Principles of Communications (通信系统原理)

Undergraduate Course

Chapter 6: Sampling, Multiplexing and PCM

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Chapter 6: Sampling, Multiplexing and PCM
Contents

1. Pulse Modulation
2. Sampling
   – Natural and Flat Topped Sampling
   – Nyquist’s Criterion
   – Aliasing
   – Practical Sampling and Reconstruction
   – Bandpass Sampling
3. Analogue Pulse Multiplexing
4. Quantized PAM and Pulse Code Modulation
   – Linear PCM
   – Companded PCM
   – Delta PCM, Differential PCM, Delta Modulation
5. Summary

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Pulse Modulation

- Pulse modulation: the process whereby the amplitude, width or position of individual pulses in a periodic *pulse train* are varied (i.e. modulated) in sympathy with the amplitude of a baseband information signal
- Pulse modulation represents an intermediate stage in the generation of digitally modulated signals
  - Pulse modulation is not a digital but an analogue technique

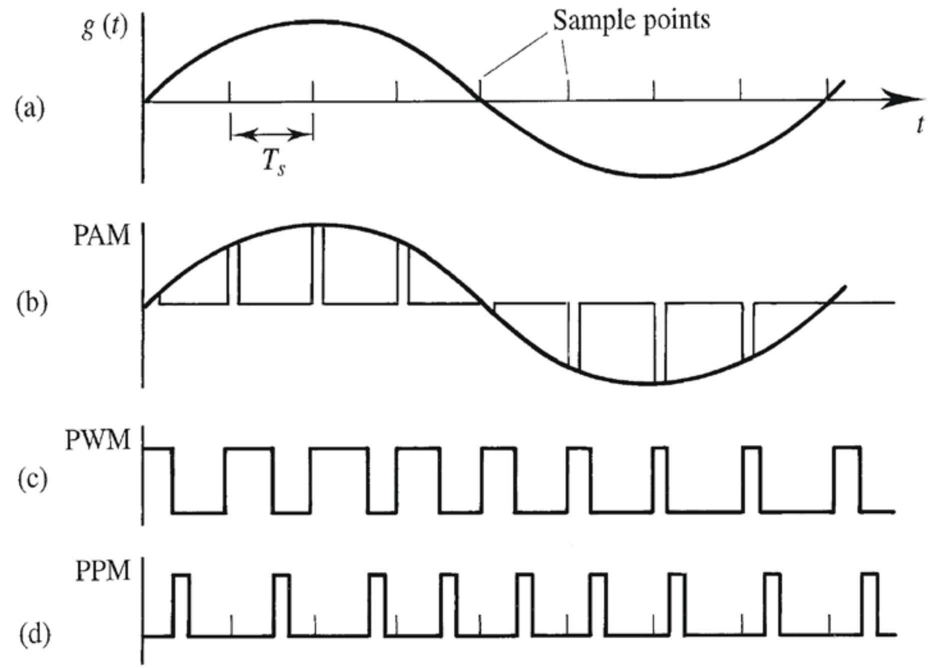
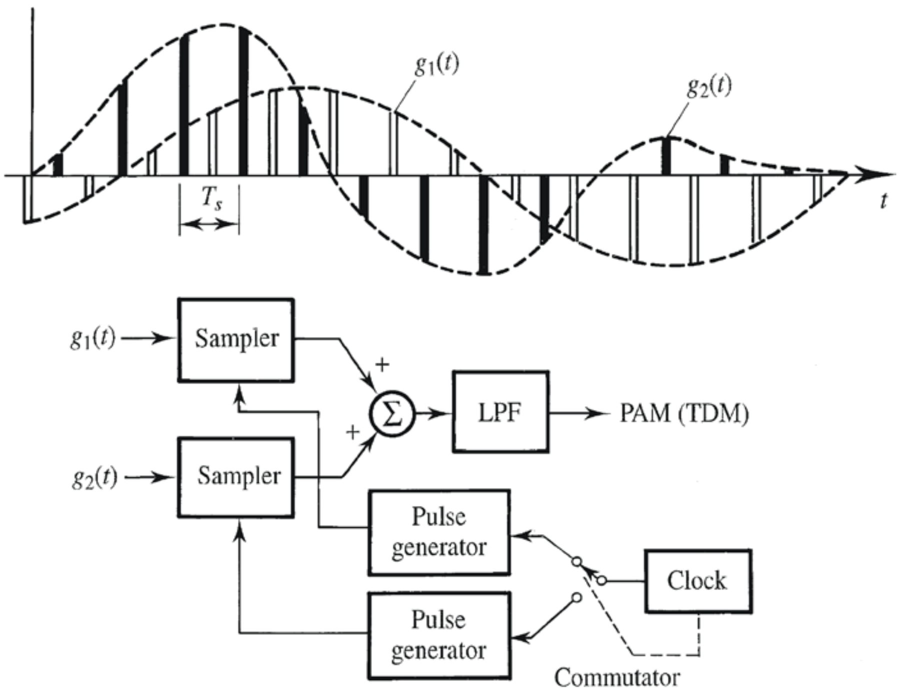


Illustration of pulse amplitude modulation (PAM), pulse width modulation (PWM) and pulse position modulation (PPM), and (a) is input (information) signal

Pulse Modulation

- The minimum pulse rate representing each information signal must be twice the highest frequency present in the signal's spectrum.
- This condition, called Nyquist sampling criterion, must be satisfied if proper reconstruction of the original continuous signal from the pulses is to be possible.
- Pulse modulation may allow many separate information carrying signals to share a single physical channel by interleaving the individual signal pulses.



Time division multiplexing of two PAM signals

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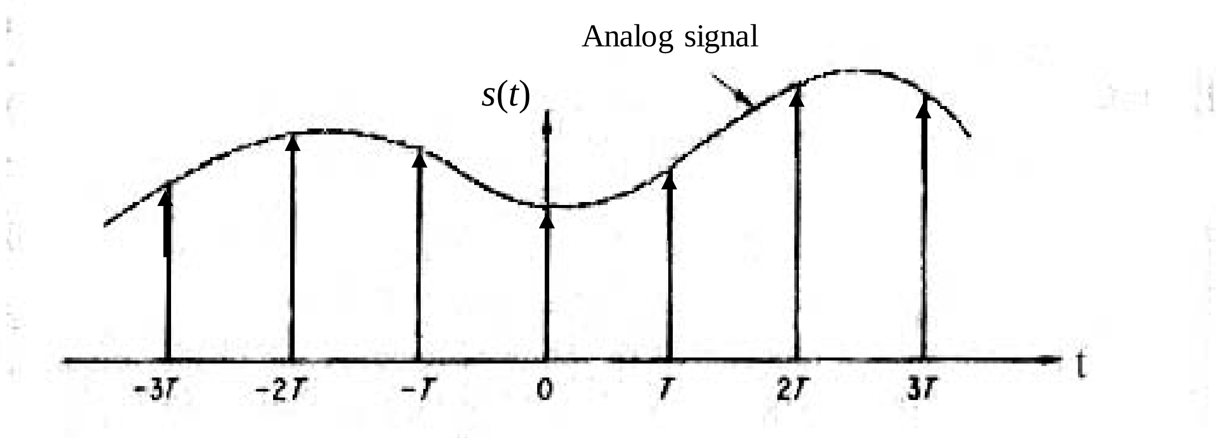
Sampling
Definitions

- The process of selecting or recording the ordinate values of a continuous (usually analogue) function at specific (usually equally spaced) values of its abscissa is called sampling.
- If the function is a signal which varies with time then the samples are called a time series.



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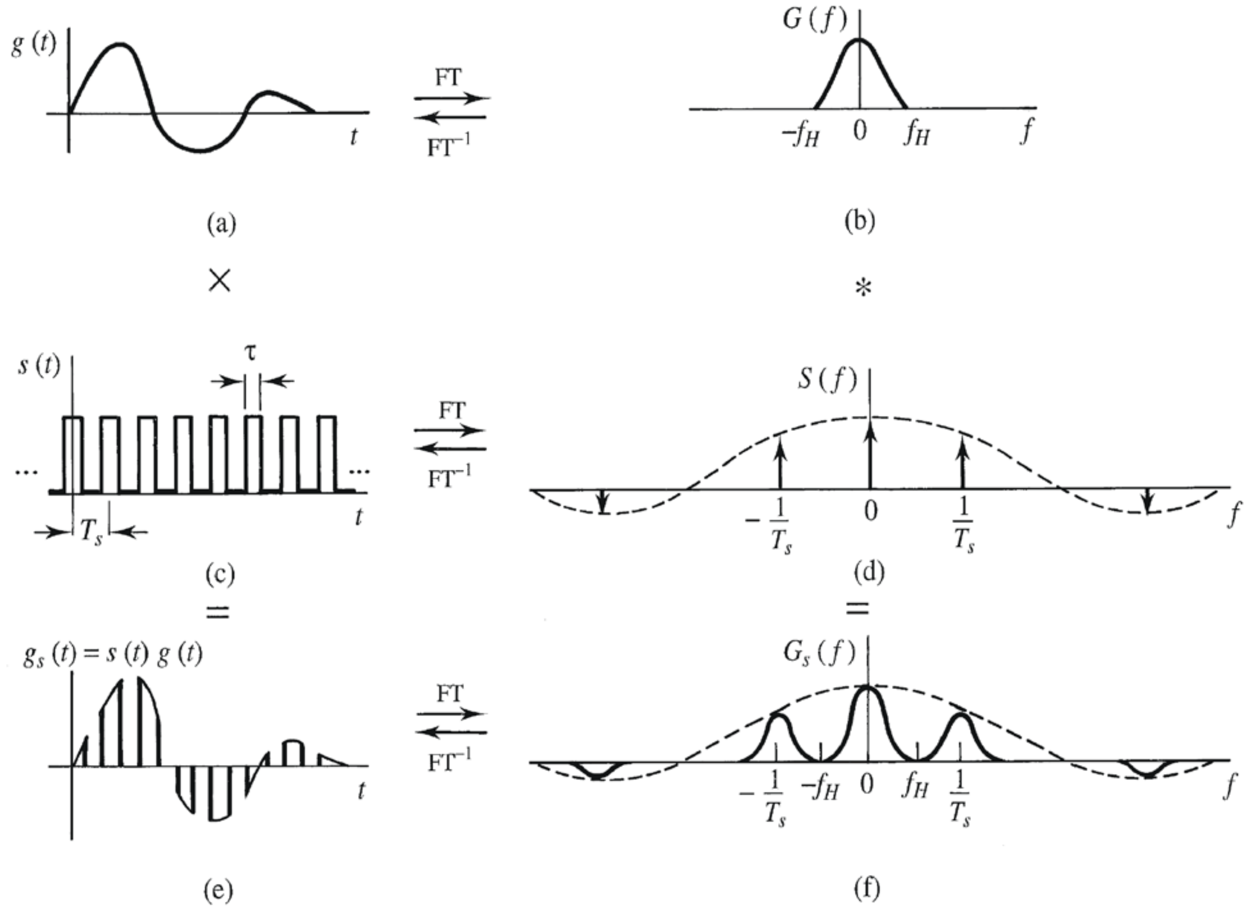
Sampling
Natural Sampling

- Consider a baseband information signal *g(t), and a* periodic pulse train *s(t)*, with width τ and period T_s.
- A naturally sampled signal is produced by multiplying *g(t)* with *s(t)*.
- The pulse tops follow the variations of the signal being sampled.
- Appropriate low pass filtering will pass only the baseband spectral version of *g(t)*
  - *g(t)* will appear undistorted at the output of the low pass filter.
- Such a filter is called a reconstruction filter.

Sampling
Natural Sampling

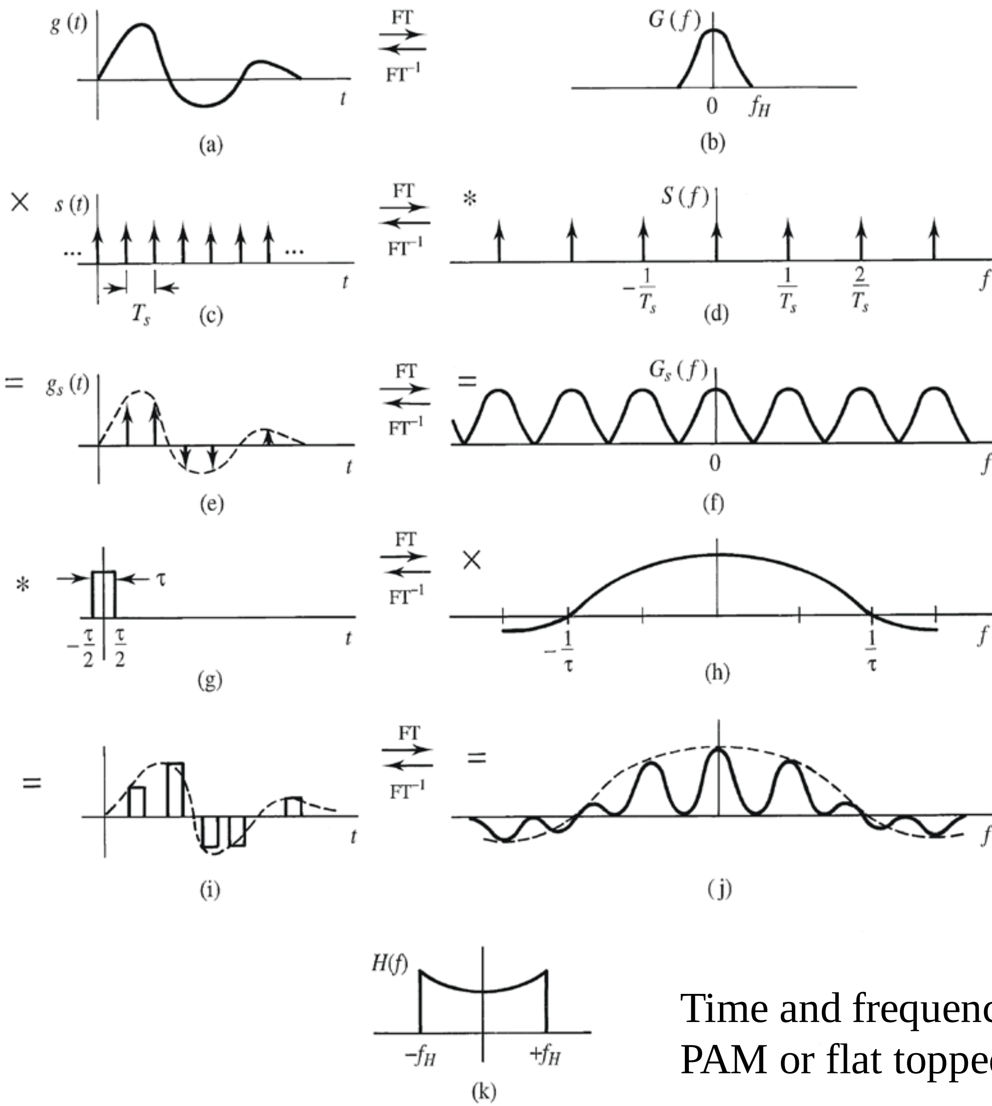


Natural sampling in time and frequency domains

Sampling
Flat Topped Sampling

- If the pulses produced by the above process are artificially flattened we have a true PAM signal or flat topped sampling.
- This can be modelled by assuming that natural sampling proceeds using a impulse train, (sometimes called impulse, or ideal, sampling) and the resulting time series of weighted impulses is convolved with a rectangular pulse.
- The resulting spectrum is that of the ideally sampled information signal multiplied with the $\mathrm{sinc}(\tau f)$ spectrum of a single rectangular pulse.
- A baseband spectral version of *g(t)* can be recovered by low pass filtering but this must then be multiplied by a function which is the inverse of the pulse spectrum $(1/\mathrm{sinc}(\tau f)$) if *g(t)* is to be restored exactly. This process is called equalization.

Sampling
Flat Topped Sampling



Time and frequency domain illustrations of PAM or flat topped sampling

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Sampling
Nyquist's Criterion

- The baseband spectrum can be recovered by low pass filtering when

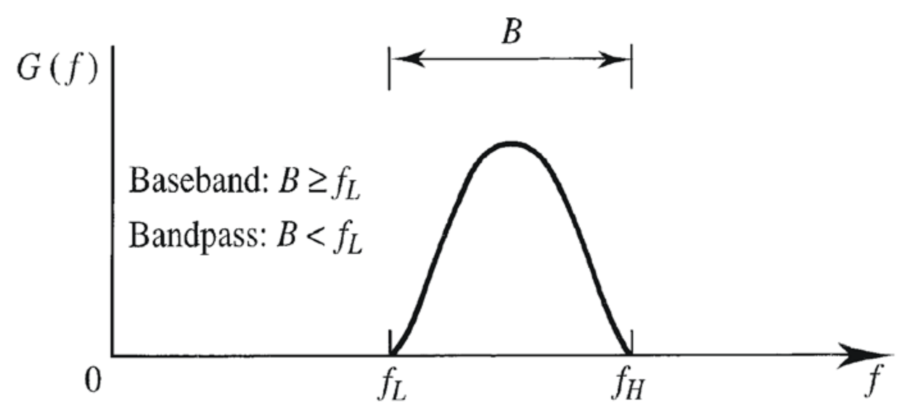
$$f_s \geq 2f_H \qquad \text{(Hz)}$$

  where f_H is the highest frequency component in the information signal.
- The Nyquist's sampling criterion:
  - ***A signal having no significant spectral components above a frequency f_H Hz is specified completely by its values at uniform spacings, no more than $1/(2f_H)$ s apart.***
- Whilst this sampling criterion is valid for any signal it is usually only used in the context of baseband signals.
- In the context of sampling, a distinction between baseband and bandpass signals can be

  For baseband signals: $B \geq f_L$ (Hz)

  For bandpass signals: $B < f_L$ (Hz)

  where B is the signal's (absolute) bandwidth.



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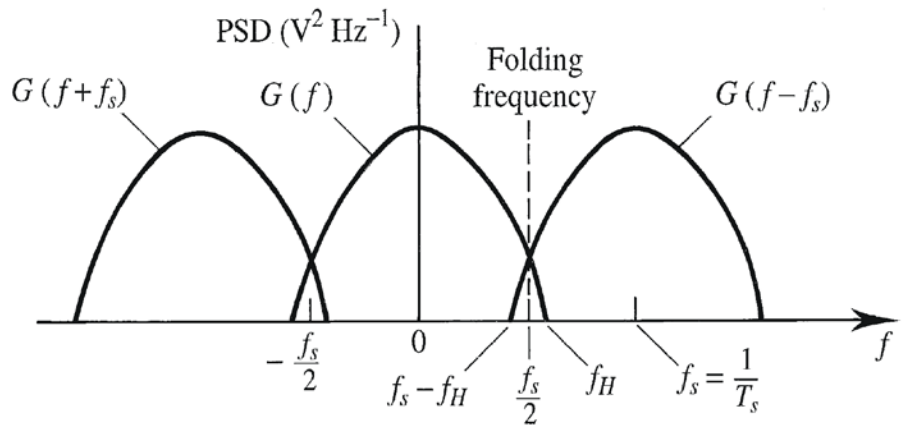
Sampling
Aliasing

- The spectrum of an undersampled baseband signal ($f_s < 2f_H$) cannot be recovered exactly, even with an ideal rectangular low pass filter.
- The best achievable would be to use a rectangular low pass filter with a cut-off frequency of $f_s/2$.
- The spectral components originally representing high frequencies now appear under the alias of lower frequencies.
- To avoid aliasing a low pass *anti-aliasing filter* with a cut-off frequency of $f_s/2$ is often placed immediately before the sampling circuit.



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Sampling

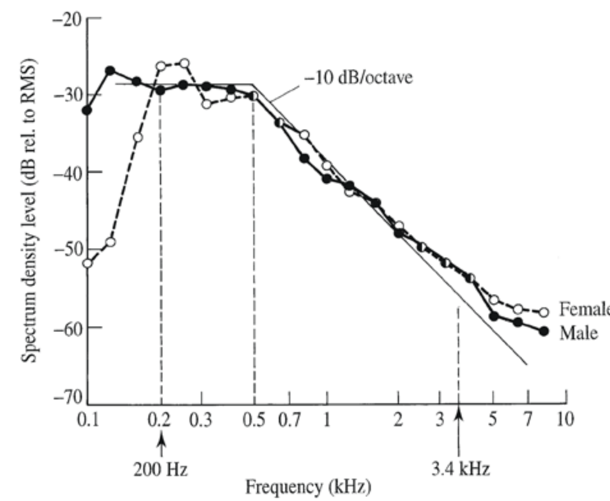
Practical Sampling and Reconstruction

- In real systems, f_H must usually be interpreted as the highest frequency component with *significant spectral amplitude*.
  - Practical signals start and stop in time and therefore, in principle, have spectra which are not bandlimited in an absolute sense.
  - For the case of voice signals, f_H in Europe is usually assumed to be 3.4 kHz.

- In practice it is necessary to sample at a slightly faster rate than $2f_H$.
  - A practical version of the baseband sampling criterion might be expressed as

$$f_s \geq 2.2 f_H$$

to allow for the transition, or roll-off, into the filter stopband.



Long-term averaged speech spectra

Sampling
Signal to Distortion Ratio (SDR)

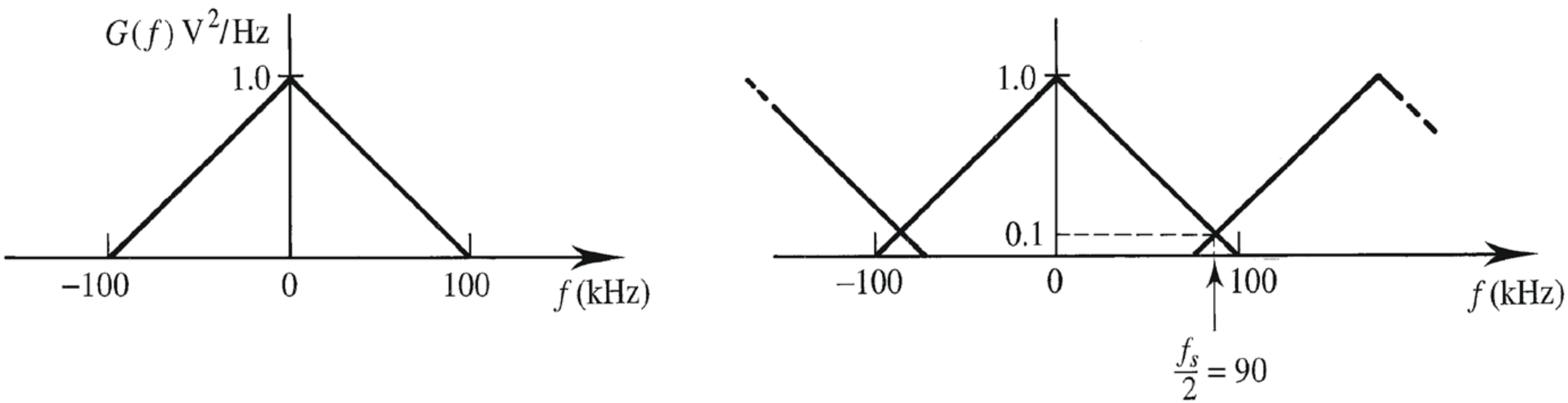
- SDR (a quantitative measure of the distortion introduced by aliasing) is defined as the ratio of unaliased to aliased power in the reconstructed signal.
- If the reconstruction filter is ideal with rectangular amplitude response then, in the absence of an anti-aliasing filter, the SDR is given by

$$SDR = \frac{\int_0^{f_s/2} G(f)df}{\int_{f_s/2}^{\infty} G(f)df}$$

where $G(f)$ is the (two-sided) power spectral density of the (real) baseband information signal *g(t)*.

Sampling
Signal to Distortion Ratio (SDR)

- **[Example 5.1]**
- Consider a signal with the power spectral density shown in the following figure. Find the alias induced SDR if the signal is sampled at 90% of its Nyquist rate and the reconstruction filter has an ideal rectangular amplitude response.



(a) Input signal spectrum (b) As (a) but illustrating folding frequency and spectral replicas

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Samping
Bandpass Sampling

- In principle, we can apply the baseband sampling criterion to sample the signal at twice the highest frequency component in a bandpass signal's spectrum.
- But usually it is possible to reconstruct the signal by sampling at a much lower rate.
- This bandpass sampling criterion can be expressed as:
  – *A bandpass signal having no spectral components below f_L Hz or above f_H Hz is specified uniquely by its values at uniform intervals spaced $T_s = 1/f_s$ s apart provided that:*

$$2B\left\{\frac{Q}{n}\right\} \le f_s \le 2B\left\{\frac{Q-1}{n-1}\right\}$$

where $B = f_H - f_L$, $Q = f_H/B$, n is a positive integer, and $n \le Q$.

Samping

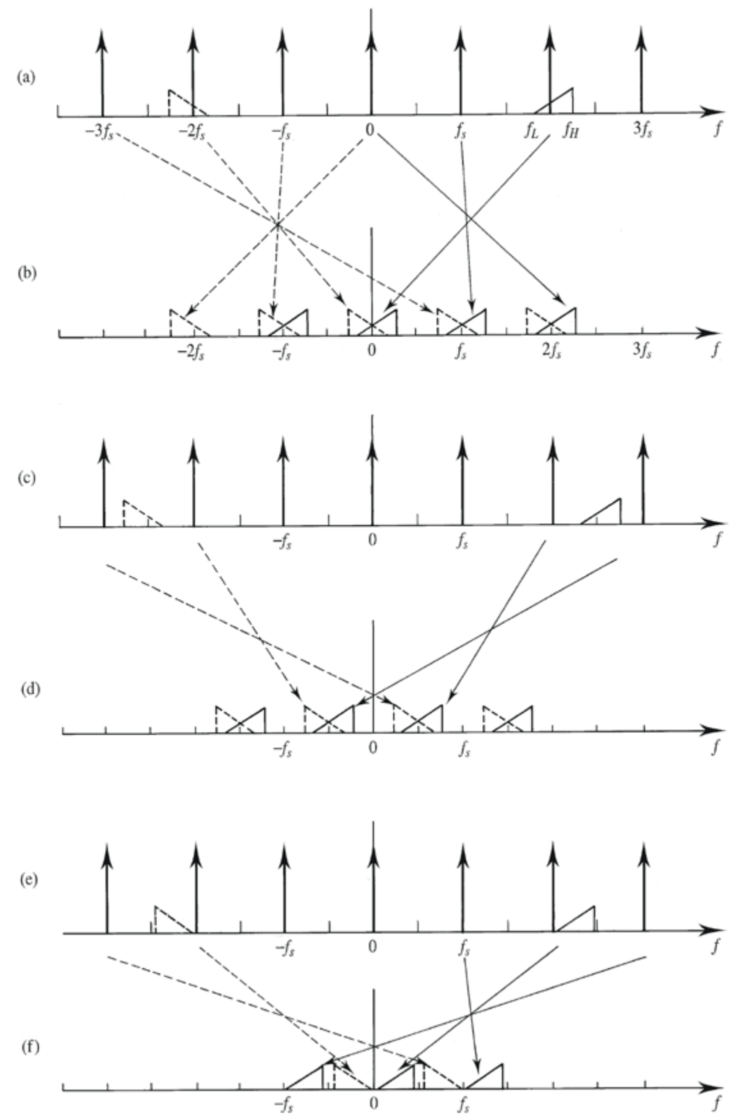
Bandpass Sampling

$$\begin{cases} f_H \le n\dfrac{f_s}{2} \quad (\text{Hz}) \\ f_L \ge (n-1)\dfrac{f_s}{2} \quad (\text{Hz}) \\ f_L = f_H - B \quad (\text{Hz}) \end{cases}$$

We have

$$\frac{2}{n} f_H \le f_s \le \frac{2}{n-1}(f_H - B)$$

Defining $Q = f_H/B$ gives us the sampling criterion.



Samping
Bandpass Sampling

- If $Q = f_H/B$ is an integer then $n \leq Q$ allows us to choose $n = Q$. In this case $f_s = 2B$ and the correct sampling frequency is exactly twice the signal bandwidth.
- If $Q = f_H/B$ is not an integer then the lowest allowed sampling rate is given by choosing $n = int(Q)$ (i.e. the next lowest integer from Q). Lower values of n will still allow reconstruction of the original signal but the sampling rate will be unnecessarily high.
- If $Q < 2$ (i.e. $f_H < 2B$ or equivalently, $f_L < B$) then $n \leq Q$ means that $n = 1$. In this case $2BQ \leq f_s \leq \infty$ (Hz) and since $BQ = f_H$ we have (the Nyquist baseband sampling criterion)

$$2f_H \leq f_s \leq \infty \qquad \text{(Hz)}$$

Samping
Bandpass Sampling

- [**Example 5.2**]
- Consider a signal with center frequency 9.5 kHz and bandwidth 1.0 kHz.
- The highest and lowest frequency components are $f_L = 9.0$ kHz and $f_H = 10.0$ kHz. The quotient Q is thus $Q = \frac{f_H}{B} = \frac{10.0}{1.0} = 10.0$

  Applying the bandpass sampling criterion:

$$2 \times 10^3 \left\{\frac{10}{n}\right\} \leq f_s \leq 2 \times 10^3 \left\{\frac{10-1}{n-1}\right\} \qquad \text{(Hz)}$$

- Since Q is an integer, the lowest allowed sampling rate is given by choosing $n = Q = 10$, i.e. $2.0 \leq f_s \leq 2.0$ (kHz). There is zero tolerance in the sampling rate if distortion is to be completely avoided.
- If n is chosen to be less than its maximum value, e.g. $n = 9$, then $2.222 \leq f_s \leq 2.250$ (kHz). The sampling rate in this case would be chosen to be 2.236 ± 0.014 kHz. The accuracy required of the sampling clock is therefore $\pm 0.63\%$.
- Now consider a signal with center frequency 10.0 kHz and bandwidth 1.0 kHz. The quotient Q=10.5 is not an integer. The lowest allowed sampling rate is therefore given by $n = int(Q) = 10.0$. The sampling rate is now bounded by $2.100 \leq f_s \leq 2.111$ kHz. This gives a required sampling rate of 2.106 ± 0.006 kHz or 2.106 kHz $\pm 0.26\%$.

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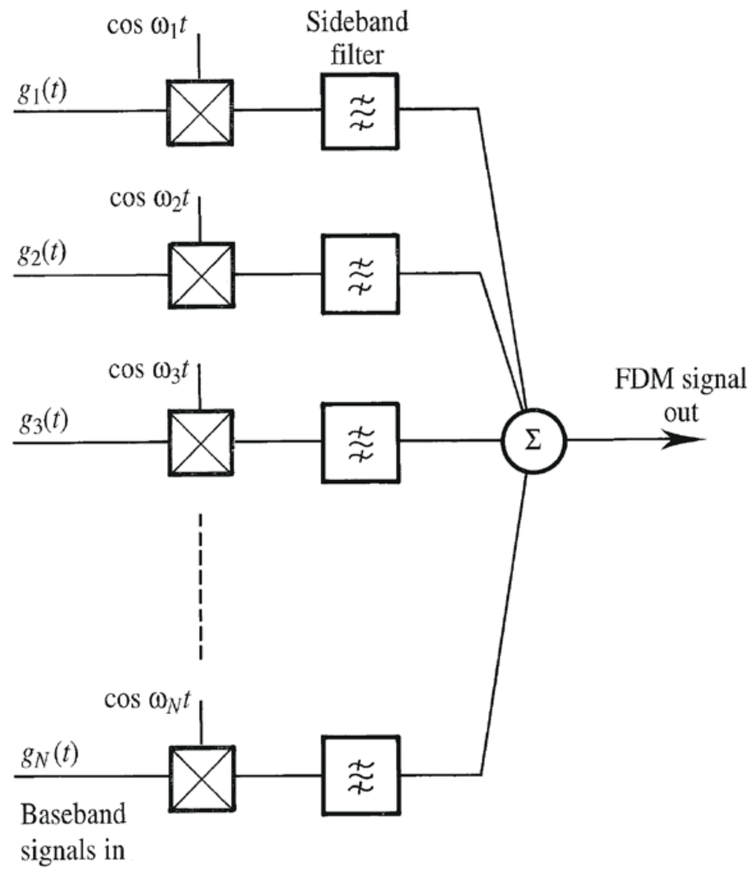
Analogue Pulse Multiplexing

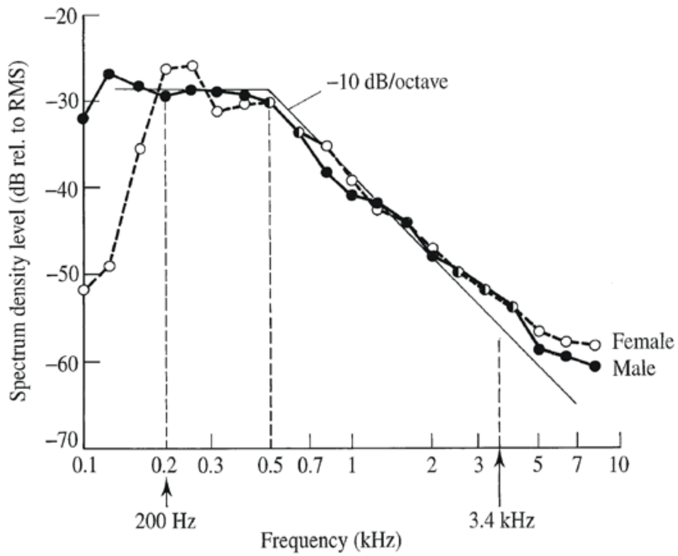
- In many communications applications different information signals must be transmitted over the same physical channel (e.g. coaxial cable, optical fiber, free space).
- To avoid crosstalk, they must be sufficiently separated.
- In traditional analogue telephony and broadcast applications, different information signals are transmitted using different carrier frequencies.
- Using different carriers, or frequency bands, to isolate signals from each other is called frequency division multiplexing (FDM)



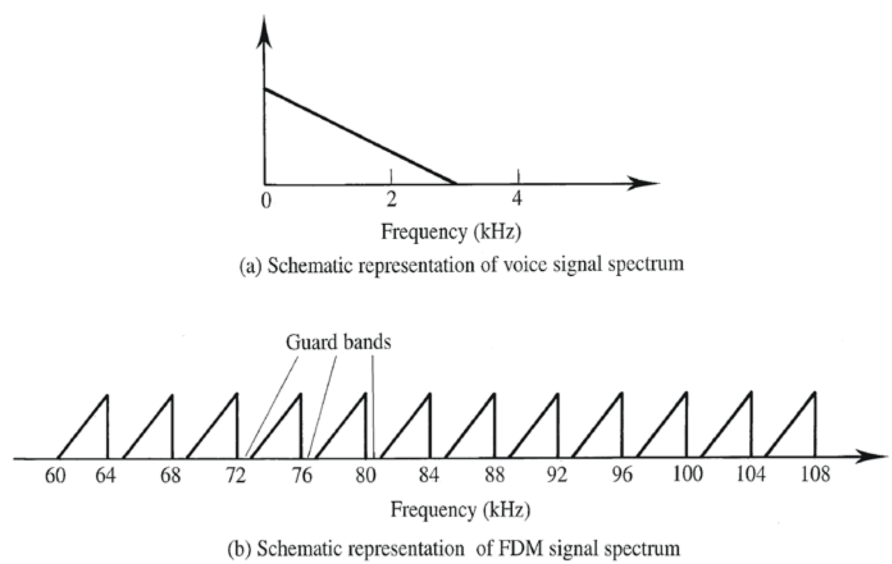
Generation of an FDM signal

Analogue Pulse Multiplexing

- In FDM telephony, 3.4 kHz bandwidth telephone signals are stacked in frequency at 4 kHz spacings with small frequency guard bands between them to allow separation using practical filters.



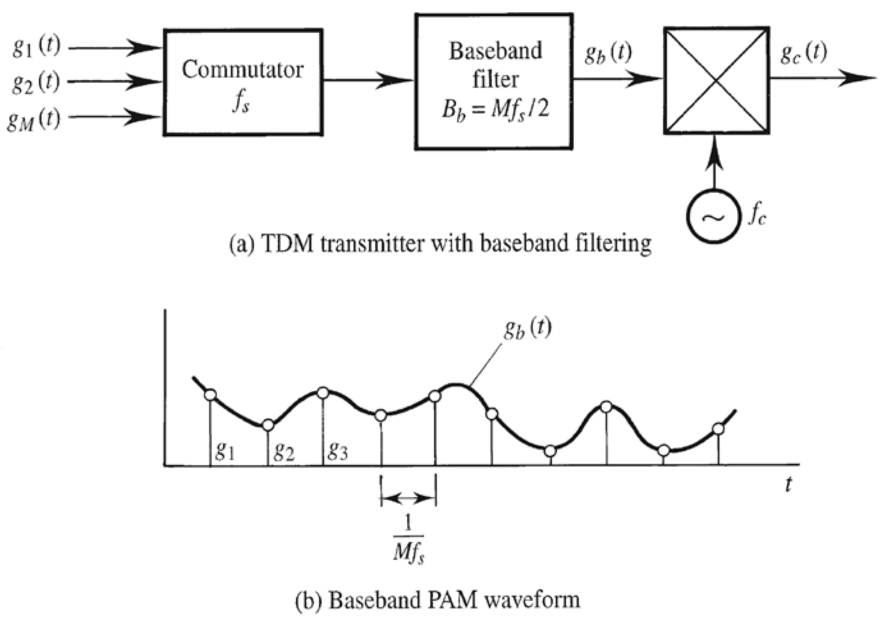
Long-term averaged speech spectra



FDM example for multiplexed speech channels

Analogue Pulse Multiplexing

- Instead of occupying separate frequency bands (as in FDM) the signals can occupy separate time slots.
- This technique is called (analogue) time division multiplexing (TDM).



Filtered TDM waveform

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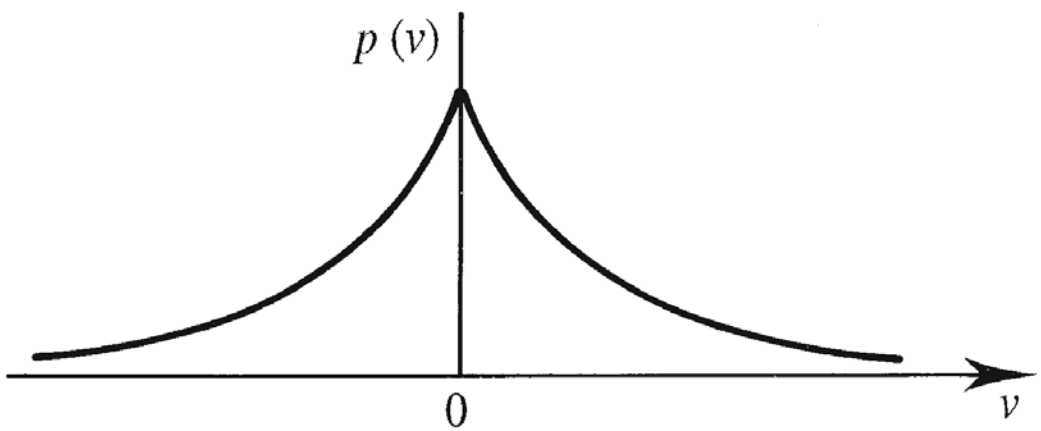
Quantized PAM and Pulse Code Modulation

Quantized PAM

- An information signal which is pulse amplitude modulated becomes discrete (in time) rather than continuous, but nevertheless remains analogue in nature.
- This is because all pulse amplitudes within a specified range are allowed, i.e. the pdf of pulse amplitudes is continuous.

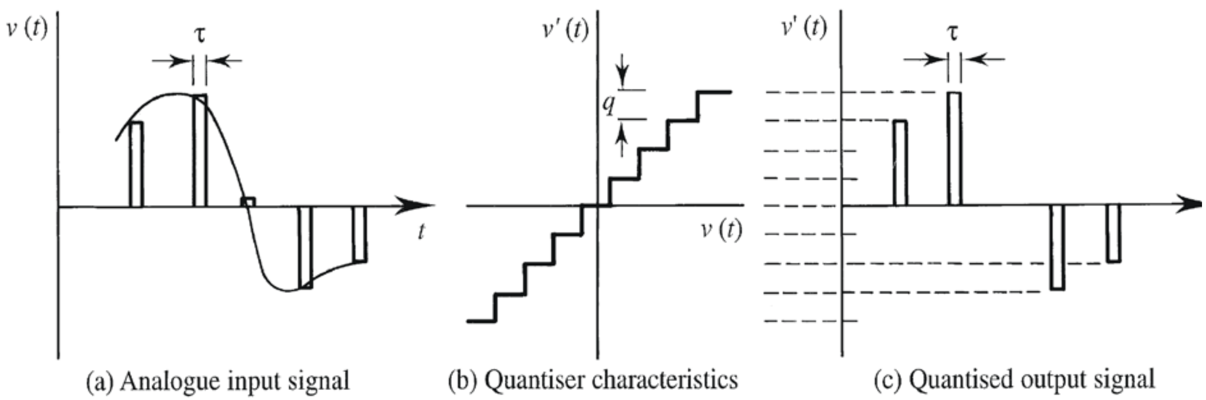


Continuous pdf of analogue PAM signal

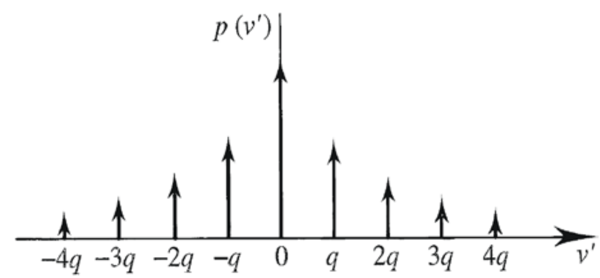
Quantized PAM and Pulse Code Modulation

Quantized PAM

- If a PAM signal is quantized, i.e. each pulse is adjusted in amplitude to coincide with the nearest of a finite set of allowed amplitudes, then the resulting signal is no longer analogue, but digital, and as a consequence has a discrete pdf.
- This digital signal can be represented by a finite set of symbols.
- The simplest and most important alphabet is the binary set.



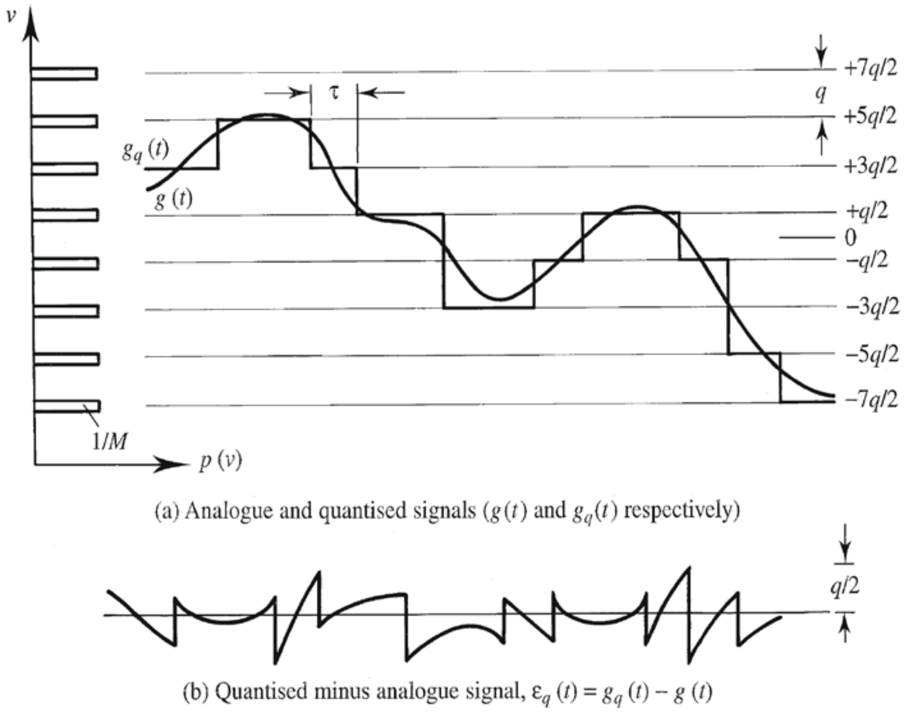
Quantization of a PAM signal



Discrete pdf of quantized PAM signal

Quantized PAM and Pulse Code Modulation
Quantization Error

- The quantization process actually degrades the quality of the information signal, since the quantized PAM signal no longer exactly represents the original, continuous analogue signal but a distorted version of it.
- The quantized signal can be decomposed into the sum of the analogue signal and the difference between the quantized and the analogue signals.
- The difference signal is essentially random and can therefore be thought of as a special type of noise process.



Quantization error interpreted as noise, i.e.

$$g_q(t) = g(t) + \epsilon_q(t)$$

Quantized PAM and Pulse Code Modulation
Signal to Quantization Noise Ratio (SN_qR)

- The power or RMS value of quantization noise can be calculated or measured, leading to the concept of signal to quantization noise ratio (SN_qR).
- Assume:
  - Linear quantization (i.e. equal increments between quantization levels)
  - Zero mean signal (i.e. symmetrical pdf about 0 V)
  - Uniform signal pdf (i.e. all signal levels equally likely)
- Let M be the (even) number of quantization levels, and q (V) be distance between the adjacent quantization levels.
- The pdf of allowed levels is given by

$$p(v) = \sum_{k=-M}^{M} \frac{1}{M} \delta\left(v - \frac{qk}{2}\right)$$

where k takes on odd values only.

Quantized PAM and Pulse Code Modulation

Signal to Quantization Noise Ratio (SN_qR)

- The mean square signal after quantization is

$$\overline{v^2} = \int_{-\infty}^{\infty} v^2 p(v) dv = \frac{2}{M}\left[\int_0^{\infty} v^2 \delta\left(v - \frac{q}{2}\right) dv + \int_0^{\infty} v^2 \delta\left(v - \frac{3q}{2}\right) dv + \cdots\right]$$

$$= \frac{2}{M}\left(\frac{q}{2}\right)^2 [1^2 + 3^2 + 5^2 + \cdots (M-1)^2] = \frac{2}{M}\left(\frac{q}{2}\right)^2 \left[\frac{M(M-1)(M+1)}{6}\right]$$

$$= \frac{M^2 - 1}{12} q^2 \quad (\mathrm{V}^2)$$

- Denoting the quantization error as ϵ_q, whose pdf is uniform:

$$p(\epsilon_q) = \begin{cases} 1/q, & -q/2 \le \epsilon_q \le q/2 \\ 0, & \text{elsewhere} \end{cases}$$

- The mean square quantization error (or noise) is

$$\overline{\epsilon_q^2} = \int_{-q/2}^{q/2} \epsilon_q^2 p(\epsilon_q) d\epsilon_q = \frac{q^2}{12} \quad (V^2)$$

Quantized PAM and Pulse Code Modulation

Signal to Quantization Noise Ratio (SN_qR)

- The SN_qR is therefore given by

$$SN_qR = \frac{\overline{v^2}}{\overline{\epsilon_q^2}} = M^2 - 1$$

- For large SN_qR the approximation $SN_qR = M^2$ is often used.
- Since the peak signal level is $Mq/2$ V then the peak signal to quantization noise (power) ratio is

$$\left(SN_qR\right)_{peak} = \frac{(Mq/2)^2}{\overline{\epsilon_q^2}} = 3M^2$$

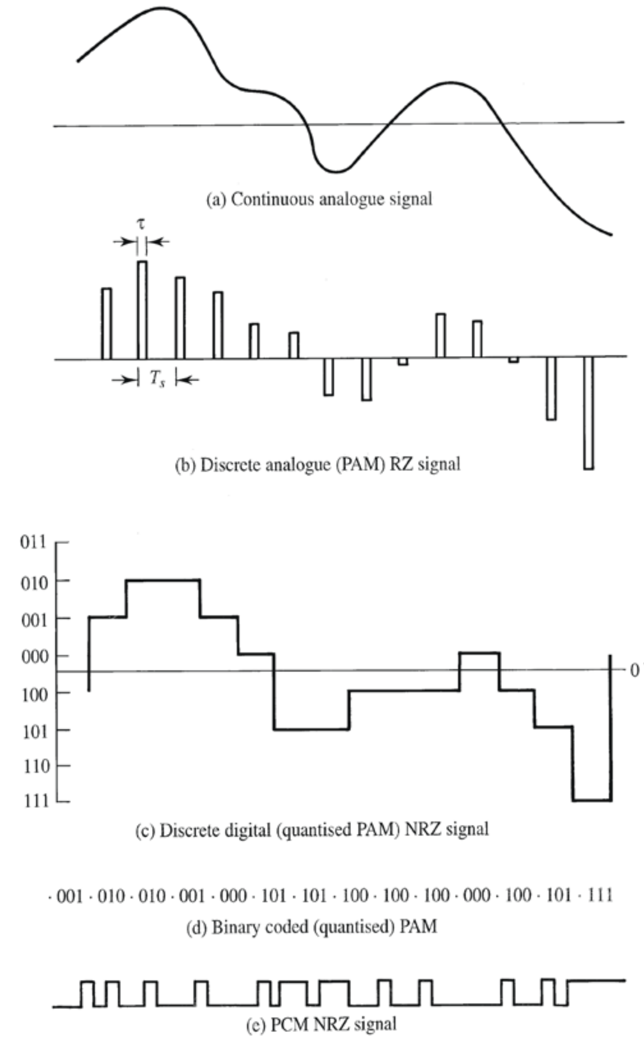
- Expressed in dB the SN_qRs are

$$SN_qR = 20\log_{10} M \quad \text{(dB)}$$

$$\left(SN_qR\right)_{peak} = 4.8 + SN_qR \quad \text{(dB)}$$

Quantized PAM and Pulse Code Modulation
Pulse Code Modulation

- After a PAM signal has been quantized the possibility exists of transmitting not the pulse itself, but a number indicating the height of the pulse.
- Usually the pulse height is transmitted as a binary number.
  - For instance, if the number of allowed quantization levels is eight then the pulse amplitudes could be represented by the binary numbers from zero (000) to seven (111).
- The binary digits are normally represented by two voltage levels (e.g. 0 V and 5 V).
- Each binary number is called a codeword and, since each quantized pulse is represented by a codeword, the resulting modulation is called pulse code modulation (PCM).



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Quantized PAM and Pulse Code Modulation
SN_qR for Linear PCM

- Despite a bandwidth penalty, the advantage of PCM is that for a given transmitted power the difference between adjacent binary 0/1 voltage levels is much greater than for quantized PAM.
  - For a given RMS noise voltage the total voltage (signal plus noise) at the receiver is less likely to be interpreted as representing a level other than that which was transmitted.
- Whilst it is true that PCM signals are more tolerant of noise than the equivalent quantized PAM signals, it is also true that both suffer the same degradation due to quantization noise.
- For a given number of quantization levels, M, the number of binary digits required for each PCM codeword is $n = \log_2 M$.

Quantized PAM and Pulse Code Modulation

SN_qR for Linear PCM

- The PCM peak signal to quantization noise ratio, $(SN_qR)_{peak}$, is

$$(SN_qR)_{peak} = 3M^2 = 3(2^n)^2$$

- If the ratio of peak to mean signal power, $v_{peak}^2/\overline{v^2}$, is denoted by α then the average SN_qR is

$$SN_qR = 3(2^{2n})(1/\alpha)$$

- Expressed in dB this becomes

$$SN_qR = 4.8 + 6n - \alpha_{dB}$$

- For a sinusoidal signal $\alpha = 2$ (or 3 dB), and for speech $\alpha = 10\ dB$.
- The SN_qR for an n-bit PCM voice system can therefore be estimated as $6(n-1)$ dB.

Quantized PAM and Pulse Code Modulation

SN_qR for Linear PCM

- [**Example 5.3**]
- A digital communications system is to carry a single voice signal using linearly quantized PCM. What PCM bit rate will be required if an ideal anti-aliasing filter with a cut-off frequency of 3.4 kHz is used at the transmitter (for a practical version of the sampling theorem) and the SN_qR is to be kept above 50 dB?

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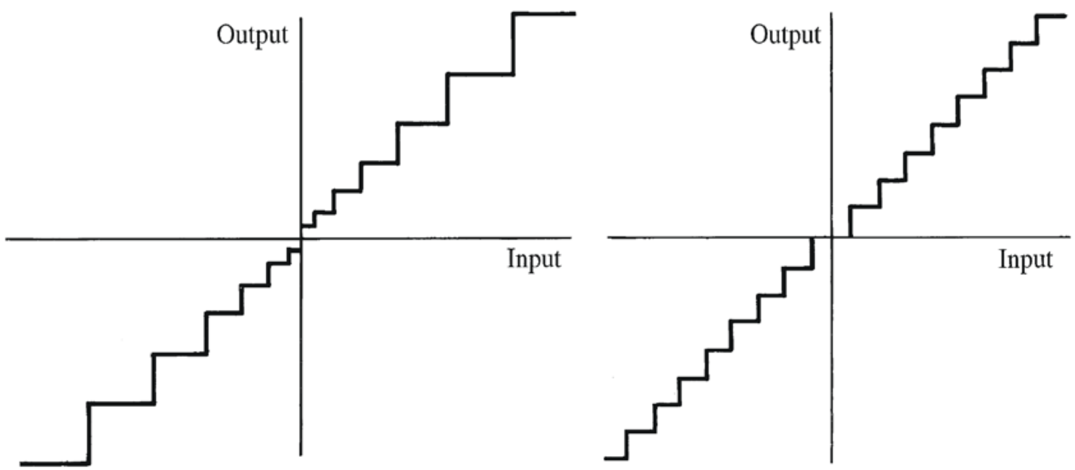
   - Companded PCM
   - Delta PCM, Differential PCM, Delta Modulation
5. Summary

Quantized PAM and Pulse Code Modulation
Companded PCM

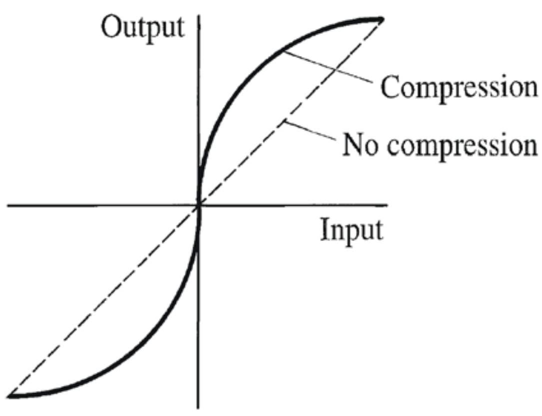
- In most communications systems, the information signal does not have a uniform pdf.
- To optimize the average SN_qR those quantization levels used most should introduce least quantization noise.
- One way to arrange for this to occur is to adopt non-linear quantization or, equivalently, companding.



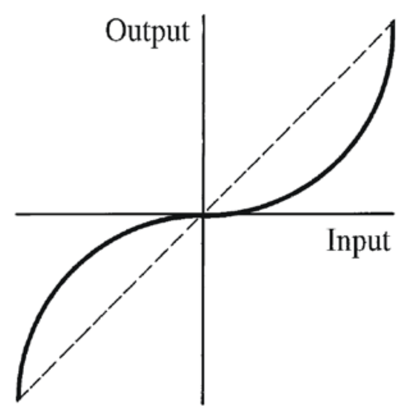
(a) Non-linear quantisation (b) Linear quantisation

Quantized PAM and Pulse Code Modulation
Companded PCM

- Companding (compressing-expanding): Compressing the information signal using a non-linear amplitude characteristic prior to linear quantization and then expanding the reconstructed information signal with the inverse characteristic.
- The companding characteristic would result in a signal with a uniform (nearly uniform in practice) pdf and therefore better SN_qR than the uncompressed signal.



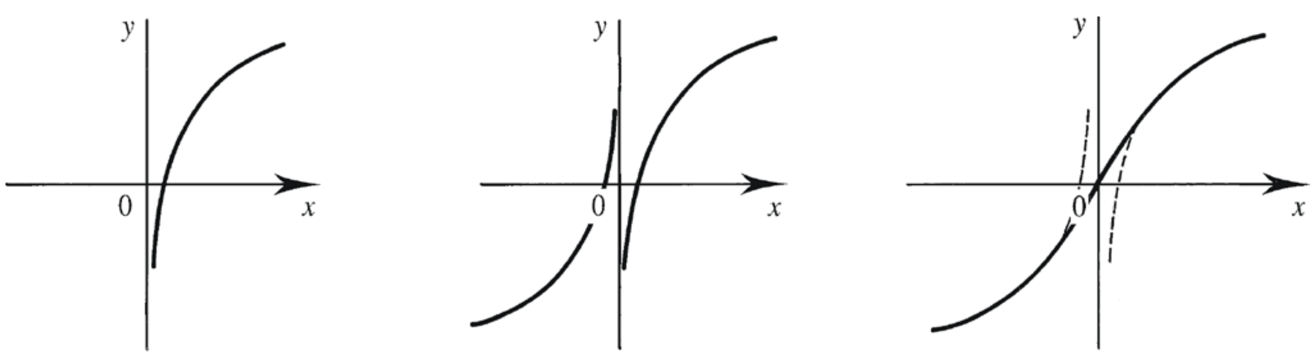
(a) Compression characteristic



(b) Expansion characteristic

Quantized PAM and Pulse Code Modulation
Companded PCM

- If the information signal has an unknown pdf or if its pdf changes with time, the companding strategy is normally to maintain, as nearly as possible, a constant SN_qR for all possible signal levels.
- The constant SN_qR compression characteristic has the form $y = \log x$.
- Since information signal can usually take both negative and positive values the logarithmic compression characteristic must be reflected to form an odd symmetric function.
- The characteristic should also be continuous across zero volts.



(a) $y = \log x$

(b) $y = \begin{cases} \log x, & x > 0 \\ -\log(-x), & x < 0 \end{cases}$

(c) Composite compression law for approximately constant SN_qR

Quantized PAM and Pulse Code Modulation
Companded PCM

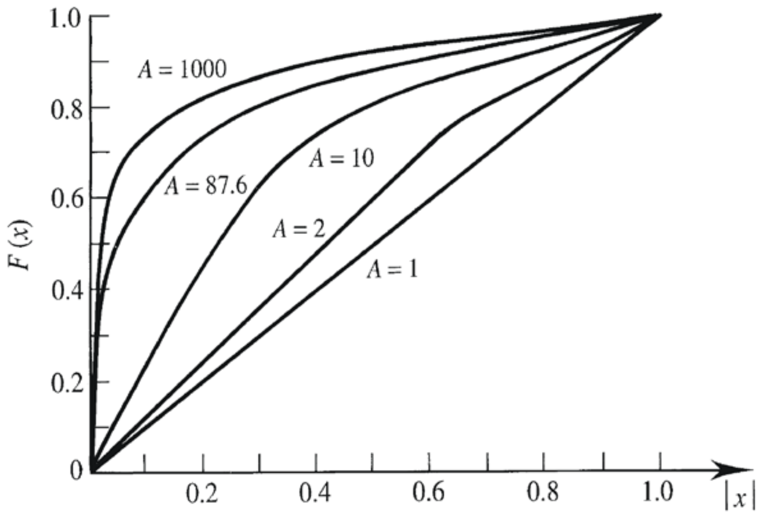
- The actual compression characteristic used in Europe and China is the A-law, defined by

$$F(x) = \begin{cases} \text{sgn}(x)\dfrac{1+\ln A|x|}{1+\ln A}, & \dfrac{1}{A} < |x| < 1 \\ \text{sgn}(x)\dfrac{A|x|}{1+\ln A}, & 0 < |x| < \dfrac{1}{A} \end{cases}$$

  - $|x| = |v/v_{peak}|$ is the normalized input signal to the compressor
  - $\text{sgn}(x)$ is the signum function which is $+1$ for $x > 0$ and -1 for $x < 0$
  - $F(x)$ is the normalized output signal from the compressor
- We then apply uniform (linear) quantization to $F(x)$

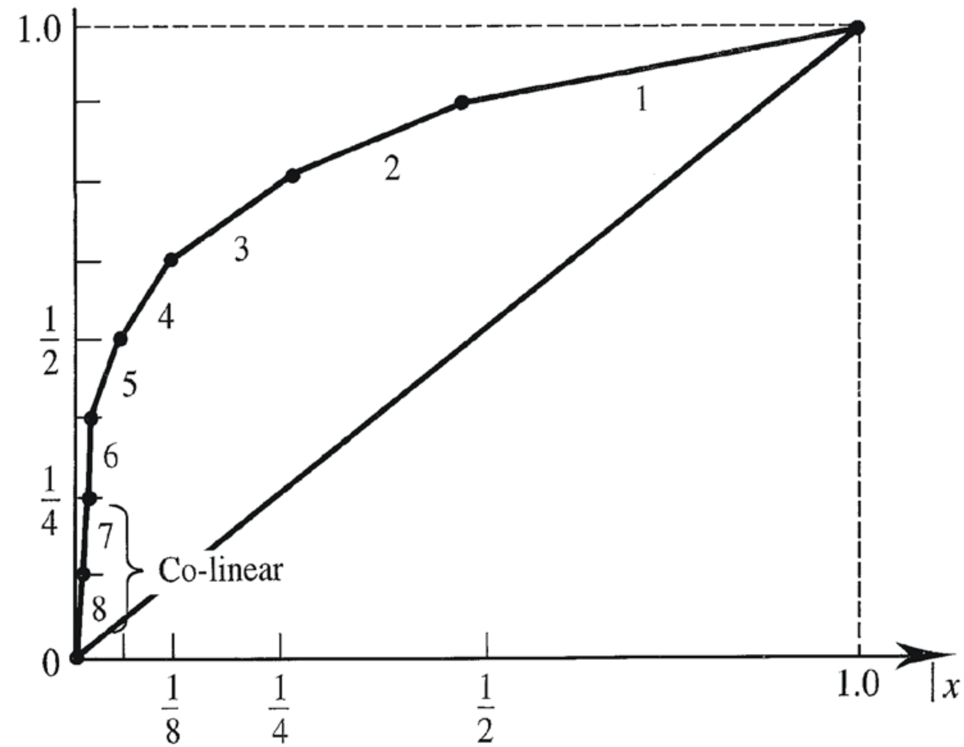
Quantized PAM and Pulse Code Modulation
Companded PCM

- The parameter A defines the curvature of the compression characteristic
  - $A = 1$ gives a linear law
  - $A = 87.6$ is commonly adopted, and gives a 24 dB improvement in SN_qR over linear PCM for small signal ($|x| < 1/A$) and an (essentially) constant SN_qR of 38 dB for large signal ($|x| > 1/A$).
- The dynamic (constant SN_qR) region is $20\log_{10}[1/(1/A)] \approx 39$ dB.
- The overall effect is to allow 11 bit (2048 level) linear PCM, which would be required for adequate voice signal quality, to be reduced to 8 bit (256 level) companded PCM.
- A 4 kHz voice channel sampled at its Nyquist rate (i.e. 8 kHz) yields a companded PCM bit rate of 64 kbit/s.



Quantized PAM and Pulse Code Modulation
Companded PCM

- The A-law characteristic is normally implemented as a 13-segment piecewise linear approximation (in practice, 16 segments but with 4 segments near origin co-linear)
- For 8 bit PCM 1 bit gives polarity, 3 bits indicate which segment the sample lies on and 4 bits provide the location on the segment.



Quantized PAM and Pulse Code Modulation
Companded PCM

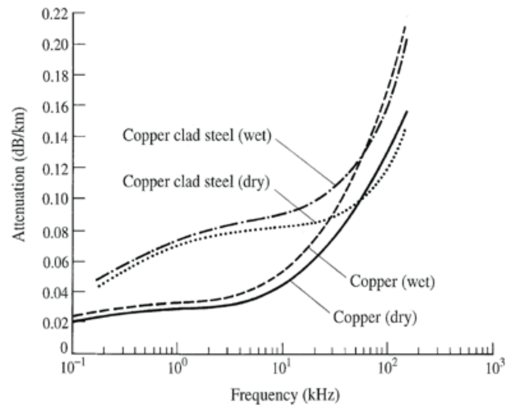
- In the USA and Japan, the μ-law is used:

$$F(x) = \mathrm{sgn}(x)\frac{\ln(1+\mu|x|)}{\ln(1+\mu)}, \qquad 0 \le |x| \le 1$$

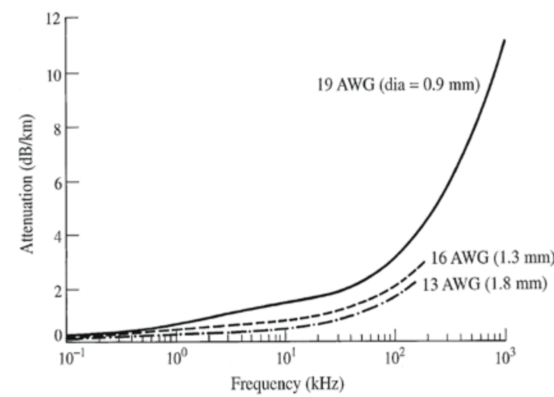
- The μ-law (with $\mu = 255$) tends to give slightly improved SN_qR for voice signals when compared with the A-law but it has a slightly smaller dynamic range.
- Like the A-law, the μ-law is usually implemented as a piecewise linear approximation (with 15 segments).
- For communications between countries using A and μ companding laws conversion from one to the other is the responsibility of the country using the μ-law.

Quantized PAM and Pulse Code Modulation
Bandwidth Reduction Techniques

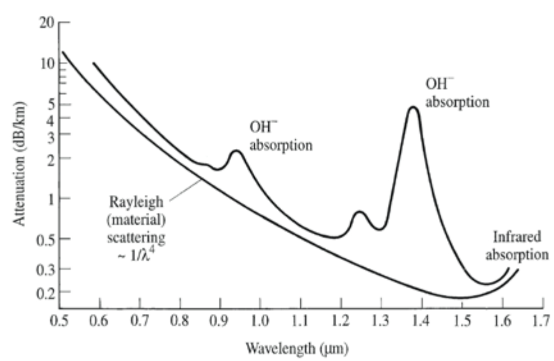
- All physical transmission lines are suitable for signaling only over a finite band of frequencies.



Typical attenuation characteristics for aerial open wire pairs



Typical attenuation characteristics for twisted pairs



Typical attenuation characteristics for optical fibres

Quantized PAM and Pulse Code Modulation
Bandwidth Reduction Techniques

- Bandwidth is even limited in the case of radio communications since the transmission properties of the earth's atmosphere are highly variable as a function of frequency.
- A given installation of transmission lines (wire-pairs, coaxial cables, optical fibers, microwave links and others) represents a finite spectral resource and adding to this installation (by e.g., laying new cables) is expensive.
- Developing spectrally efficient (i.e. reduced bandwidth) signaling techniques is highly desired.

Chapter 6: Sampling, Multiplexing and PCM
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   - Nyquist's Criterion
   - Aliasing
   - Practical Sampling and Reconstruction
   - Bandpass Sampling
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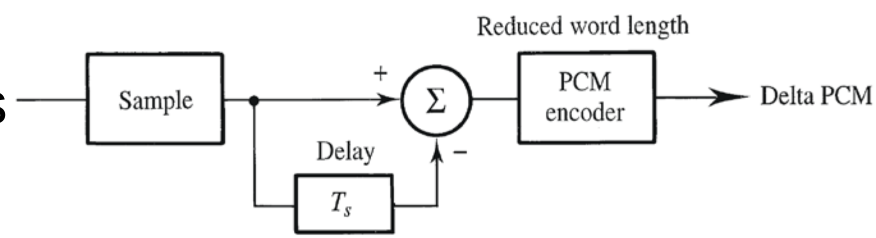
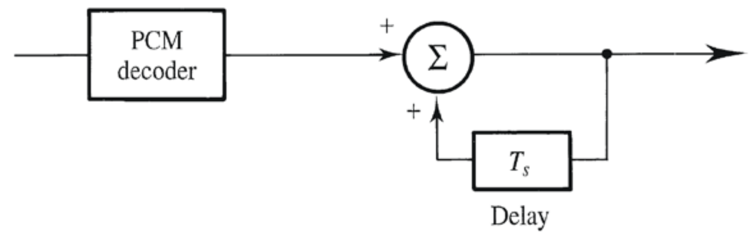
Quantized PAM and Pulse Code Modulation
Delta PCM

- Delta PCM: Transmits the *difference* between adjacent samples as conventional PCM codewords.
- The difference between adjacent samples is generally significantly less than the actual sample values, which allows the differences to be coded using fewer binary symbols per word than conventional PCM would require.
- The reduced number of bits per PCM codeword translates directly into a saving of bandwidth.
- The delta PCM system cannot, however, accommodate rapidly varying transient signals as well as conventional PCM systems.



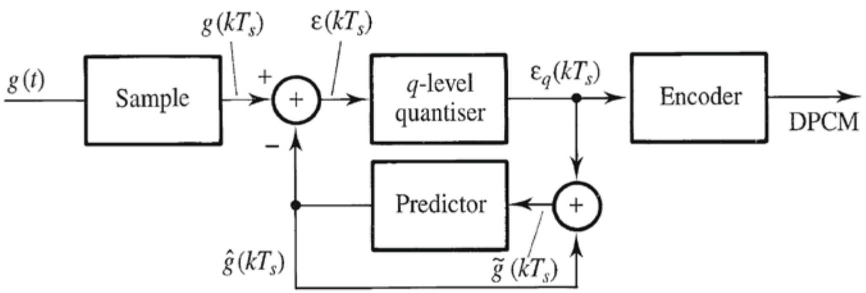
(a) Delta PCM encoder



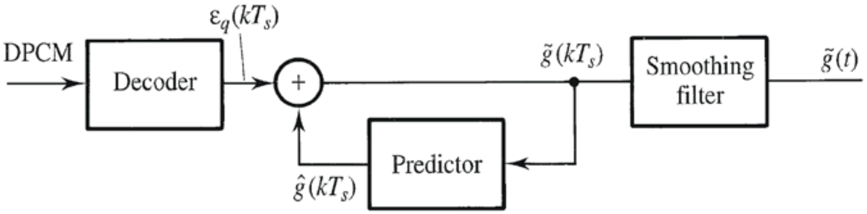
(b) Delta PCM decoder

Quantized PAM and Pulse Code Modulation
Differential PCM

- Differential PCM (DPCM) uses an algorithm to predict an information signal’s future value, and then transmits a signal which represents a correction to the predicted value.
- DPCM thus reduces the redundancy in a signal and allows the information contained in it to be transmitted using fewer symbols, less spectrum, shorter time and/or lower signal power.
- $g(kT_s)$: sampled version of information signal $g(t)$.
- $\epsilon(kT_s)$: error between the actual value of $g(kT_s)$ and $\hat{g}(kT_s)$, predicted from previous samples
- $\epsilon(kT_s)$ is quantized and encoded to give the DPCM signal.



(a) DPCM encoder

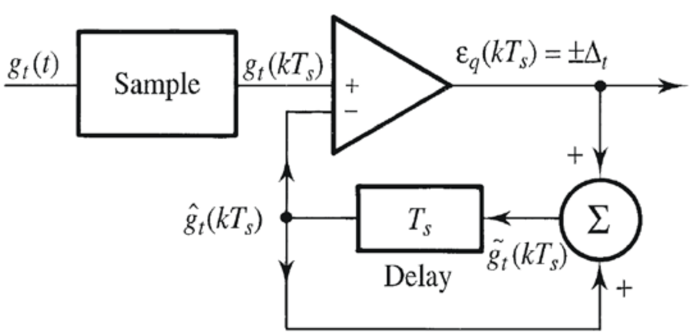


(b) DPCM decoder

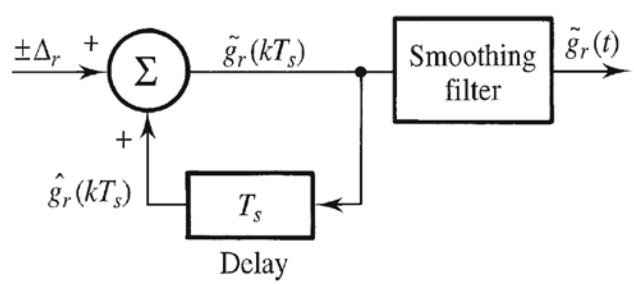
Quantized PAM and Pulse Code Modulation

Delta Modulation

- If the quantizer of a DPCM system is restricted to 1 bit (i.e. two levels only, $\pm\Delta$) then the resulting scheme is called delta modulation (DM).
- This can be implemented by replacing the DPCM differencing block and quantizer with a comparator.
- A simple prediction algorithm assumes that the next sample value is the same as the last sample value, i.e. $\hat{g}(kT_s) = \tilde{g}[(k-1)T_s]$



(a) DM coder

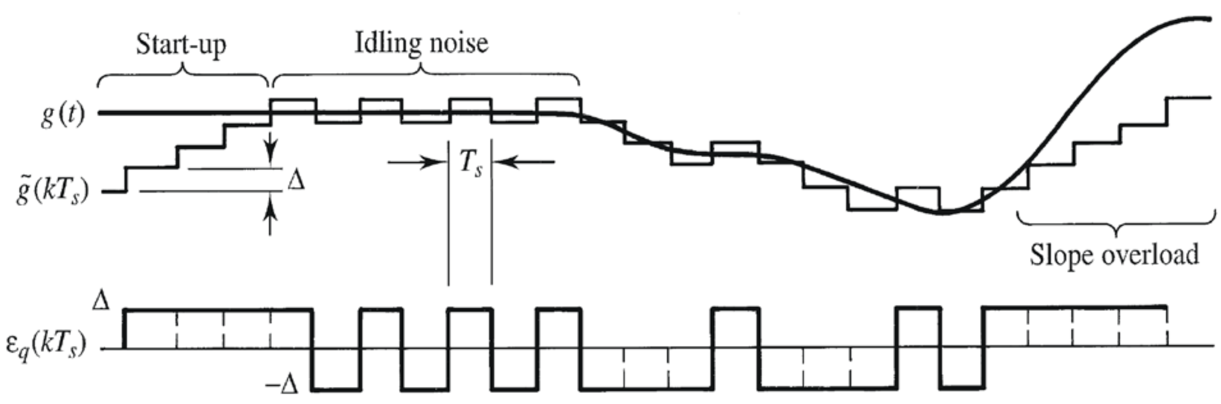


(b) DM decoder

Quantized PAM and Pulse Code Modulation

Delta Modulation

- Slope overload noise occurs when $g(t)$ changes too rapidly for $\tilde{g}(kT_s)$ to follow faithfully.
- There is a potential conflict between the requirements for acceptable quantization noise and acceptable slope overload noise.
- One way of keeping both types of noise within acceptable limits is to make Δ small, but sample much faster than the normal minimum (i.e. Nyquist) rate.



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Summary

- Continuous analogue information signals are converted to discrete analogue signals by the process of sampling.
- The minimum sampling rate required for an information signal is given by Nyquist's sampling theorem, if the signal is baseband, and by the bandpass sampling theorem if the signal is bandpass.
- Pulse modulated signals can be multiplexed together and thus allow a single physical channel to carry many real time tributary signals.
- The quantization process converts an analogue signal to a digital signal.
- The quantization noise decrease as the quantization level increases.
- PCM replaces M quantization levels with M codewords each comprising $n = \log_2 M$ binary digits.
- Companding of signals prior to PCM encoding increases the SN_qR of the decoded signal.
- Redundancy in transmitted PCM signals can be reduced by using DPCM techniques and its variants.

Principles of Communications (通信系统原理)

Undergraduate Course

Chapter 6: Sampling, Multiplexing and PCM

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