

# 机器学习作业三

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1.

(1) 色泽：

根据西瓜数据集 3.0 列出下表

色泽	青绿	乌黑	浅白
好瓜数	3	4	1
坏瓜数	3	2	4

设 $p_1 = p(\text{青绿}|\text{好瓜})$ ,  $p_2 = p(\text{乌黑}|\text{好瓜})$ ,  $p_3 = p(\text{浅白}|\text{好瓜})$

$$\text{则} P(x_{\text{色泽}}|\text{好瓜}) = p_1^3 \cdot p_2^4 \cdot p_3^1 = p_1^3 \cdot p_2^4 \cdot (1 - p_1 - p_2)^1$$

$$\text{设} L(p_1, p_2) = \ln(P(x_{\text{色泽}}|\text{好瓜})) = 3\ln(p_1) + 4\ln(p_2) + \ln(1 - p_1 - p_2)$$

$$\text{令 } \frac{\partial L}{\partial p_1} = 0, \frac{\partial L}{\partial p_2} = 0, \text{ 可得}$$

$$\frac{3}{p_1} + \frac{-3}{3-3p_1-3p_2} = 0, \frac{4}{p_2} + \frac{-1}{1-p_1-p_2} = 0.$$

$$\text{解得 } p_1 = \frac{3}{8}, p_2 = \frac{1}{2}, p_3 = \frac{1}{8}$$

$$\text{即 } p(\text{青绿}|\text{好瓜}) = \frac{3}{8}, p(\text{乌黑}|\text{好瓜}) = \frac{1}{2}, p(\text{浅白}|\text{好瓜}) = \frac{1}{8}$$

$$\text{同理求得 } p(\text{青绿}|\text{坏瓜}) = \frac{1}{3}, p(\text{乌黑}|\text{坏瓜}) = \frac{2}{9}, p(\text{浅白}|\text{坏瓜}) = \frac{4}{9}$$

(2) 根蒂：

根据西瓜数据集 3.0 列出下表

根底	蜷缩	稍蜷	硬挺
好瓜数	5	3	0
坏瓜数	3	4	2

设 $p_1 = p(\text{蜷缩}|\text{好瓜})$ ,  $p_2 = p(\text{稍蜷}|\text{好瓜})$ ,  $p_3 = p(\text{硬挺}|\text{好瓜})$

$$\text{则 } P(x_{\text{根蒂}}|\text{好瓜}) = p_1^5 \cdot p_2^3 \cdot p_3^0 = p_1^5 \cdot p_2^3$$

$$\text{设 } L(p_1, p_2) = \ln(P(x_{\text{根蒂}}|\text{好瓜})) = 5\ln(p_1) + 3\ln(p_2)$$

$$\text{令 } \frac{\partial L}{\partial p_1} = 0, \frac{\partial L}{\partial p_2} = 0, \text{ 可解得}$$

$$p_1 = \frac{5}{8}, p_2 = \frac{3}{8}, p_3 = \frac{0}{8}$$

$$\text{即 } p(\text{蜷缩}|\text{好瓜}) = \frac{5}{8}, p(\text{稍蜷}|\text{好瓜}) = \frac{3}{8}, p(\text{硬挺}|\text{好瓜}) = \frac{0}{8}$$

$$\text{同理求得 } p(\text{蜷缩}|\text{坏瓜}) = \frac{1}{3}, p(\text{稍蜷}|\text{坏瓜}) = \frac{4}{9}, p(\text{硬挺}|\text{坏瓜}) = \frac{2}{9}$$

(3) 敲声:

根据西瓜数据集 3.0 列出下表

敲声	浊响	沉闷	清脆
好瓜数	6	2	0
坏瓜数	4	3	2

设 $p_1 = p(\text{浊响}|\text{好瓜})$ ,  $p_2 = p(\text{沉闷}|\text{好瓜})$ ,  $p_3 = p(\text{清脆}|\text{好瓜})$

$$\text{则 } P(x_{\text{敲声}}|\text{好瓜}) = p_1^6 \cdot p_2^2 \cdot p_3^0 = p_1^6 \cdot p_2^2$$

$$\text{设 } L(p_1, p_2) = \ln(P(x_{\text{敲声}}|\text{好瓜})) = 6\ln(p_1) + 2\ln(p_2)$$

$$\text{令 } \frac{\partial L}{\partial p_1} = 0, \frac{\partial L}{\partial p_2} = 0, \text{ 可解得}$$

$$p_1 = \frac{3}{4}, p_2 = \frac{1}{4}, p_3 = \frac{0}{8}$$

$$\text{即 } p(\text{浊响}|\text{好瓜}) = \frac{3}{4}, p(\text{沉闷}|\text{好瓜}) = \frac{1}{4}, p(\text{清脆}|\text{好瓜}) = \frac{0}{8}$$

$$\text{同理求得 } p(\text{浊响}|\text{坏瓜}) = \frac{4}{9}, p(\text{沉闷}|\text{坏瓜}) = \frac{1}{3}, p(\text{清脆}|\text{坏瓜}) = \frac{2}{9}$$

2.

同父结构:

1.若 $x_1$ 已知:

$$p(x_1, x_3, x_4) = p(x_1)p(x_3|x_1)p(x_4|x_1)$$

$$p(x_3, x_4|x_1) = \frac{p(x_1, x_3, x_4)}{p(x_1)} = p(x_3|x_1)p(x_4|x_1)$$

$$\therefore p(x_3, x_4|x_1) = p(x_3|x_1)p(x_4|x_1)$$

即 $x_3$ 与 $x_4$ 关于 $x_1$ 独立,  $(x_3 \perp x_4)|x_1$

2.若 $x_1$ 未知:

$$p(x_1, x_3, x_4) = p(x_1)p(x_3|x_1)p(x_4|x_1)$$

$$p(x_3, x_4) = \sum_{x_1} p(x_1, x_3, x_4) = \sum_{x_1} p(x_1)p(x_3|x_1)p(x_4|x_1)$$

$\therefore x_1$ 未知,  $\therefore$ 不可推出 $p(x_3, x_4) = p(x_3)p(x_4)$

$\therefore$ 在同父结构中, 若 $x_1$ 取值未知, 则 $x_3 \perp x_4$ 不成立

顺序结构:

1.若 $x$ 已知:

$$p(x, y, z) = p(z)p(x|z)p(y|z)$$

$$p(y, z|x) = \frac{p(x, y, z)}{p(x)} = \frac{p(z)p(x|z)p(y|z)}{p(x)} = p(z|x)p(y|x)$$

$$\therefore p(y, z|x) = p(y|x)p(z|x)$$

即 $y$ 与 $z$ 关于 $x$ 独立,  $(y \perp z)|x$

2.若 $x$ 未知:

$$p(x, y, z) = p(z)p(x|z)p(y|z)$$

$$p(y, z) = \sum_x p(x, y, z) = \sum_x p(z)p(x|z)p(y|x)$$

$\therefore x$ 未知,  $\therefore$ 不可推出 $p(y, z) = p(y)p(z)$

$\therefore$ 在顺序结构中,  $(y \perp z)|x$ 成立,  $y \perp z$ 不成立。