Principles of Communications (通信系统原理)

Undergraduate Course

Chapter 7: Baseband Transmission and Line Coding

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Chapter 7: Baseband Transmission and Line Coding

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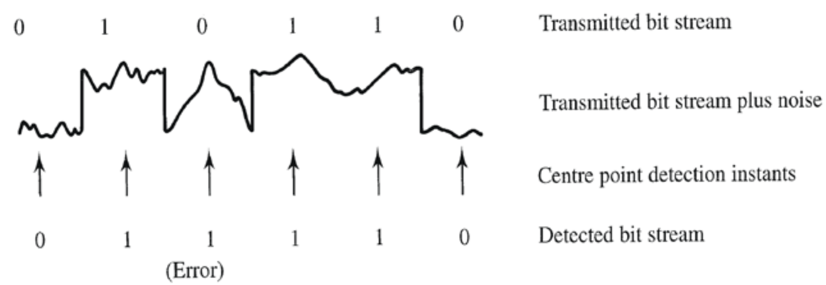
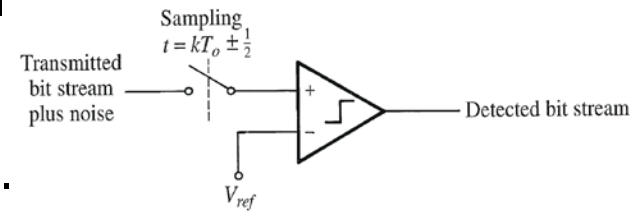
Baseband Center Point Detection
Basics

- The detection of digital signals involves two processes
  - Reduction of each received voltage pulse (i.e. symbol) to a single numerical value.
  - Comparison of this value with a reference voltage (or, for multi-symbol signaling, a set of reference voltages) to determine which symbol was transmitted.

- For symbols represented by different voltage levels they can be achieved by sampling the received signal plus noise and then sending the samples to one or multiple comparators.
- Sampling the instantaneous signal plus noise voltage near the middle of the symbol period is called center point detection.
- The noise present during detection is often either Gaussian or approximately Gaussian.



(a) Schematic illustration of centre point detection



(b) Principle of centre point detector

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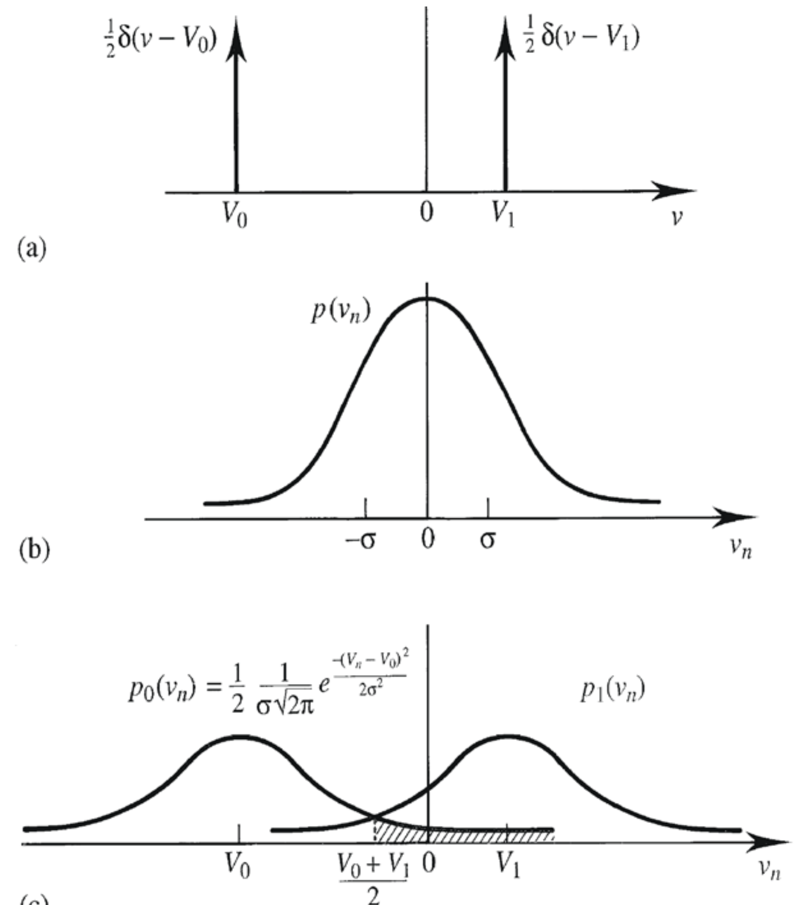
Baseband Center Point Detection

Baseband Binary Error Rates in Gaussian Noise

- For equiprobable symbols the optimum decision level is set at $(V_0 + V_1)/2$.
- Given that the symbol 0 is transmitted (i.e. a voltage level V_0) then the probability, P_{e1}, that the signal plus noise will be above the threshold at the decision instant is given by

$$P_{e1} = \int_{(V_0+V_1)/2}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v_n - V_0)^2}{2\sigma^2}} dv_n$$



Pdf of (a) binary symbols, (b) noise, (c) signal plus noise

Baseband Center Point Detection

Baseband Binary Error Rates in Gaussian Noise

- Using the change of variable $x = \frac{v_n - V_0}{\sqrt{2}\sigma}$ this becomes

$$P_{e1} = \frac{1}{\sqrt{\pi}} \int_{(V_1 - V_0)/2\sqrt{2}\sigma}^{\infty} e^{-x^2} dx$$

- This integral cannot be evaluated analytically but can be recast as a complementary error function, $\mathrm{erfc}(z)$, defined by

$$\mathrm{erfc}(z) \triangleq \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} e^{-x^2} dx$$

- Thus $P_{e1} = \frac{1}{2}\mathrm{erfc}\left(\frac{V_1 - V_0}{2\sigma\sqrt{2}}\right)$.
- Alternatively, since $\mathrm{erfc}(z)$ and the error function, $\mathrm{erf}(z)$, are related by $\mathrm{erfc}(z) \equiv 1 - \mathrm{erf}(z)$, then $P_{e1} = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{V_1 - V_0}{2\sigma\sqrt{2}}\right)\right]$

Baseband Center Point Detection

Baseband Binary Error Rates in Gaussian Noise

- If the digital symbol one is transmitted (i.e. a voltage level V_1) then the probability, P_{e0}, that the signal plus noise will be below the threshold is

$$P_{e0} = \int_{-\infty}^{(V_0+V_1)/2} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v_n - V_1)^2}{2\sigma^2}} dv_n$$

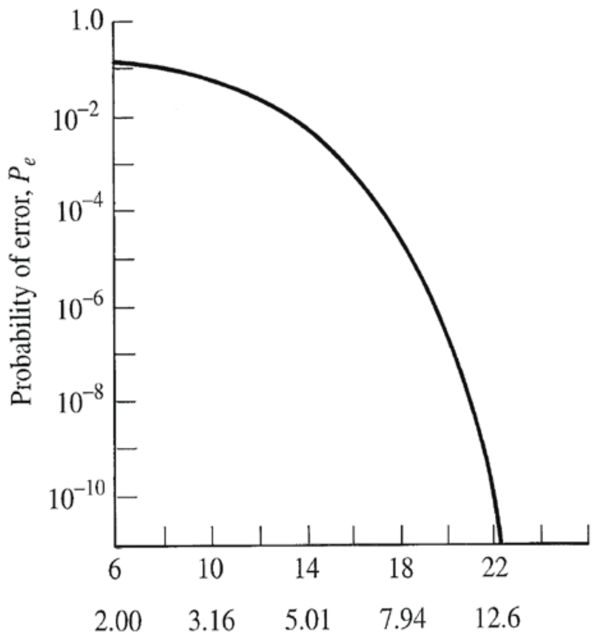
- From the symmetry of this problem, we can have $P_{e0} = P_{e1} = P_e$, where P_e is the probability of error, irrespective of whether a one or zero was transmitted.
- P_e depends on only symbol voltage difference $\Delta V = V_1 - V_0$:

$$P_e = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2}}\right)\right]$$

Baseband Center Point Detection

Baseband Binary Error Rates in Gaussian Noise

- The expression $P_e = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2}}\right)\right]$ is valid for both unipolar signaling (i.e. symbols represented by voltages of 0 and $\Delta\mathrm{V}$) and polar signaling (i.e. symbols represented by voltages of $\pm\Delta\mathrm{V}/2$).



P_e versus $\Delta V/\sigma$
Polar NRZ: SNR = $20\log_{10}(\Delta V/\sigma) - 6$ dB,
Unipolar NRZ: SNR = $20\log_{10}(\Delta V/\sigma) - 3$ dB

Baseband Center Point Detection

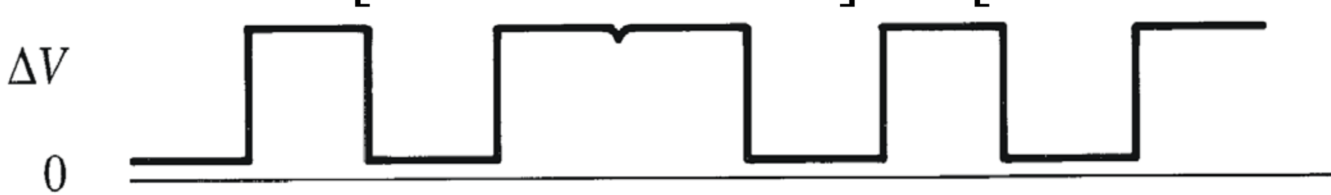
Baseband Binary Error Rates in Gaussian Noise

- For NRZ (non-return to zero), unipolar, rectangular pulse signaling, the normalized peak signal power is $S_{peak} = \Delta V^2$ and the average signal power is $S = \Delta V^2/2$. The normalized Gaussian noise power is $N = \sigma^2$. We have

$$\frac{\Delta V}{\sigma} = \left(\frac{S}{N}\right)_{peak}^{\frac{1}{2}} = \sqrt{2}\left(\frac{S}{N}\right)^{\frac{1}{2}}$$

- Therefore

$$P_e = \frac{1}{2}\left[1 - \operatorname{erf}\frac{1}{2\sqrt{2}}\left(\frac{S}{N}\right)_{peak}^{\frac{1}{2}}\right] = \frac{1}{2}\left[1 - \operatorname{erf}\frac{1}{2}\left(\frac{S}{N}\right)^{\frac{1}{2}}\right]$$



Baseband Center Point Detection

Baseband Binary Error Rates in Gaussian Noise

- For NRZ, polar, rectangular pulse signaling with the same voltage spacing as in the unipolar case, the peak and average signal powers are identical, i.e. $S_{peak} = S = \left(\frac{\Delta V}{2}\right)^2$. We have

$$\frac{\Delta V}{\sigma} = 2\left(\frac{S}{N}\right)_{peak}^{\frac{1}{2}} = 2\left(\frac{S}{N}\right)^{\frac{1}{2}}$$

- Therefore

$$P_e = \frac{1}{2}\left[1 - \operatorname{erf}\frac{1}{\sqrt{2}}\left(\frac{S}{N}\right)^{\frac{1}{2}}\right]$$

Baseband Center Point Detection

Baseband Binary Error Rates in Gaussian Noise

- For statistically independent, equiprobable binary symbols, each symbol carries 1 bit of information. The probability of symbol error, P_e, is identical to the probability of bit error, P_b.
- The symbol error rate (SER) or bit error rate (BER) are the number of symbol or bit errors occurring per unit time (usually 1 second).
- The SER is related to the probability of symbol error by

$$SER = P_e R_s$$

where R_s is the symbol rate in symbol/s or baud.

- The BER is related to the probability of bit error by

$$BER = P_b R_s n = P_b R_b$$

where n is the number of binary digits in each symbol and R_b is the bit rate.

Baseband Center Point Detection

Baseband Binary Error Rates in Gaussian Noise

- [**Example 6.1**]
- Find the BER of a 100 kbaud, equiprobable, binary, polar, rectangular pulse signaling system assuming ideal center point decision, if the measured SNR at the detector input is 12.0 dB.

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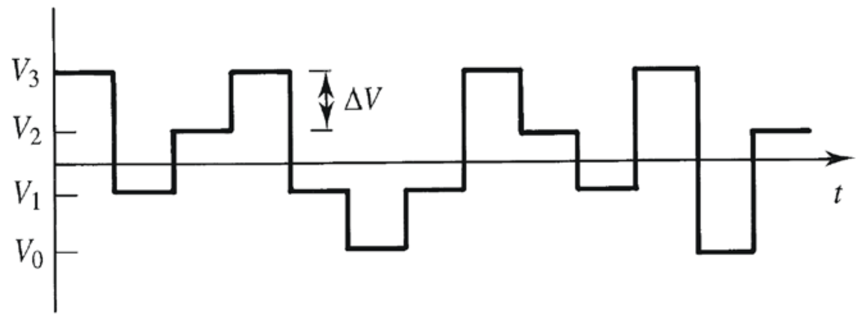
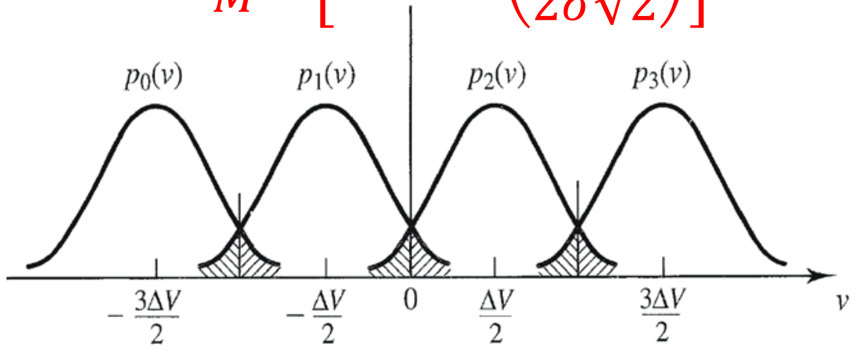
Baseband Center Point Detection
Multilevel Baseband Signaling

- For an equally spaced, M-level signal, the probability of symbol error for the $M-2$ inner symbols is twice that in the binary case, i.e. $P_{eM}|_{inner\ symbols} = 2P_e$
- The symbol error of the two outer levels is identical to that for the binary case, i.e., $P_{eM}|_{outer\ symbols} = P_e$
- For equiprobable symbols, the average probability of symbol error is

$$P_{eM} = \frac{M-2}{M}2P_e + \frac{2}{M}P_e = \frac{2(M-1)}{M}P_e = \frac{M-1}{M}\left[1 - \mathrm{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2}}\right)\right]$$



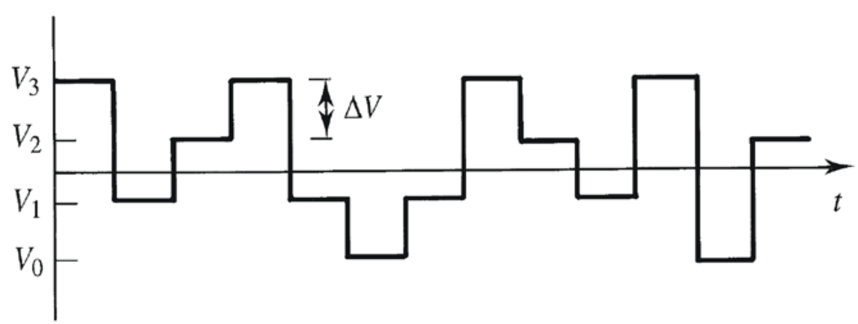
Waveform for 4-level baseband signalling



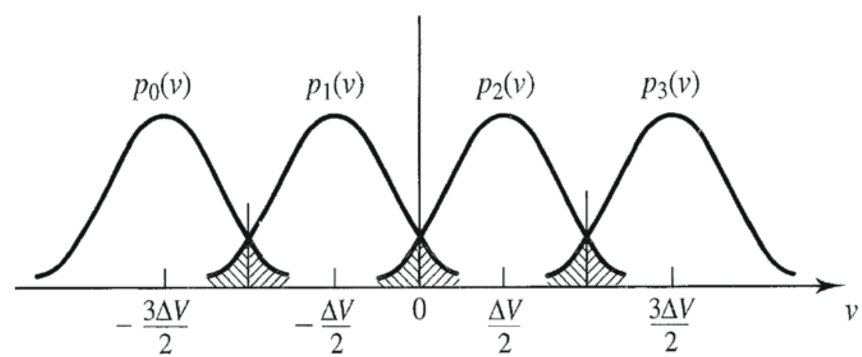
Conditional pdfs for 4-level baseband signalling

Baseband Center Point Detection
Multilevel Baseband Signaling

- [**Example 6.2**]
- A four-level, equiprobable, baseband signaling system uses NRZ rectangular pulse. The attenuation between transmitter and receiver is 15 dB and the noise power at the 50 Ω input of an ideal center point decision detector is 10 μW. Find the average signal power which must be transmitted to maintain a symbol error probability of 10^{-4}.



Waveform for 4-level baseband signalling



Conditional pdfs for 4-level baseband signalling

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Error Accumulation Over Multiple Hops
Basics

- All signal transmission media (e.g. cables, waveguides, optical fibers) attenuate signals to a greater or lesser extent.
- For long communication paths attenuation might be so severe that the sensitivity of normal receiving equipment would be inadequate to detect the signal.
- In such cases the signal is boosted in amplitude at regular intervals along the transmitter-receiver path.
- The equipment which boosts the signal is called a repeater and the path between adjacent repeaters is called a hop.
- A repeater along with its preceding hop is called a section.
- Long distance communication is usually achieved using multiple hops.
- Repeaters can be classified as amplifying repeaters and regenerative repeaters.

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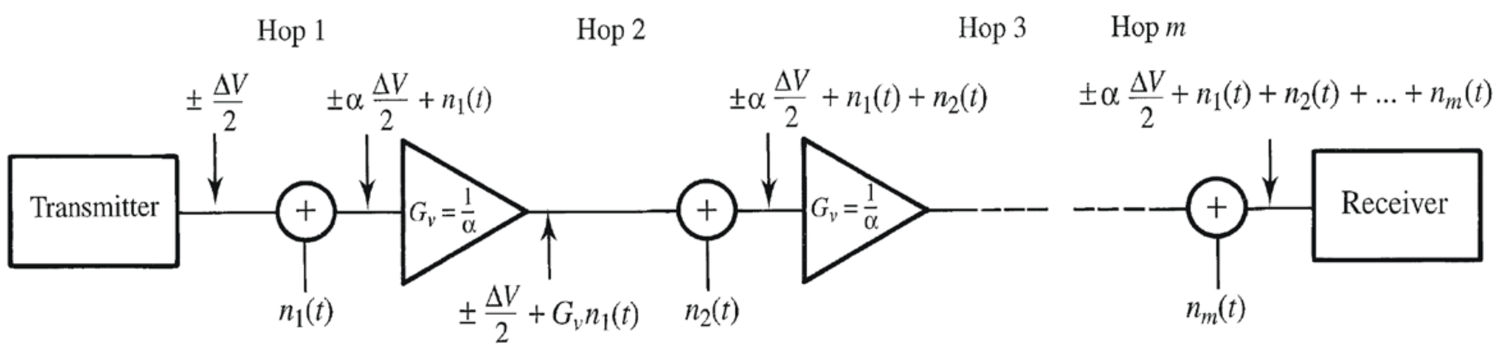
Error Accumulation Over Multiple Hops

Amplifying Repeaters

- If a binary signal with voltage levels $\pm\frac{\Delta V}{2}$ is transmitted then the voltage received at the input of the first amplifying repeater is $\pm\frac{\alpha\Delta V}{2}+n_1(t)$ where $\alpha<1$ is the linear voltage attenuation factor and $n_1(t)$ is the random noise (with standard deviation σ).
- In a well designed system the voltage gain of the repeater $G_V=1/\alpha$.
- At the output of the fist repeater the signal is restored to its original level (i.e. $\pm\frac{\Delta V}{2}$) but the noise signal is also amplified to a level $G_V n_1(t)$ and will now have an RMS value of $G_V\sigma$.

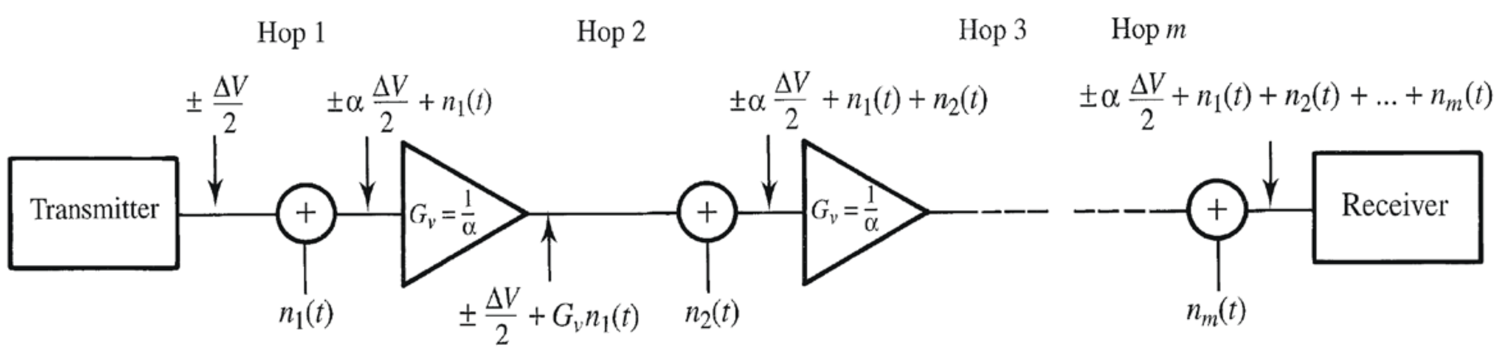


m-hop link utilising linear amplifiers as signal boosters

Error Accumulation Over Multiple Hops
Amplifying Repeaters

- Assuming each hop to incur the same attenuation, the signal voltage at the input to the second repeater will be $\pm\frac{\alpha\Delta V}{2}$ and the noise voltage from the first hop will be $n_1(t)$. A similar noise voltage $n_2(t)$, which is statistically independent from $n_1(t)$, will be added.
- Therefore, the noise power after m hops will be m times the noise power after one hop whilst the signal voltage at the receiver will be essentially the same as at the input to the first repeater.
- The probability of error for an m-hop link is therefore given by

$$P_e\Big|_{m\ hops} = \frac{1}{2}\left[1 - \text{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2m}}\right)\right]$$



m-hop link utilising linear amplifiers as signal boosters

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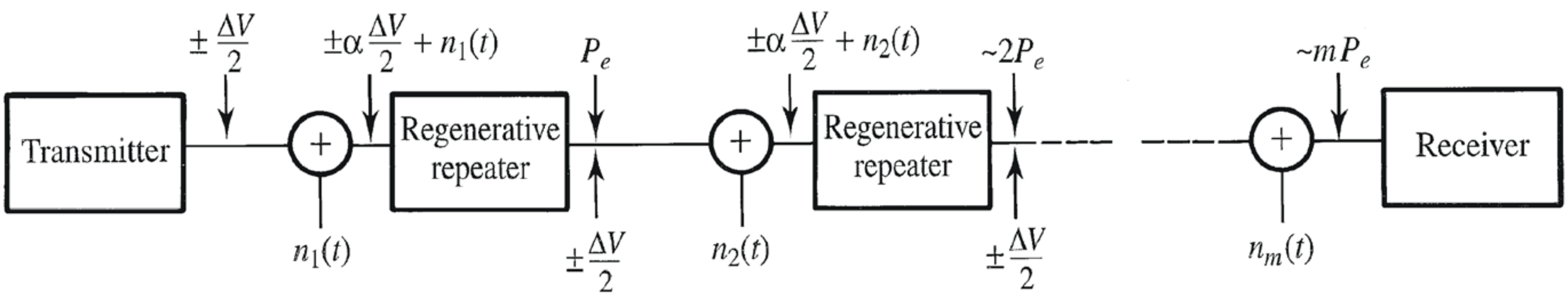
Error Accumulation Over Multiple Hops
Regenerative Repeaters

- A regenerative repeater uses a decision process to establish whether a digital 0 or 1 is present at its input and a new, and noiseless, pulse is generated for transmission to the next repeater.
- Noise does not accumulate from repeater to repeater.
- Symbols will be detected in error at each repeater with a probability P_e.
- Providing $mP_e \ll 1$, the probability of any given symbol being detected in error (and therefore inverted) more than once over the m hops of the link can be neglected.



m-hop link utilising regenerative repeaters as signal boosters

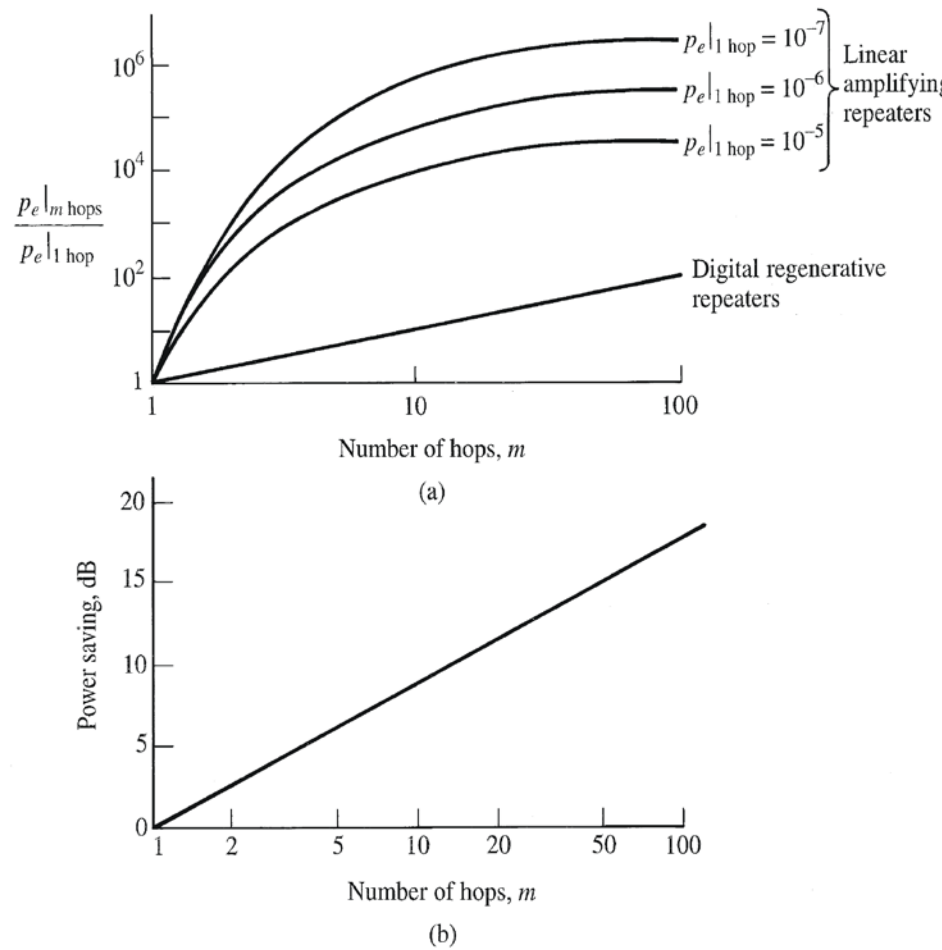
Error Accumulation Over Multiple Hops
Regenerative Repeaters

- The probability of error (rather than the noise power) accumulates linearly over the hops and after m hops we have

$$P_e \Big|_{m\ hops} = mP_e$$

$$= \frac{m}{2}\left[1 - \operatorname{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2}}\right)\right]$$

where P_e is the one-hop error probability.



Probability of error degradation due to multiple hops and power saving using digital repeaters instead of amplifying repeaters for $P_e = 10^{-5}$

Error Accumulation Over Multiple Hops
Examples

- [**Example 6.3**]
- If 15 link sections, each identical to that described in Example 6.1 (i.e. an equiprobable, binary, polar, rectangular pulse signaling system, with the measured SNR at the detector input 12.0 dB), are cascaded to form a 15-hop link, find the probability of bit error when the repeaters are implemented as
  i. linear amplifiers
  ii. digital regenerators

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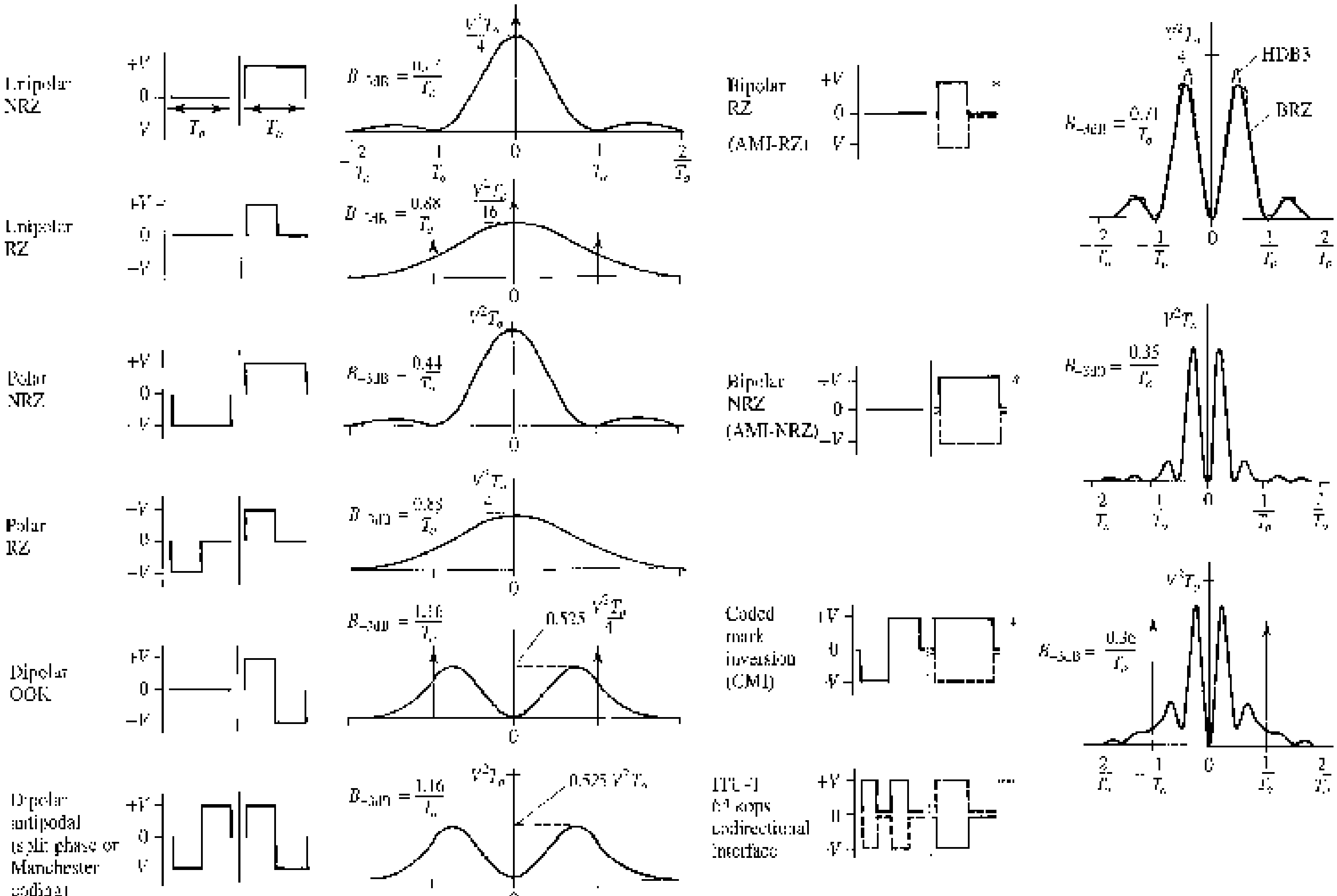
Line Coding
Basics

- Binary data can be transmitted using many pulse types, other than simply unipolar, and polar, rectangular pulses.
- The choice of a particular pair of pulses to represent the symbols 1 and 0 is called line coding.
- The selection is usually made, by considering
  – Presence or absence of a DC level
  – Power spectral density – particularly its value at 0 Hz
  – Spectral occupancy (i.e. bandwidth)
  – BER performance (i.e. relative immunity from noise)
  – Ease of clock signal recovery for symbol synchronization
  – Presence or absence of inherent error detection properties
  – Etc.
- Line coding is usually thought of as the selection, or design, of pulse pairs which retain sharp transitions between voltage levels.

Line Coding
Basics



* Alternate mark inversion

** Alternate symbol inversion

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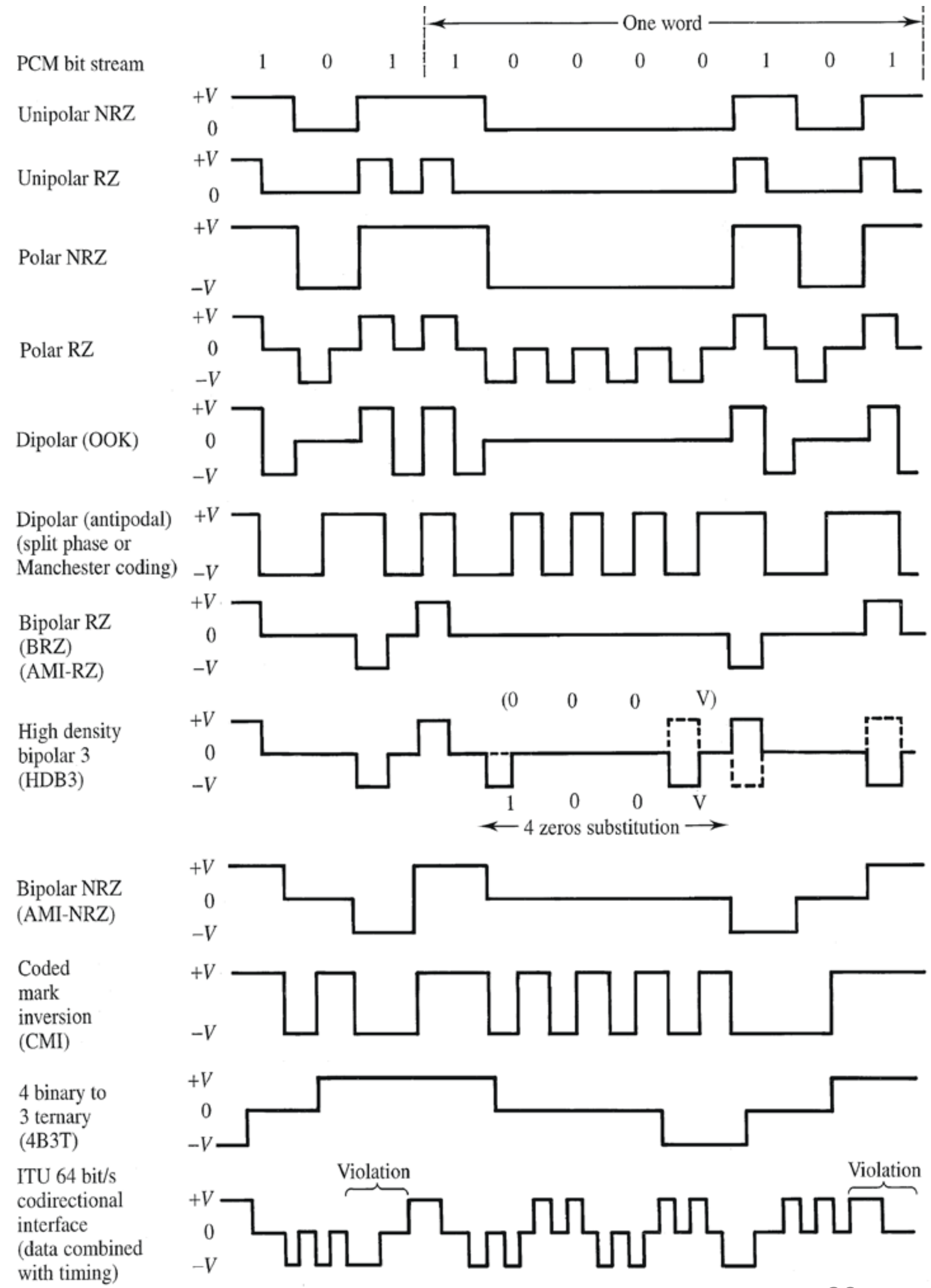
- Unipolar signaling (also called on-off keying, OOK) refers to a line code in which one binary symbol is represented by the absence of a pulse and the other is by the presence of a pulse.
- Non-return to zero (NRZ) signaling: The duration (τ) of the pulse is equal to the duration (T_0) of the symbol slot.
- Return to zero (RZ) signaling: τ is less than T_0.
- Typically RZ pulses fill only the first half of the time slot (i.e., the duty cycle $\frac{\tau}{T_0} = 0.5$)
- The PSD of both NRZ and RZ signals have a $\left[\frac{\sin x}{x}\right]^2$ shape where $x = \pi\tau f$.



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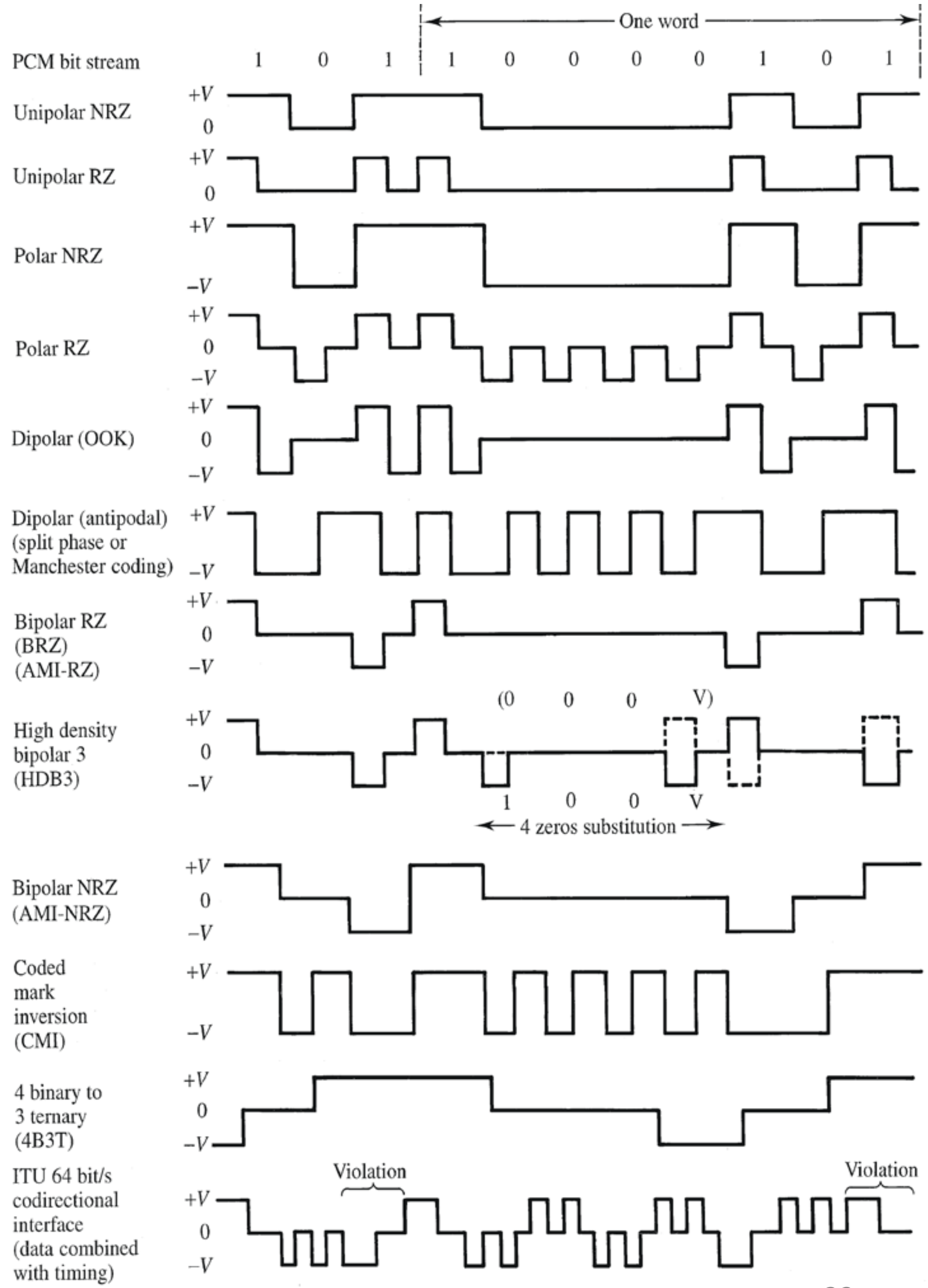
- In polar signaling system a binary one is represented by a pulse $g_1(t)$ and a binary zero by the opposite (or antipodal) pulse $g_0(t) = -g_1(t)$.
- The NRZ and RZ forms of polar signals have identically shaped spectra to the NRZ and RZ forms of unipolar signals except that, due to the opposite polarity of the one and zero symbols, neither contain any spectral lines.
- Polar signals have the same bandwidth requirements as their equivalent unipolar signals.
- Polar signaling has a significant power (or alternatively BER) advantage over unipolar signaling, since the average (or DC) level transmitted by the latter contains no information.



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Multiplex Telephony

- PCM is used in conjunction with TDM to realize multi-channel digital telephony.
- The internationally agreed European ITU standard provides for the combining of 30 speech channels, together with two subsidiary channels for signal and system monitoring.
- Each speech channel signal is sampled at 8 kHz and non-linearly quantized (companded) into 8 bit words.
- The binary symbol rate speech channel is thus 64 kbit/s, and for the composite 30+2 channel signal multiplex is $32 \times 64 = 2.048$ Mbit/s, which is often referred to as a 2 Mbit/s signal.
- A key advantage of the 2 Mbit/s TDM multiplex is that it is readily transmitted over 2 km sections of twisted pair (copper) cables which originally carried only one analogue voice signal.
- In the USA and Japan the multiplex combines fewer speech channels into a 1.5 Mbit/s signal.

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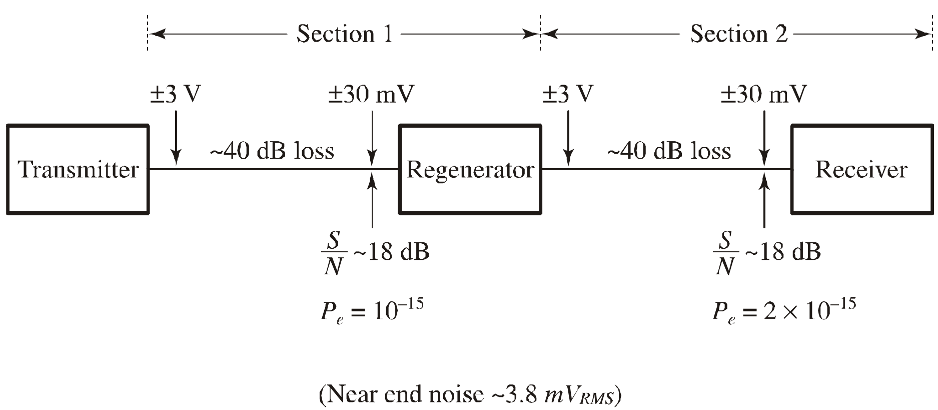
Digital Signal Regeneration
Basics

- Regeneration allows an increase in overall communications path length with negligible decrease in message quality provided each regenerator operates at an acceptable point on the error rate curve.
- Provided that the error rate on each section is acceptable then the cumulative or summed rate for the link is low, compared with the error rate when there is no regeneration and the single section loss is very high.
- Regeneration thus permits long distance transmission with high message quality provided the link is properly sectioned with the appropriate loss on each section.



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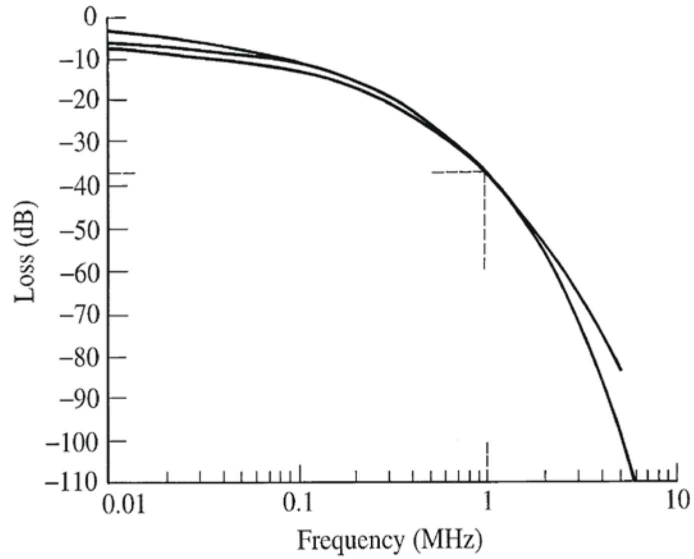
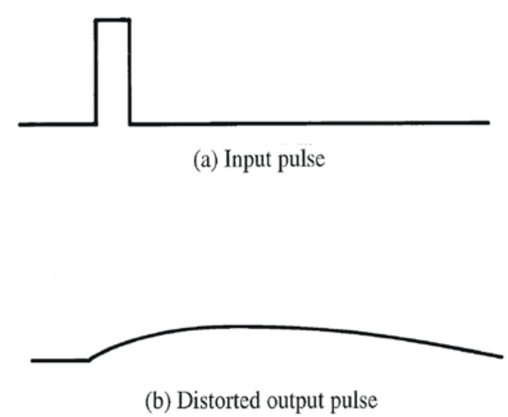
Digital Signal Regeneration
Inter-symbol interference and equalization

- A significant problem in PCM cable, and many other, communication systems is that considerable amplitude and phase distortion may be introduced by the transmission medium.
- For a 2 Mbit/s PCM cable system the RZ bipolar pulse has a width of ¼ μs and hence a bandwidth of approximately 2 MHz.
- When this is compared with typical metallic cable characteristics, it can be seen that the received pulse will be heavily distorted and attenuated.
- A potentially serious consequence of this distortion is that the pulse will be stretched in time.



Typical frequency responses for 2 km length of cable

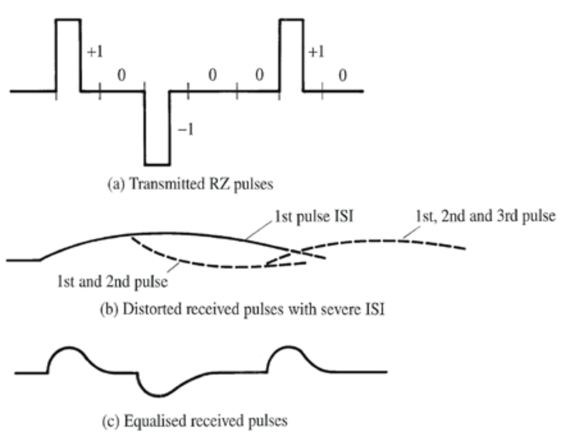


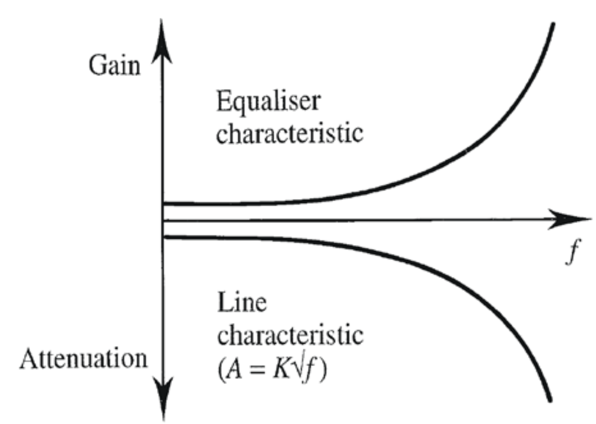
Input and output 2 Mbit/s pulse for a 2 km length of cable

Digital Signal Regeneration

Inter-symbol interference and equalization

- When we move from considering individual pulses to a data stream, the long time domain tails from the individual received symbols cause inter-symbol interference (ISI).
- This is overcome by applying an equalizing filter in the receiver which has the inverse frequency response to the raw line or channel characteristic.
- Cascading the effect of the line with the equalizer provides a flat overall response which reduces the distortion.





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Summary

- The simplest form of baseband digital detection uses center point sampling to reduce the received symbol plus noise to a single voltage, and comparators to test this voltage against appropriate references.
- The probability of error after m identical hops is m times greater than that after a single hop providing that regenerative repeaters are used between hops.
- The choice of a particular pair of pulses to represent binary data is called line coding.
- Distortion is caused due to cable frequency response and can be compensated via equalization.

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