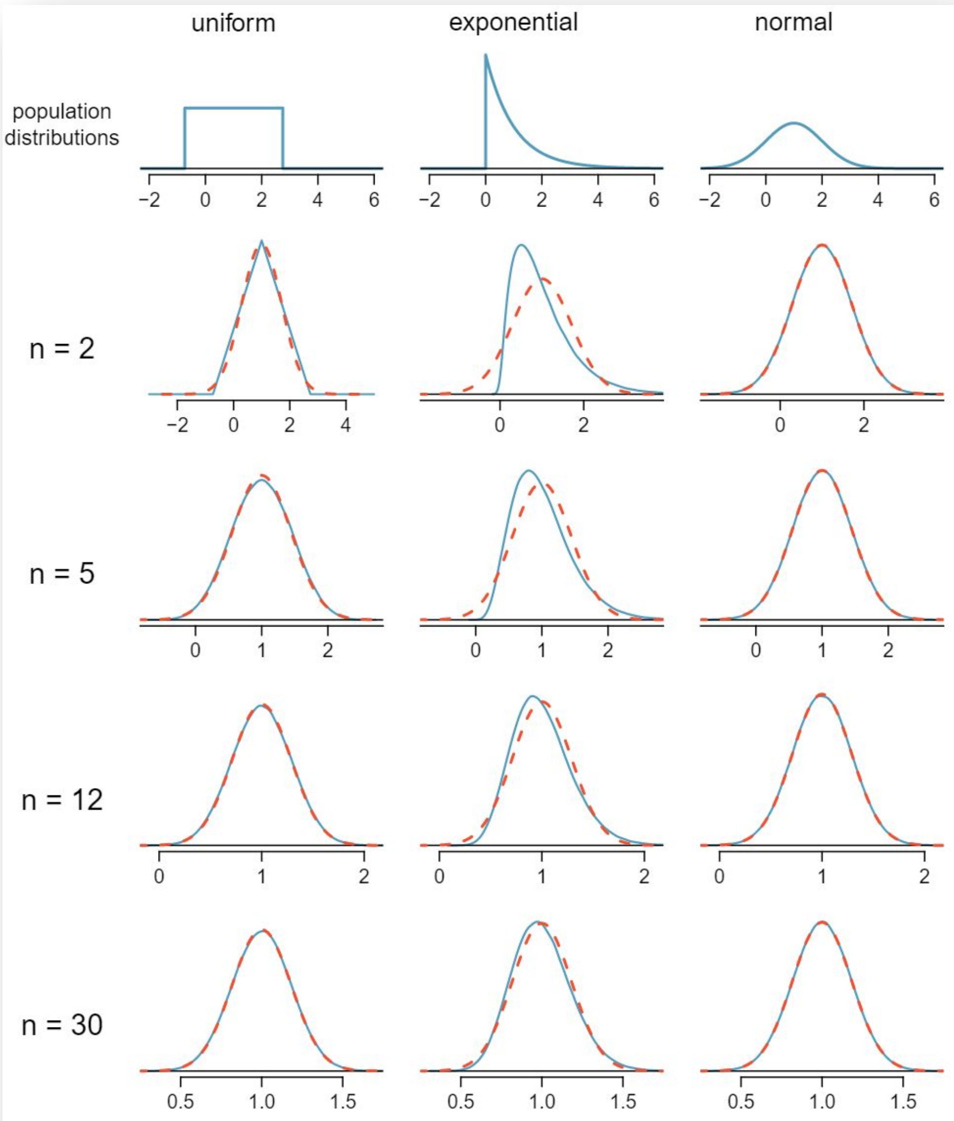
t TEST

CENTRAL LIMIT THEOREM



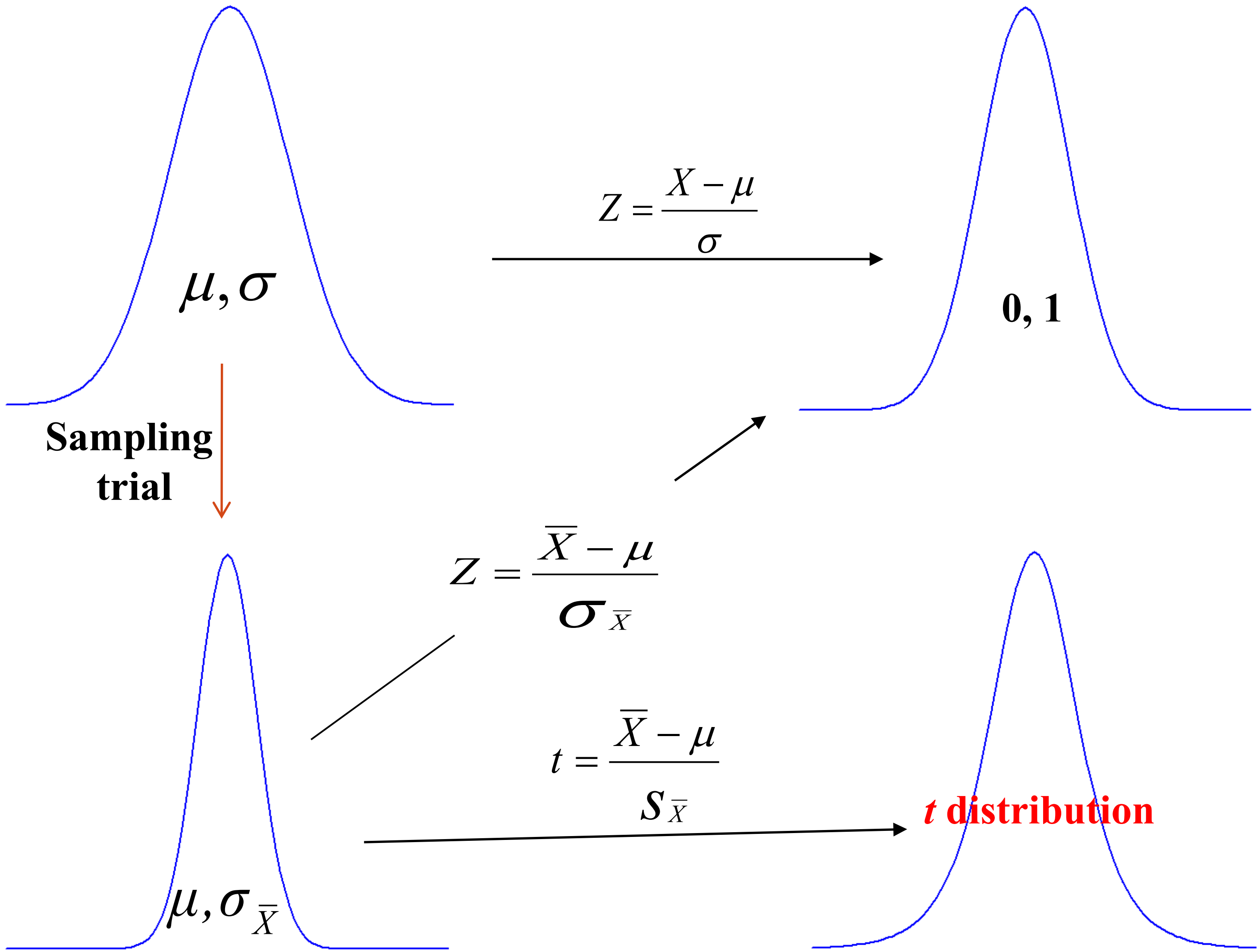
Sample mean will always be normally distributed, as long as the sample size is large enough.

If you take samples from a normal distribution, the samples' means will be normally distributed.

$$x \sim N(\mu, \sigma^2), \text{ then } \bar{X} \sim N(\mu, \sigma_{\bar{X}}^2)$$

If you take sufficiently samples (n>30) from a population (mean=μ, SD=σ), the samples' means will be normally distributed, even if the population isn't normally distributed.

Student *t* distribution



PROPERTIES OF *T* DISTRIBUTION CURVE

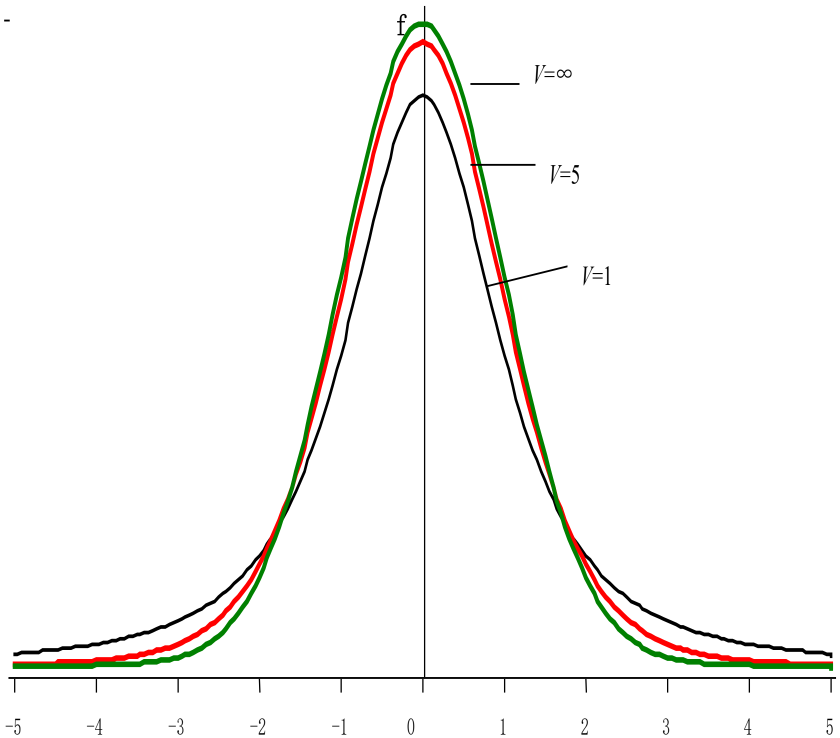


Figure 4-3 t-distribution graphs in different df

- **It is symmetrical by y axis and has one apex;**
- **Only one parameter, ν(degree of freedom, ν=n-1) determines the shape of t-distribution.**
- **The total area under the t-distribution equals to 1. The area under the curve between any of interval (t1 and t2) can be found by calculus**

Who introduced *t* test?

t* test, also known as Student's *t* test, was invented by *W. S. Gossett [1876-1937] to handle small samples for quality control in brewing.

What is t test used for?

A common hypothesis test for inferring whether two population means are equal, *especially in the cases of small sample sizes.*

Contents

1. ONE SAMPLE *t* TEST

1. One sample *t* test

One sample *t*-test is suitable for comparing the sample mean with the known population mean.

The testing condition includes the assumption of

- **Normal distribution, especially when n is small**

Example:

A doctor measured the hemoglobin concentrations in 36 workers engaged in lead operations, with

$$\overline{X} = 130.83 g / L, \quad S = 25.74 g / L$$

Is the mean of hemoglobin concentrations of lead workers smaller than that of the healthy individuals (μ_0=140g/L)?

assume the underlying distribution is normal

QUESTION

μ_0=140g/L

?

$\neq$

μ=?

n=36

$\bar{X}$ =130.83g/l

S=25.74g/l

healthy

workers

We find that hemoglobin concentrations of lead workers obey normal distribution:

(1) Establish the hypothesis and determine the significance level

(a) H_0:

μ=140, the mean of hemoglobin concentrations of lead workers is equal to that of the healthy individuals (μ_0=140g/L).

(b) H_1:

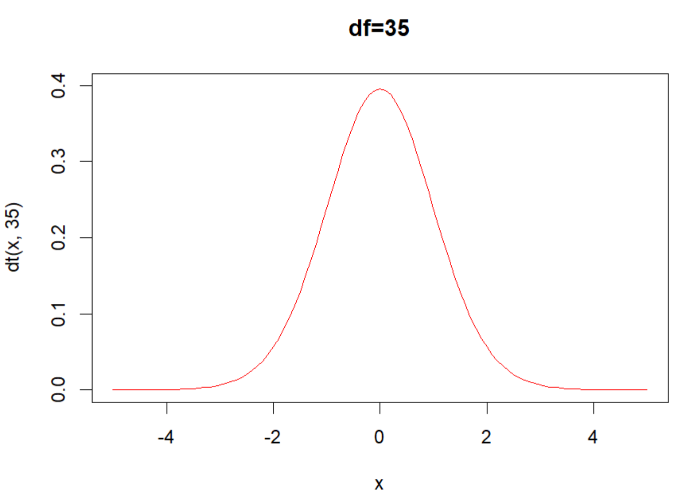
μ<140, the mean of hemoglobin concentrations of lead workers is smaller than that of the healthy individuals (μ_0=140g/L).

α =0.05

UNDER HO

The test statistic is $t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}}$ $\qquad \nu = n - 1$

Where S is the sample standard deviation, *n* is the sample size, *v* is the degree of freedom.



(2) Calculate the *t* test statistic

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = -2.138 \quad \nu = n\text{-}1 = 36\text{-}1 = 35$$

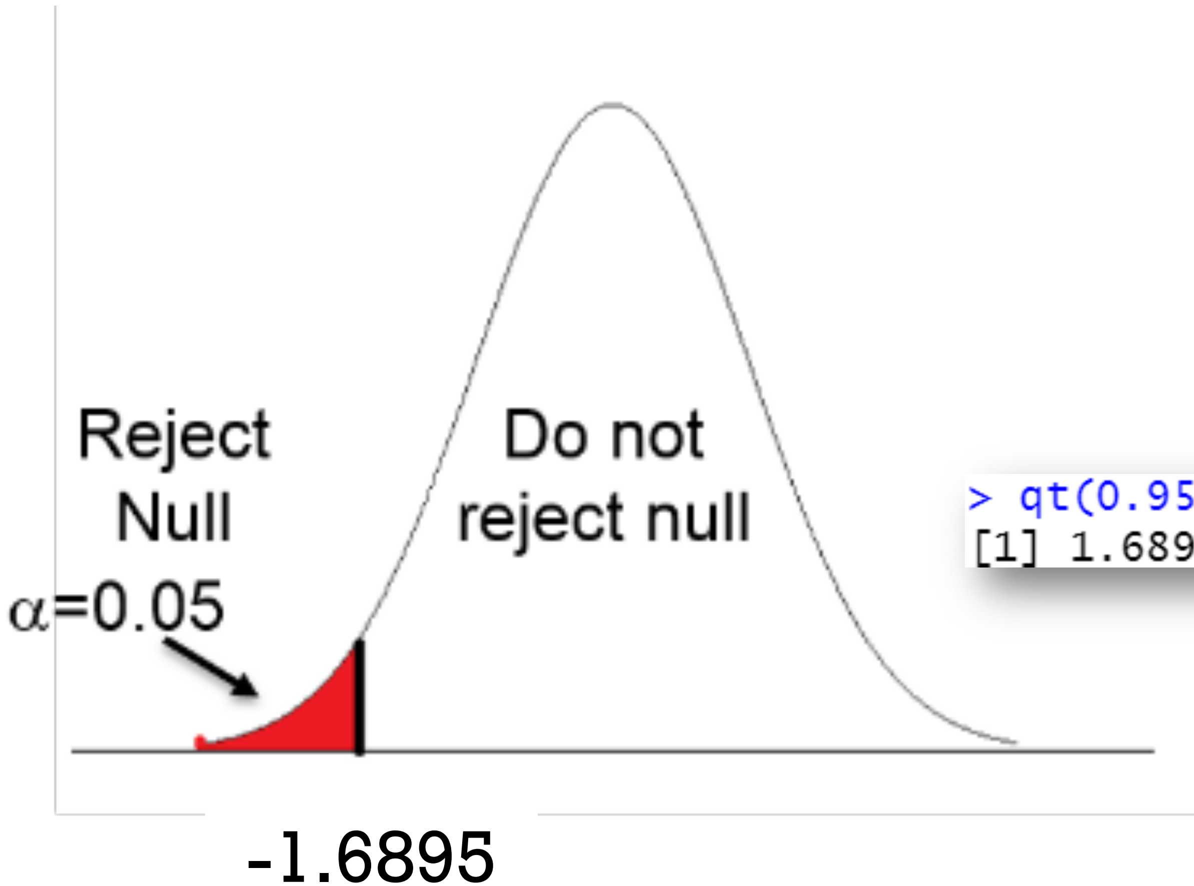
(3) Determine the *P* value and make a conclusion

Because $t_{0.05,35}$ **=1.690,** So **p<0.05**

Conclusion: **Reject *H_o* and accept *H_1*, the mean of hemoglobin concentrations of these workers is smaller than that of the healthy individuals (140g/L).**

t-test table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										




```
> qt(0.95,df=35,lower.tail=T)
[1] 1.689572
```

```
> pt(-2.138,df=35,lower.tail=T)
[1] 0.01978844
```

P=0.0198

2. PAIRED t TEST

2. Paired t test

Paired *t*-test is suitable for comparing two sample means of a paired design.

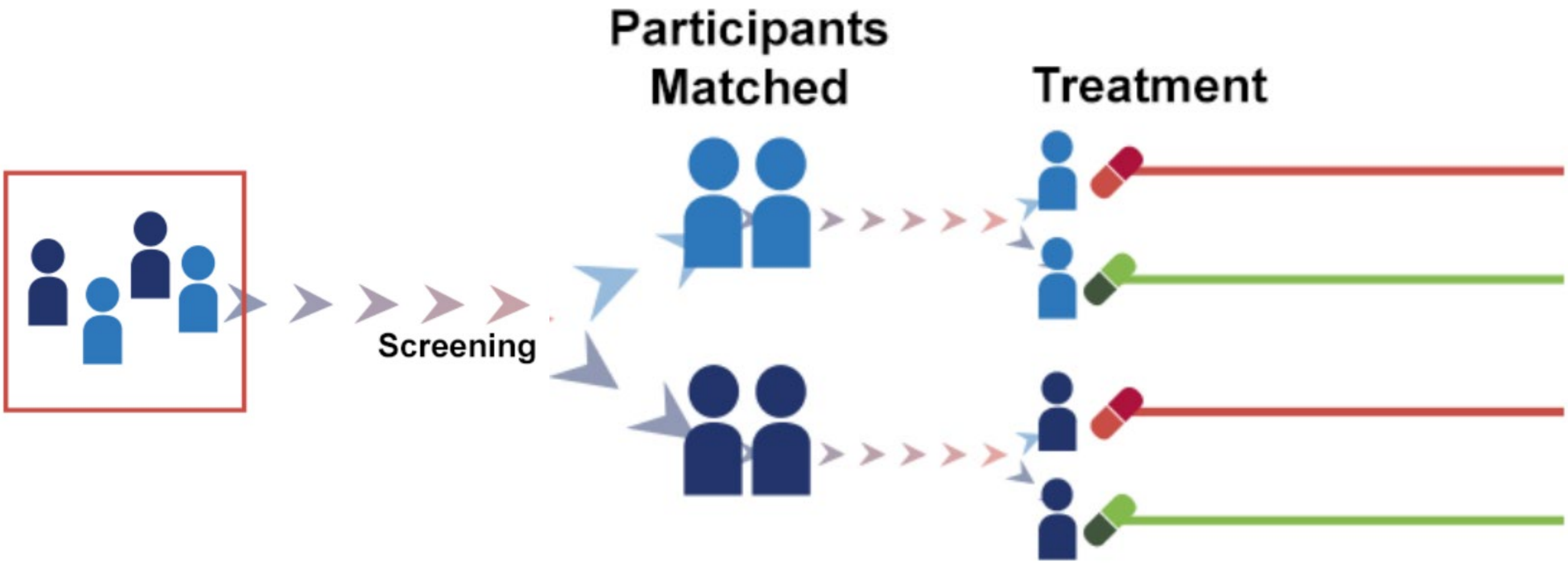
Can you give any examples of a paired design?

Self-match

Measuring the same object at two different time points

Different parts of the same object

Measured the same object with two different methods



Testing condition:

The testing condition of paired *t*-test includes the assumption of normal distribution of the differences.

H0: population mean of difference =0

The test statistic of a paired *t* test is

$$t = \frac{\bar{d} - 0}{S_d / \sqrt{n}} \qquad \nu = n\text{-}1$$

Where $\bar{d}$ is the sample mean of paired measurement differences, S_d is the standard deviation of the differences, *n* is the number of the pairs and *v* is the degree of freedom.

$$s_d = \sqrt{\left[\sum_{i=1}^{n} d_i^2 - \left(\sum_{i=1}^{n} d_i\right)^2 \Big/ n\right] \Big/ (n-1)}$$

n = number of matched pairs

Example:

same object

We collected the systolic blood pressure levels in 10 women

different time point

before and after using oral contraceptive (Table 1).

Question??

Is there any difference of systolic blood pressure levels in women before and after using oral contraceptive ?

Table 1 Systolic blood pressure levels in 10 women before and after using oral contraceptive

i	Before	After	d_i
1	115	128	13
2	112	115	3
3	107	106	-1
4	119	128	9
5	115	122	7
6	138	145	7
7	126	132	6
8	105	109	4
9	104	102	-2
10	115	117	2

We find that differences d_i obey normal distribution:

In practice, you should conduct normality test (especially for n<30)

(1) Establish the hypothesis and determine the significance level

(a) H_0:

$\mu_d = 0$, there is no difference of systolic blood pressure levels in women before and after using oral contraceptive.

(b) H_1:

$\mu_d \neq 0$, the systolic blood pressure levels in women before using oral contraceptive are different from those after using oral contraceptive .

$\alpha = 0.05$

(2) Calculate the *t* test statistic

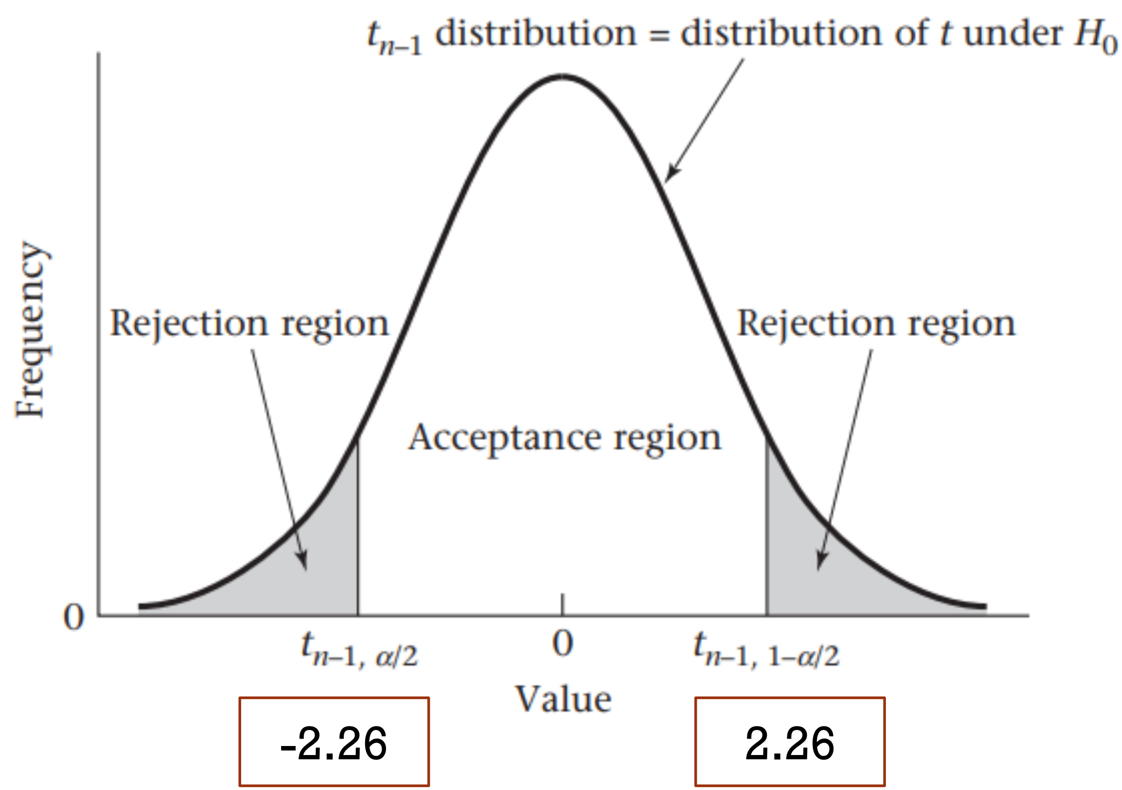
$$\bar{d} = 4.8,\ S = 4.566$$

$$t = \frac{\bar{d}}{S/\sqrt{n}} = 3.32 \qquad \nu = n\text{-}1 = 10\text{-}1 = 9$$

(3) Determine the *P* value and make a conclusion

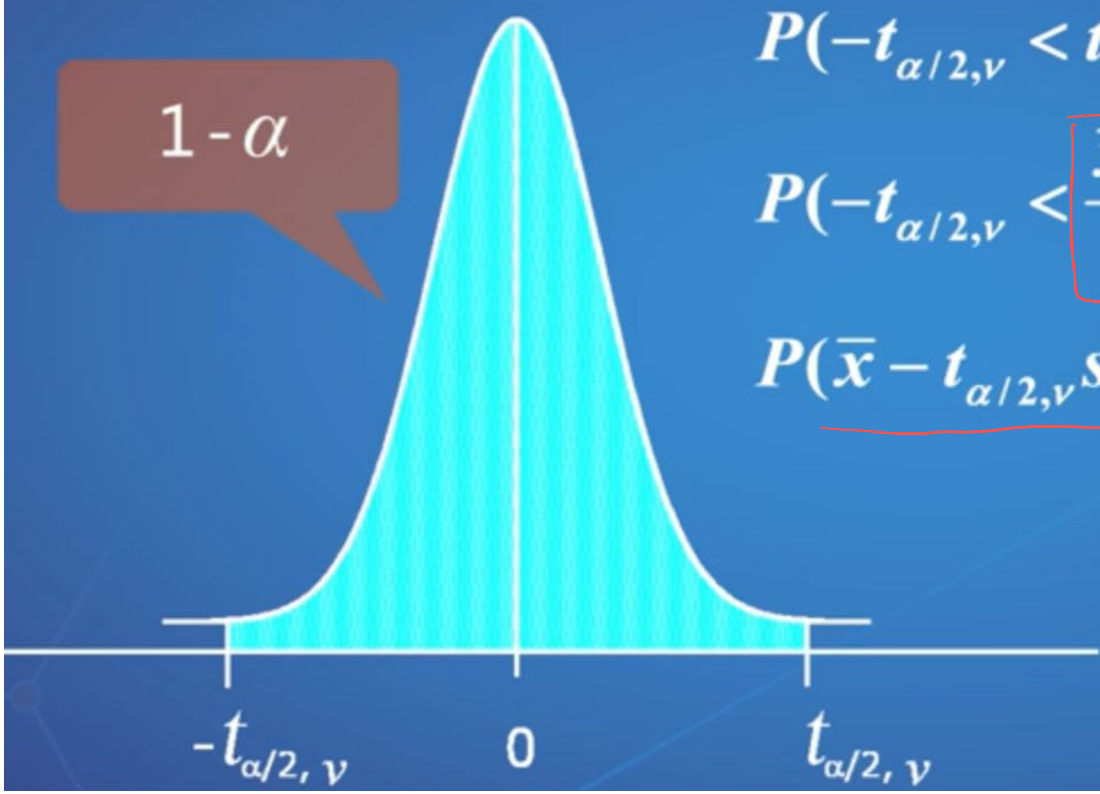
Because $t_{0.05/2,\ 9}$ **=2.262,** So **p<0.05**

Conclusion: **Reject H_o and accept H_1, the systolic blood pressure levels in women before using oral contraceptive are different from those after using oral contraceptive .**




```
> qt(0.975,df=9,lower.tail=T)
[1] 2.262157
```

```
> pt(-3.32,df=9,lower.tail=T)
[1] 0.004470159
```



$$P(|t| > t_{\alpha/2,\nu}) = \alpha$$

$$P(-t_{\alpha/2,\nu} < t < t_{\alpha/2,\nu}) = 1 - \alpha$$

$$P\left(-t_{\alpha/2,\nu} < \boxed{\frac{\bar{x} - \mu}{s_{\bar{x}}}} < t_{\alpha/2,\nu}\right) = 1 - \alpha$$

$$P(\underline{\bar{x} - t_{\alpha/2,\nu} s_{\bar{x}}} < \mu < \underline{\bar{x} + t_{\alpha/2,\nu} s_{\bar{x}}}) = 1 - \alpha$$

$$\bar{x} - t_{0.05/2,\nu} \cdot s_{\bar{x}} < \mu < \bar{x} + t_{0.05/2,\nu} \cdot s_{\bar{x}}$$

Interval Estimation For The Comparison Of Means From Two Paired Samples

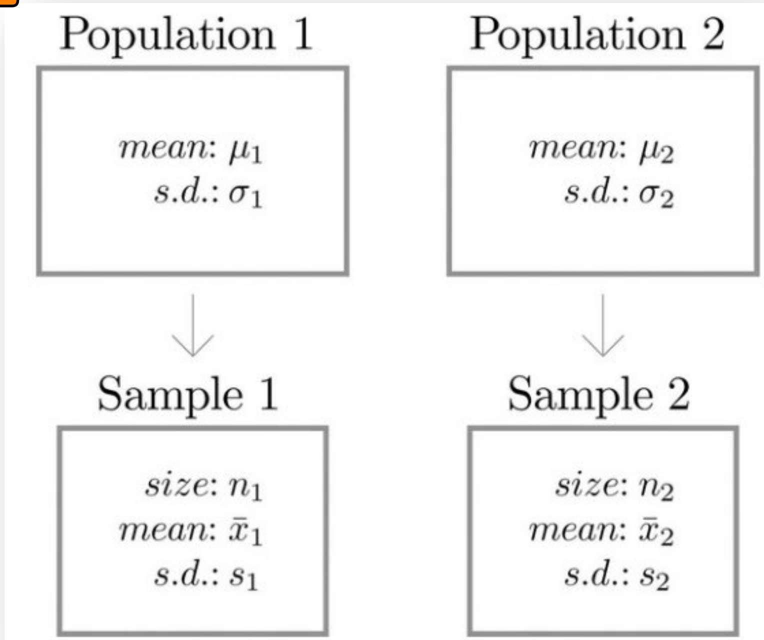
$$\bar{d} - t_{0.05/2,\nu} \cdot s_{\bar{d}} < \mu < \bar{d} + t_{0.05/2,\nu} \cdot s_{\bar{d}}$$

$$\bar{d} - t_{0.05/2,\nu} \cdot {}^{S}\!/_{\sqrt{n}} < \mu < \bar{d} + t_{0.05/2,\nu} \cdot {}^{S}\!/_{\sqrt{n}}$$

(1.357, 8.243)

Doesn’t contain 0 (H0)

3. *t* TEST FOR 2 INDEPENDENT SAMPLES



Independent Sampling from Two Populations

Additivity principle

If X and Y are independent,

$X \sim N(\mu_1, \sigma_1^2),\ Y \sim N(\mu_2, \sigma_2^2),$

then X-Y $\sim (\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$

$$\bar{X}_1 \sim N(\mu_1, \sigma_1^2 / n_1),\ \bar{X}_2 \sim N(\mu_2, \sigma_2^2 / n_2),$$

$$\text{then } \bar{X}_1\text{-}\bar{X}_2 \sim (\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$$

$$z = \frac{(\bar{X}_1\text{-}\bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

- **Overview:**

If X~N(0,1), Y~X^2_n, and X and Y are independent

$$T = \frac{X}{\sqrt{Y/n}}$$

Then T follows a t distribution with df=n, we write

$$T \sim t_n$$

If $Y_1 \sim x_m^2, Y_2 \sim x_n^2$, and Y_1 and Y_2 are independent

Then $Y_1 + Y_2 \sim x_{m+n}^2$

$$x_n^2 = \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2 \approx \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{\sigma^2}$$

$$x^2 \approx \frac{(n-1) * S^2}{\sigma^2} \sim x_{n-1}^2$$

$$S^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{n-1}$$

$$\sum_{i=1}^{n} (X_i - \bar{X})^2 = (n-1) * S^2$$

$$z=\frac{(\bar{X}_1\text{-}\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

$$\frac{(n_1-1)S_1^2}{\sigma_1^2}\sim x^2_{n1-1},\ \text{and}\ \frac{(n_2-1)S_2^2}{\sigma_2^2}\sim x^2_{n2-1},\text{then}\ \frac{(n_1-1)S_1^2}{\sigma_1^2}+\frac{(n_2-1)S_2^2}{\sigma_2^2}\sim x^2_{n1+n2-2}$$

Construct a t distribution with df=n1+n2-2???

$$\frac{\dfrac{(\bar{X}_1\text{-}\bar{X}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}}{\sqrt{\left(\frac{(n_1-1)S_1^2}{\sigma_1^2}+\frac{(n_2-1)S_2^2}{\sigma_2^2}\right)\Big/ n_1+n_2-2}}\sim t(n_1+n_2-2)$$

σ1 and σ2 are not known in practice, the above equation can only be simplified if σ1= σ2

EQUAL VARIANCE?

Suppose $\sigma_1^2 = \sigma_2^2 = \sigma^2$

$$\frac{\dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}}{\sqrt{\left(\dfrac{(n_1-1)S_1^2}{\sigma_1^2} + \dfrac{(n_2-1)S_2^2}{\sigma_2^2}\right) \Big/ (n_1+n_2-2)}}$$

$$= \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right) \boxed{\dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}}} \sim t(n_1+n_2-2)$$

pooled estimate of the variance

EQUAL VARIANCE

If $\sigma_1^2 = \sigma_2^2$

$$\bar{X}_1 \text{-} \bar{X}_2 \,\square\, (\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}) \,\square\, \left[\mu_1 - \mu_2, \sigma^2 (\frac{1}{n_1} + \frac{1}{n_2}) \right])$$

Under H_0, we know that $\mu_1 = \mu_2$.

$$\bar{X}_1 - \bar{X}_2 \sim N\left[0, \sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right]$$

$$\frac{\bar{X}_1 - \bar{X}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0,1)$$

σ in general is unknown and must be estimated from the data

How can σ be best estimated in this situation?

$$s_{\bar{X}_1-\bar{X}_2} = \sqrt{s_C^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

S_C^2 Is the pooled estimate of the variance

$(n_1-1)/(n_1+n_2-2)$ W_1

$(n_2-1)/(n_1+n_2-2)$ W_2

$$s_C^2 = W_1 \times s_1^2 + W_2 \times s_2^2$$

$$\Downarrow$$

$$s_C^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$$

3.1 *t* test for two independent samples (equal variance)

t test for two independent samples is suitable for comparing two sample means of a completely randomized design, and its purpose is to infer whether two population means are the same.

The testing conditions include the assumptions of

✓normal distribution (especially for small samples)

✓equal variances

The test statistic for independent two-sample *t* test is:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - 0}{S_{\bar{X}_1 - \bar{X}_2}} = \frac{\bar{X}_1 - \bar{X}_2}{S_{\bar{X}_1 - \bar{X}_2}} \qquad \nu = n_1 + n_2 - 2$$

$$Where \quad S_{\bar{X}_1 - \bar{X}_2} = \sqrt{S_c^{\ 2}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$When \quad S_c^{\ 2} = \frac{(n_1 - 1)S_1^{\ 2} + (n_2 - 1)S_2^{\ 2}}{n_1 + n_2 - 2}$$

where n_1 and n_2 are the sample sizes of the two groups respectively, and S_c^2 is the pooled variance of the two groups.

Example:

To compare the effects of two drugs on the blood sugar reduction, 40 volunteer patients were randomly assigned to two groups receiving two drugs, respectively. Data of blood sugar reduction were collected as follows. Are the two drugs different in terms of lowering blood sugar?

Drug 1:
-0.70 -5.60 2.00 2.80 0.70 3.50 4.00 5.80 7.10 -0.50 2.50 -1.60 1.70 3.00 0.40 4.50 4.60 2.50 6.00 -1.40

Drug 2:
3.70 6.50 5.00 5.20 0.80 0.20 0.60 3.40 6.60 -1.10 6.00 3.80 2.00 1.60 2.00 2.20 1.20 3.10 1.70 -2.00

Suppose our data meet the assumptions of normal distribution and equal variances.

(1) Establish the hypothesis and determine the significance level

(a) H_0:

$\mu_1=\mu_2$, the blood sugar reduction by drug 1 is equal to that by drug 2.

(b) H_1:

$\mu_1\neq\mu_2$, the blood sugar reduction by drug 1 is different from that by drug 2 .

$\alpha=0.05$

(2) Calculate the *t* test statistic

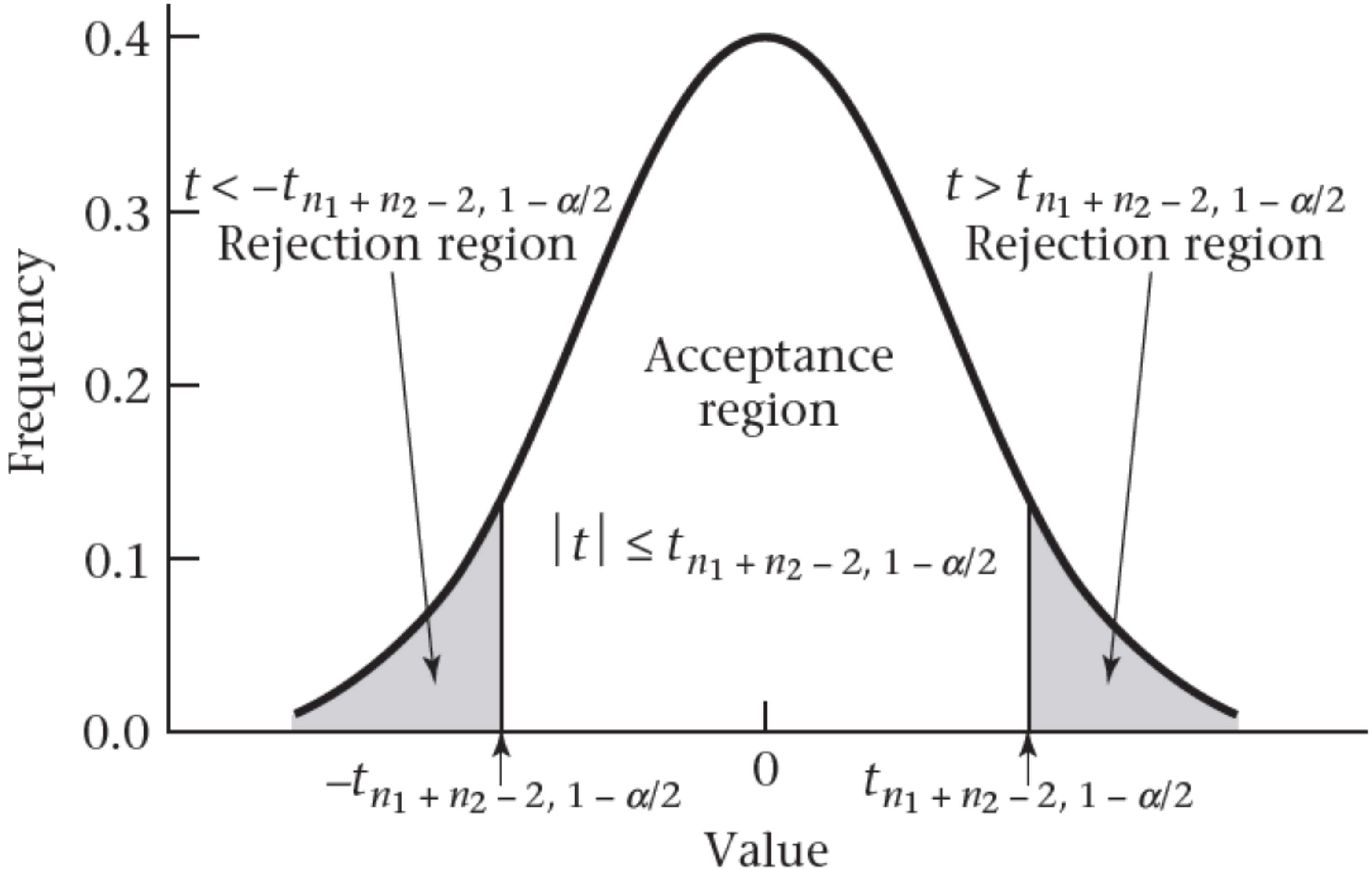
$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_{\bar{X}_1 - \bar{X}_2}} = -0.642 \qquad \nu = n_1 + n_2 - 2 = 38$$

(3) Determine the *P* value and make a conclusion

Because $t_{0.05/2,38}$ **=2.024,** So **p>0.05**

Conclusion: Can not **reject H_o. We cannot conclude that the blood sugar reduction by drug 1 is different from that by drug 2.**

```
> pt(-0.642,df=38,lower.tail=T)
[1] 0.2623637
```



If $t \le 0$, $p = 2 \times$ (area to the left of t under a $t_{n_1+n_2-2}$ distribution).

If $t > 0$, $p = 2 \times$ (area to the right of t under a $t_{n_1+n_2-2}$ distribution).

Two-Sample *t* Test for Independent Samples with Equal Variances

Suppose we wish to test the hypothesis H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 \neq \mu_2$ with a significance level of α for two normally distributed populations, where σ^2 is assumed to be the same for each population.

Compute the test statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $s = \sqrt{\left[(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2\right]/(n_1 + n_2 - 2)}$

If $t > t_{n_1 + n_2 - 2, 1-\alpha/2}$ or $t < -t_{n_1 + n_2 - 2, 1-\alpha/2}$

then H_0 is rejected.

If $-t_{n_1 + n_2 - 2, 1-\alpha/2} \le t \le t_{n_1 + n_2 - 2, 1-\alpha/2}$

then H_0 is accepted.

INTERVAL ESTIMATION (EQUAL VARIANCE CASE)

$$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim N(0,1)$$

$$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}$$

$$\left(\bar{X}_1 - \bar{X}_2 - t_{n_1+n_2-2,1-\alpha/2}\, s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}},\ \bar{X}_1 - \bar{X}_2 + t_{n_1+n_2-2,1-\alpha/2}\, s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

$$\left(\bar{x}_1 - \bar{x}_2 - t_{n_1+n_2-2,1-\alpha/2}\, s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}},\ \bar{x}_1 - \bar{x}_2 + t_{n_1+n_2-2,1-\alpha/2}\, s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\right)$$

3.2 If the variances of the two populations are not equal,

Behrens-Fisher problem

$$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

Under H0, μ1= μ2

$$\bar{X}_1 - \bar{X}_2 \sim N\left(0, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

If σ_1^2 and σ_2^2 were known,

$$z = (\bar{x}_1 - \bar{x}_2) \Big/ \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$t = (\bar{x}_1 - \bar{x}_2) \Big/ \sqrt{s_1^2/n_1 + s_2^2/n_2}$$

The exact distribution of *t* under H_0 is difficult to derive.

approximate *t* test is recommended.

- **Welch's *t* test**

$$t' = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^{\ 2}}{n_1} + \frac{S_2^{\ 2}}{n_2}}}$$

$$v' = \frac{(S_1^2/n_1 + S_2^2/n_2)^2}{\frac{(S_1^2/n_1)^2}{n_1+1} + \frac{(S_2^2/n_2)^2}{n_2+1}} - 2$$

- **Scatterthwaite's test**

$$t' = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_1^{\ 2}}{n_1} + \frac{S_2^{\ 2}}{n_2}}}$$

Compute the approximate degrees of freedom

$$v' = \frac{(S_1^2/n_1 + S_2^2/n_2)^2}{\frac{(S_1^2/n_1)^2}{n_1-1} + \frac{(S_2^2/n_2)^2}{n_2-1}} - 2$$

USE TWO-SAMPLE t TEST WITH EQUAL VARIANCES

- inference concerning mean?
- Compare between populations?

objective

Study design

- Two sample problem? Independent?

- Normal?
- Central-limit theorem?
- Equal variance?

Data properties

t-test can also be used for large sample size