INDENG240 Mini Project

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1 Problem Setup

Madison Electric Co., based in Warren, MI, is a leading manufacturer of electronic devices. As the holiday season approaches, the company is planning its production schedule for three primary products: **smartphones**, **tablets**, and **laptops**. The objective is to **maximize profits** while considering constraints on **labor**, **materials**, **production capacity**, and **quality control**.

Note: The company information for this project was obtained from real company websites, but the following data is reasonably fictitious due to the unavailability of actual relevant data.

Constraints
Labor: A fixed number of assembly hours are available.
Quality Control (QC): Each product requires a certain amount of QC time.
Production Capacity: Each production line can produce a maximum number of units per month.
Material Availability: Limited components are available for production.
Production Quotas: A certain percentage range of the total production must be allocated to each product type.

	Smartphones	**Tablets**	**Laptops**
Profit Margin per Unit	\$150	\$100	\$200
Assembly Time per Unit	0.5 hours	0.75 hours	1.5 hours
Quality Control Time per Unit	1 hour	1 hour	1.5 hours
Maximum Production Capacity	10,000 units	7,000 units	3,000 units
Total Available Assembly Hours for December: 24,000 hours.			
Total Available QC Hours for December: 4,000 hours.			

Table 1: Production Capacity and Time Allocation

Material Availability	
Micro-processors	15,000 units (used in smartphones and tablets)
Screens	10,000 units (used in all products)
Batteries	12,000 units (used in all products)
Casings	15,000 units (used in all products)
CPUs	5,000 units (used in laptops)
Keyboards	5,000 units (used in laptops)

Table 2: Material Availability

	Minimum	**Maximum**
Smartphones	40%	70%
Tablets	20%	50%
Laptops	10%	40%

Table 3: Production Quotas (as a percentage of total production)

2 Solution

Variables

- x_S: The number of smartphones (Model S) produced.
- x_T: The number of tablets (Model T) produced.
- x_L: The number of laptops (Model L) produced.

Objective Function:

$$\text{Maximize } Z = 150x_S + 100x_T + 200x_L$$

s.t.

Labor Constraint:

$$0.5x_S + 0.75x_T + 1.5x_L \leq 24000$$

Quality Control Constraint:

$$x_S + x_T + 1.5x_L \leq 4000$$

Production Capacity Constraints:

$$\begin{cases} x_S \leq 10000, \\ x_T \leq 7000, \\ x_L \leq 3000, \end{cases}$$

Material Constraints:

$$\begin{cases} x_S + x_T \leq 15000 & \text{(MicroProcessors)} \\ x_S + x_T + x_L \leq 10000 & \text{(Screens)} \\ x_S + x_T + x_L \leq 12000 & \text{(Batteries)} \\ x_S + x_T + x_L \leq 15000 & \text{(Casings)} \\ x_L \leq 5000 & \text{(CPUs)} \\ x_L \leq 5000 & \text{(Keyboards)} \end{cases}$$

Production Quotas Constraints:

$$\begin{cases} x_S \geq 0.4(x_S + x_T + x_L) \\ x_S \leq 0.7(x_S + x_T + x_L) \\ x_T \geq 0.2(x_S + x_T + x_L) \\ x_T \leq 0.5(x_S + x_T + x_L) \\ x_L \geq 0.1(x_S + x_T + x_L) \\ x_L \leq 0.4(x_S + x_T + x_L) \end{cases}$$

Optimal Solution

From the results of the Gurobi code run (see Appendix 1), we can see that this company would need to produce $2,666.67$ smartphones, 761.9 tablets, and 380.95 laptops to make a maximum profit of $\$552,380.95$.

Sensitivity Analysis (Code See Appendix 2)

2.1 Sensitivity Analysis:

Smartphones (Model S) The reduced cost of $0.00 for smartphones indicates their inclusion in the optimal production plan. The profit per unit can decrease by up to $135.71 before the smartphone would be considered for removal from production. Conversely, the profit per unit could increase by up to $242.86 before the production plan would shift, suggesting that the current production of smartphones is well within a profitable range.

Tablets (Model T) Tablets also have a reduced cost of $0.00, reflecting their presence in the optimal solution. The allowable decrease in profit per unit is up to $100.00, which is less flexible compared to smartphones. The profit per unit could increase to $128.26, indicating that the production of tablets is also profitable but with a narrower margin for changes in profit per unit.

Laptops (Model L) Laptops share a reduced cost of $0.00, confirming their role in the optimal production mix. The profit per unit can be reduced by as much as $167.50 before impacting the optimal solution, which provides a moderate buffer against decreases in profitability. The profit per unit could rise by as much as $220.00, showing that there is room for improvement in profit margins for laptops.

2.2 Constraint Sensitivity Analysis:

Labor Constraint: The shadow price associated with the labor constraint is currently zero, which indicates that the company has more labor hours available than needed to maintain optimal production levels. The labor constraint could allow for an additional 2476.19 hours before impacting the profitability, reflecting unused labor capacity.

Quality Control Constraint: The shadow price of $138.10 for the quality control constraint is significant, signifying that quality control time is a scarce and valuable resource in the production process. Currently, the constraint is binding, and the maximum available hours are being utilized. Any increase in quality control time up to 10500 hours would directly increase the overall profit, emphasizing the importance of this resource.

Production Capacity Constraints: The shadow prices for the production capacities of all products are zero. This indicates that the company is not currently producing at the maximum capacity for Smartphones, Tablets, and Laptops. Therefore, increasing the production limit of these products will not result in increased profits under the current production model.

Material Constraints: Material constraints all have a shadow price of zero. The implication here is that there is an adequate supply of Microprocessors, Screens, Batteries, Casings, CPUs, and Keyboards. The current production plan does not fully exhaust these material resources, and thus, having additional quantities of these materials would not translate into additional profit.

Quota Constraints: The negative shadow prices on the QuotaMax constraints for Smartphones and Tablets suggest that the imposed quotas are causing a reduction in potential profit. The company is producing a higher quantity of these products than would be otherwise optimal, likely due to contractual or policy obligations. It would be beneficial for the company to consider renegotiating these terms to better align production with profit maximization objectives.

Appendix 1: Gurobi Code Output

```python
import gurobipy as gp
from gurobipy import GRB
m = gp.Model("Mini Project")

xS = m.addVar(vtype=GRB.CONTINUOUS, name="Smartphones")
xT = m.addVar(vtype=GRB.CONTINUOUS, name="Tablets")
xL = m.addVar(vtype=GRB.CONTINUOUS, name="Laptops")

m.setObjective(150*xS + 100*xT + 200*xL, GRB.MAXIMIZE)

# Add labor constraint
m.addConstr(0.5*xS + 0.75*xT + 1.5*xL <= 24000, "Labor")

# Add quality control constraint
m.addConstr(xS + xT + 1.5*xL <= 4000, "QualityControl")

# Add production capacity constraints
m.addConstr(xS <= 10000, "CapacitySmartphones")
m.addConstr(xT <= 7000, "CapacityTablets")
m.addConstr(xL <= 3000, "CapacityLaptops")

# Add material constraints
m.addConstr(xS + xT <= 15000, "MicroProcessors")
m.addConstr(xS + xT + xL <= 10000, "Screens")
m.addConstr(xS + xT + xL <= 12000, "Batteries")
m.addConstr(xS + xT + xL <= 15000, "Casings")
m.addConstr(xL <= 5000, "CPUs")
m.addConstr(xL <= 5000, "Keyboards")

# Add production quota constraints
total_production = xS + xT + xL
m.addConstr(xS >= 0.4 * total_production, "QuotaMinSmartphones")
m.addConstr(xS <= 0.7 * total_production, "QuotaMaxSmartphones")
m.addConstr(xT >= 0.2 * total_production, "QuotaMinTablets")
m.addConstr(xT <= 0.5 * total_production, "QuotaMaxTablets")
m.addConstr(xL >= 0.1 * total_production, "QuotaMinLaptops")
m.addConstr(xL <= 0.4 * total_production, "QuotaMaxLaptops")

m.optimize()

if m.status == GRB.OPTIMAL:
    print(f"Optimal production of Smartphones: {xS.x}")
    print(f"Optimal production of Tablets: {xT.x}")
    print(f"Optimal production of Laptops: {xL.x}")
else:
    print("No optimal solution found")
```

```
Restricted license - for non-production use only - expires 2025-11-24
Gurobi Optimizer version 11.0.0 build v11.0.0rc2 (linux64 - "Ubuntu 22.04.3 LTS")

CPU model: Intel(R) Xeon(R) CPU @ 2.80GHz, instruction set [SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 17 rows, 3 columns and 40 nonzeros
Model fingerprint: 0x606d02a6
Coefficient statistics:
  Matrix range     [1e-01, 2e+00]
  Objective range  [1e+02, 2e+02]
  Bounds range     [0e+00, 0e+00]
  RHS range        [3e+03, 2e+04]
Presolve removed 9 rows and 0 columns
Presolve time: 0.00s
Presolved: 8 rows, 3 columns, 24 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    6.0000000e+05   6.000000e+02   0.000000e+00      0s
       2    5.5238095e+05   0.000000e+00   0.000000e+00      0s

Solved in 2 iterations and 0.01 seconds (0.00 work units)
Optimal objective  5.523809524e+05
Optimal production of Smartphones: 2666.6666666666665
Optimal production of Tablets: 761.9047619047619
Optimal production of Laptops: 380.952380952381
```

Figure 1: Optimal Solution

Appendix 2: Sensitivity Analysis Output

```
print('\nVariable Sensitivity Analysis:')
print(f"{'Name':<20} {'Reduced Cost':>20} {'Min Obj Coef':>20} {'Max Obj Coef':>20}")
for v in m.getVars():
    print(f"{v.VarName:<20} {v.RC:>20.2f} {v.SAObjLow:>20.2f} {v.SAObjUp:>20.2f}")

print('\nSensitivity Analysis:')
print(f"{'Name':<20} {'Shadow Price':>20} {'Slack':>20} {'Min RHS':>20} {'Max RHS':>20}")
for c in m.getConstrs():
    print(f"{c.ConstrName:<20} {c.Pi:>20.2f} {c.Slack:>20.2f} {c.SARHSLow:>20.2f} {c.SARHSUp:>20.2f}")
```

```
Variable Sensitivity Analysis:
Name                          Reduced Cost        Min Obj Coef        Max Obj Coef
Smartphones                           0.00              135.71              242.86
Tablets                               0.00             -100.00              128.26
Laptops                               0.00              167.50              220.00

Sensitivity Analysis:
Name                          Shadow Price               Slack             Min RHS             Max RHS
Labor                                 0.00            21523.81             2476.19                 inf
QualityControl                      138.10                0.00                0.00            10500.00
CapacitySmartphones                   0.00             7333.33             2666.67                 inf
CapacityTablets                       0.00             6238.10              761.90                 inf
CapacityLaptops                       0.00             2619.05              380.95                 inf
MicroProcessors                       0.00            11571.43             3428.57                 inf
Screens                               0.00             6190.48             3809.52                 inf
Batteries                             0.00             8190.48             3809.52                 inf
Casings                               0.00            11190.48             3809.52                 inf
CPUs                                  0.00             4619.05              380.95                 inf
Keyboards                             0.00             4619.05              380.95                 inf
QuotaMinSmartphones                   0.00            -1142.86                -inf             1142.86
QuotaMaxSmartphones                  19.05                0.00            -1000.00                0.00
QuotaMinTablets                     -30.95                0.00             -695.65                0.00
QuotaMaxTablets                       0.00             1142.86            -1142.86                 inf
QuotaMinLaptops                       0.00               -0.00                -inf                0.00
QuotaMaxLaptops                       0.00             1142.86            -1142.86                 inf
```

Figure 2: Sensitivity Analysis Results