Principles of Communications (通信系统原理)

Undergraduate Course

Chapter 2: Periodic and Transient Signals

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Chapter 2: Periodic and Transient Signals

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Introduction
Signals and Waveforms

- A signal is defined as “any sign, gesture, token, etc., that serves to communicate information.”
  - When applied to electronic communications, the word “signal” implies an electrical quantity (e.g. voltage) possessing some characteristic (e.g. amplitude) which varies unpredictably.
- A waveform is defined as “the shape of a wave or oscillation obtained by plotting the value of some changing quantity against time.”
  - In electronic communications the term waveform implies an electrical quantity which varies periodically, and therefore predictably.
- A waveform can be adapted to convey information by varying one or more of its parameters in sympathy with a signal.
  - Such waveforms are called carriers and typically consist of a sinusoid or pulse train modulated in amplitude, phase, or frequency.

Introduction
Classification of Signals

- Periodic and aperiodic signals
  - A periodic signal, if shifted by an appropriate time interval, is unchanged.
    - In this context the term periodic signal is synonymous with waveform.
  - An aperiodic signal does not possess such a property.
    - A transient signal is one which has a well defined location in time.
- Deterministic and random signals
  - The signal parameters (amplitude, shape, and phase in the case of a periodic signal, amplitude, shape and location in the case of a transient signal) of a deterministic signal are known.
  - A random signal is not deterministic and must be described using probability theory.

Chapter 2: Periodic and Transient Signals

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Periodic Signals
Basic Definitions

- A periodic signal has the property

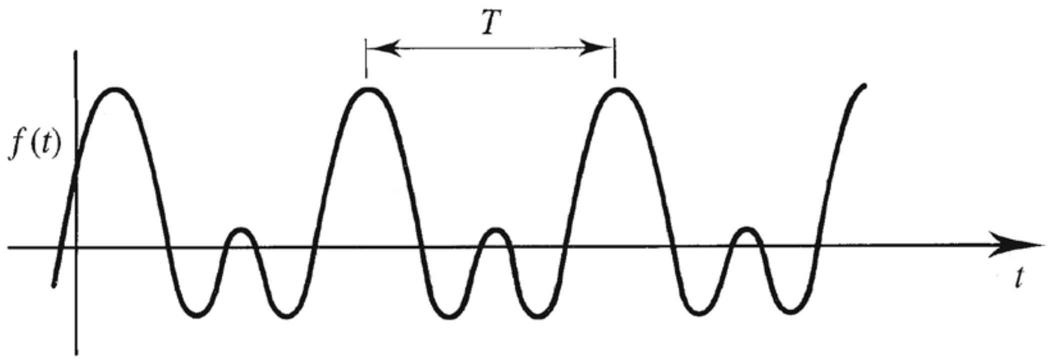
$$f(t) = f(t \pm nT)$$

where n is any integer and T is the (repetition) period.

- The normalized power is defined as

$$P = \frac{1}{T}\int_{t}^{t+T} |f(t)|^2 dt \quad (\mathrm{V}^2)$$

where the integral is the normalized energy per period.



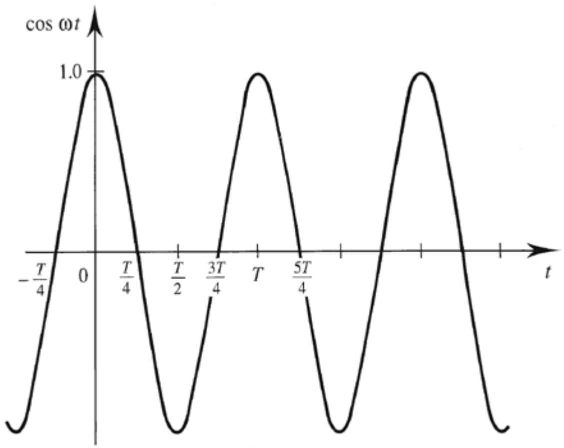
Periodic Signals
Basic Definitions

- The total energy is

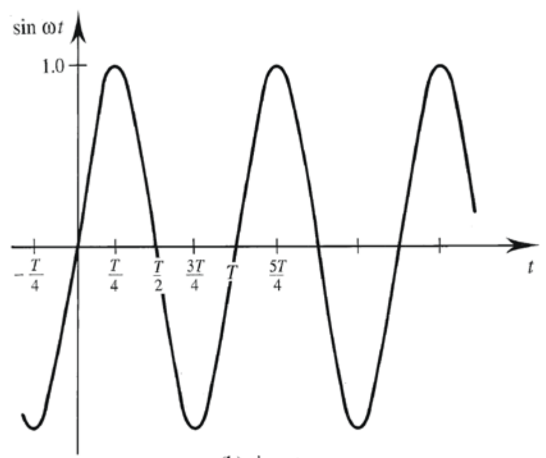
$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \infty \quad (\mathrm{V^2 s})$$

- Since *P* is finite, periodic signals are called power signals.

- A simple and useful set of periodic signals is the set of sinusoids.



(a) cos ωt



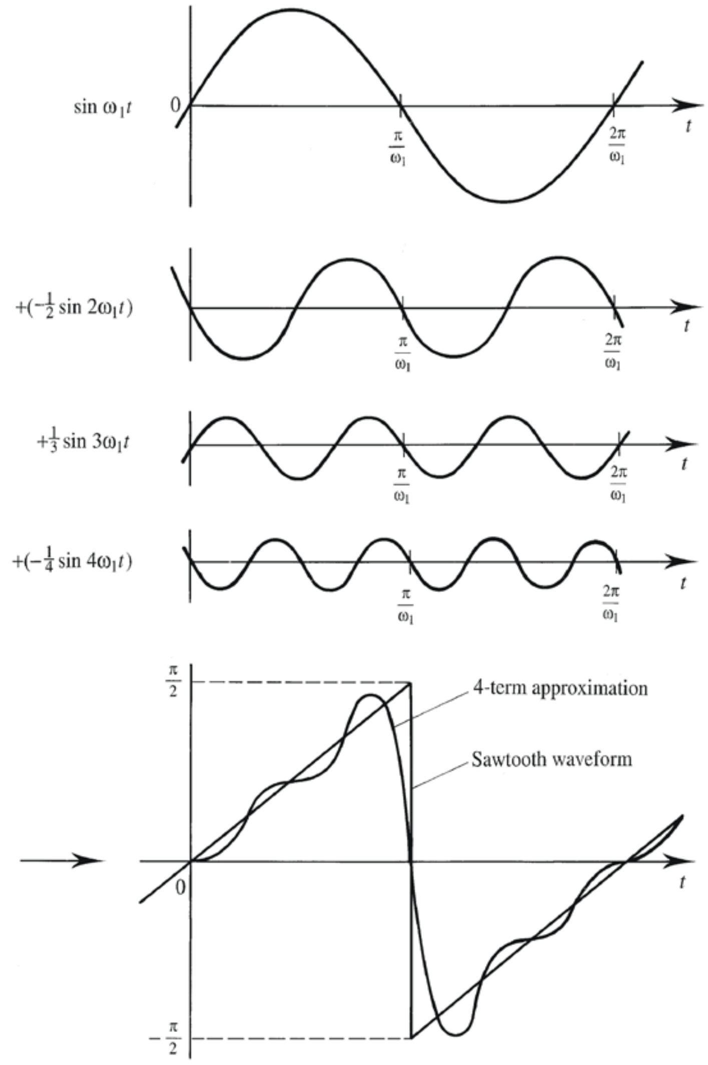
(b) sin ωt

Periodic Signals
Fourier Series

- A periodic signal can be approximated by adding together sinusoids with the correct frequencies, amplitudes and phases.
- The error between approximated and actual waveforms can be made as small as desired by including enough sinusoids.
- Only one sinusoid at each integer multiple of the fundamental frequency is required.
- The fundamental frequency, f_1, is the reciprocal of the waveform's period, T, i.e.

$$f_1 = 1/T$$

- The sinusoid with frequency $f_n = nf_1$ is called the nth harmonic of the fundamental.



Periodic Signals
Fourier Series

- If the waveform has a non-zero mean value, then a 0 Hz, constant or DC term must be included.
- The sinusoidal sum, which is called a Fourier series, is

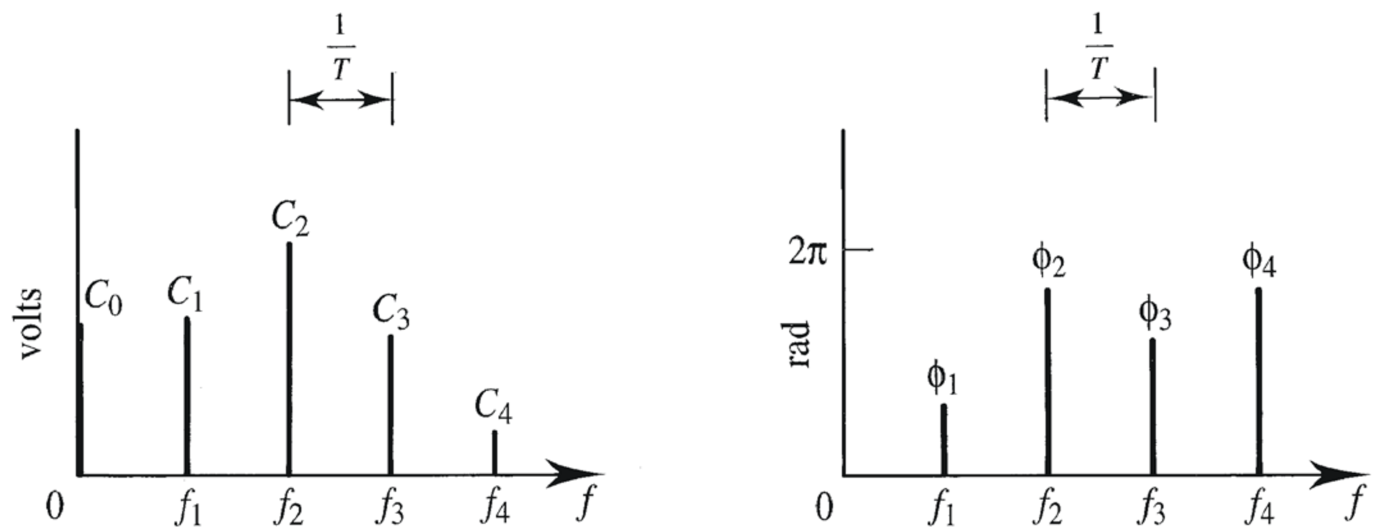
$$v(t) = C_0 + C_1 \cos(\omega_1 t + \emptyset_1) + C_2 \cos(\omega_2 t + \emptyset_2) + \cdots$$

  - C_0 (V): DC term
  - $\omega_1 = 2\pi/T$ (rad/s): the fundamental frequency
  - $\omega_2 = 2(2\pi/T)$ (rad/s): the second harmonic frequency
- The series may be truncated after a finite number of terms or may extend indefinitely.
- The Fourier series can also be expressed by

$$v(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_1 t + \emptyset_n)$$

Periodic Signals
Discrete Spectrum

- If the amplitude, C_n, of the Fourier series is plotted against frequency, $f_n = \omega_n/2\pi$ (Hz), the result is called a discrete, or line, amplitude spectrum.
- If $\emptyset_n$ is plotted against f_n the result is a discrete phase spectrum.



(a) Amplitude (b) Phase

Periodic Signals

Calculation of Fourier Coefficients

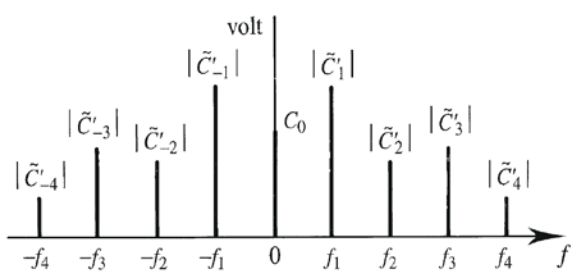
- The Fourier coefficients

$$\tilde{C}'_n = \frac{1}{T}\int_{t}^{t+T} v(t)e^{-j\omega_n t}dt$$

- The signal is

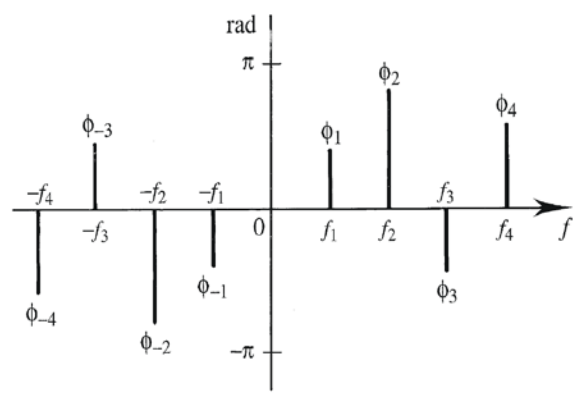
$$v(t) = \sum_{n=-\infty}^{\infty} \tilde{C}'_n e^{j\omega_n t}$$

Even symmetry ($|\tilde{C}'_1| = |\tilde{C}'_{-1}|$ for real $v(t)$)



(a) Double sided amplitude spectrum

Odd symmetry ($\phi_{-1} = -\phi_1$ for real $v(t)$)



(b) Double sided phase spectrum

Periodic Signals
Double-Sided Spectrum

- If the amplitudes ($|\tilde{C}'_n|$) and phase angles of the cisoids are plotted against frequency f_n, the results are called a double (or two-)sided amplitude spectrum and phase spectrum, respectively.
- If $v(t)$ is *real*, the double sided amplitude spectrum has *even* symmetry about 0 Hz, and the double sided phase spectrum has *odd* symmetry about 0 Hz, i.e.

$$|\tilde{C}'_n| = |\tilde{C}'_{-n}| \quad \text{and} \quad \arg(\tilde{C}'_n) = -\arg(\tilde{C}'_{-n})$$

- The single sided amplitude spectrum (positive frequencies only) can be found from the double sided spectrum by folding over the negative frequencies of the latter and adding them to the positive frequencies.

Periodic Signals

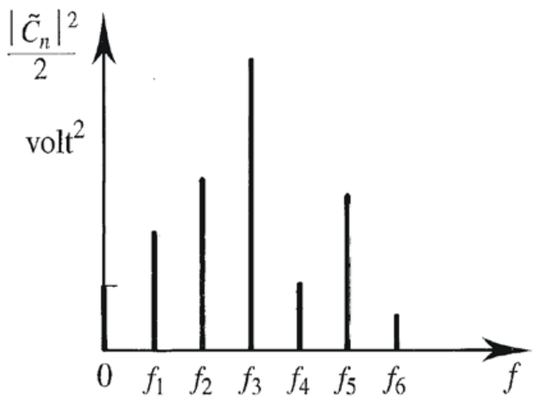
Double-Sided Spectrum and Parseval Theorem

- Double sided power spectrum

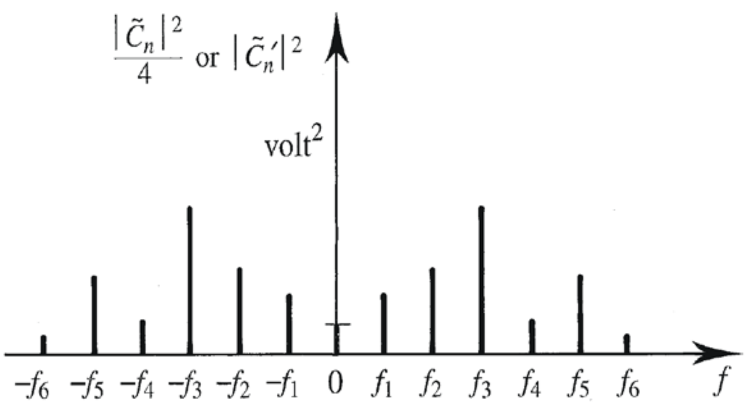
$$P_{n2} = \left|\tilde{C}'_n\right|^2$$

- Parseval's theorem: The total power in an entire line spectrum is the sum of the powers in each individual line.

$$P = \frac{1}{T}\int_{t}^{t+T} |v(t)|^2 dt = \sum_{n=-\infty}^{\infty} \left|\tilde{C}'_n\right|^2$$



(a) Single sided power spectrum



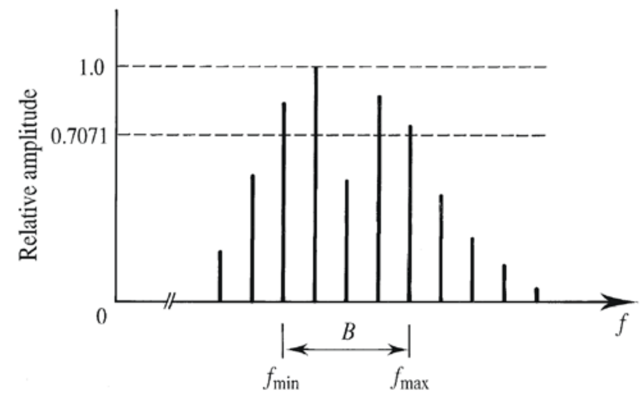
(b) Double sided power spectrum

Periodic Signals
Bandwidth

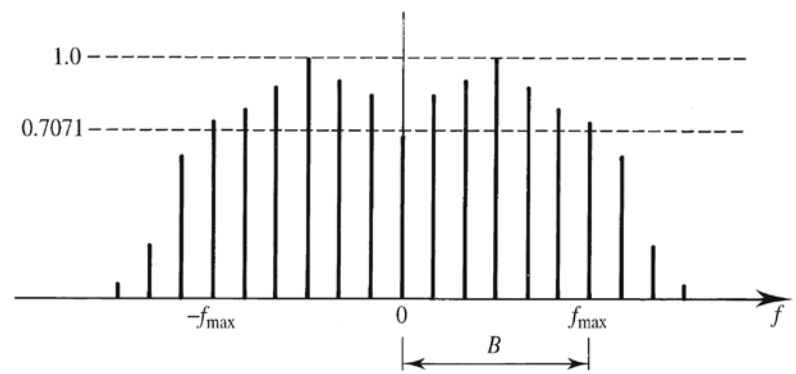
- The bandwidth of a signal:

$$B = f_{max} - f_{min} \quad \text{(Hz)}$$

- f_{max} and f_{min} are, respectively, the frequencies above and below which the spectral components are *small*.
- Half power (or 3 dB) bandwidth: f_{max} and f_{min} are defined as the frequencies at which spectral components fall to $1/\sqrt{2}$ of the peak spectral component.
- For baseband signals (i.e. signals with significant spectral components all the way down to their fundamental frequency or even DC), $f_{min} = 0$ Hz, not $-f_{min}$ Hz.



(a) 3 dB bandwidth of a (bandpass) periodic signal



(b) 3 dB bandwidth of a (baseband) periodic signal shown on a double sided spectrum

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Transient Signals
Basic Definitions

- Transient signals are essentially localized in time, including
  - signals with a well defined start and stop time
  - signals with no start time, stop time, or either, but tending to zero as time tends to $\pm\infty$ and containing finite total energy.
- Since the power of such signals averaged over all time is zero, they are sometimes called energy signals.
- Since transient signals are not periodic they cannot be represented by an ordinary Fourier series.
- Fourier transformation technique is used to find a frequency domain, or spectral, description of such signals.

Transient Signals
Fourier Transforms

- Fourier transform

$$V(f) = \int_{-\infty}^{\infty} v(t) e^{-j2\pi f t}\, dt$$

- $V(f)$ has units of V/Hz, and is called a voltage spectral density.
- Inverse Fourier transform

$$v(t) = \int_{-\infty}^{\infty} V(f) e^{j2\pi f t} df$$

- A one-sided amplitude spectrum can be formed by folding the negative frequency components of the two-sided spectrum onto the positive frequencies and adding.
- If $v(t)$ is even (and real): $V(f) = 2\int_0^{\infty} v(t)\cos 2\pi f t\, dt$
- If $v(t)$ is odd (and real): $V(f) = -2j\int_0^{\infty} v(t)\sin 2\pi f t\, dt$

Transient Signals
Fourier Transforms

- **[Example 1.1]**
- Find and sketch the amplitude and phase spectrum of the transient signal $v(t) = 2e^{-|t|/\tau}$ (V).
  - (Given that $\int e^{ax} \cos bx \, dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2+b^2}$)

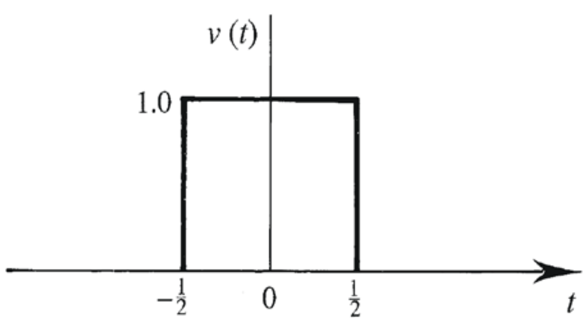
Transient Signals
Fourier Transform Pairs

- The Fourier transform (for transient functions) and Fourier series (for periodic functions) provide a link between quite different ways of describing signals, i.e. time domain description and frequency domain description (amplitude and phase).
- These two descriptions are equivalent in the sense that
  - There is one, and only one, amplitude and phase spectrum pair for each possible time plot.
  - Given a complete time domain description, the frequency domain description can be obtained exactly and vice versa.

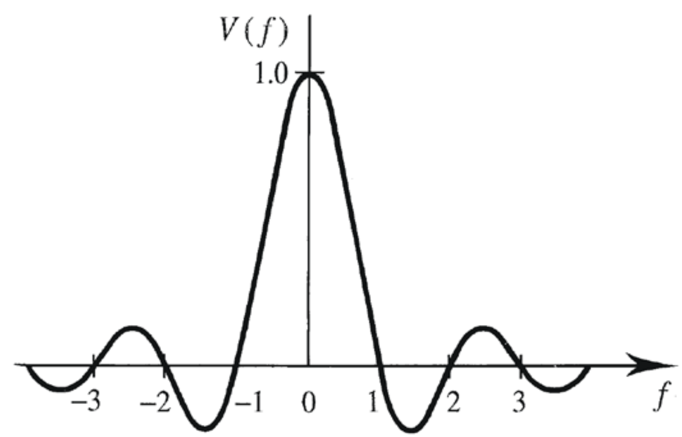
Transient Signals
Fourier Transform (Rectangular Pulse)

- The unit rectangular pulse is defined by

$$\Pi(t) \triangleq \begin{cases} 1.0, & |t| < 1/2 \\ 0.5, & |t| = 1/2 \\ 0, & |t| > 1/2 \end{cases}$$



(a) Unit rectangular pulse, $\Pi(t)$



(b) Fourier transform of unit rectangular pulse centred on $t = 0$

Transient Signals
Fourier Transform (Rectangular Pulse)

- The voltage spectrum is given by its Fourier transform:

$$V_{\Pi}(f) = \int_{-\infty}^{\infty} \Pi(t) e^{-j2\pi f t} dt = \int_{-1/2}^{1/2} e^{-j2\pi f t} dt$$

$$= \left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{-1/2}^{1/2} = \frac{1}{j2\pi f}\left[e^{j\pi f} - e^{-j\pi f}\right]$$

$$= \frac{j2\sin(\pi f)}{j2\pi f} = \frac{\sin(\pi f)}{\pi f} \triangleq \text{sinc}(f)$$

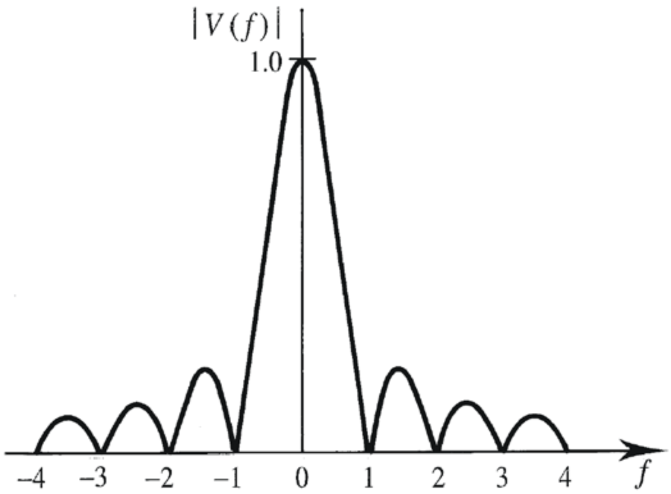
- The unit rectangular pulse and unit sinc function form a Fourier transform

$$\Pi(t) \overset{FT}{\Leftrightarrow} \text{sinc}(f)$$

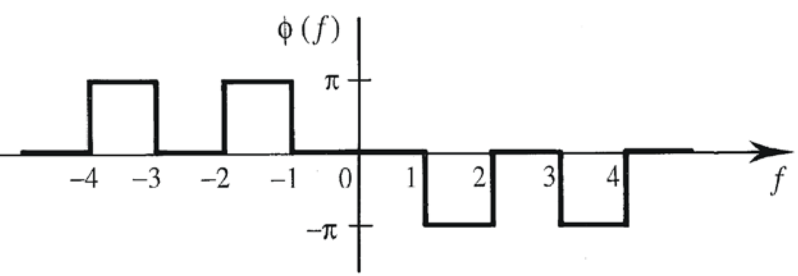
Transient Signals
Fourier Transform (Rectangular Pulse)

- Whilst in this case the voltage spectrum can be plotted as a single curve, in general the voltage spectrum of a transient signal is complex and must be plotted as amplitude and phase spectra.
- The (complex) voltage spectrum of $\Pi\left[\frac{(t-T)}{\tau}\right]$ where T is the location of the center of the pulse and τ is its width is given by

$$\Pi\left[\frac{(t-T)}{\tau}\right] \overset{FT}{\Leftrightarrow} \tau \text{sinc}(\tau f) e^{-j2\pi f T}$$



(a) Amplitude spectrum

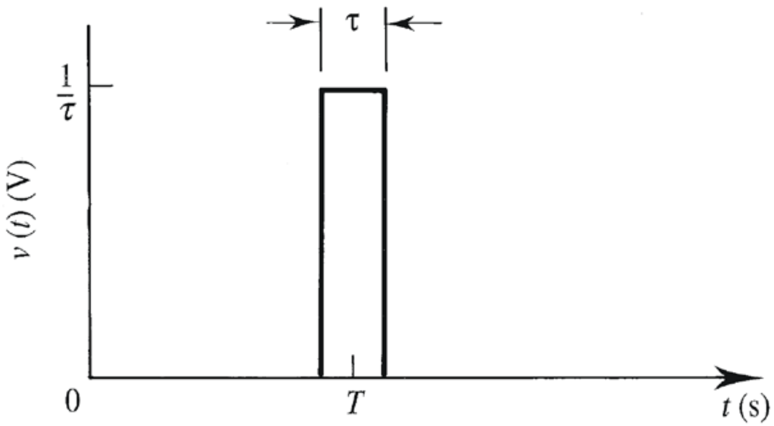


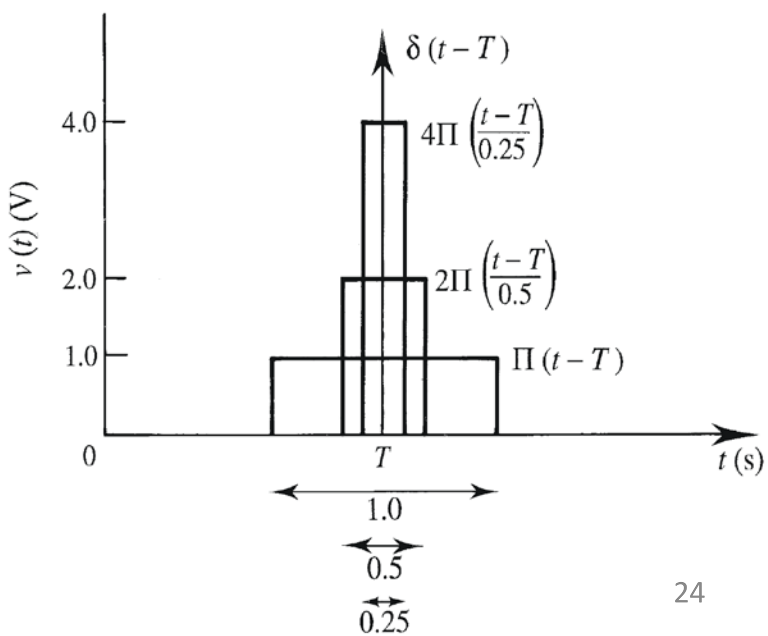
(b) Phase spectrum

Transient Signals
Fourier Transform (Impulse Function)

- Consider a tall, narrow, rectangular voltage pulse of width τ (s) and amplitude $1/\tau$ (V) occurring at time $t = T$.
- The area under the pulse (also called its strength) is 1.0 V s.
- The impulse function (also called the Dirac delta function) is defined as the limit of this rectangular pulse as τ tends to zero:

$$\delta(t-T) = \lim_{\tau \to 0} \left(\frac{1}{\tau}\right) \Pi\left[\frac{(t-T)}{\tau}\right]$$





Transient Signals
Fourier Transform (Impulse Function)

- Whatever the value of τ, the strength of the pulse remains unity.
- Mathematically, the impulse is described by

$$\delta(t-T) = \begin{cases} \infty, & t = T \\ 0, & t \neq T \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t-T)dt = \int_{T^-}^{T^+} \delta(t-T)dt = 1.0$$

- More strictly, the impulse is defined by its sampling property:

$$\int_{-\infty}^{\infty} \delta(t-T)f(t)dt = f(T)$$

- Proof:

$$\int_{-\infty}^{\infty} \delta(t-T)f(t)dt = \int_{T^-}^{T^+} \delta(t-T)f(t)dt = \int_{T^-}^{T^+} \delta(t-T)f(T)dt = f(T)\int_{T^-}^{T^+} \delta(t-T)dt = f(T)$$

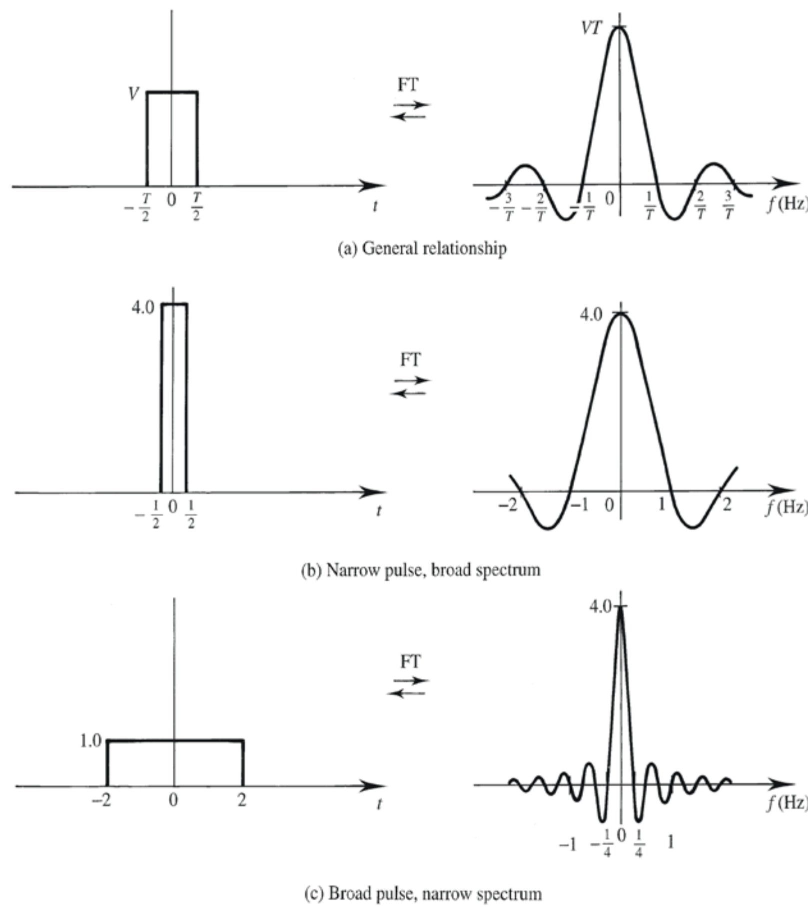
Transient Signals
Fourier Transform (Impulse Function)

- As a rectangular pulse gets narrower its Fourier transform (i.e. a sinc function) gets wider.
- Using $\Pi\left[\frac{(t-T)}{\tau}\right] \overset{FT}{\Leftrightarrow} \tau\mathrm{sinc}(\tau f)e^{-j2\pi fT}$ it is seen that as $\tau \to 0$ then $\tau f \to 0$ and $\mathrm{sinc}(\tau f) \to 1.0$. It follows that

$$\lim_{\tau\to 0} \mathrm{FT}\left\{\frac{1}{\tau}\Pi\left[\frac{(t-T)}{\tau}\right]\right\} = e^{-j2\pi fT}$$

i.e.:

$$\delta(t-T) \overset{FT}{\Leftrightarrow} e^{-j2\pi fT}$$



(a) General relationship

(b) Narrow pulse, broad spectrum

(c) Broad pulse, narrow spectrum

Transient Signals
Fourier Transform (Impulse Function)

- For an impulse occurring at the origin this reduces to

$$\delta(t) \overset{FT}{\Leftrightarrow} 1$$

- The amplitude spectrum of an impulse function is therefore a constant (measured in V/Hz if $\delta(t)$ has units of V).
- Such a spectrum is sometimes referred to as white, since all frequencies are present in equal quantities. This is analogous to white light.

Transient Signals
Properties of Fourier Transform

- Since signals can be fully described in either the time or frequency domain, it follows that any operation on a signal in one domain has a precisely equivalent operation in the other domain.

Function	Time domain	Frequency domain
Linearity	$av(t) + bw(t)$	$aV(f) + bW(f)$
Time delay	$v(t-T)$	$V(f)e^{-j\omega T}$
Change of scale	$v(at)$	$V(f/a)/\|a\|$
Time reversal	$v(-t)$	$V(-f)$
Time conjugation	$v^*(t)$	$V^*(-f)$
Frequency translation	$v(t)e^{j\omega_c t}$	$V(f-f_c)$
Convolution	$v(t) * w(t)$	$V(f)W(f)$
Multiplication	$v(t)w(t)$	$V(f) * W(f)$

Transient Signals

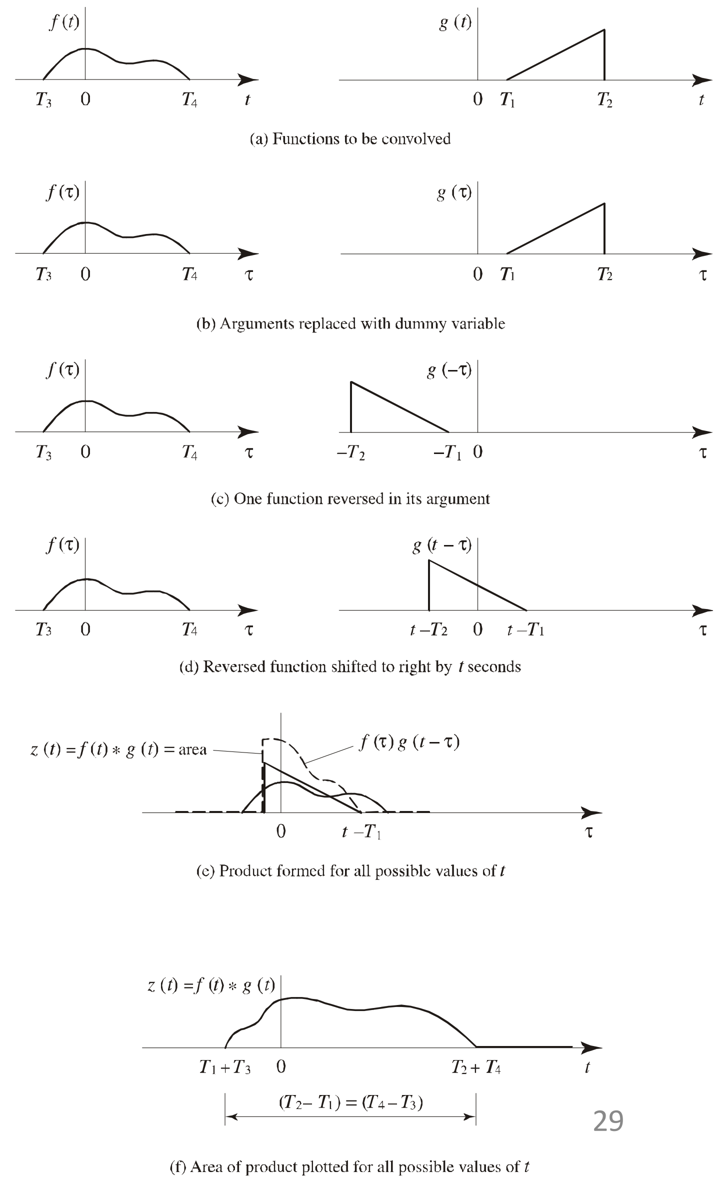
Convolution

- Convolution applied to two time functions $z(t) = f(t) * g(t)$ is defined by

$$z(t) = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

- Convolution is not restricted to the time domain. It can be applied to functions of any variable, e.g. frequency (or space)

$$Z(f) = F(f) * G(f)$$
$$= \int_{-\infty}^{\infty} F(\emptyset) G(f - \emptyset) d\emptyset$$



Transient Signals
Convolution

- The unitary operator for convolution is $\delta(t)$:

$$f(t) * \delta(t) = \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)d\tau = f(t)$$

$$f(t) * \delta(t-T) = f(t-T)$$

- Convolution in the time domain corresponds to multiplication in the frequency domain and vice versa.
- Convolution is commutative, associative and distributive:

$$f * g = g * f$$

$$f * (g * h) = (f * g) * h$$

$$f * (g + h) = f * g + f * h$$

- The derivative of a convolution:

$$\frac{d}{dt}[v(t) * w(t)] = \frac{dv(t)}{dt} * w(t) = v(t) * \frac{dw(t)}{dt}$$

Transient Signals
Convolution

- **[Example 1.2]**
- Convolve the two transient signals, $x(t) = \Pi\left[\frac{(t-1)}{2}\right]\sin \pi t$ and $h(t) = 2\Pi\left[\frac{(t-2)}{2}\right]$.

- **[Example 1.3]**
- Convolve the function $\Pi\left(t - \frac{1}{2}\right)$ with itself and show that the Fourier transform of the result is the square of the Fourier transform of $\Pi\left(t - \frac{1}{2}\right)$.
  - (Given that for triangular function of base width 2τ, denoted by $\Lambda\left(\frac{t}{\tau}\right)$, we have $\Lambda\left(\frac{t}{\tau}\right) \overset{FT}{\Leftrightarrow} \tau\mathrm{sinc}^2(\tau f)$)

Transient Signals
Rectangular Pulse Train

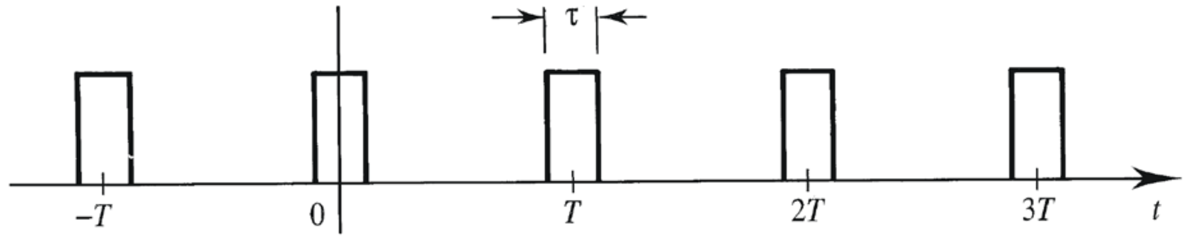
- Using the impulse function a Fourier transform of a periodic function can be defined.
- A periodic rectangular pulse stream $\sum_{n=-\infty}^{\infty} \prod[(t-nT)/\tau]$ can be represented by the convolution of a transient signal (corresponding to the single period given by $n=0$) with the periodic impulse train $\sum_{n=-\infty}^{\infty} \delta(t-nT)$, i.e.:

$$\sum_{n=-\infty}^{\infty} \Pi\left[\frac{t-nT}{\tau}\right] = \Pi\left(\frac{t}{\tau}\right) * \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

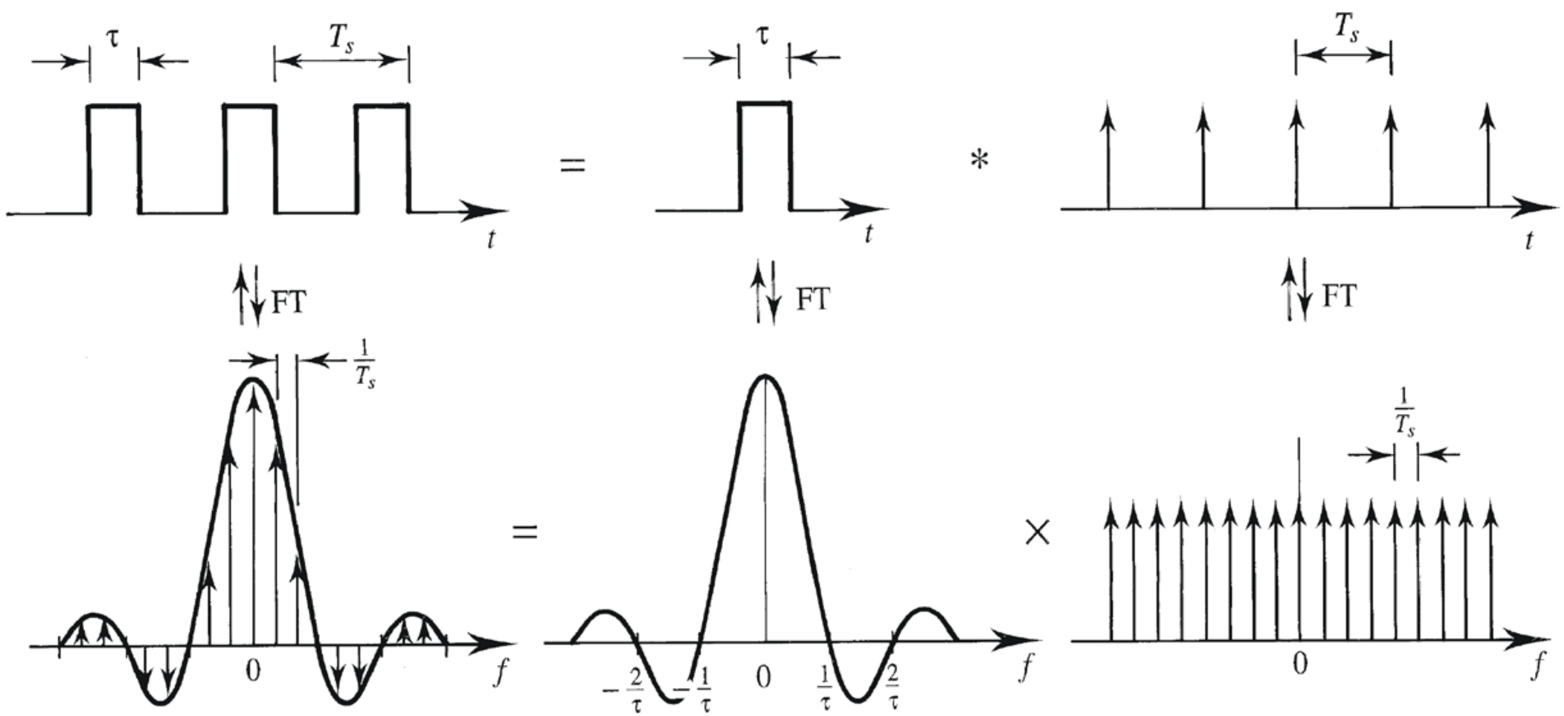
- The Fourier transform of an impulse train is another impulse train:

$$\sum_{k=-\infty}^{\infty} \delta(t-kT_s) \overset{FT}{\Leftrightarrow} f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

where T_s is the time domain period, f_s is the frequency domain period, and $T_s = 1/f_s$.



(a) Periodic pulse train



(b) Time and frequency domain representation of a periodic pulse train showing spectral lines arising from periodicity and spectral envelope arising from pulse shape

Transient Signals
Rectangular Pulse Train

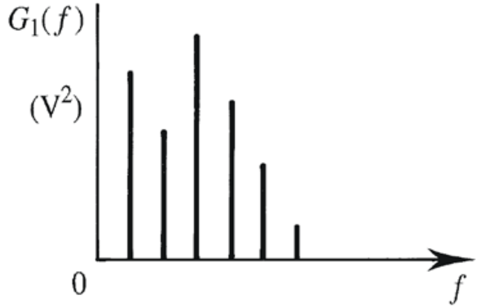
- The voltage spectrum of a rectangular pulse train is thus
$$\mathrm{FT}\left\{\sum_{n=-\infty}^{\infty}\Pi\left[\frac{t-nT}{\tau}\right]\right\}=\mathrm{FT}\left\{\Pi\left(\frac{t}{\tau}\right)\right\}\cdot\mathrm{FT}\left\{\sum_{n=-\infty}^{\infty}\delta(t-nT)\right\}$$
$$=\tau\text{sinc}(\tau f)\frac{1}{T}\sum_{n=-\infty}^{\infty}\delta\left(f-\frac{n}{T}\right)$$
- This shows that the spectrum is given by a periodic impulse train (i.e. a line spectrum) with impulse separation of $1/T$ and impulse strength of $\left(\frac{\tau}{T}\right)\text{sinc}(\tau f)$.
- $\left(\frac{\tau}{T}\right)\text{sinc}(\tau f)$ is usually called the spectrum envelope and, although real here, is potentially complex.

Transient Signals
Rectangular Pulse Train

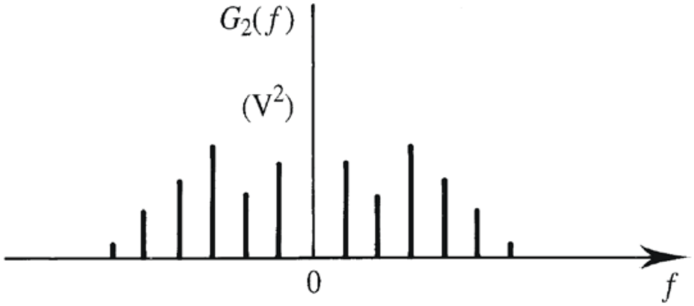
- **[Example 1.4]**
- Sketch the Fourier spectra for a rectangular pulse train comprising pulses of amplitude A V and width 0.05 s with the following pulse repetition periods: (a) $\frac{1}{4}$ s; (b) $\frac{1}{2}$ s; and (c) 1 s.

Transient Signals
Power and Energy Spectra

- For periodic signals, as an alternative to plotting voltage against frequency, the following quantity can be plotted:
$$G_1(f) = |V_{RMS}(f)|^2 \quad (\mathrm{V}^2)$$
- This is a (single sided, denoted by the subscript 1) line spectrum representing normalized power.
- The two sided spectrum can be defined by half the power in each spectral line with a negative frequency.
- The total power in a signal is the sum of the powers in all its spectral lines.



(a) One sided spectrum, $G_1(f)$



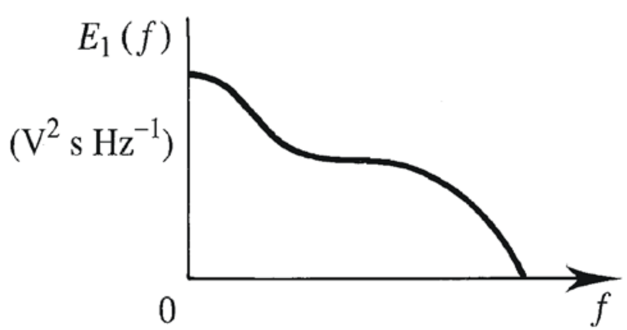
(b) Two sided spectrum, $G_2(f) = \frac{G_1(f) + G_1(-f)}{2}$

Transient Signals
Power and Energy Spectra

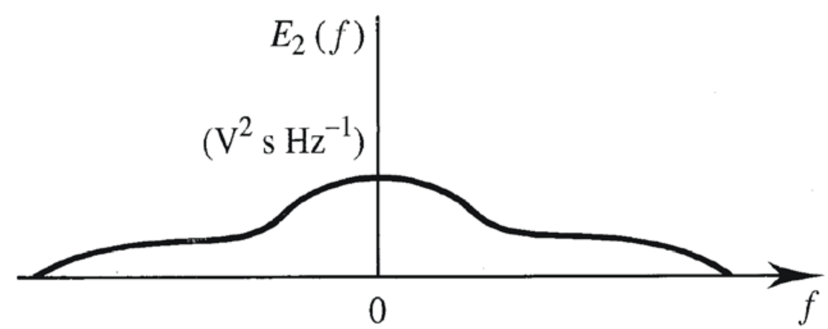
- For a transient signal the energy spectral density is defined as

$$E_2(f) = |V(f)|^2$$

- Like power spectra, energy spectra can be presented as two- or one-sided.
- Energy spectra are always continuous whilst power spectra can be either discrete (as is the case for periodic waveforms) or continuous (as is the case for random signals).



(a) One sided spectrum, $E_1(f)$



(b) Two sided spectrum, $E_2(f) = \frac{E_1(f) + E_1(-f)}{2}$

Chapter 2: Periodic and Transient Signals

Contents

Correlation Functions
Cross-Correlation

- The cross correlation of real transient signals $v(t)$ and $w(t)$ is defined as

$$R_{vw}(\tau) = \int_{-\infty}^{\infty} v(t)w(t-\tau)dt$$

- $R_{vw}(\tau)$ is a measure of the similarity between $v(t)$ and a time shifted version of $w(t)$.
- The value of $R_{vw}(\tau)$ depends not only on the similarity of the signals, but also on their magnitude. The latter dependence can be removed by normalizing both functions such that their associated normalised energies are unity, i.e.:

$$\rho_{vw}(\tau) = \frac{\int_{-\infty}^{\infty} v(t)w(t-\tau)dt}{\sqrt{\left(\int_{-\infty}^{\infty}|v(t)|^2 dt\right)}\sqrt{\left(\int_{-\infty}^{\infty}|w(t)|^2 dt\right)}}$$

Correlation Functions
Cross-Correlation

- $\rho_{vw}(\tau)$ has the following properties
  - $-1 \leq \rho_{vw}(\tau) \leq 1$
  - $\rho_{vw}(\tau) = -1$ if, and only if, $v(t) = -kw(t-\tau)$
  - $\rho_{vw}(\tau) = 1$ if, and only if, $v(t) = kw(t-\tau)$
  - $\rho_{vw}(\tau) = 0$ indicates that the signals are orthogonal and hence have no similarity whatsoever.
- For real periodic waveforms, $p(t)$ and $q(t)$, the cross correlation function is defined by

$$R_{pq}(\tau) = \lim_{T' \to \infty} \frac{1}{T'} \int_{-T'/2}^{T'/2} p(t)q(t-\tau)dt$$

- The normalized cross correlation function, $\rho_{pq}(\tau)$ can be defined similarly.

Correlation Functions
Cross-Correlation

- **[Example 1.5]**
- Find the normalized cross correlation function of the sinusoid waveform $q(t) = \cos 100\pi t$ and the square waveform $p(t) = \sum_{n=-\infty}^{\infty}\left[\Pi\left(\frac{t-0.02n-0.005}{0.01}\right) - \Pi\left(\frac{t-0.02n+0.005}{0.01}\right)\right]$.

Correlation Functions
Autocorrelation

- If the two signals being correlated are identical then the result is called the autocorrelation function, denoted by $R_{vv}(\tau)$ or $R_v(\tau)$.
- For real transient signals:

$$R_v(\tau) = \int_{-\infty}^{\infty} v(t)v(t-\tau)dt$$

- For real periodic signals:

$$R_p(\tau) = \frac{1}{T}\int_{t}^{t+T} p(t)p(t-\tau)dt$$

Correlation Functions
Properties of the Autocorrelation

- $R_x(\tau)$ is real
- $R_x(\tau)$ has even symmetry about $\tau = 0$, i.e. $R_x(\tau) = R_x(-\tau)$
- $R_x(\tau)$ has a maximum (positive) magnitude at $\tau = 0$, i.e. $|R_x(\tau)| \leq R_x(0)$, for any $\tau \neq 0$
- If $x(t)$ is periodic and has units of V then $R_x(\tau)$ is also periodic (with the same period as $x(t)$) and has units of V^2 (i.e. normalized power)
- If $x(t)$ is transient and has units of V then $R_x(\tau)$ is also transient and has units of $V^2\ s$ (i.e. normalized energy)
- The autocorrelation function of a transient signal and its (two-sided) energy spectral density are a Fourier transform pair, i.e. $R_v(\tau) \overset{FT}{\Leftrightarrow} E_v(f)$
- The autocorrelation function of a periodic signal and its (two-sided) power spectral density (represented by a discrete set of impulse functions) are a Fourier transform pair, i.e. $R_p(\tau) \overset{FT}{\Leftrightarrow} G_p(f)$

Correlation Functions
Autocorrelation Functions

- **[Example 1.6]**
- What is the autocorrelation function, and decorrelation time, of the rectangular pulse $\mathrm{v}(\mathrm{t}) = 2\Pi(t - 1.5)$? From a knowledge of its autocorrelation function find the pulse's energy spectral density.

Chapter 2: Periodic and Transient Signals

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   – Signals and waveforms
2. Periodic signals
   – Fourier series
   – Spectrum and power spectrum
3. Transient signals
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   – Spectrum density and energy spectrum density
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5. Summary

Summary

- Deterministic signals can be periodic or transient.
- Periodic waveforms
  - They have discrete (line) spectra.
  - They can be expressed as a sum of harmonically related sinusoids.
  - For real signals the two-sided amplitude spectrum and phase spectrum have even and odd symmetry about 0 Hz, respectively.
  - If the power associated with each sinusoid in a Fourier series is plotted against frequency the result is a power spectrum with units of V^2 or W.
  - The total power in a waveform is the sum of the powers in each spectral line (Parseval's theorem)
  - Bandwidth refers to the width of the frequency band in a signal's spectrum that contains significant power (energy for transient signals)

Summary

- Transient signals
  - The voltage spectrum is continuous and is given by the Fourier transform. Since the units are V/Hz, it is referred to as a voltage spectral density.
  - The square of the amplitude spectrum has units of V^2s/Hz and is called an energy spectral density.
  - The total energy in a transient signal is the integral over all frequencies of the energy spectral density.
  - Fourier transform pairs are uniquely related. Operation (e.g. convolution) in one domain is equivalent to manipulation (e.g. multiplication) in the other domain.
- Correlation
  - It is a measure of signal similarity.
  - The energy and power spectral densities of transient and periodic signals respectively are the Fourier transforms of their autocorrelation functions.

Principles of Communications (通信系统原理)

Undergraduate Course

Chapter 2: Periodic and Transient Signals

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