

Announcements

Midterm 2:

- 3/20, 8-10 PM in various rooms.
- Covers material through 3/16 (next Friday).
- Study using study guides.
 - THE KEY IS METACOGNITION: Reflect on your problem solving strategies and those of your fellow students.
 - Understanding a handful of solutions to old midterm problems is less helpful than you might think -- look at answers as late as possible.
- There is an alternate 61C midterm from 6 - 8 in 1 LeConte.

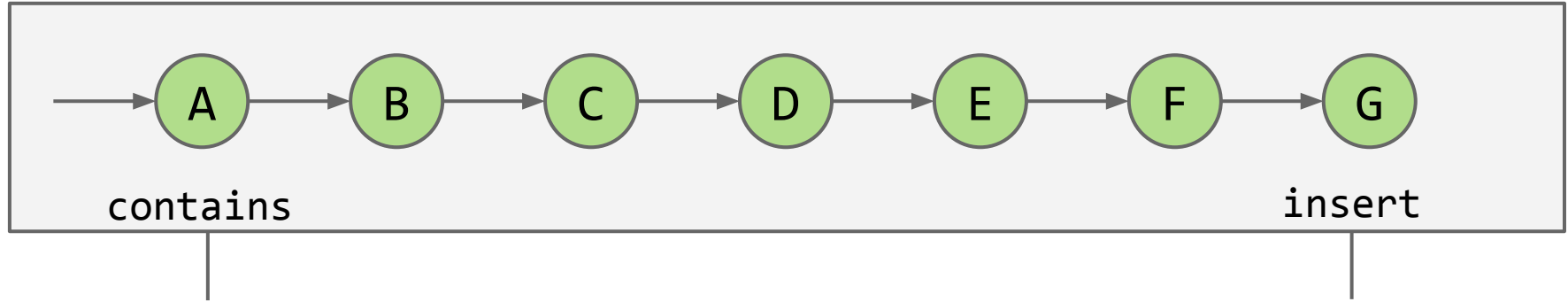
CS61B

Lecture 23: Hashing

- Set Implementations, DataIndexedIntegerSet
- Binary Representations, DataIndexedSet
- Handling Collisions
- Hash Functions



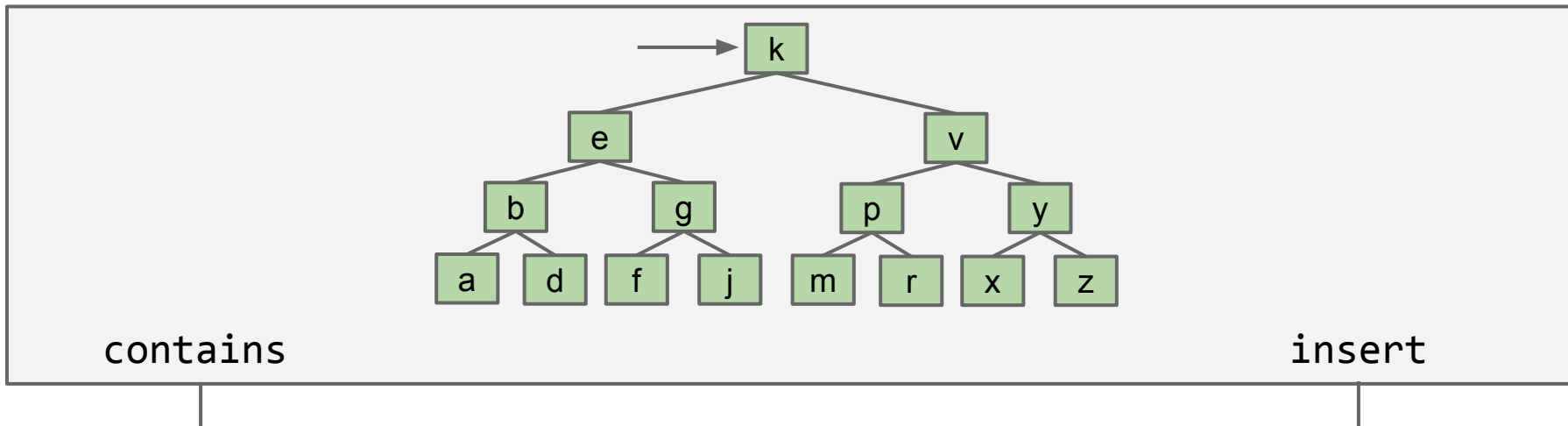
Techniques for Storing Data: Ordered Linked List



	contains(x)	insert(x)
Linked List	$\Theta(N)$	$\Theta(N)$

Worst case runtimes

Techniques for Storing Data: Bushy BST



Limitations:

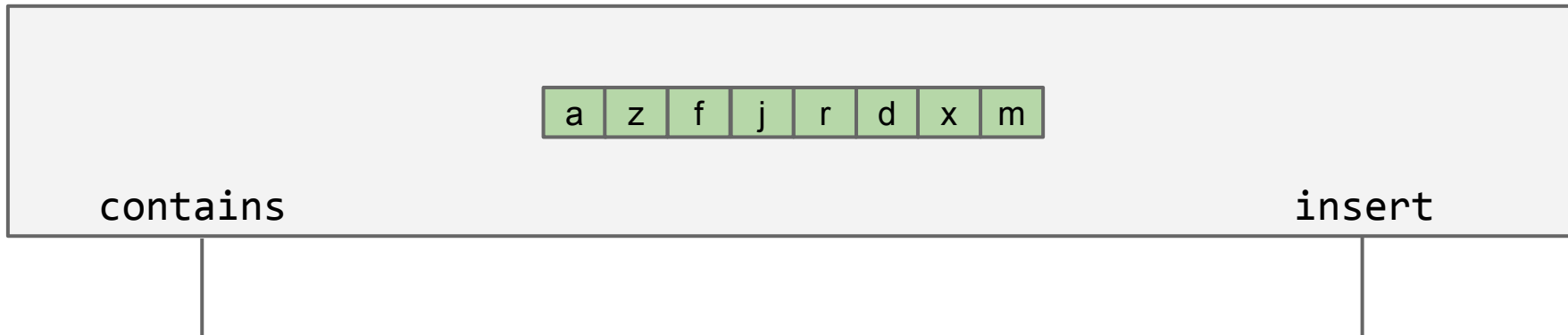
- Items must be comparable.
- Maintaining bushiness is non-trivial.
- $\Theta(\log N)$, can we do better?

	contains(x)	insert(x)
Linked List	$\Theta(N)$	$\Theta(N)$
Bushy BSTs	$\Theta(\log N)$	$\Theta(\log N)$

Worst case runtimes

Note: log is pretty good. One billion items yields tree height of only 30.

Techniques for Storing Data: Unordered Array



	contains(x)	insert(x)
Linked List	$\Theta(N)$	$\Theta(N)$
Bushy BSTs	$\Theta(\log N)$	$\Theta(\log N)$
Unordered Array	$\Theta(N)$	$\Theta(N)$

Worst case runtimes

Unordered arrays
are terrible (and so
are ordered ones).

But a third type of
array...

Using data as an Index

One extreme approach: All data is really just bits.

- Use data itself as an array index.
- Store true and false in the array.

Downsides of this approach (that we can maybe fix):

- Extremely wasteful of memory. To support checking presence of all positive integers, we need 2 billion booleans.
- Need some way to generalize beyond integers.

```
DataIndexedIntegerSet diis = new DataIndexedIntegerSet();  
diis.insert(0);  
diis.insert(5);  
diis.insert(10);  
diis.insert(11);
```

T	0
F	1
F	2
F	3
F	4
T	5
F	6
F	7
F	8
F	9
T	10
T	11
F	12
F	13
F	14
F	15

...

Set containing 0, 5, 10, 11

DataIndexedIntegerSet Implementation

```
public class DataIndexedIntegerSet {  
    boolean[] present;  
  
    public DataIndexedIntegerSet() {  
        present = new boolean[16];  
    }  
  
    public insert(int i) {  
        present[i] = true;  
    }  
  
    public contains(int i) {  
        return present[i];  
    }  
}
```

T	0
F	1
F	2
F	3
F	4
T	5
F	6
F	7
F	8
F	9
T	10
T	11
F	12
F	13
F	14
F	15

...

Set containing 0, 5, 10, 11

DataIndexedIntegerSet Implementation

```
public class DataIndexedIntegerSet {
    boolean[] present;

    public DataIndexedIntegerSet() {
        present = new boolean[100000];
    }

    public insert(int i) {
        present[i] = true;
    }

    public contains(int i) {
        return present[i];
    }
}
```

	contains(x)	insert(x)
Linked List	$\Theta(N)$	$\Theta(N)$
Bushy BSTs	$\Theta(\log N)$	$\Theta(\log N)$
Unordered Array	$\Theta(N)$	$\Theta(N)$
DataIndexedArray	$\Theta(1)$	$\Theta(1)$

Worst case runtimes

Binary Representations

DataIndexedSet

Generalizing the DataIndexedIntegerSet Idea

Suppose we want to insert("cat")

The key question:

- What is the catth element of an array?
- One idea: Use the first letter of the word as an index.

0	F	
1	F	a
2	F	b
3	T	c
4	F	d

...

What's wrong with this approach?

- 0 never changes (so a tiny bit of wasted space).
- Other words start with c.
 - contains("chupacabra") : YES
- Can't store "=98yae98fwyawef"

Refined Approach

Treat the string as a n-digit base 27 number.

- c: 3rd letter of alphabet, a: 1st letter, t: 20th letter
- Thus the index of “cat” is $3 * 27^2 + 1 * 27 + 20$

Why this specific pattern?

- Let's review how numbers are represented in decimal.

F	0
F	1

...

T	2234
---	------

...

The Decimal Number System

In the decimal number system, we have 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Want numbers larger than 9? Use a sequence of digits.

Example: 7091 in base 10

- $7091_{10} = (7 \times 10^3) + (0 \times 10^2) + (9 \times 10^1) + (1 \times 10^0)$

Binary (Base 2)

In the binary number system, we have two digits: 0, 1.

Want larger numbers than 1? Use a sequence of digits.

Example: What is 1110_2 in base 10?

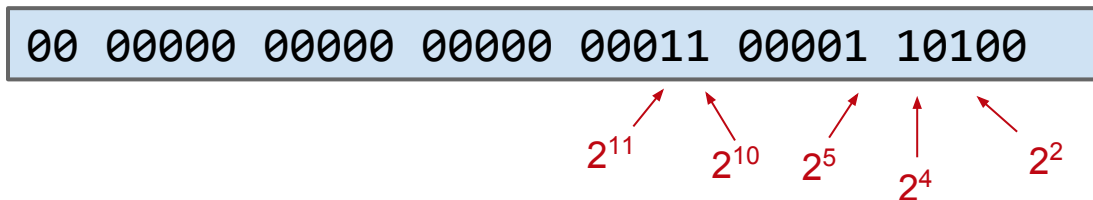
- $$\begin{aligned} 1110_2 &= (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ &= (8) + (4) + (2) + (0) \\ &= 14_{10} \end{aligned}$$

Base 10	Base 8	Base 2
0	0	0
1	1	1
2	2	10
3	3	11
4	4	100
5	5	101
6	6	110
7	7	111
8	10	1000
9	11	1001

Binary (Base 2): Larger Example

Suppose we have the 32 bit binary number below.

- What is its decimal? Sum the 2nd, 4th, 5th, 10th, and 11th powers of 2.



- $(1 \times 2^{11}) + (1 \times 2^{10}) + (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^2)$
 $= (2048) + (1024) + (32) + (16) + (4)$
 $= 3124_{10}$

Generalizing to Words

There are many ways to represent cat.

Base 27: Our approach before: cat $\rightarrow 3 * 27^2 + 1 * 27 + 20 = 2234$

00 00000 00000 00000 00010 00101 11010 2234

Base 32: $3 * 32^2 + 1 * 32 + 20 = 3124$

- Nice feature: Each letter is exactly 5 bits. Good for lecture.

00 00000 00000 00000 00011 00001 10100 3124

c
3
a
1
t
20

Why does this work?
Multiplying by 32 is equivalent
to shifting right by 5 places.

F 0
F 1

...

T 2234

F 0
F 1

...

T 3124

Generalizing to Words

Suppose we're storing multiple words.

- Convert each word to unique integer representation.
- As on previous slide: Use 5 bits per letter. ← Equivalent to treating like a base 32 number.
 - example: "cat" becomes 3124.
 - c: 3rd letter of alphabet, a: 1st, t: 20th

00 00000 00000 00000 00011 00001 10100

c a t

```
DataIndexedWordSet diws = new DataIndexedWordSet();  
diws.put("cat");      → 3124  
diws.put("dog");      → 4583  
diws.put("potato");   → 553256591  
diws.put("snack");    → 20382827
```

F 0
F 1

...

T 3124

...

T 4583

...

T 20382827

...

F 524555300

...

T 553256591

F 553256592

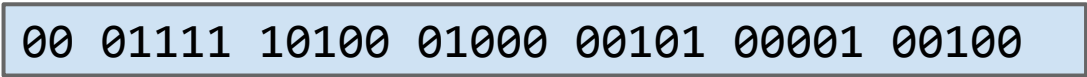
...

Generalizing to Words

What about longer strings? Have to tolerate either:

- A maximum string (seems lame).
- Ambiguity: e.g. “hothead” vs. “pothead”.
 - Both represented by 523878693

Switching to base 27 from 32 won't save us (more soon).



F	0
F	1

...

T	3124
---	------

...

T	4583
---	------

...

T	20382827
---	----------

...

F	524555300
---	-----------

...

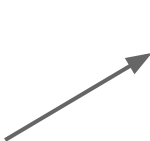
T	553256591
---	-----------

F	553256592
---	-----------

...

bottom bits of p
bottom bits of h

hothead?
pothead?
othead?



DataIndexedWordSet Implementation

```
public void insert(String s) {  
    int intRep = convertToInt(s);  
    present[intRep] = true;  
}  
  
public boolean contains(String s) {  
    int intRep = convertToInt(s);  
    return present[intRep];  
}
```

F	0
F	1

...

T	3124
---	------

...

T	4583
---	------

...

T	20382827
---	----------

...

F	524555300
---	-----------

...

T	553256591
---	-----------

F	553256592
---	-----------

...

DataIndexedWordSet Implementation

```
/** Converts ith character of String to a Letter number.  
 * e.g. 'a' -> 1, 'b' -> 2, 'z' -> 26 */
```

```
public static int letterNum(String s, int i) {  
    int ithChar = s.charAt(i);  
    if ((ithChar < 'a') || (ithChar > 'z'))  
        { throw new IllegalArgumentException();  
    }  
    return ithChar - 'a' + 1;  
}  
  
public static int convertToInt(String s) {  
    int intRep = 0;  
    for (int i = 0; i < s.length(); i++) {  
        intRep = intRep << 5; // same as intRep * 32;  
        intRep = intRep + letterNum(s, i);  
    }  
    return intRep;  
}
```

F	0
F	1

...

T	3124
---	------

...

T	4583
---	------

...

T	20382827
---	----------

...

F	524555300
---	-----------

...

T	553256591
---	-----------

F	553256592
---	-----------

...

DataIndexedArray

Two fundamental challenges:

- How do we resolve ambiguity (“grosspie” vs. “bosspie”)?
 - We’ll call this ***collision handling***.
- How do we convert arbitrary data to an index?
 - We’ll call this ***computing a hashCode***.
 - For Strings, this was relatively straightforward (treat as a base 27 or base 32 number).
 - Note: Java requires that EVERY object provide a method that converts itself into an integer: hashCode()
 - More on what makes a good hashCode() later.

F	0
F	1

...

T	3124
---	------

...

T	4583
---	------

...

T	20382827
---	----------

...

F	524555300
---	-----------

...

T	553256591
F	553256592

...

Hat Giveaway



North America's leader in Track and Transit & Systems construction and maintenance services

Handling Collisions

Resolving Ambiguity

Biggest array in Java is 2 billion entries.

- [Pigeonhole principle](#) tells us that if there are more than 2 billion possible items, multiple items will share the same box.
- Example: More than 2 billion possible Planets.
 - Each has mass, xPos, yPos, xVel, yVel, imgName.
- Example: More than 2 billion possible strings.
 - “one”, “two”, ... “four billion and six”, ...



Resolving Ambiguity

Pigeonhole principle tells us that if there are more than 2 billion possible things, multiple items will share the same box.

Suppose N items have the same hashcode h :

- Instead of storing `true` in position h , store a list of these N items at position h .

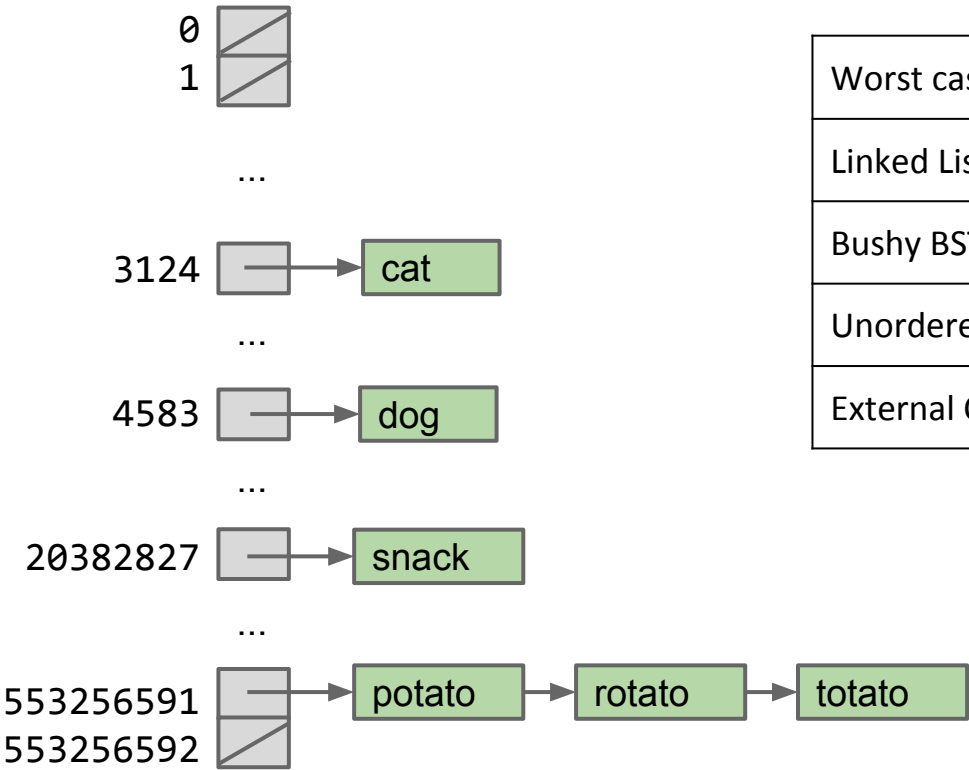
How to implement list?

- Easiest way: Linked list.
- But any list would do (ArrayList, etc.)
- (... if you wanted, could use a set instead)



External Chaining

External Chaining: Storing all items that map to h in a linked list.



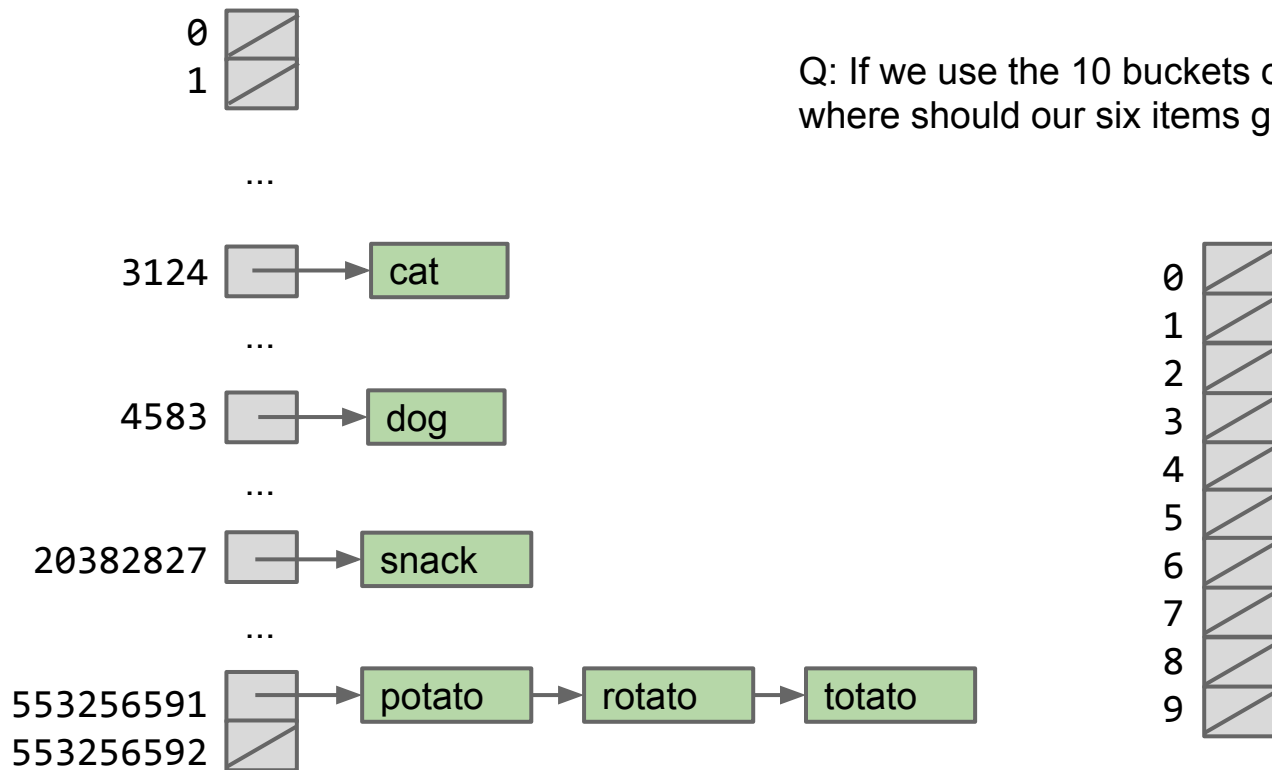
Worst case time	contains(x)	insert(x)
Linked List	$\Theta(N)$	$\Theta(N)$
Bushy BSTs	$\Theta(\log N)$	$\Theta(\log N)$
Unordered Array	$\Theta(N)$	$\Theta(N)$
External Chaining	$\Theta(Q)$	$\Theta(Q)$

Why Q and not 1?

Q: Length of longest list

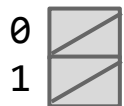
External Chaining

Observation: We don't really need 2 billion buckets.



External Chaining

Observation: Can use modulus of hashcode to reduce bucket count.



...



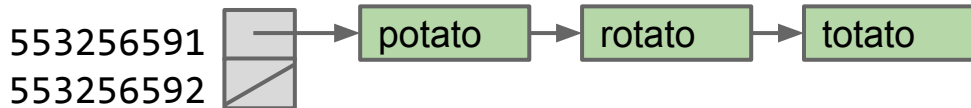
...



...

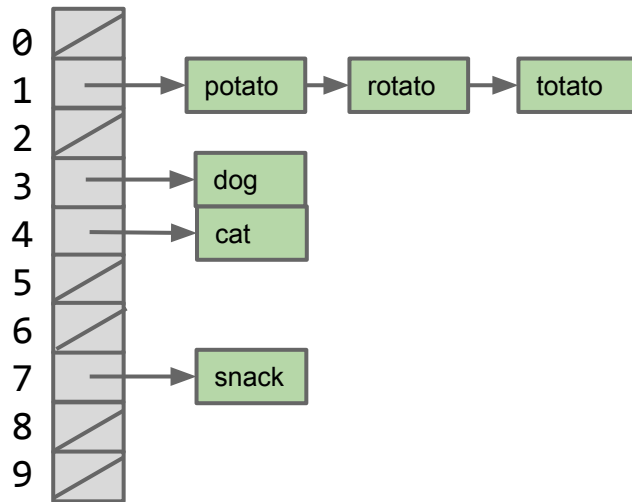


...



Q: If we use the 10 buckets on the right, where should our six items go?

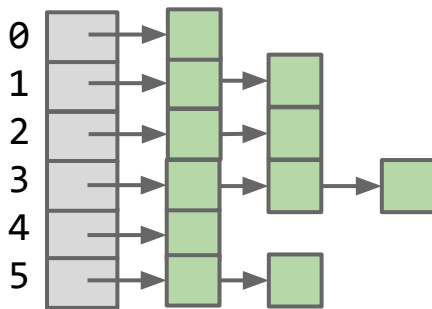
- Put in bucket = hashCode % 10



External Chaining Performance

Depends on the number of items in the 'bucket'.

- If N items are distributed across M buckets, average time grows with $N/M = L$, also known as the **load factor**.
 - Average runtime is $\Theta(L)$.



Load factor: $11/6 = 1.83$

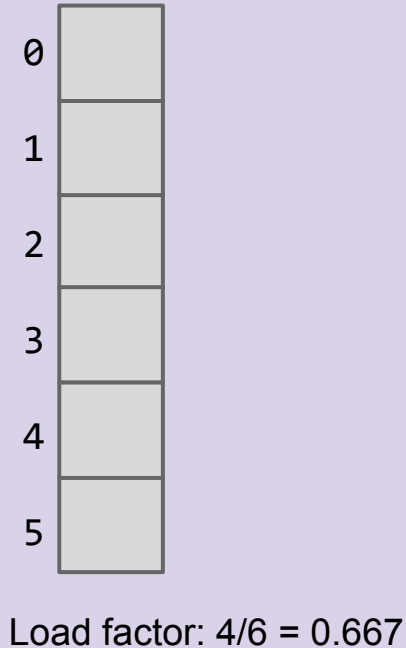
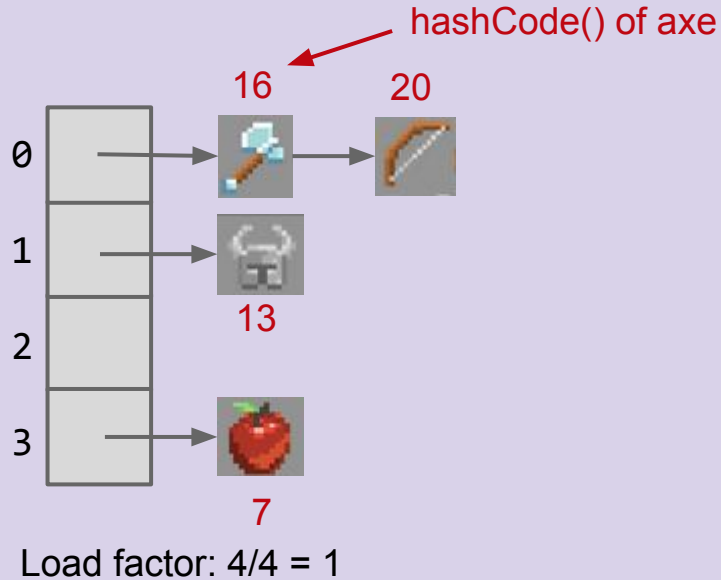
Obvious observation: If L is small, our data structure will be very fast.

Question: As N grows, what can we do to ensure that L stays small?

Array Resizing: <http://yellkey.com/skin>

Whenever $L=N/M$ exceeds some number, increase M by resizing.

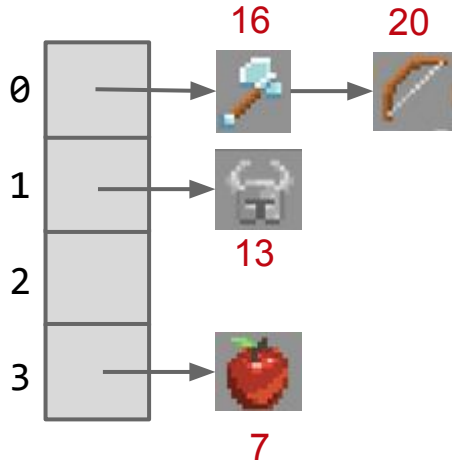
- Question: In which bin will the apple appear after resizing?



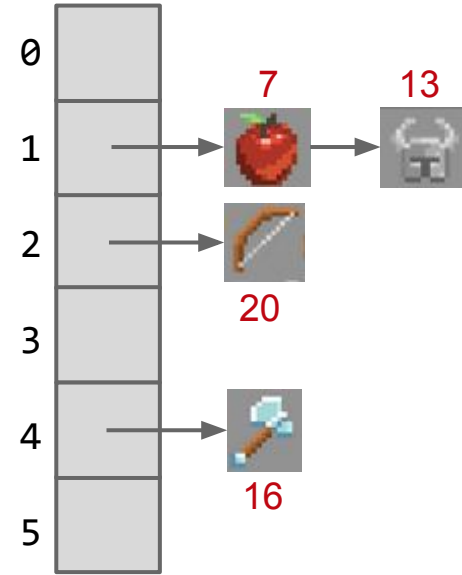
Array Resizing

Whenever $L=N/M$ exceeds some number, increase M by resizing.

- Question: In which bin will the apple appear after resizing?



Load factor: $4/4 = 1$



Load factor: $4/6 = 0.667$

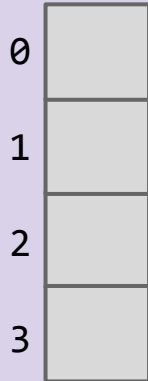
$7 \% 6 = 1$

Using Negative .hashCode(): <http://yellkey.com/medical>



Suppose that `.hashCode()` returns -1.

- Philosophically, into which bucket is it most natural to place this item?

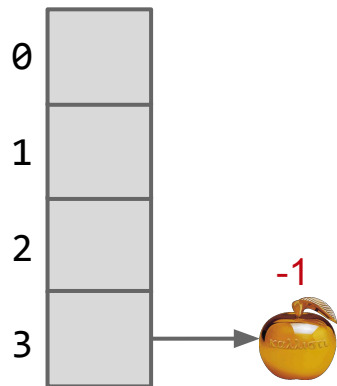


Using Negative .hashCode()



Suppose that `.hashCode()` returns -1.

- Philosophically, into which bucket is it most natural to place this item?
 - I say 3, since $-1 \rightarrow 3$, $0 \rightarrow 0$, $1 \rightarrow 1$, $2 \rightarrow 2$, $3 \rightarrow 3$, $4 \rightarrow 0$, ...

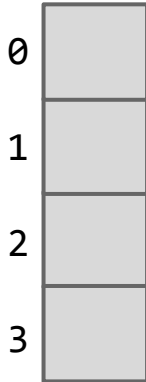


Using Negative .hashCode() in Java



Suppose that `.hashCode()` returns `-1`.

- Unfortunately, $-1 \% 4 = -1$. Will result in index errors!
- Use `Math.floorMod` instead.



```
public class ModTest {  
    public static void main(String[] args) {  
        System.out.println(-1 % 4);  
        System.out.println(Math.floorMod(-1, 4));  
    }  
}
```

```
$ java ModTest  
-1  
3
```

Hash Table Definition and Key Implementation Details

This data structure we've designed is called a *hash table*:

- Every item is mapped to a bucket number using a hash function.
 - Typically, computing hash function consists of two steps:
 1. Computing a hashCode (integer between -2^{31} and $2^{31} - 1$).
 2. Computing index = hashCode modulo M.
 - If $L = N/M$ gets too large, increase M.
- Be careful, negative numbers % M won't work.

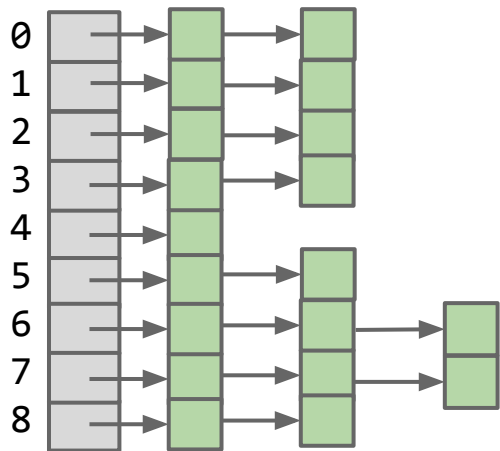
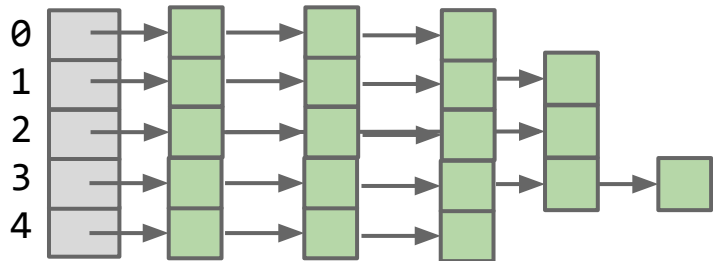
If multiple items map to the same bucket, we have to resolve ambiguity somehow.

Two common techniques:

- External Chaining (creating a list for each bucket, the technique we just used).
- Open Addressing (a little stranger, not necessarily better, see extra slides).
 - May come up at job interviews.

External Chaining Performance

Assuming items are spread out (e.g. not all in the same bucket):



Average case time	contains(x)	insert(x)
External Chaining, Fixed Size	$\Theta(L)$	$\Theta(L)$
External Chaining With Resizing	$\Theta(L)$	$\Theta(L)$
Balanced BST	$\Theta(\log N)$	$\Theta(\log N)$

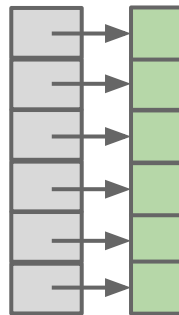
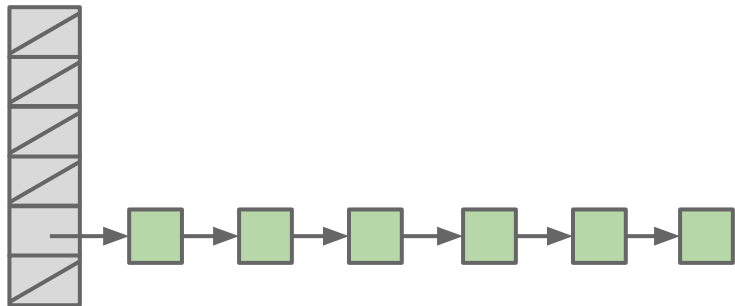
	Load Factor L
External Chaining, Fixed Size	$\Theta(N)$
External Chaining With Resizing	$\Theta(1)$

Amortized!

One Last Little Detail

Performance depends on the number of items in each 'bucket'.

- Given load factor of N/M .
 - Average runtime is $\Theta(L)$.
 - Average isn't the whole story. Want balanced buckets. Analogous to maintaining bushiness in a BST, but conceptually much easier to solve.

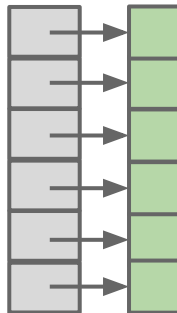
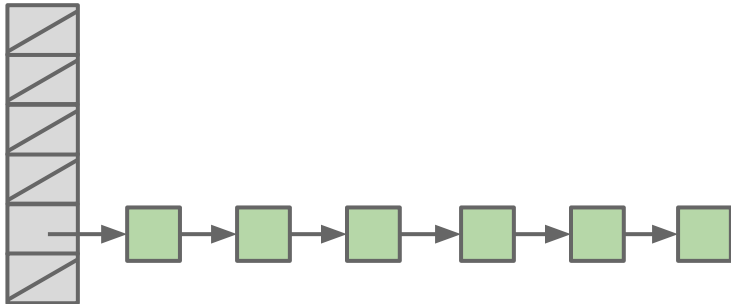


Hash Functions

What Makes a good .hashCode()?

Goal: We want hash tables that look like the table on the right.

- Want a hashCode that spreads things out nicely on real data.
 - Example #1: return 0 is a bad hashCode function.
 - Example #2: Our convertToInt function for strings was bad. Top bits were ignored, e.g. “potato” and “give me a potato” have same hashCode.
- Writing a good hashCode() method can be tricky.



Example: String hashCode function

Our convertToInt function:

- $$h(s) = (s_0 - 'a' + 1) \times 32^{n-1} + (s_1 - 'a' + 1) \times 32^{n-2} + \dots + (s_{n-1} - 'a' + 1)$$

Problems:

- Intended for lower case strings only.
- Top bits are totally ignored.

Example: String hashCode function

Improved convertToInt function:

- $$h(s) = s_0 \times 32^{n-1} + s_1 \times 32^{n-2} + \dots + s_{n-1}$$

Problems:

- Intended for lower case strings only: Fix by removing - 'a'
- Top bits are totally ignored.
 - Why? Because multiplying by 32 is equivalent to left shifting by 5 bits.
Result: Top characters get pushed out completely.
 - How can we fix?

Example: String hashCode function

Java's actual hashCode() function for Strings:

- $$h(s) = s_0 \times 31^{n-1} + s_1 \times 31^{n-2} + \dots + s_{n-1}$$

Problems:

- Intended for lower case strings only: Fix by removing - 'a'
- Top bits are totally ignored.
 - Why? Because multiplying by 32 is equivalent to left shifting by 5 bits. Result: Top characters get pushed out completely.
 - How can we fix: Multiply by powers of 31!
 - In convertToInt, we tried to have the kth character contribute to specific bits of the hashCode(). Nice for understanding lecture, but exactly the wrong idea for avoiding hashCode collisions!

Example: String hashCode Function Example

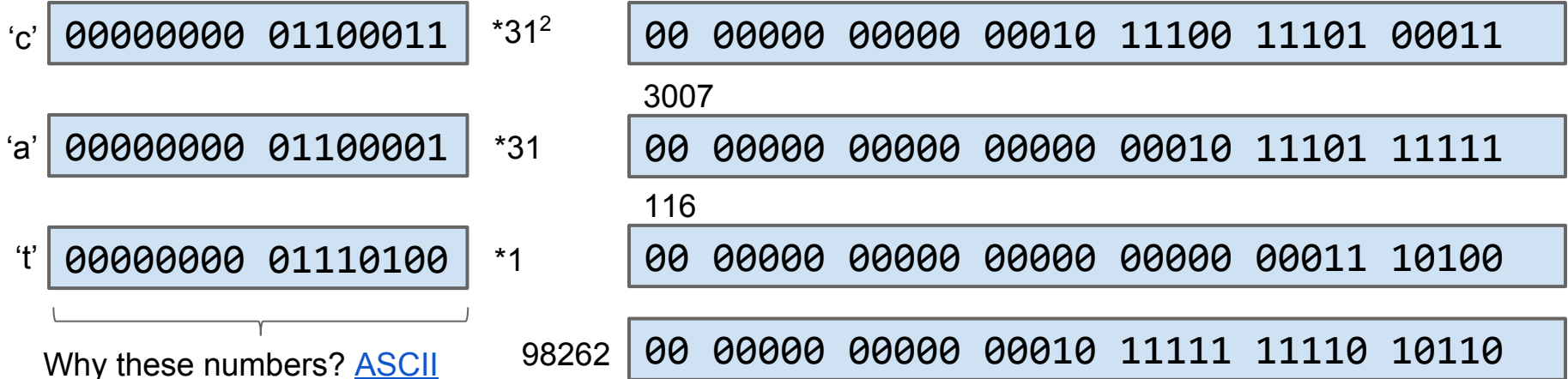
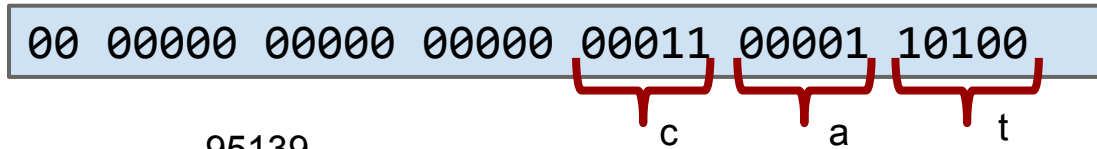
Java's hashCode() function:

- $$h(s) = s_0 \times 31^{n-1} + s_1 \times 31^{n-2} + \dots + s_{n-1}$$

Food for thought: Hash tables with a number of buckets equal to a multiple of 31 will not work very well with this hashCode(). Why?

Consider 'cat':

convertToInt()



Hashing

How do you make hashbrowns?

- Chopping potato into nice predictable segments? No way!
- This is a hashbrown:



Example: Hashing a Collection

Lists are a lot like strings: Collection of items each with its own hashCode:

```
@Override
public int hashCode() {
    int hashCode = 1;
    for (Object o : this) {
        hashCode = hashCode * 31;
        hashCode = hashCode + o.hashCode();
    }
    return hashCode;
}
```

elevate/smear the current hash code

add new item's hash code

To save time hashing: Look at only first few items.

- Higher chance of collisions but things will still work.

Example: Hashing a Recursive Data Structure

Computation of the hashCode of a recursive data structure involves recursive computation.

- For example, binary tree hashCode (assuming sentinel leaves):

```
@Override
public int hashCode() {
    if (this.value == null) {
        return 0;
    }
    return this.value.hashCode() +
        31 * this.left.hashCode() +
        31 * 31 * this.right.hashCode();
}
```

Default hashCode()

All Objects have hashCode() function.

- Default: returns this (i.e. address of object).
 - Can have strange consequences: "hello".hashCode() is not the same as ("h" + "ello").hashCode()
- Can override for your type.
- Hash tables (HashSet, HashMap, etc.) are so important that Java requires that all objects implement hashCode().

HashSets and HashMaps

Java provides a hash table based implementation of sets and maps.

- Idea is very similar to what we've done in lecture.
- Warning: Never store mutable objects in a HashSet or HashMap!
- Warning #2: Never override `equals` without also overriding `hashCode`.
 - Why these warnings? See study guide.

In lab 9, you'll get a chance to implement a hash map.

Summary

With good hashCode() and resizing, operations are $\Theta(1)$ amortized.

- No need to maintain bushiness (but still need good hashCode).
- Store and retrieval does not require items to be comparable.

	contains(x)	insert(x)
Linked List	$\Theta(N)$	$\Theta(N)$
Bushy BSTs	$\Theta(\log N)$	$\Theta(\log N)$
Unordered Array	$\Theta(N)$	$\Theta(N)$
Hash Table	$\Theta(1)$	$\Theta(1)$

Used by: TreeSet

Used by: HashSet,
Python dictionaries

Worst case runtimes

Collision Resolution (Extra)

Open Addressing: An Alternate Disambiguation Strategy (Extra)

If target bucket is already occupied, use a different bucket, e.g.

- Linear probing: Use next address, and if already occupied, just keep scanning one by one.
 - Demo: <http://goo.gl/o5EDvb>
- Quadratic probing: Use next address, and if already occupied, try looking 4 ahead, then 9 ahead, then 16 ahead, ...
- Many more possibilities. See the optional reading for today (or CS170) for a more detailed look.

In 61B, we'll settle for external chaining.

Citations

<http://www.nydailynews.com/news/national/couple-calls-911-forgotten-mcdonalds-hash-browns-article-1.1543096>

http://en.wikipedia.org/wiki/Pigeonhole_principle#mediaviewer/File:TooManyPigeons.jpg

https://cookingplanit.com/public/uploads/inventory/hashbrown_1366322674.jpg