Problem 7 - Stochastic Inventory Control: A retailer faces stochastic demand for a product and needs to determine the optimal reordering strategy over the next 10 days (planning horizon). At the end of the planning horizon, the new generation of the product will replace the current one, and any unsold inventory will be salvaged at a reduced price to a secondary market (e.g. an outlet store, Groupon, etc.). The assumptions and problem data are as follows:

- **Assumptions:**
  - There is no lead time (orders are received instantaneously)
  - Demand is stationary (same distribution for every day, no seasonality)
  - No backlogging (any unsatisfied demand is lost)
- **Problem data:**
  - β = discount factor = 0.975
  - p = per unit selling price = \$250
  - c = per unit purchasing cost = \$170
  - f = fixed ordering cost = \$200
  - h = per unit holding cost per day = \$5
  - v = per unit salvage value at the end of the horizon = \$150
  - D = random daily demand $\sim$ Binomial(20,0.7)
  - M = shelf-space/warehouse capacity = 20

(a) Formulate the inventory control problem as an MDP. Clearly define the state space, the action space, the reward function, the transition probabilities, and set up the Bellman equation.

(b) Solve the problem in MATLAB and visualize the optimal policy. Submit a copy of your script.

(c) Figure 1 is a heat map of the optimal controls as a function of time (n) and state (i). Identify the "Order" and "No Order" regions.

(d) Table 1 summarizes the optimal controls as a function of time (n) and state (i). Discuss briefly (qualitatively and quantitatively) the optimal re-ordering policy.

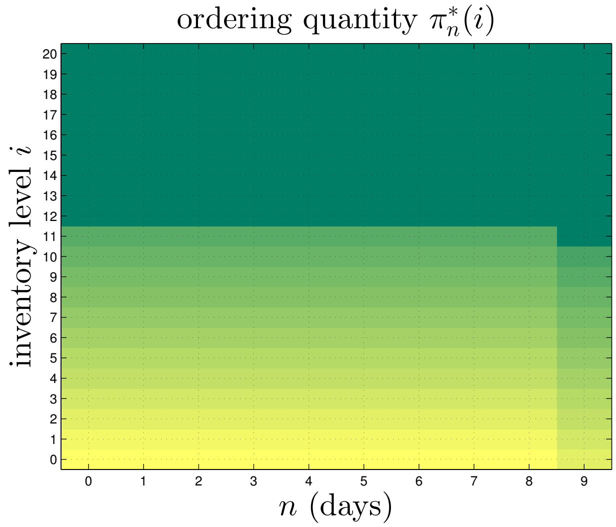

Figure 1: Optimal Controls

Optimal Controls $\pi_n^*(i)$

i \ n	0	1	2	3	4	5	6	7	8	9
20	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0
11	6	6	6	6	6	6	6	6	6	0
10	7	7	7	7	7	7	7	7	7	5
9	8	8	8	8	8	8	8	8	8	6
8	9	9	9	9	9	9	9	9	9	7
7	10	10	10	10	10	10	10	10	10	8
6	11	11	11	11	11	11	11	11	11	9
5	12	12	12	12	12	12	12	12	12	10
4	13	13	13	13	13	13	13	13	13	11
3	14	14	14	14	14	14	14	14	14	12
2	15	15	15	15	15	15	15	15	15	13
1	16	16	16	16	16	16	16	16	16	14
0	17	17	17	17	17	17	17	17	17	15

Table 1: Optimal Controls