INDENG 240 Case Study

Group Members

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In [1]:
```
pip install gurobipy
```

```
Requirement already satisfied: gurobipy in /Users/ellasdigits/anaconda3/lib/pytho
n3.11/site-packages (10.0.2)
Note: you may need to restart the kernel to use updated packages.
```

In [2]:
```
import gurobipy as gp
from gurobipy import GRB
```

In [3]:
```
m = gp.Model("CaseStudy1")

x1 = m.addVar(lb = 0.0, vtype = GRB.CONTINUOUS, name = "x1")
x2 = m.addVar(lb = 0.0, vtype = GRB.CONTINUOUS, name = "x2")
x3 = m.addVar(lb = 0.0, vtype = GRB.CONTINUOUS, name = "x3")
x4 = m.addVar(lb = 0.0, vtype = GRB.CONTINUOUS, name = "x4")
x5 = m.addVar(lb = 0.0, vtype = GRB.CONTINUOUS, name = "x5")
x6 = m.addVar(lb = 0.0, vtype = GRB.CONTINUOUS, name = "x6")
x7 = m.addVar(lb = 0.0, vtype = GRB.CONTINUOUS, name = "x7")
x8 = m.addVar(lb = 0.0, vtype = GRB.CONTINUOUS, name = "x8")
x9 = m.addVar(lb = 0.0, vtype = GRB.CONTINUOUS, name = "x9")

m.setObjective(20*x1+30*x2+40*x3+50*x4+80*x5+125*x6+100*x7+250*x8+325*x9, GRB.MA

m.addConstr (x1>=750, "c0")
m.addConstr (x2>=1500, "c1")
m.addConstr (x3>=100, "c2")
m.addConstr (x4>=400, "c3")
m.addConstr (x5>=1500, "c4")
m.addConstr (x6>=100, "c5")
m.addConstr (x7>=25, "c6")
m.addConstr (x8>=150, "c7")
m.addConstr (x9>=50, "c8")

m.addConstr (14*x1+24*x2+30*x3<=65000, "c9")
m.addConstr (8*x1+15*x2+24*x3<=60000, "c10")
m.addConstr (22*x4+40*x5+55*x6+25*x7+45*x8+60*x9<=175000, "c11")
m.addConstr (1.5*x1+2*x2+2.5*x3<=96000, "c12")
m.addConstr (x1+x2+x3+x4+x5+x6<=96000, "c13")
m.addConstr (3*x4+4*x5+5*x6+3*x7+4*x8+5*x9<=19200, "c14")

m.addConstr (0.2*(x1+x2+x3+x4+x5+x6+x7+x8+x9)<=(x1+x2+x3), "c15")
m.addConstr ((x1+x2+x3)<=0.5*(x1+x2+x3+x4+x5+x6+x7+x8+x9), "c16")
m.addConstr (0.4*(x1+x2+x3+x4+x5+x6+x7+x8+x9)<=(x4+x5+x6), "c17")
m.addConstr ((x4+x5+x6)<=0.6*(x1+x2+x3+x4+x5+x6+x7+x8+x9), "c18")
m.addConstr (0.1*(x1+x2+x3+x4+x5+x6+x7+x8+x9)<=(x7+x8+x9), "c19")
```

```python
m.addConstr ((x7+x8+x9)<=0.2*(x1+x2+x3+x4+x5+x6+x7+x8+x9), "c20")
m.addConstr (0.2*(x1+x2+x3+x4+x5+x6+x7+x8+x9)<=(x1+x4+x7), "c21")
m.addConstr ((x1+x4+x7)<=0.35*(x1+x2+x3+x4+x5+x6+x7+x8+x9), "c22")
m.addConstr (0.4*(x1+x2+x3+x4+x5+x6+x7+x8+x9)<=(x2+x5+x8), "c23")
m.addConstr ((x2+x5+x8)<=0.7*(x1+x2+x3+x4+x5+x6+x7+x8+x9), "c24")
m.addConstr (0.05*(x1+x2+x3+x4+x5+x6+x7+x8+x9)<=(x3+x6+x9), "c25")
m.addConstr ((x3+x6+x9)<=0.15*(x1+x2+x3+x4+x5+x6+x7+x8+x9), "c26")

m.addConstr (15*x1+17*x2+19*x3+23*x4+28*x5+32*x6+76*x7+93*x8+110*x9<=230400, "c2

m.optimize()

for v in m.getVars():
    print('%s %g' % (v.VarName, v.X))

print ('obj:%g' % m.ObjVal)
```

```
Restricted license - for non-production use only - expires 2024-10-28
Gurobi Optimizer version 10.0.2 build v10.0.2rc0 (mac64[arm])

CPU model: Apple M1
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 28 rows, 9 columns and 153 nonzeros
Model fingerprint: 0x6e4c8b64
Coefficient statistics:
  Matrix range     [5e-02, 1e+02]
  Objective range  [2e+01, 3e+02]
  Bounds range     [0e+00, 0e+00]
  RHS range        [2e+01, 2e+05]
Presolve removed 11 rows and 0 columns
Presolve time: 0.01s
Presolved: 17 rows, 9 columns, 135 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    7.9669841e+05   3.344941e+03   0.000000e+00      0s
      13    6.1211338e+05   0.000000e+00   0.000000e+00      0s

Solved in 13 iterations and 0.01 seconds (0.00 work units)
Optimal objective  6.121133832e+05
x1 750
x2 1500
x3 100
x4 525.537
x5 1657.5
x6 825.403
x7 25
x8 1069.24
x9 50
obj:612113
```

In [4]:
```python
print('\nVariable Sensitivity Analysis:')
print('Name\tReduced Cost\tMin Obj Coef\tMax Obj Coef')
for v in m.getVars():
    print(f"{v.VarName}\t{round(v.RC, 2)}\t\t{round(v.SAObjLow, 2)}\t\t{round(v.


print('\nSensitivity Analysis:')
print('Name\t\tShadow Price\tSlack\tMin RHS\tMax RHS')
```

```
for c in m.getConstrs():
    print(f"{c.ConstrName}\t\t{round(c.Pi, 2)}\t\t{round(c.Slack, 2)}\t{round(c.
```

```
Variable Sensitivity Analysis:
Name    Reduced Cost    Min Obj Coef    Max Obj Coef
x1      0.0             -inf            24.06
x2      0.0             -inf            42.05
x3      0.0             -inf            78.34
x4      0.0             45.4            61.67
x5      0.0             72.03           83.06
x6      0.0             121.22          171.26
x7      0.0             -inf            188.36
x8      0.0             246.89          270.17
x9      0.0             -inf            328.77

Sensitivity Analysis:
Name            Shadow Price    Slack   Min RHS Max RHS
c0              -4.06           0.0     438.77  892.19
c1              -12.05          0.0     1187.61 2145.83
c2              -38.34          0.0     -0.0    616.67
c3              0.0             -125.54 -inf    525.54
c4              0.0             -157.5  -inf    1657.5
c5              0.0             -725.4  -inf    825.4
c6              -88.36          0.0     -0.0    151.28
c7              0.0             -919.24 -inf    1069.24
c8              -3.77           0.0     -0.0    778.39
c9              0.0             15500.0 49500.0 inf
c10             0.0             29100.0 30900.0 inf
c11             0.23            0.0     169646.44       195939.58
c12             0.0             91625.0 4375.0  inf
c13             0.0             90641.56        5358.44 inf
c14             0.0             2264.39 16935.61        inf
c15             0.0             1049.46 -1049.46        inf
c16             0.0             901.34  -901.34 inf
c17             0.0             407.37  -407.37 inf
c18             0.0             893.17  -893.17 inf
c19             0.0             493.98  -493.98 inf
c20             0.0             156.29  -156.29 inf
c21             12.79           0.0     -281.92 114.49
c22             0.0             975.4   -975.4  inf
c23             0.0             1625.67 -1625.67        inf
c24             0.0             325.13  -325.13 inf
c25             0.0             650.27  -650.27 inf
c26             31.09           0.0     -331.81 114.9
c27             2.6             0.0     200442.85       239324.73
```

Report

Variables:

X_1 be the number of desk for student size in economy line

X_2 be the number of desk for standard size in economy line

X_3 be the number of desk of executive size in economy line

X_4 be the number of desk for student size in basic pine line

X_5 be the number of desk for standard size in basic pine line

X_6 be the number of desk of executive size in basic pine line

X_7 be the number of desk for student size in hand-craft line

X_8 be the number of desk for standard size in hand-craft line

X_9 be the number of desk of executive size in hand-craft line

Objective Function:

Max

$$20X_1 + 30X_2 + 40X_3 + 50X_4 + 80X_5 + 125X_6 + 100X_7 + 250X_8 + 325X_9$$

s.t.

Outstanding Orders Constrain:

$$\begin{cases} X_1 \geq 750 & c_0 \\ X_2 \geq 1500 & c_1 \\ X_3 \geq 100 & c_2 \\ X_4 \geq 400 & c_3 \\ X_5 \geq 1500 & c_4 \\ X_6 \geq 100 & c_5 \\ X_7 \geq 25 & c_6 \\ X_8 \geq 150 & c_7 \\ X_9 \geq 50 & c_8 \end{cases}$$

Aluminum Constrain:

$$14X_1 + 24X_2 + 30X_3 \leq 65000 \text{.................} c_9$$

Practice board Constrain:

$$8X_1 + 15X_2 + 24X_3 \leq 60000 \ldots\ldots\ldots\ldots\ldots c_{10}$$

Pine sheets Constrain:

$$22X_4 + 40X_5 + 55X_6 + 25X_7 + 45X_8 + 60X_9 \leq 175000 \ldots\ldots c_{11}$$

Production Line Constrain:

$$\begin{cases} 1.5X_1 + 2X_2 + 2.5X_3 \leq 96000 & c_{12} \\ X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \leq 96000 & c_{13} \\ 3X_4 + 4X_5 + 5X_6 + 3X_7 + 4X_8 + 5X_9 \leq 19200 & c_{14} \end{cases}$$

Sept. Order Constrain:

$$\begin{cases} \sum_{i=1}^{3} X_i \leq 0.5 \sum_{i=1}^{9} X_i & c_{15} \\ \sum_{i=1}^{3} X_i \geq 0.2 \sum_{i=1}^{9} X_i & c_{16} \\ \sum_{i=4}^{6} X_i \geq 0.4 \sum_{i=1}^{9} X_i & c_{17} \\ \sum_{i=4}^{6} X_i \leq 0.6 \sum_{i=1}^{9} X_i & c_{18} \\ \sum_{i=7}^{9} X_i \geq 0.1 \sum_{i=1}^{9} X_i & c_{19} \\ \sum_{i=4}^{9} X_i \leq 0.2 \sum_{i=1}^{9} X_i & c_{20} \\ X_1 + X_4 + X_7 \leq 0.35 \sum_{i=1}^{9} X_i & c_{21} \\ X_1 + X_4 + X_7 \geq 0.2 \sum_{i=1}^{9} X_i & c_{22} \\ X_2 + X_5 + X_8 \leq 0.7 \sum_{i=1}^{9} X_i & c_{23} \\ X_2 + X_5 + X_8 \geq 0.4 \sum_{i=1}^{9} X_i & c_{24} \\ X_3 + X_6 + X_9 \leq 0.15 \sum_{i=1}^{9} X_i & c_{25} \\ X_3 + X_6 + X_9 \geq 0.05 \sum_{i=1}^{9} X_i & c_{26} \end{cases}$$

Labor Constrain:

$$((0.15) * 2 + 10)X_1 + ((2 + 1) * 2 + 11)X_2 + ((2.5 + 1) * 2 + 12)X_3 + ((1 + 3) \\ + (3 * 2 + 20 + 50)X_7 + (4 * 2 + 25 + 60)X_8 + (5 * 2$$

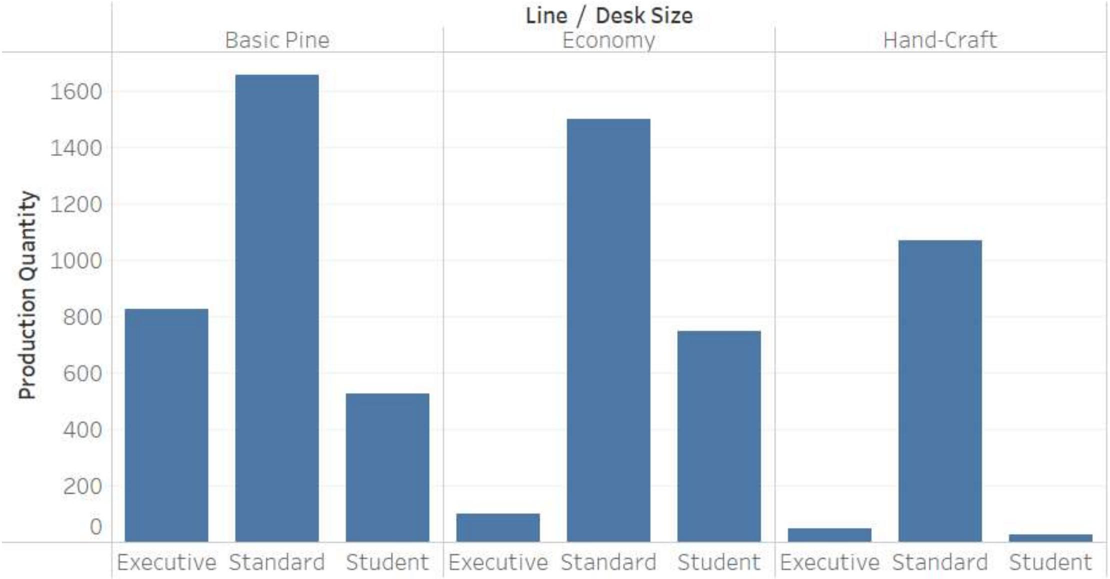
Optimal Solution

Result

From the Output above, we can see that the optimal solution is producing 750 economy desks for student size, 1500 economy desks for standard size, 100 economy desks for executive size; 525.537 basic pine desks for student size, 1657.5 basic pine desks for standard size, 825.403 basic pine desks for executive size; 525.537 basic pine desks for student size, 1657.5 basic pine desks for standard size, 825.403 basic pine desks for executive size; 25 hand-craft desks for student size, 1069.24 hand-craft desks for standard size, 50 hand-craft desks for executive size. And the profit for this optimal solution is 612113.

Graph of result

Sheet 1



Sum of Production Quantity for each Desk Size broken down by Line.

Observations

- The Basic Pine Line has the highest production numbers, especially for the Standard and Executive size desks. This implies its importance in our profit-making strategy.
- While the Economy Line doesn't reach the production numbers of the Basic Pine Line, it's still essential, especially for the Standard size desks.
- The Hand-Craft Line is more niche, focusing mainly on the Standard size desks.
- All three lines produce the most table sizes in Standard sizes.

Analysis

- Both the Economy Line and Basic Pine Line have substantial production numbers, suggesting that these are the most profitable lines given our constraints. The Hand-Craft Line, although more artisanal, contributes less to the optimal production mix.
- Rounding Considerations: Some desk numbers, especially for the Basic Pine Line, had to be rounded. In real-world settings, we can't produce a fraction of a desk. However, these fractional values provide insights into how close we are to producing one more unit of a particular product and might suggest that small changes could lead to slightly higher profits.

Sensitivity Analysis

From the above sensitivity analysis, we can see that all the decision variables are basic since $x_j \neq 0$ and $r_j = 0$ for all j = 1, 2 ... 9.

And Constraint c_3, c_4, c_5, c_7, c_9, c_{10}, c_{12}, c_{13}, c_{14}, c_{15}, c_{16}, c_{17}, c_{18}, c_{19}, c_{20}, c_{22}, c_{23}, c_{24}, c_{25} are not binding since $\sum_{j=1} na_{ij}x_j > b_i$, in other words y_i = 0 by looking at the shadow price; Also, Constraint c_0, c_1, c_2, c_6, c_8, c_{11}, c_{21}, c_{26} are binding since $\sum_{j=1} na_{ij}x_j = b_i$, in other words s_i = 0 by looking at the slack;

Shadow Price

- Shadow Price reflects how much the objective function value would change if the right-hand side (RHS) of a constraint is increased by one unit, keeping all else constant.
- Most of the constraints have a shadow price of 0, indicating that an increase in the right-hand side of these constraints won't have an impact on the objective function.
- Constraints like c0, c1, c2, c6, c8, c11, c21, c26, and c27 have non-zero shadow prices, meaning they have an impact on the objective function. Among them, constraints c2, c6, and c26 have relatively high absolute values, making them potentially more influential.

Slack

- Slack measures the amount by which the left-hand side (LHS) of the constraint is away from the right-hand side (RHS) of the constraint.
- Constraints like c0, c1, c2, c6, c8, c11, c21, c26, c27 have a slack of 0, indicating they are binding constraints or are being met exactly.

Min RHS and Max RHS

- These two show the range within which the right-hand side (RHS) of a constraint can vary without changing the current solution's basic variables.
- inf represents infinity, suggesting that the constraint's right-hand side can increase indefinitely without changing the optimal basis. On the other hand, -inf suggests that the constraint's right-hand side can decrease indefinitely.
- Some constraints, like c9, c10, c11, and c12, have very large values, which suggest significant flexibility in those constraints.