Principles of Communications (通信系统原理)

Undergraduate Course

Chapter 10: Bandpass Modulation

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Chapter 10: Bandpass Modulation

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Introduction

- Modulation refers to *the modification of one signal's characteristics in sympathy with another signal*.
- The signal being modified is called the carrier.
- The signal doing the modifying is called the information signal.
- In *intermediate or radio frequency (IF or RF) bandpass modulation*, the carrier is a sinusoid and the characteristics adjusted are amplitude, frequency or phase.

Introduction

- The principal reason for employing IF modulation is to transform information signals (which are usually generated at baseband) into signals with more convenient (bandpass) spectra.
- This allows:
  - Signals to be matched to the characteristics of transmission lines or channels.
  - Signals to be combined using frequency division multiplexing and subsequently transmitted using a common physical transmission medium.
  - Efficient antennas of reasonable physical size to be constructed for radio communication systems.
  - Radio spectrum to be allocated to services on a rational basis and regulated so that interference between systems is kept to acceptable levels.

Introduction
Spectral and Power Efficiency

- Different modulation schemes can be compared on the basis of their spectral and power efficiencies.
- Spectral efficiency is a measure of information transmission rate per Hz of bandwidth used, with units of bit/s/Hz.
- Carrier-to-noise ratio (CNR) or equivalently the ratio of symbol energy to noise power spectral density (NPSD), E/N_0, is used to compare the power efficiencies of different schemes.
  - Normally, different digital communications systems are compared on the basis of the relative signal power needed to support a given received information rate, assuming identical noise environments.
  - In practice, this usually means comparing the signal power required by different modulation schemes to sustain identical BERs for identical transmitted information rates.

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2. Binary IF modulation
   – Binary amplitude shift keying (and on-off keying)
   – Binary phase shift keying (and phase reversal keying)
   – Binary frequency shift keying
3. Spectrally-efficient modulation techniques
   – M-symbol phase shift keying
   – Amplitude/phase keying and quadrature amplitude modulation
4. Power-efficient modulation techniques
   – Multi-dimensional signaling and MFSK
5. Summary

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Binary IF Modulation
Binary Amplitude Shift Keying

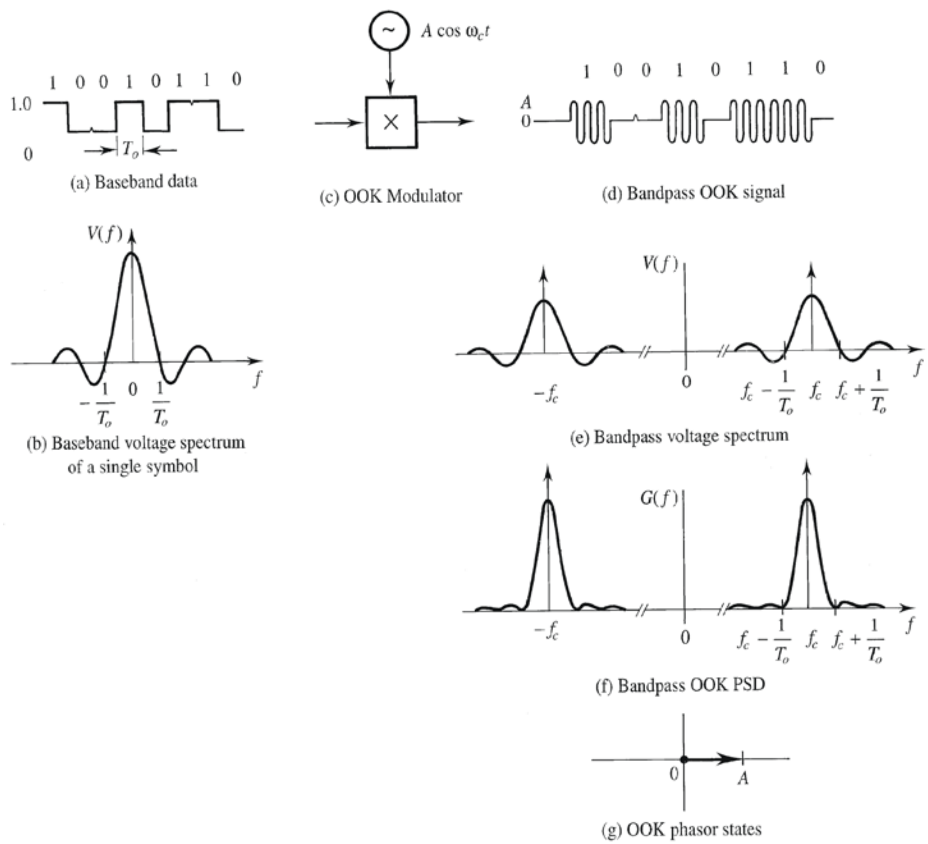
- In binary amplitude shift keying (BASK) systems the two digital symbols, zero and one, are represented by pulses of a sinusoidal carrier (frequency, f_c) with two different amplitudes A_0 and A_1.
- In practice, one of the amplitudes, A_0, is chosen to be zero, resulting in on-off keying (OOK) IF modulation, i.e.:

$$f(t) = \begin{cases} A_1 \Pi(t/T_0) \cos 2\pi f_c t\,, & \text{for a digital 1} \\ 0, & \text{for a digital 0} \end{cases}$$

where T_0 is the symbol duration and $\Pi(\cdot)$ is the rectangular pulse function.

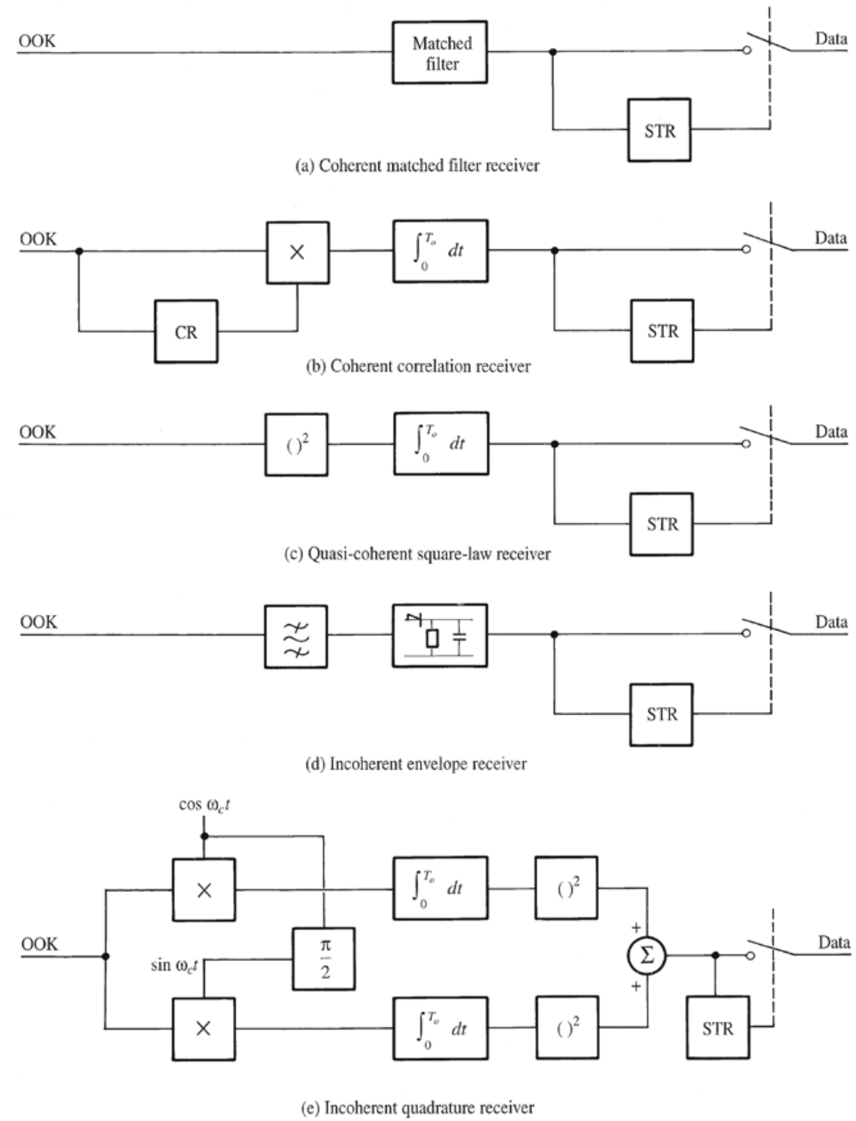
Binary IF Modulation
Binary Amplitude Shift Keying

- An OOK modulator can be implemented either as a simple switch, which keys a carrier on and off, or as a double balanced modulator (or mixer) which is used to multiply the carrier by a baseband unipolar OOK signal.
- The modulated signal has a double sideband (DSB) spectrum centered on $\pm f_c$ and, since a constant carrier waveform is being keyed, the OOK signal has two phasor states, 0 and $A = A_1$.



Binary IF Modulation
Binary Amplitude Shift Keying

- Detection of IF OOK signals can be coherent or incoherent.
- In coherent detection, a matched filter or correlator is used prior to sampling and decision thresholding.
- In incoherent detection, envelope detection is used to recover the baseband digital signal followed by center point sampling or integrate and dump (I+D) detection.



Binary IF Modulation
Binary Amplitude Shift Keying

- The decision instant voltage, $f(nT_0)$, at the output of an OOK matched filter or correlation detector is

$$f(nT_0) = \begin{cases} kE_1, & \text{digital 1} \\ 0, & \text{digital 0} \end{cases}$$

where E_1 $(\mathrm{V}^2 s)$ is the normalized energy contained in symbol 1 and k has units of Hz/V.

- The normalized noise power, σ^2 (V^2), at the detector output is

$$\sigma^2 = k^2 E_1 N_0 / 2$$

where N_0 $(\mathrm{V}^2/\mathrm{Hz})$ is the normalized one-sided NPSD at the matched filter, or correlator, input.

Binary IF Modulation
Binary Amplitude Shift Keying

- The post-filtered decision process is identical to the baseband binary decision process.
- $P_e = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{\Delta V}{2\sigma\sqrt{2}}\right)\right]$ can be used with $\Delta V = k(E_1 - 0)$ and $\sigma^2 = k^2 E_1 N_0/2$ to give the probability of symbol error:

$$P_e = \frac{1}{2}\left[1 - \mathrm{erf}\frac{1}{2}\left(\frac{\mathrm{E}_1}{N_0}\right)^{\frac{1}{2}}\right]$$

- This can be expressed in terms of the time averaged energy per symbol, $\langle E\rangle = \frac{1}{2}(E_1 + E_0)$, where, for OOK, $E_0 = 0$, i.e.:

$$P_e = \frac{1}{2}\left[1 - \mathrm{erf}\frac{1}{\sqrt{2}}\left(\frac{\langle E\rangle}{N_0}\right)^{\frac{1}{2}}\right]$$

Binary IF Modulation
Binary Amplitude Shift Keying

- The above equations can be expressed in terms of received carrier-to-noise ratios (C/N) using the following relations:

$$C = \langle E \rangle / T_0 \quad (\mathrm{V}^2)$$
$$N = N_0 B \quad (\mathrm{V}^2)$$
$$\langle E \rangle / N_0 = T_0 B C/N$$

where C is the received carrier power averaged over all symbol periods and N is the normalized noise power in a bandwidth B Hz.

- This gives

$$P_e = \frac{1}{2}\left[1 - \mathrm{erf}\frac{(T_0 B)^{\frac{1}{2}}}{\sqrt{2}}\left(\frac{C}{N}\right)^{\frac{1}{2}}\right]$$

- For minimum bandwidth (i.e. Nyquist) pulses $T_0 B = 1.0$ and $\langle E \rangle / N_0 = C/N$.

Binary IF Modulation

Binary Amplitude Shift Keying

- **[Example 9.1]**
- An OOK IF modulated signal is detected by an ideal matched filter receiver. The non-zero symbol at the matched filter input is a rectangular pulse with am amplitude 100 mV and a duration of 10 ms. The noise at this point is known to be white and Gaussian, and has an RMS value of 140 mV when measured in a noise bandwidth of 10 kHz. Calculate the probability of bit error.

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Binary IF Modulation
Binary Phase Shift Keying

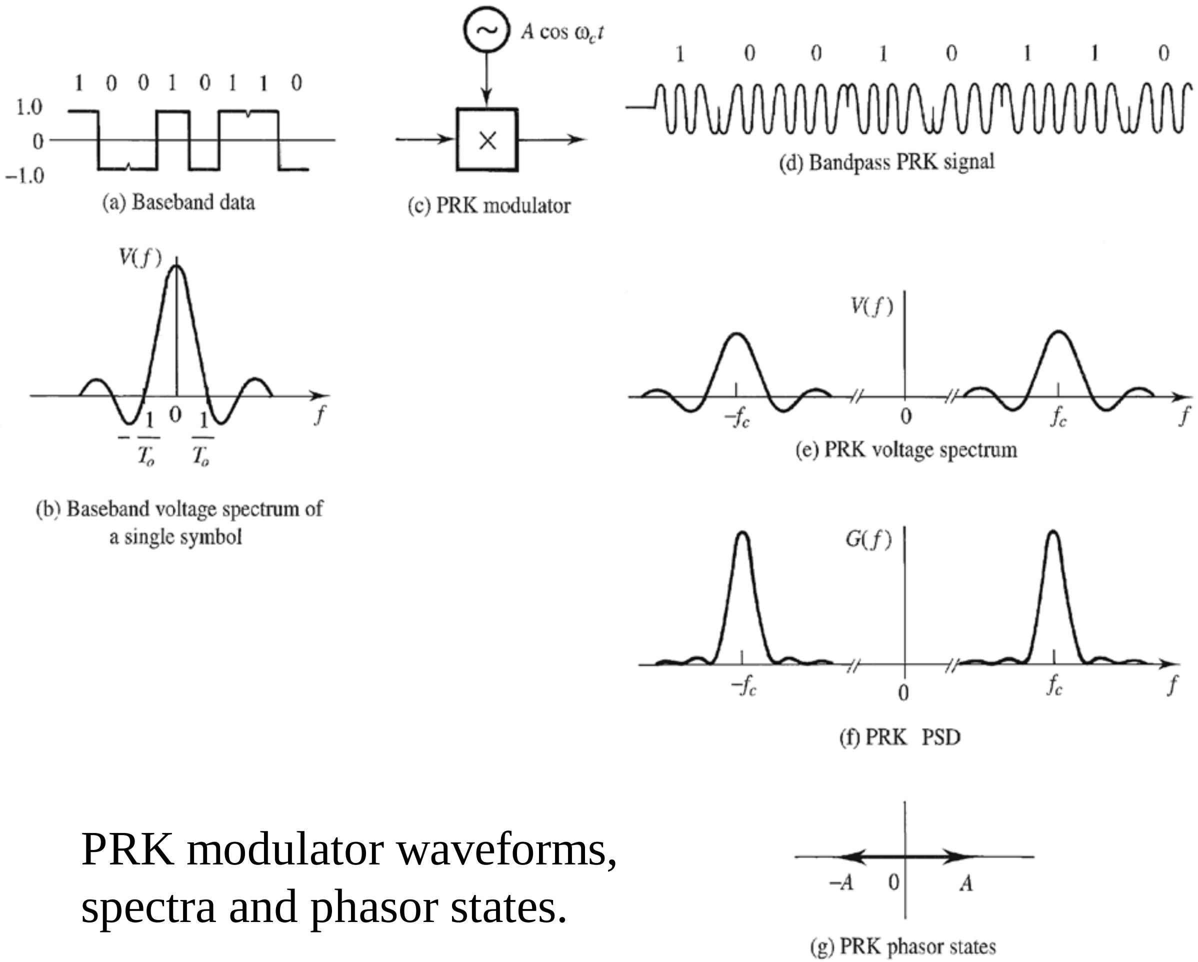
- Binary phase shift keying (BPSK) impresses baseband information onto a carrier, by changing the carrier's phase in sympathy with the baseband digital data, i.e.:

$$f(t) = \begin{cases} A\Pi(t/T_0)\cos 2\pi f_c t & \text{for a digital 1} \\ A\Pi(t/T_0)\cos(2\pi f_c t + \phi) & \text{for a digital 0} \end{cases}$$

- In principle, any two phasor states can be used to represent the binary symbols but usually antipodal states are chosen (i.e. states separated by $\phi = 180^{\circ}$). This type of modulation is called phase reversal keying (PRK).

Binary IF Modulation

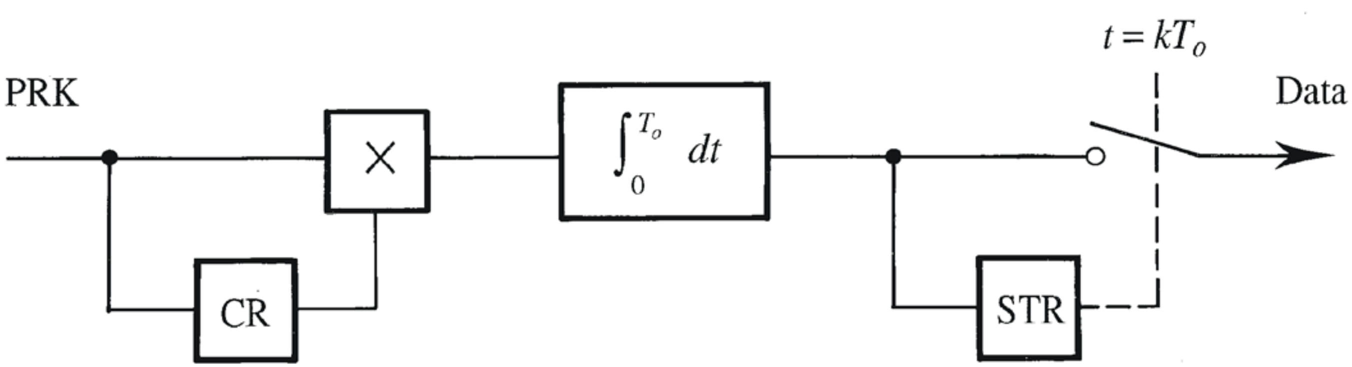
Binary Phase Shift Keying



PRK modulator waveforms, spectra and phasor states.

Binary IF Modulation
Binary Phase Shift Keying

- PRK systems must employ coherent detectors which can be implemented as either matched filters or correlators.
- Since the zero and one symbols are antipodal, only one receiver channel is needed.



Binary IF Modulation
Binary Phase Shift Keying

- The post-filtered decision instant voltages are $\pm kE$ (V) where E (V^2s) is the normalized energy residing in either symbol.
- The normalized noise power, σ^2 (V^2), at the filter output is the same as the BASK case.
- $\Delta V = 2kE$ and $\sigma^2 = k^2 E N_0/2$ lead to

$$P_e = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{E}{N_0}\right)^{\frac{1}{2}}\right]$$

- Since in this case $E = \langle E \rangle$, the PRK probability of symbol error can be expressed as

$$P_e = \frac{1}{2}\left[1 - \mathrm{erf}(T_0 B)^{\frac{1}{2}}\left(\frac{C}{N}\right)^{\frac{1}{2}}\right]$$

Binary IF Modulation
Binary Phase Shift Keying

- **[Example 9.2]**
- A 140 Mbit/s ISI-free PRK signaling system uses pulse shaping to constrain its transmission to the double sideband Nyquist bandwidth. The received signal power is 10 mW and the one-sided noise power spectral density is 6.0 pW/Hz. Find the BER expected at the output of an ideal matched filter receiver.

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Binary IF Modulation
Binary Frequency Shift Keying

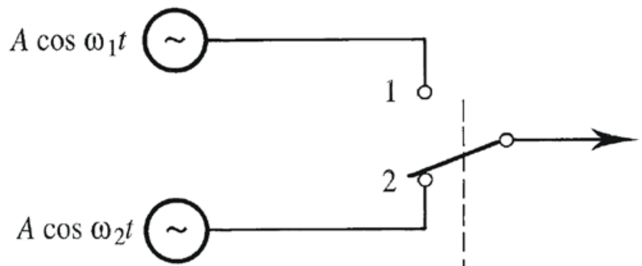
- Binary frequency shift keying (BFSK) represents digital ones and zeros by carrier pulses with two distinct frequencies, f_1 and f_2, i.e.:

$$f(t) = \begin{cases} A\Pi(t/T_0)\cos 2\pi f_1 t & \text{for a digital 1} \\ A\Pi(t/T_0)\cos 2\pi f_2 t & \text{for a digital 0} \end{cases}$$

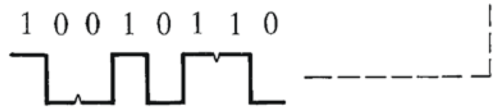
- In practice, the BFSK modulator would normally be implemented as a numerically controlled oscillator.
- The voltage spectrum of the BFSK signal is the superposition of two OOK spectrum, one representing the baseband data stream modulated onto a carrier with frequency f_1 and one representing the inverse data stream modulated onto a carrier with frequency f_2.

Binary IF Modulation

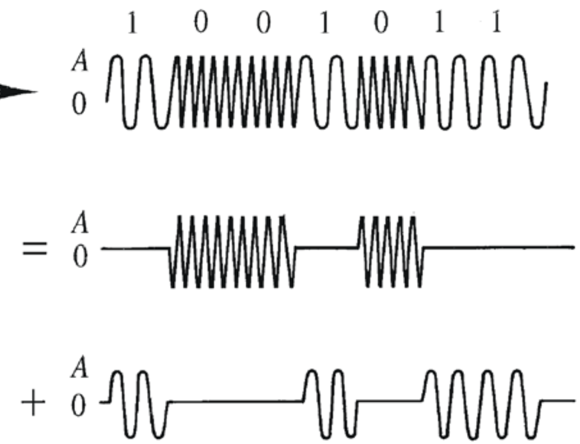
Binary Frequency Shift Keying



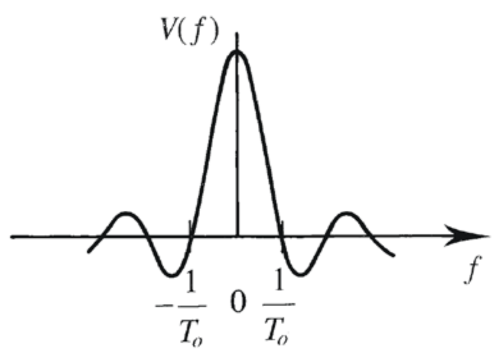
(c) BFSK modulator



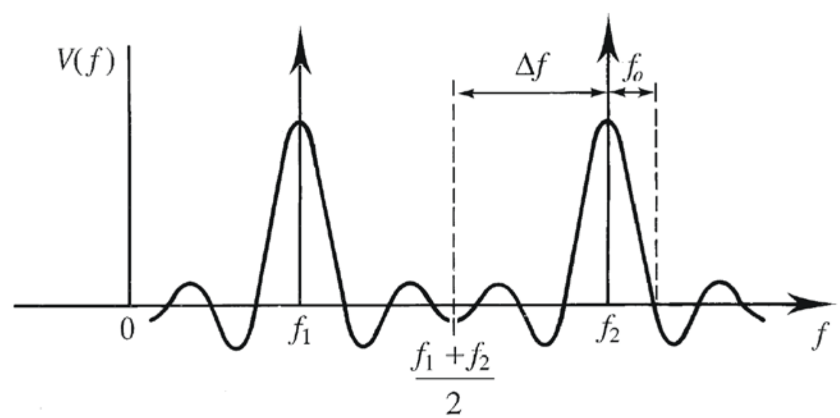
(a) Baseband data



(d) BSFK signal and two, component, bandpass OOK signals

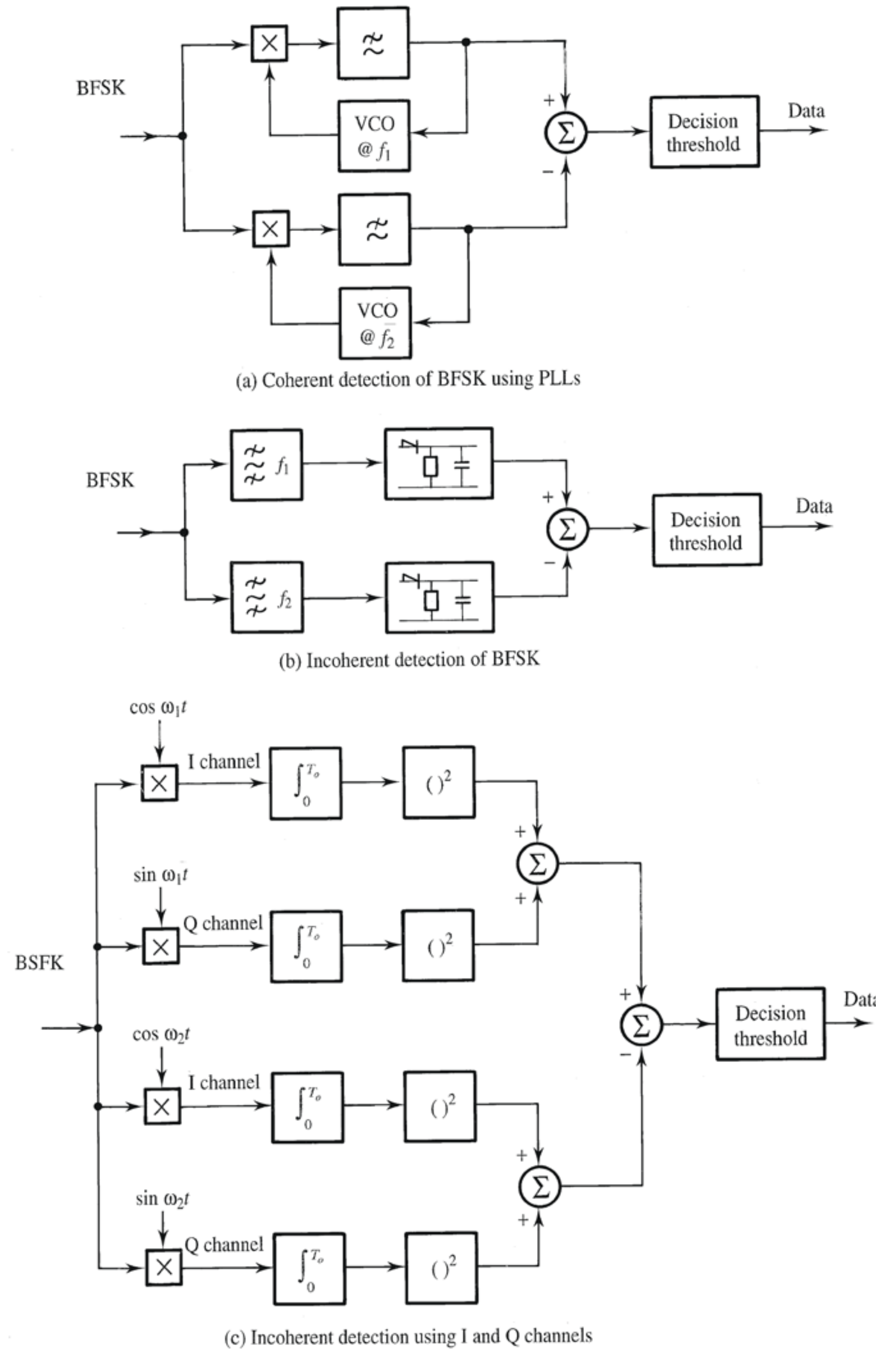


(b) Baseband voltage spectrum of a single symbol



(e) BFSK voltage spectrum (of two symbols, 0 and 1). Note overlapping OOK spectra

- Detection of BFSK can be coherent or incoherent.



Binary IF Modulation
Binary Frequency Shift Keying

- Define BFSK carrier frequency, f_c, as
$$f_c = \frac{f_1 + f_2}{2} \quad \text{(Hz)}$$
and BFSK frequency deviation, Δf, as
$$\Delta f = \frac{f_1 - f_2}{2} \quad \text{(Hz)}$$
- Use the first zero crossing points in the BFSK voltage spectrum to define its bandwidth B:
$$B = 2\Delta f + 2 f_0$$
- Here $f_0 = 1/T_0$ is both the nominal bandwidth and the baud rate of the baseband data stream

Binary IF Modulation
Binary Frequency Shift Keying

- If the binary symbols of a BFSK system are orthogonal:

$$\int_{0}^{T_0} \cos(2\pi f_1 t)\cos(2\pi f_2 t)\, dt = 0$$

then, when the output of one channel of a coherent BFSK receiver is a maximum, the output of the other channel will be zero.

- After subtracting the post-filtered signals arising from each receiver channel the orthogonal BFSK decision instant voltage is

$$f(nT_0) = \begin{cases} kE, & \text{for a digital } 1 \\ -kE, & \text{for a digital } 0 \end{cases}$$

Binary IF Modulation
Binary Frequency Shift Keying

- If the one-sided NPSD at the BFSK receiver input is N_0 (V^2/Hz), then the noise power, $\sigma_1^2 = k^2 E N_0 / 2$, received via channel 1 and the noise power, $\sigma_2^2 = k^2 E N_0 / 2$, received via channel 2 will add power-wise.
- The total noise power at the receiver output will be
$$\sigma^2 = \sigma_1^2 + \sigma_2^2 = k^2 E N_0 \quad (\mathrm{V}^2)$$
- $\Delta V = 2kE$ and $\sigma = k\sqrt{EN_0}$ give the probability of error for coherently detected orthogonal BFSK:
$$P_e = \frac{1}{2}\left[1 - \mathrm{erf}\frac{1}{\sqrt{2}}\left(\frac{E}{N_0}\right)^{\frac{1}{2}}\right]$$
- Since $E = \langle E \rangle$, the BFSK expression for P_e in terms of CNR is identical to that with OOK.

Binary IF Modulation
Comparison of Techniques

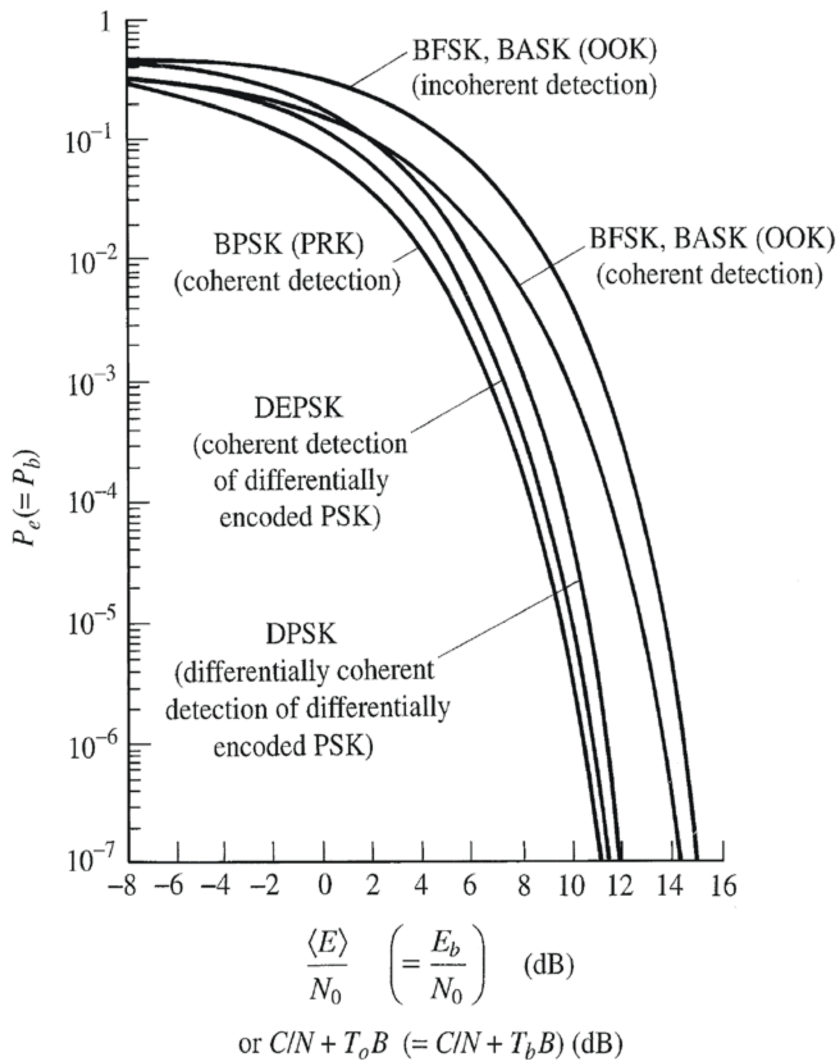
		P_e	
Baseband signaling	Unipolar (OOK)	$\frac{1}{2}\operatorname{erfc}\sqrt{\frac{1}{2}\frac{E}{N_0}}$	$\frac{1}{2}\operatorname{erfc}\sqrt{\frac{1}{4}\frac{S}{N}}$
	Polar	$\frac{1}{2}\operatorname{erfc}\sqrt{\frac{E}{N_0}}$	$\frac{1}{2}\operatorname{erfc}\sqrt{\frac{1}{2}\frac{S}{N}}$
IF/RF signaling	OOK	$\frac{1}{2}\operatorname{erfc}\sqrt{\frac{1}{2}\frac{E}{N_0}}$	$\frac{1}{2}\operatorname{erfc}\sqrt{\frac{T_0 B}{2}\frac{C}{N}}$
	BFSK (orthogonal)	$\frac{1}{2}\operatorname{erfc}\sqrt{\frac{1}{2}\frac{E}{N_0}}$	$\frac{1}{2}\operatorname{erfc}\sqrt{\frac{T_0 B}{2}\frac{C}{N}}$
	PRK	$\frac{1}{2}\operatorname{erfc}\sqrt{\frac{E}{N_0}}$	$\frac{1}{2}\operatorname{erfc}\sqrt{T_0 B\frac{C}{N}}$

Binary IF Modulation
Comparison of Techniques

- OOK and orthogonal FSK systems are equally power efficient, i.e. have the same probability of symbol error for the same $\langle E \rangle / N_0$, since they are both orthogonal signalling schemes.
- PRK signalling is antipodal and more power efficient, i.e. it has a better P_e performance or, alternatively, a power saving of 3 dB.
- If comparison are made on a peak power basis then orthogonal FSK requires 3 dB more power than PRK but also requires 3 dB less power than OOK since all the energy of the OOK transmission is squeezed into only one type of symbol.

	Bandpass OOK	Orthogonal BFSK	PRK	
$\frac{E_1}{N_0}$	4	2	1	
$\frac{\langle E \rangle}{N_0}$	2	2	1	Relative power efficiencies

Binary IF Modulation
Comparison of Techniques



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Spectrally-Efficient Modulation

Basics

- Spectral efficiency, η_s, depends on symbol (or baud) rate, R_s, signal bandwidth, B, and entropy, H, i.e.:

$$\eta_s = \frac{R_s H}{B} \quad \text{(bit/s/Hz)}$$

- Since $R_s = 1/T_0$ and $H = \log_2 M$, for statistically independent, equiprobable symbols, then η_s is

$$\eta_s = \frac{\log_2 M}{T_0 B} \quad \text{(bit/s/Hz)}$$

- Clearly, spectral efficiency is maximized by making the symbol alphabet size, M, large and the $T_0 B$ product small.
- This is the strategy employed by spectrally efficient modulation techniques.

Spectrally-Efficient Modulation
Basics

- For baseband signals, the minimum $T_0 B$ product (avoiding ISI) is limited by

$$T_0 B \geq 0.5$$

and the minimum bandwidth $B = 1/(2T_0)$ is called the single sided Nyquist bandwidth.

- In the context of IF modulation (i.e. bandpass signals) the modulation process results in a double sideband (DSB) signal.
- The minimum ISI-free $T_0 B$ product is then given by

$$T_0 B \geq 1$$

and the minimum bandwidth is now $B = 1/T_0$, sometimes called the double sided Nyquist bandwidth.

Spectrally-Efficient Modulation
Basics

- Dramatic increases in η_s come from increased alphabet size.
- Operational systems currently exist with $M = 64, 128, 256$ and 1024.
- Although in principle such multi-symbol signaling can lead to increased spectral efficiency of MASK, MPSK and MFSK systems, only MASK, MPSK and combinations of them are used in practice.
- This is because MFSK signals are normally designed to retain orthogonality between all symbol pairs. In this case increasing M results in an approximately proportional increase in B which actually results in a decrease in spectral efficiency.
- MASK and MPSK sacrifice orthogonality when $M > 4$.

Spectrally-Efficient Modulation
Channel Capacity

- The Shannon-Hartley channel capacity theorem states that the maximum rate of information transmission for a single transmit antenna and signal receive antenna link, R_{max}, over a channel with bandwidth B and signal-to-noise ratio S/N is given by:

$$R_{max} = B\log_2\left(1 + \frac{S}{N}\right) \quad \text{(bit/s)}$$

- Thus as B increases S/N can be decreased to compensate.
- In a 3.2 kHz wide audio channel with an SNR of 1000 (30 dB) the theoretical maximum bit rate, R_{max}, is slightly in excess of 30 kbit/s. The corresponding maximum spectral efficiency, R_{max}/B, for a channel with 30 dB SNR is 10 bit/s/Hz.

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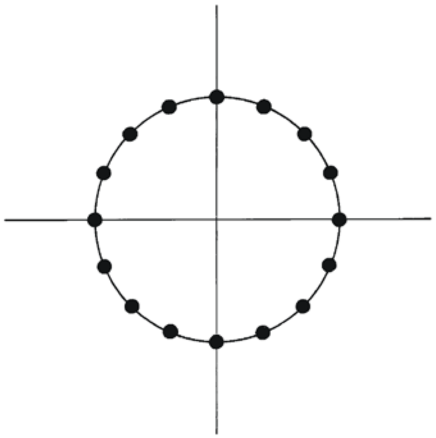
Spectrally-Efficient Modulation
M-symbol Phase Shift Keying

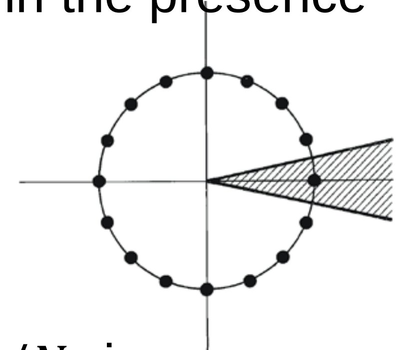
- In the context of PSK signaling, M-symbol PSK (i.e. MPSK) implies the extension of the number of allowed phasor states from 2 to 4, 8, 16, … (i.e. 2^n).
- The phasor diagram (also called the constellation diagram) for 16-PSK is shown below, where there are now 16 distinct states.
- The constellation diagram plots the complex baseband representation of the MPSK modulated signal vector $f(t)$ in terms of its real and imaginary baseband components, as an extension of the BPSK representation.
- As the signal amplitude is constant these states all lie on a circle in the complex plane.



Spectrally-Efficient Modulation
M-symbol Phase Shift Keying

- The probability of symbol error for MPSK system is found by integrating the 2-dimensional pdf of the noise centered on the tip of each signal phasor in turn over the corresponding error region and averaging the results.
- The error region for phasor state 0 is shown below.
- The exact expression of the symbol error probability in the presence of Gaussian noise is complicated.
- A simple approximation for $M \geq 4$ is

$$P_e \approx 1 - \operatorname{erf}\left[\sin\left(\frac{\pi}{M}\right)\left(\frac{E}{N_0}\right)^{1/2}\right]$$



- This approximation becomes better as both M and E/N_0 increase.
- Using $C = E/T_0$ and $N = N_0 B$ leads to:

$$P_e \approx 1 - \operatorname{erf}\left[(T_0 B)^{1/2}\sin\left(\frac{\pi}{M}\right)\left(\frac{C}{N}\right)^{1/2}\right]$$

Spectrally-Efficient Modulation

M-symbol Phase Shift Keying

- Multi-symbol signaling can be thought of as a coding or bit mapping process in which n binary symbols (bits) are mapped into a single M-ary symbol (here each symbol is an IF pulse).
- A detection error in a single symbol can therefore translate into several errors in the corresponding decoded bit sequence.
- The probability of bit error, P_b, depends not only on the probability of symbol error, P_e, and the symbol entropy, $H = \log_2 M$, but also on the code or bit mapping used and types of error which occur.
- The most probable type of error involves a given phasor state being detected as an adjacent state.
- If a Gray code is used to map binary symbols to phasor states this type of error results in only a single decoded bit error.

Spectrally-Efficient Modulation
M-symbol Phase Shift Keying

- Providing that the probability of other errors is negligible, then

$$P_b = \frac{P_e}{\log_2 M}$$

- In order to compare the performance of different modulation schemes on an equitable basis, P_b is expressed as a function of average energy per bit, E_b.
- Since $E_b = \frac{E}{\log_2 M}$, we have

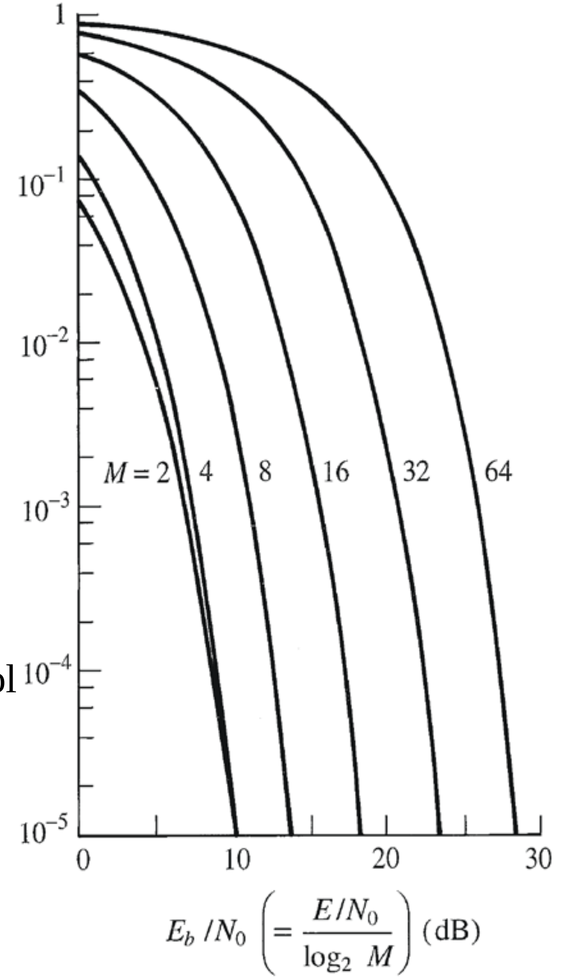
$$P_b = \frac{1}{\log_2 M}\left\{1 - \operatorname{erf}\left[\sin\left(\frac{\pi}{M}\right)\sqrt{\log_2 M}\left(\frac{E_b}{N_0}\right)^{\frac{1}{2}}\right]\right\}$$

- In terms of CNR this becomes

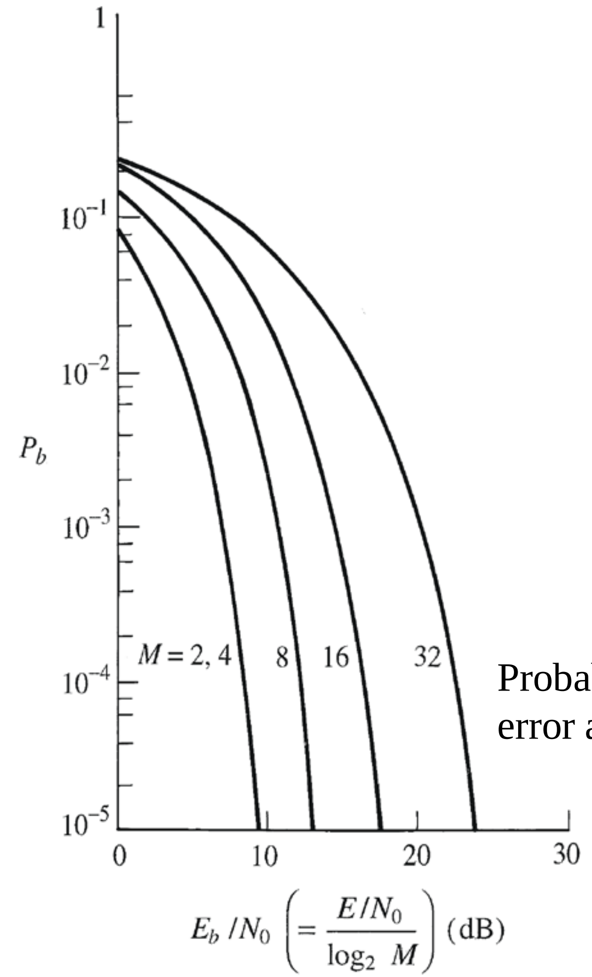
$$P_b = \frac{1}{\log_2 M}\left\{1 - \operatorname{erf}\left[(T_0 B)^{\frac{1}{2}}\sin\left(\frac{\pi}{M}\right)\left(\frac{C}{N}\right)^{\frac{1}{2}}\right]\right\}$$

Spectrally-Efficient Modulation

M-symbol Phase Shift Keying



(a)



(b)

Probability of symbol error against E_b/N_0

Probability of bit error against E_b/N_0

Spectrally-Efficient Modulation
M-symbol Phase Shift Keying

- Since each symbol of an MPSK signal has an identical amplitude spectrum to all the other symbols the spectral occupancy of MPSK depends only on baud rate and pulse shaping and is independent of M.
- For unfiltered MPSK (i.e. MPSK with rectangular pulses) the nominal (i.e. main lobe null-to-null) bandwidth is $2/T_0$ Hz. In this case, the spectral efficiency is
$$\eta_s = 0.5 \log_2 M \quad \text{(bit/s/Hz)}$$
- The maximum possible, ISI-free, spectral efficiency occurs when pulse shaping is such that signaling takes place in the double sided Nyquist bandwidth $B = 1/T_0$ Hz, i.e.
$$\eta_s = \log_2 M \quad \text{(bit/s/Hz)}$$
- Thus we usually say BPSK has an efficiency of 1 bit/s/Hz and 16 PSK 4 bit/s/Hz.

Spectrally-Efficient Modulation

M-symbol Phase Shift Keying

- **[Example 9.3]**
- An MPSK, ISI-free, system is to operate with 2^N PSK symbols over a 120 kHz channel. The minimum required bit rate is 900 kbit/s. What minimum CNR is required to maintain reception with a P_b no worse than 10^{-6}?

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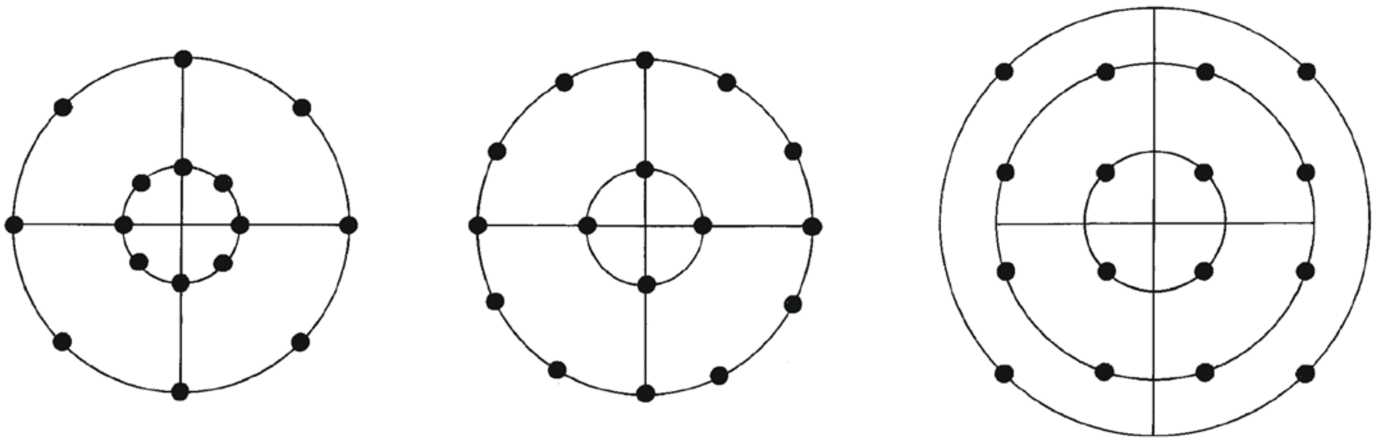
Spectrally-Efficient Modulation

Amplitude/Phase Keying and QAM

- It is possible to introduce amplitude as well as phase modulation to give an improved distribution of signal states in the signal constellation.
- These modulation schemes are called amplitude/phase keying (APK).
- Since a square constellation APK signal can be interpreted as a pair of multilevel ASK (ASK) signals modulated onto quadrature carriers it is normally called quadrature amplitude modulation (QAM).



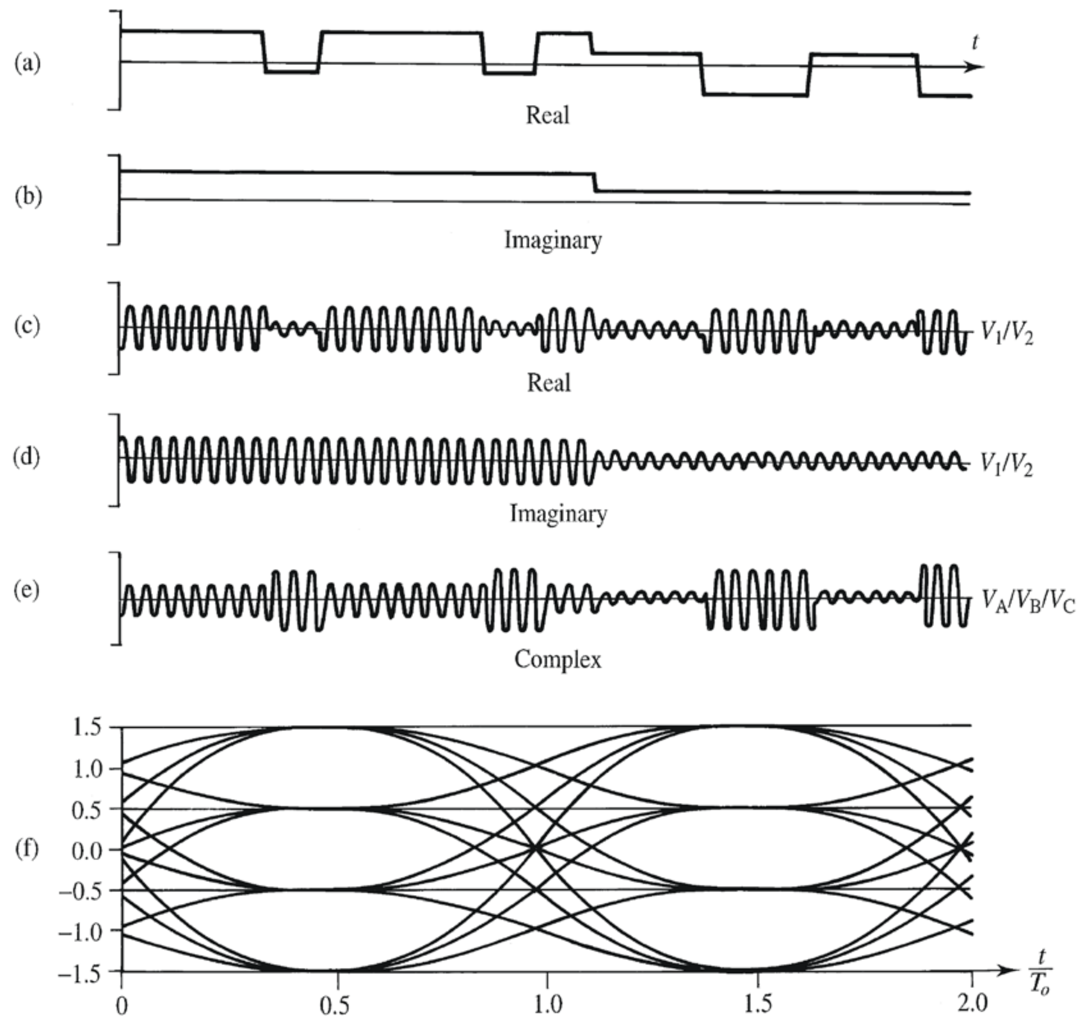
(a) 16 APK (8,8) constellation

(b) 16 APK (4,12) constellation

(c) 16 APK (4,8,4) constellation (or 16 QAM)

Spectrally-Efficient Modulation
Amplitude/Phase Keying and QAM

- The 16-state QAM signal: (a) four-level baseband signals in the inphase and (b) quadrature branches; (c), (d) corresponding four-level modulated complex signals; (e) resulting combined complex (three-level) QAM signal; (f) demodulated signal eye diagram over two symbols (in the I or Q channels)



Spectrally-Efficient Modulation
Amplitude/Phase Keying and QAM

- A 16-QAM signal has two amplitude levels and two distinct phase states in both the real and imaginary channels.
- Two bits at a time are taken by each of the I and Q channels which use low resolution (2 bit) digital to analogue converters to generate the appropriate bipolar drive signals, for the four-quadrant multipliers.
- The final summation results in the three-level, 12-phase signal, i.e. the constellation.
- As there are separate I and Q channels, and 2 bit converters in each of these, then 4 input bits define each of the 16 symbols giving a spectral efficiency of 4 bit/s/Hz if ideal filtering (or pulse shaping) is employed prior to the I and Q channel multipliers.

Spectrally-Efficient Modulation
Amplitude/Phase Keying and QAM

- A simple approximation for the probability of symbol error for MQAM (M even) signaling in Gaussian noise is

$$P_e = 2\left\{\frac{M^{1/2}-1}{M^{1/2}}\right\}\left[1-\mathrm{erf}\sqrt{\frac{3}{2(M-1)}}\left(\frac{\langle E\rangle}{N_0}\right)^{1/2}\right]$$

  where $\langle E\rangle$ is the average energy per QAM symbol.

- For equiprobable rectangular pulse symbols $\langle E\rangle$ is given by

$$\langle E\rangle = \frac{1}{3}\left(\frac{\Delta V}{2}\right)^2 (M-1)T_0$$

  where ΔV is the voltage separation between adjacent inphase or quadrature MASK levels and T_0 is the symbol duration.

Spectrally-Efficient Modulation
Amplitude/Phase Keying and QAM

- Using $C = \langle E \rangle / T_0$ and $N = N_0 B$, we have

$$P_e = 2\left\{\frac{M^{1/2}-1}{M^{1/2}}\right\}\left[1 - \operatorname{erf}\sqrt{\frac{3T_0 B}{2(M-1)}\left(\frac{C}{N}\right)}^{1/2}\right]$$

- For Gray code mapping of bits along the inphase and quadrature axes of the QAM constellation the probability of bit error is approximately $P_b = \frac{P_e}{\log_2 M}$, i.e.

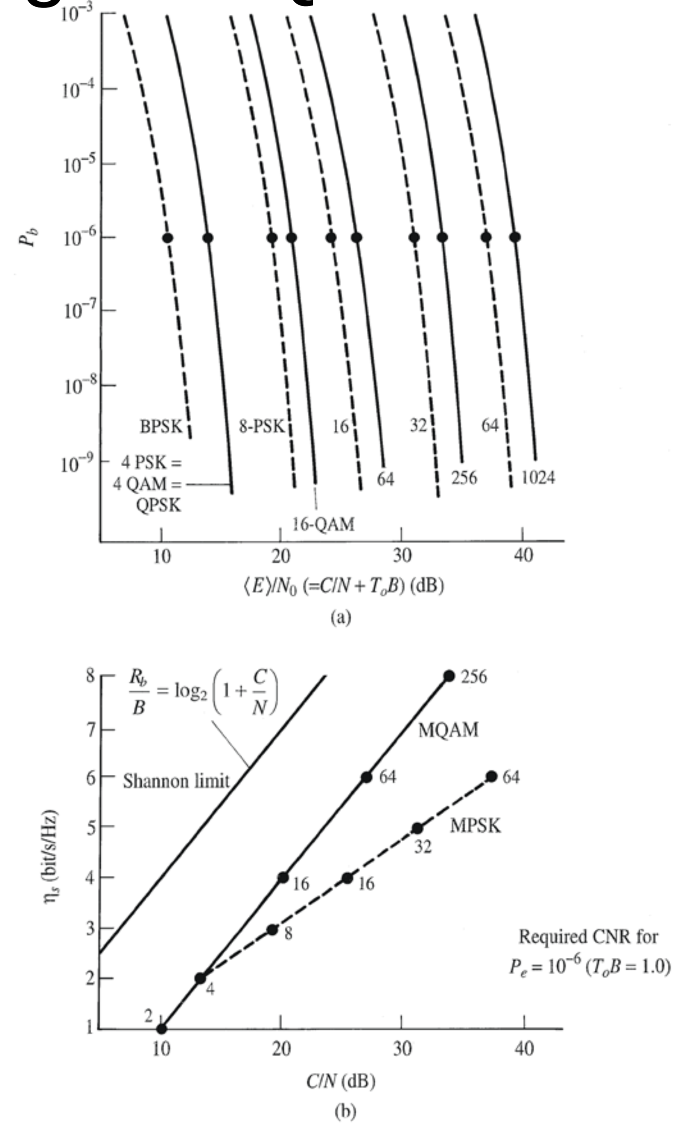
$$P_b = \frac{2}{\log_2 M}\left\{\frac{M^{1/2}-1}{M^{1/2}}\right\}\left[1 - \operatorname{erf}\sqrt{\frac{3\log_2 M}{2(M-1)}\left(\frac{E_b}{N_0}\right)}^{1/2}\right]$$

since the average energy per bit $E_b = \langle E \rangle / \log_2 M$, and

$$P_b = \frac{2}{\log_2 M}\left\{\frac{M^{1/2}-1}{M^{1/2}}\right\}\left[1 - \operatorname{erf}\sqrt{\frac{3T_0 B}{2(M-1)}\left(\frac{C}{N}\right)}^{1/2}\right]$$

Spectrally-Efficient Modulation
Amplitude/Phase Keying and QAM

- Like MPSK all the symbols in a QAM (or APK) signal occupy the same spectral space.
- The spectral efficiency is therefore identical to MPSK and is given (for statistically independent equiprobables symbols) by $\eta_s = \frac{\log_2 M}{T_0 B}$ bit/s/Hz.
- Unfiltered and Nyquist filtered APK signals therefore have (nominal and maximum) spectral efficiencies given by $\eta_s = 0.5 \log_2 M$ bit/s/Hz and $\eta_s = \log_2 M$ bit/s/Hz, respectively.
- The superior constellation packing in QAM over MPSK gives a lowered required E_b/N_0 for the same P_e value.



Spectrally-Efficient Modulation
Amplitude/Phase Keying and QAM

Modulation	C/N ratio (dB)	E_b/N_0 (dB)
PRK	10.6	10.6
QPSK	13.6	10.6
4-QAM	13.6	10.6
8-PSK	18.8	14.0
16-PSk	24.3	18.3
16-QAM	20.5	14.5
32-QAM	24.4	17.4
64-QAM	26.6	18.8

Comparison of various digital modulation schemes ($P_b = 10^{-6}, T_0B = 1.0$)

Spectrally-Efficient Modulation
Amplitude/Phase Keying and QAM

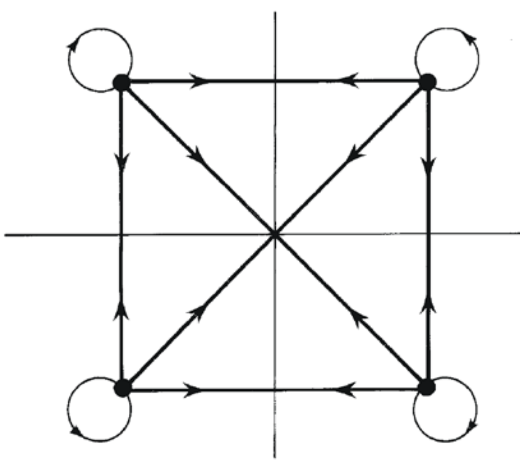
- **[Example 9.4]**
- Find the maximum spectral efficiency of ISI-free 16 QAM. What is the noise induced probability of symbol error in this scheme for a received CNR of 24.0 dB if the maximum spectral efficiency requirement is retained? What is the Gray coded probability of bit error?

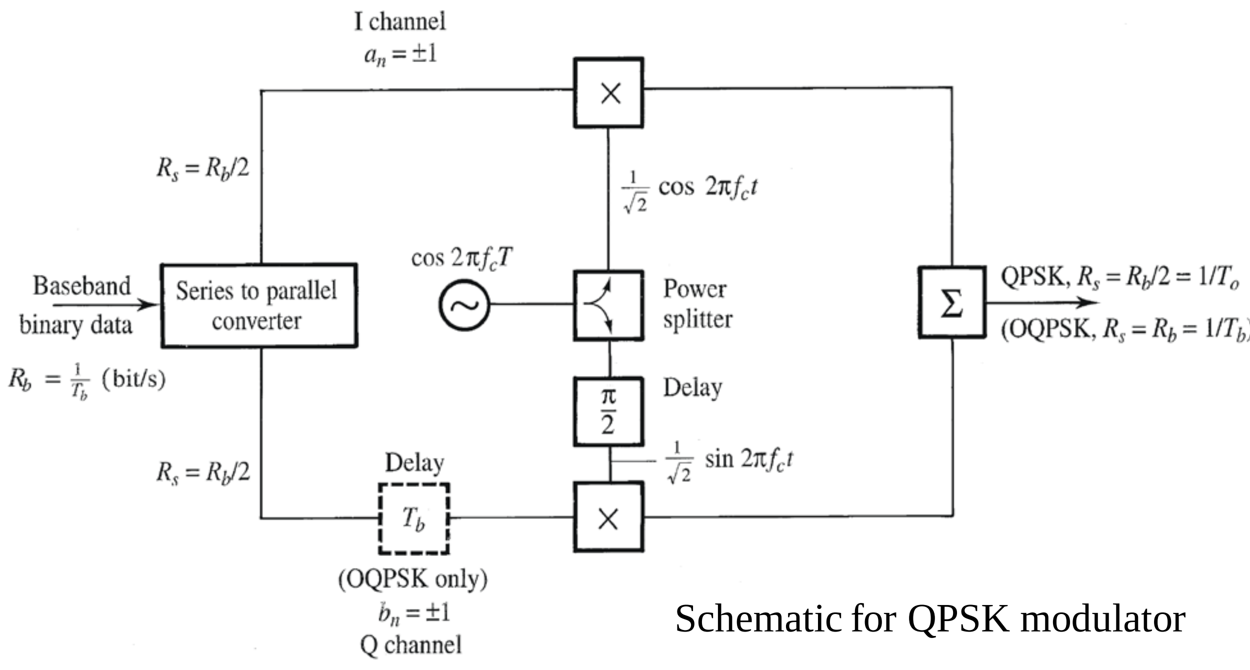
- **[Example 9.5]**
- Repeat Example 9.3 as an MQAM system, recalculating the new value for the minimum CNR required to maintain reception with a P_b no worse than 10^{-6} and compare this new result with that attained in Example 9.3.

Spectrally-Efficient Modulation

Quadrature Phase Shift Keying (QPSK)

- Quadrature phase shift keying (QPSK) can be interpreted either as 4-PSK with carrier amplitude A (i.e. quaternary PSK) or as a superposition of two (polar) BASK signals with identical 'amplitudes' $\pm A/\sqrt{2}$ and quadrature carriers $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$, i.e. 4-QAM.

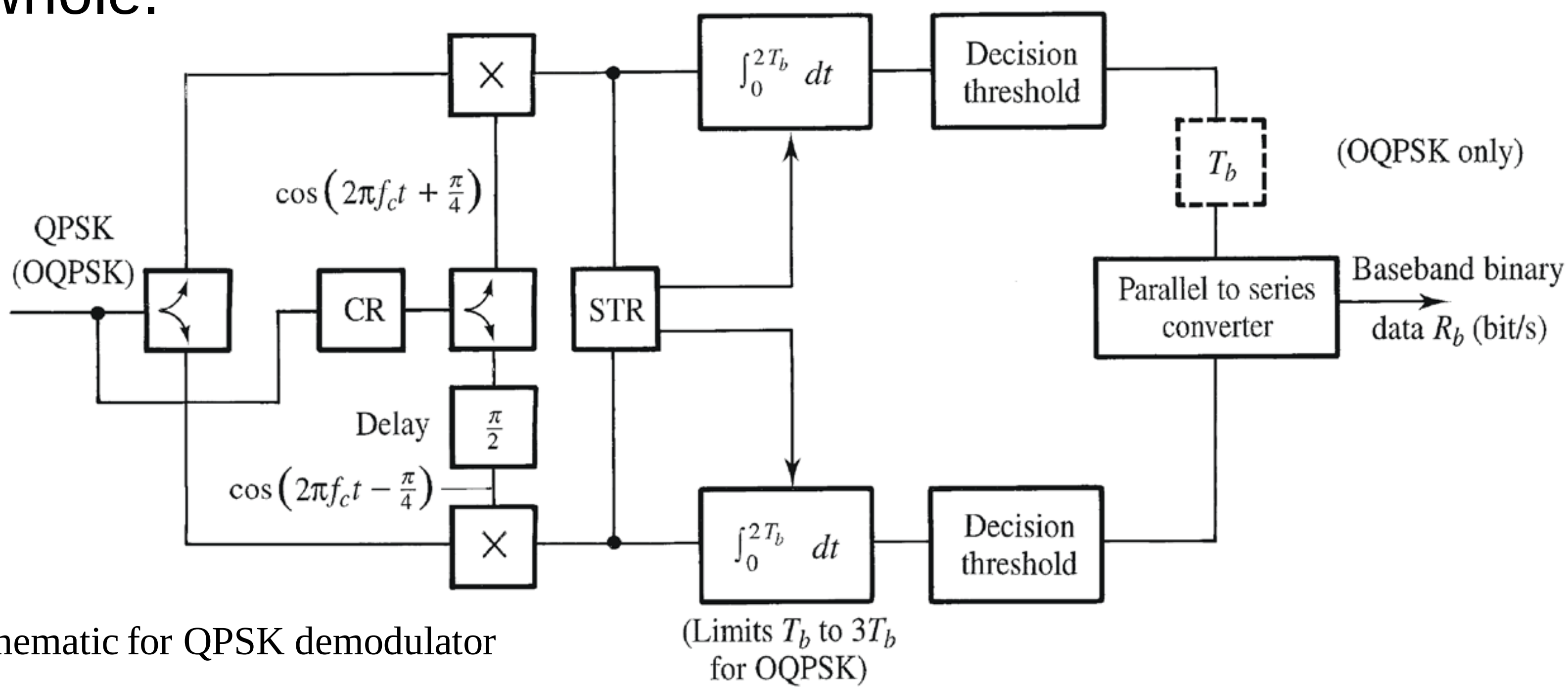




Schematic for QPSK modulator

Spectrally-Efficient Modulation
Quadrature Phase Shift Keying (QPSK)

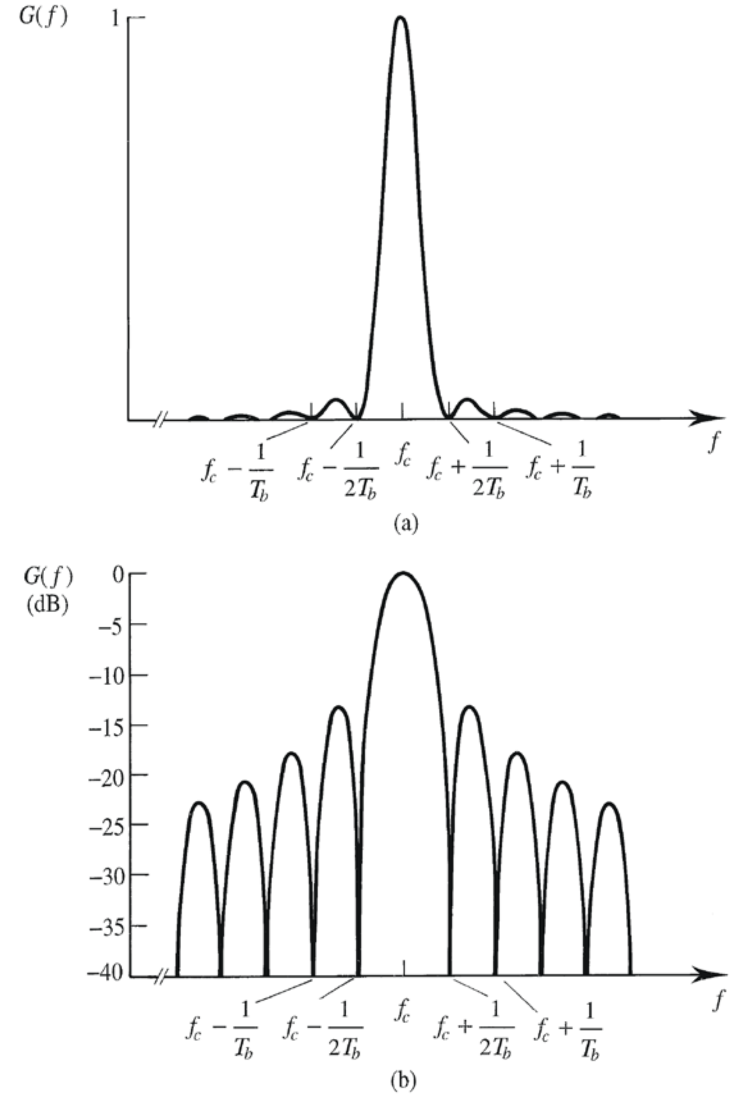
- The transmitter and receiver are effectively two PRK transmitters and receivers arranged in phase quadrature, the inphase (I) and quadrature (Q) channels each operating at half the bit rate of the QPSK system as a whole.

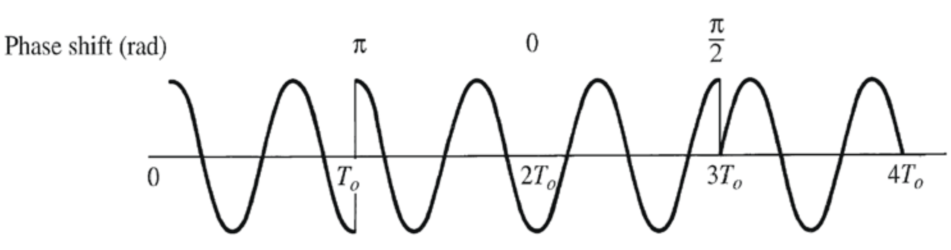


Schematic for QPSK demodulator

Spectrally-Efficient Modulation
Quadrature Phase Shift Keying (QPSK)

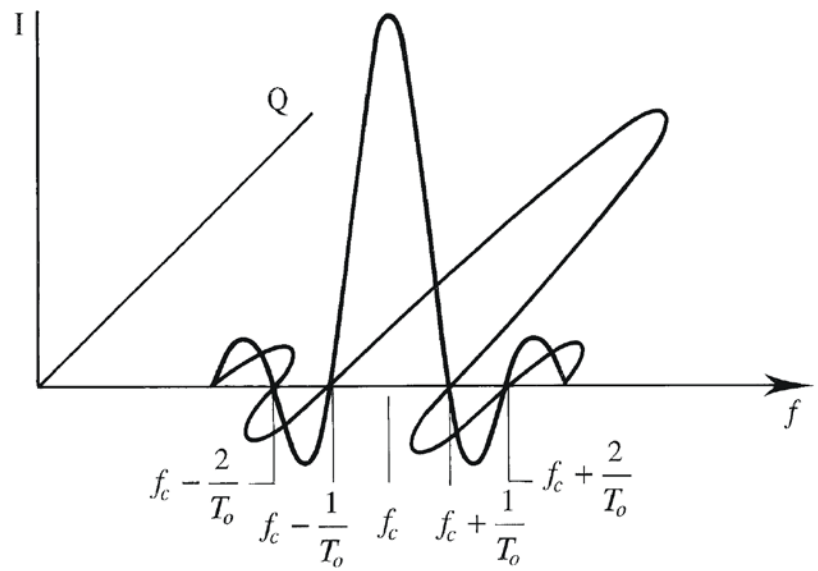
- If pulse shaping and filtering are absent the signal is said to be unfiltered, or rectangular pulse, QPSK.
- The bottom figure shows an example sequence of unfiltered QPSK data and the right figure show the corresponding PSD with $T_0 = 2T_b$





Spectrally-Efficient Modulation
Quadrature Phase Shift Keying (QPSK)

- The spectral efficiency of QPSK is twice that for BPSK, because the symbols in each quadrature channel occupy the same spectral space and have half the spectral width of a BPSK signal with the same data rate as the QPSK signal.



QPSK orthogonal I and Q voltage spectra

Spectrally-Efficient Modulation
Quadrature Phase Shift Keying (QPSK)

- The P_e performance of QPSK systems is worse than that of PRK systems, since the decision regions on the constellation diagram are reduced from half spaces to quadrants.
- The P_b performances of QPSK and PRK systems are identical, since the I and Q channels of the QPSK system are independent of (orthogonal to) each other.
- Because each QPSK I or Q channel symbol is twice the duration and half the power of the equivalent PRK symbols, the total message energy (and therefore the energy per bit, E_b) is the same in both the QPSK and PRK cases.

Spectrally-Efficient Modulation
Quadrature Phase Shift Keying (QPSK)

- The P_b performance of ideal QPSK signalling is therefore given by

$$P_b = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{E_b}{N_0}\right)^{\frac{1}{2}}\right]$$

- In terms of CNR:

$$P_b = \frac{1}{2}\left[1 - \operatorname{erf}(T_b B)^{1/2}\left(\frac{C}{N}\right)^{\frac{1}{2}}\right]$$

where the bit period T_b is half the QPSK symbol period, T_0.

- The probability of symbol error, P_e, is given by

$$P_e = P_b(1 - P_b) + (1 - P_b)P_b + P_b P_b = 2P_b - P_b^2$$

Chapter 10: Bandpass Modulation
Contents

1. Introduction
2. Binary IF modulation
   - Binary amplitude shift keying (and on-off keying)
   - Binary phase shift keying (and phase reversal keying)
   - Binary frequency shift keying
3. Spectrally-efficient modulation techniques
   - M-symbol phase shift keying
   - Amplitude/phase keying and quadrature amplitude modulation
4. Power-efficient modulation techniques
   - Multi-dimensional signaling and MFSK
5. Summary

Power-efficient Modulation Techniques
Basics

- Some communication systems operate in environments where large bandwidths are available but signal power is limited, and thus rely on power efficient modulation schemes to achieve acceptable bit error and data rates.
- Dara rate can be improved by increasing the number of symbols (i.e. the alphabet size) at the transmitter.
- In order not to degrade P_e the enlarged alphabet of symbols must remain at least as widely spaced in the constellation as the original symbol set.
- This can be achieved without increasing transmitted power by adding orthogonal axes to the constellation space, a technique resulting in multi-dimensional signaling.
- Power can also be conserved by carefully optimizing the arrangement of points in the constellation space.

Chapter 10: Bandpass Modulation
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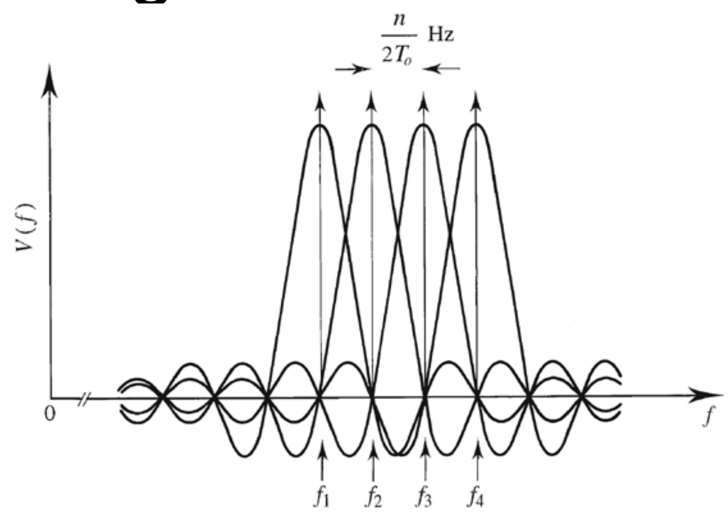
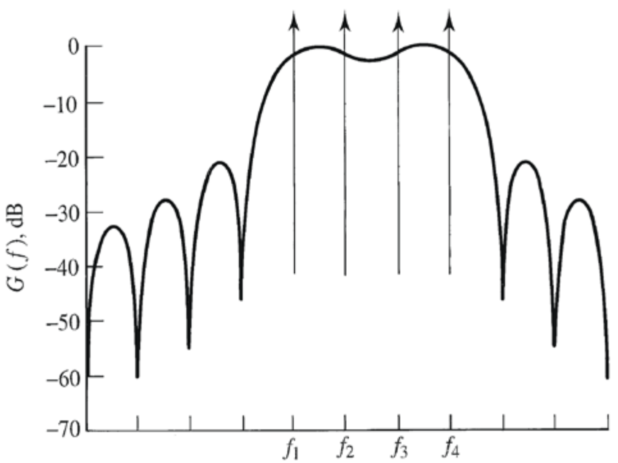
Power-efficient Modulation Techniques
Multi-dimensional Signaling and MFSK

- Multiple frequency shift keying (MFSK) is a good example of a power efficient, multi-dimensional, modulation scheme if its symbols are designed to be mutually orthogonal.
- The RHS figure (a) shows the voltage spectrum of an orthogonal MFSK signal ($M = 4$) as a superposition of OOK signals and (b) shows the power spectrum plotted in dB.



(a)



(b)

Power-efficient Modulation Techniques

Multi-dimensional Signaling and MFSK

- The increased data rate realized by MFSK signaling is achieved entirely at the expense of increased bandwidth.
- Since each symbol (for equiprobable, independent, symbol systems) conveys $H = \log_2 M$ bits of information then the nominal spectral efficiency of orthogonal MFSK is given by

$$\eta_s = \frac{\log_2 M}{(n/2)(M-1)+2} \quad \text{(bit/s/Hz)}$$

where n is the selected zero crossing point on the $\rho - T_0$ diagram and the (nominal) signal bandwidth is defined by the first spectral nulls above and below the highest and lowest frequency symbols respectively.

- For incoherent detection ($n \geq 2$) the maximum spectral efficiency ($n = 2$) is given by $\eta_s = \frac{\log_2 M}{M+1}$ (bit/s/Hz).

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Summary

- IF or RF modulation is used principally to shift the spectrum of a digital information signal into a convenient frequency band, in order to
  - match the spectral band occupied by a signal to the passband of a transmission line,
  - allow frequency division multiplexing of signals, or
  - enable signals to be radiated by antennas of practical size.
- Two performance measures: Spectral efficiency and power efficiency.
- Three generic IF modulation techniques for digital data: ASK, PSK, and FSK.
- MPSK, APK, and QAM are spectrally efficient modulation schemes.
- MFSK is usually operated as an orthogonal modulation scheme and is therefore power efficient.

Principles of Communications (通信系统原理)

Undergraduate Course

Chapter 10: Bandpass Modulation

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