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Three Dynamic Pricing Schemes for Resource Allocation of Edge Computing for IoT Environment

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***Abstract*—With the widespread use of Internet of Things (IoT), edge computing has recently emerged as a promising technology to tackle low-latency and security issues with personal IoT data. In this regard, many works have been concerned with computing resource allocation of edge computing server, and some studies have conducted to the pricing schemes for resource allocation additionally. However, few works have attempted to address the comparison among various kinds of pricing schemes. In addition, some schemes have their limitations such as fairness issues on differentiated pricing schemes. To tackle these limitations, this paper considered three dynamic pricing mechanisms for resource allocation of edge computing for IoT environment with a comparative analysis: BID-PRoportional Allocation Mechanism (BID-PRAM), UNIform PRIcing Mechanism (UNI-PRIM), and FAIrness-seeking Differentiated PRIcing Mechanism (FAID-PRIM). BID-PRAM is newly proposed to overcome the limitation of the auction-based pricing scheme; UNI-PRIM is a basic uniform pricing scheme; FAID-PRIM is newly proposed to tackle the fairness issues of differentiated pricing scheme. BID-PRAM is formulated as a non-cooperative game. UNI-PIM and FAID-PRIM are formulated as a single-leader-multiple-followers Stackelberg game. In each mechanism, the Nash equilibrium (NE) or Stackelberg equilibrium (SE) solution is given with the proof of existence and uniqueness. Numerical results validate the proposed theorems and present a comparative analysis of three mechanisms. Through these analyses, the advantages and disadvantages of each model are identified, to give edge computing service providers guidance on various kinds of pricing schemes.**

***Index Terms*—Edge Computing, Internet of Things, Dynamic Pricing, Bid-Proportional Pricing, Fairness-seeking Differentiated Pricing**

I. Introduction

In recent years, thanks to the rapid advancement of artificial intelligence (AI) technology for data analytics of Internet of Things (IoT) [1]–[3] and the emergence of IoT dedicated networks such as LoRa, NB-IoT LTE-M [4], [5], the number of IoT devices such as mobiles, wearable devices, tablets, smart metering devices, and smart speaker has been exponentially increasing. According to "Erriccson Mobility Report 2018", the number of IoT devices worldwide is expected to exceed 4.1 billion in 2024, increasing at a compound annual growth rate of 33% since 2013 [6]. Under these circumstances, lack of computation power from handling this explosive growth of IoT data, and security issue of personal IoT data forced IoT devices to use an edge computing for the quality of experience (QoE) [7]–[13].

In this regard, lots of works have been concerned with optimal resource allocation of edge servers for computation offloading for IoT or mobile devices [14]–[27]. In these studies, the optimization problems have been suggested, and solved to achieve optimal resource allocation in order to maximize energy efficiency or minimize latency where resource can be considered as computing power, transmission power, and bandwidth, etc. However, most of these studies only focus on improving the overall system wide performance, causing many fairness problems of IoT devices. In this manner, there has been a lot of research to resolve this issue [28]–[37]. Nevertheless, those works devoted to improve the fairness issue cannot properly account for some users who may have a stronger desire to use edge server even with a higher price (simply called non-pricing edge computing system). Accordingly, the pricing mechanism can be a good option to differentiate the resource allocation considering both system level performance and different user desire levels.

A. Related Works

From this point of view, several studies have been recently conducted to optimize the pricing schemes for edge computing servers' resources [38]–[43]. In [38], uniform pricing is considered in edge computing systems of which users consider the energy efficiency and cost. In [38], the authors formulated Stackelberg game model among the users and edge seller, and determined an optimal unit price to maximize the seller's revenue. Also, in [39], the authors suggested an algorithm, which finds Nash equilibrium (NE) among users by considering energy efficiency, cost, and delay under the given fixed-price policy. In [40], the Stackelberg game model between users considering delay and cost of the sellers is proposed. In addition, they proposed two algorithms that can maximize the seller's revenue in the case of using a 1) uniform and 2) differentiated pricing policy. In addition, [41] proposed a differentiated pricing scheme which maximized the overall happiness of both the users and the seller. In [42],

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TABLE I
SUMMARY OF THE WORKS ON PRICING SCHEMES FOR EDGE COMPUTING

	[38], [39]	[40]	[41], [42]	[43]	ours
uniform pricing	O	O	X	X	O
differentiated pricing	X	O	O	X	O
: considering fairness	X	X	X	X	O
bid-based mechanism	X	X	X	O	O
comparative analysis	X	O	X	X	O

differentiated pricing is considered in the system model with the coalition between device users and edge servers. Finally, [43] introduced a bid-based auction style pricing mechanism by considering joint communication and computation resource allocation.

Some studies also considered pricing schemes of edge computing on specific situations. In [44]–[46], the authors formulated a case where mobile users run blockchain applications using edge computing to mine blocks and considering both the uniform and the differentiated pricing. Also, in [47], the Double auction system for edge computing is discussed. The authors proposed two different systems, breakeven based Double auction and dynamic pricing based Double auction, and analyzed their efficiencies. Moreover, in [48], they formulated resource management as a Double auction game and analyzed it. In [49], a system with mobile users, edge clouds and remote clouds are considered. In [50], the concept of general equilibrium is suggested within systems that are composed of edge nodes with capacity limitations and services with budget limitations. Further, in [51], the authors focused on a certain situation where sellers offer the edge-cloud based caching service to users. As providers purchase the caching space at a fixed price, Stackelberg equilibrium between the sellers and users is considered.

All the aforementioned studies have concentrated on designing the optimal pricing mechanisms with given specific scenarios or metrics such as energy efficiency, delay, fairness, etc for edge computing system. However, none of the above have provided comparative analysis among several pricing policies. Furthermore, most of studies on edge computing considered only a single pricing scheme. Thus, it is difficult to measure the advantages and disadvantages of various pricing schemes. Table I-A gives a summary of the studies on pricing schemes for edge computing from this perspective.

B. Contributions

To this end, this paper considers three dynamic pricing mechanisms for edge computing, and provides a comparative analysis among them: BID-PRoportional Allocation Mechanism (BID-PRAM), UNIform PRIcing Mechanism (UNI-PRIM), and FAIrness-seeking Differentiated PRIcing Mechanism (FAID-PRIM). Based on the analysis, it is emphasized that the detailed modeling of pricing schemes should be recognized, and that a detailed pricing scheme should also be studied in depth for efficient edge computing operation.

The contributions of this paper are summarized as follows.

- This paper clearly defines three dynamic pricing schemes for edge computing for IoT environment where all the data are transmitted to be processed: BID-PRAM, UNI-PRIM, and FAID-PRIM.
- We rigorously analyze all three in a game theoretical approach. BID-PRAM is formulated as a non-cooperative game. UNI-PIM and FAID-PRIM are formulated as a single-leader-multiple-followers Stackelberg game. In each mechanism, the Nash equilibrium (NE) or Stackelberg equilibrium (SE) solution is given with the proof of existence and uniqueness.
- BID-PRAM is proposed based on the auction-based models in [43]. Specifically, the auction-based pricing model has limitations in markets requiring low latency such as edge computing, because the final deal is not well formed when there are few sellers and buyers. Accordingly, BID-PRAM is newly proposed to overcome these limitations. In this method, when there are one seller and several buyers, the transaction is always, quickly made based on the buyers' bid price.
- To tackle fairness issues on differentiated pricing schemes for edge computing, FAID-PRIM is newly proposed based on the previous mechanisms [40]–[42]. In this mechanism, the pricing-fairness factor is introduced so that the price difference does not exceed a certain level.
- In particular, this FAID-PRIM provides a more rigorous analysis of how much the fairness factor should be set for service providers to earn more than UNI-PRIM. This analytic guideline is expected to be useful to the edge service providers; they can predict how much revenue will be generated compared to UNI-PRIM, as long as users do not complain.
- From the performance analysis and numerical analysis, we compare advantages and disadvantages of three pricing schemes. Through this analysis, service providers can choose an appropriate pricing scheme based on their IoT operational environment.
- This study will emphasize the necessity and importance of pricing schemes for edge computing for IoT, and will provide a basis for studying various pricing policies.

The rest of the paper is organized as follows. Section II presents an overview of the proposed pricing mechanisms. Section III formulates each mechanisms as a non-cooperative game model, and derives NE solution of each model with the proof of existence and uniqueness. Section IV gives several numerical and experimental results of the proposed models to validate the proposed theorems and to give comparison of the average performances of each of them. Section V provides discussions about realization, game model, and future work. Finally, Section VI concludes the paper.

II. System Model

In this section, system models for the allocation of CPU cycles of the edge computing are proposed with the cost function of device and the revenue of the edge computing server. Also, the following three pricing schemes for resource allocation are provided: BID-PRAM, UNI-PRIM, and FAID-PRIM.

Assume that there is an edge computing server with N IoT devices to use the edge server at certain time. Let the set of IoT devices $\mathcal{I}$, denoted by $\{1, 2, \cdots, N\}$. Every IoT device $k \in \mathcal{I}$ has to use the edge computing server to process its offloading data with size R_k (bits); denote the set of all the devices' offloading data by $\mathcal{R} = \{R_1, R_2, \cdots, R_N\}$. There are total F CPU cycles per a second available in the edge computing server that will be allocated to the IoT devices by some allocation mechanisms; three allocation mechanisms are proposed in the remainder of this section.

Now, let F_k be the allocated CPU cycles to the kth device with a set of all the devices' allocated CPU cycles, $\mathcal{F} = \{F_1, F_2, \cdots, F_N\}$. Also, let w_k be the payment of the kth device to use CPU cycles of edge computing server with a set of all the devices' payment $\mathbf{w} = [w_1, w_2, \cdots, w_N]$ ($w_i > 0,\ \forall i \in \mathcal{I}$). Then the computation time of each data R_i is R_iC_i/F_i, where $C_i \in \{C_1, C_2, \cdots, C_N\}$ denotes the needed CPU cycles when the edge computing computes 1 bit of data of ith device.

For each IoT device, it is beneficial to reduce the computation time R_kC_k/F_k while minimizing the cost w_k. Now, to analyze each device's happiness, the cost function of the ith IoT device can be defined as a real-valued function from all the devices' strategies to its cost, which proportionally increases when the computation time and the cost increase.

Definition 1 (Cost Function of the kth IoT Device)**.** *The cost function of the kth IoT device, $u_i : \mathbb{R}^N \to \mathbb{R}$ is defined by*

$$u_i(w_i, \mathbf{w}_{-i}) := \frac{R_iC_i}{F_i} + \mu_i w_i. \tag{1}$$

Here, w_i the kth device's strategy of the payment and $\mathbf{w}_{-i}$ is the others' strategies, that is, $\mathbf{w}_{-i} = \{w_1, \cdots, w_{k-1}, w_{k+1}, \cdots, w_N\}$. μ_i is a weight factor indicating the tendency of the kth IoT device to think about how much the payment cost is more important than the delay time.

Lastly, for the convenience of calculation, the utility function of the kth IoT device v_i is defined as follows.

Definition 2 (Utility Function of the kth IoT Device)**.** *The utility function of the kth IoT device, $v_i : \mathbb{R}^N \to \mathbb{R}$ is defined by*

$$\begin{aligned} v_i(w_i, \mathbf{w}_{-i}) &:= -\,u_i(w_i, \mathbf{w}_{-i}) \\ &= -\frac{R_iC_i}{F_i} - \mu_i w_i. \end{aligned} \tag{2}$$

The utility function of the device seems to be only as a function of w_i. Based on the CPU allocation mechanism, F_i can be expressed as a function of all the strategies $\mathbf{w}$, that is, $F_i(w_i, \mathbf{w}_{-i})$. Then the strategy of all the other devices will ultimately affect its utility v_i, which formulates a non-cooperative game among IoT devices. As will be discussed in more detail in the following subsections, in summary, only BID-PRAM among the three mechanisms is formulated as a non-cooperative game.

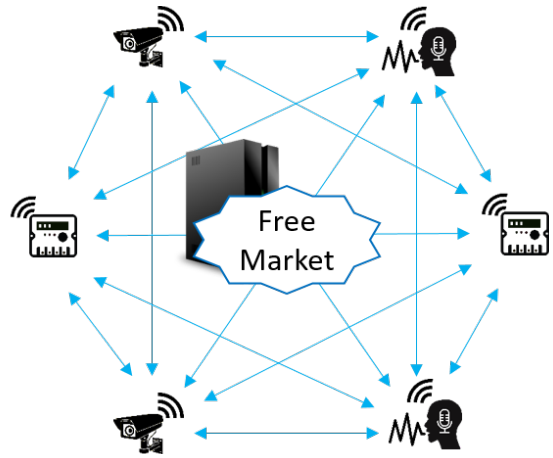

Fig. 1. System Model of BID-PRAM.

A. BID-PRAM

First in this subsection, BID-PRAM is proposed to resolve the problem that it takes too long for transactions to be completed for all computing resources in the auction-based pricing mechanism, and is formulated as a non-cooperative game among IoT devices.

In BID-PRAM, each IoT device user bids some amount of money to the edge computing server to get more CPU cycles. Let w_i be the amount of bids to the provider of the edge computing to get more CPU cycles for faster computation. Then the provider allocates certain portions of the cycles to the users in exact proportion to their bids, that is, the distributed CPU cycles are given by,

$$F_i = F\frac{w_i}{\sum_{i\in\mathcal{I}} w_i}. \tag{3}$$

Based on this utility function, every device in $\mathcal{I}$ is going to choose its best strategy to maximize its own utility function. Then, the computing resource competition among the devices is formulated as a non-cooperative strategic form game $G = \left(\mathbf{W}, v_i\right)_{i\in\mathcal{I}}$, where $\mathbf{W}$ is the strategy domain for all the IoT devices, given by $\mathbf{W} = \prod_{k=1}^{N}[0, \infty)$. In this non-cooperative game, the market price is determined automatically, which is going to be analyzed in the subsection III-A.

In the auction-based mechanism, the edge server may have to participate in the market many times to sell all of its computing resources. On the other hand, in BID-PRAM, all users have to bid up to one time in order to deal with all resources. In this respect, BID-PRAM achieves the final transaction for all resources very quickly compared to the auction-based mechanism.

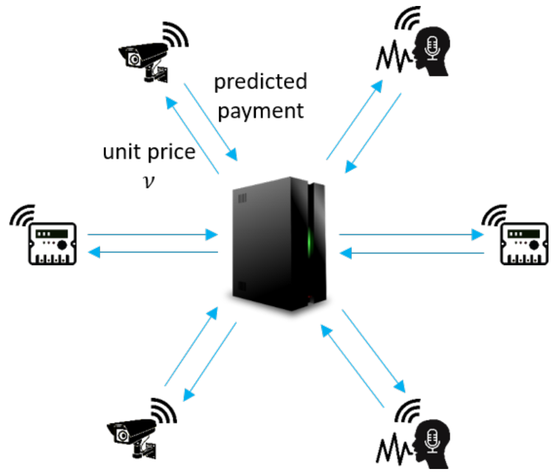

Fig. 2. System Model of UNI-PRIM.

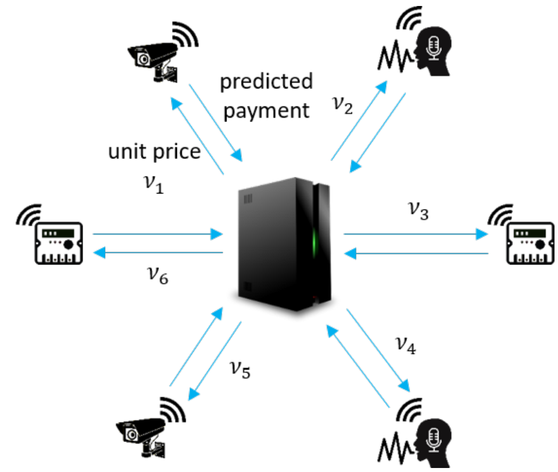

Fig. 3. System Model of FAID-PRIM.

B. UNI-PRIM

This subsection describes the simple procedure of UNI-PRIM. In UNI-PRIM, at first, the edge server declares a uniform unit price, p, to all devices. Then each device, decides the amount of payment, w_i to get CPU cycles $F_i = \frac{w_i}{p}$, maximizing its utility v_i.

Note that the edge server predicts the costs of the users and determines the unit price to use all CPU cycles in the edge computing in BID-PRIM. Thus, UNI-PRIM is formulated as a single-leader-multiple-followers Stackelberg game model, where the edge server role is a leader and IoT devices roles are multiple followers.

C. FAID-PRIM

In this subsection, FAID-PRIM is newly proposed based on the previous mechanisms to resolve fairness issues on differentiated pricing scheme for edge computing [40]–[42]. In FAID-PRIM, the edge server can differentiate the unit price of each user. Each user decides their payments to maximize their utility function, and the edge server changes the unit price of each user to allocate all CPU cycles and to maximize the revenue based on exact prediction. Thus, FAID-PRIM is also formulated as a single-leader-multiple-followers Stackelberg game model, where the edge server roles as a leader and IoT devices role as multiple followers.

However, if the unit prices have huge differences with each other, there are some possibilities that the users could feel a sense of unfairness. Thus a fairness factor f is proposed to restrict the portion between the highest and lowest unit prices.

$$1 \leq \frac{\max_{k \in \mathcal{I}} p_k}{\min_{k \in \mathcal{I}} p_k} \leq f. \tag{4}$$

III. Performance Analysis

In this subsection, a Nash euqilibrium of BID-PRAM is defined and the unique NE solution is provided with the existence and uniqueness. Also, each of Stackelberg equilibrium of UNI-PRIM and FAID-PRIM is defined, and unique SE solution is provided with the existence and uniqueness. Finally, a comparative analysis among the three models is provided, which concludes this section.

A. BID-PRAM

In BID-PRAM, the market price of CPU cycles in the edge computing is totally determined by strategies of device users, without any participation of the seller. We assume that the users are independent and choose their strategies to maximize each user's own utility function.

In this non-cooperative game of BID-PRAM, the stable market price may exist at the NE point among users, defined as follows:

Definition 3 (Nash equilibrium (NE) of BID-PRAM)**.** *The NE point of BID-PRAM among the users,* $\mathbf{w}^* = [w_1^*, w_2^*, \cdots, w_N^*]$*, is defined as*

$$v_i(w_i^*, \mathbf{w}_{-i}^*) \geq v_i(w_i, \mathbf{w}_{-i}^*), \quad \forall w_i > 0, i \in \mathcal{I}.$$

Whether there exist a NE point of the game or not is crucial to analyze the behaviors of the users in the system. We suggest mathematical approaches to prove the uniqueness of the NE of BID-PRAM.

Lemma 1. *The utility function* $v_i(w_i, \mathbf{w}_{-i}^*)$ *is strictly concave and continuously differentiable in* $w_i > 0$.

Proof. By taking first and second partial derivatives of $v_i(w_i, \mathbf{w}_{-i}^*)$ with respect to w_i,

$$\frac{\partial}{\partial w_i} v_i(w_i, \mathbf{w}_{-i}^*) = \frac{R_i C_i}{F} \frac{\sum_{k \in \mathcal{I} \setminus \{i\}} w_k^*}{w_i^2} - \mu_i; \tag{5}$$

$$\frac{\partial^2}{\partial w_i^2} v_i(w_i, \mathbf{w}_{-i}^*) = -\frac{2 R_i C_i}{F} \frac{\sum_{k \in \mathcal{I} \setminus \{i\}} w_k^*}{w_i^3}. \tag{6}$$

we can see that the second derivative is negative for all $w_i > 0$, proving the concavity of the utility function.

Also, the first derivative is in a rational form with respect to w_i, proving the continuous differentiability of the utility function. □

Lemma 2. *$\mathbf{w}^* = [w_1^*, w_2^*, \cdots, w_N^*]$ is a NE point among the users if and only if the following condition holds:*

$$\frac{\sum_{k\in\mathcal{I}\setminus\{i\}} w_k^*}{w_i^{*2}} = \frac{F\mu_i}{R_iC_i}. \tag{7}$$

Proof. Let $\mathbf{w}^*$ be a NE among the users. $v_i(w_i, \mathbf{w}_{-i}^*)$ is strictly concave and continuously differentiable in $w_i > 0$ by Lemma 1. Thus, by Definition 1, $v_i(w_i, \mathbf{w}_{-i}^*)$ has the unique local and global maximizer w_i^* over $w_i > 0$. By Fermat's Theorem, the following condition holds:

$$\frac{\partial}{\partial w_i} v_i(w_i^*, \mathbf{w}_{-i}^*) = 0 \tag{8}$$

which is equivalent to the equation (7).

Conversely, suppose that the condition (7) holds for $\mathbf{w}^*$. The utility function $v_i(w_i, \mathbf{w}_{-i}^*)$ is strictly concave and continuously differentiable $w_i > 0$ by Lemma 1. Also, the first derivative of the utility function at w_i^* is 0, meaning $v_i(w_i, \mathbf{w}_{-i}^*)$ has the unique local and global maximizer w_i^* in $w_i > 0$. By Definition 1, $\mathbf{w}^*$ is a NE. □

Lemma 3. *Consider a problem finding $\mathbf{e}^* = [e_1^*, e_2^*, \cdots, e_N^*]$ and scalar p that satisfies the following conditions for all $i \in \mathcal{I}$:*

$$\begin{cases} \dfrac{R_iC_i}{\mu_iF}\dfrac{F - e_i^*}{e_i^{*2}} - p = 0; \\ \displaystyle\sum_{k\in\mathcal{I}} e_k^* - F = 0; \\ e_i^*, p > 0. \end{cases} \tag{9}$$

Then this problem has the unique solution, given by

$$e_i^* = \frac{-R_iC_i + \sqrt{R_i^2C_i^2 + 4\mu_iF^2R_iC_ip}}{2\mu_iFp} \tag{10}$$

where p is a real number satisfying $\sum_{k\in\mathcal{I}} e_k^* = F$.

Proof. By the first condition of (9),

$$\mu_iFpe_i^{*2} + R_iC_ie_i^* - FR_iC_i = 0$$

The discriminant of this quadratic equation, $4\mu_iF^2R_iC_ip + R_i^2C_i^2$, is obviously positive, proving the existence of e_i^*.

By quadratic formula and the third condition of (9), the solution is given by

$$e_i^* = \frac{-R_iC_i + \sqrt{R_i^2C_i^2 + 4\mu_iF^2R_iC_ip}}{2\mu_iFp}$$

showing the uniqueness of e_i^*.

Also, by the second condition of (9),

$$\sum_{k\in\mathcal{I}} \frac{-R_iC_i + \sqrt{R_i^2C_i^2 + 4\mu_iF^2R_iC_ip}}{2\mu_i} - F^2p = 0.$$

Let the left-handed side be $f(p)$. Taking first and second derivative, we get

$$\frac{\partial f(p)}{\partial p} = \sum_{k\in\mathcal{I}} \frac{F^2R_iC_i}{\sqrt{R_i^2C_i^2 + 4\mu_iF^2R_iC_ip}} - F^2 \quad \text{and}$$

$$\frac{\partial^2 f(p)}{\partial p^2} = -\sum_{k\in\mathcal{I}} \frac{2\mu_iF^4R_i^2C_i^2}{(R_i^2C_i^2 + 4\mu_iF^2R_iC_ip)^{3/2}}.$$

Since $\frac{\partial f(p)}{\partial p}|_{p=0} > 0$ and $\frac{\partial^2 f(p)}{\partial p^2} < 0$ for all $p > 0$, there exists a unique $p > 0$ that satisfies (9). □

Lemma 4. *If $\mathbf{w}^*$ is a NE of noncooperative game among users, then the vector $\mathbf{e}^* = (\frac{w_k^*}{p})_{k\in\mathcal{I}}$ and the scalar p defined by $\frac{\sum_{k\in\mathcal{I}} w_k^*}{F}$ are the unique solution to (9).*

Proof. By Lemma 2, $\mathbf{w}^*$ satisfies the condition (7). Substituting $w_i^* = pe_i^*$ and $p = \frac{\sum_{k\in\mathcal{I}} w_k^*}{F}$, we get

$$\frac{F - e_i^*}{pe_i^{*2}} = \frac{\mu_iF}{R_iC_i} \tag{11}$$

which shows that $\mathbf{w}^*$ satisfies the first condition of (9). Similarly, we can easily show that $\sum_{k\in\mathcal{I}} e_k^* - F = 0$, proving that newly defined vector $\mathbf{e}^*$ and the scalar p are a solution to (9).

Also, Lemma 3 ensures that the newly defined solution of (9) is unique. □

Lemma 5. *If $(\mathbf{e}^*, p)$ satisfies the condition (9), then the vector $\mathbf{w}^* = (pe_k^*)_{k\in\mathcal{I}}$ is a NE.*

Proof. It can be shown by the reverse argument of the proof of Lemma 4. Also by Lemma 3, such $(\mathbf{e}^*, p)$ is unique. □

Theorem 1. *There exists the unique NE $\mathbf{w}^*$ of noncooperative game among device users in $\mathcal{I}$, which is given by*

$$w_i^* = \frac{-R_iC_i + \sqrt{R_i^2C_i^2 + 4\mu_iF^2R_iC_ip}}{2\mu_iF} \tag{12}$$

where p is a real number satisfying $\sum_{k\in\mathcal{I}} w_k^* = pF$.

Proof. Lemma 3 and Lemma 5 proves the existence of NE, and Lemma 4 proves the uniqueness of the equilibrium. By substituting $\mathbf{w}^* = (pe_k^*)_{k\in\mathcal{I}}$ to the (10), it is easily shown that the unique NE among users is given by (12). □

Theorem 1 states that there exist a unique NE point and we can calculate the point through (12). This result shows that the payments of users will tend to be fixed after competitions between the users, regardless of the initial payments. Therefore, the edge seller can predict the strategies of IoT devices and determine the amount of CPU cycles for users with Theorem 1.

B. UNI-PRIM

In the Stackelberg game of UNI-PRIM, the stable market price may exist at the SE point, defined as follows.

Definition 4 (Stackelberg equilibrium (SE) of UNI-PRIM)**.** *A SE point of UNI-PRIM is a set of strategies $\{\{w_i^*\}_{i\in\mathcal{I}}, p^*\}$ satisfying the following conditions.*

Condition 1: $v_i(w_i^*) \geq v_i(w_i), \forall w_i \in [0, \infty), i \in \mathcal{I}$.

Condition 2: $\sum_{i\in\mathcal{I}} F_i(w_i^*, p^*) = \sum_{i\in\mathcal{I}} w_i^*/p^* = F$.

Since UNI-PRIM is the same as FAID-PRIM with a fairness factor of 1, a following theorem is provided to find the SE of UNI-PRIM, which concludes this subsection.

Theorem 2. *The SE solution of UNI-PRIM is uniquely given by*

$$w_i^* = \sqrt{\frac{R_i C_i p^*}{\mu_i}} \qquad \text{and} \tag{13}$$

$$p^* = \frac{\left(\sum_{k\in\mathcal{I}} \sqrt{\frac{R_k C_k}{\mu_k}}\right)^2}{F^2}. \tag{14}$$

Proof. Suppose that the provider declares a uniform unit price p^* to all users. Then, the user's utility function is given by

$$v_i(w_i) = -\frac{R_i C_i p^*}{w_i} - \mu_i w_i. \tag{15}$$

Taking first and second derivative, we get

$$\frac{d}{dw_i} v_i(w_i) = \frac{R_i C_i p^*}{w_i^2} - \mu_i \quad \text{and} \tag{16}$$

$$\frac{d^2}{dw_i^2} v_i(w_i) = -\frac{2R_i C_i p^*}{w_i^3}. \tag{17}$$

The second derivative is negative over $w_i > 0$, proving the concavity of the utility function. Then if there exists w_i that makes the first derivative 0, it is the unique maximizer of the function. Thus the optimal point w_i^* is uniquely given by (13).

Meanwhile, the provider exactly predicts how much the users pay and decides the unit price to sell all of the CPU cycles in edge computing. Then p^* is determined to satisfy $\sum_{k\in\mathcal{I}} w_k^* = p^* F$. Thus $Fp^* - \left(\sum_{k\in\mathcal{I}} \sqrt{\frac{R_k C_k}{\mu_k}}\right)\sqrt{p^*} = 0$, proving the uniqueness of p^* and (14). □

Theorem 2 shows that the users' payments and the unit price to sell all CPU cycles in edge computing are uniquely determined when the IoT devices choose their strategies to maximize their own utilities.

C. FAID-PRIM

In the Stackelberg game of FAID-PRIM, the stable market price may exist at the SE point, defined as follows similar to UNI-PRIM.

Definition 5 (Stackelberg equilibrium (SE) of FAID-PRIM)**.** *A SE point of FAID-PRIM is a set of strategies $\{\{w_i^*\}_{i\in\mathcal{I}}, \{p_i^*\}_{i\in\mathcal{I}}\}$ satisfying the following conditions.*

Condition 1: $v_i(w_i^*) \geq v_i(w_i), \forall w_i \in [0,\infty), i \in \mathcal{I}$.

Condition 2: $\sum_{i\in\mathcal{I}} F_i(w_i^*, p_i^*) = \sum_{i\in\mathcal{I}} w_i^*/p_i^* = F$.

To find the optimal unit prices to maximize the seller's revenue in FAID-PRIM, following definition and lemma are provided.

Definition 6. *The vector of the optimal differentiated unit prices, $\boldsymbol{p} = [p_1, p_2, \cdots, p_N]$ ($p_i \in [0, p_{max}]$, $\forall i \in \mathcal{I}$), are defined as*

$$\boldsymbol{p} = \arg\max_{\boldsymbol{p}} \sum_{k\in\mathcal{I}} w_i \tag{18}$$

, where $w_i = \arg\max_{w_i} v_i(w_i, p_i) = \arg\max_{w_i}(-\frac{R_i C_i p_i}{w_i} - \mu_i w_i)$, $\sum_{k\in\mathcal{I}} F_k = F$, *and the condition* (4) *holds.*

Lemma 6. *The vector $\boldsymbol{p} = [p_1, p_2, \cdots, p_N]$ ($p_i > 0$, $\forall i \in \mathcal{I}$) is the optimal differentiated unit price vector if and only if it is the solution to the following optimization problem,*

$$\max_{\boldsymbol{p}} \sum_{k\in\mathcal{I}} \sqrt{\frac{R_k C_k p_k}{\mu_k}}$$
$$\text{subject to} \begin{cases} \sum_{k\in\mathcal{I}} \sqrt{\frac{R_k C_k}{\mu_k p_k}} = F \\ 1 \leq \frac{\max_{k\in\mathcal{I}} p_k}{\min_{k\in\mathcal{I}} p_k} \leq f \end{cases} \tag{19}$$

Proof. The utility function, $v_i(w_i) = -\frac{R_i C_i p_i}{w_i} - \mu_i w_i$, is strictly concave and continuously differentiable. Thus it has a unique maximizer $w_i = \sqrt{\frac{R_i C_i p_i}{\mu_i}}$, making its first derivative 0. Substituting w_i to (18), we get (19). □

To solve this optimization problem, more simplified problem is proposed.

Problem 1.

$$\max_{\mathbf{x}} \sum_{i=1}^{N} \frac{a_i}{x_i}$$
$$\text{subject to} \begin{cases} \sum_{i=1}^{N} a_i x_i = s \\ 1 \leq \frac{\max_{i\in\mathcal{I}} x_i}{\min_{i\in\mathcal{I}} x_i} \leq K \\ a_i, x_i > 0, \forall i \in \mathcal{I} \end{cases} \tag{20}$$

Lemma 7. *If $\mathbf{x} = [x_1, x_2, \cdots, x_N]$ is the solution to the Problem 1, then $x_i \in \{\max_{k\in\mathcal{I}} x_k, \min_{k\in\mathcal{I}} x_k\}$ for every $i \in \mathcal{I}$.*

Proof. The proof is in Appendix A □

Theorem 3. *Suppose that two sets $A = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ and $B = \{\beta_1, \beta_2, \cdots, \beta_{N-n}\}$ satisfy $A \cap B = \varnothing, A \cup B = \{a_1, a_2, \cdots, a_N\}$, and $|\sum_{x\in A} x - \sum_{x\in B} x|$ is the smallest among all divisions of the set $\{a_1, a_2, \cdots, a_N\}$. Here, let $\sum_{x\in A} x = \alpha$ and $\sum_{x\in B} x = \beta$.*

Then the solution to the Problem 1 is uniquely given by

$$x_i = \begin{cases} \frac{s}{\alpha + K\beta}, & \text{if } a_i \in A \\ \frac{Ks}{\alpha + K\beta}, & \text{if } a_i \in B \end{cases} \tag{21}$$

Proof. By Lemma 7, the solution to the Problem 1 consists of 2 different real numbers, denoted by x_{max} and x_{min}. Suppose that $\mathbf{x}$ is the solution to the problem. Then define

$A = \{a_i | x_i = x_{min}\}$ and $B = \{a_i | x_i = x_{max}\}$. Also, denote that $\frac{x_{max}}{x_{min}} = k$. According to the equality condition,

$$x_{min} = \frac{s}{\sum_{x\in A} x + k\sum_{x\in B} x}$$
$$x_{max} = \frac{ks}{\sum_{x\in A} x + k\sum_{x\in B} x}$$

Denote $\sum_{x\in A} = \alpha$, $\sum_{x\in B} = \beta$, and $\alpha + \beta = \gamma$. Then the following equality is satisfied:

$$\begin{aligned}\sum_{i=1}^{N}\frac{a_i}{x_i} &= \sum_{x\in A}\frac{x(\alpha+k\beta)}{s} + \sum_{x\in B}\frac{x(\alpha+k\beta)}{ks}\\ &= \frac{\alpha(\alpha+k\beta)}{s} + \frac{\beta(\alpha+k\beta)}{ks}\\ &= \frac{(\alpha+\beta)^2 + (k+\frac{1}{k}-2)\alpha\beta}{s}\\ &= \frac{\gamma^2 + (k+\frac{1}{k}-2)\alpha(\gamma-\alpha)}{s}\end{aligned}$$

Here, it is a quadratic function of α, which has a negative leading coefficient and the axis $x = \gamma/2$. Since $\mathbf{x}$ is the solution to the Problem 1, α is the closest to $\gamma/2$ among all other possible divisions of the coefficient set. Therefore, it satisfies (21).

Conversely, we can prove that $\mathbf{x}$ satisfying (21) is the solution to the Problem 1 in same way. □

Theorem 4. *Suppose that two sets $A = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ and $B = \{\beta_1, \beta_2, \cdots, \beta_{N-n}\}$ satisfy $A \cap B = \varnothing, A \cup B = \{\sqrt{\frac{R_iC_i}{\mu_i}} | i = 1, 2, \cdots, N\}$, and $|\sum_{x\in A} x - \sum_{x\in B} x|$ is the smallest among all divisions of the set $\{\sqrt{\frac{R_iC_i}{\mu_i}} | i = 1, 2, \cdots, N\}$. Here, let $\sum_{x\in A} x = \alpha$ and $\sum_{x\in B} x = \beta$.*

Then the optimal differentiated unit price vector $\boldsymbol{p}$ is uniquely given by,

$$p_i = \begin{cases}\left(\frac{\alpha+\beta f^{1/2}}{F}\right)^2, & \text{if } \sqrt{\frac{R_iC_i}{\mu_i}} \in A\\ \left(\frac{\alpha+\beta f^{1/2}}{f^{1/2}F}\right)^2, & \text{if } \sqrt{\frac{R_iC_i}{\mu_i}} \in B\end{cases} \tag{22}$$

Proof. By substituting $p_i = \frac{1}{x_i^2}$, $a_i = \sqrt{\frac{R_iC_i}{\mu_i}}$, and $K = \sqrt{f}$ to (21), we can easily get the optimal differentiated unit price as (22). □

D. Comparison of Three Models

In this subsection, the mathematical comparisons among proposed three models is provided.

Theorem 5. *The seller's total revenue of FAID-PRIM is strictly increasing with $f \geq 1$. Especially, it is identical to UNI-PRIM when $f = 1$.*

Proof. In FAID-PRIM,

$$\sum_{k\in\mathcal{I}} w_k = \frac{\alpha^2 + \beta^2 + (f^{1/2} + f^{-1/2})\alpha\beta}{F} \tag{23}$$

In UNI-PRIM,

$$\sum_{k\in\mathcal{I}} w_k = \frac{(\alpha+\beta)^2}{F} \tag{24}$$

The difference of total revenues between these two models is

$$\frac{(f^{1/2} + f^{-1/2} - 2)\alpha\beta}{F} \tag{25}$$

which is strictly increasing with f. Especially, when $f = 1$, the difference equals to 0, showing that UNI-PRIM is identical to FAID-PRIM when $f = 1$. □

Corollary 1. *The ratio of the difference of the revenue between the second and FAID-PRIM to the revenue of UNI-PRIM approximately equals to*

$$\frac{f^{1/2} + f^{-1/2} - 2}{4}$$

when α and β are close enough.

Proof. The ratio equals to

$$\frac{(f^{1/2} + f^{-1/2} - 2)\alpha\beta}{(\alpha+\beta)^2} \tag{26}$$

When α and β are close enough, we can assume that $\alpha \approx \beta$. Therefore,

$$\begin{aligned}\frac{(f^{1/2} + f^{-1/2} - 2)\alpha\beta}{(\alpha+\beta)^2} &\approx \frac{(f^{1/2} + f^{-1/2} - 2)\alpha^2}{(2\alpha)^2}\\ &= \frac{f^{1/2} + f^{-1/2} - 2}{4}\end{aligned}$$

□

Corollary 1 suggests an approximate formula about how much increase in revenue the seller can expect by choosing FAID-PRIM instead of UNI-PRIM. Taking $f = 3.47$, the edge seller can expect 10% increase on their revenue.

Theorem 6. *The seller's total revenue of UNI-PRIM is always higher than the total revenue of BID-PRAM.*

Proof. Consider the first and second allocation models for same users $\mathcal{I} = \{1, 2, \cdots, N\}$. Let the unit prices of the two models be p_1 and p_2. Denote that $[w_1^*, w_2^*, \cdots, w_N^*]$ and $[w_1^*, w_2^*, \cdots, w_N^*]$ represent the costs users pay in first and second models. Then the unit prices and costs satisfy the following equalities:

$$\sum_{k\in\mathcal{I}} w_k^* = p_1 F$$
$$\sum_{k\in\mathcal{I}} w_k^* = p_2 F$$

These equalities yield another equation relating the unit prices and costs.

TABLE II
ROLES OF EACH FILE IN GITHUB CODES

Filename	Role
buyer.py	A buyer's class
manager1.py	An optimal algorithm in BID-PRAM
manager2.py	An optimal algorithm in UNI-PRIM
manager3.py	An optimal algorithm in FAID-PRIM
main.py	Generating Fig. 4, 5, 6, 7
main2.py	Generating Fig. 9
main3.py	Generating Fig. 10, 11, 12
main4.py	Generating Fig. 8

$$\sum_{k\in\mathcal{I}}\left(\frac{w_k^*}{p_1}-\frac{w_k^*}{p_2}\right) = \sum_{k\in\mathcal{I}}\left(\frac{-R_kC_k+\sqrt{R_k^2C_k^2+4\mu_kF^2R_kC_kp_1}}{2\mu_kFp_1}-\sqrt{\frac{R_kC_k}{\mu_kp_2}}\right) = 0 \tag{27}$$

Let $f_k(x) = \frac{-R_kC_k+\sqrt{R_k^2C_k^2+4\mu_kF^2R_kC_kx}}{2\mu_kFx}$ and $g_k(x) = \sqrt{\frac{R_kC_k}{\mu_kx}}$. Then both are positive over $x>0$. Also,

$$g_k^2(x)-f_k^2(x) = \frac{R_kC_k}{2\mu_k^2F^2x^2}\left(\sqrt{R_k^2C_k^2+4\mu_kF^2R_kC_kx}-R_kC_k\right) > 0$$

which means $f_k(x) < g_k(x)$.

In addition, both functions are decreasing on $x>0$. Assume that $p_1 \geq p_2$. Then

$$\sum_{k\in\mathcal{I}}\left(\frac{w_k^*}{p_1}-\frac{w_k^*}{p_2}\right) = \sum_{k\in\mathcal{I}}(f_k(p_1)-g_k(p_2)) \leq \sum_{k\in\mathcal{I}}(f_k(p_2)-g_k(p_2)) < 0$$

which contradicts (27).

Therefore, p_1 is smaller than p_2, proving that the total revenue of UNI-PRIM, p_2F, is always higher than the revenue of BID-PRAM, p_1F. □

Theorem 5 and 6 show that the seller's revenue decreases in this order: FAID-PRIM, UNI-PRIM, and BID-PRAM. However, the seller using FAID-PRIM has to deal with complaints about price discrimination of users, so he or she should adopt one of the three trading system models considering each model's strengths and weaknesses.

IV. NUMERICAL RESULTS

In this section, several numerical results of the proposed three allocations mechanisms are given to validate the proposed theorems and to analyze the performances of them. All simulations used in this section are based on Python 3.5, and the programmed codes are provided in GitHub [52]. The role of each file is explained in Table II.

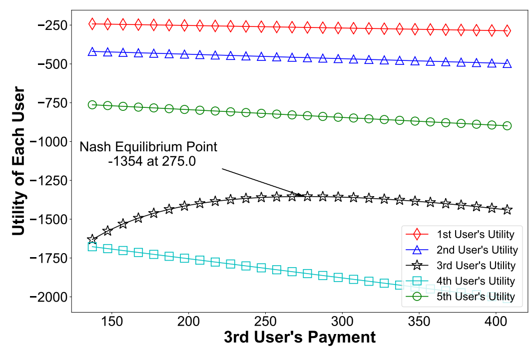

Fig. 4. Utility of each users when the payment of the 3rd user varies from 137.5 to 412.5.

Let the set of users be $\{1,2,3,4,5\}$, and consider the situation that 2 users (user 1 and 2) are trying to run voice recognition algorithms, 2 users (user 3 and 4) are running face recognition programs, and 1 device (user 5) is playing a video game. Assume the set of the sizes of their data be $(a_1,a_2,a_3,a_4,a_5) = (1,2,8,10,15)$ MB. Also, assume that the constants μ_i of each user are given by $(\mu_1,\mu_2,\mu_3,\mu_4,\mu_5) = (1,1.5,2,3,5)$. The total CPU cycles of the edge computing is set to be 8GHz. In [53], the processing densities of face recognition algorithms and 400 frame video game are 31680 and 2640 cycles per a bit, so it is reasonable to let the needed CPU cycles per 1 bit for user 3, 4 and 5 be 31680, 31680 and 2640. Likewise, we assumed that the processing density of voice recognition programs is 20000 cycles per a bit.

A. Validation of Theorems

First, validation of Theorem 1 will be proposed. For the situation suggested above, the NE point is given by

$$\mathbf{w}^* = [122.4, 139.7, 275.0, 255.2, 79.23] \tag{28}$$

which can be obtained by Theorem 1. To verify the validity of this result, it is sufficient to check that the utility of the 3rd user is maximized when $w_3^* = 275.0$.

From Figure 4, when the 3rd user's payment varies from $0.5w_3^*(137.5)$ to $1.5w_3^*(412.5)$, the utility of the 3rd user reaches the highest value -1354 at $w_3 = 275.0$, which is equivalent to the calculated NE point from Theorem 1.

Second, Theorem 2 will be verified numerically. For same users suggested in the beginning of the section, Theorem 2 proposes that the unit price is determined as 0.1444 to allocate all of the CPU Cycles in UNI-PRIM. For validation of this result, the unit price will be changed from 0 to theoretical unit price.

From Figure 5, we can see that when the unit price changes from 0 to 0.1444, the total distributed CPU cycles keep decreasing. Also, the distributed CPU cycles reach 8GHz, the

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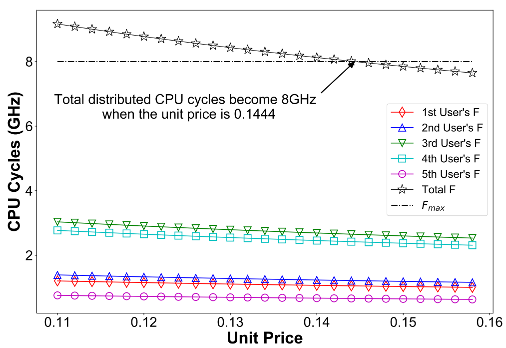

Fig. 5. Distributed CPU Cycles and their sum when the unit price varies from 0.11 to 0.16.

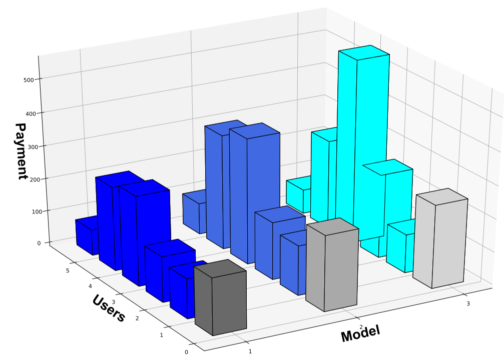

Fig. 7. Each user's payment in each model.

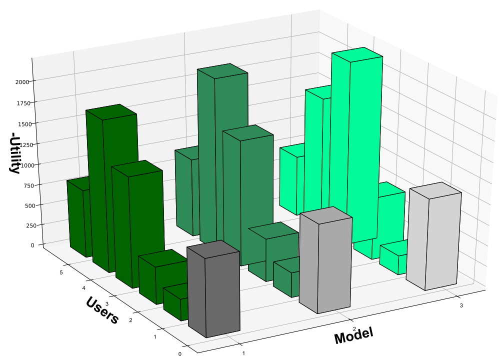

Fig. 6. Each user's absolute value of utility function in each model.

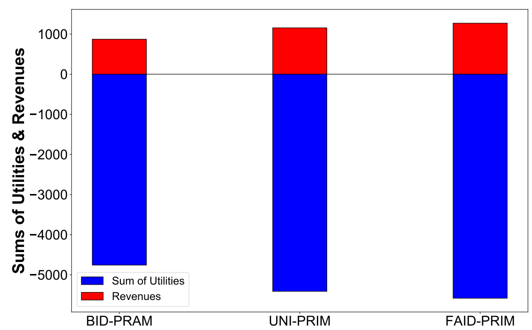

Fig. 8. Sum of utilities of mobile users and total revenues of the edge seller.

total CPU cycles of the edge computing, at 0.1444, certifying Theorem 2.

B. Comparison Among Three Models

We first compare the value of utility function and payments of each user in three models. Assumptions for properties of device users and the cloud are equivalent to the previous section.

In Figure 6, the absolute values of the utility function are suggested through the 3D graph, and the average value is demonstrated in gray bars. It is shown that the absolute value of the utility function has a tendency to increase in the order of BID-PRAM, UNI-PRIM, and FAID-PRIM, meaning the real value decreases in the same order.

Also, Figure 7, showing the payments of users in three models, demonstrates the tendency of increasing payments in order of BID-PRAM, UNI-PRIM, and FAID-PRIM. This result is consistent with theorems in subsection III-D, giving theoretical comparison between the models. These tendencies of utilities and payments are summarized in Figure 8. The absolute values of sum of utilities and total revenues of the seller are both increasing in order of BID-PRAM, UNI-PRIM, and FAID-PRIM.

In addition, a simulation containing randomly generated device users is designed for realization of real-life situations. Assume the total CPU cycles of the edge computing is 100, the needed CPU cycles per 1 bit is 10, and the fairness factor is 3.5. Also, each user is generated, taking μ and the size of its data randomly between 0 to 10. We differentiated the number of device users and plotted the unit prices of three models in each number of users. Figure 9 effectively demonstrates the result.

In Figure 9, we can easily see the increasing value of unit prices as the number of users increases, showing the principle of supply and demand. Also, the relations between each model demonstrated in Theorem 5 and 6 are verified even in the systems of more users.

C. Changes of Systems As CPU Cycles Increase

With same mobile users of previous sections, we analyzed the systems of three models, increasing the total CPU cycles of the edge computing. We can intuitively predict that the utilities of users will increase and the total delays of users will decrease as CPU cycles increase. Also, we can anticipate that the payments will decrease, according to the principle of supply and demand.

Increasing the CPU cycles of the edge computing from 2GHz to 8GHz, the changes of average processing times, payments, and utilities are shown in Figure 10, 11, and 12. Figure 10 and 11 show that the average processing times

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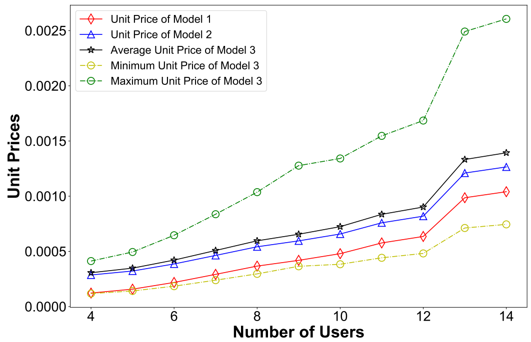

Fig. 9. Trend of unit prices as the number of users increase.

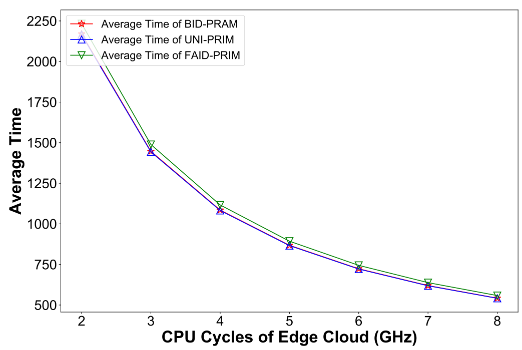

Fig. 10. Average processing time of mobile users as CPU cycles increase.

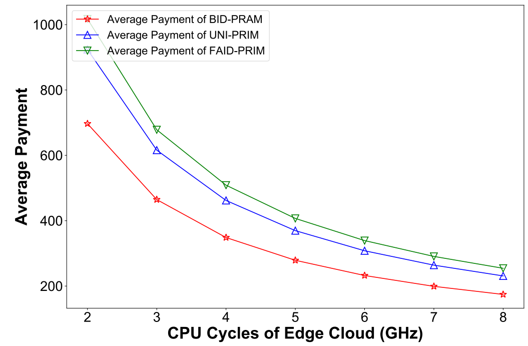

Fig. 11. Average payment of mobile users as CPU cycles increase.

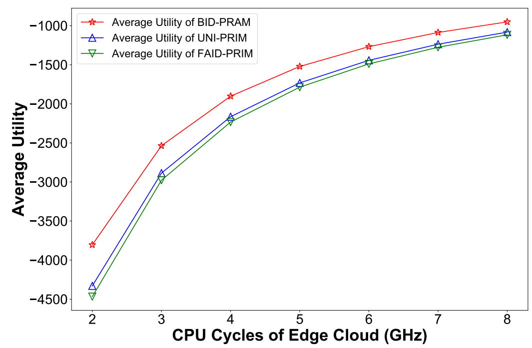

Fig. 12. Average utility of mobile users as CPU cycles increase.

and payments of mobile users tends to decrease as the total CPU cycles increase, and they follow the tendencies similar to inversely proportional relations. Hence, the average utility of users tends to increase as CPU cycles of the edge computing increase, meaning that the total CPU cycles of the edge computing can be improved to make mobile users more satisfied. These numerical results certify that the CPU cycles of the edge computing are directly related to overall happiness of the systems.

V. Discussions

In this section, we discuss potential applications of the proposed algorithms to real-world, justification of the proposed game models, and the future direction of this research.

Amazon Web Services (AWS) provides Spot Instances pricing scheme for Amazon Elastic Compute Cloud (Amazon EC2) [54], which is a service that provides scalable computing in the Amazon Web Services (AWS) cloud [55]. Compared to the On-demand pricing scheme, the Spot Instances pricing scheme reduces the cost by 90% and it varies real-time according to Amazon EC2 usage. This is a good example of how the dynamic pricing scheme is utilized in the real-world.

In this manner, IoT edge servers may also implement dynamic pricing policies in order to make the best use of their resources. In order to do so, an edge server needs to determine a pricing mechanism and a corresponding price optimization algorithm, such as the proposed mechanisms and algorithms in this paper, to maximize profit. These algorithms may work online by predicting the IoT users' optimal offloading strategies for a game-theoretic approaches. They may also be recommended for edge servers to provide optimal offloading strategies to all IoT users in order to lower the probability of IoT users behaving unpredictably.

In terms of game theory, the resource trading mechanism of edge computing for IoT may have various trading models in addition to the three types proposed in this paper. The edge server may choose one of the resource trading mechanisms, based on the status of the edge server. One of the most widely used methods for resource trading is the Double auction mechanism, which takes a little longer to complete a transaction. Therefore, the Double auction mechanism is suitable for edge computation offloading, such as a batch process, where the delay limit is not strict. For real-time offloading applications, however, the Double auction method can hardly be used due to the long transaction time. This paper proposes alternative and more suitable models for real-time offloading applications since they approach the Nash Equilibrium quickly.

An important point to note here is that, for real-time applications, an optimal resource allocation algorithm should perform fast enough to run online on an edge computing

TABLE III
AVERAGE ELAPSED TIME FOR THE PROPOSED ALGORITHMS

Proposed Algorithm	Average Elapsed Time
For BID-PRAM	0.0105 seconds
For UNI-PRIM	0.0010 seconds
For FAID-PRIM	0.0010 seconds

server. In order to see the proposed algorithms' performance in this respect, we evaluated the average runtime for each algorithm. Experiments were performed on a computer with 4.00GHz i7-6700K CPU and 36GB RAM. The experimental results are shown in Table III.

As the research progresses, we expect the following topics to be addressed. An edge resource trading model that considers not only the delay of IoT device offloading but also the energy efficiency will be proposed, and its equilibrium will be analyzed. In addition, we will propose and analyze a multiple-leader-multiple-follower edge resource trading model in which one or more edge servers compete with each other. An edge resource trading model for the network between IoT devices and edge servers can also be proposed and analyzed more precisely. Lastly, an edge resource trading model that considers applications, networks, and offloading models in a more realistic manner will be proposed, and the performance of the trading system will be compared and analyzed using the same analysis as this paper.

VI. CONCLUSIONS

This paper has examined the issues of how to price the computing resource of the edge computing server for edge computing in an IoT environment. To this end, three dynamic pricing mechanisms are proposed and rigorously analyzed to compare to each other: BID-PRAM, UNI-PRIM, and FAID-PRIM. We modeled each pricing scheme from a game theoretical perspective and derived NE solutions from existence to uniqueness. Numerical results verified our theorems, and presented statistical results to compare the three mechanisms. Through these results, we have identified the advantages and disadvantages of each model to give edge computing service providers guidance on pricing schemes. This study will emphasize the necessity and importance of pricing scheme of edge computing for IoT, and will provide a basis for studying various pricing policies.

APPENDIX

A. The proof of Lemma 7

Assume that the solution to the Problem 1, $\mathbf{x}$, has some elements that do not belong to the set $\{\max_{k\in\mathcal{I}} x_k, \min_{k\in\mathcal{I}} x_k\}$.

We can rearrange the elements of the vector $\mathbf{x}$ in descending power with the coefficients of the elements, a_i. Let the rearranged vector be $\mathbf{y} = [y_1, y_2, \cdots, y_N]$, and the rearranged coefficients be $\mathbf{b} = [b_1, b_2, \cdots, b_N]$. Also, denote that n elements, $y_1, y_2, \cdots, y_n$, are equivalent to $\max_{k\in\mathcal{I}} x_k$, and m elements, $y_{N-m+1}, y_{N-m+2}, \cdots, y_N$, equal to $\min_{k\in\mathcal{I}} x_k$. Thus $\mathbf{y}$ satisfies the following inequality.

$$y_1 = y_2 = \cdots = y_n > y_{n+1} \geq \\ \cdots \geq y_{N-m} > y_{N-m+1} = y_{N-m+2} = \cdots = y_N \quad (29)$$

Let $\dfrac{\max_{i\in\mathcal{I}} x_i}{\min_{i\in\mathcal{I}} x_i} = k$.

If $\frac{k}{y_1^2(k+1)} - \frac{b_{n+1}}{y_{n+1}^2} + \frac{1}{y_N^2 k} > 0$, take $\epsilon > 0$ satisfying the following inequality:

$$y_1 - \frac{f}{(f+1)\sum_{i=1}^{n} b_i}\epsilon > y_{n+1} + \epsilon \quad (30)$$

Let y', y'' be

$$y' = y_1 - \frac{f}{(f+1)\sum_{i=1}^{n} b_i}\epsilon$$
$$y'' = y_N - \frac{1}{(f+1)\sum_{i=N-m+1}^{N} b_i}\epsilon$$

Define a new vector $\mathbf{y}' = [y'_1, y'_2, \cdots, y'_N]$, such that

$$y'_i = \begin{cases} y', & \text{if } i = 1, 2, \cdots, n \\ y_{n+1} + \epsilon, & \text{if } i = n+1 \\ y_i, & \text{if } i = n+2, n+2, \cdots, N-m \\ y'', & \text{if } i = N-m+1, N-m+2, \cdots, N \end{cases}$$

Then $\mathbf{y}'$ satisfies the condition (20), maintaining the ratio of maximum to minimum k. Let $f(\epsilon) = \sum_{i=1}^{N} \frac{b_i}{y'_i}$. Taking the first derivative, we get

$$\frac{df(\epsilon)}{d\epsilon} = \frac{k}{y_1'^2(k+1)} - \frac{b_{n+1}}{y_{n+1}'^2} + \frac{1}{y_N'^2 k} \quad (31)$$

Also,

$$\frac{df(\epsilon)}{d\epsilon}\Big|_{\epsilon=0} = \frac{k}{y_1^2(k+1)} - \frac{b_{n+1}}{y_{n+1}^2} + \frac{1}{y_N^2 k} > 0$$
$$\frac{d^2 f(\epsilon)}{d\epsilon^2} > 0, \forall \epsilon > 0$$

showing that $f(\epsilon)$ is increasing on $\epsilon > 0$. It is contradictory to the assumption that $\mathbf{x}$ is the solution to the optimization problem.

If $\frac{k}{y_1^2(k+1)} - \frac{b_{n+1}}{y_{n+1}^2} + \frac{1}{y_N^2 k} < 0$, take ϵ satisfying $y_{n+1} - \epsilon > y_\alpha$, where y_α is the biggest element among the elements less than y_{n+1}.

Let y', y'' be

$$y' = y_1 + \frac{f}{(f+1)\sum_{i=1}^{n} b_i}\epsilon$$
$$y'' = y_N + \frac{1}{(f+1)\sum_{i=N-m+1}^{N} b_i}\epsilon$$

Define a new vector $\mathbf{y}' = [y'_1, y'_2, \cdots, y'_N]$, such that

$$y'_i = \begin{cases} y', & \text{if } i = 1, 2, \cdots, n \\ y_n - \epsilon, & \text{if } i = n \\ y_i, & \text{if } i = n+1, n+2, \cdots, N-m \\ y'', & \text{if } i = N-m+1, N-m+2, \cdots, N \end{cases}$$

Let $f(\epsilon) = \sum_{i=1}^{N} \frac{b_i}{y'_i}$. Similarly, by taking the first derivative, we can easily prove that $f(\epsilon)$ is increasing on $\epsilon > 0$, which contradicts the assumption that $\mathbf{x}$ is the solution to the optimization problem.

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