EPFL Fall 2022
Master level
MATH-458
September 22, 2023

Programming Concepts in Scientific Computing

Eigen library

The goal of the present exercises is to employ Eigen in order to perform linear algebra operations which can be helpful in computational sciences.

Exercise 1: *Wrapping*

- Declare an array with three columns with random coordinates. You are invited to read the Eigen::Array documentation. The result should be structured like:

$$\begin{bmatrix} p_{1x} & p_{1y} & p_{1z} \\ \vdots & \vdots & \vdots \\ p_{nx} & p_{ny} & p_{nz} \end{bmatrix} \tag{1}$$

- Loop over each point stored in the array and compute the cross product with the vector $[1, 1, 1]$. You should use an auto-range loop of the kind:

```
for (auto & e : array){
  // ...
}
```

  You should read the Reduction documentation and in particular have a care for the **rowwise()** method.

- Modify your code to store the result in another array.
- Reshape the result into an array with a single column (that task can be useful when needing to resolve a linear system)
- What was the storage convention ? (row-major or col-major ?)
- Create now an array with 12 columns. Make a loop over the rows and interpret for each row the first 9 columns as a 3×3 matrix M and the last 3 columns a vector V of size 3.
- Construct an array with each row containing the flattened product $M \cdot V$.

Exercise 2: *Solving a linear system*

- Construct a matrix with the following shape:

$$A_{n\times n} = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & & & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix} \tag{2}$$

- This system is the stiffness matrix of a system of n springs: Compute the determinant to show inversibility.
- Solve the system $Ax = b$ where b is a vector of your choice. Your should read Eigen::solving.

- Verify that the obtained result is the solution.
- Construct A as a sparse matrix.
- Solve again the system with a sparse solver. Eigen::sparseSolvers.
- Which of the two approaches is the fastest ? why ?