A set of criteria for the prediction of initiation and propagation of transverse cracks

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Abstract

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1. Normalization function

$$G_0 = G_0\left(\varepsilon_0, V_f, E_{1f}, E_{2f}, E_m, \nu_{12f}, \nu_{23f}, \nu_m, G_{12f}, G_{23f}\right) \tag{1}$$

Given the elastic properties of the transversely isotropic UD ply $E_1, E_2, nu_{12}, nu_{23}$, for a 90° ply under transverse tension the cross section along the direction of the load coincides with the plane of transversal isotropy. It is thus possible, for a system in plane strain, to define equivalent isotropic Young's modulus and Poisson's ratio as follows. The effective Young's modulus and Poisson's ratio in plane strain in the plane of isotropy are defined as

$$E^* = \frac{E_2}{1-\nu_{21}\nu_{12}} \qquad \nu^* = \frac{\nu_{23}+\nu_{21}\nu_{12}}{1+\nu_{23}} \tag{2}$$

$$\begin{aligned} G_0 &= \frac{\sigma_0^2}{E^*}\pi R_f \quad \text{for a stress or force controlled test} \\ G_0 &= E^*\varepsilon_0^2\pi R_f \quad \text{for a strain or displacement controlled test} \end{aligned} \tag{3}$$

2. Boundary conditions

The ratio of maximum radial and tangential crack displacements with respect to the free case (single repeating element or single fiber layer ply?) can be considered as proxies for the effect of boundary conditions

$$\frac{u^{BC}_{r,max}}{u^{free}_{r,max}}, \frac{u^{BC}_{\theta,max}}{u^{free}_{\theta,max}} \tag{4}$$

3. Initiation of fiber-matrix debonds

Following Asp,

$$U_{\nu,m} = \frac{1-2\nu}{6E}\left(\sigma_{1,m}+\sigma_{2,m}+\sigma_{3,m}\right) \tag{5}$$

$$U_{\nu,m} \geq U^{cr}_{\nu,m} \tag{6}$$

$$\theta_0 = \max_{\theta} U_{\nu,m}, \quad U_{\nu,m} \geq U^{cr}_{\nu,m} \tag{7}$$

4. Propagation of fiber-matrix debonds

$$\frac{G_I}{G_0} = \begin{cases} A_\delta\left(V_f\right)\log\left(\delta\right) & +A_{\Delta\theta}\left(V_f, \frac{u^{BC}_{r,max}}{u^{free}_{r,max}}, \frac{u^{BC}_{\theta,max}}{u^{free}_{\theta,max}}\right)\sin\left(B_{\Delta\theta}\Delta\theta + C_{\Delta\theta}\right) + D \\ & B_{\Delta\theta}\Delta\theta_{max}\left(V_f, \frac{u^{BC}_{r,max}}{u^{free}_{r,max}}, \frac{u^{BC}_{\theta,max}}{u^{free}_{\theta,max}}\right) + C_{\Delta\theta} = \frac{\pi}{2} \\ & B_{\Delta\theta}\Delta\theta_{CZ}\left(\frac{u^{BC}_{r,max}}{u^{free}_{r,max}}, \frac{u^{BC}_{\theta,max}}{u^{free}_{\theta,max}}\right) + C_{\Delta\theta} = \pi \\ & for\ \Delta\theta < \Delta\theta_{CZ}\left(\frac{u^{BC}_{r,max}}{u^{free}_{r,max}}, \frac{u^{BC}_{\theta,max}}{u^{free}_{\theta,max}}\right) \\ 0 & otherwise \end{cases} \tag{8}$$

$$\frac{G_{II}}{G_0} = \begin{cases} E_\delta\left(V_f\right)\log\left(\delta\right) & +F_{\Delta\theta}\left(V_f, \frac{u^{BC}_{r,max}}{u^{free}_{r,max}}, \frac{u^{BC}_{\theta,max}}{u^{free}_{\theta,max}}\right)\sin\left(G_{\Delta\theta}\Delta\theta + H_{\Delta\theta}\right) + \\ & +I_{\Delta\theta}\left(V_f, \frac{u^{BC}_{r,max}}{u^{free}_{r,max}}, \frac{u^{BC}_{\theta,max}}{u^{free}_{\theta,max}}\right)\sin\left(2G_{\Delta\theta}\Delta\theta + H_{\Delta\theta}\right) + L \\ & G_{\Delta\theta}\Delta\theta_{max}\left(V_f, \frac{u^{BC}_{r,max}}{u^{free}_{r,max}}, \frac{u^{BC}_{\theta,max}}{u^{free}_{\theta,max}}\right) + H_{\Delta\theta} = \frac{\pi}{2} \\ & G_{\Delta\theta}\Delta\theta_{CZ}\left(\frac{u^{BC}_{r,max}}{u^{free}_{r,max}}, \frac{u^{BC}_{\theta,max}}{u^{free}_{\theta,max}}\right) + H_{\Delta\theta} = \pi \end{cases} \tag{9}$$

5. Fracture toughness

$$G_c = G_{Ic}\left(1+\tan^2\left(\left(1-\lambda\right)\psi\right)\right) \qquad \psi = \tan^{-1}\left(\sqrt{\frac{G_{II}}{G_I}}\right) \tag{10}$$

Hypothesis

$$p\left(\Delta\theta\right) = p\left(\Delta\theta|\varepsilon\right) \sim \frac{1}{\sqrt{2\pi}\sigma_{\Delta\theta}\left(\varepsilon\right)} e^{\left(\frac{\Delta\theta-\overline{\Delta\theta}}{\sigma_{\Delta\theta}}\left(\varepsilon\right)\right)} \tag{11}$$

→ *Verified by measuring debond's size at different strain levels (see preliminary experimental work)*

$$\begin{cases} G_{TOT}\left(\Delta\theta\right) & = G_{Ic}\left(1+\tan^2\left(\left(1-\lambda\right)\psi\right)\right) \\ \psi & = \tan^{-1}\left(\sqrt{\frac{G_{II}(\Delta\theta)}{G_I(\Delta\theta)}}\right) \end{cases} \qquad \forall\Delta\theta : p(\Delta\theta) \neq 0 \tag{12}$$

$$p\left(\Delta\theta\right) = p\left(\Delta\theta|\varepsilon\right) \sim \frac{1}{\sqrt{2\pi}\sigma_{\Delta\theta}\left(\varepsilon\right)} e^{\left(\frac{\Delta\theta-\overline{\Delta\theta}}{\sigma_{\Delta\theta}}\left(\varepsilon\right)\right)} \tag{13}$$

6. Transition to collective mesoscopic behavior

$$\begin{cases} \frac{G_{TOT}}{G_0}\Big|_{debond} & > \frac{G_{TOT}}{G_0}\Big|_{transverse\ crack} \\ & \rightarrow \textit{Propagation of debonds at fiber/matrix interface level will occur, discrete events, "debonds' regime"} \\ \frac{G_{TOT}}{G_0}\Big|_{debond} & < \frac{G_{TOT}}{G_0}\Big|_{transverse\ crack} \\ & \rightarrow \textit{Propagation of transverse cracks will occur, collective behavior of debonds} \\ & \quad \textit{inter-fiber matrix cracks propagating and coalescing, "transverse cracks' regime"} \end{cases} \tag{14}$$

7. Global propagation function

Hypothesis

$$\frac{G_{TOT}}{G_0}\left(a, \frac{t_{0^\circ}}{t_{90^\circ}}\right) = -A \cdot \left(\frac{t_{0^\circ}}{t_{90^\circ}} - \frac{t_{0^\circ}}{t_{90^\circ}}|_{ref}\right)^{2n+1} \sqrt{a} + \frac{G_{TOT}}{G_0}|_0 \qquad (15)$$