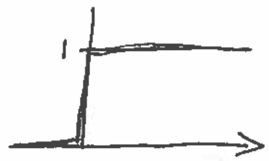
Problem Set 10 - Jiaying Wei

1. Use the laplace transform to verify that the step responsal $\dot{y}+y=x$ is $y(t)=(1-e^{-t})u(t)$.

$u(t) =$ 

$$\begin{tikzcd} u(t) \arrow[r] \arrow[d, "\mathcal{L}"] & \boxed{h(t)} \arrow[r] & y(t)=(1-e^{-t})u(t) \\ U(s)=\frac{1}{s} \arrow[r] & \boxed{H(s)} \arrow[r] & Y(t) \arrow[u, "\mathcal{L}^{-1}"] \end{tikzcd}$$

$$\mathcal{L}\{\dot{y}+y=x\} = sY(s)+Y(s)=X(s) \qquad U(s)=X(s)$$

$$sY(s)+Y(s)=\frac{1}{s}$$

$$Y(s)(s+1)=\frac{1}{s}$$

$$Y(s)=\frac{1}{s(s+1)} \longrightarrow \frac{A_1}{s}+\frac{A_2}{s+1}$$

$$\left.\frac{\cancel{s}\cdot 1}{\cancel{s}(s+1)}\right|_{s=0} = \left.\frac{sA_1}{s}+\frac{sA_2}{s+1}\right|_{s=0}$$

$$1\cancel{0} = A_1 + 0$$

$$\boxed{A_1=1}$$

$$\left.\frac{\cancel{(s+1)}}{s\cancel{(s+1)}}\right|_{s=-1} = \left.\frac{(s+1)A_1}{s}+\frac{\cancel{(s+1)}A_2}{\cancel{(s+1)}}\right|_{-1}$$

$$\frac{1}{-1} = \frac{0\cdot \cancel{A_1}}{\cancel{-1}} + A_2 \qquad \boxed{A_2=-1}$$

$$Y(s)=\frac{1}{s}+\frac{-1}{s+1}$$

$$y(t) = 1\cdot e^{-0\cdot t}u(t) + -1e^{-1\cdot t}u(t)$$

$$= u(t)-e^{-t}u(t)$$

$$\boxed{y(t)=(1-e^{-t})u(t)}$$

## Problem 2

4. Find the DC gain of the system $\frac{Y(s)}{Y_{sp}(s)}$ if you use an integral controller $K(s) = \frac{K}{s}$ for any $H(s)$


$$\frac{Y}{Y_{sp}} = \frac{KH}{1+KH} = \frac{K_I/s \cdot H}{1+K_I/s \cdot H}$$

, now we have to solve this for when $s=0$ because that will be the first coefficient to $e^0$ thus giving us the DC offset of the system

<u>But</u> $K_I/0$ is going to ERROR so we need to take the $\lim_{s\to 0} \frac{K_I}{s}$

<u>But</u> if we do, we run into $\frac{\infty}{\infty}$ which is also bad so we need to use L'Hospital's Rule

L'Hopital said $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$

so:

$$\frac{Y}{Y_{sp}} = \lim_{s\to 0} \frac{d}{ds}\left(\frac{K_I/s \cdot H}{1+K_I/s \cdot H}\right) = \lim_{s\to 0} \frac{-H\cancel{K_I}/s^2}{-H\cancel{K_I}/s^2} = \boxed{1 \leftarrow \text{DC offset of the system is 1}}$$

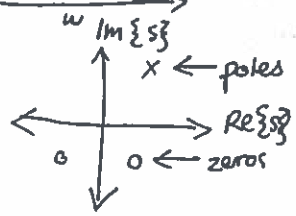
## Problem 3

Analyze the behavior of the below systems using a Bode plot, a pole-zero map, & step response

Bode plot →


: How does the system react to different ω

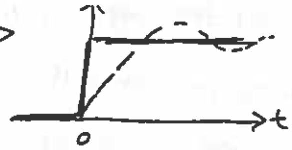
Pole-zero map → (s-plane)




: Shows the poles and zeros of the transfer function

<u>Poles</u> - Values of s that cause the denomination to go to 0, which if you take the limit of that $\frac{Y}{Y_{sp}}$ goes to ∞ so it tells you when your system will go off into infinity

<u>Zeros</u> - Values of s that cause the numerator to go to 0, which means that your system is at a stable point where it is not changing. Not nessesarily at the set point though.

Step Response $\rightarrow$ 

How the system responds in time to a change in input/setpoint $u(t)$

A. $\frac{Y}{Y_{sp}} = \frac{s}{s+1}$

1. It appears that the step response of this system shows that it will jump almost instantaneously to the set point and then ~~exponent~~ exponentially to zero.
2. The Bode plot seems to indicate that it is more sensitive to more rapid change.
3. The poles tells us the system is unstable @ $s = -1 + 0j$ and stable @ $s = 0 + 0j$

B. $\frac{Y}{Y_{sp}} = \frac{s}{s^2 + 100s + 1}$

1. This particular system was much slower that the 1st system to getting to the setpoint, but also started decreasing to 0 after reaching the sp though at a much slower rate.
2. The bode plot comfirms this, showing greater magnitudes at lower frequencies
3. poles @ $s = -100 + 0j$ ; $s = -1 + 0j$
   zero @ $s = 0 + 0j$

C. $\frac{Y}{Y_{sp}} = \frac{s}{s^2 + s + 1}$

1. The step response shows that even before it ~~star~~ reaches the setpoint the system has an ossilating decay
2. It has a similar Bode plot to B
3. poles @ $s = -0.5 + \approx .8j$ ; $s = -0.5 - \approx .8j$
   zero @ $s = 0 + 0j$ appearing to be complex conjugates
   ↳ maybe something to do w/ ossillation

D. $\frac{Y}{Y_{sp}} = \frac{s}{s^2 + 0.1s + 1}$

1. Has a similar ossilating decay to C, only w/ larger ossilation / slower decay
2. Similar Bode plot to D
3. poles @ $s = -0.1 + 1j$ ; $s = -0.1 - 1j$
   zero @ $s = 0 + 0j$ does appear that complex conjugates do have something to do w/ ossillation

★ Maybe a larger j value means larger ossillations?
Maybe a smaller ___ s value also causes that?

E. $\dfrac{Y}{Y_s} = \dfrac{s^2 - 0.01s + 1}{s^2 + 0.01s + 1}$

A. For the first time it seems that the resultant y(t) is following the set point, albiet some ossillations

B. The magnitude Bode plot seems to indicate that it treats fast and slow inputs equally

C. I can't really tell by eyeballing it where the poles ; zeros are since they're overlapping kinda. But both appear to be in complex conjugates pairs.
Not sure what having complex conjugate zeros do

F. $\dfrac{Y}{Y_s} = \dfrac{s^2 + 0.1s + 1}{s^2 + 0.11s + 1}$

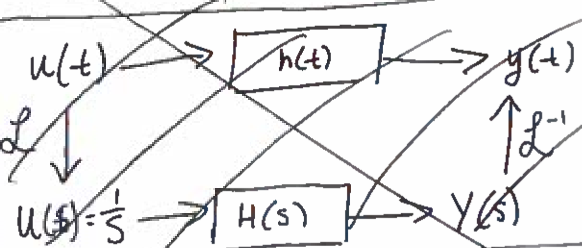
A. The system follows the setpoint pretty spot on. There appears to be just a little ossillation but my eyes can't really tell.

B. Bodeplot shows that it reacts greater to higher frequency changes

C. Similar looking poles and zeros to E

Problem 4 - Stabilize

plant! $\longrightarrow H(s) = \dfrac{1}{s^2 - 0.01s + 1}$




$Y(s) = U(s)H(s) = \dfrac{1}{s^3 - 0.01s^2 + s} = \dfrac{1}{s(\underbrace{s^2 - 0.01s + 1}_{\text{quadratic formula this}})}$

$s = \dfrac{0.01 \pm \sqrt{(-.01)^2 - 4(1 \cdot 1)}}{2 \cdot 1}$

$s = \dfrac{0.01 \pm \sqrt{0.0001 - 4}}{2} = \dfrac{0.01}{2} \pm \dfrac{\sqrt{-3.9999}}{2}$

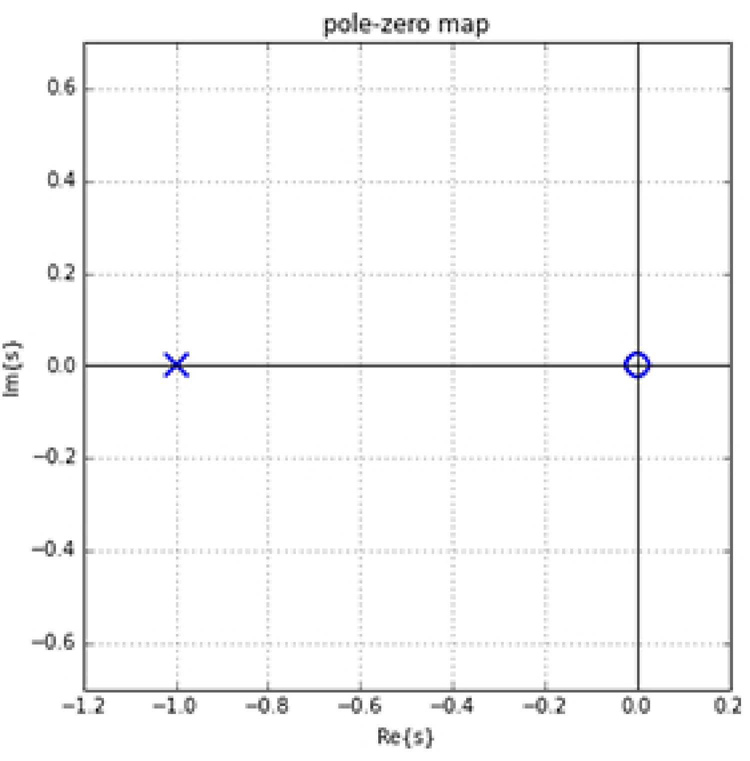
$Y(s) = \dfrac{1}{s(0.005}$

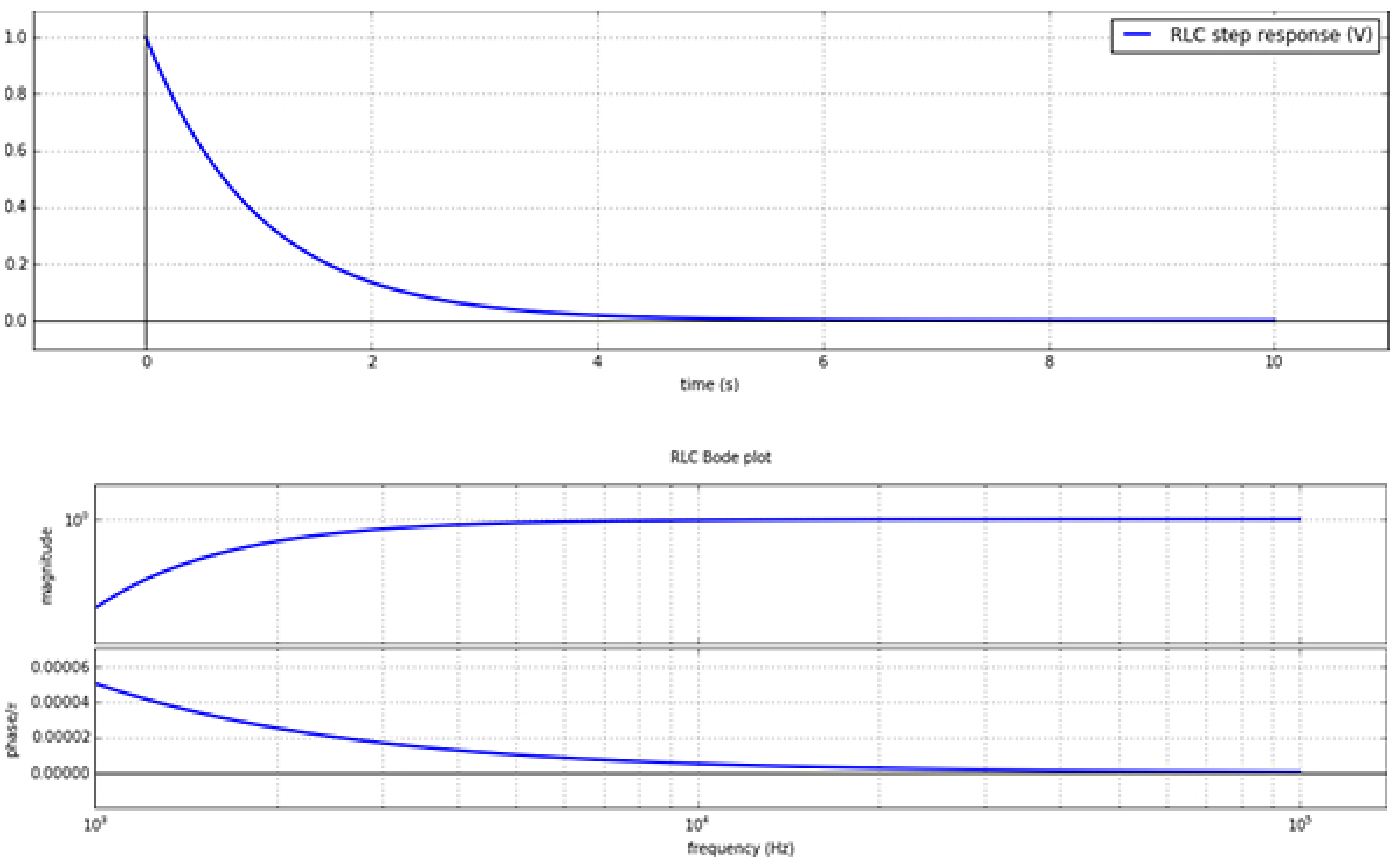
So I wasn't supposed to d pane A by hand...


$\dfrac{Y}{Y_{sp}} = \dfrac{KH}{1 + KH}$

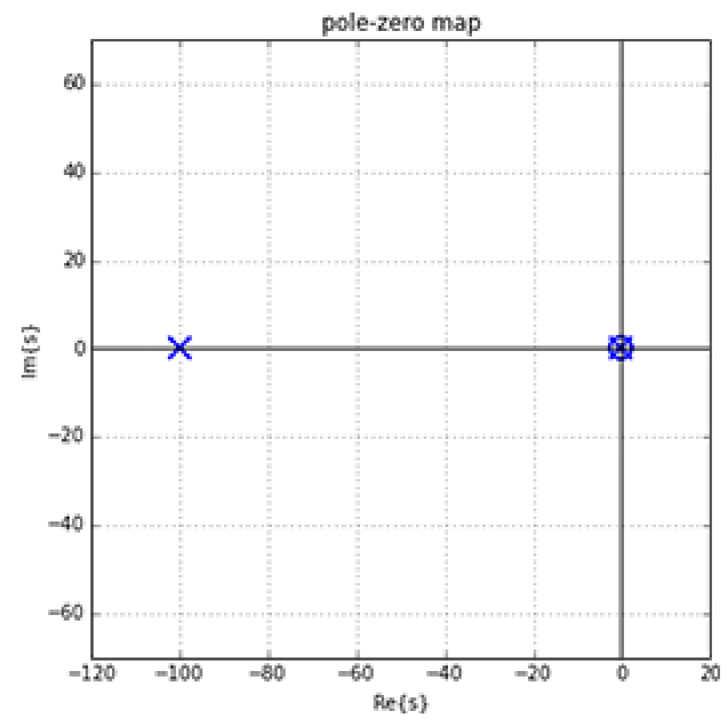
# Problem 3

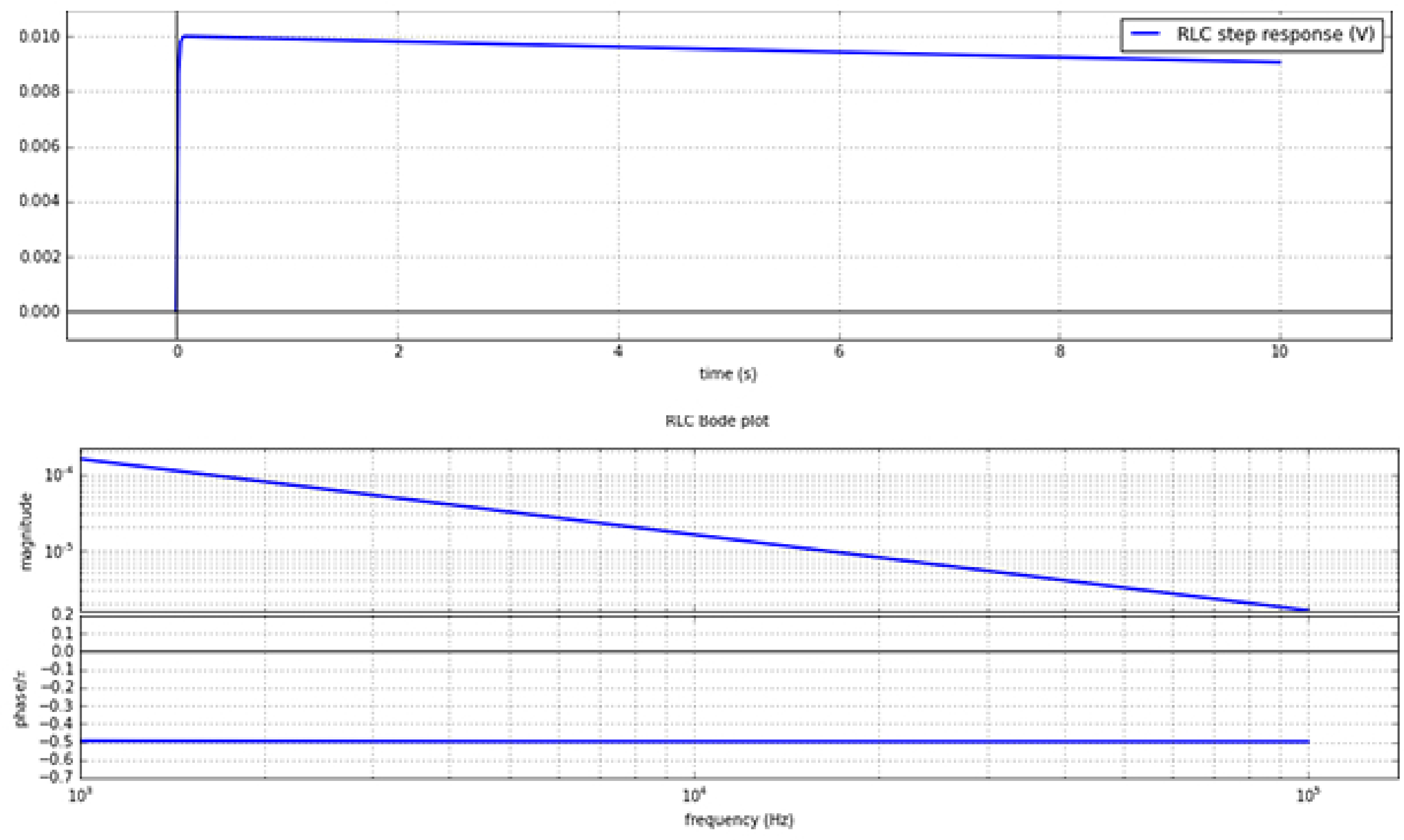
## System A



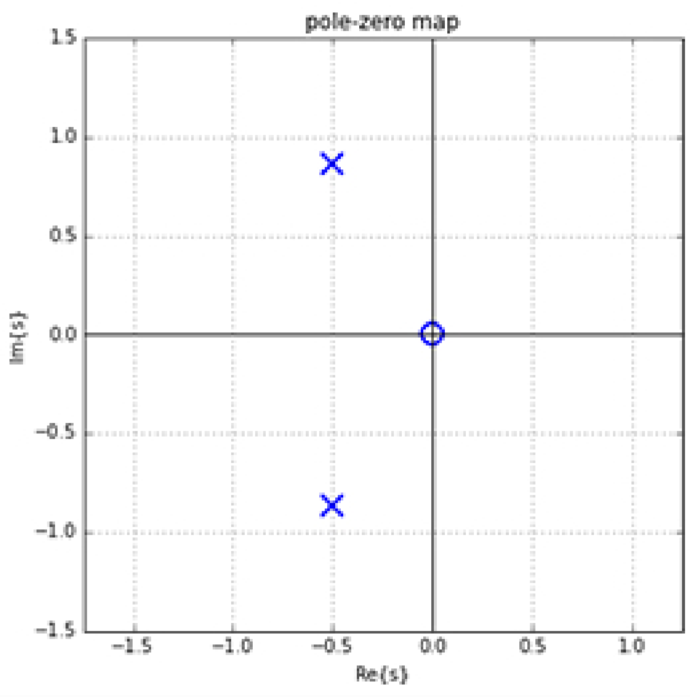



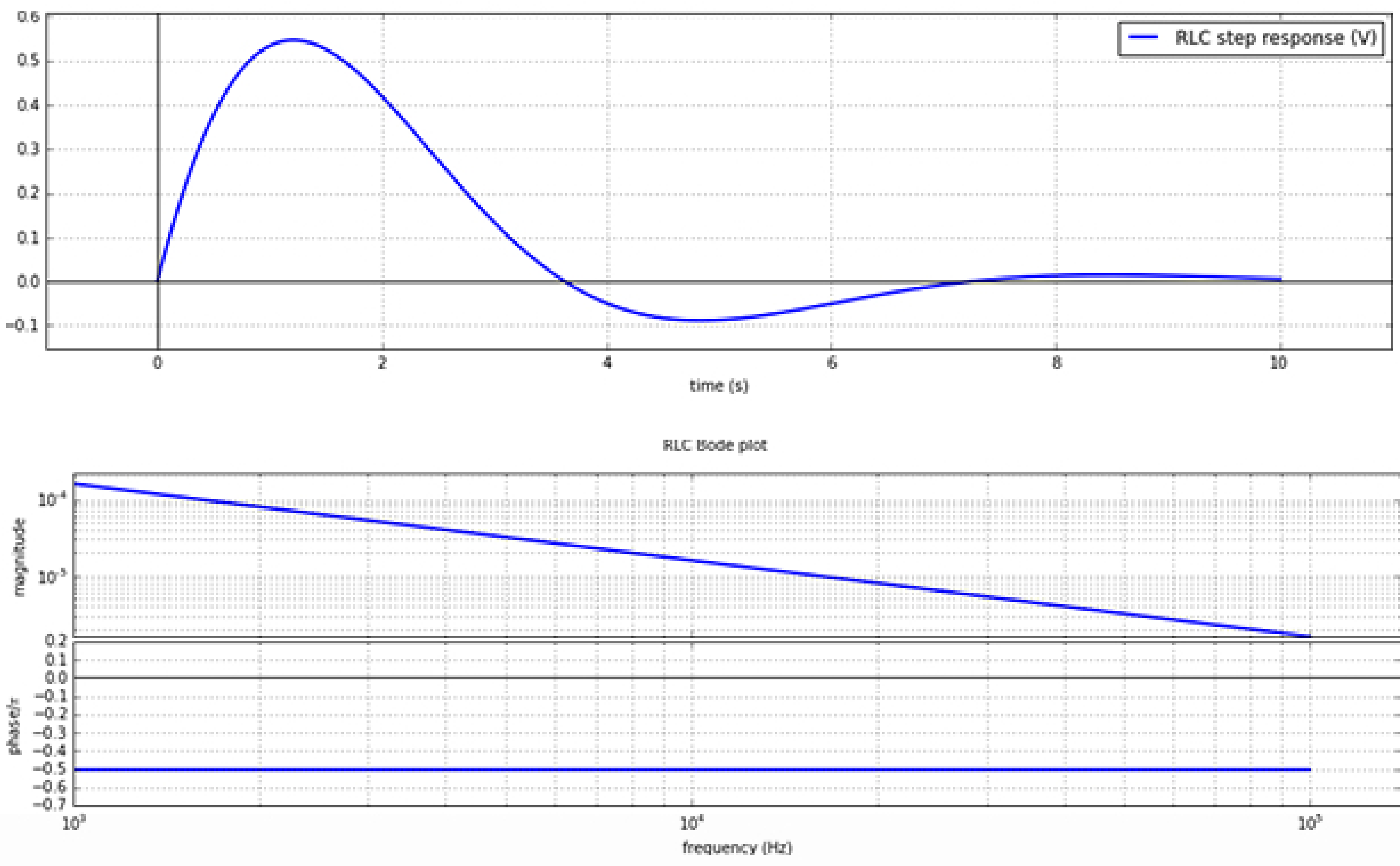

## System B



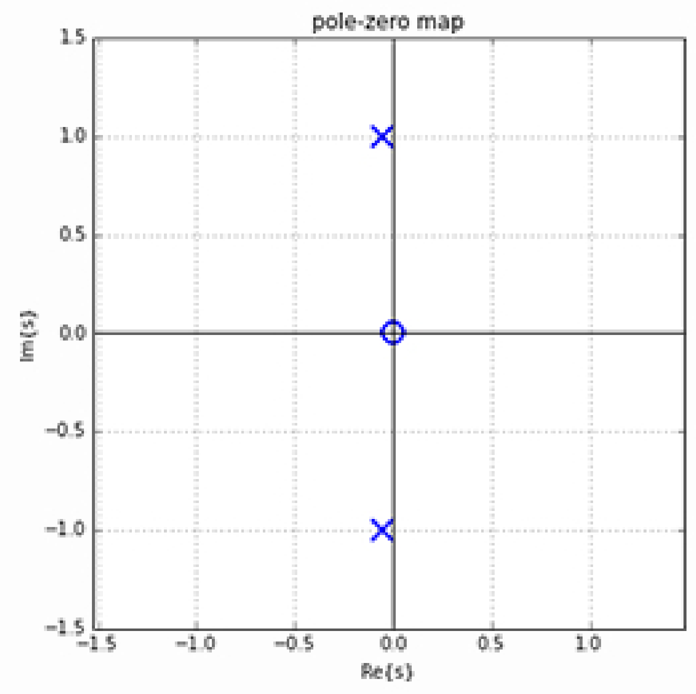



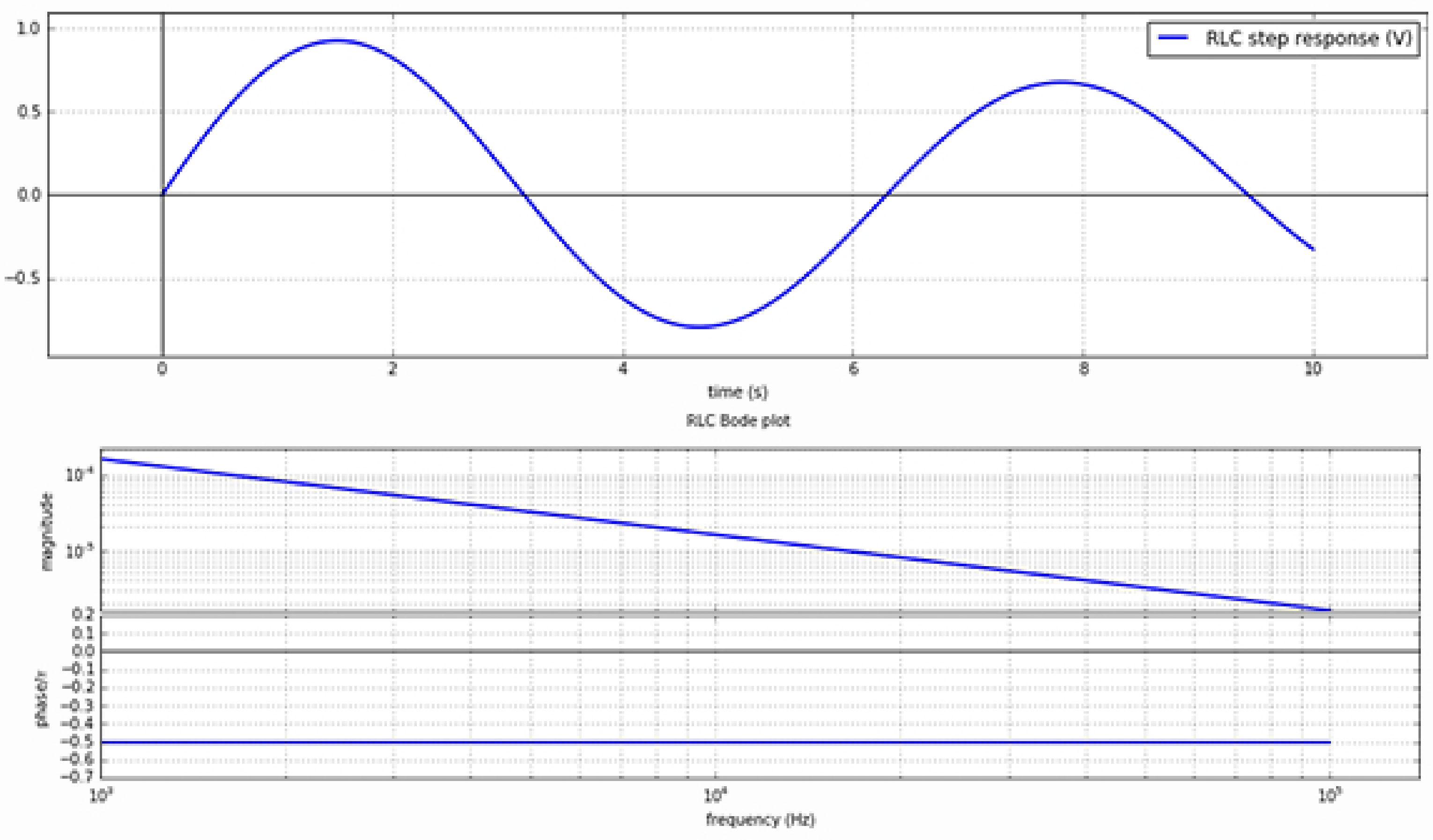

## System C



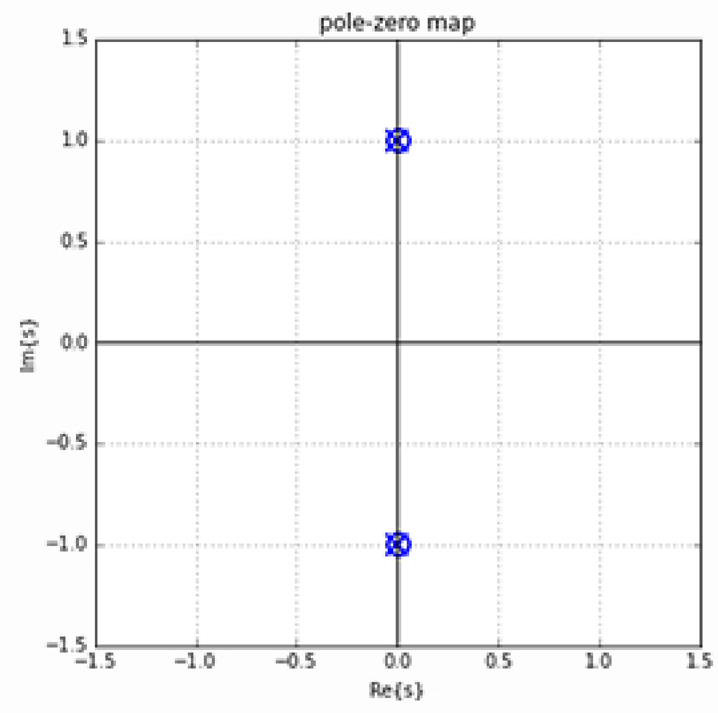



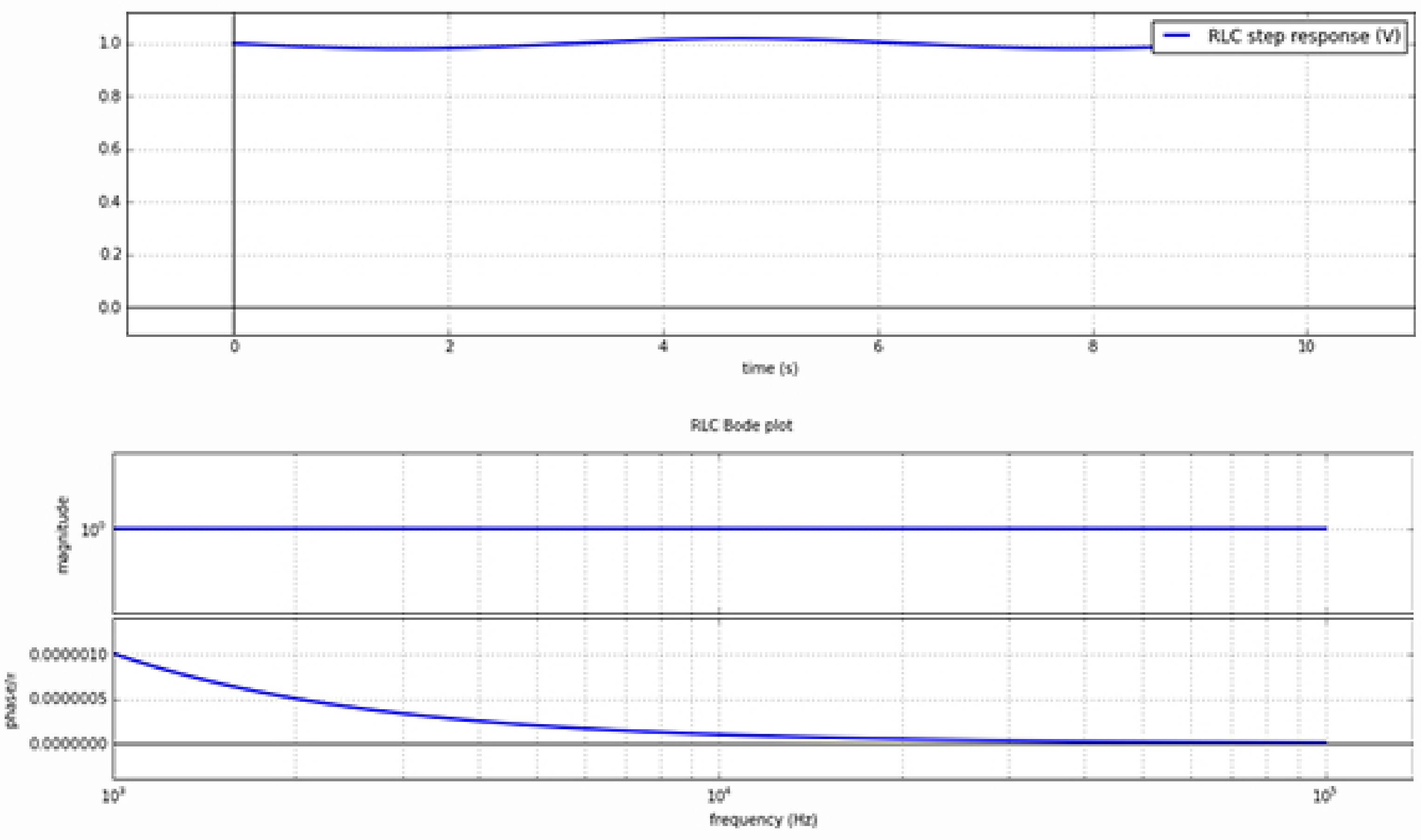

## System D



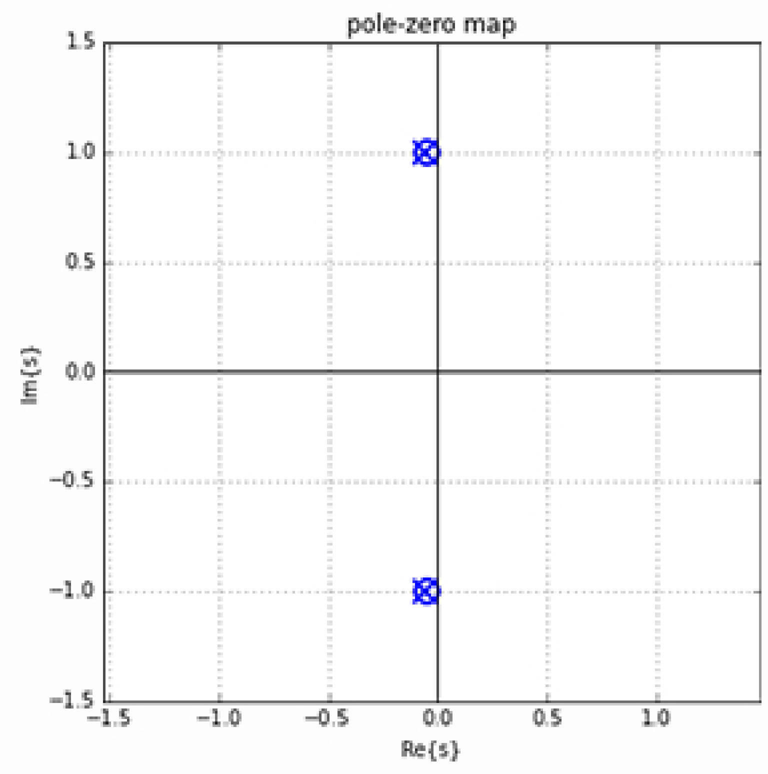



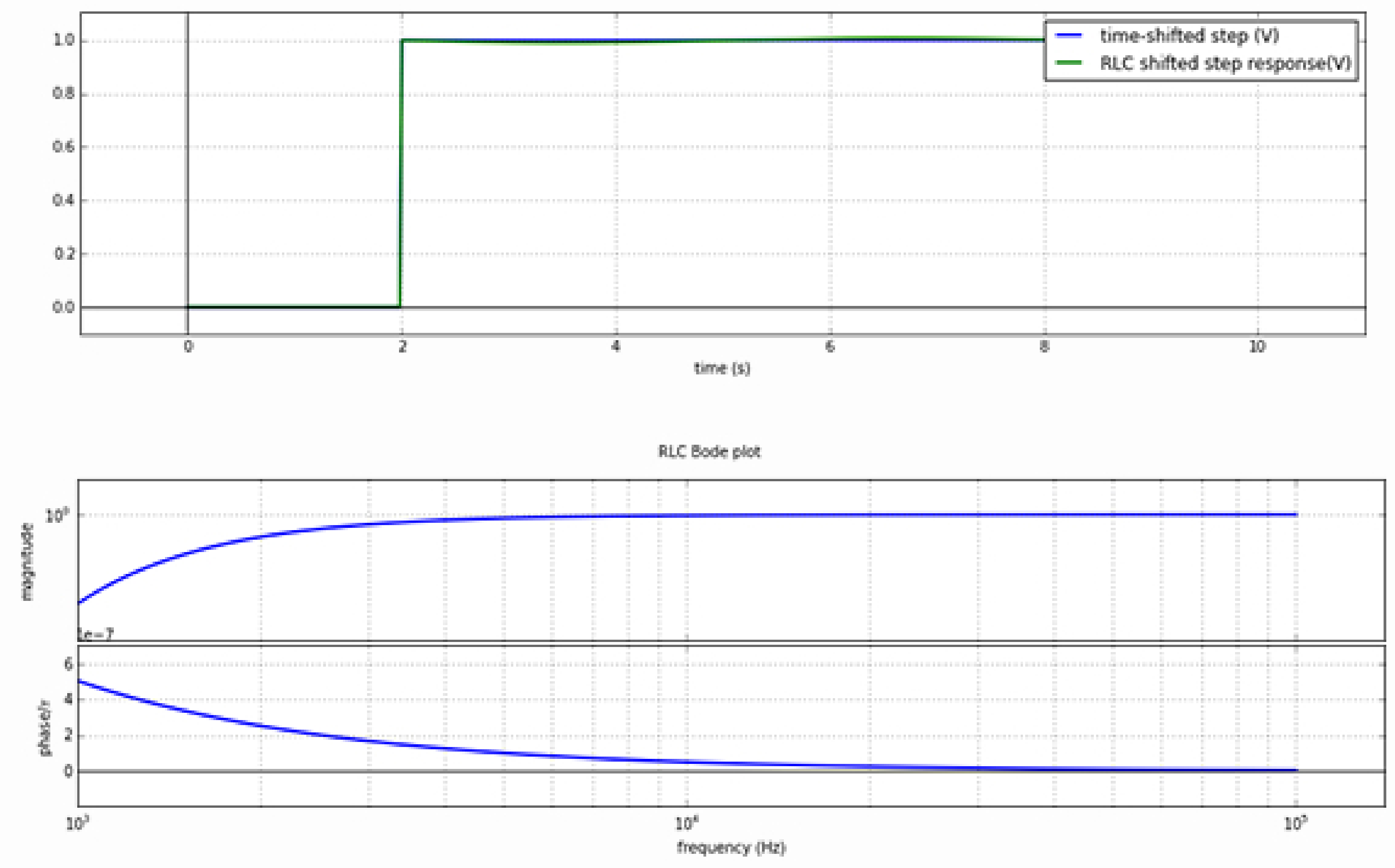

## System E



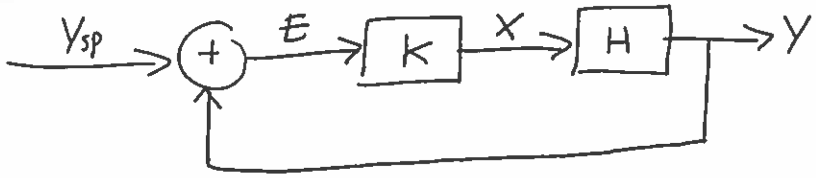




## System F



Problem 4




$$H(s) = \frac{1}{s^2 - 0.01s + 1}$$

A. Plot the poles & ~~system~~ step response (see graphs)

System is naturally wanting to ossillate -1 to 1

B. Show the effects of using proportional control on the system

$$\frac{Y}{Y_{sp}} = \frac{K_p H}{1 + K_p H} = \frac{K_p\left(\cancel{\frac{1}{s^2-0.01s+1}}\right)}{1 + K_p\left(\frac{1}{s^2-0.01s+1}\right)} \frac{\left(\cancel{s^2-0.01s+1}\right)}{\left(s^2-0.01s+1\right)} = \frac{K_p}{K_p + s^2 - 0.01s + 1}$$

So it doesn't look like you can stabilize this system using porportional control. You just get various frequencies of ossillations.

C. Show the effects of using integral control $K(s) = K_I/s$

$$\frac{Y}{Y_{sp}} = \frac{K_I/s \cdot H}{1 + K_I/s \cdot H} = \frac{K_I/s \cdot \left(\cancel{\frac{1}{s^2-0.01s+1}}\right)}{1 + K_I/s \cdot \left(\frac{1}{s^2-0.01s+1}\right)} \frac{\left(\cancel{s^2-0.01s+1}\right)}{\left(s^2-0.01s+1\right)} = \frac{K_I/\cancel{s}}{K_I/s + s^2 - 0.01s + 1} \frac{(\cancel{s})}{(s)}$$

$$\frac{Y}{Y_{sp}} = \frac{K_I}{K_I + s^3 - 0.01s^2 + s}$$

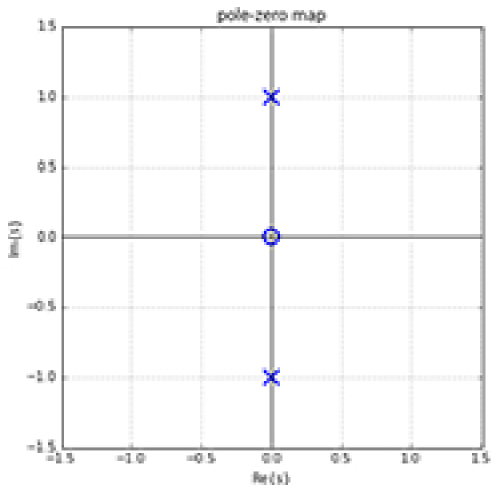
It looks like integral control doesn't help much either

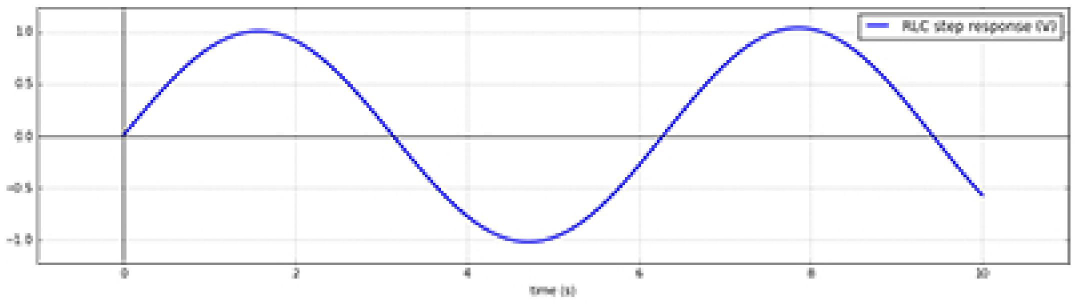
D. Show the effects of using derivative control $K(s) = K_D s$

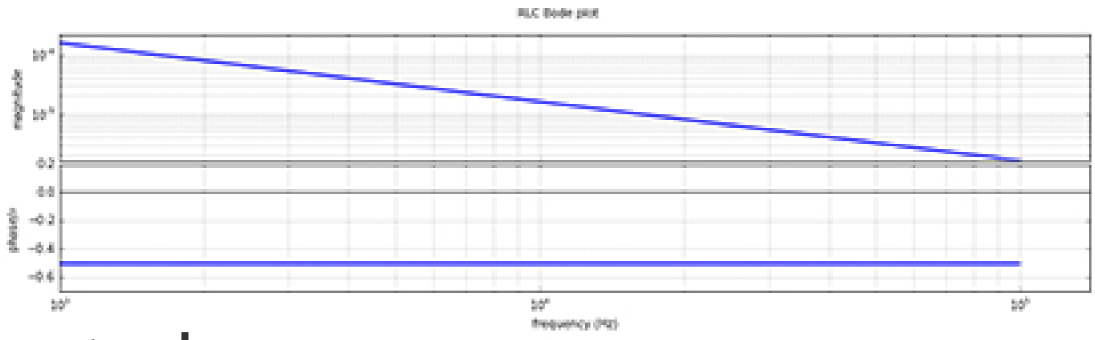
$$\frac{Y}{Y_{sp}} = \frac{K_D s H}{1 + K_D s H} = \frac{K_D s \left(\cancel{\frac{1}{s^2-0.01s+1}}\right)}{1 + K_D s\left(\frac{1}{s^2-0.01s+1}\right)} \frac{\left(\cancel{s^2-0.01s+1}\right)}{\left(s^2-0.01s+1\right)} = \frac{K_D s}{K_D s + s^2 - 0.01s + 1}$$

It looks like you need to use derivative control for this system. For high values of $K_D$ it basically does/follows the step.

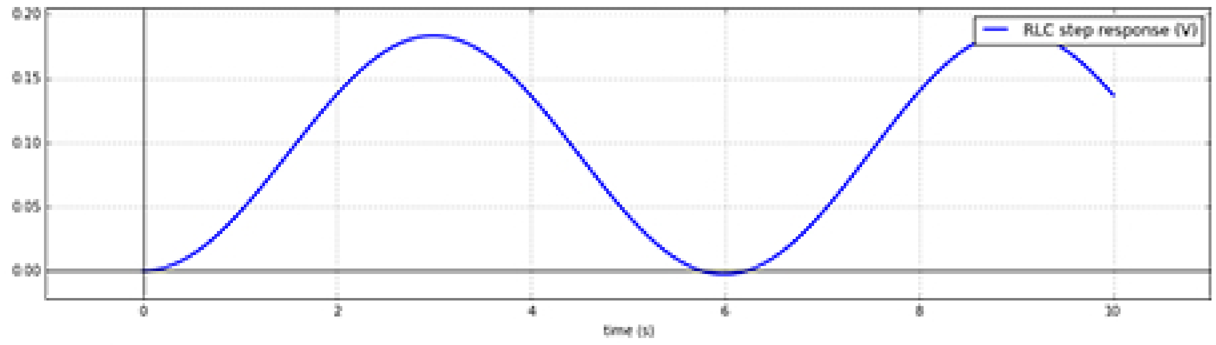
# Original System - Step response & Poles



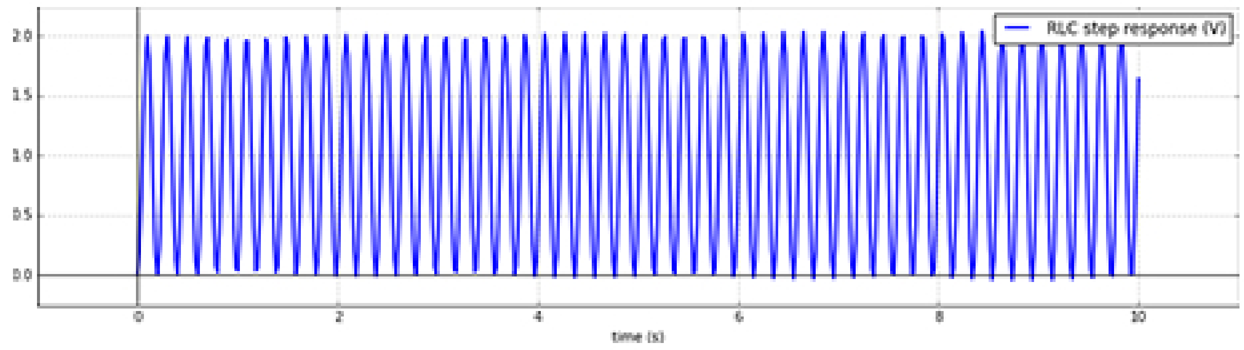




# Proportional Control

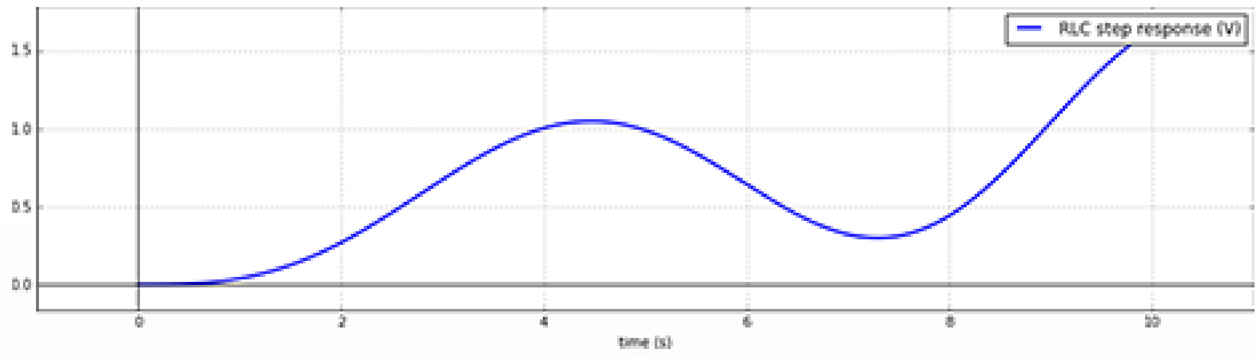



Kp = 1

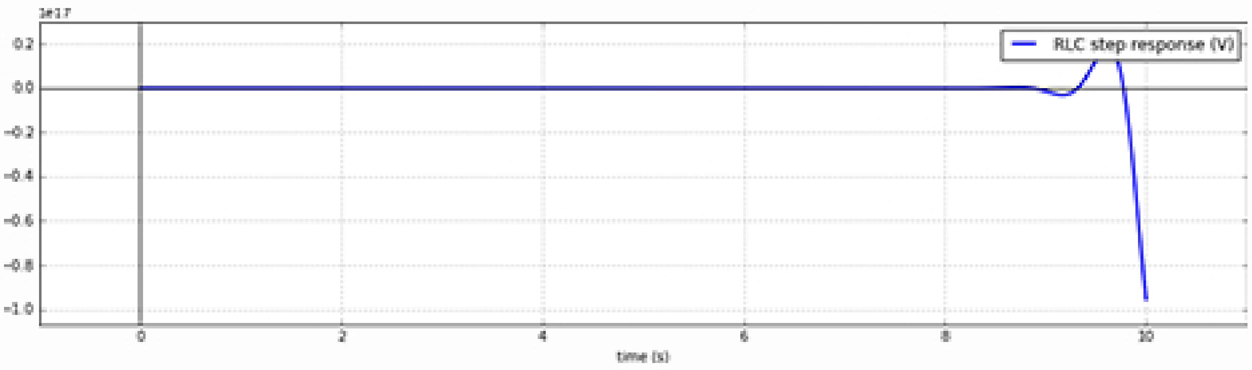



Kp = 1000

# Integral Control

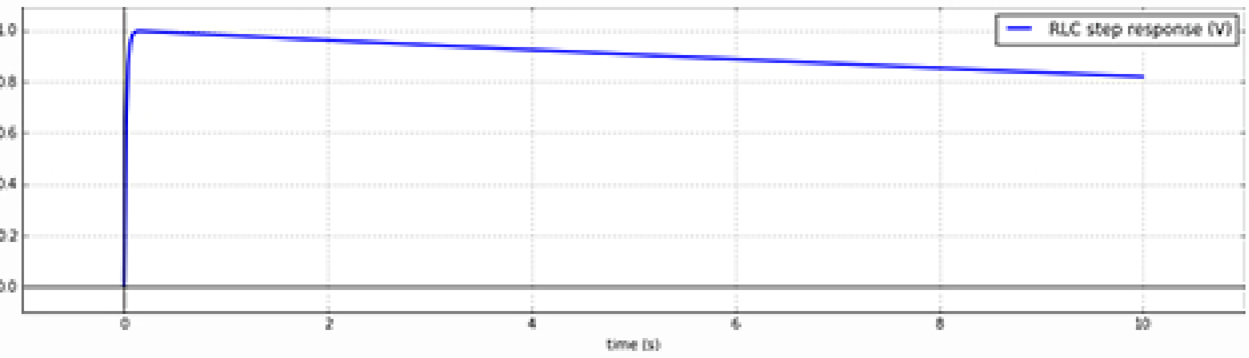



KI = 25

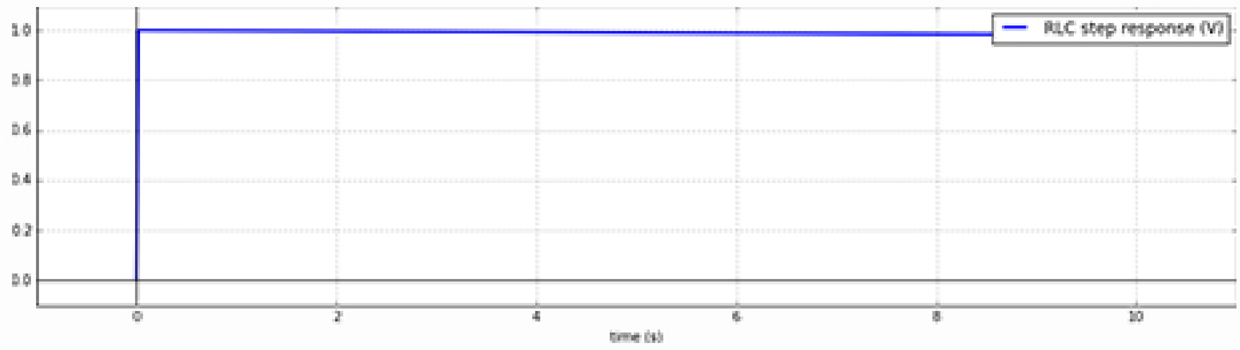



KI = 500

# Derivative Control




KD = 50




KD = 500