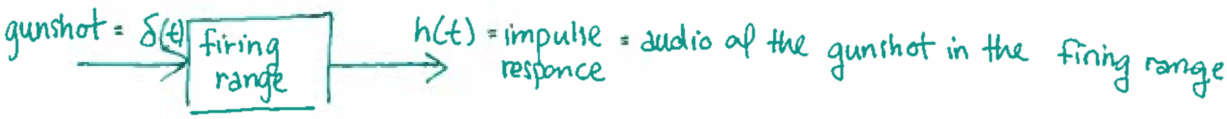
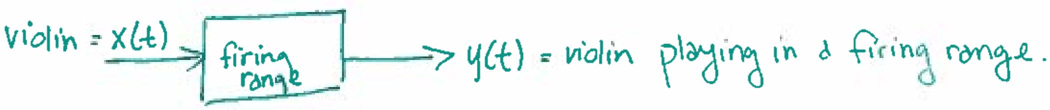
Sig Sys - Problem set 6 Ver 2

1. Impulse - a infinantly tall rectangle w/ area A (1 for unit impulses) that happens almost instantly

Impulse response - the result after you send the impulse through the system.





In order to find how the system changes the violin, you need to convolve the violin signal with the impulse response.

$$x(t) * h = y(t)$$

As you are convolving the 2, you are picking $x(\text{☺})$ and then multiplying and shifting based on your impulse response. The result is $y(t)$ which is how the violin sounds in the shooting range.

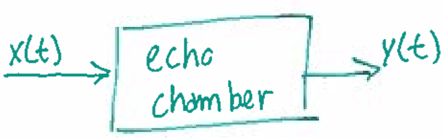
2. $y(t) = \frac{1}{2}x(t-1) + \frac{1}{4}x(t-10)$ ← this is a modle for an echo chamber

It moves the original signal to $t=1$ w/ an amplitude of $\frac{1}{2}$

It moves another copy to $t=10$ w/ an amplitude of $\frac{1}{4}$

It's an echo chamber because the signal "bounces" with decreasing amplitudes as it loses energy from the bounces.

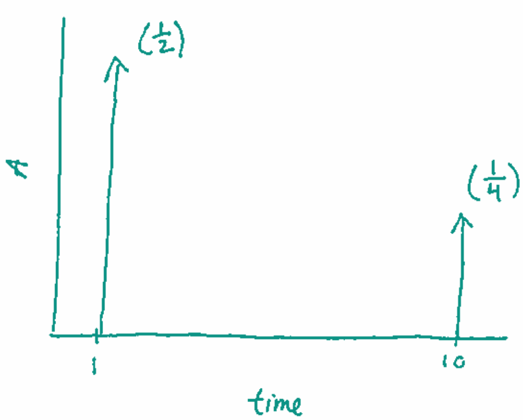
* We know that when we convolve with an impulse we multiply by the area of the impulse and move it to the location of the impulse.



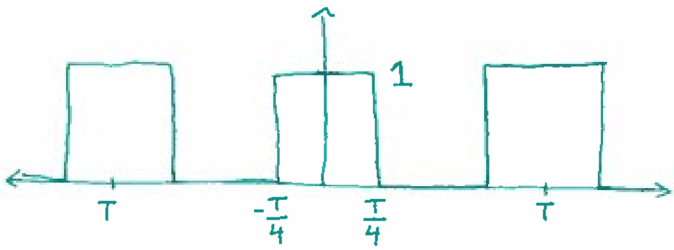
$$y(t) = x * h(t)$$

this impulse response must shift 2 copies of the signal and decrease the amplitudes

So impulse responce looks like:



3. Find the Fourier series rep for this square wave



$$x(t) = \begin{cases} 1 & @\ -\frac{T}{4} < t < \frac{T}{4} \\ 0 & @\ \text{else} \end{cases}$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$$ ← we assume this is true

$$\frac{1}{T}x(t)e^{-j\frac{2\pi}{T}mt} = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}\frac{1}{T} e^{-j\frac{2\pi}{T}mt}$$ ← we multiply both sides by $\frac{1}{T}e^{-j\frac{2\pi}{T}mt}$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}(k-m)t}$$ ← concatenate exponents

Integrate both sides w/ respect to t

$$\frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-j\frac{2\pi}{T}mt} = \frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}(k-m)t}\,dt$$

Switch the summation & the integral probably b/c adding is communative

(m is like a particular coefficiant in a set of all the coefficiants k)

$$= \sum_{k=-\infty}^{\infty}\frac{C_k}{T}\int_{-T/2}^{T/2} e^{j\frac{2\pi}{T}(k-m)t}\,dt$$

★ So when you do out the math you find that for any k that is not = m the value is 0 so the summation value only depends on when k = m

$$\frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-j\frac{2\pi}{T}mt} = \frac{C_m}{T}\int_{-T/2}^{T/2} e^{j\frac{2\pi}{T}(0)t}\,dt \qquad e^0 = 1$$

$$= \frac{C_m}{T}\cdot 1\Big|_{-T/2}^{T/2} = \frac{C_m}{T}\cdot\frac{T}{2}(1) - -\frac{T}{2}(1) = \frac{C_m}{T}\cdot$$

$$= C_m$$ ← equation for the mth Foureir series coefficiant

Finding an expression for C_k

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-j\frac{2\pi}{T}kt}\,dt \qquad x(t) = \begin{cases} 1 & @\ -\frac{T}{4} < t < \frac{T}{4} \\ 0 & @\ \text{else} \end{cases}$$

$$C_k = \frac{1}{T}\left(\int_{-T/2}^{-T/4} 0\cdot e^{-j\frac{2\pi}{T}kt}\,dt + \int_{-\frac{T}{4}}^{\frac{T}{4}} 1\cdot e^{-j\frac{2\pi}{T}kt}\,dt + \int_{+T/4}^{T/2} 0\cdot e^{-j\frac{2\pi}{T}kt}\,dt\right)$$

$$C_k = \frac{1}{T}\left(0 + \frac{1}{-j\frac{2\pi}{T}k}e^{-j\frac{2\pi}{T}kt}\Big|_{-\frac{T}{4}}^{\frac{T}{4}} + 0\right)$$

3 cont'

$$C_k = \frac{1}{T}\left(\frac{1}{-j\frac{2\pi}{T}k} e^{-j\frac{2\pi}{T}k\frac{T}{4}} - \frac{1}{-j\frac{2\pi}{T}k} e^{+j\frac{2\pi}{T}k\frac{T}{4}}\right)$$

$\theta = -\frac{\pi}{2}k$

$$C_k = \frac{1\cdot \cancel{T}}{-\cancel{T}\cdot 2\pi jk} e^{-j\frac{\pi}{2}k} - \frac{1\cdot \cancel{T}}{-j2\pi k\cancel{T}} e^{j\frac{\pi}{2}k}$$

$$\boxed{\sin(\theta) = \frac{1}{2j}e^{j\theta} - \frac{1}{2j}e^{-j\theta}}$$

$$C_k \frac{1}{-\pi k}\left(\frac{1}{2j}e^{j\theta} - \frac{1}{2j}e^{-j\theta}\right)$$

$$\boxed{C_k = \frac{1}{-\pi k}\sin\left(-\frac{\pi}{2}k\right) \qquad C_k = \begin{cases} 0 & \text{if even} \\ \frac{1}{-\pi k}\sin\left(-\frac{\pi}{2}k\right) & \text{if odd} \end{cases}}$$

c) In the three Fourier approximations of the square wave I notice that as I increase the # of Fourier coefficients the "square wave" becomes more defined. However it does appear that the approximate signal cannot jump from 0 to 1 immediately, that there must be some slope to make it a continuous function and that no matter what the Gibbs phenomenon ( ) will always be present. Even though the spikes will never go away, higher approximations of the wave still result in a more better approximation because the Gibbs spike lose area as they get smaller.

4. Suppose $x(t)$ is a periodic signal w/ fundamental period T, and has a Fourier series representation w/ coefficient C_k. Consider a new signal $y(t) = x(t-T_1)$ where $|T_1| < T$ such that $y(t)$ is a delayed version of $x(t)$. Find the C_ks of $y(t)$ in terms of C_k

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t-T_1)e^{-\frac{2\pi}{T}jkt}\,dt \qquad \tau = t - T_1$$

$$\frac{d\tau}{dt} = 1 \qquad d\tau = 1\,dt$$

$$C_{k_{new}} = \frac{1}{T}\int_{-T/2-T_1}^{T/2-T_1} x(\tau)e^{-\frac{2\pi}{T}jk(\tau+T_1)}\,d\tau$$

4 con't

* Because shifting the window of integration doesn't change the C_k for a periodic signal

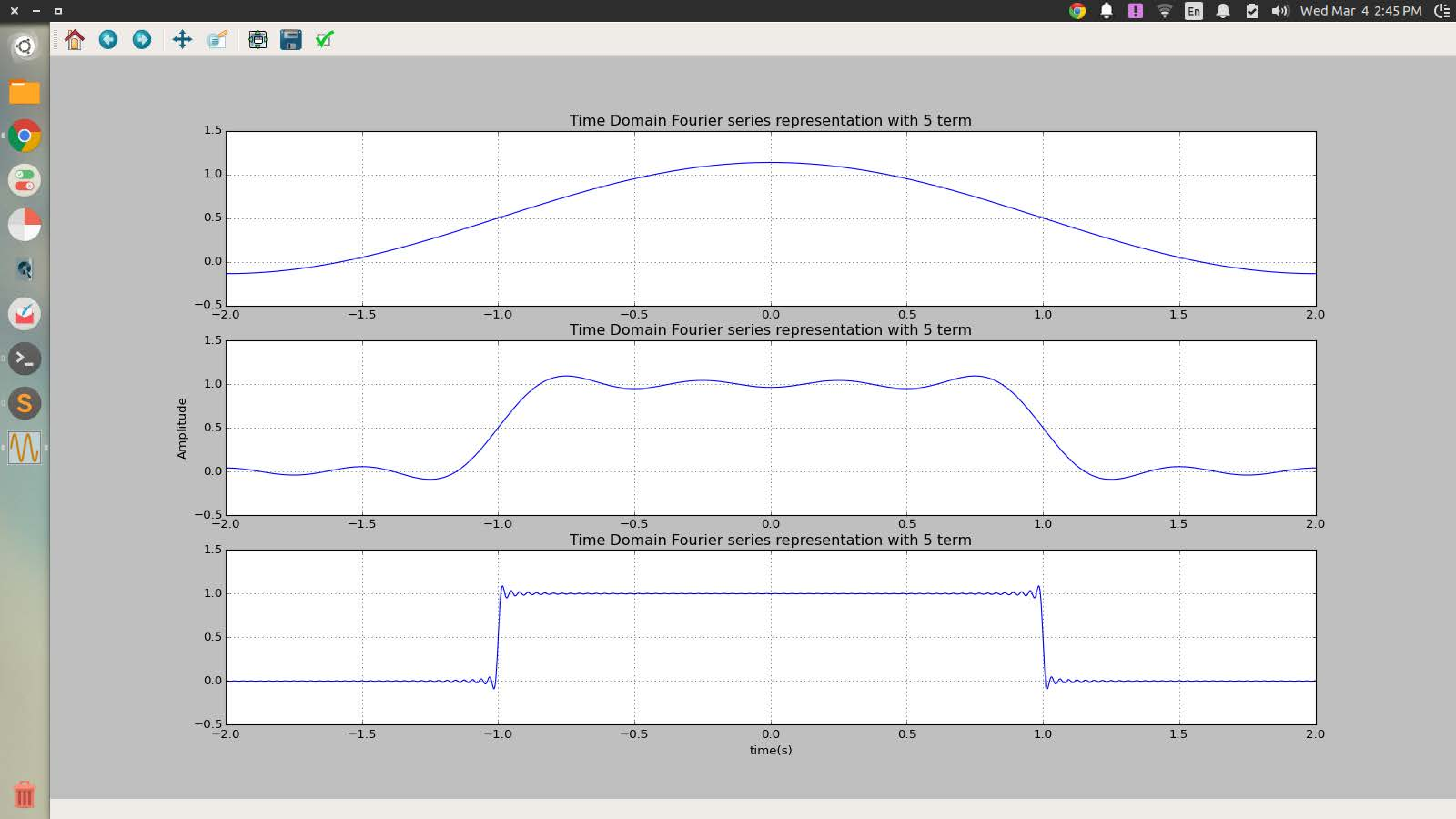
$$C_{k_{new}} = \frac{1}{T}\int_{-T/2}^{T/2} x(\tau)\, e^{-j\frac{2\pi}{T}k(\tau+T_1)}\, d\tau$$

that's just a phase shift

$$C_{k_{new}} = \frac{1}{T}\int_{-T/2}^{T/2} x(\tau)\, e^{-j\frac{2\pi}{T}k\tau}\, \underbrace{e^{-j\frac{2\pi}{T}kT_1}}_{\text{constant}}\, d\tau$$

$$C_{k_{new}} = \frac{1}{e^{j\frac{2\pi}{T}kT_1}} \cdot \underbrace{\frac{1}{T}\int_{-T/2}^{T/2} x(\tau)\, e^{-j\frac{2\pi}{T}k\tau}\, d\tau}_{=C_k}$$

$$\boxed{C_{k_{new}} = \frac{1}{e^{j\frac{2\pi}{T}kT_1}}\, C_k}$$


En
Wed Mar 4 2:45 PM
Time Domain Fourier series representation with 5 term
1.5
1.0
0.5
0.0
−0.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
Time Domain Fourier series representation with 5 term
Amplitude
Time Domain Fourier series representation with 5 term
time(s)

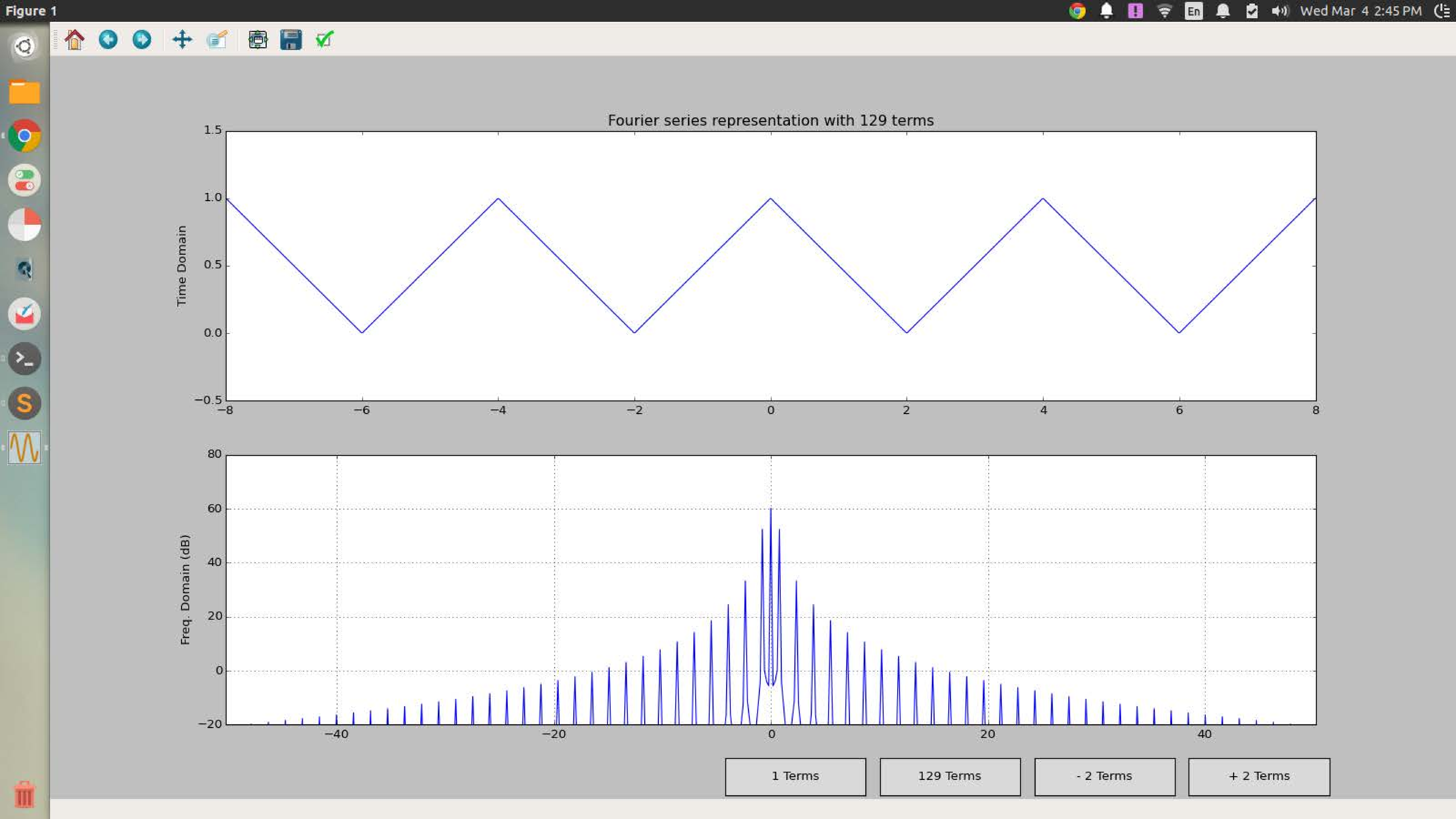


Figure 1
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Fourier series representation with 129 terms
Time Domain
Freq. Domain (dB)
1 Terms
129 Terms
- 2 Terms
+ 2 Terms