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Dynamic Pricing for Resource-Quota Sharing in Multi-Access Edge Computing

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Abstract—In this paper, we analyze resource allocation in Multi-Access Edge Computing (MEC) from the perspective of revenue and profit management. Current coarse-grained pricing and resource plans have not made full use of the extreme heterogeneity of computing usage levels across users in the Internet of Everything (IoE), leading to wastage. Some users would have excess un-utilized resource quota while others might have reserved insufficient resources. Therefore, we introduce a novel sharing economy-inspired business model, following the trend towards the decentralized provision and sharing of digital resources. In our model, a platform facilitates the sharing of excess resource quota among users, leading to a more efficient usage of resources. Based on our model, we design and analyse two dynamic pricing mechanisms for resource quota sharing, which maximizes the social welfare or profit of the platform respectively. In our dynamic pricing mechanisms, the platform coordinates the sharing of compute resource quota in real-time, without knowledge of users' payoffs or control over users' decisions. We prove the optimality and convergence of the proposed mechanisms, and provide simulations to illustrate the convergence and robustness of our mechanisms to changes in demand and supply, as well as provide insights under changes in exogenous conditions.

Index Terms—Network Economics, Sharing Economy, Dynamic Pricing, Multi-Access Edge Computing

1 Introduction

More than 50 billion end devices will be connected to the Internet by 2020, based on Cisco's prediction [1]. Everything is expected to be connected with everything, anywhere and all the time, creating a new distributed ecosystem named the Internet of Everything (IoE) [2]. These end devices will not only host traditional voice communication services, but also new multimedia services such as facial recognition, natural language processing and augmented reality. These novel applications require huge amounts of computational resources, and possess stringent delay requirements. As mobile devices typically do not have the processing power needed to run these applications [3], Multi-Access Edge Computing (MEC) is an emerging paradigm [4]. In MEC, users can offload their computational jobs to the network edge (e.g. base station, wifi-access point, etc.), which are equipped with virtualized computational resources. [5] Having computing resources near the end users at the network edge helps to avoid wide area network (WAN) delay, which would occur had users offloaded their jobs via the core network and the internet, to remote cloud servers [6], [7].

Resource allocation is a key challenge in MEC. Researchers have been actively investigating the computation offloading problem in MEC [8], balancing the objectives of minimizing energy consumption and low latency under various scenarios. For example when there is a single user [9]–[11], multiple users [12]–[14], multiple APs [15], D2D sharing of resources [16] and energy harvesting involved [17], [18]. As pricing is a key tool for aligning user incentive with system-wide objectives in resource allocation, it has been used to aid in solving the computation offloading problem [19], [20]. Besides resource allocation schemes, solutions like model compression and model partitioning have been proposed to decrease the resource consumption of artificial intelligence applications [21].

While there has been plenty of work on the technical challenges of enabling computation offloading, there has been less work from the perspective of profit and revenue management, developing business models for the monetization of computing resources at the edge. (Note that the above works which involved pricing used it to facilitate the design of computation offloading solutions). With billions of devices being connected to the IoE, there is a huge market opportunity. It is of interest to investigate and design resource plans and pricing mechanisms in MEC from the perspective of revenue and resource management [22]–[24], which will also provide regulators with insights. In particular, [22] proposed incentive mechanisms for profit maximization, [23] proposed auction-based profit maximization for a hierarchical cloudlet model, and [24] modelled bargaining games between the network operator and application service providers.

Users in the future IoE market have a wide range of applications such as augmented reality, facial recognition, temperature and humidity sensors, smart farming IoT, etc. This indicates that users consume varying amounts of computing resources [25]. A revenue management scheme in MEC would need to balance and satisfy the diverse usage levels of the various applications [26].

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According to [27], in the mobile data pricing market, pricing strategies like usage-based pricing, shared data plans, and sponsored data pricing, fail to make full use of the heterogeneous demands across all users. Furthermore, the unused portion will be cleared away each month. Likewise, for edge computing resources: with course grained resource plans, heavy users would have reserved an insufficient amount of resources, while light users would have excess un-utilized resource quota. Even if several resource pricing plans are posted, they are still unable to take full advantage of the heterogeneous usage requirements across users. The current resource management schemes [22]–[24] and the current industrial practices [28] are insufficient. A more fine-grained resource scheme should be introduced to reduce the partial wastage caused by coarse-grained resource plans.

Resource sharing and a decentralized provision of resources has been a recent trend in other markets, with the use of 1) mobile data trading platforms [27], in which users with excess monthly data quota sell the quota to other users, and 2) crowdsourced wireless community networks [29]. In terms of edge computing resources, the new paradigms of personal volunteer computing [30] and user-provided fog infrastructure [31] has been proposed, where users share the computing resources of their personal devices. Furthermore, [26] posits that heterogeneity may give rise to interesting market structures involving middlemen and brokers facilitating the transaction of resources. In light of this trend, we take inspiration from the mobile data trading platforms to deal with the partial wastage scenario cause by coarse grained pricing schemes. Specifically, we propose a novel resource quota-sharing business model, where a third-party sharing platform facilitates the sharing of computing resource quota among users. Our model combines fixed resource plans from the Mobile Network Operator (MNO) with a sharing economy [32] inspired scheme, catering to the heterogeneity of usage levels across the IoE. Users who use more computational resources would buy resource plans directly from the MNO, and rent their unused quota via the sharing platform, to others who consume less, resulting in a more efficient usage of resources.

In this work, we focus on resource sharing under both the social welfare and profit maximizing scenarios. Investigating the social welfare problem helps to give insight to regulatory bodies, while investigating profit maximizing is in line with the platform's aim to profit maximize. Optimizing the social welfare and profit is challenging because:

- Users are strategic, making decisions to maximize individual welfare. As such, the sharing platform is typically unable to optimize social welfare or profit centrally: It cannot control the decision variables centrally and directly.
- The platform does not have information on individual user utilities and payoffs, and will be unable to centrally compute the optimal price.

In light of this, to maximize social welfare and profit, we propose decentralized dynamic pricing mechanisms DRP-SW and DRP-P respectively, wherein we use pricing as a demand and supply management tool to incentivize users to make optimal decisions.

The operations of the mechanism are as follows: the platform posts the rental price and an inconvenience measure to users. Users then decide whether to purchase resource plans directly from the MNO or rent their quota from the platform. The platform observes the resulting supply and demand or profit and revises the rental price and inconvenience measure iteratively. At equilibrium, the platform and participants would have arrived at the optimal point. Theoretical analysis show that the dynamic pricing mechanisms DRP-SW and DRP-P converge to the socially optimal and profit maximizing resource sharing points respectively. With dynamic pricing, our model is able to handle on-demand resource allocation, e.g. the transition from non-peak to peak period.

Our contributions are summarized as follows:

- We investigate resource allocation in MEC from the perspective of revenue management. To fully make use of the heterogenous usage levels of users in the IoE, we propose a sharing economy-inspired business model, where the sharing platform facilitates resource quota sharing among users. This reduces the resource wastage caused by coarse grained resource schemes.
- For our resource sharing model, we propose a dynamic pricing mechanism DRP-SW which maximizes social welfare. We prove via a Lyapunov argument that DRP-SW converges (Theorem 1), and that its' equilibrium point is the optimal point of the non-convex social welfare problem (Theorem 2), and provide conditions under which our proofs hold.
- For resource sharing, we propose a dynamic pricing mechanism DRP-P which maximizes profit. We prove that DRP-P terminates at a point within ϵ of the optimal price (Theorem 3), and provide conditions under which our proof holds.
- Numerical results show that our mechanisms converge to optimality, and illustrate that DRP-SW is able to handle on-demand resource allocation, being robust to changes in job arrival rate. Besides this, we show that the efficiency loss at the profit maximizing point is negligible when the resource plan cost is low, among other insights on the two-sided market under changes in exogeneous conditions.

2 System Model

We consider an edge computing system with multiple devices/users and a set of access points such as base station, wifi-access points. Besides serving its conventional role as an access point to the network, each access point is equipped with a computing server, creating a virtualised pool of computing resources. Users running computationally heavy jobs are able to offload their job computation to the edge, to overcome the limitations of their insufficient battery energy and computing power. The mobile network operator (MNO) charges users for reservation and usage of the edge's resources, by selling computing resource plans at a service fee of c.

As mentioned earlier, users in the future IoE market are extremely heterogeneous in their demand for computing resources. This results in the partial wastage scenario caused

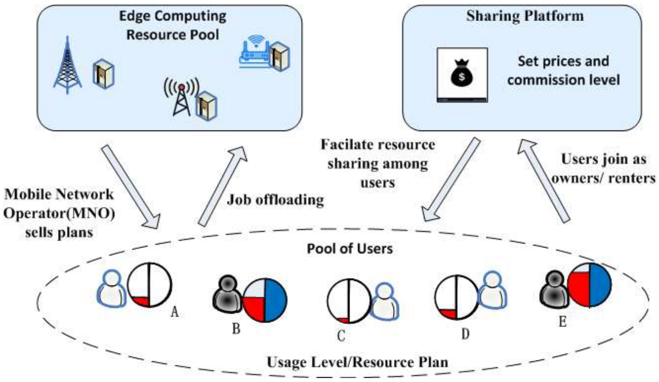

Fig. 1. The platform facilitates the sharing of resource quota among users in the IoE.

by coarse-grained resource schemes. Light users would have excess un-utilized resource quota, and heavy users would have reserved an insufficient amount of resources, calling for a more fine-grained revenue and resource management solution, introduced in the following:

2.0.1 The Sharing Platform

Consider a third-party platform which oversees and facilitates the sharing of compute resource quota among devices - see Fig. 1. It sets the rental price p for the sharing/transaction of resource between users, and collects a commission of δ.

Every individual user/device i has a usage level ξ_i representing the amount of compute resource (CPU cycles) it requires. Without loss of generality, the population's usage levels are normalized to $[0,1]$, where $\xi_i = 0$ represents no usage and $\xi_i = 1$ represents the maximum usage. The usage levels in the population follow a distribution with probability density function (PDF) $f(\xi_i) \geq 0$. A user derives a utility $u(\xi_i)$ from using ξ_i of compute resources, where $u()$ is an increasing and concave function. In our prior work [33], [34], we combined physical layer considerations (the job arrival rate, the data rate, distance from edge resource etc) with economic analysis, and showed that in the offloading problem, the users' utility function follows the law of diminishing marginal returns (a concave utility function). Alternatively, in this work we focused solely on the economics aspects of edge resource sharing. Thus, we simplified the physical layer considerations with an abstract concave utility function. The more a user offloads to the AP, the more edge computing resource it consumes and the higher benefits it obtains from offloading. Furthermore, as the amount consumed increases, the marginal utility of the next unit consumed decreases, which indicates that this utility function can be modelled by the law of diminishing marginal returns [35].

We next describe the two classes of platform users: Owners and Renters, following which we describe how a user chooses its class.

2.0.2 Owners

An owner i is a user who has purchased the compute resource plan from the MNO directly, at unit cost c. As the owner only requires ξ_i of the plan, it makes the excess offloading quota of $1-\xi_i$ available for rent. It rents this plan portion to other users (Renters) via the platform, earning an additional income. For example, as seen in Fig. 1, User B is running computationally intensive tasks and has a high usage level (indicated by the red semicircle), hence it buys a compute resource plan from the MNO (indicated by the blue semicircle). Meanwhile, in comparison to its' usage level, it still has some excess offloading quota, which it is willing to sell to other users if given the appropriate economic incentive.

2.0.3 Renters

A renter j does not directly buy a computing resource plan from the MNO. It rents its required ξ_j of the resource plan at p per unit from owners, via the platform. The rental price p is set by the platform, who also collects a commission fee of δ per transaction. δ indicates the proportion of renters' payment p which goes to the platform. $(1-\delta)p$ will go to the owner. As seen in Fig. 1, User A is running computationally light tasks (indicated by the red semicircle) and therefore is not willing to buy compute resource plans from the MNO due to the relative high cost. Instead, it would obtain its quota for offloading by renting a portion of the plan from other users such as User B.

The sharing platform is a two sided market. The pool of users of the can freely decide between being a renter or owner, and can role switch.

2.0.4 Payoffs

Users decide whether to be an owner or renter of the resource plan by weighing which role gives it a higher payoff.

The payoff function of an owner with usage ξ_i is given by:

$$\pi_o(p,\alpha,\xi_i) = u(\xi_i) + (1-\delta)(1-\xi_i)\alpha p - c, \tag{1}$$

which is the utility of it's own job offloading $u(\xi_i)$ plus the income $(1-\delta)(1-\xi_i)\alpha p$ from renting $1-\xi_i$, it's unused quota, less c, the cost of plan ownership it paid the MNO. The platform commission δ is taken into account. α is the probability that an owner can successfully find a renter via the platform. The sharing economy contains matching frictions, as aggregate supply and demand might not be equal. Matching friction and the inconvenience caused by waiting are captured by the owner's matching probability α.

The payoff of a renter with usage ξ_i is given by:

$$\pi_r(p,\beta,\xi_i) = u(\beta\xi_i) - p\beta\xi_i, \tag{2}$$

which is the utility of job offloading $u(\beta\xi_i)$ minus the rental cost $p\beta\xi_i$ of renting a portion of the plan via the platform. As mentioned, the sharing economy contains matching frictions because aggregate supply and demand might not be equal. β is the probability that a renter will be able to successfully rent a portion of a plan via the platform. α and β can also be interpreted as measures of inconvenience, being proportional to the inverse of the waiting time.

TABLE 1
Summary of key Notations

Parameter	Definition
ξ_i	User i's resource usage level
$f(\xi_i)$	The population's usage level distribution
$u(\xi_i)$	Utility Function
c	Ownership cost of resource plans from MNO
p	Per unit rental price set by platform for plan sharing
δ	The platform's commission
$\beta(\theta)$	Matching probability for renters
$\alpha(\theta)$	Matching probability for owners
$\pi_o(p, \alpha, \xi_i)$	Owner's payoff function
$\pi_r(p, \beta, \xi_i)$	Renter's payoff function
θ	Threshold separating owners from renters
$S(\theta)$	Supply
$D(\theta)$	Demand

2.0.5 Decision Making

Users with an identical offloading level ξ_i make the same choice of being an owner or renter. We assume that devices are rational and choose between being owners or renters based on which gives a higher payoff. In other words, a device ξ_i would choose to be an owner if $\pi_o(p, \alpha, \xi_i) \geq \pi_r(p, \beta, \xi_i)$, and vice versa.

Sharing of compute resources takes place if, for some users being a renter gives a higher payoff, while for other users being an owner gives a higher payoff (as opposed to having only owners or only renters in the system). This occurs when there exists a threshold $\theta \in (0, 1)$ such that

$$\pi_o(p, \alpha, \xi_i) < \pi_r(p, \beta, \xi_i)\, \forall \xi_i \in [0, \theta) \tag{3}$$

and

$$\pi_o(p, \alpha, \xi_i) > \pi_r(p, \beta, \xi_i)\, \forall \xi_i \in (\theta, 1]. \tag{4}$$

(3) and (4) mean that for users with usage ξ_i below the threshold θ, the payoff of being a renter is higher. They would become renters, renting their required quota from other users via the platform. And for users with usage levels ξ_i above θ, the payoff of being an owner is higher. They would become owners, buying resource plans from the MNO directly. The threshold θ seperates owners from renters, based on their offloading level ξ_i. When $\xi_i = \theta$, the payoffs are equal. $\pi_o(p, \alpha, \theta) = \pi_r(p, \beta, \theta)$.

In this paper we assume that (3) and (4) hold, and in this paragraph we explain why this assumption is valid. Firstly, this assumption is equivalent to assuming that $\pi_o(p, \alpha, \xi_i) - \pi_r(p, \beta, \xi_i)$ is monotonically increasing in ξ_i, by the Intermediate Value Theorem. For $\pi_o(p, \alpha, \xi_i) - \pi_r(p, \beta, \xi_i)$ to be monotonically increasing, the conditions $u'(\xi_i) \geq \beta u'(\beta \xi_i)$ and $\beta \geq (1 - \delta)\alpha$ are required. $u'(\xi_i) \geq \beta u'(\beta \xi_i)$ is equivalent to $\frac{\partial u(\xi_i)}{\partial \xi_i} \geq \frac{\partial u(\beta \xi_i)}{\partial \xi_i}$ which indicates that the marginal utility is greater when there is no inconvenience as opposed to when there is (e.g. waiting time). This condition is satisfied by common utility functions such as linear and logarithmic functions.

(3) and (4) agrees with intuition, that devices with higher computational requirements would prefer to purchase full resource plans from the MNO. At the same time, devices with lower computational usage have less incentive to buy full resource plans, preferring to rent the their required amount from other users.

2.0.6 Supply and Demand

In this sharing economy, the supply of plan portions is provided by owners while the demand comes from renters. As owners have usage levels $\xi_i > \theta$ and renters have usage levels $\xi_i < \theta$ the total supply and demand are as follows:

$$S(\theta) = \int_{\theta}^{1} (1 - \xi_i) f(\xi_i) d\xi_i, \tag{5}$$

$$D(\theta) = \int_{0}^{\theta} \xi_i f(\xi_i) d\xi_i, \tag{6}$$

which are functions of the threshold θ separating owners from renters.

2.0.7 Matching Dynamics

In equilibrium, the total satisfied demand equals the total rented supply. Recall that matching frictions exist in the system, hence we have

$$\alpha S(\theta) = \beta D(\theta). \tag{7}$$

Note that α and β are endogeneous, that is, their values depend on the actual supply and demand which in turn depend on the fraction of owners and renters in the economy.

The short-term matching dynamics of this resource plan sharing economy can be modelled using a multi-server loss queuing system [36], where the owners' matching probability α corresponds to the server utilization and $1 - \beta$ (probability of the renter not being matched) corresponds to the blocking probability. Letting t be the mean rental time, we define the arrival rate of renter requests as $\lambda = \frac{D(\theta)}{t}$ and the rate of renter requests being fulfilled (service rate) as $\mu = \frac{S(\theta)}{t}$. The ratio of request arrival rate against the service rate is therefore $\rho = \frac{\lambda}{\mu} = \frac{D(\theta)}{S(\theta)}$. By Eqn (7) it follows that

$$\rho = \frac{\alpha}{\beta} \rightarrow \alpha = \rho\beta. \tag{8}$$

By Sobel's upper bound approximation on the blocking probability [37], $1 - \beta \approx 1 - \frac{1}{1+\rho}$. Therefore,

$$\beta \approx \frac{\mu}{\mu + \lambda} = \frac{S(\theta)}{S(\theta) + D(\theta)}, \tag{9}$$

i.e. the renters' matching probability can be approximated as the ratio of supply over the total supply and demand. Likewise from Equation (8), α can be approximated as

$$\alpha \approx \frac{\rho}{1 + \rho} = \frac{D(\theta)}{D(\theta) + S(\theta)}, \tag{10}$$

i.e. the owners' matching probabillity can be approximated as the ratio of demand over the total supply and demand. From Equations (9) and (10) we infer that

$$\alpha = 1 - \beta, \tag{11}$$

which we use in the remainder of the paper. When combined with (7), this indicates that (7) can be simplified to:

$$\beta D(\theta) = (1 - \beta) S(\theta). \tag{12}$$

3 The Social Welfare and Profit Maximization Problems

3.1 Social Welfare Maximization

We first consider the case of commission-free platforms, setting $\delta = 0$. This is relevant for publicly owned platforms and regulatory bodies. For the commission free platform, the aim is to optimally allocate the pool of users into owners vs renters, such that the resulting supply and demand ratio maximizes social welfare.

Social welfare is the total sum of owners' and renters' individual payoffs. Rental payments p (transactions from renters to owners) are not included, by definition of the social welfare function, because there is no net payment either in or out of the system. The social welfare function is thus defined as follows:

$$SW(\beta, \theta) = \int_0^\theta u(\beta \xi_i) f(\xi_i) d\xi_i + \int_\theta^1 (u(\xi_i) - c) f(\xi_i) d\xi_i. \tag{13}$$

The platform's social welfare maximizing problem is:

$$\begin{aligned} \mathbf{S}: & \max_{\theta \in [0,1],\ \beta(\theta) \in [0,1]} SW(\beta, \theta), \\ & \text{s.t.} \quad \beta D(\theta) = (1-\beta) S(\theta). \end{aligned}$$

The constraint $\beta D(\theta) = (1-\beta)S(\theta)$ comes from (12) and ensures that the total satisfied demand equals the total rented-out supply. The first decision variable is θ, ie. the threshold separating owners from renters. Hence, the maximization is performed over the allocation of owners vs renters. The other decision variable β (the renters' matching probability) is an endogenous variable, dependent on the proportion of renters and owners in the system. β might have exogenous factors as well, such as the rate of job requests differing over different parts of the 24-hour cycle, etc. In Section 6, our model handles on-demand resource allocation, accounting for the exogenous factors.

While the problem is seemingly tractable, the key difficulty the platform faces is its lack of control and information to centrally optimize the system. The platform is unable to control the decision variables centrally and directly, as they depend on mobile device users' decisions of being owners or renters. This is because firstly, computational usage levels are exogeneously determined for the service applications. Secondly, users are inherently selfish and need to be incentivised to work towards the global optimal. Furthermore, even if they were cooperative, their information is limited to their local information. Therefore, we propose using prices as a supply and demand management tool to incentivize users to make globally optimal decisions. However, the platform operator has incomplete information. It does not have knowledge of users' payoff functions to centrally compute an optimal price. This motivates a decentralized control mechanism, where prices iteratively signal users towards the optimal thereshold θ^*.

3.2 Profit Maximization

We now consider the case where the platform wishes to maximize its profit. It takes a commission $\delta > 0$ from every transaction. The platform aims to allocate the pool of users into owners and renters, such that the resulting supply and demand ratio maximizes its' profit. The profit function is as follows:

$$PF(\theta, \beta) = p\delta(1-\beta)S(\theta), \tag{14}$$

which is the price multiplied by the commission, multiplied by the amount successfully transacted $(1-\beta)S(\theta)$. The platform's profit maximizing problem is:

$$\mathbf{P}: \max_{\theta \in [0,1], \beta(\theta) \in [0,1]} PF(\theta, \beta)$$

$$\text{s.t. } \beta D(\theta) = (1-\beta)S(\theta) \tag{15}$$

$$\text{and } \pi_r(\theta) = \pi_o(\theta) \tag{16}$$

Once again, the decision variables are θ (the threshold level) and β (the renters' matching probability), which is endogenous. The first constraint ensures that the total satisfied demand equals the total rented supply. The second constraint requires that at the threshold level θ, the payoff of owners and renters are equal.

Similar to the social welfare problem, the difficulty the platform faces in optimizing profit lies is its lack of control and information to centrally optimize the system. We likewise propose another decentralized pricing mechanism which induces users to make decisions such that optimality is attained at equilibrium.

With the platform's lack of control and information, how do we *design pricing mechanisms that align users' objectives with the platform's objective and learn the optimal price at equilibrium?* We answer this in the following sections.

4 The Socially Optimal Dynamic Pricing Mechanism

In this section, we study the case where the platform's commission factor $\delta = 0$ and it aims to maximize the social welfare ie. solve problem **S**.

4.1 Social Welfare Maximizing Mechanism DRP-SW

To deal with the platform's lack of control and information, we propose a decentralised mechanism named Distributed Rental Pricing - Social Welfare (DRP-SW) to solve **S**. In this mechanism, the platform uses pricing signals and the inconvenience measure β to iteratively signal users towards the optimal social welfare. Fig. 2 illustrates the operations of our mechanism. At each timeslot t, the platform calculates the rental price p^t and matching probability β^t. It then broadcasts the rental price p^t and matching probability β^t to the devices, who individually choose whether to be owners or renters based which maximizes their payoff. This determines the set of renters, and thus the threshold θ^{t+1}. Based on the observed supply $S(\theta^{t+1})$ and demand $D(\theta^{t+1})$, the platform once again calculates and updates the rental price p^{t+1} and matching probability β^{t+1}, and the entire process repeats until convergence. Please refer to Algorithm 1 for more details.

In the following, we describe the construction of the mechanism DRP-SW, showing that p, β and θ are updated based on the maximization of the Lagrangian. Subsequently, we will prove the convergence and optimality DRP-SW.

Firstly, we construct the Lagrangian of the social welfare maximization problem **S**. Let p denote the dual variable

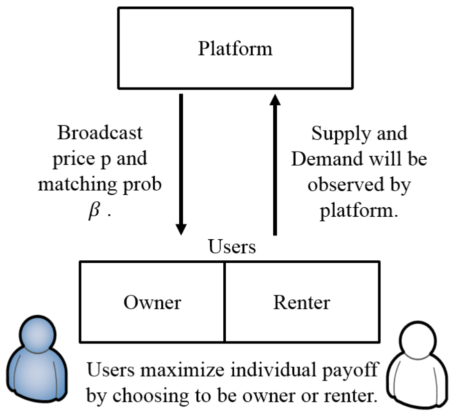

Fig. 2. Schematic view of the information flow and structure of Distributed Rental Pricing (DRP-SW).

associated with the constraint $\beta D(\theta) = (1-\beta)S(\theta)$. We will explain later how this is associated with the rental price. The Lagrangian of **S** is given by:

$$L(\theta, \beta, p) := \int_0^\theta u(\beta\xi_i) f(\xi_i) d\xi_i + \int_\theta^1 (u(\xi_i) - c) f(\xi_i) d\xi_i \tag{17}$$
$$+ p\left[(1-\beta)S(\theta) - \beta D(\theta)\right],$$

Next, by definition, the dual function of (17) is given by maximizing over the Lagrangian:

$$L_D(p) := \max_{\theta \in [0,1], \beta \in [0,1]} L(\theta, \beta, p), \tag{18}$$

and the dual problem is given by

$$\min_{p \geq 0} L_D(p). \tag{19}$$

Note that the Lagrangian can be rewritten as the sum of the payoffs of owners and renters in the system as follows (recall that $\alpha = 1 - \beta$ by (11)):

$$L(\theta, \beta, p) = \int_0^\theta \pi_r(\xi, p, \beta) f(\xi_i) d\xi_i + \int_\theta^1 \pi_o(\xi_i, p, \beta) f(\xi_i) d\xi_i$$

which also explains our decision to associated the dual variable with the rental price. Thus, the Lagrangian is maximized over θ (ie, the dual function is solved), when each user maximizes his/her individual payoff by weighing payoffs and choosing to be either an owner or renter, as follows:

$$\pi_r(\xi_i, p, \beta) \underset{\text{owner}}{\overset{\text{renter}}{\gtreqless}} \pi_o(\xi_i, p, \beta) \tag{20}$$

This, combined with (3) and (4), indicates that

$$\theta^t = \max_{\xi_i} \{\xi_i | \pi_r(\xi_i, p^{t-1}, \beta^{t-1}) \geq \pi_o(\xi_i, p^{t-1}, \beta^{t-1})\} \tag{21}$$

where θ^t is the resulting threshold seperating owners from renters, after users make their individual-welfare-optimizing decisions. The platform is not able to observe the threshold θ^t, but it can observe the resulting supply and demand.

The platform's pricing update aims to incentivize users to make socially optimal decisions. They aim to induce an equilibrium such that the solution to (21) also solve the social welfare problem **S**. The pricing update is obtained via gradient descent on the dual problem (19):

$$p^t = p^{t-1} - \gamma \frac{\partial L_D(p)}{\partial p}, \tag{22}$$

where the derivative of the dual function $\frac{\partial L_D(p)}{\partial p} = (1 - \beta^{t-1})S(\theta^t) - \beta^{t-1}D(\theta^t)$. This is the difference between the rented supply and satisfied demand, which is observable to the platform operator. The supply-demand imbalance of our two sided market is used as a control feedback measure, signalling to the platform how it should adjust prices in order to reach optimality. This pricing update agrees with the law of supply and demand. When supply is greater than demand, the price decreases, and vice versa when supply is less than demand.

The update for β^t under linear utility $u(\xi_i) = \xi_i$ is obtained as follows:

$$\frac{\partial L(\theta, \beta, p)}{\partial \beta} = 0 \tag{23}$$
$$\iff \left[D(\theta^t) - p^t\left(D(\theta^t) + S(\theta^t)\right)\right] = 0$$
$$\iff \beta^t = 1 - p^t$$

Algorithm 1 DRP-SW: The Social Welfare Maximizing Mechanism

Input: maxItr
Initialize: $t \leftarrow 1, \theta^t, p^t \leftarrow \text{rand}, \beta^t \leftarrow 1 - p^t$
for $t = 2$ to maxItr **do**
  $\theta^t \leftarrow$Users choose to be owner/renter ▷ By Eq. (21)
  $S^t \leftarrow \int_{\theta^t}^1 (1-\xi_i) f(\xi_i) d\xi_i$ ▷ By Eq. (5)
  $D^t \leftarrow \int_0^{\theta^t} \xi_i f(\xi_i) d\xi_i$ ▷ By Eq. (6)
  Platform observes the supply and demand S^t, D^t
  $p^t \leftarrow p^{t-1} - \gamma[(1-\beta^{t-1})S^t - \beta^{t-1}D^t]$ ▷ By Eq. (22)
  $\beta^t \leftarrow 1 - p^t$ ▷ By Eq. (23)
  Platform broadcasts p^t and β^t
end for
end

4.2 Convergence and Optimality of Algorithm DRP-SW

We prove that, under linear utility functions and uniform distribution of usage levels, DRP-SW converges to the optimal solution of **S** (Theorems 1 and 2). In Section 6, we provide simulations to illustrate the optimality of DRP-SW under more general distributions.

To help us prove algorithm DRP-SW's convergence, note that DRP-SW can be viewed as a dynamical system, where the dynamics are given by the following equations:

DRP-SW:

$$\theta = \max_{\xi_i} \{\xi_i | \pi_r(\xi_i, p, \beta) \geq \pi_o(\xi_i, p, \beta)\},$$
$$\dot{p} = -\gamma\left((1-\beta)S(\theta) - \beta D(\theta)\right),$$
$$\beta = 1 - p$$

where γ is a strictly positive step size.

As mechanism DRP-SW is in the form of a dynamical system, in Theorem 1 we use a Lyapunov stability argument to prove its' convergence by proving that it is stable. Stability indicates that the system eventually settles down in it's "lowest energy" state, ie. it's equilibrium point. We now introduce the definition of stability:

Definition 1. *The equilibrium point $\hat{p}$ of DRP-SW is said to be Lyapunov stable if, $\forall \epsilon > 0, \exists \delta(\epsilon) > 0$ s.t. $|p^t - \hat{p}| \leq \epsilon$ if $|p^0 - \hat{p}| \leq \delta, \forall t > 0$. [38]*

This indicates that for every given ϵ, there exists a δ such that for all initial values within δ of the equilibrium point $\hat{p}$, all trajectories of the system p^t will remain within ϵ from the equilibrium point $\hat{p}$. This also indicates stability in spite of perturbations to the system. Next, we present our result showing the convergence of DRP-SW:

Theorem 1. *Suppose that $f(\xi_i) = 1$, that is, usage is uniformly distributed, and that utility is linear. The equilibrium point (p^*, β^*) of the dynamical system DRP-SW is locally stable.*

Proof. See Appendix A □

The basic idea of the proof is as follows. We show that our dynamical system is stable by proving that it satisfies Lyapunov's theorem for stability. We obtain a candidate function $W(p)$ which fulfill the conditions $W(p) > 0$ for all p, $W(p) = 0$. Next, we show that the dynamical system decreases with respect to time, until it reaches the equilibrium, by showing that the Lyapunov function satisfies $\frac{\partial W}{\partial t} \leq 0$. In particular, $V(p) = (p - \hat{p})\gamma^{-1}(p - \hat{p})$ is used as our candidate Lyapunov function. We use a linear transformation $x = p - \hat{p}$, given that our equilibrium point $\hat{p}$ does not occur at 0. Using a result from [39], we show that $\frac{\partial W(p)}{\partial t} \leq 0$ by proving that $\frac{\partial \dot{p}}{\partial p}\gamma^{-1} + \gamma^{-1}\frac{\partial \dot{p}}{\partial p} \leq 0$.

We do not employ the Lipschitz convergence technique (bounding the difference between updates $||p^{t+1} - p^t||$) as our problem does not possess the typical network utility maximization structure: Because θ is a threshold separating owners from renters based on their offloading level ξ_i, by equations (3) and (4), the price (dual variable) and primal variable have the following relationship: $\theta = \frac{c-p(1-\beta)}{1-\beta-p+2p\beta}$. Therefore $||p^{t+1} - p^t|| = \frac{\partial L_D}{\partial p} = \frac{c^2(1-2p)+2cp^2+p^3(-4+5p-2p^2)}{4(p-1)^2p^2}$.

Remark 1: Theorem 1 states that the mechanism DRP-SW will be able to converge to equilibrium, dependent on the system's starting state. Nevertheless, in our simulations (Section 6), DRP-SW is actually able to converge regardless of the initialisation.

Next, we present our result showing that any equilibrium point of mechanism DRP-SW is the optimal solution of the social welfare problem:

Theorem 2. *Suppose that $f(\xi) = 1$, that is, usage is uniformly distributed, and that utility is linear. Let (p^*, β^*, θ^*) denote an equilibrium point of $\mathbf{DRP-SW}$. Then (β^*, θ^*) is an optimal solution to $\mathbf{S}$.*

Proof. See Appendix B. □

The basic idea of the proof is as follows. The social welfare problem $\mathbf{S}$ (whose dual we base our pricing mechanism on) is not concave as the Hessian of it's objective function is $\begin{bmatrix} \beta - 1 & \theta \\ \theta & 0 \end{bmatrix}$, where $\beta, \theta > 0$. We convert the problem into a concave form and obtain the new problem's first order conditions. Finally, we show that the equilibrium conditions of DRP-SW satisfy the first order conditions.

emphRemark 2: While our theoretical results state uniform distribution as a condition, in Section 6 we simulate to show that DRP-SW is able to converge to optimality when the users' usage levels follow Beta Distribution, a more general condition.

Our algorithm bears resemblance to the primal dual algorithm [40], which has been used in areas such as network utility maximization. Our work differs from the existing literature in the following ways: Firstly, the primal dual algorithm is traditionally applied to a one sided market with price directly inducing quantity demanded. Due to the homogeneous user property of the sharing economy market model, in our algorithm price does not induce quantity demanded. Instead, price induces participants to make a binary decision - that of being owners or renters. Furthermore, due to network externalities and the supply demand mismatch (both unique characteristics of the sharing economy), we have to deal with an endogeneous variable β (characterising supply demand mismatch) as the decision variable. This variable does not stay constant with respect to time, and is a function of θ, hence the social welfare problem does not possess the typical NUM structure of $\max_{x \geq 0} \sum_i U_i(x_i)$ s.t. $Rx \leq C$ Finally, unlike majority of existing work, our problem is not convex, and therefore a direct application of the primal dual algorithm does not suffice.

5 The Profit Maximizing Dynamic Pricing Mechanism

In this section, we study the scenario where the platform collects a commission ($\delta > 0$) and aims to maximize its profit.

5.1 Profit Maximizing Mechanism DRP-P

Just as in the social welfare maximizing case, the platform is unable to control the decision variables of $\mathbf{P}$ centrally and directly, as they depend on devices' decisions of being owners vs renters. Because the rental price determines devices' decisions and hence the proportion of renters/owners, we propose another decentralized pricing mechanism called Distributed Rental Pricing - Profit (DRP-P), for the scenario where utility is linear and users' usage follows a uniform distribution. In this mechanism, the platform uses pricing signals to influence demand and supply. The resulting profit level is used as feedback to update prices. The entire process iterates until the system arrives at an allocation that solves $\mathbf{P}$.

The the idea behind our proposed mechanism DRP-P (Algorithm 2) is for the platform to search for the optimal price, in incremental steps (line 19 in Algorithm 2). Although the platform will not know the optimal point, it can observe the resulting profit level every time it adjusts the price. If the observed profit has decreased compared to the previous profit level, the platform would change the direction of the price search (lines 16-18). Otherwise, the platform would continue the current direction of price movement. Analogous to how the supply demand imbalance was used as a control feedback measure in DRP-SW, here profit is used as the feedback measure, signalling to the platform how it should adjust prices in order to attain optimality.

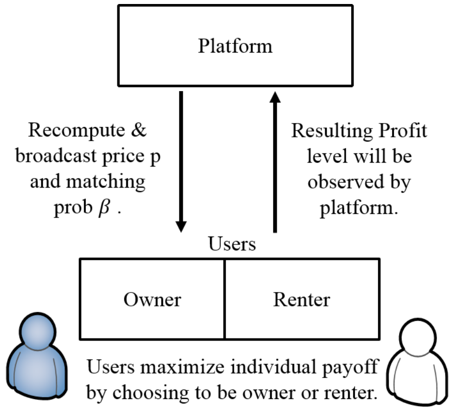

Fig. 3. Schematic view of the information flow and structure of Distributed Rental Pricing (DRP-P).

The operations of our mechanism are illustrated in Fig 3. The platform computes and broadcasts the price p^t and matching probability β^t (lines 19-21). It computes the matching probability as such: For linear utility and when the usage levels come from a uniform distribution $U[a,b]$, solving the two constraints of the profit maximization problem (Eqns. (15) and (16)) gives this relationship between p and θ:

$$\begin{aligned} p &= h(\theta) \\ &= \frac{-\theta^3 + 2c\theta^2 + (a^2 - 2c)\theta + (2b - b^2 - a^2)c}{\delta\theta^3 + (-1-\delta)\theta^2 + (2b - b^2 + a^2 - \delta\theta^2)\theta - a^2(1-\delta)} \end{aligned} \tag{24}$$

Remark 3: Note that $U[a,b]$ can serve as an approximation for a general distribution. Based on the historical demand and supply levels, the platform is able to estimate the distribution parameters a and b.

The following Proposition shows that given any p^t, under some conditions on the uniform distribution's limits a and b, the platform is able to compute $h^{-1}(p^t)$ and hence it's corresponding matching probability $f(h^{-1}(p^t))$, where $\beta = f(\theta)$ is calculated from the definition of the matching probability (problem constraint Equation (15)).

Proposition 1. *Given p^t, and when a, b satisfy the condition $-(p+\delta p+2c)^2+3(p\delta+1)[p(2b-b^2+a^2-\delta a^2)-a^2+2c] > 0$, $h^{-1}(p^t)$ exists and is unique.*

Proof. See Appendix C. □

The basic idea of the proof is as follows. Firstly, we let $h(\theta)-p=0$, to obtain a cubic equation. To show that $h^{-1}(p)$ exists and is unique, we show that the cubic equation has only 1 real root for all p. We prove by contradiction, first assuming that the cubic equation has 2 or more real roots. By Rolle's Theorem, this implies that there exists a point with derivative of zero. Next, we show that the derivative of the cubic equation is bounded above zero, hence contradicting the fact the cubic equation has 2 or more real roots.

After p^t and β^t are posted, the devices will choose whether to be owners or renters, by weighing payoffs as follows, according to (20). After devices make their decision, the platform will observe the demand D^t, supply S^t, matching probability β^t_{true} by (9), and hence the profit level PT^t. It continues searching over p: If the profit has decreased (lines 16-18), it will inverse the direction of price adjustment. Else, it would remain searching in the current direction of price adjustment. The platform will post the new price p^t and matching probability β^t. The process repeats until convergence, ie. the platform observes oscillation in profit levels (lines 12-14). The convergence condition checks if the profit is equal to the profit level four timeslots ago. The rationale behind this: when the direction of price adjustment is inversed, the mechanism oscillates among 3 prices within 2ϵ of the optimal price. An illustration of this is provided in Appendix D. For a more rigorous explanation, we refer the reader to the proof of Theorem 3. For more details on the mechanism, refer to Algorithm 2.

Algorithm 2 DRP-P: The Profit Maximizing Mechanism

1: **Initialize:** $t \leftarrow 1, p^t \leftarrow \text{rand}, \beta^t \leftarrow \text{rand}$
2: Platform broadcasts p^t, β^t
3: $\theta^t \leftarrow$ Users choose to be owner/renter ▷ By Eq. (21)
4: $S^t \leftarrow \int_{\theta^t}^{b}(1-\xi)f(\xi)d\xi$ ▷ By Eq. (5)
5: $D^t \leftarrow \int_{a}^{\theta^t}\xi f(\xi)d\xi$ ▷ By Eq. (6)
6: $\beta^t_{\text{true}} \rightarrow \frac{D^t}{D^t+S^t}$ ▷ By Eq. (9)
7: $PF^t(\theta^t, \beta^t_{\text{true}}) \rightarrow \delta p^t(1-\beta^t_{\text{true}})S^t$ ▷ By Eq. (14)
8: Platform observes the resulting profit PF^t
9: $t \leftarrow t+1, p^t \leftarrow p^{t-1}+\epsilon, \beta^t \leftarrow f(h^{-1}(p^t))$
10: Repeat Steps 3-8
11: **while** True **do**
12:     **if** $t > 4$ and $PF^t == PF^{t-4}$ **then**
13:         Exit while loop
14:     **end if**
15:     $t \leftarrow t+1$
16:     **if** $PF^{t-1} - PF^{t-2} < 0$ **then**
17:         $\epsilon \leftarrow -\epsilon$ ▷ Change direction of price adjustment
18:     **end if**
19:     $p^t \leftarrow p^{t-1}+\epsilon$
20:     $\beta^t \leftarrow f(h^{-1}(p^t))$ ▷ By Eq. (24) and (15)
21:     Platform broadcasts p^t, β^t
22:     $\theta^t \leftarrow$ Users choose to be owner/renter ▷ By Eq. (21)
23:     $S^t \leftarrow \int_{\theta^t}^{b}(1-\xi)f(\xi)d\xi$ ▷ By Eq. (5)
24:     $D^t \leftarrow \int_{a}^{\theta^t}\xi f(\xi)d\xi$ ▷ By Eq. (6)
25:     $\beta^t_{\text{true}} \rightarrow \frac{D^t}{D^t+S^t}$ ▷ By Eq. (9)
26:     $PF^t(\theta^t, \beta^t_{\text{true}}) \rightarrow \delta p^t(1-\beta^t_{\text{true}})S^t$ ▷ By Eq. (14)
27:     Platform observes the resulting profit PF^t
28: **end while**
29: **Return:** $PF \leftarrow \max(PF^t, PF^{t-1}, PF^{t-2}), \omega \leftarrow \text{argmax}_{\text{i}}(PF^{t-i}, i=0,1,2), p \leftarrow p^w$

5.2 Convergence and Optimality of DRP-P

In the following theorem, we prove that DRP-P converges to within ϵ of the optimal point of **P**.

Theorem 3. *Suppose that $f(\xi) = 1$, that is, usage is uniformly distributed, and utility is linear. DRP-P converges to a price $p \in [p^* - \epsilon, p^* + \epsilon]$, where p^* is the optimal price.*

Proof. See Appendix E. □

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The basic idea of the proof is as follows. First we establish that the profit function is quasiconcave. Next, using the definition of quasiconcavity, we prove the following two statements by contradiction: R1) Given $p^{t+1} > p^t, PF^t \leq PF^{t+1} \Rightarrow p^t < p^{t+1} < p^*$or $p^t < p^* < p^{t+1}$and R2) Given $p^{t+1} > p^t, PF^t \geq PF^{t+1} \Rightarrow p^t \leq p^* \leq p^{t+1}$ or $p^* \leq p^t \leq p^{t+1}$. This indicates that once the condition $PF^t \geq PF^{t+1}$ occurs, the platform knows that it has overshot the optimal. Finally, we show that DRP-P will oscillate among prices within ϵ of the optimal price p^*.

6 Numerical Results

In this section, we illustrate that both mechanisms DRP-SW and DRP-P converge to optimality. We show that DRP-SW is robust when users' usage follow a more general distribution, and is robust to disruptions in the job arrival rate. Next, we provide insights on the two sided market's equilibrium state. Finally, we compare our solution to the scenarios where there is no sharing involved, and when the MNO posts multiple resource plans.

6.1 Social Welfare Maximizing and the Demand Disruption Scenario

We consider an MEC system with 10000 devices with usage levels ξ uniformly distributed in $[0, 1]$. The devices have a linear utility function $u(\xi) = \xi$. The cost of ownership $c = 0.2$. We use stepsize $\gamma = 0.1$ and choose random initial values $\theta(0) \in [0, 1]$, and $p(0) \in [0, 1]$. Fig. 4 a) shows how social welfare changes with time. Observe that the system converges to the optimal social welfare.

In real world scenarios, demand for computing resources does not remain constant, but surges or drops at various times. For example during the transition from night to day, the devices would start having more jobs to compute. With this in mind, we explore the robustness of mechanism DRP-SW to changes and disturbances in system demand, via modelling an abrupt disruption in the distribution of the devices' usage types ξ. We assume that usage types follow a beta distribution with PDF $f(x; \alpha, \beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$, where the normalisation constant $\frac{1}{B(\alpha,\beta)} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$. We initialise the usage distribution to be Beta(2,4), representing a lower demand. Following this, we model an increase in demand at 100 iterations by changing the usage distribution abruptly to Beta(4,2). As seen in Fig. 5 a), the system evolves and converges to the new optimal allocation after the change in demand. Note also that, although the Beta distribution $f(x; \alpha, \beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ does not satisfy the conditions in Theorems 1 and 2, the system nevertheless converges to the new optimal allocation.

6.2 The Profit Maximizing Scenario

Here we consider the case where the platform collects a commission and aims to maximize profit. It implements Algorithm DRP-P of Section 5. We run the algorithm for a scenario where users' usage follows $U[0.1, 0.8]$, the external resource plan cost $c = 0.2$ and the platform's commission level is $\delta = 0.2$, Fig. 4 b) shows that profit converges to the optimal value.

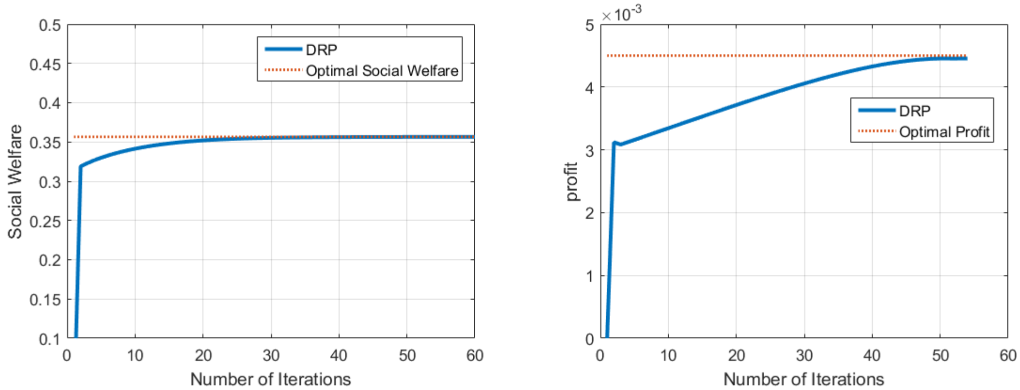
6.3 Insights at Equilibrium

Fig. 5 b) illustrates how the equilibrium's optimal threshold θ varies with the MNO's resource plan fee c. Recall that above θ, users become owners, and under θ, users are renters. We see that as the cost c increases, the payoff for owners decreases and therefore θ increases, i.e. there are more renters. There is a noticeable difference in the optimal thresholds for the social welfare versus profit maximizing scenarios. This is because only the social welfare problem takes the MNO's plan cost $-c$ into account, and it'll optimize such that there are more renters. From Fig. 5 c) we can see that the platform's profit increases as the external plan cost c increases. When c increases, the threshold θ increases, and more renters results in the renters' matching probability β decreasing. With a higher demand and lower supply, renters will have to pay a higher rental price p to rent, explaining the increase in profit.

Fig. 5 d) illustrates that it is easier to jointly optimize social welfare and profit when the resource plan cost c is low. At this point, both the social welfare and profit maximizing platforms have a common aim of balancing the ratio of owners vs renters, as it helps to maximize their objectives. One way to jointly optimize profit and social welfare is the use of a weighted average objective function, with a parameter controlling the priority. However, the challenges lie in the implementation, of designing a solution to the weighted objective. For social welfare maximization, it is easier to align the individual users' objectives with the platform's social welfare objective due to the structure of the SW objective, as seen in the design of Algorithm DRP-SW. For the profit maximizing problem it is tougher to align incentives. Another challenge is that vastly different optimal prices are required to enable social welfare and profit maximization.

6.4 Comparison to Scenario Without Sharing

In Figures 5 e) and f) we evaluate the performance of mechanisms DRP-SW and DRP-P in comparison with the scenario where there is no sharing platform. Under the no-sharing scenario, all users buy the MNO's single plan to reserve computational resources at the edge, at cost c. In Figure 5 e), we plot the social welfare per unit of resource consumed, for $c = 0.1$ and $c = 0.3$. Under the sharing scheme, less resources are reserved as only the owners users buy plans from the MNO - the rest get their quota by renting plans. We can see that the social welfare per unit of resource consumed is higher under our proposed mechanisms, because via sharing quotas to offload, users collectively reserve less resources, and pay less. Next, we analyse the payoffs for individual users in Fig 5 f). It can be seen that for users with higher usage levels, their individual welfare is higher compared to other users, under both scenarios. When there is no sharing, users with low usage levels do not stand to gain as the MNO's cost c is too high with respect to their low usage. With sharing, they can rent the offloading quota they need, at a lower payment level, hence improving their individual payoffs.



(a) Convergence of DRP-SW to optimal social welfare. (b) Convergence of DRP-P to optimal profit.

Fig. 4. Convergence of the proposed dynamic pricing mechanisms

6.5 Comparison to MNO Posting Multiple Plans

Now we evaluate the performance of mechanisms DRP-SW and DRP-P in comparison with the scenario where the MNO posts two resource plans A and B at (quota, price) = (ξ_A, c_A) and (ξ_B, c_B), and there is no sharing. For a fair comparison, the plan (ξ_A, c_A) is equivalent to the MNO's plan (cost c) under our setup, and $\xi_B < \xi_A$, $c_B < c_A$. We compare the individual welfare across users of different usage levels. Fig 5 g) shows that sharing gives light users (those with usage below θ) and heavy users (those with usage above ξ_B) a higher welfare, while multiple (fixed) plans gives medium users (those with usage below ξ_B and above θ) a higher welfare. This is because under fixed plans, light users would be paying more for quota they do not use, ie "subsidizing the heavy users". Heavy users benefit under sharing because they get to earn extra income. Meanwhile medium users (owners with usage below ξ_B) obtain a higher welfare under the lower quota plan as compared to being owners. Therefore, having multiple plans can be combined with sharing to reap the benefits of both solutions. Besides this, we compare the social welfare obtained. Fig 5 h) shows that sharing leads to higher welfare, unless the cost of the second plan is low and the quota given by Plan B is high.

7 Conclusion

We investigate resource allocation in MEC from the perspective of revenue management. To reduce the partial wastage cause by coarse grained plans and to fully make use of the heterogenous usage levels across the IoE market, we propose a novel sharing economy-inspired model, where the platform facilitates resource quota sharing among users. We study the scenarios where the platform's aim is to maximize social welfare and profit. And we present online dynamic pricing mechanisms which the platform could implement in spite of its imperfect knowledge and control, to achieve its social welfare and profit maximizing aims. For both our mechanisms, we prove that they converge to optimality. We also provide simulations to illustrate convergence under more general conditions, robustness to changes in the job arrival rate, and provide insights on the two-sided market under changes in exogeneous conditions.

The focus of this work is on how computing resource quota can be shared among users with heterogeneous usage levels to reduce wastage, and not on the technical challenges of enabling edge computing. In future work, we can combine the economic analysis with the technical aspects of computing resource provision. For example, we can analyse how the profit maximizing prices would be explicitly affected by the distance between the mobile user and edge location, the CPU requirements, etc.

In this paper, our model and dynamic pricing mechanisms focus on the behaviour and dynamics of system averages, e.g. the user population can be interpreted as the average of user arrivals and departures. Therefore one limitation is that our proposed approaches are only applicable at the timescale where system averages can be sufficiently well estimated. Nevertheless, we believe that our proposed approaches provide important insights into the problem as it is extremely challenging to design pricing algorithms that are amenable at the timescale of point arrivals and departures. In future work, we will investigate the practical consequences of the timescale limitations of our algorithm as well as develop extensions for operating on faster timescales.

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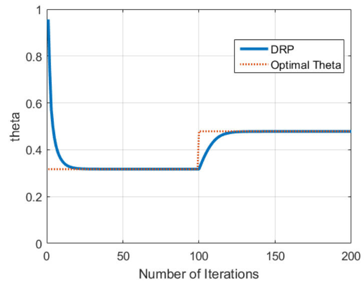
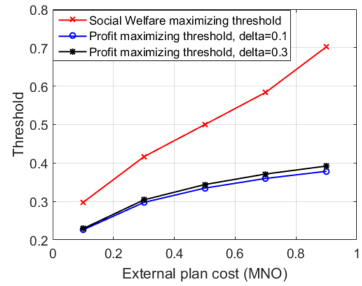
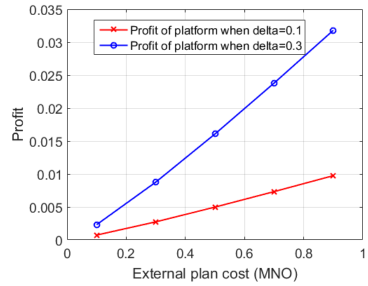


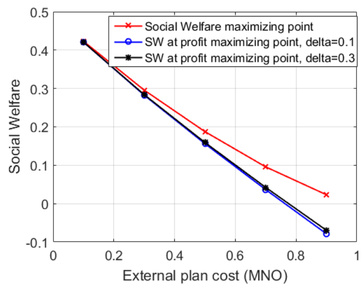
(a) Effect of Demand Disruption on Theta



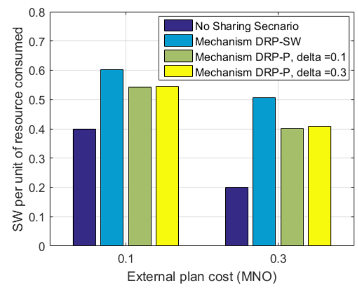
(b) Equilibrium Threshold against MNO's Resource Plan Cost c



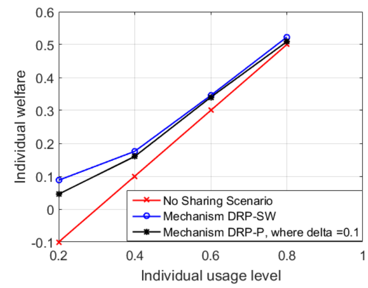
(c) Platform's Profit as MNO's Plan Cost is varied



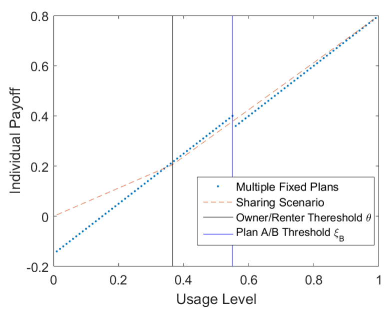
(d) Social Welfare loss under the Profit Maximizing Scenario



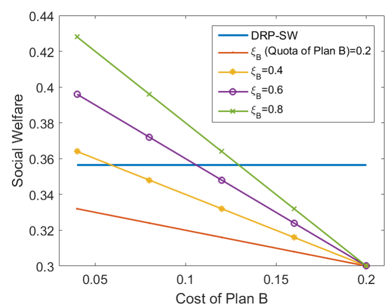
(e) Comparison with the No-Platform Scenario - Social Welfare



(f) Comparison with the No-Platform scenario - Individual Welfare, c=0.35



(g) Comparison to MNO posting Multiple Plans - Individual Welfare, c=0.2



(h) Comparison to MNO posting Multiple Plans - Social Welfare, c=0.2

Fig. 5.

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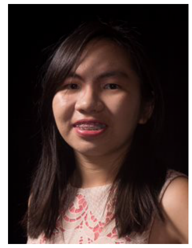

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