Principles of Communications (通信系统原理)

Undergraduate Course

Chapter 9: Optimum Filtering for Transceivers

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Chapter 9: Optimum Filtering for Transceivers

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Chapter 9: Optimum Filtering for Transceivers

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Introduction

- There are two signal filtering techniques which are of basic importance in digital communications.
- Pulse shaping for optimum transmission is concerned with filtering at the transmitter in order to minimize signal bandwidth.
- Pulse filtering for optimum reception is concerned with filtering at the receiver in order to maximize the SNR at the decision instant (and consequently minimize the probability of symbol error).

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Pulse Shaping for Optimum Transmission

Basics

- Letting R_s be the symbol rate, H be entropy (the average amount of information, measured in bits, conveyed per symbol), and B be the occupied bandwidth, then spectral efficiency, η_s, is defined as the rate of information transmission per unit of occupied bandwidth:

$$\eta_s = R_s H / B \quad \text{(bits/s/Hz)}$$

- The same term is also sometimes used for the quantity R_s/B (symbol/s/Hz or baud/Hz).
- Since spectrum is a limited resource, it is often desirable to minimize the bandwidth occupied by a signal of given baud rate.

Chapter 9: Optimum Filtering for Transceivers

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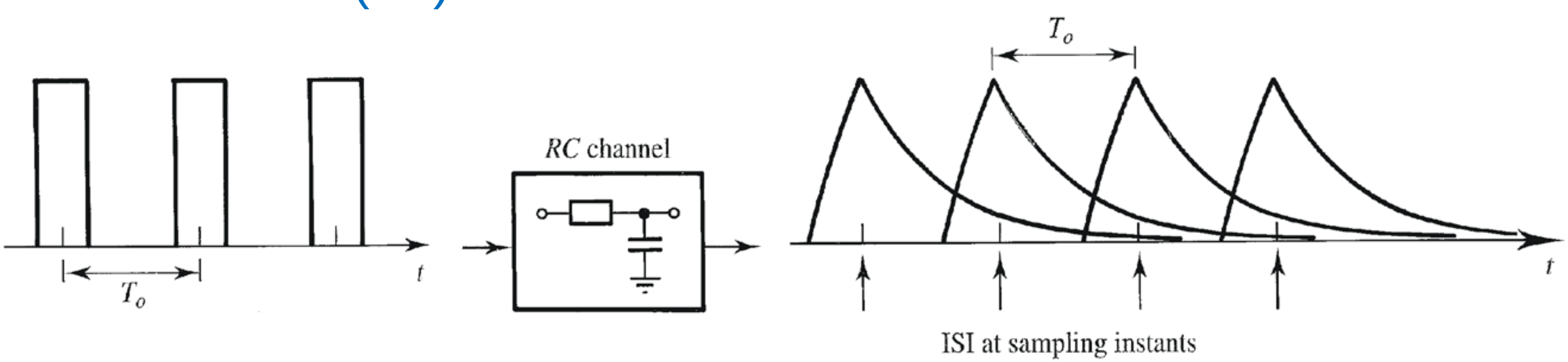
Pulse Shaping for Optimum Transmission
Inter-symbol Interference (ISI)

- Rectangular pulse signaling, in principle, has infinite bandwidth.
- In practice, rectangular pulses can be transmitted over channels with finite bandwidth if a degree of distortion can be tolerated.
- If distortion is severe enough, then pulses may overlap in time.
- The decision instant voltage might then arise not only from the current symbol pulse but also from one or more preceding pulses.
- The smearing of one pulse into another is called inter-symbol interference (ISI).



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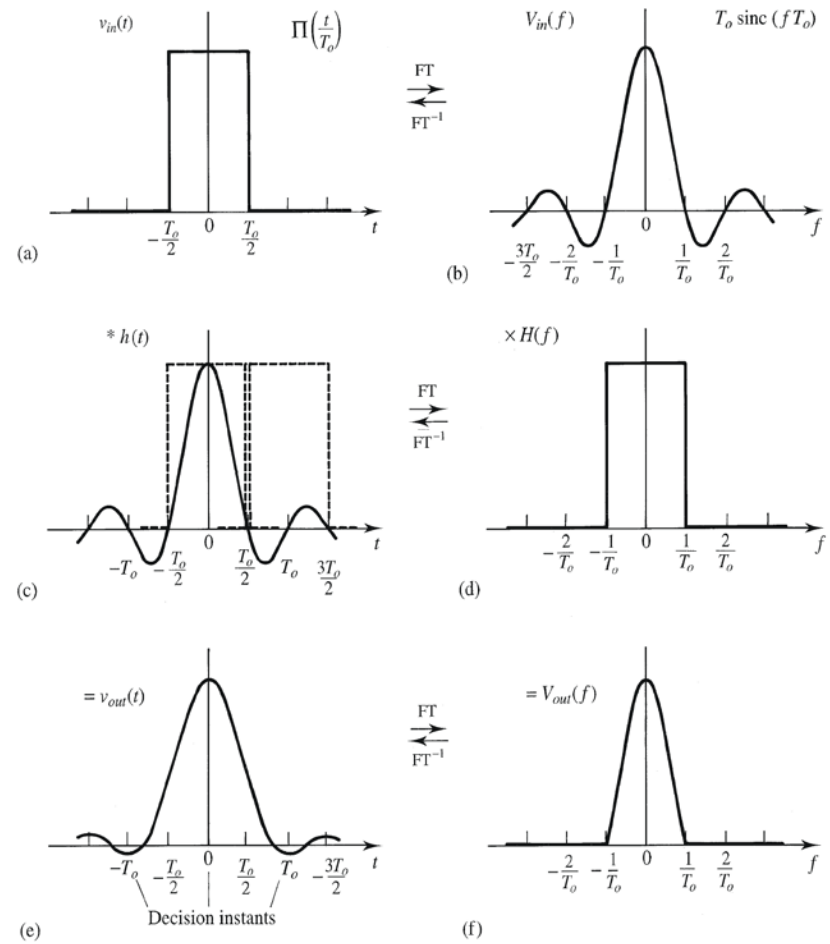
Pulse Shaping for Optimum Transmission
Bandlimiting of Rectangular Pulses

- The nominal bandwidth of a baseband, unipolar, NRZ signal with baud rate $R_s = 1/T_0$ symbol/s is $B = 1/T_0$ Hz.
- The RHS figure shows the effect of limiting a rectangular pulse to this bandwidth before transmission.
- At the center point sampling instants $(\ldots, -T_0, 0, T_0, 2T_0, \ldots)$ receiver decisions would be based not only on that pulse which should be considered but also, erroneously, on contributions from adjacent pulses.
- These unwanted contributions have the potential to degrade BER performance.



NRZ rectangular pulse distortion due to rectangular frequency response filtering

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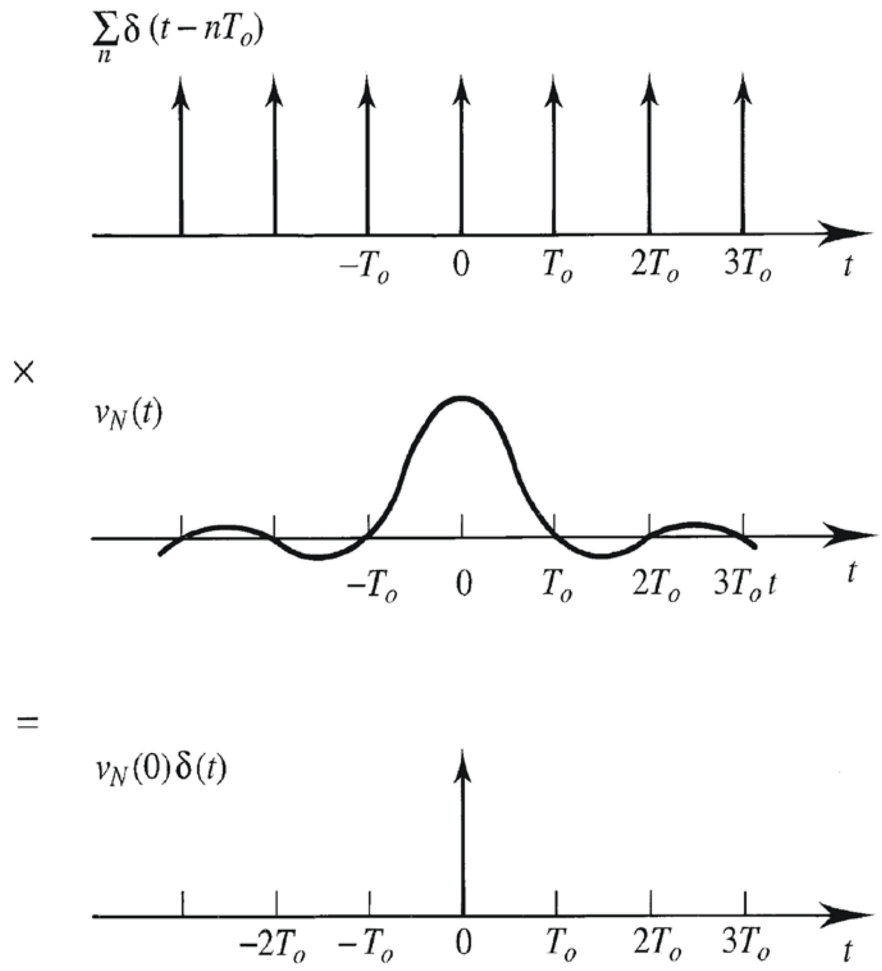
Pulse Shaping for Optimum Transmission

ISI-free Signals

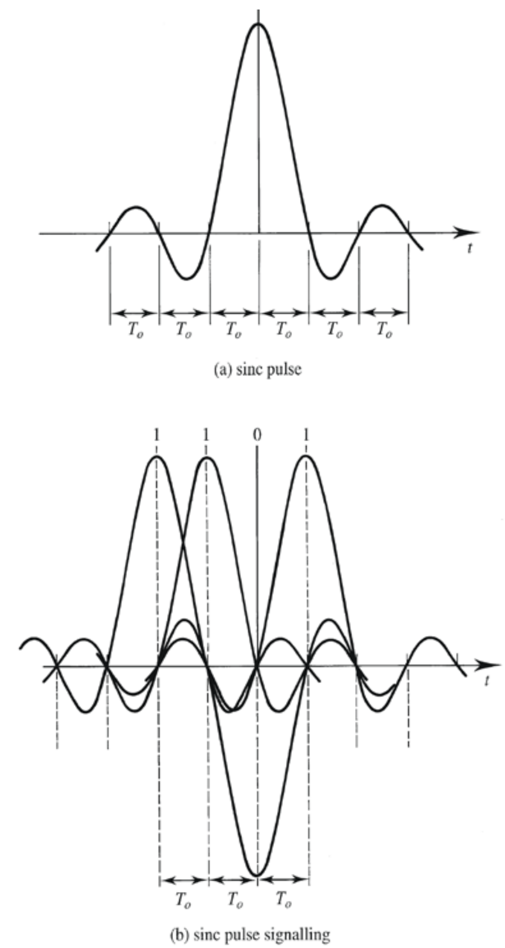
- Only decision instant ISI is relevant to the performance of digital communications systems. ISI occurring at times other than the decision instants does not matter.
- *An ISI-free signal is any signal which passes through zero at all but one of the sampling instants.*
- Denoting the ISI-free signal by $v_N(t)$ and the sampling instants by nT_0 (n is an integer and T_0 is the symbol period):

$$v_N(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = v_N(0)\delta(t)$$



Pulse Shaping for Optimum Transmission
ISI-free Signals

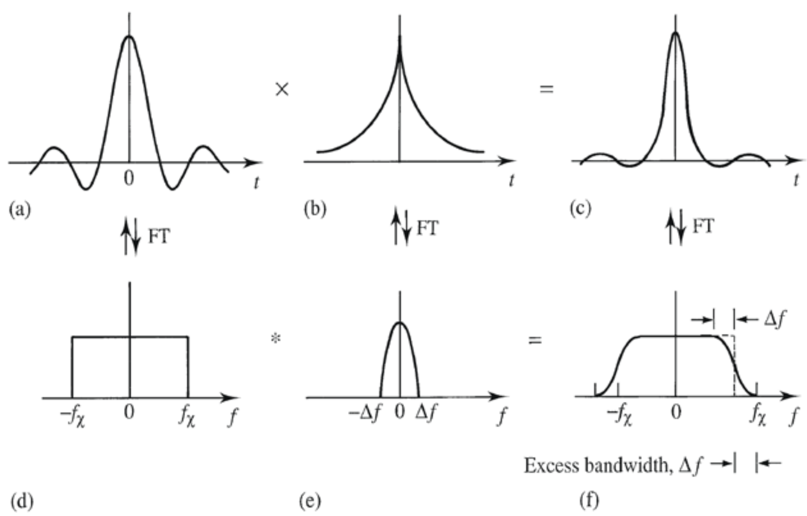
- A good example of an ISI-free signal is the sinc pulse.
- These pulses have a peak at one decision instant and are zero at all other decision instants as required.
- An OOK, multilevel, or analogue PAM system could, in principle, be implemented using sinc pulse signaling.
- In practice, there are two problems:
  - sinc pulses are not physically realizable
  - sinc pulse sidelobes (and their rates of change at the decision instants) are large and decay only with $1/t$. Thus extremely accurate decision timing would be required.



ISI free transmission using sinc pulses

Pulse Shaping for Optimum Transmission
ISI-free Signals

- A more practical signal pulse shape would retain the desirable feature of sinc pulses (i.e. regularly spaced zero crossing points) but have an envelope with much more rapid roll-off.
- This can be achieved by multiplying the sinc pulse with a rapidly decaying monotonic function.
- As long as the decaying function is real and even its spectrum will be real and even, which implies that the modified pulse spectrum will have *odd symmetry* about the sinc pulse's cut-off frequency, f_x.



Suppression of sinc pulse sidelobes and its effect on pulse spectrum

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Pulse Shaping for Optimum Transmission
Nyquist Vestigial Symmetry Theorem

- If the amplitude response of a low pass rectangular filter with linear phase and bandwidth f_x is modified by the addition of a real valued function having odd symmetry about the filter's cut-off frequency, then the resulting impulse response will retain at least those zero crossings presented in the original $\mathrm{sinc}(2f_x t)$ response, i.e. it will be an ISI-free signal.

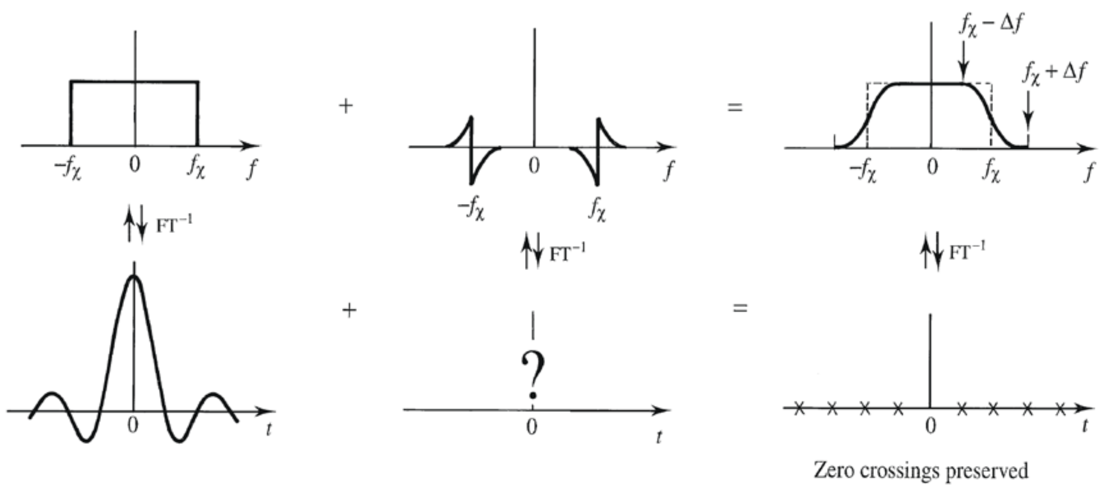

Illustration of Nyquist's vestigial symmetry theorem

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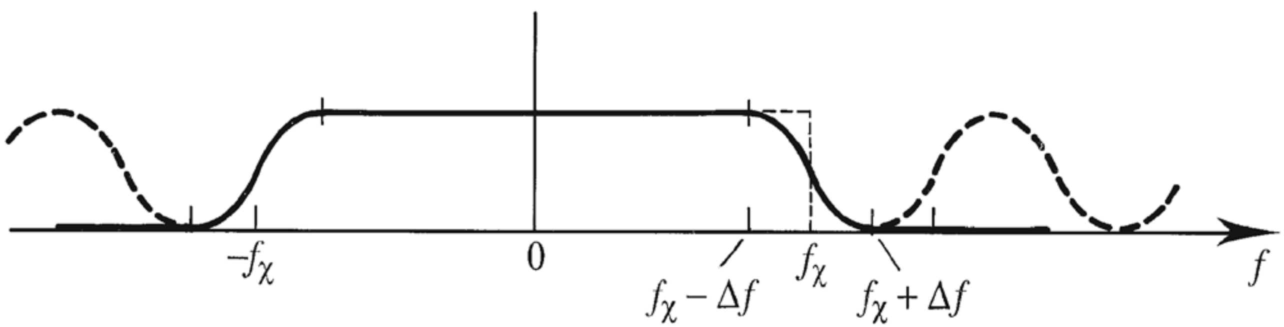
Pulse Shaping for Optimum Transmission
Raised Cosine Filtering

- The family of raised cosine filters is an important and popular subset of family of Nyquist filters.
- Their (low pass) amplitude response therefore has the following piecewise form:

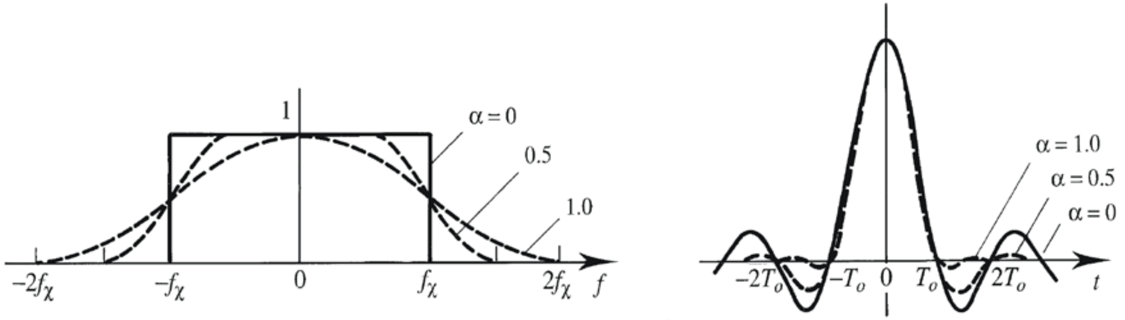
$$|H(f)| = \begin{cases} 1.0, & |f| \le f_x - \Delta f \\ \frac{1}{2}\left\{1 + \sin\left[\frac{\pi}{2}\left(1 - \frac{|f|}{f_x}\right)\frac{f_x}{\Delta f}\right]\right\}, & f_x - \Delta f < |f| < f_x + \Delta f \\ 0, & |f| \ge f_x + \Delta f \end{cases}$$



Amplitude response of (linear phase) raised cosine filter ($f_x = R_s/2$)

Pulse Shaping for Optimum Transmission
Raised Cosine Filtering

- f_x is the cut-off frequency of the prototype rectangular low pass filter, and is related to the symbol period, T_0, by $f_x = \frac{R_s}{2} = \frac{1}{2T_0}$.
- Δf is the excess bandwidth of the filter over the rectangular low pass prototype.
- The normalized excess bandwidth $\alpha = \frac{\Delta f}{f_x}$ is called the roll-off factor and can take any value between 0 and 1.



(a) Frequency response (b) Impulse response

Responses of raised cosine filters with three different roll-off factors

Pulse Shaping for Optimum Transmission
Raised Cosine Filtering

- When $\alpha = 1$ the characteristic is said to be a full raised cosine and the amplitude response is

$$|H(f)| = \begin{cases} \frac{1}{2}\left[1 + \cos\left(\frac{\pi f}{2f_x}\right)\right], & |f| \le 2f_x \\ 0, & |f| > 2f_x \end{cases} = \begin{cases} \cos^2\left(\frac{\pi f}{4f_x}\right), & |f| \le 2f_x \\ 0, & |f| > 2f_x \end{cases}$$

- The impulse response of a full raised cosine filter is

$$h(t) = 2f_x \frac{\sin 2\pi f_x t}{2\pi f_x t} \frac{\cos 2\pi f_x\, t}{1 - (4f_x t)^2}$$

- The first part represents the sinc impulse response of the prototype rectangular filter. The second part modifies this with extra zeros and faster decaying envelope.
- The absolute bandwidth of a baseband filter is $B = \frac{1}{2T_0}(1 +$

Pulse Shaping for Optimum Transmission

Raised Cosine Filtering

- **[Example 8.1]**
- What absolute bandwidth is required to transmit an information rate of 8.0 kbit/s using 64 level baseband signaling over a raised cosine channel with a roll-off factor of 40%?

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Pulse Shaping for Optimum Transmission
Nyquist Filtering for Rectangular Pulses

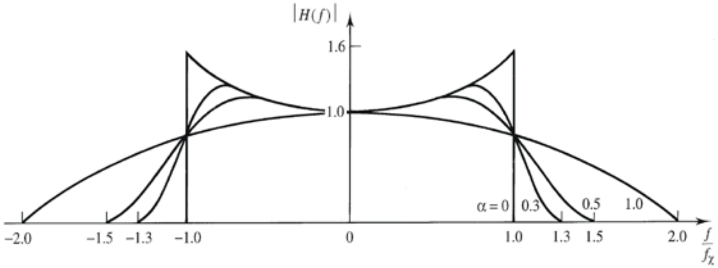
- In addition to generate ISI-free signals by shaping impulses with Nyquist filters, a more usual requirement is to generate such signals by shaping rectangular pulses.
- The pulse shaping filter then has a frequency response:

$$H(f) = \frac{V_N(f)}{\mathrm{sinc}(\tau f)}$$

where $V_N(f)$ is the voltage spectrum of an ISI-free pulse and $\mathrm{sinc}(\tau f)$ is the frequency response of a hypothetical filter which converts impulses into rectangular pulses with width τ.

- If $V_N(f)$ is chosen to have a full raised cosine shape, then

$$H(f) = \frac{\pi f \tau}{\sin(\pi f \tau)} \cos^2\left(\frac{\pi f}{4 f_X}\right)$$



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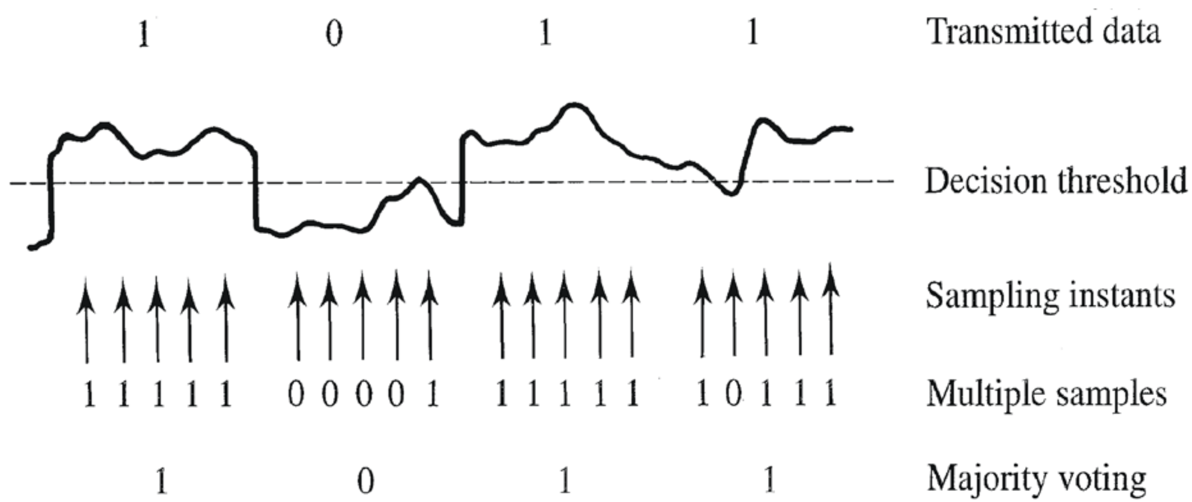
Pulse Shaping for Optimum Reception

Basics

- The center point decision process compares a single sample value of signal plus noise with an appropriate threshold.
- If several samples of the signal plus noise voltage are examined at different time instants within the duration of a single symbol, is it possible to obtain a more reliable decision?



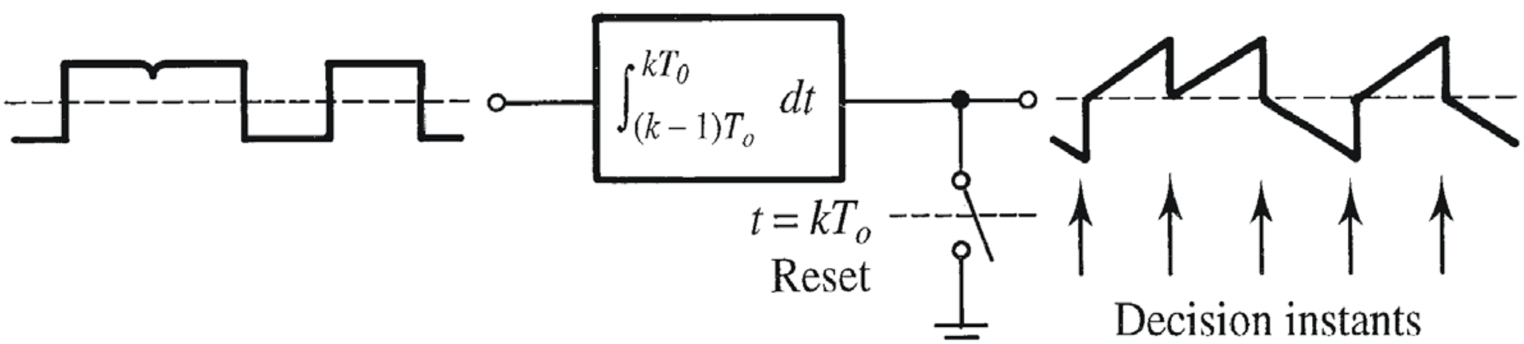
Pulse Shaping for Optimum Reception
Basics

- In addition to majority voting, if n samples were examined, the samples could be added together and compared with n times the appropriate threshold for a single sample.
- If this idea is extend to its limit (i.e. $n \to \infty$) then the discrete summation of symbol plus noise samples becomes continuous integration of the symbol plus noise voltage.
- The post-integration decision threshold then becomes $\frac{1}{2}\left(\int_0^{T_0} v_0 dt + \int_0^{T_0} v_1 dt\right)$ where v_0 and v_1 are the voltage levels representing binary zeros and ones respectively.
- After each symbol the integrator output would be reset to zero ready for the next symbol.

Pulse Shaping for Optimum Reception

Basics

- This signal processing technique is called integrate and dump (I+D) detection, and is the optimum detection process for baseband rectangular pulses in that the resulting probability of error is a minimum.
- I+D is a special case of a general and optimum type of detection process, which can be applied to any pulse shape, called matched filtering.



Integrate and dump detection for rectangular pulses

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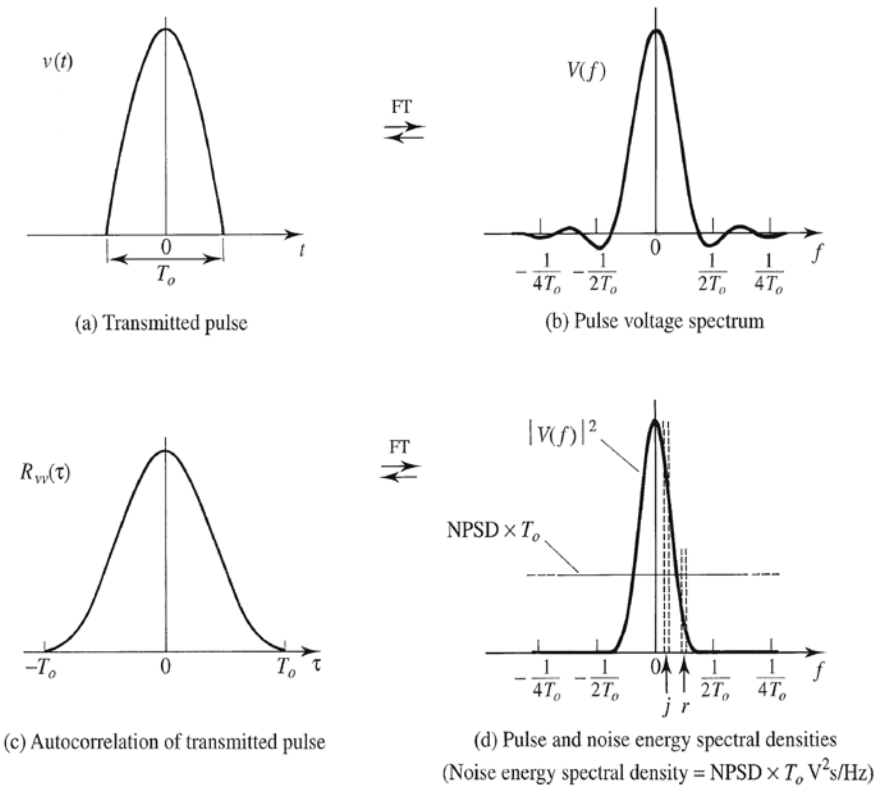
Pulse Shaping for Optimum Reception

Matched Filtering

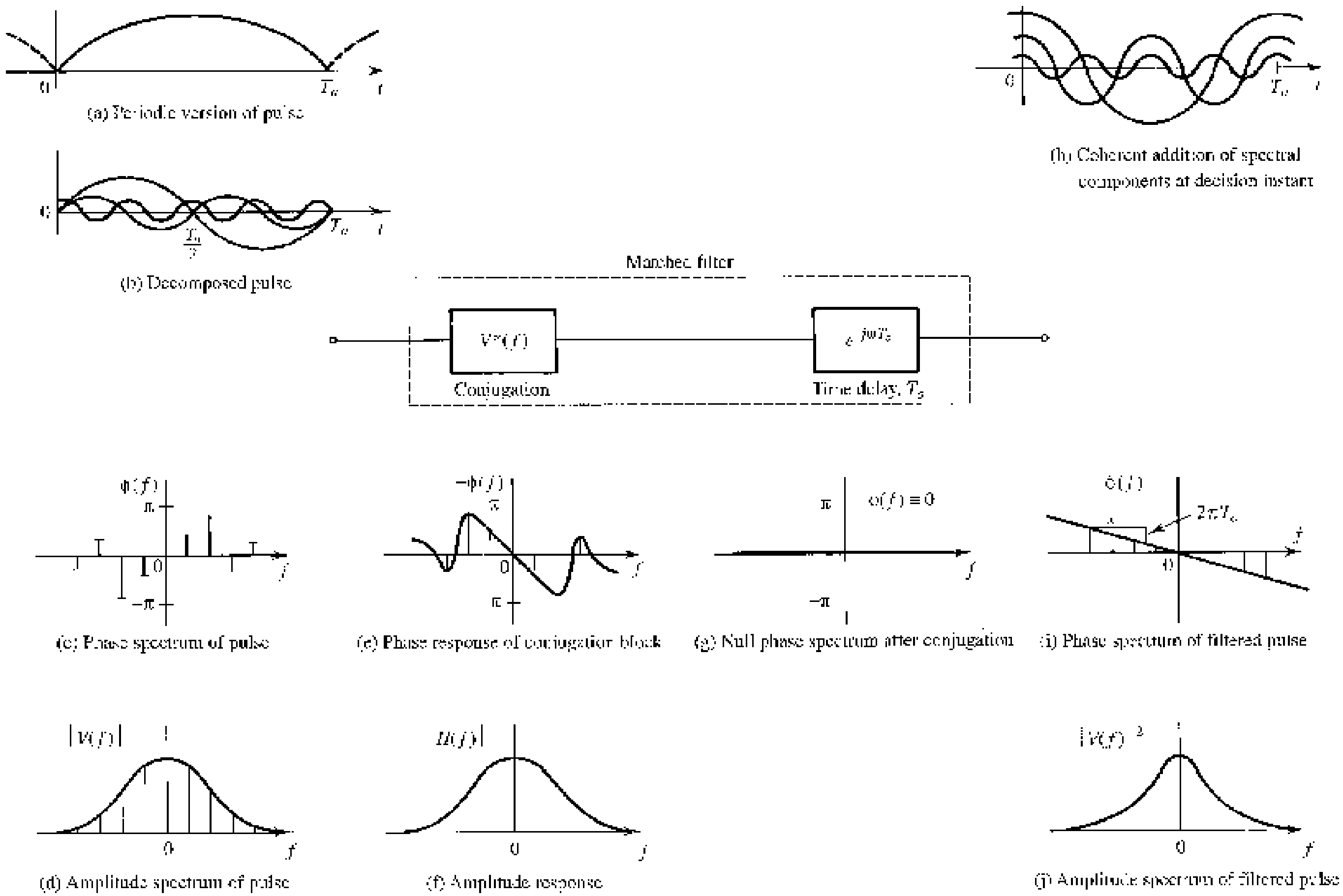
- A matched filter can be defined as:
  - A filter which immediately precedes the decision circuit in a digital communications receiver is said to be matched to a particular symbol pulse, if it maximizes the output SNR at the sampling instant when that pulse is present at the filter input.



The square of the amplitude response of a matched filter has the same shape as the energy spectral density of the pulse to which it is matched.

Pulse Shaping for Optimum Reception

Matched Filtering



The phase response of a matched filter is the negative of the phase spectrum of the pulse to which it is matched plus an additional linear phase of $-2\pi f T_0$ rad.

Pulse Shaping for Optimum Reception

Matched Filtering

- The matched filtering amplitude and phase criteria can be expressed mathematically as:

$$|H(f)|^2 = k^2|V(f)|^2$$

$$\phi(f) = -\phi_v(f) - 2\pi T_0 f \quad \text{(rad)}$$

where $\phi_v(f)$ is the phase spectrum of the expected pulse and k is a constant.

- The above equations can be combined into a single criterion:

$$H(f) = kV^*(f)e^{-j\omega T_0}$$

where the superscript $*$ indicates complex conjugation.

- For the case with non-white noise:

$$|H(f)| = \frac{k|V(f)|}{\sqrt{G_n(f)}}$$

where $G_n(f)$ is the NPSD. The phase response is identical to that for white noise.

Pulse Shaping for Optimum Reception
Matched Filtering

- **[Example 8.2]**
- Find the frequency response of the filter which is matched to the triangular pulse $\Lambda(t-1)$.

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Pulse Shaping for Optimum Reception
Correlation Detection

- The impulse response of a filter is related to its frequency response by the inverse Fourier transform:

$$h(t) = \int_{-\infty}^{\infty} H(f)e^{j2\pi ft}df$$

- This can be used to transform the matched filtering criterion into the time domain

$$h(t) = \int_{-\infty}^{\infty} kV^*(f)e^{j2\pi f(t-T_0)}df = k\left[\int_{-\infty}^{\infty} V(f)e^{j2\pi f(T_0-t)}df\right]^*$$

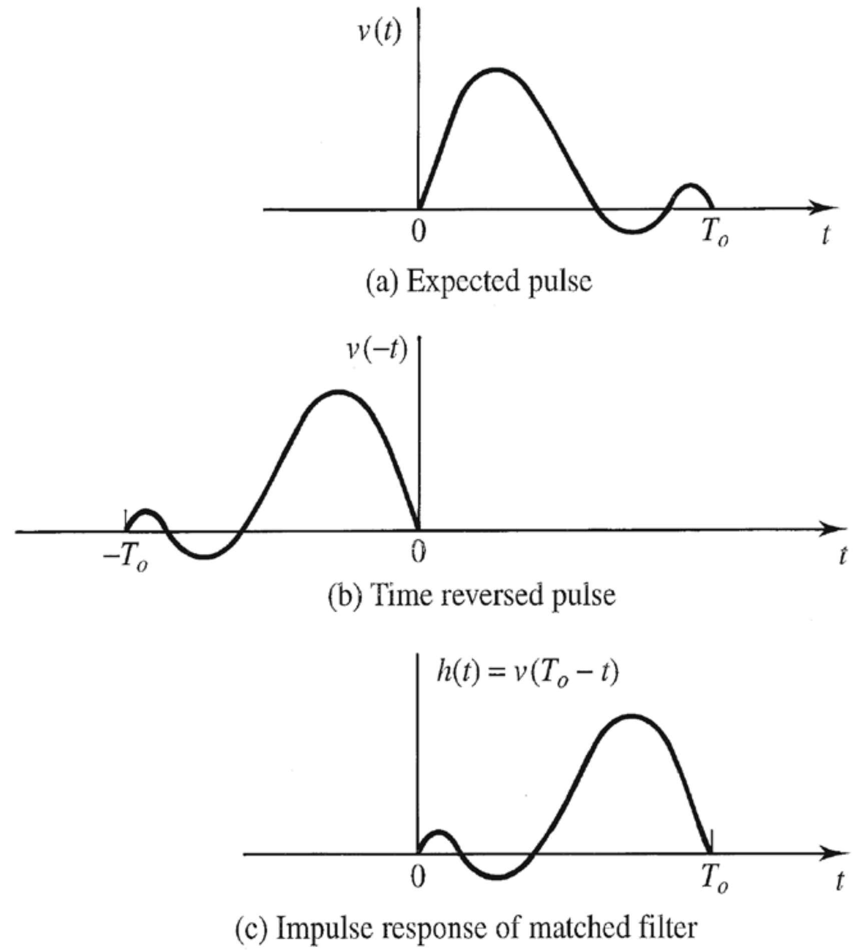
i.e.

$$h(t) = kv^*(T_0 - t)$$

Pulse Shaping for Optimum Reception

Correlation Detection

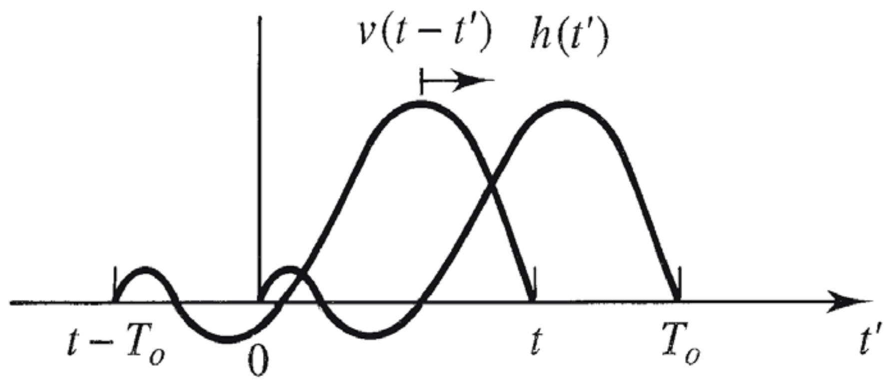
- For a purely real pulse, *the impulse response of a matched filter is a time reversed version of the pulse to which it is matched, delayed by a time equal to the duration of the pulse.*
- The time delay T_0 is needed to ensure causality and corresponds to the need for the linear phase factor.



Relationship between expected pulse and impulse response of matched filter

Pulse Shaping for Optimum Reception
Correlation Detection

- The output of any time invariant linear filter is its input convolved with it impulse response. The convolution process involves reversing one of the functions, sliding the reversed over the non-reversed function and integrating the product.
- The output of a matched filter is thus given by the integrated sliding product of either the input or the impulse response with an unreversed version of itself, i.e. the autocorrelation.



Pulse Shaping for Optimum Reception
Correlation Detection

- Let $v_{in}(t)$, $v_{out}(t)$, and $h(t)$ be the input, output and impulse response of a filter. Then by convolution: $v_{out}(t) = v_{in}(t) * h(t)$
- If the filter is matched to $v_{in}(t)$ then

$$v_{out}(t) = v_{in}(t) * kv_{in}(T_0 - t) = k\int_{-\infty}^{\infty} v_{in}(t')v_{in}(T_0 + t' - t)dt'$$

- Putting $T_0 - t = \tau$:

$$v_{out}(t) = v_{out}(T_0 - \tau) = k\int_{-\infty}^{\infty} v_{in}(t')v_{in}(t' + \tau)dt' = kR_{v_{in}v_{in}}(\tau)$$
$$\left(= kR_{hh}(\tau)\right)$$

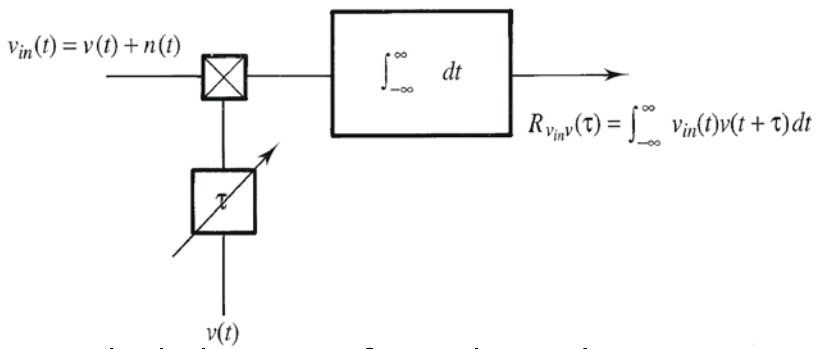
- Therefore, *the output of a filter driven by, and matched to, a real input pulse is, to within a multiplicative constant,* k*, and a time shift,* T_0*, the autocorrelation of the input pulse.*

- **[Example 8.3]**
- What will be the output of a filter matched to rectangular input pulses with width 1.0 ms?

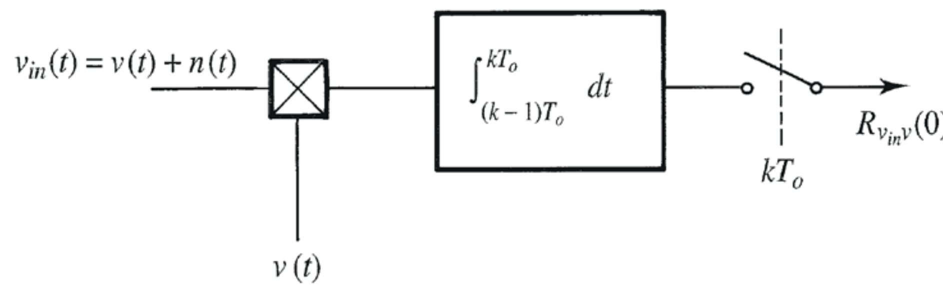
Pulse Shaping for Optimum Reception

Correlation Detection

- The correlation property of a matched filter can be realized directly in the time domain.



Block diagram of signal correlator



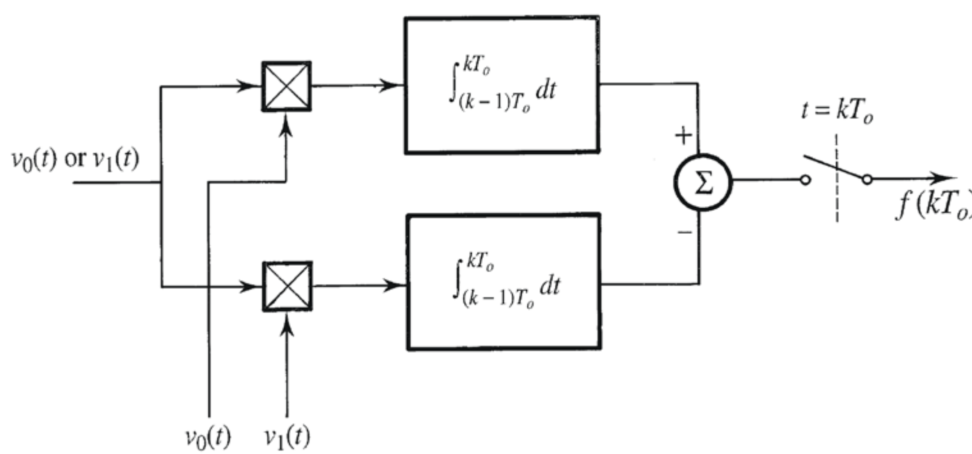
Signal correlator for digital communications receiver.

- In digital communications the variable delay τ is usually unnecessary since the pulse arrival times are normally known.
- Only the peak value of the correlation function, $R_{v_{in}v}(0)$, is important.
- The correlator output reaches this maximum value at the end of the input pulse, i.e. after T_0 seconds, which represents the correct sampling instant (leading to an optimum/maximum decision SNR).

Pulse Shaping for Optimum Reception

Correlation Detection

- A single matched filter is adequate as a detector in the case of on-off keyed (OOK) systems since the output will be a maximum when a pulse is present and essentially zero (ignoring noise) when no pulse is present.
- For binary systems employing two non-zero pulses a possible implementation would include two filters or correlators, one matched to each pulse type.



Pulse Shaping for Optimum Reception

Correlation Detection

- If the filter output is denoted by $f(t)$ then the possible sampling instant output voltages are:

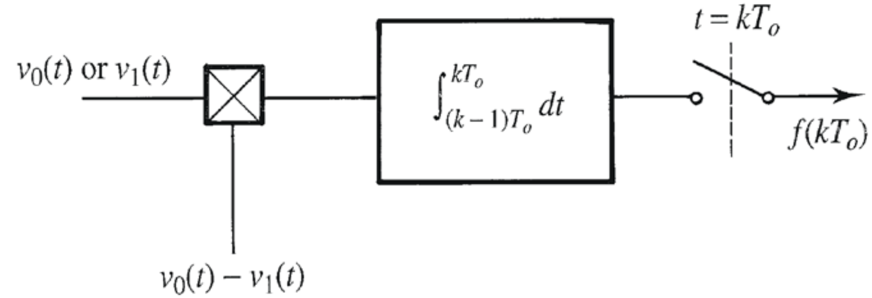
$$f(kT_0) = \begin{cases} \int_0^{T_0} v_0^2(t)dt - \int_0^{T_0} v_0(t)v_1(t)dt & \text{if symbol 0 is present} \\ -\int_0^{T_0} v_1^2(t)dt + \int_0^{T_0} v_1(t)v_0(t)dt & \text{if symbol 1 is present} \end{cases}$$

- If the signal pulses $v_0(t)$ and $v_1(t)$ are orthogonal but contain equal normalized energy $E_s\ V^2$s, then the sampling instant voltages will be $\pm E_s$.
- If the pulses are antipodal (i.e. $v_1(t) = -v_0(t)$) then the sampling instant voltages will be $\pm 2E_s$.

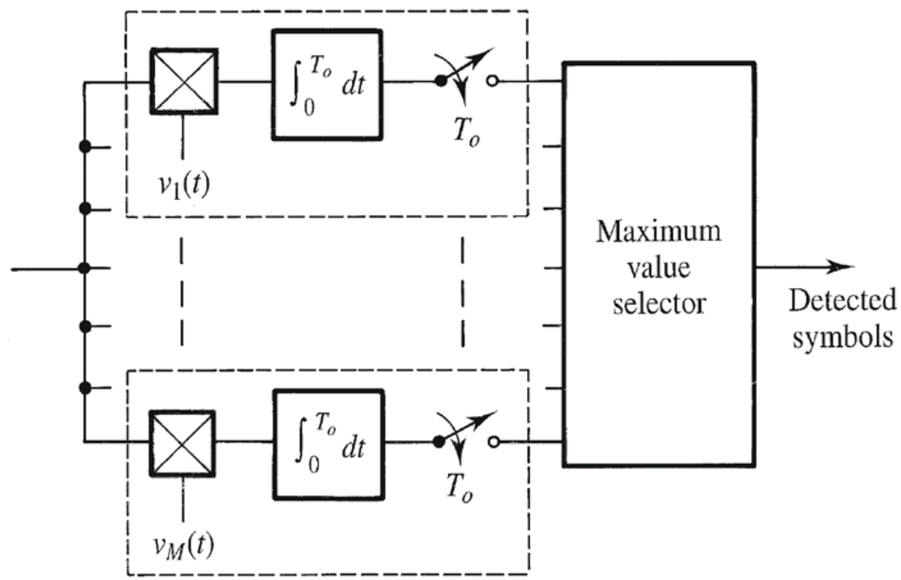
Pulse Shaping for Optimum Reception
Correlation Detection

- The same output voltages can be generated for all (orthogonal, antipodal or other) binary pulse systems using only one filter or correlator by matching to the pulse difference signal, $v_1(t) - v_0(t)$.

- For multisymbol signaling the number of channels in the matched filter or correlation receiver can also be extended.



One channel binary symbol correlator



Multichannel correlator for reception of M-ary signals

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Pulse Shaping for Optimum Reception

Decision Instant SNR

- Considering the two-channel, binary symbol correlator with orthogonal signal pulses, since the correlator output voltage is $f(kT_0) = E_s$ (V), the sampling instant normalized signal power is

$$|f(kT_0)|^2 = E_s^2 \ \ (V^2)$$

- At the input to the multiplier the autocorrelation function (ACF) of noise $n(t)$ is

$$R_{nn}(\tau) = \langle n(t)n(t+\tau)\rangle \ \ (V^2)$$

- Assuming that $n(t)$ is white with double sided PSD $N_0/2$ (V^2/Hz) then its ACF can be calculated, by taking the inverse Fourier transform, as $R_{nn}(\tau) = \frac{N_0}{2}\delta(\tau)$ (V^2).
- The ACF of the noise after multiplication with $v(t)$ is

$$R_{xx}(\tau) = \langle x(t)x(t+\tau)\rangle = \langle n(t)v(t)n(t+\tau)x(t+\tau)\rangle \ \ (V^2)$$

where $x(t) = n(t)v(t)$.

Pulse Shaping for Optimum Reception
Decision Instant SNR

- Since $n(t)$ and $v(t)$ are independent processes, we have

$$R_{xx}(\tau) = \langle n(t)n(t+\tau)\rangle\langle v(t)v(t+\tau)\rangle = \frac{N_0}{2}\delta(\tau)R_{vv}(\tau)$$

$$= \frac{N_0}{2}\delta(\tau)R_{vv}(0) = \frac{N_0}{2}\delta(\tau)\frac{1}{T_0}\int_0^{T_0} v^2(t)dt = \frac{N_0}{2}\frac{E_s}{T_0}\delta(\tau)\ (\mathrm{V}^2)$$

- Using the Wiener-Kintchine theorem, the two-sided PSD of $x(t) = n(t)v(t)$ is the Fourier transform of $R_{xx}(\tau)$, i.e.:

$$G_x(f) = \frac{N_0}{2}\frac{E_s}{T_0} \quad (\mathrm{V}^2/\mathrm{Hz})$$

Decision Instant SNR

- The impulse response of a device which integrates from 0 to T_0 seconds is a rectangle of unit height and T_0 seconds duration.
- The frequency response of this time windowed integrator is the Fourier transform of its impulse response, i.e. $H(f) = T_0 \text{sinc}(T_0 f) e^{-j\omega T_0/2}$ and its amplitude response is therefore

$$|H(f)| = T_0 |\text{sinc}(T_0 f)|$$

- The NPSD at the integrator output is

$$G_y(f) = G_x(f)|H(f)|^2 = \frac{N_0}{2} E_s T_0 \text{sinc}^2(T_0 f)$$

- The total noise power at the correlator output is

$$N = \frac{N_0}{2} E_s T_0 \int_{-\infty}^{\infty} \text{sinc}^2(T_0 f) df = \frac{N_0}{2} E_s \ (\text{V}^2)$$

Pulse Shaping for Optimum Reception

Decision Instant SNR

- The standard deviation of the noise at the correlator output is

$$\sigma = \sqrt{N} = \sqrt{\left(\frac{N_0}{2} E_s\right)} \quad (V)$$

- The decision instant signal to RMS noise voltage ratio is

$$\frac{f(T_0)}{\sigma} = \sqrt{\left(\frac{2E_s}{N_0}\right)}$$

- The decision instant signal to noise power ratio is

$$\frac{S}{N} = \frac{|f(T_0)|^2}{\sigma^2} = \frac{2E_s}{N_0}$$

- *The decision instant SNR at the output of a correlation receiver (or matched filter) depends only on pulse energy and input NPSD. It is independent of pulse shape.*

Pulse Shaping for Optimum Reception

Decision Instant SNR

- **[Example 8.4]**
- What is the sampling instant SNR at the output of a filter matched to a triangular pulse of height 10 mV and width 1.0 ms, if the noise at the input to the filter is white with a PSD of 10 nV^2/Hz?

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Pulse Shaping for Optimum Reception

BER Performance of Optimum Receivers

- Considering a single channel correlator matched to the binary symbol difference, $v_1(t) - v_0(t)$, when a binary 1 is present at the receiver input the decision instant voltage at the output is given by:

$$f(kT_0) = \int_{(k-1)T_0}^{kT_0} v_1(t)[v_1(t) - v_0(t)]dt = E_{s1} - \int_{(k-1)T_0}^{kT_0} v_1(t)v_0(t)dt$$

  where E_{s1} is the binary 1 symbol energy.

- When a binary 0 is present:

$$f(kT_0) = -E_{s0} + \int_{(k-1)T_0}^{kT_0} v_1(t)v_0(t)dt$$

  where E_{s0} is the binary 0 symbol energy.

- The second term represents the correlations between symbols.

Pulse Shaping for Optimum Reception

BER Performance of Optimum Receivers

- Defining the normalized correlation coefficient to be

$$\rho = \frac{1}{\sqrt{E_{s0}E_{s1}}} \int_{(k-1)T_0}^{kT_0} v_1(t)v_0(t)dt$$

then

$$f(kT_0) = \begin{cases} E_{s1} - \rho\sqrt{E_{s0}E_{s1}} & \text{for binary 1} \\ -E_{s0} + \rho\sqrt{E_{s0}E_{s1}} & \text{for binary 0} \end{cases}$$

- The difference in decision instant voltages representing binary 1 and 0 is

$$\Delta V = E_{s1} + E_{s0} - 2\rho\sqrt{E_{s0}E_{s1}}$$

- The energy, E_s', in the reference pulse of the single channel correlator is

$$E_s' = \int_0^{T_0} [v_1(t) - v_0(t)]^2 dt = \Delta V$$

Pulse Shaping for Optimum Reception

BER Performance of Optimum Receivers

- The RMS noise voltage at the output of the receiver is

$$\sigma = \sqrt{\left(\frac{N_0}{2}E_s'\right)} = \left[\frac{N_0}{2}\left(E_{s1} + E_{s0} - 2\rho\sqrt{E_{s1}E_{s0}}\right)\right]^{\frac{1}{2}}$$

- Hence for binary symbols of equal energy, E_s:

$$\frac{\Delta V}{\sigma} = \left[\frac{2}{N_0}\left(E_{s1} + E_{s0} - 2\rho\sqrt{E_{s1}E_{s0}}\right)\right]^{\frac{1}{2}} = 2\sqrt{\frac{E_s}{N_0}(1-\rho)}$$

- Substituting it into the formula of symbol error probability of the center point sampling, $P_e = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2}}\right)\right]$, gives

$$P_e = \frac{1}{2}\left\{1 - \operatorname{erf}\sqrt{\frac{E_s}{2N_0}(1-\rho)}\right\}$$

- For orthogonal signaling schemes (including OOK), $\rho = 0$. For antipodal schemes ($v_1(t) = -v_0(t)$), $\rho = -1$.

Pulse Shaping for Optimum Reception
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- **[Example 8.5]**
- A baseband binary communications system transmits a positive rectangular pulse for digital ones and a negative triangular pulse for digital zeros. If, for both pulses at an ideal correlation receiver, the widths are 1.0 ms, the peak pulse voltages are 10 mV, and noise power spectral density is 10 nV^2/Hz, find the probability of bit error.

Chapter 9: Optimum Filtering for Transceivers

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Pulse Shaping for Optimum Reception

Comparison of Baseband Matched Filtering and Center Point Detection

- For unipolar NRZ transmission, the detection instant $\Delta V/\sigma$ after matched filtering is related to that for simple center point detection (CPD) by

$$\left(\frac{\Delta V}{\sigma}\right)_{MF} = \left(\frac{2E_{s1}}{N_0}\right)^{\frac{1}{2}} = \left(\frac{2\Delta V^2 T_0}{\sigma^2/B}\right)^{\frac{1}{2}} = \sqrt{2}(T_0 B)^{1/2}\left(\frac{\Delta V}{\sigma}\right)_{CDP}$$

where T_0 is the rectangular pulse duration and B is the CPD predetection (rectangular) bandwidth.

- The saving of transmitter power that matched filtering provides for compared with CPD is therefore

$$\frac{(\Delta V/\sigma)_{MF}}{(\Delta V/\sigma)_{CPD}} = \sqrt{2}(T_0 B)^{1/2} = 3.0 + (T_0 B)_{dB} \quad \text{(dB)}$$

- A CPD predetection bandwidth of three times the baud rate ($B = 3/T_0$), for example, gives a power saving of 7.8 dB.

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Summary

- Two types of optimum filtering are important to digital communications.
  - Nyquist filtering constrains the bandwidth of a signal whilst avoiding sampling instant ISI at the decision circuit input.
  - Matched filtering maximizes the sampling instant SNR at the decision circuit input.
  - Both types of filtering are optimum, in the sense that they minimize the probability of bit error.
- The amplitude response of a Nyquist filter has odd symmetry about its -6 dB frequency points and its phase response is linear.
- The amplitude response of matched filter is proportional to the amplitude spectrum of the symbol to which it is matched and its phase response is, to within a linear phase factor, opposite to the phase spectrum of the symbol. Correlation detection is matched filtering implemented in the time domain.

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Chapter 9: Optimum Filtering for Transceivers

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