Introduction to Aerial Robotics
A Robust Control Approach

Jiayao Wang and Vinzent Rudolf

Institute for Systems Theory and Automatic Control
University of Stuttgart

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Goal

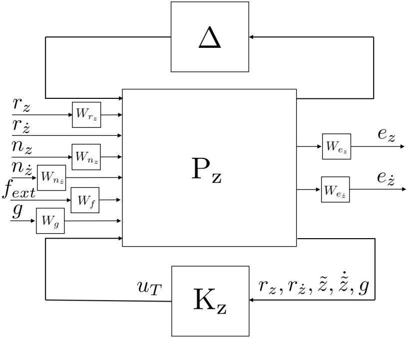
Project Target

apply *Robust Control Theory* to a *Multirotor Aerial Vehicle* (MAV), so that the control loop is robust stable and still achieves good performance regarding

- model uncertainties
- measurement noise
- external disturbances

Theoretical Background - The General Framework

Robust Control Theory is well known for linear time invariant systems



Theoretical Background - Theoretical Guarantees

$$P \star K = N = \begin{pmatrix} M & N_{12} \\ N_{21} & N_{22} \end{pmatrix}$$

$$\Delta \star N = N_{22} + N_{21}\Delta(I - M\Delta)^{-1}N_{12}$$

starting from this representation, following theorem can be proved

Theorem

Suppose M is a proper and stable transfer matrix. If

$$\det(I - M(i\omega)\Delta) \neq 0 \quad \forall \Delta \in \mathbf{\Delta},\ \omega \in \mathbb{R} \cup \{\infty\}$$

then $I - M\Delta$ has a proper and stable inverse for all $\Delta \in \mathbf{\Delta}$ [2].

major advantage of this theorem:

- suffices to check imaginary axes for stability and not the full right-half plane!
- can be solved very efficiently
- able to take the structure of Δ into account

Theoretical Background - Theoretical Guarantees

- famous *Structured Singular Value* (SSV) is a simple test for theorem 1, which evaluates M and Δ at specific frequencies
- fortunately, one can calculate a *lower* and *upper* bound for the SSV pretty efficiently [2]
- lower bound indicates robust stability for the closed-loop for all $\|\Delta\|_\infty < \frac{1}{\gamma_l}$
- upper bound implies that there exists a destabilizing uncertainty with $\|\Delta\|_\infty < \frac{1}{\gamma_u}$ [3]
- uncertainty weights W_Δ are used to normalize the input and output of Δ
- an additional SSV test can be used to check robust performance, this is known as the *Main Loop Theorem* [2]

Theoretical Background - H_∞ and DK Synthesis

nominal controller for P is minimizing the H_∞-norm of $P \star K$

$$\begin{aligned} \min_K \quad & \|P \star K\|_\infty \\ \text{s.t.} \quad & K \text{ stabilizes } P \end{aligned} \tag{1}$$

- performance weights W_P are used to weight frequencies differently
- H_∞ problem does not take uncertainties into account, hence so called *DK-Synthesis* is used
- approach uses so called *D-Scalings* in the controller design, which are used to compute better upper bounds of the SSV test

Content

System Dynamics

dynamics of a MAV are given by [5]

$$u = \begin{pmatrix} u_T & \tau_x & \tau_y & \tau_z \end{pmatrix}^T \in \mathbb{R}^4$$

$$\dot{x} = \begin{pmatrix} \dot{p} \\ \ddot{p} \\ \dot{\eta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} \dot{p} \\ g e_z \\ T(\eta)\omega \\ -J_B^{-1}[\omega]_\wedge J_B \omega \end{pmatrix} + \begin{pmatrix} 0 \\ -\frac{1}{m} R(\eta) e_z u_T \\ 0 \\ J_B^{-1}\tau \end{pmatrix} \in \mathbb{R}^{12} \qquad (2)$$

$\Rightarrow$ dynamics are nonlinear. Therefore, the problem to design a robust controller for (2) is splitted up into different sub tasks

1. decouple dynamics of (2) into altitude and attitude dynamics
2. design robust controller for altitude and attitude independently from each other
3. design cascaded position controller for x and y

System Dynamics - Approximated System

$$\dot{x} = \begin{pmatrix} \dot{p}_z \\ \ddot{p}_z \\ \dot{\eta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta_{T(\eta)} \\ 0 & 0 & 0 & -J_B^{-1}\Delta_{[\omega]_\wedge J_B} \end{pmatrix} \begin{pmatrix} p_z \\ \dot{p}_z \\ \eta \\ \omega \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{m}(1+W_m\Delta_m)(1+W_{u_T}\Delta_{u_T}) & -\frac{1}{m} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_B^{-1}(1+W_\tau\Delta_\tau) & J_B^{-1} \end{pmatrix} \begin{pmatrix} u_T \\ f_{Ext} \\ g \\ \tau \\ \tau_{Ext} \end{pmatrix}$$

$$\tilde{x} = \begin{pmatrix} \tilde{p}_z \\ \dot{\tilde{p}}_z \\ \tilde{\eta} \\ \tilde{\omega} \end{pmatrix} = \begin{pmatrix} p_z \\ \dot{p}_z \\ \eta \\ \omega \end{pmatrix} + \begin{pmatrix} n_z \\ n_{\dot{z}} \\ n_\eta \\ n_\omega \end{pmatrix}$$

System Dynamics - Approximated System

approximated system contains following simplifications and extensions:

1. difference between commanded torques / thrusts and applied torques / thrusts is covered by dynamic and parametric uncertainties
2. uncertainties for nonlinear terms are introduced
3. all measurements are noisy
4. external torques and external forces can also occur as inputs
5. gravity vector g is used as additional input to the system

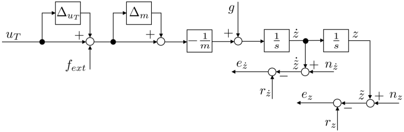


Fig.: MAV altitude dynamic model with uncertainties.

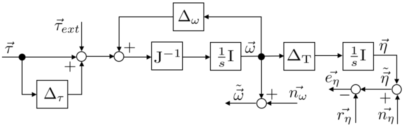


Fig.: MAV rotational dynamic model with uncertainties.

System Dynamics - Actuator Uncertainties

Mass Uncertainty

- altitude dynamics consist of a real parametric uncertainty Δ_m, which describes a non-dynamic relative mass deviation $\frac{\hat{m}-m}{\hat{m}}$

$$\frac{1}{\hat{m}} = \frac{1}{m}(1 + W_m \Delta_m) \Leftrightarrow \quad W_m \Delta_m = \frac{\hat{m} - m}{\hat{m}}$$

- it is assumed that the nominal mass m does not differ more than 10% from the real (actual / uncertain) mass $\hat{m} \Rightarrow W_m = 0.1$

System Dynamics - Actuator Uncertainties

Motor Uncertainty

- direct feedthrough of the thrust u_T and torques τ are assumed
- in practice, the desired force can not be applied exactly and motors have internal dynamics

$$G = \frac{1+k}{Ts+1} = 1 + W_{u_T}\Delta_{u_T} \quad \Leftrightarrow \quad W_{u_T}\Delta_{u_T} = \frac{k - Ts}{Ts+1}$$

- internal dynamics are designed as PT1 filter with a maximum time constant T
- gain k describes a fixed relation between desired and actual force, which can be compared to a tilting of the characteristic curve of the motor
- thrust: $T = 10\text{ms}$, $|k| < 0.5$
- torques: $T = 40\text{ms}$, because multiple motors have to create thrust, so that a torque around the body axes is applied; $|k| < 0.5$

System Dynamics - Nonlinear Terms

- *Robust Control Theory* can only be applied to linear systems, the system dynamics (2) have to be brought into a linear form
- nonlinear terms are $R(\eta)$, $T(\eta)$ and $[\omega]_\wedge J_B \omega$
- in order to not simply ignore these terms, they are introduced as uncertain systems into the uncertain system dynamics

$R(\eta)$

- no additional uncertainty object is introduced
- uncertainty of this term is already implicitly captured with Δ_m and Δ_{u_T}

System Dynamics - Nonlinear Terms

$T(\eta)$

- $T(\eta)$ is approximated by $\Delta_{T(\eta)}$.
- therefore, $T(\eta)$ is linearized around $\begin{pmatrix}\phi & \theta\end{pmatrix} = \begin{pmatrix}0 & 0\end{pmatrix}$
- new linearized variables $\Delta\phi$ and $\Delta\theta$ are used as dynamic uncertainties
- maximum roll and pitch angle of $|\phi|, |\theta| < 10^\circ$ is assumed, which is used as output weight W_η.

$$\begin{aligned}\Delta_{T(\eta)} =& T(\eta)\big|_{\phi,\theta=0} + \left.\frac{\partial T(\eta)}{\partial(\phi,\theta)}\right|_{\phi,\theta=0} \begin{pmatrix} W_\eta \Delta_\phi \\ W_\eta \Delta_\theta \end{pmatrix} \\ =& \begin{pmatrix} 1 & 0 & W_\eta \Delta_\theta \\ 0 & 1 & -W_\eta \Delta_\phi \\ 0 & W_\eta \Delta_\phi & 1 \end{pmatrix}\end{aligned} \qquad (3)$$

System Dynamics - Nonlinear Terms

$[\omega]_\wedge J_B \omega$

- inertia matrix J_B is assumed to have only entries on the main diagonal, whereby $J_{xx} \equiv J_{yy}$

$$
\begin{aligned}
[\omega]_\wedge J_B \omega &= \begin{pmatrix} (J_{zz} - J_{yy})\omega_z\omega_y \\ (J_{xx} - J_{zz})\omega_z\omega_x \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} 0 & (J_{zz} - J_{yy})W_{\omega_z}\Delta_{\omega_z} & 0 \\ (J_{xx} - J_{zz})W_{\omega_z}\Delta_{\omega_z} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \omega \qquad (4) \\
&= \Delta_{[\omega]_\wedge J_B} \omega
\end{aligned}
$$

- nonlinear term can be rearranged, so that the inner matrix only depends on ω_z, which is exchanged with a dynamic uncertainty Δ_{ω_z}
- output weight $W_{\omega_z} = \frac{2}{s+1}$: 2 is the assumed max. body rate $|\omega_z|$, PT1 filter describes correlation between ω_z and $\omega_{x,y}$: two body rates do not change simultaneously very fast

System Dynamics - Nonlinear Terms

Time Variance of Uncertainties

- dynamic uncertainties of the nonlinear terms are very unspecific, so that nearly any possible (stable) transfer function could be obtained by them
- approach has the advantage that all possible situations are covered under the assumption that the maximum values fit
- however, these uncertainties are in general time variant, because the exact transfer function depends on the current state vector x, which is formally not covered in Robust Control Theory
- nevertheless, **simulation shows satisfactory results**

System Dynamics - Noisy Measurements

- to create a system simulation close to real world applications noise is added to the feedback channels of p_z, $p_{\dot{z}}$, Euler-angles η and body rates ω
- taken into account in the controller design by introducing additional performance channels from input noise to controller error
- accuracy values for p_z, $\dot{p}_z$ and the heading are taken from the datasheet of a GNSS receiver [4]
- due to missing data, the accuracy of ϕ and θ is estimated
- variance of the normal distributed noise n_ω is taken from the IMU datasheet [1]

System Dynamics - External Inputs

- two additional input channels are added into the system structure to take external forces and external torques like those created by wind, into account
- for simulation purposes following external inputs are assumed

$$f_{Ext} = 1 \cdot \sin(1t), \quad \tau_{Ext} = 1 \cdot \sin(0.1t)$$

System Dynamics - Position Controller for x and y

- it is assumed, that the MAV is in a state in which almost all Euler-angles are close to zero, hence altitude and the rotation can be controlled separately
- position controller for x and y is implemented in the same way as for the near hovering controller
- unfortunately, it is not possible to apply the robust control theory to this structure
- nevertheless, possible extension is proposed in our paper to be able to apply robust control theory also to position controller

Content

Controller Design

final robust controllers are each a LTI system with following properties:

1. The altitude controller consists of 16 states, 5 inputs (g, p_{des_z}, $\dot{p}_{des_z}$, $\tilde{p}_z$, $\dot{\tilde{p}}_z$) and 1 output (thrust u_T).
2. The attitude controller has got 35 states, 9 inputs (r_η, $\tilde{\eta}$, $\tilde{\omega}$) and 3 outputs (torques τ)

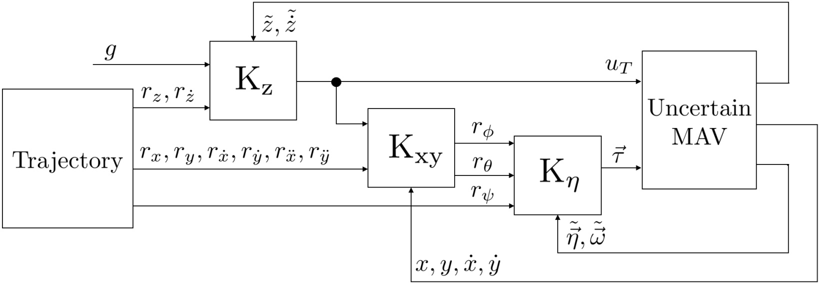


Fig.: Sketch of the robust control loop

Controller Design - General Framework

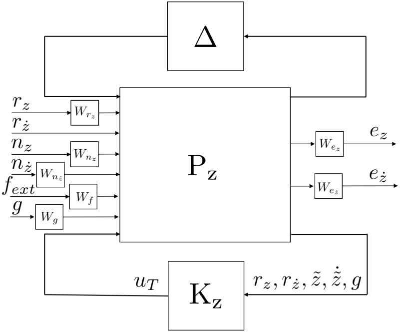


Fig.: Δ-P-K structure for altitude control

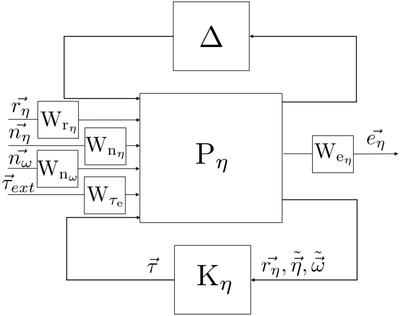


Fig.: Δ-P-K structure for attitude control

Results - Robust Performance

Robust Performance

- during weight design it turns out, that the *dksyn* command is able to compute better controllers, if different performance weights than the desired dynamic behavior are used
- performance has only been analyzed during simulation studies and using the bode plot

Results - Robust Performance

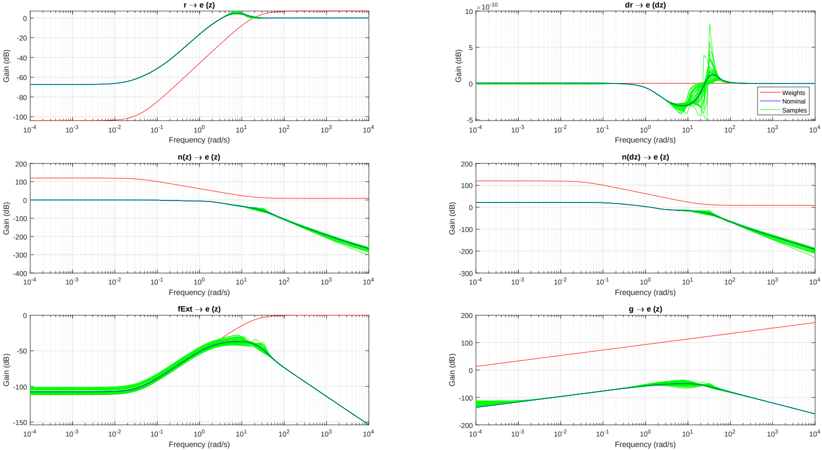


Fig.: Bode Plot of Altitude Controller

Results - Robust Performance

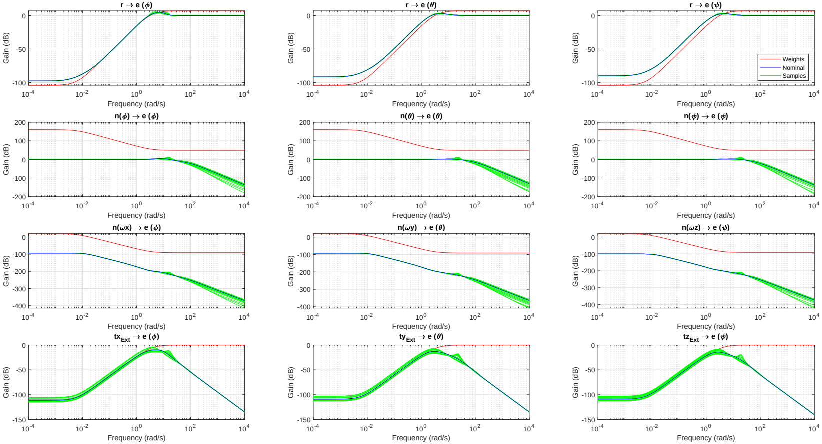


Fig.: Bode Plot of Attitude Controller

Results - Robust Stability

Robust Stability

- robust stability does not depend on the weights of the performance channels
- stability of linear systems does not depend on the inputs (in contrast to nonlinear systems)
- Therefore, it is possible to give stability margins $r = \frac{1}{\gamma}$

	lower bound	upper bound
Altitude Controller	0.97966	0.98165
Attitude Controller	1.00184	1.00387

Fig.: Robust Stability Margins

Results - Robust Stability

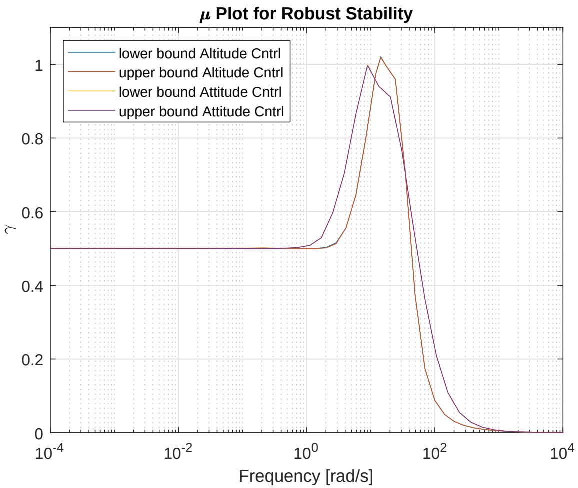


Fig.: μ Plot for Robust Stability

Results - Simulation Study

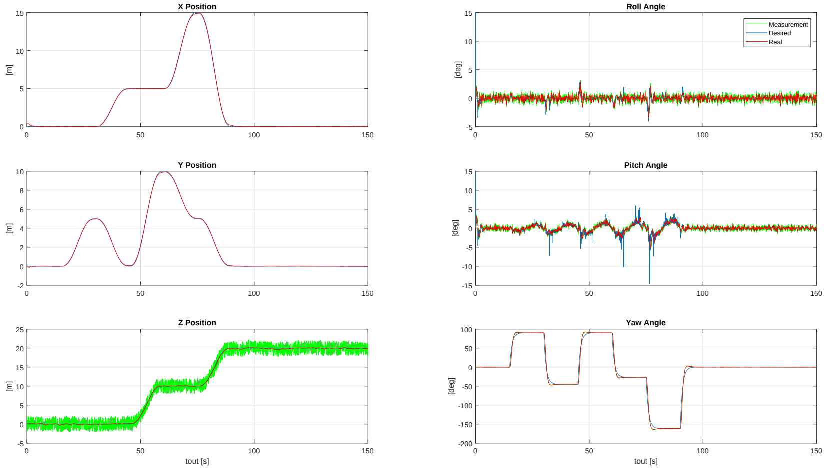


Fig.: Simulation Results: Robust Controller

Results - Simulation Study

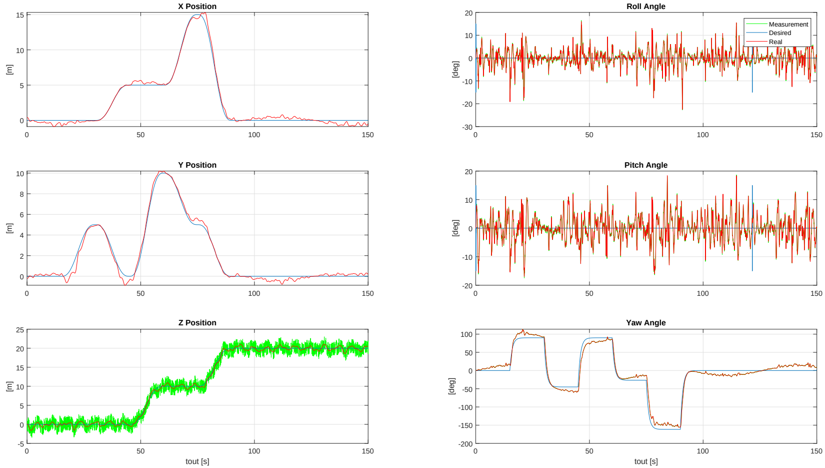
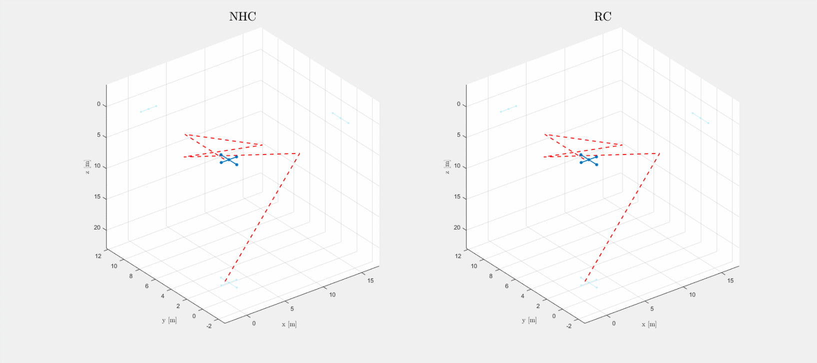


Fig.: Simulation Results: NHC

Results - Simulation Study



Conclusion and Outlook

Conclusion

- Approach is promising but extendable
- Robust Controller is derived
- Introduced uncertainties are justified
- RC can compete with NHC
- Stability guarantees can be given

Outlook

- Implementation of further performance channels, e.g. reference $\rightarrow$ controller output
- Implementation of x and y controller as robust controller
  - Handling of noise and external inputs on these values possible
  - Weights have to be found
- **Adding more weights will get more difficult!**

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