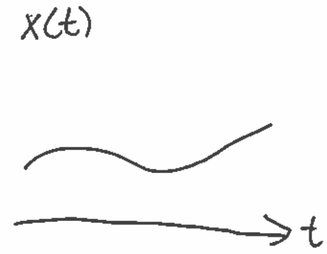
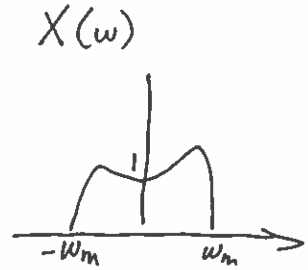
Problem Set 8 – Jiaying Wei

1. Consider a signal $x(t)$ which is bandwidth limited to ω_m



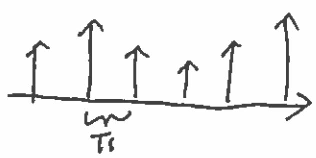


$p(t)$ is an impulse train w/ impulses seperated by T_s

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

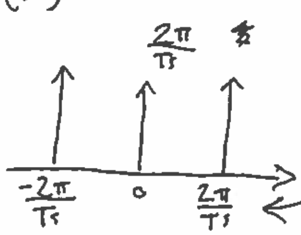
$$x_p(t) = x(t)p(t)$$

a) Sketch $x_p(t)$



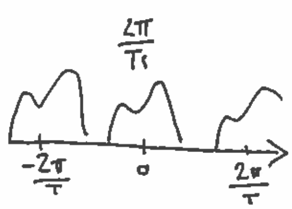
← impulse train w/ shape of $x(t)$ like discrete sampling

b) Sketch $P(\omega)$



frequency is one over period

c) Sketch $X_p(\omega)$

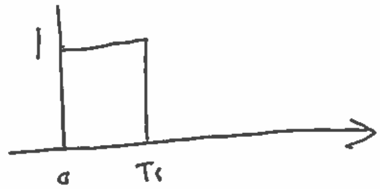


d) To ensure that you have all the information in $X_p(\omega)$ to be able to recover $X(\omega)$ you need to sample $\frac{2\pi}{T_s}$ of the original otherwise they will overlap and you can't unbake the cake.

e) You bandpass it

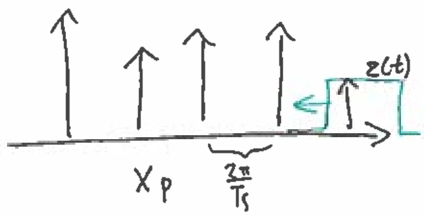
and then inverse CFFT it

f) Consider $z(t)$

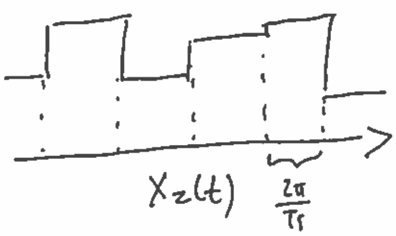


★ $x_z(t)$ is a zero order hold reconstruction of the sampled signal $x_p(t)$

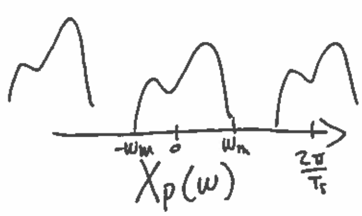
g) Sketch $x_z(t) = x_p * z(t)$

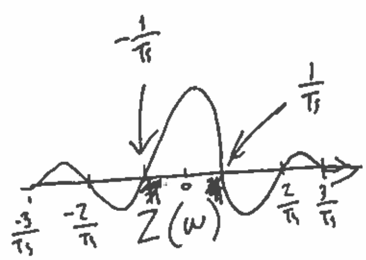


As $z(t)$ travels ← ; convolving w/ $x_p(t)$ it takes on the value of the impulse its covering

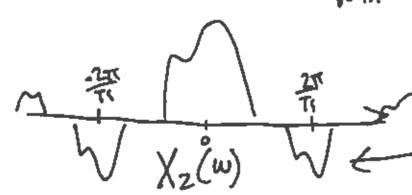


h) Sketch $X_z(\omega)$ which should be = to $X_p(\omega)Z(\omega)$



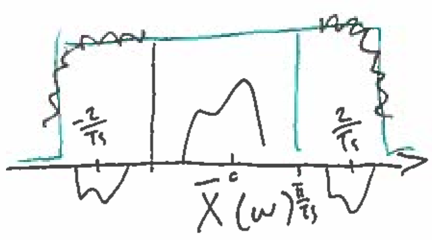


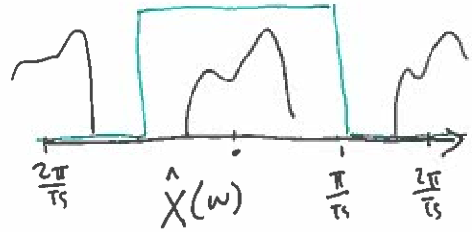
assuming that $w_m < \frac{1}{T_s}$

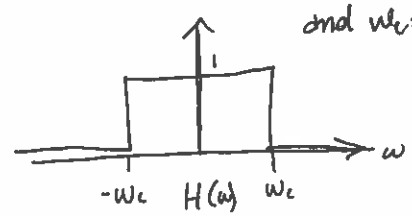


← not nessesarily upsidedown

i) Sketch ~~$\bar{X}$~~ $\bar{X}(\omega) = X_z(\omega)H(\omega)$ and $\hat{X}(\omega) = X_p(\omega)H(\omega)$ where $H(\omega)$ is







and $w_c = \frac{\pi}{T_s}$

j) ↑ this ones more distorted b/c

↑ this ones a little less distorted

sync

k) What is the ratio of $\bar{X}(w_m)$ to $\hat{X}(w_m)$

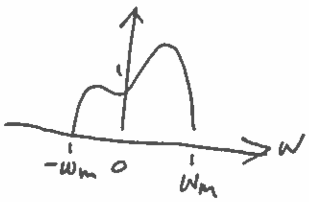
$w_m = \frac{\pi}{T_s}$

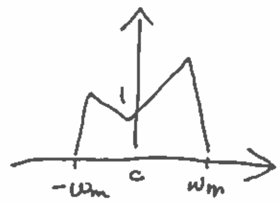
The ratio is 1:1

$\bar{X}(w_m) = 0$

$\hat{X}(w_m) = 0$

2. Consider $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$ $\quad \underbrace{X_1(\omega) = 0 \quad X_2(\omega) = 0}_{\text{if } |\omega| > \omega_m}$

$X_1(\omega)$ 

$X_2(\omega)$ 

assume

$\omega_1 >> \omega_m$

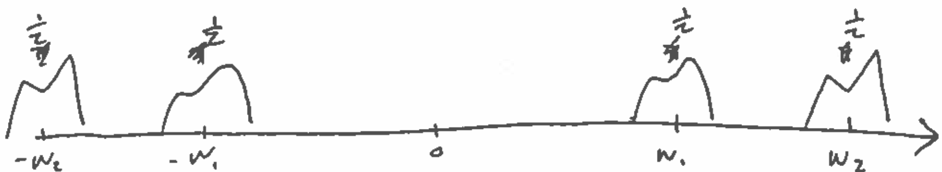
$\omega_2 >> \omega_m$

$\omega_1 + 2\omega_m < \omega_2$ ← they don't overlap

$\omega_2 > \omega_1$

a. Sketch $Y(\omega)$

$$Y(\omega) = X_1(\omega)\left(\pi\delta(\omega - \omega_1) + \pi\delta(\omega + \omega_1)\right)$$
$$+ X_2(\omega)\left(\pi\delta(\omega - \omega_2) + \pi\delta(\omega + \omega_2)\right)$$

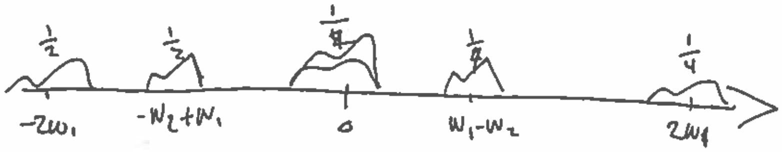


$Y(\omega)$

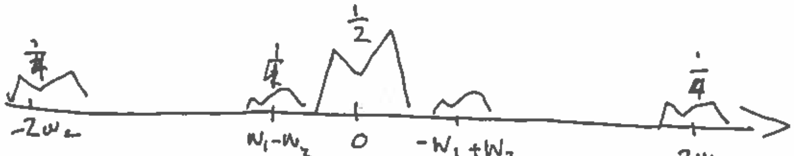
b. Sketch the Fourier Transforms of $y(t)\cos(\omega_1 t)$ and $y(t)\cos(\omega_2 t)$

$$FT\{y(t)\cos(\omega_1 t)\} = \begin{pmatrix} X_1(\omega)(\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1)) \\ + X_2(\omega)(\pi\delta(\omega-\omega_2) + \pi\delta(\omega+\omega_2)) \end{pmatrix}(\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1))$$

$$= X_1(\omega)(\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1))^2$$
$$+ X_2(\omega)(\pi\delta(\omega-\omega_2) + \pi\delta(\omega+\omega_2))(\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1))$$

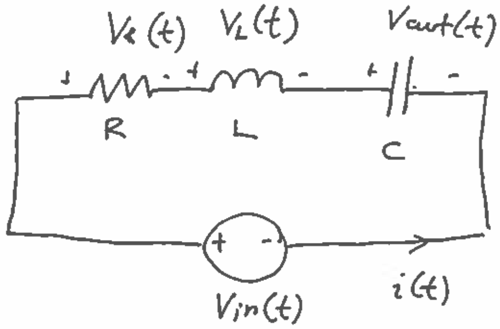


$FT\{y(t)\cos(\omega_1 t)\}$



c. We can recover $x_1(t)$ by bandpassing $FT\{y(t)\cos(\omega_1 t)\}$ and multiplying by 2. We can recover $x_2(t)$ by bandpassing $FT\{y(t)\cos(\omega_2 t)\}$ and multiplying by 2.

3. RLC System



$$i(t) = C\frac{d}{dt}V_{out}(t)$$

$$V_L(t) = L\frac{d}{dt}i(t)$$

a. Write a differential equation relating Vout and Vin(t)

$$V_R(t) + V_L(t) + \underline{V_{out}(t)} = \underline{V_{in}(t)}$$

$$iR + L\frac{d}{dt}i + V_{out}(t) = V_{in}(t)$$

$$C\frac{d}{dt}V_{out}(t)R + L\frac{d}{dt}C\frac{d}{dt}V_{out}(t) + V_{out} = V_{in}(t)$$

$$LC\frac{d^2}{dt}V_{out}(t) + RC\frac{d}{dt}V_{out}(t) + V_{out} = V_{in}(t)$$

b Find $H(\omega)$ the transfer function that relates $\frac{\text{input (fq)}}{\text{output (fq)}}$

$$LCj^2w^2V_{out}(w) + RCjwV_{out}(w) + V_{out}(w) = V_{in}(w)$$

$$V_{out}(w)\left(LCj^2w^2 + RCjw + 1\right) = V_{in}(w)$$

$$H(w) = \frac{V_{out}(w)}{V_{in}(w)} = \frac{1}{LCj^2w^2 + RCjw + 1} = \frac{1}{-LCw^2 + RCjw + 1}$$

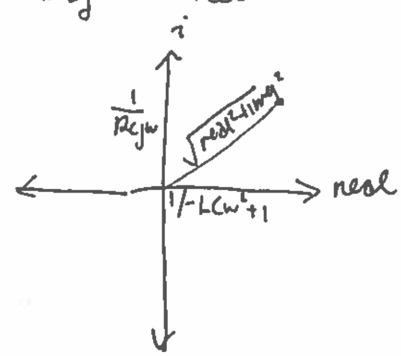
$$= \frac{1}{\underbrace{RCjw}_{imag} \underbrace{- LCw^2 + 1}_{real}}$$

c. Find the magnitude of $H(w)$

$$|H(w)| = \frac{1}{\sqrt{(RCjw)^2 + (-LCw^2+1)^2}}$$

$$\cancel{= \frac{1}{\sqrt{-R^2C^2w^2}}}$$

* You don't include the j in there because it's the axis kinda like if $-LCw^2+1$ real had a real attached to it



a. As a function of R L C find the value of w that maximizes $|H(w)|$

$$|H(w)| = \frac{1}{\sqrt{(RCw)^2 + (-LCw^2+1)^2}} = ((RCw)^2 + (-LCw^2+1)^2)^{-\frac{1}{2}}$$

$$\frac{d}{dw} H(w) = \cancel{\frac{-1}{2}(2RCw - 2 \cdot 2LC)^{-\frac{3}{2}}} \quad 2(-LCw^2+1) \cdot -2LCw$$

$$= -\frac{1}{2}((RCw)^2 + (-LCw^2+1)^2)^{-\frac{3}{2}}(2RCw - 4LC)$$

$$= -\frac{1}{2}((RCw)^2 + (-LCw^2+1)^2)^{-\frac{3}{2}}(2RCw + -4LCw(-LCw^2+1))$$

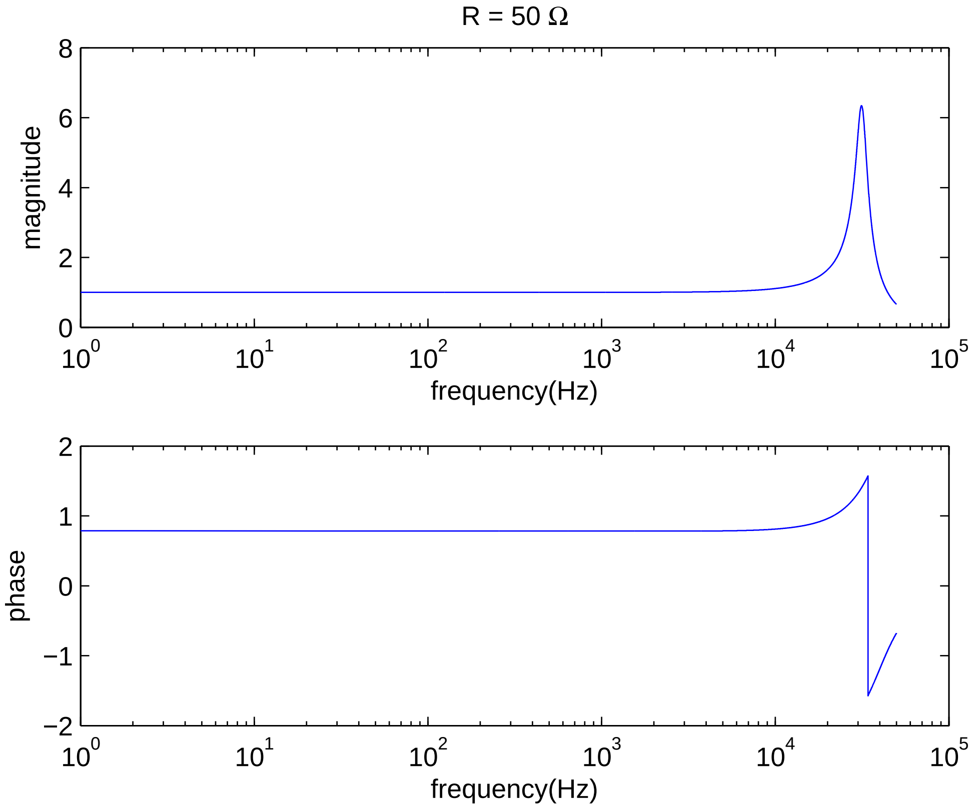
* So using the calculus

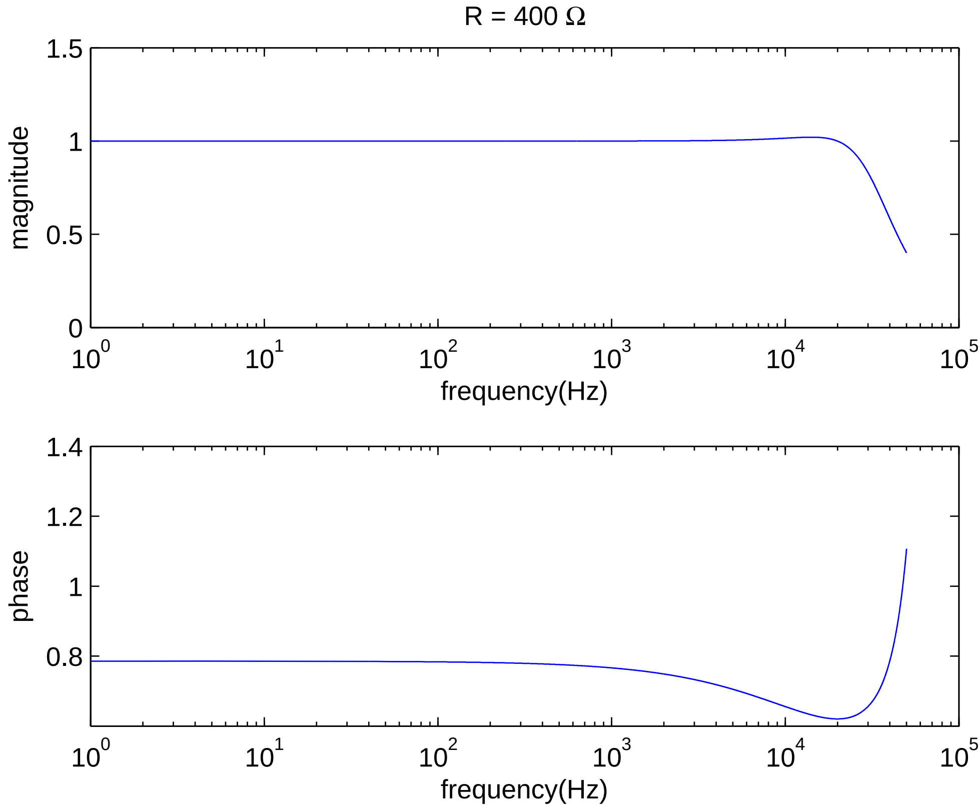
↳ when ~~theres a~~ the 1st derivative $=0$ theres a min/max

↳ then using Wolfram Alpha to find the root

$$\frac{d}{dw} H(w) = 0 \quad \text{when} \quad w = \frac{\sqrt{2L-R}}{\sqrt{2}\sqrt{C}L} \quad \text{or} \quad w = -\frac{\sqrt{2L-R}}{\sqrt{2}\sqrt{C}L} \quad \text{or} \quad w = 0$$

when $\sqrt{C}L \neq 0$ | when $\sqrt{C}L \neq 0$ | when $CL \neq 0$


R = 50 Ω
magnitude
frequency(Hz)
phase
frequency(Hz)


R = 400 Ω
1.5
1
0.5
0
magnitude
10^0 10^1 10^2 10^3 10^4 10^5
frequency(Hz)
1.4
1.2
1
0.8
phase
10^0 10^1 10^2 10^3 10^4 10^5
frequency(Hz)