_\ell^m(\theta,\varphi) Y_\ell^{m,*}(\theta_\mathrm{s},\varphi_\mathrm{s}), \tag{8}$$

- Circular piston source, linear radiation [1–2]
- Circular piston source, Westervelt far field [3]
- Circular piston source, near field [4]
- Circular source with an **arbitrary profile** (e.g., steerable PALs), near field [5]

Pros:

- Valid in the full field
- 100–500 times faster than the DIM **without loss of accuracy**

Cons:

- Inefficient for rectangular sources

1. T. D. Mast et al. In: **J. Acoust. Soc. Am.** 118.6 (2005), pp. 3457–3464
2. J. Zhong et al. In: **J. Theor. Comp. Acoust.** 28 (2020), p. 2050026
3. J. Zhong et al. In: **J. Acoust. Soc. Am.** 147.5 (2020), pp. 3502–3510
4. J. Zhong et al. In: **J. Acoust. Soc. Am.** 149.3 (2021), pp. 1524–1535
5. J. Zhong et al. In: **J. Acoust. Soc. Am.** 152.4 (2022), pp. 2296–2308

Decomposing 2D Green's functions into **cylindrical harmonics** [1]

$$H_0(k_\mathrm{u}|\boldsymbol{\rho}-\boldsymbol{\rho}_\mathrm{s}|)=\sum_{m=-\infty}^{\infty} J_m(k_\mathrm{u}\rho_{\mathrm{s},<})H_m(k_\mathrm{u}\rho_{\mathrm{s},>})\mathrm{e}^{\mathrm{i}m(\varphi-\varphi_\mathrm{s})} \tag{9}$$

Pros:

- Valid in the full field
- Faster than the 2D DIM without loss of accuracy
- Arbitrary profile (e.g., steerable PALs)

Cons:

- Applicable only to 2D physical problems

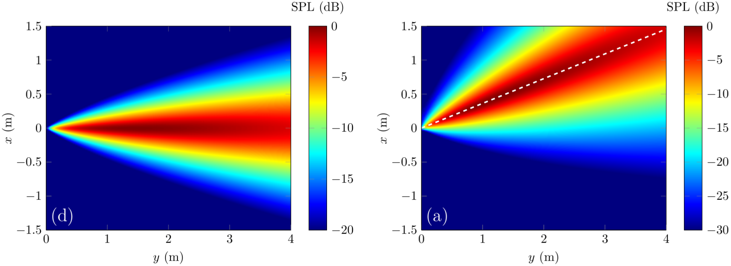


Fig 10. Audio SPL generated by a PAL with a (left) uniform profile and a (right) steerable profile

[1] J. Zhong et al. In: J. Acoust. Soc. Am. 150.5 (2021), pp. 3797–3806

Gaussian Beam Expansion (GBE) Method

Paraxial approximation and decomposing source profiles into **Gaussian functions**

- Circular piston source, linear radiation [1]
- Circular piston source, PAL [2]
- Rectangular piston source, PAL [3]
- Rectangular steerable source, steerable PAL [4]

Pros:

- Faster than DIM and SWE

Cons:

- Valid in the paraxial region of Westervelt far field
- Inaccurate at low audio frequencies, small aperture sizes

1. J. J. Wen et al. In: **J. Acoust. Soc. Am.** 83.5 (1988), pp. 1752–1756
2. M. Červenka et al. In: **J. Acoust. Soc. Am.** 134.2 (2013), pp. 933–938
3. J. Yang et al. In: **IEEE Trans. Ultrason., Ferroelect., Freq. Contr.** 52.4 (2005), pp. 610–618
4. T. Zhuang et al. In: **J. Acoust. Soc. Am.** 153.1 (2023), pp. 124–136

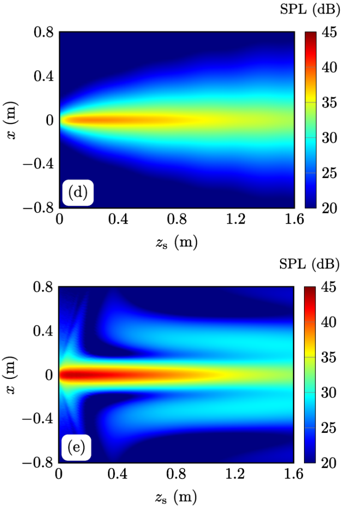


Fig 11. Audio SPL obtained by (top) exact solution and (bottom) GBE at 500 Hz

Convolution Directivity Method (CDM)

CDM: Audio sound directivity = **convolution** of ultrasound directivities and Westervelt directivity

- Westervelt directivity [1]

$$\mathcal{D}_{\mathrm{W}}(\varphi) = \frac{1}{1 - \mathrm{i}k_{\mathrm{a}}\alpha_{\mathrm{u}}^{-1}\sin^2(\varphi/2)} \tag{10}$$

- 2D problem: linear convolution ($\mathcal{D}_1$ and $\mathcal{D}_2$ are ultrasound directivities) [2]

$$\mathcal{D}_{\mathrm{a}}(\varphi) = (\mathcal{D}_1^*\mathcal{D}_2 * \mathcal{D}_{\mathrm{W}})(\varphi) \tag{11}$$

- 3D problem: spherical convolution [3]

$$\mathcal{D}_{\mathrm{a}}(\theta,\varphi) = (\mathcal{D}_1^*\mathcal{D}_2 \circledast \mathcal{D}_{\mathrm{W}})(\theta,\varphi) \tag{12}$$

Pros: Very fast
Cons: Applicable only in the inverse-law far field

1. P. J. Westervelt. In: **J. Acoust. Soc. Am.** 35.4 (1963), pp. 535–537
2. C. Shi et al. In: **J. Acoust. Soc. Am.** 137.2 (2015), pp. 777–784
3. J. Zhong et al. In: **J. Acoust. Soc. Am.** 153.3 (2023), pp. 1439–1451

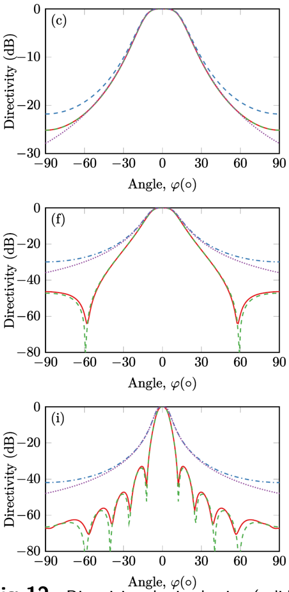


Fig 12. Directivity obtained using (solid) exaction solution, (dashed) modified CDM, (dash-dotted) direct CDM, and (dotted) Westervelt directivity at 400 Hz, 1 kHz, and 4 kHz [3]

Summary of existing computational models

Select appropriate computational models for a specific problem

Model	Valid region	Computational speed
FEM (Finite Element Method)	Near field Westervelt far field	Very slow
DIM (Direct Integration Method)	Full field	Slow
SWE (Spherical Wave Expansion)	Full field	Fast
CWE (Cylindrical Wave Expansion)	Full 2D field	Fast
GBE (Gaussian Beam Expansion)	Paraxial region in Westervelt far field	Very fast
CDM (Convolution Directivity Method)	Inverse-law far field	Very fast

Table 1: Summary of existing computational models

Conclusions and Future Work

Conclusions

- Modeling of the PAL is complicated due to the nonlinear process
- Near field, Westervelt far field, inverse-law far field
- Existing models: DIM, SWE, CWE, GBE, FEM, CDM

Future work

- Modeling a **rectangular PAL** with an arbitrary profile
- Modeling in the **time domain**
  - Wide band signals
  - Distortion
- **Intense ultrasound** (quasilinear approximation is invalid, e.g., $>$130 dB)
- PAL **array processing** (total sound field of the array $\neq$ superposition of the audio sound by each array element)
  - Binaural audio
  - Sound field reproduction
  - Wave field synthesis

Thanks

Thank you!
Any Questions?

Scan to evaluate my presentation