

Project #1

Part I:

Consider the *FitzHugh-Nagumo oscillator model* which is a two-dimensional simplification of the *Hodgkin-Huxley model* of spike generation in neurons.

$$\begin{aligned}\dot{x} &= \alpha \left[y + x - \frac{x^3}{3} + z \right] \\ \dot{y} &= -\frac{1}{\alpha} \left[\omega^2 x - a + by \right]\end{aligned}$$

Where x is the excitability variable, y is the refractoriness variable, and z is the stimulus intensity.

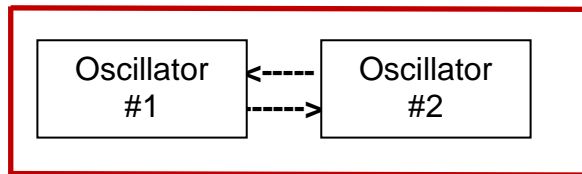
α, ω^2, a, b are model parameters

Choose $a=0.7$, $b=0.8$ and $\omega^2=1$

(a) Investigate the intrinsic frequency of the oscillator output by plotting it as a function of “stimulus $z=\text{constant}=k$ ” for the special case $\alpha=3$.

Part II:

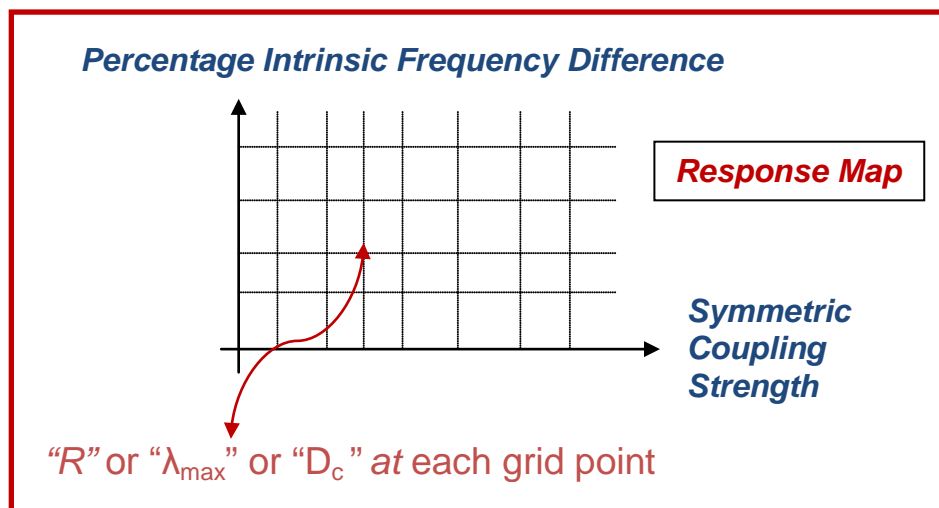
Consider two bidirectionally coupled *FitzHugh-Nagumo* oscillators.



(b) Investigate the phase coherence index “R” of the coupled oscillators as a function of their percentage intrinsic frequency difference and symmetric coupling strength.

Hints:

- For bidirectional coupling use $z_1 = k_1 + c x_2$ and $z_2 = k_2 + c x_1$ where “ k_1 ” and “ k_2 ” are associated with intrinsic frequency of oscillators #1&2 respectively, and “ c ” is the symmetric coupling strength.
- The intrinsic frequency is estimated from Part I when $c=0$.
- Use **response maps** to indicate different regions of *system behaviour*, in your investigations of coupled oscillators in Parts II and III.



Part III:

(c) Investigate two **complexity measures** the maximum Lyapunov exponent “ λ_{\max} ” and the correlation dimension “ D_c ”, using the oscillator output time series $x_1(t)$ & $x_2(t)$, for each grid point in the depicted response map.

Hints:

- This will produce 6 response maps (3 for each oscillator).
- Use a Rosenstein's algorithm to compute “ λ_{\max} ” from short time series data.
- Use a Grassberger-Procaccia algorithm to compute “ D_c ”.