Principles of Communications (通信系统原理)

Undergraduate Course

Chapter 5: Analog Modulation

José Rodríguez-Piñeiro (Xose)
j.rpineiro@tongji.edu.cn

Wireless Channel Research Laboratory
Department of Information and Communication Engineering

College of Electronics and Information Engineering
China-Deutsch Center for Intelligent Systems

Tongji University

Chapter 5: Analog Modulation

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   - Amplitude Modulation (AM)
   - Double-Sideband Modulation (DSB)
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   - Phase Modulation
   - Frequency Modulation
4. Summary

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Introduction

- Modulation refers to *the modification of one signal's characteristics in sympathy with another signal*.
- Analog modulation: Modulation of a carrier by utilizing a source baseband analog signal
- Carrier: A deterministic periodic waveform, e.g.:

$$c(t) = A\cos(\omega_0 t + \varphi_0)$$

where A is amplitude, ω_0 is angular frequency, and φ_0 is initial phase.

- Modulation enables a parameter of the carrier, $c(t)$, to vary with the source signal, $m(t)$, i.e. the value of $m(t)$ is represented by the value of a parameter of $c(t)$.

Introduction

- The source signal $m(t)$ is called modulating signal.
- The carrier after being modulated, $s(t)$, is called modulated signal.
- The device for modulation is called modulator.
- Purposes of modulation
  - To realize frequency spectrum movement: Accommodating the requirement of channel transmission or conducting multi-channel transmission
  - To improve the anti-jamming ability
- Classification
  - Linear modulation: The frequency spectrum structure of modulated signal remains the same as that of modulating signal.
  - Nonlinear modulation (also called angle modulation): The frequency spectrum structures of modulated and modulating signals are different (new frequency components are generated)

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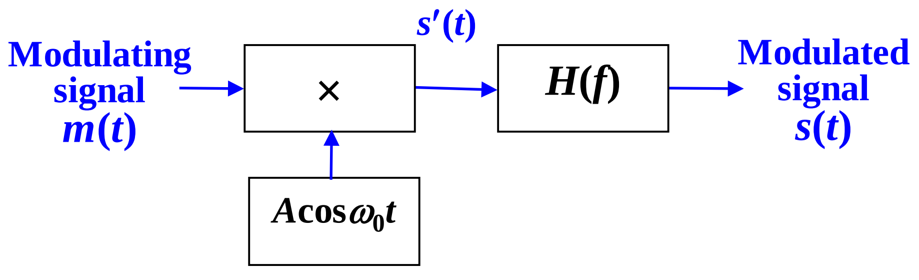
Linear Modulation
Definitions

- Let the carrier signal be $c(t) = A\cos\omega_0 t = A\cos 2\pi f_0 t$, where A is amplitude (V), f_0 is frequency (Hz), $\omega_0 = 2\pi f_0$ is angular frequency (rad/s).
- The principle model of a linear modulator is to pass the multiplication of $m(t)$ and $c(t)$, i.e.

$$s'(t) = m(t)A\cos\omega_0 t$$

through a bandpass filter with transfer function $H(f)$.

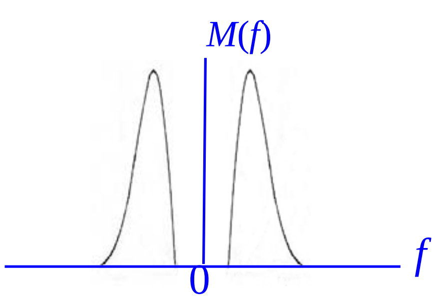


Linear Modulation
Definitions

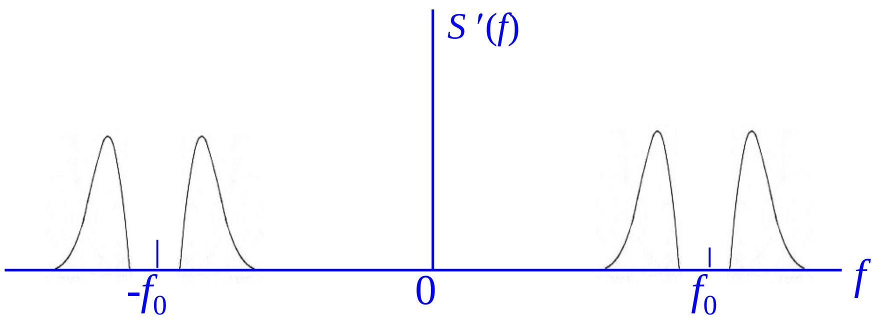
- Assume $m(t)$ to be an energy signal with frequency spectral density $M(f)$, i.e. $m(t) \overset{\text{FT}}{\Leftrightarrow} M(f)$. We have:

$$m(t)A\cos\omega_0 t \overset{\text{FT}}{\Leftrightarrow} S'(f) = \frac{A}{2}[M(f - f_0) + M(f + f_0)]$$

- The signal $s'(t)$ is cosinusoidal wave, whose amplitude is proportional to $m(t)$, i.e. the amplitude of the carrier waveform is modulated.
- $S'(f)$ is the displacement result of $M(f)$.



(a) Frequency spectral density of input signal



(b) Frequency spectral density of output signal

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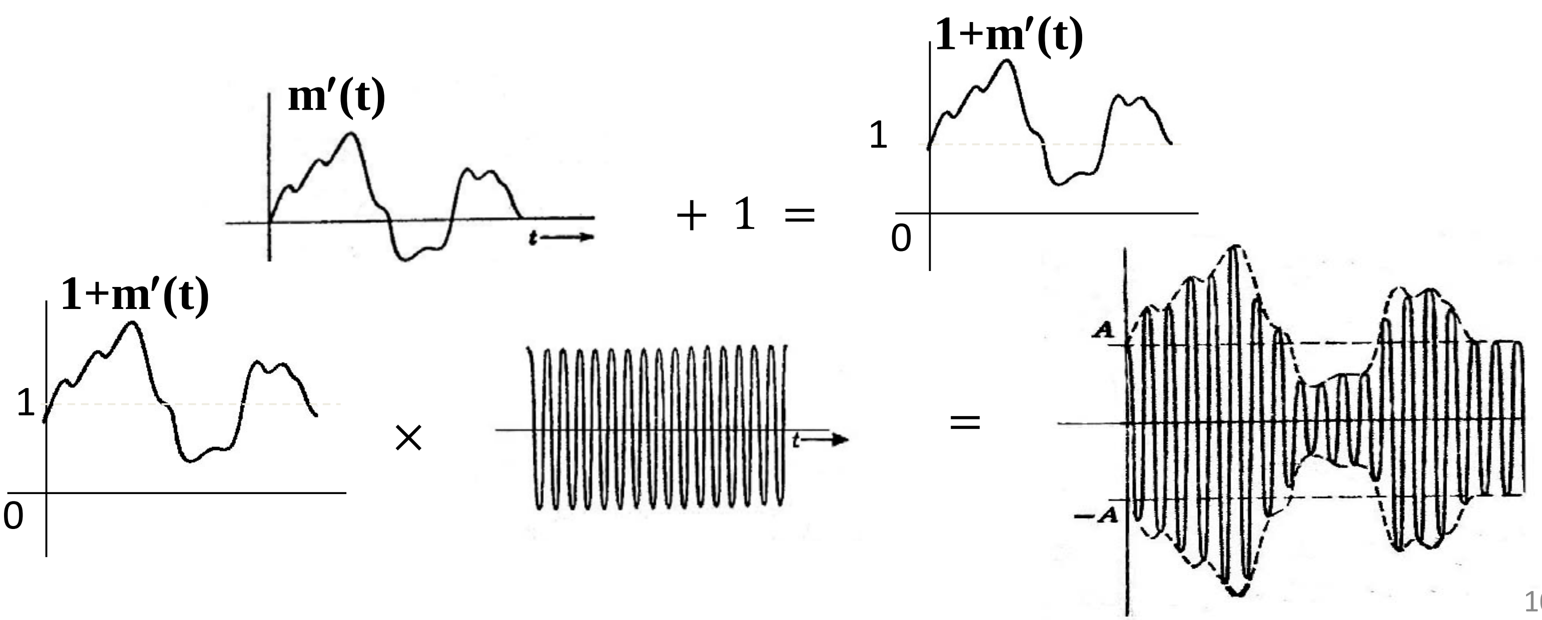
Linear Modulation
Amplitude Modulation (AM)

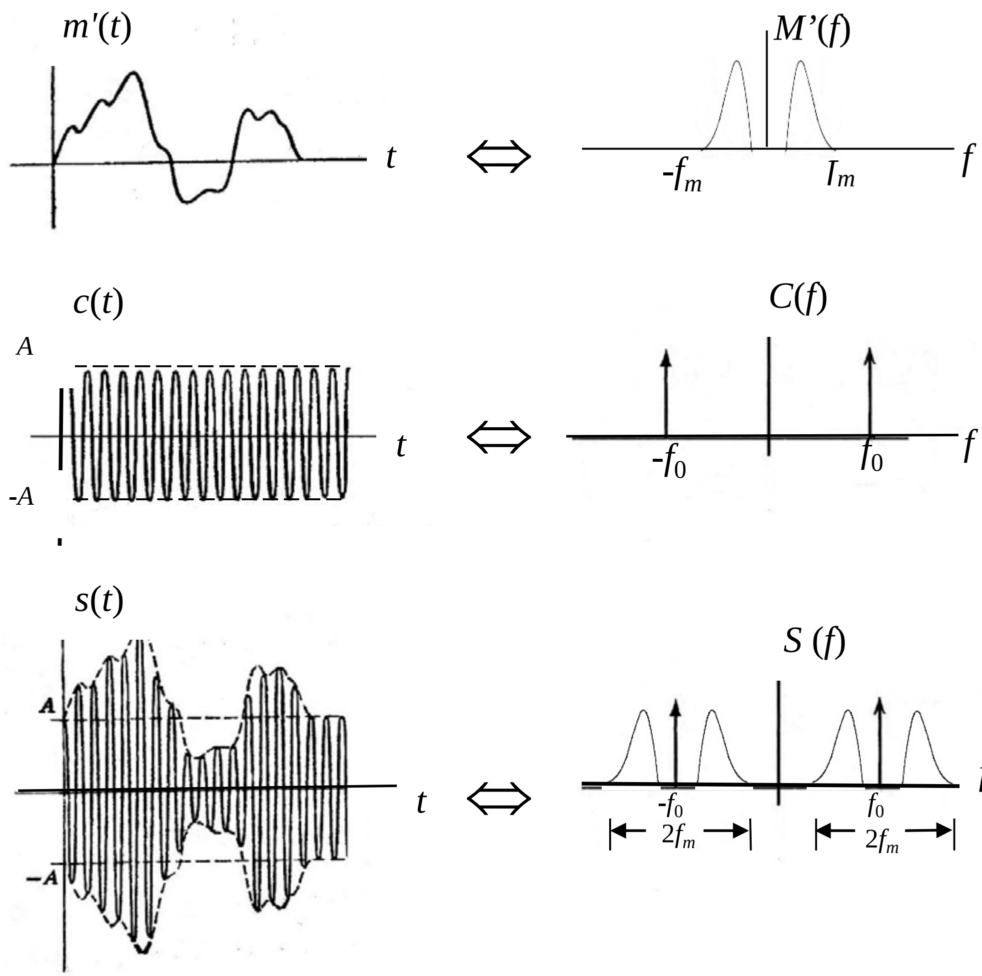
- Let $m(t) = 1 + m'(t)$, $|m'(t)| \le 1$
  - 1 can be seen as the DC component, $m'(t)$ is the AC component
  - The maximum of $|m'(t)|$ is called modulation index m ($m \le 1$)
- The output of the multiplier is

$$s'(t) = [1 + m'(t)]A\cos\omega_0 t$$



Linear Modulation
Amplitude Modulation (AM)

- The frequency spectral density contains discrete carrier components
- The bandwidth is $2f_m$, where f_m is the highest frequency of the baseband signal
  - Spectral efficiency of AM is low
- Most part of the power is occupied by the carrier signal, which does not contain information
  - Power efficiency of AM is low.



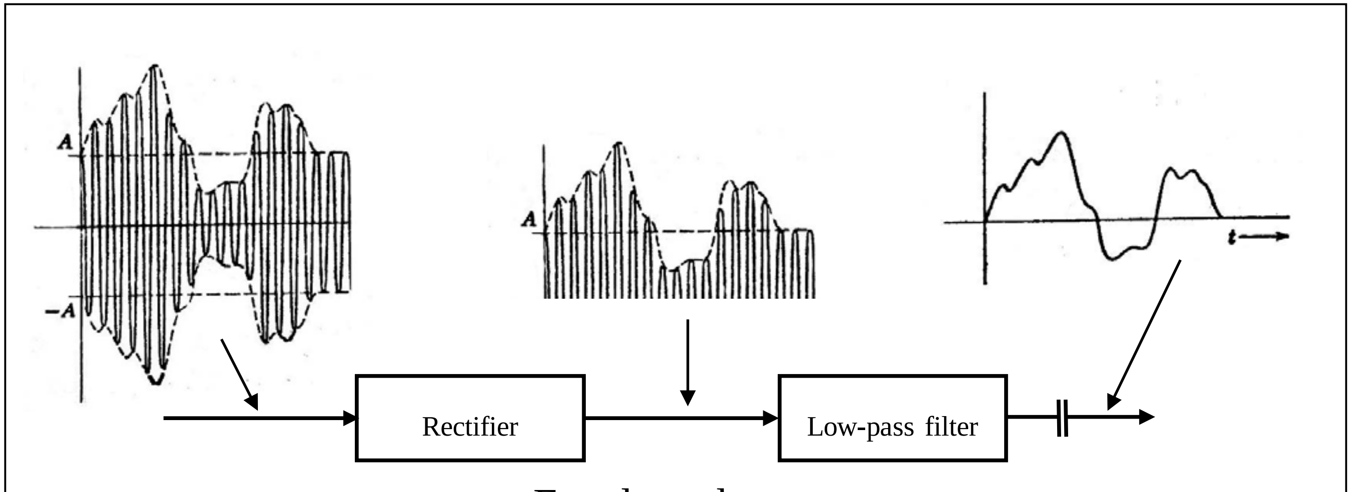
Waveform and spectrum of modulated signal

Linear Modulation
Amplitude Modulation (AM)

- **[Example 4.1]**
- Let the modulating signal $m(t) = 1 + m\cos\Omega t$, calculate, when $m = 1$ (or 100%) and $m = 0.5$ (or 50%), the ratio of the sum of the two sideband powers to the total modulated signal power.

Linear Modulation
Demodulation of AM Signals

- An envelope detector is normally used to restore the original modulating signal.
- The envelope detector consists of a rectifier and a low-pass filter.
- A capacitor is used to separate the DC component



Envelope detector

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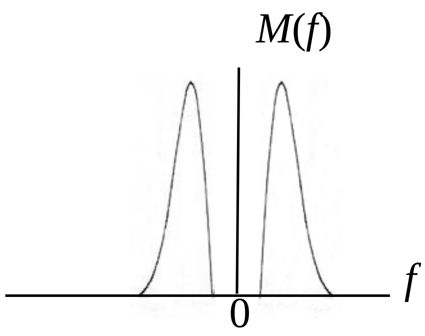
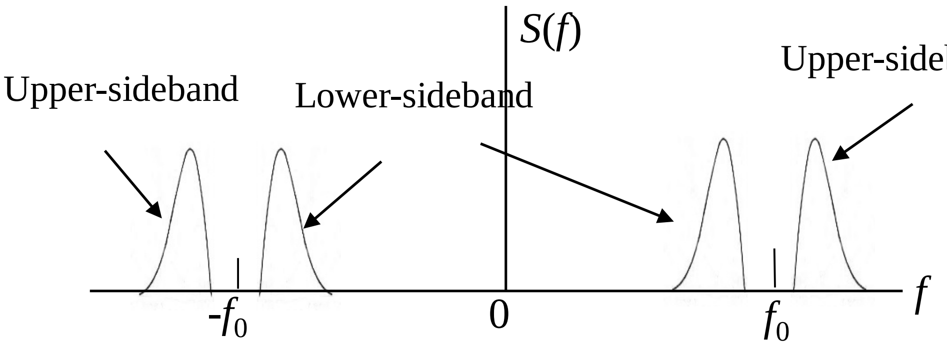
Linear Modulation
Double-sideband Modulation (DSB)

- When the modulating signal $m(t)$ does not contain DC component, there is no carrier in the output signal of the multiplier.
- This kind of modulation is called DSB, since its frequency spectrum contains two sidebands.
- The two sidebands are called upper-sideband and lower-sideband respectively, which contain the same information.



(a) Frequency spectral density of modulating signal

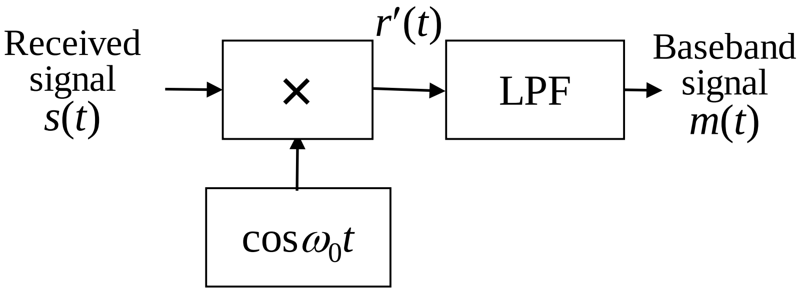


(b) Frequency spectral density of modulated signal

Linear Modulation
Demodulation of DSB Signals

- Assume that the received DSB signal is $m'(t)\cos\omega_0 t$, then
$$r'(t) = m'(t)\cos\omega_0 t\cos\omega_0 t = \frac{1}{2}m'(t)(1+\cos 2\omega_0 t)$$
- The output of the low-pass filter is $\frac{1}{2}m'(t)$
- If the frequency and phase of the local carrier have errors, i.e. assume its expression is $\cos\big((\omega_0+\Delta\omega)t+\varphi\big)$, we have
$$r'(t) = \frac{1}{2}m'(t)\Big(\cos(\Delta\omega t+\varphi)+\cos\big((2\omega_0+\Delta\omega)t+\varphi\big)\Big)$$

output: $\frac{1}{2}m'(t)\cos(\Delta\omega t+\varphi)$



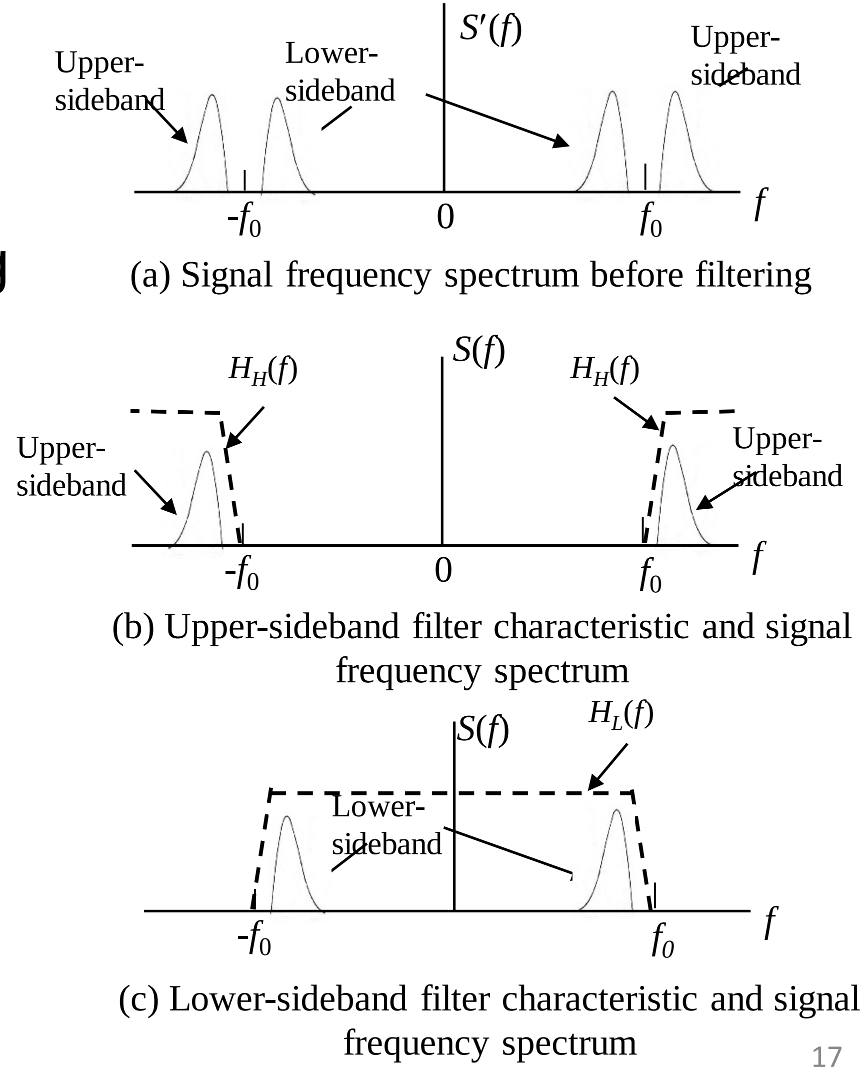
Linear Modulation
Single-sideband Modulation (SSB)

- SSB transmits only one sideband of the DSB signal.
- To allow the filter to separate the sidebands, the modulating signal should not contain very low frequency components.
- The filter

$$H_H(f) = \begin{cases} 1 & |f| \geq f_0 \\ 0 & |f| < f_0 \end{cases}$$

$$H_L(f) = \begin{cases} 1 & |f| \leq f_0 \\ 0 & |f| > f_0 \end{cases}$$

- Both power efficiency and spectrum efficiency of SSB signals are higher than AM and DSB signals.



Linear Modulation
Demodulation of SSB Signals

- Demodulation: A carrier is multiplied to the received signal
- Time-domain and frequency-domain operations

$$x(t) * y(t) \overset{\mathrm{FT}}{\Leftrightarrow} X(\omega)Y(\omega)$$

$$x(t)y(t) \overset{\mathrm{FT}}{\Leftrightarrow} X(\omega) * Y(\omega)$$

- The multiplication of carrier $\cos \omega_0 t$ with the received signal is equivalent to the convolution of the carrier frequency spectrum with the signal frequency spectrum in the frequency domain.
- [Example] The demodulation of an upper-sideband signal.

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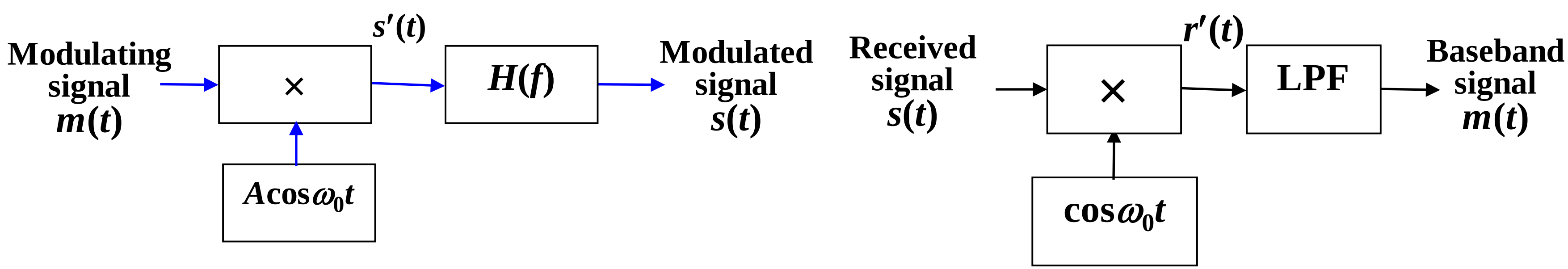
Linear Modulation

Vestigial Sideband Modulation (VSB)

- Despite the high power and spectrum efficiency, SSB requires high system complexity
  - The local carrier at the demodulator must have the same frequency and phase as that in the transmitter
  - The modulator should have a very high-quality filter to attain SSB from DSB or AM signals.
- VSB allows the modulating signal to contain very low frequency and DC components



Linear Modulation
Vestigial Sideband Modulation (VSB)

- Frequency spectrum expression of the output signal of the multiplier is $S'(f) = \frac{A}{2}[M(f - f_0) + M(f + f_0)]$
- After being filtered, the frequency spectrum of VSB signal $s(t)$ is:

$$S(f) = \frac{A}{2}[M(f - f_0) + M(f + f_0)]H(f)$$

- What condition should be satisfied, to realize demodulation without distortion?
- The frequency spectrum of the output signal of the multiplier at the demodulator, $r'(t)$, is

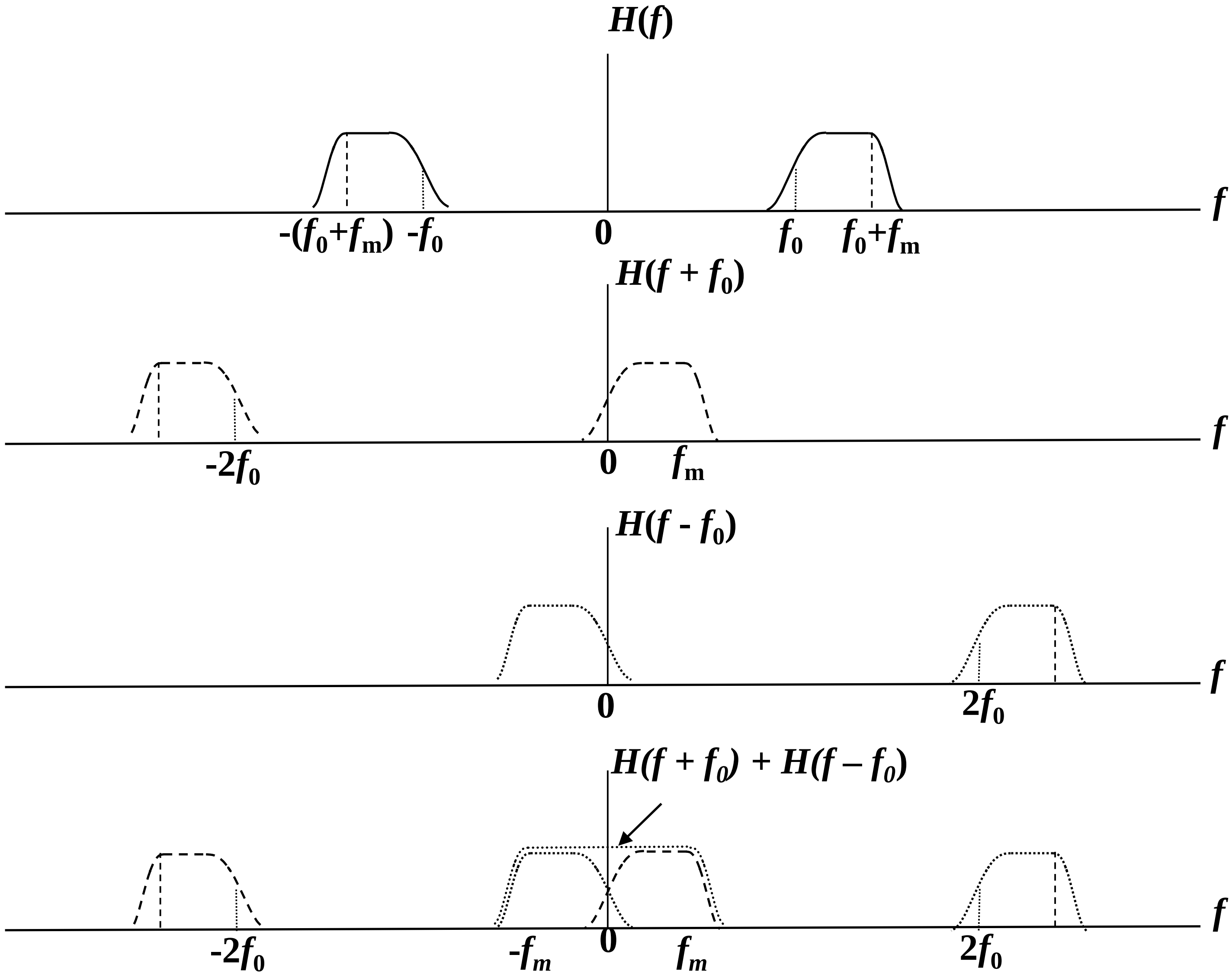
$$\frac{1}{2}[S(f + f_0) + S(f - f_0)]$$
$$= \frac{A}{4}\{[M(f + 2f_0) + M(f)]H(f + f_0) + [M(f - 2f_0) + M(f)]H(f - f_0)\}$$

Linear Modulation
Vestigial Sideband Modulation (VSB)

- The output demodulated signal after the low-pass filter is
$$\frac{A}{4}M(f)[H(f+f_0)+H(f-f_0)]$$
- Transmission without distortion requires
$$H(f+f_0)+H(f-f_0)=C$$
where C is a constant.
- Since $M(f)$ is the frequency spectral density function of the baseband modulating signal, let its highest frequency component be f_m, i.e. $M(f)=0$ when $|f|>f_m$.
- We have
$$H(f+f_0)+H(f-f_0)=C \qquad |f|\le f_m$$

Linear Modulation

Vestigial Sideband Modulation (VSB)



Linear Modulation
Vestigial Sideband Modulation (VSB)

- The cut-off characteristic of the filter is complementary symmetry with respect to the carrier frequency f_0.
- In addition to the whole frequency spectrum of SSB signal, a part of the carrier frequency component and a small part of another sideband frequency spectrum are reserved in the frequency spectrum of VSB modulated signal.
- The filter in the modulator can be easily manufactured.
- The main disadvantage is that the occupied frequency band is a little wider than that of the SSB signal.

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Non-linear Modulation
Definitions

- Nonlinear modulation is also called angle modulation, which enables *frequency* and *phase* of the carrier to vary with the modulating signal.
- Let a carrier signal be expressed as

$$c(t) = A\cos\varphi(t) = A\cos(\omega_0 t + \varphi_0)$$

where φ_0 is the initial phase, $\varphi(t) = \omega_0 t + \varphi_0$ is the instantaneous phase, and $\omega_0 = d\varphi(t)/dt$ is the angular frequency of the carrier.

- After being modulated, the signal's instantaneous frequency $\omega_i(t)$ becomes a function of time, i.e.

$$\omega_i(t) = \frac{d\varphi(t)}{dt}$$

- Equivalently

$$\varphi(t) = \int \omega_i(t)dt + \varphi_0$$

- Angle modulation: $\varphi(t)$ varies with the modulating signal $m(t)$.

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Non-linear Modulation
Phase Modulation

- Phase modulation: Phase $\varphi(t)$ linearly varies with $m(t)$, i.e.

$$\varphi(t) = \omega_0 t + \varphi_0 + k_p m(t)$$

where k_p is a constant.

- The modulated signal can be expressed as

$$s_p(t) = A\cos\left[\omega_0 t + \varphi_0 + k_p m(t)\right]$$

- The instantaneous frequency of the modulated signal is

$$\omega_i(t) = \omega_0 + k_p \frac{d}{dt} m(t)$$

- The instantaneous frequency in phase modulation varies with the derivative of the modulating signal.

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Non-linear Modulation
Frequency Modulation

- Frequency modulation: Instantaneous frequency linearly varies with the modulating signal, i.e.

$$\omega_i(t) = \omega_0 + k_f m(t)$$

  where k_f is a constant.

- The phase of the modulated signal is:

$$\varphi(t) = \int \omega_i(t)dt + \varphi_0 = \omega_0 t + k_f \int m(t)dt + \varphi_0$$

- The modulated signal is

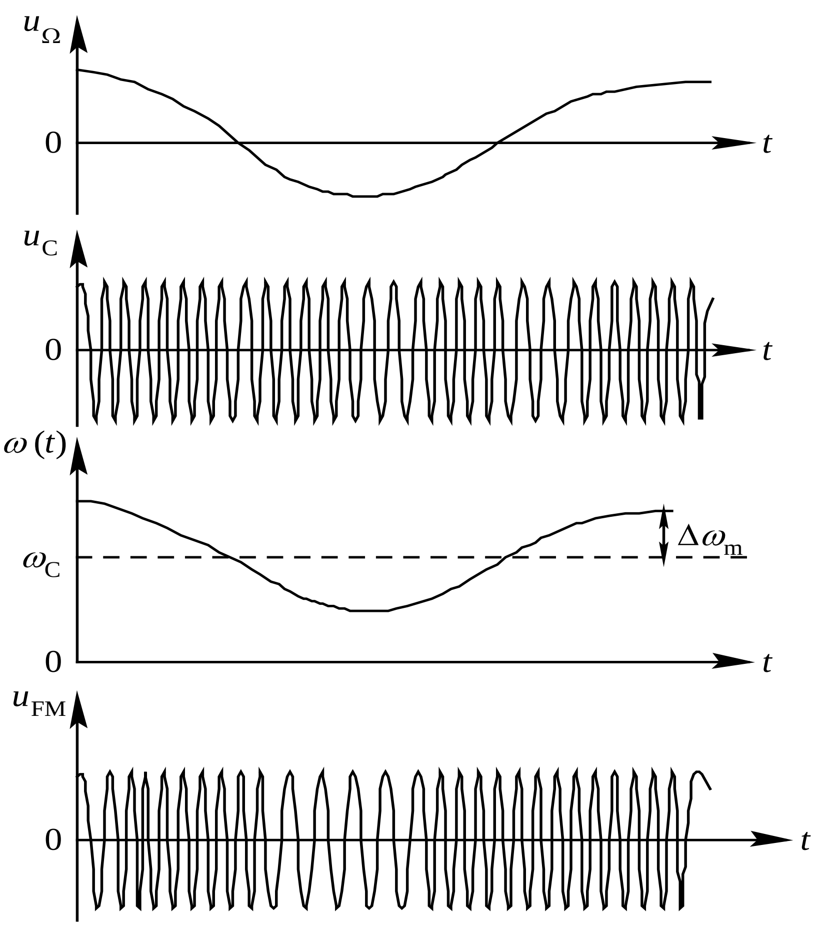
$$s_f(t) = A\cos\left[\omega_0 t + \varphi_0 + k_f \int m(t)dt\right]$$

Non-linear Modulation
Phase and Frequency Modulations

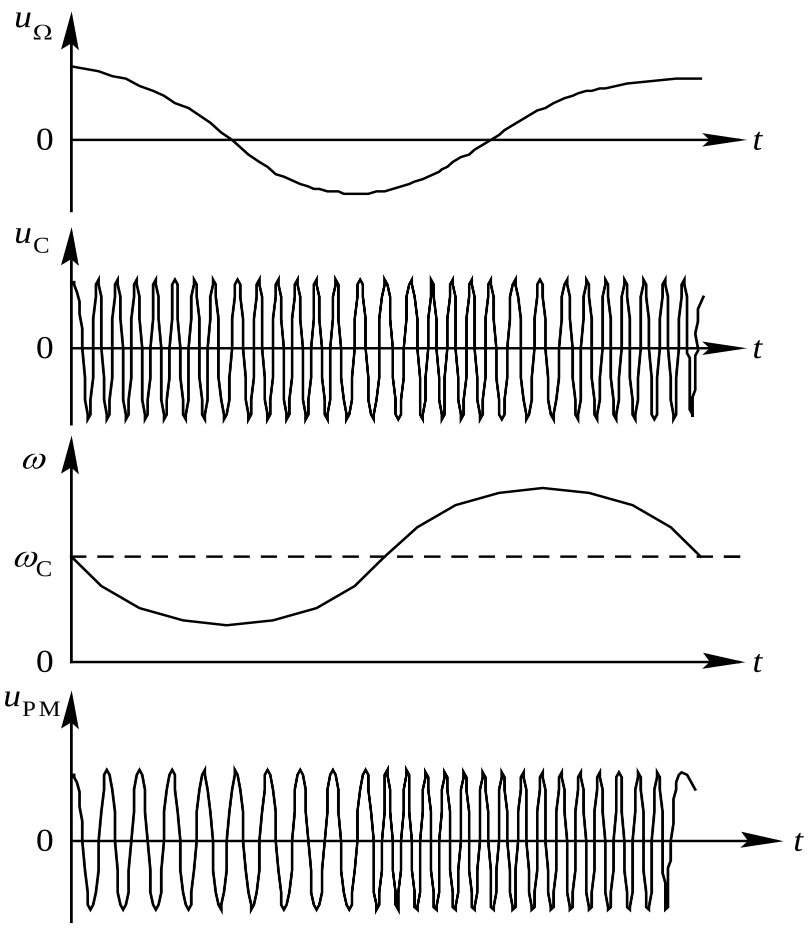
- The phase $\varphi(t)$ of a phase modulated signal varies linearly with modulating signal $m(t)$.
- The phase $\varphi(t)$ of a frequency modulated signal varies linearly with the integration of modulating signal $m(t)$.
- Essentially there is no difference between them.
  - If $m(t)$ is integrated first, and the phase of the carrier is modulated, then a frequency modulated signal is obtained.
  - If $m(t)$ is differentiated first, and the frequency of the carrier is modulated, then a phase modulated signal is obtained.
  - In both cases, the amplitude of a modulated signal is a constant.
  - It is not possible to distinguish them by modulated signal waveforms: The difference between them lies only in the relationship between the modulated and modulating signals.

Non-linear Modulation

Phase and Frequency Modulations



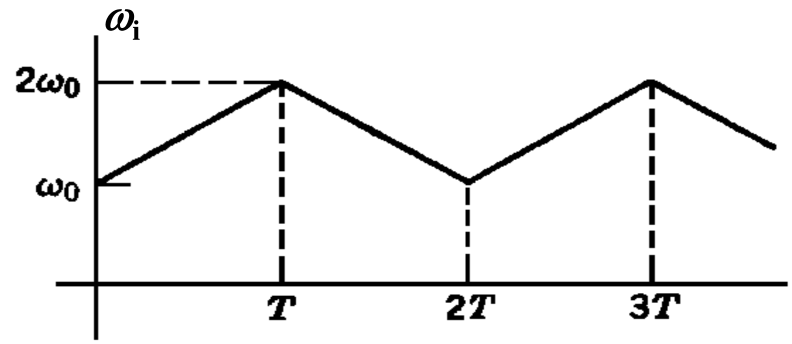
Frequency modulation

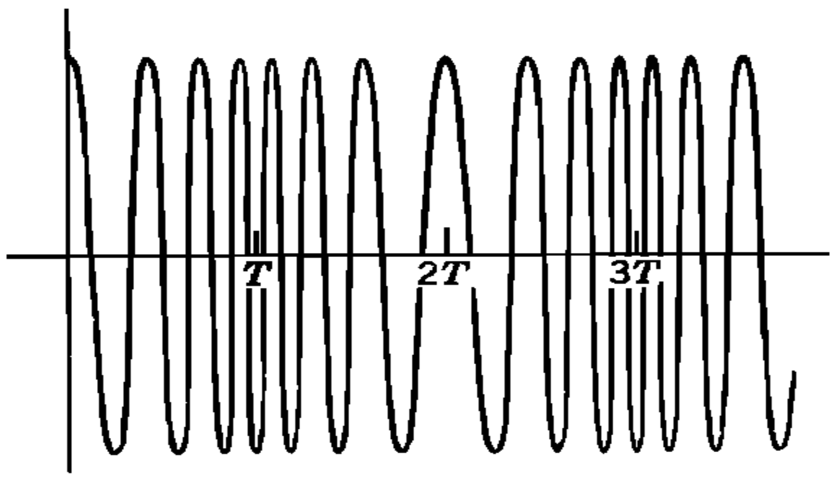


Phase modulation

Non-linear Modulation
Phase and Frequency Modulations

- The instantaneous frequency varies linearly between ω_0 and $2\omega_0$.
- If the waveform of $m(t)$ varies along straight lines as the instantaneous frequency, then the modulated signal is a frequency modulated signal.
- If the waveform of $m(t)$ varies with t^2 (the instantaneous frequency varies along straight lines), then the modulated signal is a phase modulated signal.





Non-linear Modulation
Frequency Spectrum and Bandwidth

- Let $m(t)$ be a cosinusoidal wave $m(t) = \cos\omega_m t$.
- The instantaneous angular frequency obtained from frequency modulation is $\omega_i(t) = \omega_0 + k_f m(t) = \omega_0 + k_f \cos\omega_m t$.
- The largest deviation of angular frequency is $\Delta\omega = k_f$ (rad/s).
- Setting $\varphi_0 = 0$, the modulated signal is expressed as

$$s_f(t) = A\cos\left(\omega_0 t + k_f \int \cos\omega_m t\, dt\right)$$

$$= A\cos[\omega_0 t + (\Delta\omega/\omega_m)\sin\omega_m t]$$

- $m_f = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m} = \frac{k_f}{\omega_m}$ is the ratio of the largest frequency deviation to the frequency of the baseband signal, and is called frequency modulation index.

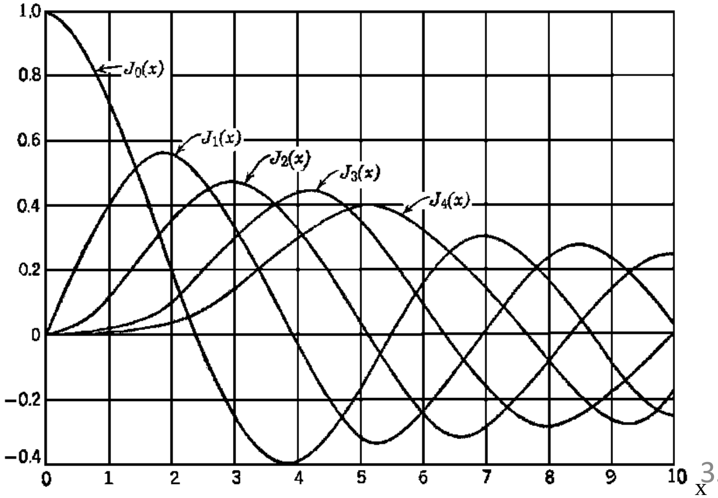
Non-linear Modulation
Frequency Spectrum and Bandwidth

- The expression of $s_f(t)$ can be expended as the following infinite series:

$$s_f(t)$$
$$= A\{J_0(m_f)\cos\omega_0 t + J_1(m_f)[\cos(\omega_0+\omega_m)t - \cos(\omega_0-\omega_m)t]$$
$$+ J_2(m_f)[\cos(\omega_0+2\omega_m)t + \cos(\omega_0-2\omega_m)t]$$

- The Bessel function has the following properties

$$J_n(m_f) = J_{-n}(m_f) \quad \text{for even } n$$
$$J_n(m_f) = -J_{-n}(m_f) \quad \text{for odd } n$$

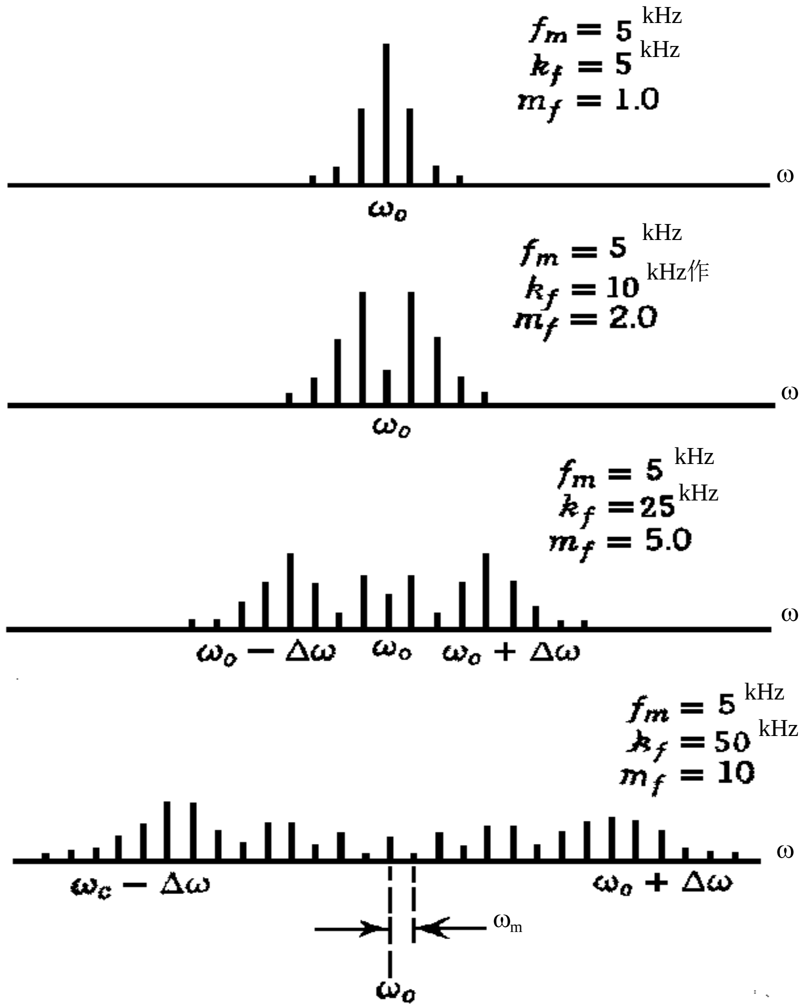


Non-linear Modulation
Frequency Spectrum and Bandwidth

- We have

$$s_f(t) = A \sum_{n=-\infty}^{\infty} J_n(m_f) \cos(\omega_0 + n\omega_m)t$$

- There are side frequencies in pair with angular frequencies $(\omega_0 \pm \omega_m)$, $(\omega_0 \pm 2\omega_m)$, …, appearing on both sides of the carrier frequency in the frequency spectrum, which means that the bandwidth of modulated signal is infinity.



Non-linear Modulation
Frequency Spectrum and Bandwidth

- However, most of the power concentrates within a limited bandwidth centered at the carrier frequency.
- When $m_f \ll 1$, the bandwidth is approximately $2\omega_m$.
- Frequency modulation with small modulation index is called narrowband frequency modulation, otherwise it is called broadband frequency modulation.
- The bandwidth increases as the modulation index increases.
- For broadband frequency modulation, if we neglect the side frequencies with amplitudes smaller than 10% of unmodulated carrier amplitude, then we can omit those $J_n(m_f)$ with $n > m_f + 1$.

Non-linear Modulation
Frequency Spectrum and Bandwidth

- The bandwidth B of the modulated signal for frequency modulation is approximately (Carson bandwidth)

$$B \approx 2(m_f + 1)\omega_m = 2(\Delta\omega + \omega_m) \qquad \text{(rad/s)}$$

$$B \approx 2(m_f + 1)f_m = 2(\Delta f + f_m) \qquad \text{(Hz)}$$

where $\Delta f = \Delta\omega/2\pi$ is frequency deviation, and f_m is the frequency of the modulating signal.

- The above example discusses single sinusoidal wave modulation. For a modulating signal with multiple frequency components, f_m in the above equation should be the frequency of the highest frequency component of the modulating signal.

Non-linear Modulation

Reception of Angular Modulation Signals

- The amplitude of an angular modulated signal is constant and does not contain information.
- The change of amplitude due to fading and noise in the channel does not affect information reception.
- The impact of fading and noise on the angle (frequency and phase) of a signal is not as significant as that on amplitude.
- The anti-jamming ability of angular modulation is stronger than that of linear modulation.
- To eliminate the impact of fading and noise, an amplitude limiter is applied before demodulation to remove amplitude variation.

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Summary

- Analog modulation can be classified as linear modulation and nonlinear modulation.
- Frequency spectrum structure of a linear modulated signal is the displacement of that of the modulating signal.
- Linear modulation includes AM, DSB, SSB, and VSB.
- Nonlinear modulation is also called angle modulation, including frequency modulation and phase modulation.
- Frequency spectrum structure of a nonlinear modulated signal is different from that of the modulating signal.
- New frequency components appear, resulting in increased bandwidth.

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