Project1

October 9, 2023

[ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

import xgboost as xgb
import statsmodels.api as sm
from sklearn import linear_model
from sklearn.decomposition import PCA
from statsmodels.tsa.api import ARIMA
import statsmodels.tsa.stattools as tsat
from scipy.stats import pearsonr, spearmanr
from sklearn.metrics import mean_squared_error, r2_score
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf


from fredapi import Fred

import warnings
warnings.filterwarnings('ignore')
```

---

1 0. Data Collection

FRED API

[ ]:
```
fred = Fred(api_key='923766e61730ee1dbda318b45ca1da35')
```

The target: S&P/Case-ShillerU.S. National Home Price Index

[ ]:
```
data = pd.DataFrame()
HPI = fred.get_series('CSUSHPISA')
data['date'] = HPI.index
data['HPIX'] = HPI.values
data = data[~np.isnan(data['HPIX'])]
data.reset_index(drop=True, inplace=True)
data
```

```
[ ]:            date     HPIX
     0   1987-01-01   63.964
     1   1987-02-01   64.424
     2   1987-03-01   64.736
     3   1987-04-01   65.132
     4   1987-05-01   65.563
     ..         …        …
     434 2023-03-01  298.396
     435 2023-04-01  300.208
     436 2023-05-01  302.685
     437 2023-06-01  304.741
     438 2023-07-01  306.720

     [439 rows x 2 columns]
```

other indicators relevant to the home price index

```
[ ]: # GDP
     GDP = fred.get_series('GDP')
     # M2 money
     M2 = fred.get_series('M2SL')
     # Consumer Price Index for All Urban Consumers
     CPI = fred.get_series('CPIAUCSL')
     # Interest rate: Federal Funds Effective Rate
     Interest = fred.get_series('FEDFUNDS')
     # Personal saving
     Psave = fred.get_series('PSAVE')
     # Employment rate
     Employ = fred.get_series('LNS12300060')
     # Population level
     Popu = fred.get_series('CNP16OV')
     # Salary level: Compensation of Employees, Received: Wage and Salary␣
      ↪Disbursements
     Salary = fred.get_series('A576RC1')
     # Nasdaq
     Nasdaq = fred.get_series('NASDAQCOM')
```

---

2 1. Data pre-processing

check if the data is time-continuous

```
[ ]: time_diff = (data['date'] - data['date'].shift(1)) / np.timedelta64(1, 'M')
     time_diff.max(), time_diff.min()
```

```
[ ]: (1.0185014065997249, 0.9199367543481386)
```

merge the dataset

```
[ ]: # since the frequency of the data might differ
     # need to merge the data
     # and make sure there is no look-ahead
     def merge_single(ori, app):
         '''
         Add app series to ori dataframe
         use searchsorted so there is no lookahead
         '''
         app = app.fillna(method='ffill')
         insertix = np.searchsorted(app.index, ori['date'])
         insertix[insertix == len(app)] = len(app)-1
         return app.values[insertix]
```

```
[ ]: data['GDP'] = merge_single(data, GDP)
     data['M2'] = merge_single(data, M2)
     data['CPI'] = merge_single(data, CPI)
     data['Interest'] = merge_single(data, Interest)
     data['Psave'] = merge_single(data, Psave)
     data['Employ'] = merge_single(data, Employ)
     data['Popu'] = merge_single(data, Popu)
     data['Salary'] = merge_single(data, Salary)
     data['Nasdaq'] = merge_single(data, Nasdaq)
```

merged dataset

```
[ ]: data.head()
```

```
[ ]:         date    HPIX       GDP      M2    CPI  Interest    Psave  Employ  \
     0 1987-01-01  63.964  4722.156  2743.9  111.4      6.43  290.040    77.7
     1 1987-02-01  64.424  4806.160  2747.5  111.8      6.10  216.931    77.9
     2 1987-03-01  64.736  4806.160  2753.7  112.2      6.13  216.931    78.0
     3 1987-04-01  65.132  4806.160  2767.7  112.7      6.37  216.931    78.2
     4 1987-05-01  65.563  4884.555  2772.9  113.0      6.85  244.481    78.4

            Popu  Salary  Nasdaq
     0  181827.0  2175.1  348.83
     1  181998.0  2193.4  397.18
     2  182179.0  2209.5  423.91
     3  182344.0  2215.2  428.34
     4  182533.0  2232.9  418.44
```

sanity check

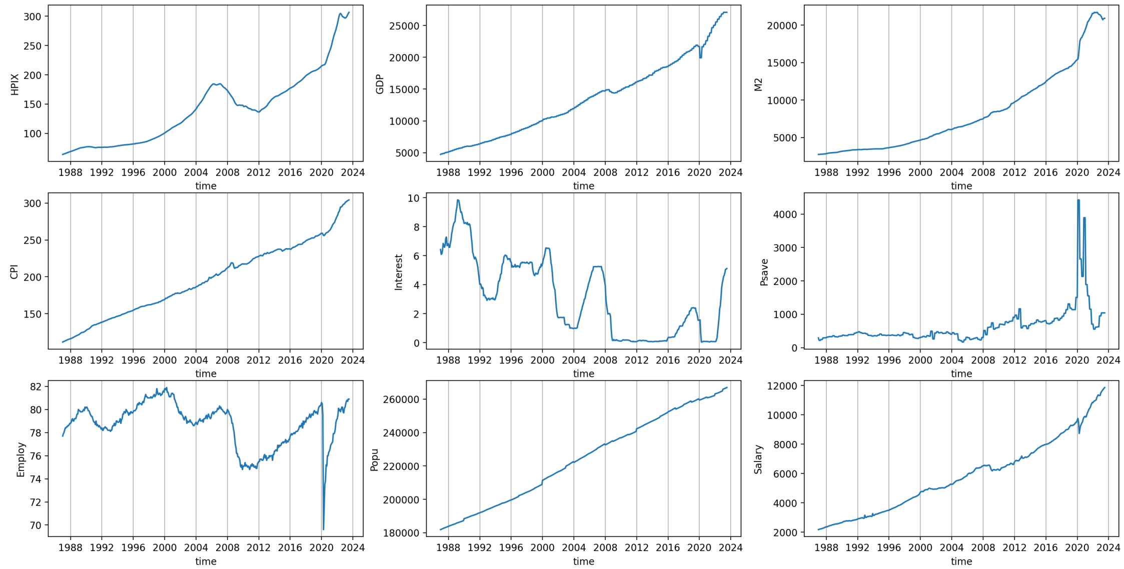
```
[ ]: data.isnull().any()
```

[ ]:
```
date        False
HPIX        False
GDP         False
M2          False
CPI         False
Interest    False
Psave       False
Employ      False
Popu        False
Salary      False
Nasdaq      False
dtype: bool
```

overview

[ ]:
```python
# target overview
plt.figure(figsize=(20,10), dpi=200)

plt.subplot(331)
plt.plot(data['date'], data['HPIX'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("HPIX", fontsize=10)

plt.subplot(332)
plt.plot(data['date'], data['GDP'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("GDP", fontsize=10)

plt.subplot(333)
plt.plot(data['date'], data['M2'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("M2", fontsize=10)

plt.subplot(334)
plt.plot(data['date'], data['CPI'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("CPI", fontsize=10)

plt.subplot(335)
plt.plot(data['date'], data['Interest'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Interest", fontsize=10)
```

```
plt.subplot(336)
plt.plot(data['date'], data['Psave'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Psave", fontsize=10)

plt.subplot(337)
plt.plot(data['date'], data['Employ'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Employ", fontsize=10)

plt.subplot(338)
plt.plot(data['date'], data['Popu'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Popu", fontsize=10)

plt.subplot(339)
plt.plot(data['date'], data['Salary'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Salary", fontsize=10)


plt.tick_params(labelsize=10)
plt.show()
```



3 3. Feature Engineering

Momentum effect

```
[ ]: # predict on the movement, instead the target itself
for col in ['HPIX', 'GDP', 'M2', 'CPI', 'Psave', 'Popu', 'Salary', 'Nasdaq']:
    data[f"mtm_{col}"] = (data[col] - data[col].shift(1)) / data[col].shift(1)
```

average line

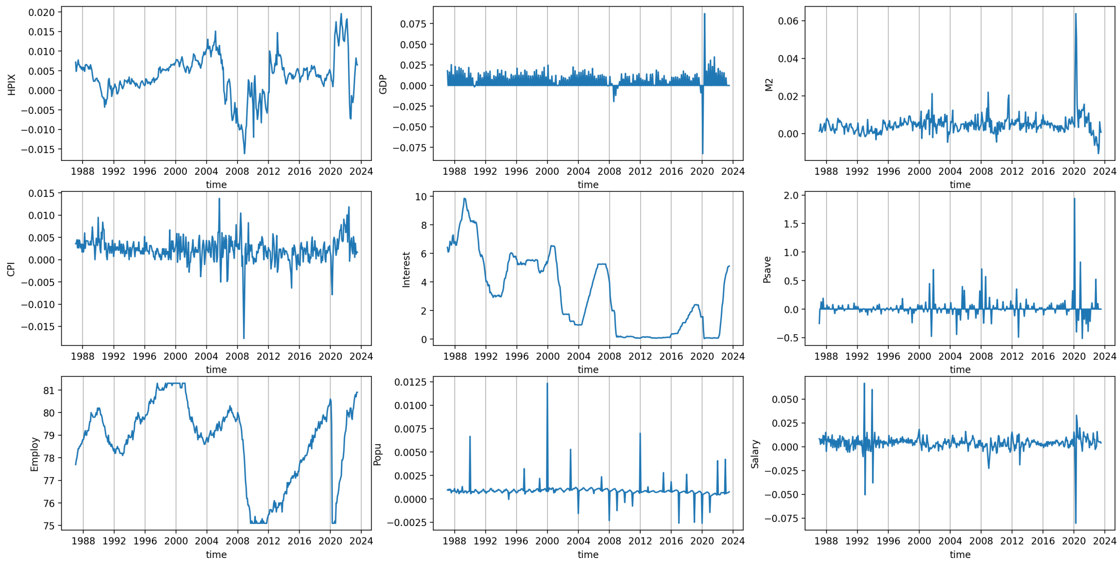
```
[ ]: for col in ['GDP', 'M2', 'CPI', 'Psave', 'Popu', 'Salary']:
    data[f'rolratio_{col}'] = data[col] / data[col].rolling(12, min_periods=1).
 ↪mean()
```

outlier handling

```
[ ]: # for the outliers
# simply cut-down
for col in ['Employ', 'GDP', 'M2', 'Popu', 'Salary', 'mtm_GDP', 'mtm_M2',␣
 ↪'mtm_Popu', 'mtm_Salary', 'mtm_Psave']:
    low_perc, up_perc = np.percentile(data[col], [5,95])
    data[col] = np.where(data[col] > up_perc, up_perc, data[col])
    data[col] = np.where(data[col] < low_perc, low_perc, data[col])
```

```
[ ]: # target overview
plt.figure(figsize=(20,10), dpi=200)

plt.subplot(331)
plt.plot(data['date'], data['mtm_HPIX'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("HPIX", fontsize=10)

plt.subplot(332)
plt.plot(data['date'], data['mtm_GDP'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("GDP", fontsize=10)

plt.subplot(333)
plt.plot(data['date'], data['mtm_M2'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("M2", fontsize=10)

plt.subplot(334)
```

```python
plt.plot(data['date'], data['mtm_CPI'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("CPI", fontsize=10)

plt.subplot(335)
plt.plot(data['date'], data['Interest'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Interest", fontsize=10)

plt.subplot(336)
plt.plot(data['date'], data['mtm_Psave'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Psave", fontsize=10)

plt.subplot(337)
plt.plot(data['date'], data['Employ'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Employ", fontsize=10)

plt.subplot(338)
plt.plot(data['date'], data['mtm_Popu'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Popu", fontsize=10)

plt.subplot(339)
plt.plot(data['date'], data['mtm_Salary'])
plt.grid(axis='x')
plt.xlabel("time", fontsize=10)
plt.ylabel("Salary", fontsize=10)


plt.tick_params(labelsize=10)
plt.show()
```



[ ]:
```python
data = data.iloc[1:].reset_index(drop=True)
data.head()
```

[ ]:
```
         date    HPIX        GDP       M2    CPI  Interest    Psave  Employ  \
0  1987-04-01  65.132  5500.0786  2978.71  112.7      6.37  216.931    78.2
1  1987-05-01  65.563  5500.0786  2978.71  113.0      6.85  244.481    78.4
2  1987-06-01  66.071  5500.0786  2978.71  113.5      6.73  244.481    78.4
3  1987-07-01  66.507  5500.0786  2978.71  113.8      6.58  244.481    78.5
4  1987-08-01  66.938  5500.0786  2978.71  114.3      6.73  290.912    78.5

       Popu   Salary  …  mtm_Psave  mtm_Popu  mtm_Salary  mtm_Nasdaq  \
0  185231.0  2504.88  …   0.000000  0.000906    0.002580    0.010450
1  185231.0  2504.88  …   0.126999  0.001037    0.007990   -0.023112
2  185231.0  2504.88  …   0.000000  0.000931    0.003896   -0.008197
3  185231.0  2504.88  …   0.000000  0.000996    0.004193    0.022771
4  185231.0  2504.88  …   0.189917  0.000640    0.011950    0.020426

   rolratio_GDP  rolratio_M2  rolratio_CPI  rolratio_Psave  rolratio_Popu  \
0           1.0          1.0      1.002223        1.000000            1.0
1           1.0          1.0      1.003255        1.081227            1.0
2           1.0          1.0      1.005760        1.059708            1.0
3           1.0          1.0      1.006723        1.047203            1.0
4           1.0          1.0      1.009272        1.196991            1.0

   rolratio_Salary
0              1.0
1              1.0
2              1.0
```

```
3              1.0
4              1.0

[5 rows x 25 columns]
```

[ ]:
```
data.columns
```

[ ]:
```
Index(['date', 'HPIX', 'GDP', 'M2', 'CPI', 'Interest', 'Psave', 'Employ',
       'Popu', 'Salary', 'Nasdaq', 'mtm_HPIX', 'mtm_GDP', 'mtm_M2', 'mtm_CPI',
       'mtm_Psave', 'mtm_Popu', 'mtm_Salary', 'mtm_Nasdaq', 'rolratio_GDP',
       'rolratio_M2', 'rolratio_CPI', 'rolratio_Psave', 'rolratio_Popu',
       'rolratio_Salary'],
      dtype='object')
```

Some other features like:

- Cross terms in the regression model
- Sometimes realization factors overplays expectation factors.
- Upstream or downstream indexes, such as building material and furniture prices.
- Other factors as market sentiment or government land policy.

Due to time limit, not implemented.

---

4 4. Model Selection

- **Baseline**: ARIMA
- **Improved**:
  - OLS & Lasso
  - Xgboost

4.0.1 ARIMA: data pre-process

ARMA requires the hypothesis that the time-series is stationary

tested by ADF test

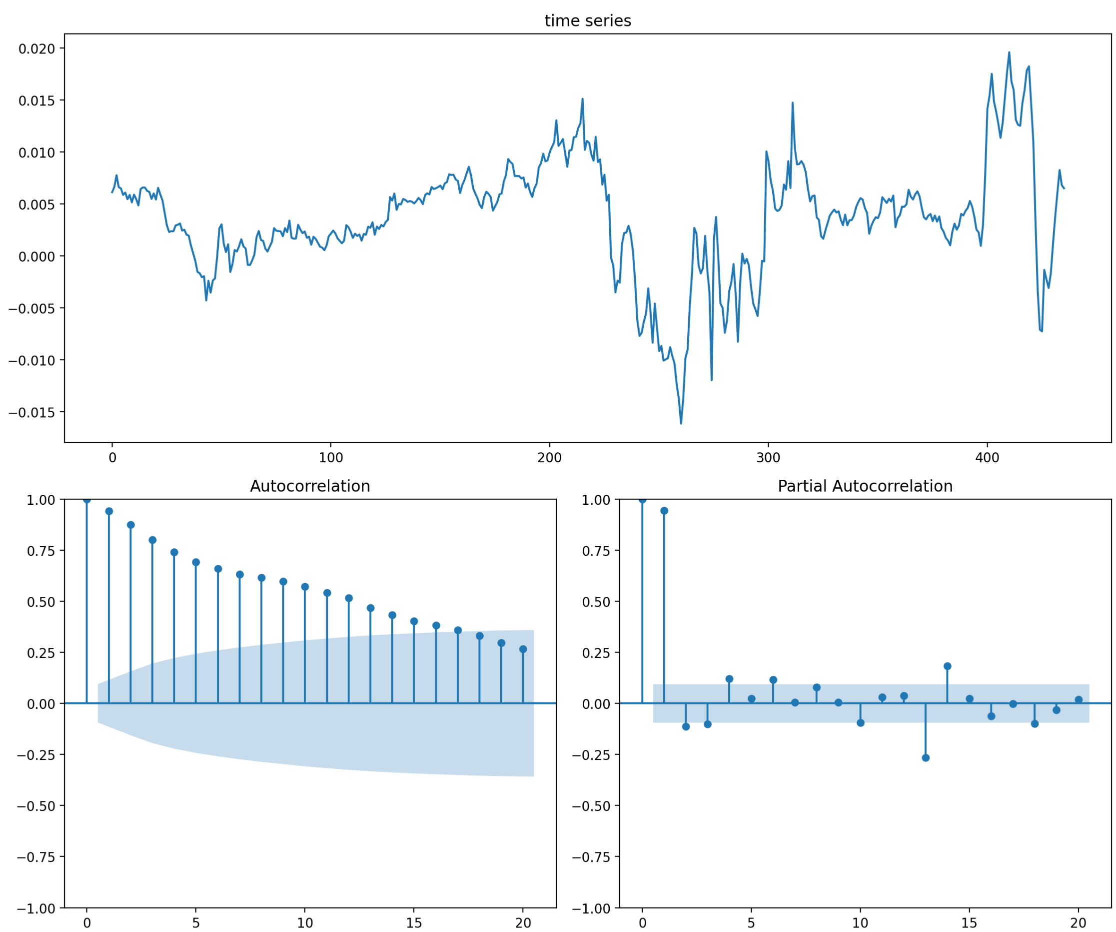
[ ]:
```
# ADF test on original time series Home price Index
print(tsat.adfuller(data['HPIX']))
# ADF test on the first-order of time series Home price Index
print(tsat.adfuller(data['mtm_HPIX']))
```

```
(1.014348659714905, 0.9944150600035891, 18, 417, {'1%': -3.446129402876608,
'5%': -2.8684960761128346, '10%': -2.570475362616382}, 228.36001000572344)
(-2.955316220668871, 0.039283289700147585, 13, 422, {'1%': -3.44594128742536,
'5%': -2.868413360220551, '10%': -2.570431271085555}, -4098.829611340794)
```

After the first-order transformation(in other words we are predicting the momentum of the HPIX), the time-series is stationary on the 5% confidence level, given by ADF test.

[ ]:

```
# ACF & PACF Plot
def plotds(xt, nlag=10):
    plt.figure(figsize=(12,10), dpi=200)
    layout = (2,2)
    ax_xt = plt.subplot2grid(layout, (0,0), colspan=2)
    ax_acf = plt.subplot2grid(layout, (1,0))
    ax_pacf = plt.subplot2grid(layout, (1,1))

    xt.plot(ax=ax_xt)
    ax_xt.set_title('time series')
    plot_acf(xt, lags=nlag, ax=ax_acf)
    plot_pacf(xt, lags=nlag, ax=ax_pacf)
    plt.tight_layout()

    plt.show()

plotds(data['mtm_HPIX'], nlag=20)
```



Notice that the auto-correlation graph is thick-tailed, while partial auto-correlation decay very fast. Which tells us ARIMA model will be better than simple ARMA model.

Grid search for the order parameter in ARIMA model

[ ]:
```
data_new = data['mtm_HPIX'].copy()

aic_value = []
for ari in range(1,5):
    for arj in range(1,5):
        arma_ij = ARIMA(data_new.values.tolist(), order=(ari,0,arj)).fit()
        aic_value.append([ari, arj, arma_ij.aic])

aic_value.sort(key=lambda x:x[2])
aic_value[0]
```

[ ]:
```
[4, 3, -4266.519840726124]
```

Parameter of Order in ARIMA: (4,3)

4.0.2 OLS: data pre-process

OLS makes basic hypothesis about the multicolinearity in the X dataset

- Need to be satisfied if we want to test hypothesis over coefficients.
- Is considered not to be that important if we want to get the inference and evalutaion.

Since we also want to test the stability of coefficients over time, we need an independent X set. - PCA method is ususally stable over a larger dataset, which means on our own dataset, if we want to conduct stability analysis, we shall not use PCA for dimension reduction while excluding look-ahead problem. - Hence for time-rolling training-prediction mechanism we choose Lasso Regression, which can eliminate the effect of multicolinearity.

[ ]:
```
data.drop(columns=['date', 'HPIX', 'mtm_HPIX']).corr()
```

[ ]:
```
                GDP        M2       CPI  Interest     Psave    Employ  \
GDP        1.000000  0.968072  0.991055 -0.703755  0.583800 -0.324164
M2         0.968072  1.000000  0.944874 -0.620685  0.647622 -0.302605
CPI        0.991055  0.944874  1.000000 -0.732193  0.557375 -0.344218
Interest  -0.703755 -0.620685 -0.732193  1.000000 -0.445498  0.686254
Psave      0.583800  0.647622  0.557375 -0.445498  1.000000 -0.358603
Employ    -0.324164 -0.302605 -0.344218  0.686254 -0.358603  1.000000
Popu       0.984317  0.917575  0.986254 -0.775045  0.573746 -0.396701
Salary     0.998258  0.966736  0.988992 -0.692140  0.597793 -0.291005
Nasdaq     0.877781  0.946332  0.838070 -0.449447  0.619686 -0.088196
mtm_GDP   -0.021482  0.000868 -0.048853  0.024622 -0.082926 -0.048814
mtm_M2     0.139499  0.137912  0.126860 -0.175594  0.462319 -0.169583
mtm_CPI   -0.026684  0.036035 -0.056189  0.186649 -0.113998  0.123456
mtm_Psave  0.027444  0.023125  0.040048  0.008503  0.253111  0.055029
mtm_Popu  -0.113046 -0.125107 -0.102363  0.092686 -0.158271  0.097004
```

mtm_Salary	-0.007424	0.010797	-0.029388	0.084334	-0.104256	0.085177
mtm_Nasdaq	0.010733	0.015571	0.007403	-0.020096	0.045047	-0.049895
rolratio_GDP	-0.060990	-0.076279	-0.089408	0.003698	-0.189721	0.068505
rolratio_M2	0.340630	0.326507	0.331217	-0.375878	0.589569	-0.290756
rolratio_CPI	-0.024994	0.079982	-0.048473	0.346165	-0.165696	0.315547
rolratio_Psave	-0.006007	-0.019757	0.020507	0.062286	0.402947	0.024350
rolratio_Popu	-0.283780	-0.405272	-0.239631	0.060996	-0.302568	0.153159
rolratio_Salary	-0.058483	-0.084859	-0.078255	0.150957	-0.074462	0.305394

	Popu	Salary	Nasdaq	mtm_GDP	…	mtm_Psave	\
GDP	0.984317	0.998258	0.877781	-0.021482	…	0.027444	
M2	0.917575	0.966736	0.946332	0.000868	…	0.023125	
CPI	0.986254	0.988992	0.838070	-0.048853	…	0.040048	
Interest	-0.775045	-0.692140	-0.449447	0.024622	…	0.008503	
Psave	0.573746	0.597793	0.619686	-0.082926	…	0.253111	
Employ	-0.396701	-0.291005	-0.088196	-0.048814	…	0.055029	
Popu	1.000000	0.981995	0.792620	-0.048853	…	0.040089	
Salary	0.981995	1.000000	0.885465	-0.041665	…	0.045699	
Nasdaq	0.792620	0.885465	1.000000	0.019153	…	0.008604	
mtm_GDP	-0.048853	-0.041665	0.019153	1.000000	…	-0.380793	
mtm_M2	0.172523	0.150408	0.119433	0.202566	…	-0.083158	
mtm_CPI	-0.090928	-0.029968	0.132359	0.029281	…	-0.033352	
mtm_Psave	0.040089	0.045699	0.008604	-0.380793	…	1.000000	
mtm_Popu	-0.104521	-0.110710	-0.097332	-0.040906	…	-0.008103	
mtm_Salary	-0.038749	-0.008096	0.064362	0.089870	…	-0.004053	
mtm_Nasdaq	0.008127	0.004816	0.045953	0.088222	…	0.013503	
rolratio_GDP	-0.080627	-0.078110	0.037503	0.418285	…	-0.293224	
rolratio_M2	0.380916	0.353497	0.307501	0.122261	…	-0.045699	
rolratio_CPI	-0.136303	-0.022039	0.226100	0.019308	…	-0.002877	
rolratio_Psave	0.029557	0.015164	-0.090168	-0.265090	…	0.561281	
rolratio_Popu	-0.194012	-0.275810	-0.379848	-0.027626	…	-0.038862	
rolratio_Salary	-0.073617	-0.049372	0.060634	-0.033163	…	-0.004156	

	mtm_Popu	mtm_Salary	mtm_Nasdaq	rolratio_GDP	rolratio_M2	\
GDP	-0.113046	-0.007424	0.010733	-0.060990	0.340630	
M2	-0.125107	0.010797	0.015571	-0.076279	0.326507	
CPI	-0.102363	-0.029388	0.007403	-0.089408	0.331217	
Interest	0.092686	0.084334	-0.020096	0.003698	-0.375878	
Psave	-0.158271	-0.104256	0.045047	-0.189721	0.589569	
Employ	0.097004	0.085177	-0.049895	0.068505	-0.290756	
Popu	-0.104521	-0.038749	0.008127	-0.080627	0.380916	
Salary	-0.110710	-0.008096	0.004816	-0.078110	0.353497	
Nasdaq	-0.097332	0.064362	0.045953	0.037503	0.307501	
mtm_GDP	-0.040906	0.089870	0.088222	0.418285	0.122261	
mtm_M2	-0.028562	-0.211574	-0.076154	-0.158200	0.639436	
mtm_CPI	0.037468	0.174704	0.133116	0.214189	-0.108461	
mtm_Psave	-0.008103	-0.004053	0.013503	-0.293224	-0.045699	

```
mtm_Popu          1.000000    0.069695    0.043557       0.032464    -0.019074
mtm_Salary        0.069695    1.000000    0.176608       0.294839    -0.024985
mtm_Nasdaq        0.043557    0.176608    1.000000       0.160037     0.060935
rolratio_GDP      0.032464    0.294839    0.160037       1.000000     0.108612
rolratio_M2      -0.019074   -0.024985    0.060935       0.108612     1.000000
rolratio_CPI      0.032671    0.122665   -0.056955       0.161007    -0.297148
rolratio_Psave   -0.036225   -0.171264   -0.028127      -0.572751     0.054915
rolratio_Popu     0.385473   -0.000209    0.029614       0.337725     0.124310
rolratio_Salary   0.073924    0.434863    0.087888       0.645088    -0.025760

                  rolratio_CPI  rolratio_Psave  rolratio_Popu  rolratio_Salary
GDP                  -0.024994       -0.006007      -0.283780        -0.058483
M2                    0.079982       -0.019757      -0.405272        -0.084859
CPI                  -0.048473        0.020507      -0.239631        -0.078255
Interest              0.346165        0.062286       0.060996         0.150957
Psave                -0.165696        0.402947      -0.302568        -0.074462
Employ                0.315547        0.024350       0.153159         0.305394
Popu                 -0.136303        0.029557      -0.194012        -0.073617
Salary               -0.022039        0.015164      -0.275810        -0.049372
Nasdaq                0.226100       -0.090168      -0.379848         0.060634
mtm_GDP               0.019308       -0.265090      -0.027626        -0.033163
mtm_M2               -0.228086        0.207696      -0.010911        -0.157317
mtm_CPI               0.574905       -0.263976      -0.110997         0.140109
mtm_Psave            -0.002877        0.561281      -0.038862        -0.004156
mtm_Popu              0.032671       -0.036225       0.385473         0.073924
mtm_Salary            0.122665       -0.171264      -0.000209         0.434863
mtm_Nasdaq           -0.056955       -0.028127       0.029614         0.087888
rolratio_GDP          0.161007       -0.572751       0.337725         0.645088
rolratio_M2          -0.297148        0.054915       0.124310        -0.025760
rolratio_CPI          1.000000       -0.226606      -0.171276         0.215829
rolratio_Psave       -0.226606        1.000000      -0.079116        -0.209825
rolratio_Popu        -0.171276       -0.079116       1.000000         0.380125
rolratio_Salary       0.215829       -0.209825       0.380125         1.000000

[22 rows x 22 columns]
```

---

5 5. Model Training and Validation

There are 2 different train-test split method usually used in financial prediction: - **Time-split**: use the first 80% time for as training set and the remaining 20% as testing & Validation Set. - **Time-Rolling /Prediction**: To capture the short-term market style and avoid high delay, we can update our model at a fixed frequency. For example, Every 3 years, we train a new model with the data of the past 3 years and make predict ion on the following 6 months. After all the training & prediction, we merge all the inference and make evaluation together.

The second method is more common in high-frequency prediction due to the limit of dataset.

On our task we mainly use the naive time-split method, and explore the second method for trial.

5.0.1 5.0 Time-Split

[ ]:
```
# features
features = [
    'GDP', 'M2', 'CPI', 'Interest', 'Psave', 'Employ', 'Popu', 'Salary',
    'Nasdaq', 'mtm_GDP', 'mtm_M2', 'mtm_CPI', 'mtm_Psave', 'mtm_Popu',
    'mtm_Salary', 'mtm_Nasdaq', 'rolratio_GDP', 'rolratio_M2', 'rolratio_CPI',
    'rolratio_Psave', 'rolratio_Popu', 'rolratio_Salary'
]
```

[ ]:
```
# time split
split_ix = int(0.8 * data.shape[0])

train_x = data.loc[:split_ix][features]
test_x = data.loc[split_ix+1:][features]
train_y = data.loc[:split_ix]['mtm_HPIX']
test_y = data.loc[split_ix+1:]['mtm_HPIX']
```

5.0.2 5.1 time-rolling prediction framework

[ ]:
```
# training and predicting batch size
train_batch_size, test_batch_size = 36, 6
batch_size = train_batch_size + test_batch_size
batches = data.shape[0] // batch_size

# batched data
batch_data = []
for i in range(batches):
    train_x_batch = data.loc[i*batch_size:␣
 ↪i*batch_size+train_batch_size-1][features]
    train_y_batch = data.loc[i*batch_size:␣
 ↪i*batch_size+train_batch_size-1]['mtm_HPIX']
    test_x_batch = data.loc[i*batch_size+train_batch_size:␣
 ↪(i+1)*batch_size-1][features]
    test_y_batch = data.loc[i*batch_size+train_batch_size:␣
 ↪(i+1)*batch_size-1]['mtm_HPIX']
    assert train_x_batch.shape[0] == train_batch_size
    assert test_x_batch.shape[0] == test_batch_size
    batch_data.append((train_x_batch, train_y_batch, test_x_batch,␣
 ↪test_y_batch))
```

5.0.3 5.2.1 ARIMA model

Time-split method

[ ]:
```
ARIMA_split1_model = ARIMA(train_y.values.tolist(), order=(4,0,3)).fit()
ARIMA_split1_inference = ARIMA_split1_model.forecast(test_y.shape[0])
```

5.0.4 5.2.2 Lasso Regression

Time-split method

[ ]:
```
LASSO_split1_model = linear_model.Lasso(alpha=0.1).fit(train_x, train_y)
LASSO_split1_inference = LASSO_split1_model.predict(test_x)
```

Rolling spliting method

[ ]:
```
# coeffients
coefs = []
inf_dates = []
test_y_real, test_y_infer = np.array([]), np.array([])
for train_x_batch, train_y_batch, test_x_batch, test_y_batch in batch_data:
    LASSO_split2_model = linear_model.Lasso(alpha=0.1).fit(train_x_batch,␣
 ↪train_y_batch)
    LASSO_split2_inference = LASSO_split2_model.predict(test_x_batch)
    coefs.append(LASSO_split2_model.coef_)
    inf_dates.append(data.loc[test_x_batch.index]['date'].values)

    test_y_real = np.hstack([test_y_real, test_y_batch])
    test_y_infer = np.hstack([test_y_infer, LASSO_split2_inference])

LASSO_split2_inference = pd.DataFrame()
LASSO_split2_inference['real'] = test_y_real
LASSO_split2_inference['predict'] = test_y_infer
LASSO_split2_inference.index = np.hstack(inf_dates)
LASSO_split2_inference.head()
```

[ ]:

	real	predict
1990-04-01	0.000984	0.000989
1990-05-01	0.000246	0.000806
1990-06-01	-0.000492	0.000609
1990-07-01	-0.001553	0.000432
1990-08-01	-0.001685	0.000223

5.0.5 5.2.3 XGBoost

[ ]:
```
# time-split method
XGB_split1_model = xgb.XGBRegressor(
    booster='gbtree',
    objective='reg:squarederror',
    eval_metric='rmse',
    n_estimators=100,
    learning_rate=0.08,
```

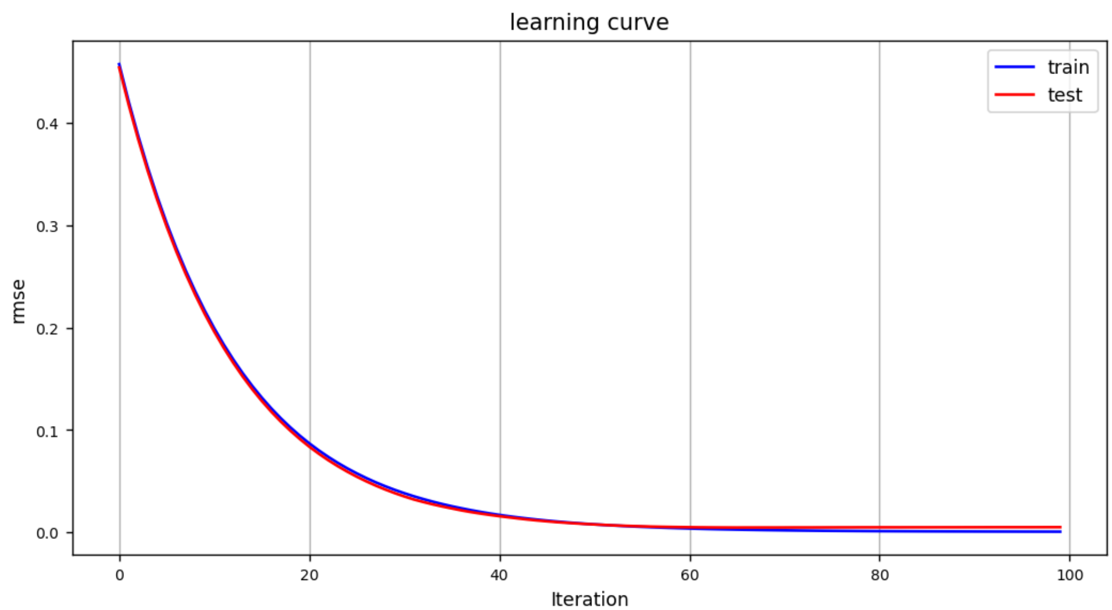
```
    subsample=0.75,
    colsample_bytree=1,
    max_depth=5,
    gamma=0,
)
XGB_split1_model.fit(train_x, train_y, eval_set=[(train_x, train_y),(test_x,␣
 ↪test_y)], verbose=1)
XGB_split1_inference = XGB_split1_model.predict(test_x)
```

```
[0]	validation_0-rmse:0.45748	validation_1-rmse:0.45418
[1]	validation_0-rmse:0.42102	validation_1-rmse:0.41772
[2]	validation_0-rmse:0.38748	validation_1-rmse:0.38418
[3]	validation_0-rmse:0.35659	validation_1-rmse:0.35329
[4]	validation_0-rmse:0.32819	validation_1-rmse:0.32489
[5]	validation_0-rmse:0.30204	validation_1-rmse:0.29874
[6]	validation_0-rmse:0.27798	validation_1-rmse:0.27468
[7]	validation_0-rmse:0.25584	validation_1-rmse:0.25254
[8]	validation_0-rmse:0.23545	validation_1-rmse:0.23216
[9]	validation_0-rmse:0.21669	validation_1-rmse:0.21339
[10]	validation_0-rmse:0.19943	validation_1-rmse:0.19614
[11]	validation_0-rmse:0.18355	validation_1-rmse:0.18025
[12]	validation_0-rmse:0.16894	validation_1-rmse:0.16565
[13]	validation_0-rmse:0.15548	validation_1-rmse:0.15219
[14]	validation_0-rmse:0.14309	validation_1-rmse:0.13980
[15]	validation_0-rmse:0.13169	validation_1-rmse:0.12841
[16]	validation_0-rmse:0.12118	validation_1-rmse:0.11790
[17]	validation_0-rmse:0.11153	validation_1-rmse:0.10825
[18]	validation_0-rmse:0.10266	validation_1-rmse:0.09939
[19]	validation_0-rmse:0.09450	validation_1-rmse:0.09123
[20]	validation_0-rmse:0.08697	validation_1-rmse:0.08370
[21]	validation_0-rmse:0.08009	validation_1-rmse:0.07683
[22]	validation_0-rmse:0.07373	validation_1-rmse:0.07048
[23]	validation_0-rmse:0.06790	validation_1-rmse:0.06465
[24]	validation_0-rmse:0.06252	validation_1-rmse:0.05928
[25]	validation_0-rmse:0.05760	validation_1-rmse:0.05437
[26]	validation_0-rmse:0.05305	validation_1-rmse:0.04982
[27]	validation_0-rmse:0.04885	validation_1-rmse:0.04563
[28]	validation_0-rmse:0.04502	validation_1-rmse:0.04181
[29]	validation_0-rmse:0.04149	validation_1-rmse:0.03829
[30]	validation_0-rmse:0.03824	validation_1-rmse:0.03506
[31]	validation_0-rmse:0.03526	validation_1-rmse:0.03209
[32]	validation_0-rmse:0.03251	validation_1-rmse:0.02963
[33]	validation_0-rmse:0.02996	validation_1-rmse:0.02734
[34]	validation_0-rmse:0.02763	validation_1-rmse:0.02525
[35]	validation_0-rmse:0.02549	validation_1-rmse:0.02334
[36]	validation_0-rmse:0.02353	validation_1-rmse:0.02147
[37]	validation_0-rmse:0.02170	validation_1-rmse:0.01982
```

```
[38]	validation_0-rmse:0.02002	validation_1-rmse:0.01837
[39]	validation_0-rmse:0.01847	validation_1-rmse:0.01703
[40]	validation_0-rmse:0.01704	validation_1-rmse:0.01578
[41]	validation_0-rmse:0.01574	validation_1-rmse:0.01463
[42]	validation_0-rmse:0.01455	validation_1-rmse:0.01353
[43]	validation_0-rmse:0.01344	validation_1-rmse:0.01251
[44]	validation_0-rmse:0.01243	validation_1-rmse:0.01159
[45]	validation_0-rmse:0.01149	validation_1-rmse:0.01079
[46]	validation_0-rmse:0.01063	validation_1-rmse:0.01000
[47]	validation_0-rmse:0.00982	validation_1-rmse:0.00935
[48]	validation_0-rmse:0.00910	validation_1-rmse:0.00876
[49]	validation_0-rmse:0.00844	validation_1-rmse:0.00820
[50]	validation_0-rmse:0.00780	validation_1-rmse:0.00773
[51]	validation_0-rmse:0.00724	validation_1-rmse:0.00727
[52]	validation_0-rmse:0.00673	validation_1-rmse:0.00683
[53]	validation_0-rmse:0.00624	validation_1-rmse:0.00644
[54]	validation_0-rmse:0.00578	validation_1-rmse:0.00614
[55]	validation_0-rmse:0.00536	validation_1-rmse:0.00587
[56]	validation_0-rmse:0.00499	validation_1-rmse:0.00561
[57]	validation_0-rmse:0.00465	validation_1-rmse:0.00542
[58]	validation_0-rmse:0.00432	validation_1-rmse:0.00526
[59]	validation_0-rmse:0.00402	validation_1-rmse:0.00512
[60]	validation_0-rmse:0.00374	validation_1-rmse:0.00501
[61]	validation_0-rmse:0.00349	validation_1-rmse:0.00490
[62]	validation_0-rmse:0.00325	validation_1-rmse:0.00483
[63]	validation_0-rmse:0.00304	validation_1-rmse:0.00478
[64]	validation_0-rmse:0.00285	validation_1-rmse:0.00474
[65]	validation_0-rmse:0.00267	validation_1-rmse:0.00471
[66]	validation_0-rmse:0.00250	validation_1-rmse:0.00473
[67]	validation_0-rmse:0.00235	validation_1-rmse:0.00474
[68]	validation_0-rmse:0.00221	validation_1-rmse:0.00475
[69]	validation_0-rmse:0.00208	validation_1-rmse:0.00474
[70]	validation_0-rmse:0.00195	validation_1-rmse:0.00476
[71]	validation_0-rmse:0.00184	validation_1-rmse:0.00480
[72]	validation_0-rmse:0.00173	validation_1-rmse:0.00480
[73]	validation_0-rmse:0.00163	validation_1-rmse:0.00481
[74]	validation_0-rmse:0.00154	validation_1-rmse:0.00483
[75]	validation_0-rmse:0.00146	validation_1-rmse:0.00485
[76]	validation_0-rmse:0.00140	validation_1-rmse:0.00488
[77]	validation_0-rmse:0.00134	validation_1-rmse:0.00489
[78]	validation_0-rmse:0.00127	validation_1-rmse:0.00492
[79]	validation_0-rmse:0.00121	validation_1-rmse:0.00492
[80]	validation_0-rmse:0.00115	validation_1-rmse:0.00492
[81]	validation_0-rmse:0.00111	validation_1-rmse:0.00492
[82]	validation_0-rmse:0.00107	validation_1-rmse:0.00491
[83]	validation_0-rmse:0.00103	validation_1-rmse:0.00493
[84]	validation_0-rmse:0.00100	validation_1-rmse:0.00494
[85]	validation_0-rmse:0.00096	validation_1-rmse:0.00493
```

```
[86]	validation_0-rmse:0.00093	validation_1-rmse:0.00494
[87]	validation_0-rmse:0.00091	validation_1-rmse:0.00495
[88]	validation_0-rmse:0.00088	validation_1-rmse:0.00495
[89]	validation_0-rmse:0.00086	validation_1-rmse:0.00497
[90]	validation_0-rmse:0.00083	validation_1-rmse:0.00498
[91]	validation_0-rmse:0.00080	validation_1-rmse:0.00498
[92]	validation_0-rmse:0.00078	validation_1-rmse:0.00500
[93]	validation_0-rmse:0.00077	validation_1-rmse:0.00501
[94]	validation_0-rmse:0.00075	validation_1-rmse:0.00502
[95]	validation_0-rmse:0.00073	validation_1-rmse:0.00502
[96]	validation_0-rmse:0.00071	validation_1-rmse:0.00502
[97]	validation_0-rmse:0.00070	validation_1-rmse:0.00503
[98]	validation_0-rmse:0.00068	validation_1-rmse:0.00504
[99]	validation_0-rmse:0.00066	validation_1-rmse:0.00505
```

[ ]:
```
# learning curve
plt.figure(figsize=(10,5), dpi=125)

plt.plot(XGB_split1_model.evals_result()['validation_0']['rmse'], 'b-',␣
 ↪label='train')
plt.plot(XGB_split1_model.evals_result()['validation_1']['rmse'], 'r-',␣
 ↪label='test')

plt.legend(fontsize=10)
plt.grid(axis='x')
plt.xlabel("Iteration", fontsize=10)
plt.ylabel("rmse", fontsize=10)
plt.tick_params(labelsize=8)
plt.title('learning curve', fontsize=12)

plt.show()
```



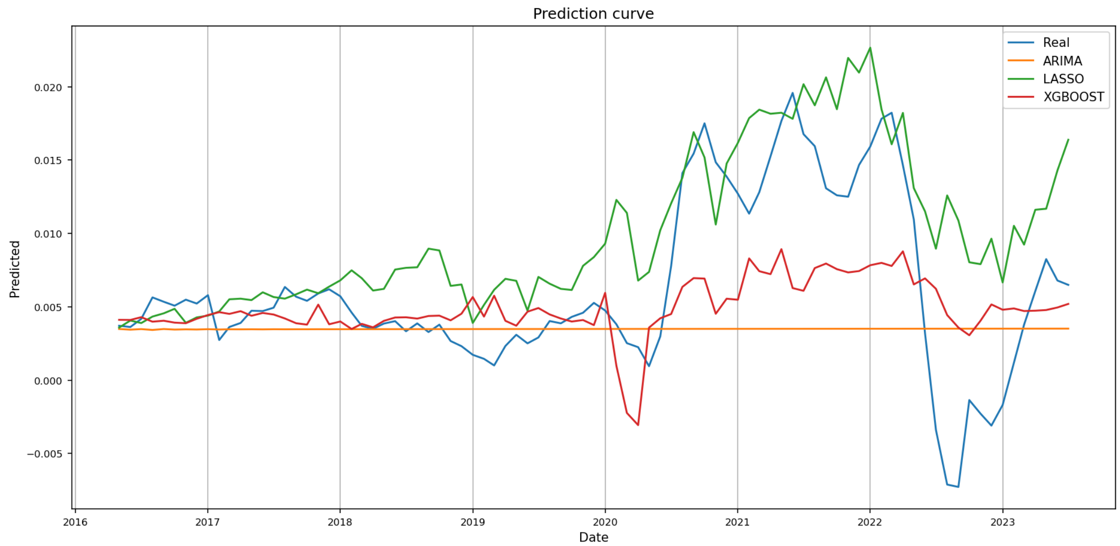
---

6 6. Evaluation

- **General metrics**: RMSE, IS/OOS R^2, Pearson/Spearman Correlation
- **Other metrics**: Stratification monotonicity and variance, time-rolling parameter stability

overview

[ ]:
```
# prediction curve
inference_dates = data.loc[test_x.index]['date']
plt.figure(figsize=(15,7), dpi=150)

plt.plot(inference_dates, test_y, label='Real')
plt.plot(inference_dates, ARIMA_split1_inference, label='ARIMA')
plt.plot(inference_dates, LASSO_split1_inference, label='LASSO')
plt.plot(inference_dates, XGB_split1_inference, label='XGBOOST')

plt.legend(fontsize=10, loc='best')
plt.grid(axis='x')
plt.xlabel("Date", fontsize=10)
plt.ylabel("Predicted", fontsize=10)
plt.tick_params(labelsize=8)
plt.title('Prediction curve', fontsize=12)

plt.show()
```



From the visual point of view, - ARIMA predicts the average target and lacks variance. - Both LASSO & XGBoost can capture some movement tendency in HPIX, but there is obvious strong lag in the prediction. - Both models inherit the feature of the significantly positive mean on the training set, so there is a **significant positive bias** after the data distribution transition of the training and test sets.

General Metrics

[ ]:
```
# RMSE
RMSE = [
    np.sqrt(mean_squared_error(y_true=test_y, y_pred=ARIMA_split1_inference)),
    np.sqrt(mean_squared_error(y_true=test_y, y_pred=LASSO_split1_inference)),
    np.sqrt(mean_squared_error(y_true=test_y, y_pred=XGB_split1_inference))
]

# Out-of-sample R^2
R2 = [
    r2_score(y_true=test_y, y_pred=ARIMA_split1_inference),
    r2_score(y_true=test_y, y_pred=LASSO_split1_inference),
    r2_score(y_true=test_y, y_pred=XGB_split1_inference)
]

# Pearson correlation
pearson = [
    pearsonr(test_y, ARIMA_split1_inference)[0],
    pearsonr(test_y, LASSO_split1_inference)[0],
    pearsonr(test_y, XGB_split1_inference)[0]
]
```

```
# spearman correlation
spearman = [
    spearmanr(test_y, ARIMA_split1_inference)[0],
    spearmanr(test_y, LASSO_split1_inference)[0],
    spearmanr(test_y, XGB_split1_inference)[0]
]
```

[ ]:
```
general_eval = pd.DataFrame()
general_eval['RMSE'] = RMSE
general_eval['R2'] = R2
general_eval['Pearson'] = pearson
general_eval['Spearman'] = spearman
general_eval.index = ['Baseline: ARIMA', 'LASSO', 'XGBoost']
for col in general_eval.columns:
    general_eval[col] = general_eval[col].apply(lambda x: '%.3g'%x)
general_eval
```

[ ]:

	RMSE	R2	Pearson	Spearman
Baseline: ARIMA	0.0064	-0.225	0.204	0.141
LASSO	0.00561	0.0585	0.718	0.503
XGBoost	0.00505	0.238	0.612	0.527

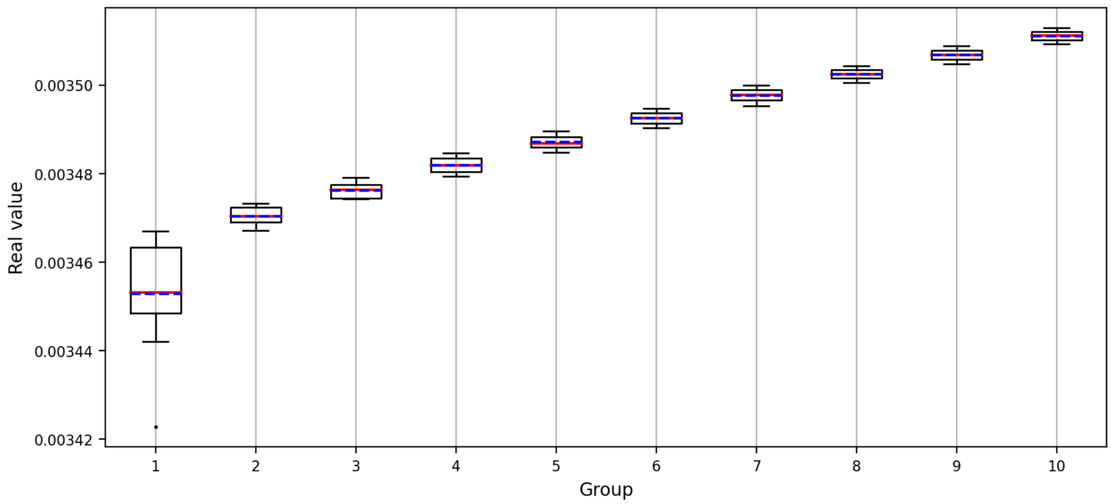
Stratification monotonicity and variance

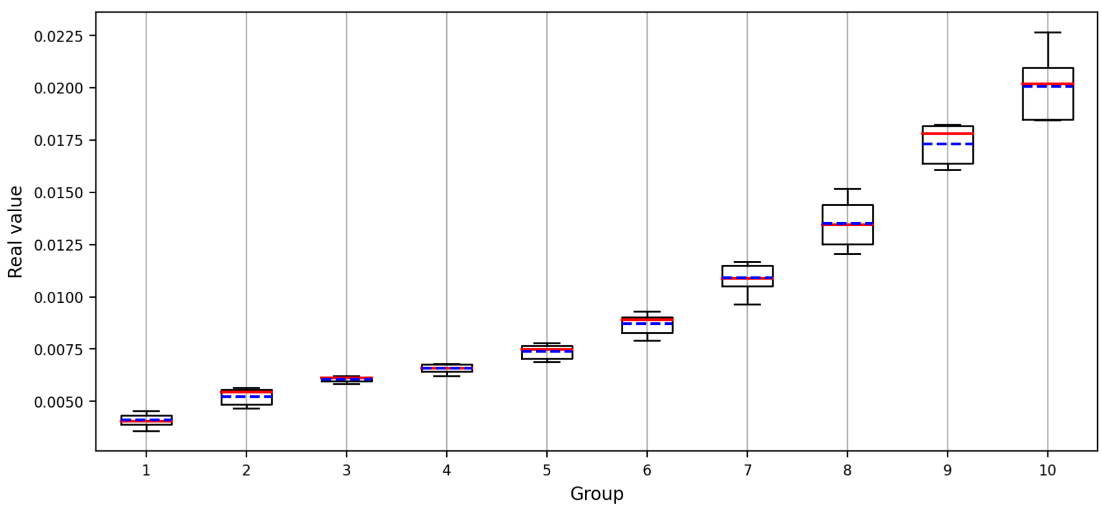
- If we match each pair of real target with the predicted target,
- split the pairs into n quantiles based on the predicted value.
- As the increase of the quantile we expect the real target within the quantile group to increase monotonously and steadily,
- which can be reflected through the monotonicity and variance of the quantile.

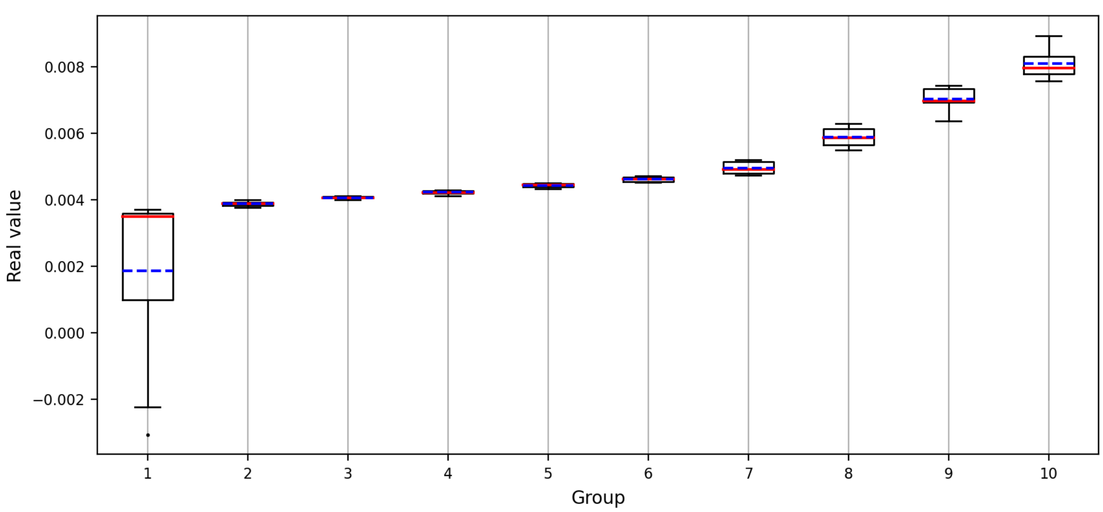
[ ]:
```
def box_plot(groupedOutput, ax, name):
    ax.boxplot(
        groupedOutput,
        medianprops={
            'color': 'red',
            'linewidth': '1.5'
        },
        meanline=True,
        showmeans=True,
        meanprops={
            'color': 'blue',
            'ls': '--',
            'linewidth': '1.5'
        },
        flierprops={
            "marker": "o",
            "markerfacecolor": "red",
```

```
            "markersize": 1
        },
    )

    plt.grid(axis='x')
    plt.xlabel("Group", fontsize=10)
    plt.ylabel("Real value", fontsize=10)
    plt.tick_params(labelsize=8)
```

[ ]:
```
temp = pd.DataFrame()
temp['real'] = test_y
plt.figure(figsize=(10, 15), dpi=200)

# ARIMA
temp['predicted'] = ARIMA_split1_inference
temp['group'] = pd.qcut(temp['predicted'], 10, labels=range(10))
grouped_predict = [output['predicted'].values for i, output in temp.
 ↪groupby('group')]

ax = plt.subplot(311)
box_plot(grouped_predict, ax, 'ARIMA')

# LASSO
temp['predicted'] = LASSO_split1_inference
temp['group'] = pd.qcut(temp['predicted'], 10, labels=range(10))
grouped_predict = [output['predicted'].values for i, output in temp.
 ↪groupby('group')]

ax = plt.subplot(312)
box_plot(grouped_predict, ax, 'LASSO')

# ARIMA
temp['predicted'] = XGB_split1_inference
temp['group'] = pd.qcut(temp['predicted'], 10, labels=range(10))
grouped_predict = [output['predicted'].values for i, output in temp.
 ↪groupby('group')]

ax = plt.subplot(313)
box_plot(grouped_predict, ax, 'XGBoost')

plt.show()
```







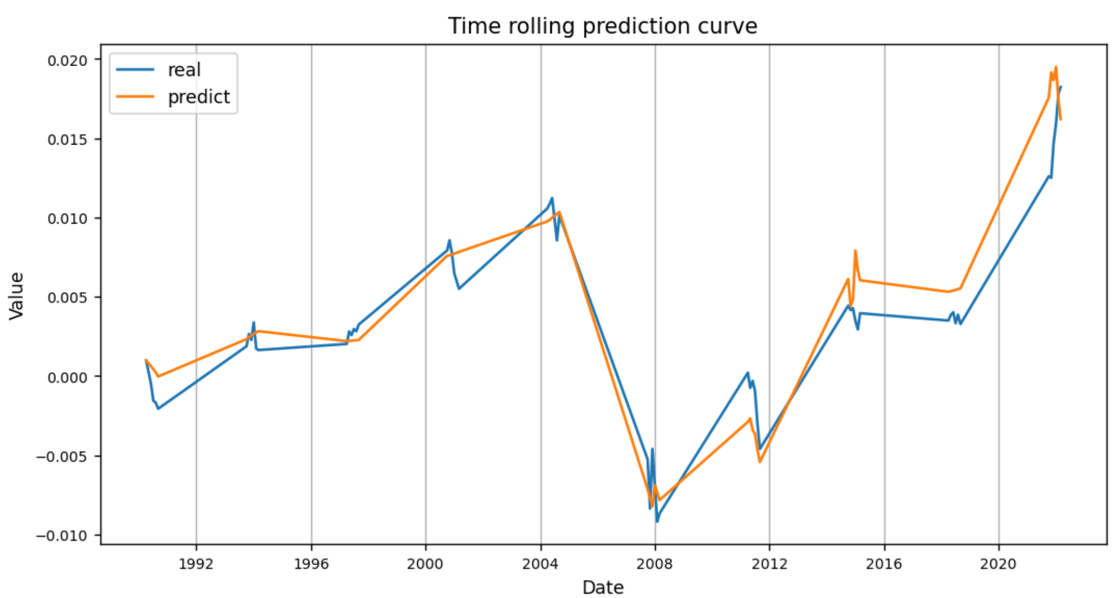
From the graph above, - ARIMA gives almost linear monotonicity, which is not realistic in most cases. While LASSO and XGBoost gives quadratic monotonicity, which is more common in quantile prediction since points with larger points can be predicted with higher confidence. - ARIMA and XGBoost perfrom worse in the first quantile, which is the data points with the most decrease. This might come from the Covid-19 crisis, creating unexpected shrink which can hardly be captured by our model and feature. - Instead, LASSO performs better on the first quantiles than the last ones. Consider information ratio as

$$\frac{\text{Mean}}{\text{Variance}}$$

The increase in the variance is nearly proportional to the increase in the mean value.

time-rolling scheme evaluation

[ ]:
```
# learning curve
plt.figure(figsize=(10,5), dpi=125)

plt.plot(LASSO_split2_inference['real'], label='real')
plt.plot(LASSO_split2_inference['predict'], label='predict')

plt.legend(fontsize=10)
plt.grid(axis='x')
plt.xlabel("Date", fontsize=10)
plt.ylabel("Value", fontsize=10)
plt.tick_params(labelsize=8)
plt.title('Time rolling prediction curve', fontsize=12)

plt.show()
```



We use LASSO only for time-rolling scheme, due to dataset limit. - Note that it is not rational to compare general metrics across different train-test split schemes. Since the time-gap in the rolling scheme will create over-estimated R^2 and pearson/spearman correlation.

[ ]:
```
# RMSE
RMSE = np.sqrt(
    mean_squared_error(y_true=LASSO_split2_inference['real'],
                       y_pred=LASSO_split2_inference['predict']))

# Out-of-sample R^2
R2 = r2_score(y_true=LASSO_split2_inference['real'],
              y_pred=LASSO_split2_inference['predict'])

# Pearson correlation
pearson = pearsonr(LASSO_split2_inference['real'],
                   LASSO_split2_inference['predict'])[0]

# spearman correlation
spearman = spearmanr(LASSO_split2_inference['real'],
                     LASSO_split2_inference['predict'])[0]
```

[ ]:
```
general_eval = pd.DataFrame()
general_eval['RMSE'] = [RMSE]
general_eval['R2'] = [R2]
general_eval['Pearson'] = [pearson]
general_eval['Spearman'] = [spearman]
general_eval.index = ['LASSO:rolling']
for col in general_eval.columns:
    general_eval[col] = general_eval[col].apply(lambda x: '%.3g'%x)
general_eval
```

[ ]:

	RMSE	R2	Pearson	Spearman
LASSO:rolling	0.00206	0.886	0.96	0.961

Significant features given by time-rolling LASSO

- Since there is high multicolinearity in the dataset, we can't simply test the stability for a given feature.
- For example, we can not use permutation test over the null hypothesis that $\hat{\beta}_0$ is a constant over time.
- It is more rational to test the coefficient of a cluster of features with similar financial meanings.

[ ]:
```
mean_coefs = np.mean(np.array(coefs), axis=0)
mean_coefs
```

[ ]:
```
array([-1.61809974e-06,  2.14541735e-08,  0.00000000e+00,  0.00000000e+00,
       -2.94930623e-08,  0.00000000e+00,  5.42369166e-07, -1.32659171e-06,
        3.91006614e-07,  0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
        0.00000000e+00,  0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
        0.00000000e+00,  0.00000000e+00,  0.00000000e+00,  0.00000000e+00,
```

```
0.00000000e+00,  0.00000000e+00])
```

[ ]:
```
np.array(features)[np.where(mean_coefs != 0)]
```

[ ]:
```
array(['GDP', 'M2', 'Psave', 'Popu', 'Salary', 'Nasdaq'], dtype='<U15')
```

The most significant & stable feature

[ ]:
```
features[7]
```

[ ]:
```
'Salary'
```

- Still, we can not say there is cause-effect relationship between the salary and the home price index.
- All we know is Salary might bring strong prediction power and such power is stable and significant over time.

---

7 7. Conclusion & Discussions

7.0.1 Conclusions

In this project, we

- Use public Fredapi to collect the S&P/Case-Shiller U.S. National Home Price Index and other indexes that might be relevant to this HPIX such as GDP, M2, Personal saving, Salary level…
- Do data cleaning & feature engineering and transformation for prediction and to satisfy the model hypothesis.
- Design two train-test split method and build 3 models for the regression task:
  - Baseline: ARIMA
  - Improved: LASSO / XGBoost
- Train / predict over the schemes and implement several evaluation metric, including general ones like RMSE, R^2 and correlation. And other metrics as stratification monotonicity and parameter stability.
- After comparison between the model performance, we tell that
  - Traditional time-series ARIMA can hardly capture the movement of the HPIX, where the predicted variance decay quickly over time and converge to the mean.
  - LASSO and XGBoost gives similar performance under general split method and metrics, reaching about 70% pearson and 50% spearman out-of-sample data.
  - Although there is over-estimated bias, the time-rolling scheme can improve the prediction power of LASSO. Possible reasons comes from the transition of in-sample & out-of-sample data distribution. **Update our model in time is likely to help to capture the change in market style.**

7.0.2 Problems & Improvements

- The pandamic in 2020 and crisis in 2008 can hardly be captured with our features.
- Such crisis can be seen as outliers and might be mitigated by least absolute error regression (L_1 error).
- Due to the limit of dataset, we can hardly build some equivalent testing set to compare the performance between the 2 split methods.
- Also due to the limit of dataset, the complexity of model is constrained, NNs are not recommended.