

# Effect of uniform distributions of bonded and debonded fibers on the growth of the fiber/matrix interface crack in thin UD laminates with different fiber contents under transverse loading

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## Abstract

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## 1. Introduction

1. We start with a few lines devoted to the spread tow technology and thin plies: what they are, what can be done, what are the possible applications.
2. By quoting the relevant references, we report on the observation that one of the main beneficial mechanisms in thin ply is the retardation of transverse crack propagation. We then enlarge by reporting the microscopical observations by Saito, in which debonds where also observed. We observe that available microscopic observations are just a few and mainly in 2D.
3. Propagation of transverse cracks has been widely investigated both analytically and numerically
4. Initiation at the level of fiber/matrix interface is instead a less researched subject.

5. cohesive elements are a possible choice, but have some drawbacks, which  
15 makes a LEFM approach valuable
6. With regard to LEFM studies of laminates under transverse loading, models can be found in the literature about: the single fiber in infinite matrix under different mode of loading, the effect of adjacent fibers on a fiber in infinite matrix under different mode of loading, the single fiber in an  
20 equivalent composite in transverse tension, the effect of adjacent fibers on a fiber in an equivalent composite in transverse tension.
7. For initiation of transverse cracking at the fiber/matrix interface in UD laminates under transverse tension, there is thus a gap regarding: the effect of fiber volume fraction; the interaction of debonded and bonded  
25 fibers in micro-structured assemblies, i.e. no homogenization. This article addresses these two points.
8. We conclude the introduction with a summary of the article's structure.

## 2. RVE models & FE discretization

### 2.1. Introduction & Nomenclature

30 In this paper, we analyze debond development in unidirectional (UD) composites subjected to in-plane transverse tensile loading. The interaction between debonds in UD composites is studied developing models of different Repeating Unit Cells (RUC) of laminates where only the central fiber in the cell has a damage in the form of a fiber/matrix interface crack (debond). The composite  
35 RUC may be repeating in the transverse direction only (representing an ultra-thin composite) or repeating also in the composite thickness direction, representing an infinite composite in a limiting case. Thus, the conditions at the UD composite's upper and lower boundaries are one of the parameters for the investigation. The used RUCs allow for the consideration of the composite  
40 with debonds as a sequence of damaged and undamaged "rows", each "row" with only one fiber in the thickness direction. Since all of these RUCs feature regular microstructures with fibers placed according to a square-packing tiling,

they are Representative Volume Elements (RVE) of composites with a certain distribution of debonds. Introducing in-plane coordinates  $x$  and  $y$ , where  $x$  is in the transverse direction of the UD composite under consideration, the strain in the  $y$ -direction due to a load in the  $x$ -direction is small, due to the very small minor Poissons ratio of the UD composite. Additionally, debonds are considered to be significantly longer in the fiber direction than in the arc direction. Therefore, we use 2D models under the assumption of plane strain, defined in the  $x - z$  section of the composite. Thus, the analysis presented applies to long debonds, with a focus on understanding the mechanisms of growth along its arc direction. The composites are subjected to transverse tensile strain, applied as a constant displacement in the  $x$ -direction along the vertical boundary of the RUC as shown in Figure 1 to 4. As the models are differentiated by the number of layers of fibers and by the spacing between debonds along the vertical and horizontal directions, the corresponding RUCs can be distinguished from each other based on the number  $n$  of fibers in the horizontal direction and  $k$  in the vertical direction. Furthermore, the horizontal surfaces can be either free or vertical displacement coupling can be applied. We thus introduce the common notation  $n \times k - free$  and  $n \times k - coupling$  to denote a RUC with  $n \times k$  fibers and, respectively, a free upper surface or kinematic coupling applied to it. The specific combinations of particular choices of  $n$ ,  $k$ , and boundary conditions are detailed in Section 2.2, together with the corresponding models of damaged composite they are representing.

## 2.2. Models of Representative Volume Element (RVE)

The first two models feature, as shown in Fig. 1, an ultra-thin UD laminate with only one “row” of fibers across its thickness,  $k = 1$ . This is quite an extreme model from the microstructural point of view; however, it allows to focus the analysis on the interaction between debonded fibers placed along the  $x$ -direction. Furthermore, as the horizontal surfaces are considered free, the interaction is stronger in this case than in any other, making the predictions of this model rather conservative. In retrospective, if only 20 years ago such a

model would have been considered too abstracted from the physical reality, the recent advancements in the spread tow technology make this approach appealing  
75 also as a limiting case for practical considerations.

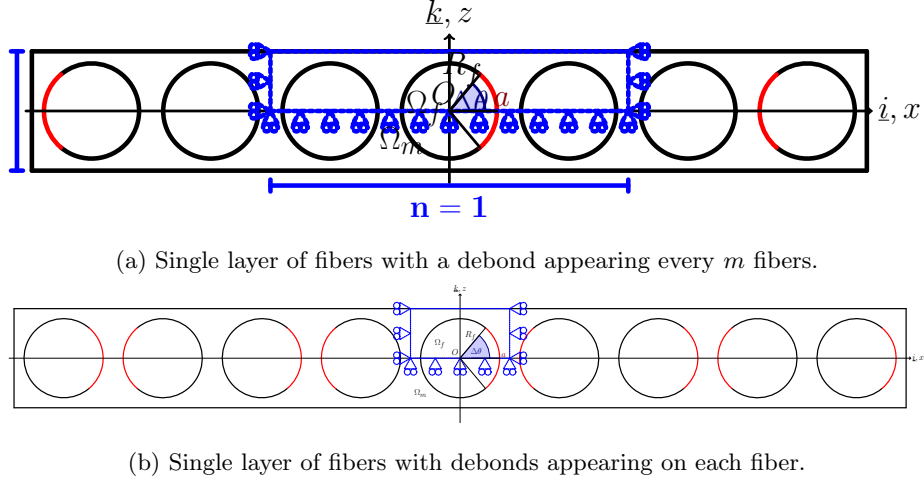
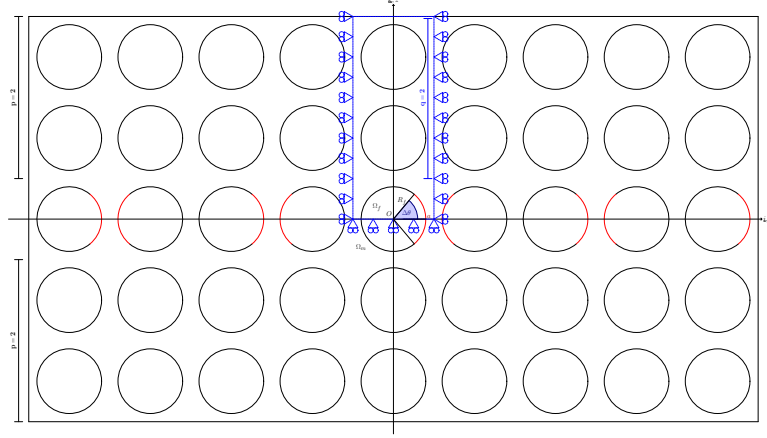


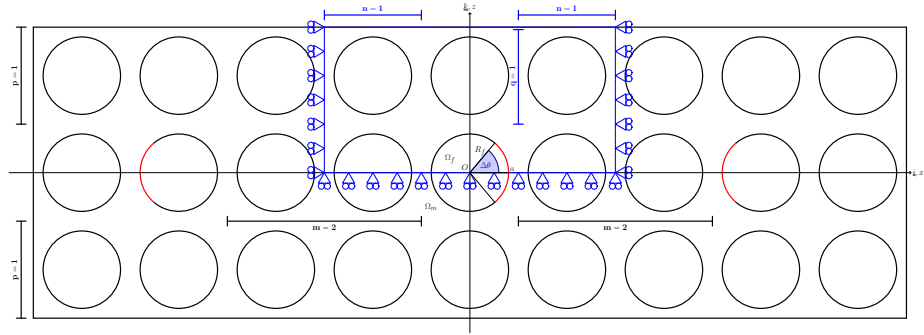
Figure 1: Models of UD laminates with a single layer of fibers and debonds repeating at different distances. The corresponding repeating element (RVE) is highlighted in blue.

In the first version of the model laminate, Fig. 1a, debonds appear in the laminate on every  $(m + 1)^{th}$  fiber on alternating sides of the partially debonded fiber. The symmetries of the model allow the use of a Repeating Unit Cell (RUC), which corresponds to the Representative Volume Element (RVE) of this  
80 microstructure, with a central debonded fiber and  $\frac{m}{2}$  fiber(s) on each side. It is highlighted by blue lines in 1a. Following the notation introduced in Section 2.1, we will refer to this model as  $(m + 1) \times 1 - free$ , where  $n = (m + 1), k = 1$ . In the second version of the single-layer-of-fibers model, 1b, a debond appears on each fiber on alternating sides and the corresponding RUC has only one  
85 debonded fiber. We will refer to this model as  $1 \times 1 - free$ , where  $n = k = 1$ .

The second set of models considers instead laminates with multiple layers of fibers across the thickness: a finite number of layers in the first two models ( 2a and 2b); an infinite number in the model of Fig. 3. In the first representative laminate (Fig. 2a), all the fibers in the central layer are debonded. The UD is



(a) Multiple layers of fibers with debonds appearing on each fiber belonging to the central layer.



(b) Multiple layers of fibers with a debond appearing every  $m$  fibers within the central layer.

Figure 2: Models of UD laminates with different layers of fibers and debonds repeating at different distances. The corresponding repeating element (RVE) is highlighted in blue.

made by  $2p + 1$  layers of fibers across the thickness, corresponding to a RUC  
 with  $p + 1$  fibers in the  $z$  direction. This model will be referred to in the  
 following as  $1 \times (p + 1) - free$ , where  $n = 1, k = (p + 1)$ . In the second model  
 (Fig. 2b), a debond appear every  $(m + 1)^{th}$  fiber in the central line of fibers in  
 a laminate with  $2p + 1$  layers. The corresponding RUC has thus  $\frac{m}{2}$  fiber(s) on  
 each side and  $p$  above the partially debonded fiber. We will refer to this model  
 as  $(m + 1) \times (p + 1) - free$ , where  $n = (m + 1), k = (p + 1)$ .



Figure 3: Model of UD laminates with an infinite number of layers of fibers and debonds appearing on each fiber. The corresponding repeating element (RVE) is highlighted in blue.

Finally, the last model considers an UD composite with an infinite number of partially debonded fibers. The corresponding RUC is made by a single partially debonded fiber and kinematic coupling conditions applied to the upper  
100 boundary. This model is referred to as  $1 \times 1$  - coupling, where  $n = k = 1$ .

### 2.3. Finite Element (FE) discretization

Each RUC is discretized using the Finite Element Method (FEM) within the Abaqus environment, a commercial FEM package [1]. The length  $l$  and height  $h$  of the model (see Fig. 4a) are determined by number of fibers  $n$  in the horizontal  
105 direction and  $k$  across the thickness (see 2.2) according to Eq. 1:

$$l = 2nL \quad h = (2k - 1)L; \quad (1)$$

where the reference length  $L$  is defined as a function of the fiber volume fraction  $V_f$  and the fibers' radius according to

$$L = \frac{R_f}{2} \sqrt{\frac{\pi}{V_f}}. \quad (2)$$

The fibers' radius  $R_f$  is assumed to be the same for each fiber present in the model and equal to  $1 \mu m$ . The relationships in Eqs. 1 and 2 thus ensure that  
110 the local and global  $V_f$  are everywhere equal.

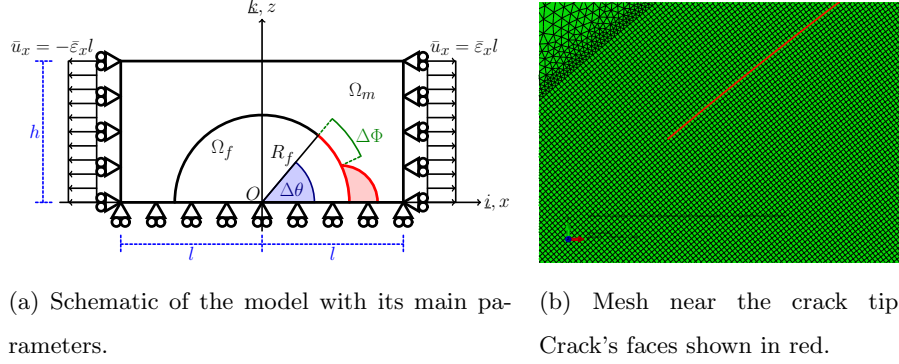


Figure 4: Details and main parameters of the Finite Element model.

The debond is placed symmetrically with respect to the  $x$  axis (in red in 4a) and has an angular size of  $\Delta\theta$  (the full debond's size is thus  $2\Delta\theta$ ). For high debond's sizes ( $\geq 60^\circ - 80^\circ$ ), a region of variable size  $\Delta\Phi$  appears at the crack tip in which the crack's faces are in contact and slide on each other. Due to its appearance, frictionless contact is considered between the two crack's faces to allow free slipping and avoid interpenetration. Symmetry with respect to the  $x$  axis is applied on the lower boundary and kinematic coupling on the left and right sides. The upper boundary is in general free, except for the model  $1 \times 1 - coupling$  (Fig. 3) which requires kinematic coupling also on the upper side. Constant transverse strain  $\bar{\varepsilon}$  equal to 1% is applied to the right and left sides by means of an imposed displacement of, respectively,  $\pm\bar{\varepsilon}l$ .

Table 1: Summary of the mechanical properties of fiber and matrix.

Material	$E$ [GPa]	$G$ [GPa]	$\nu$ [-]
Glass fiber	70.0	29.2	0.2
Epoxy	3.5	1.25	0.4

The model is meshed using second order, 2D, plane strain triangular (CPE6) and rectangular (CPE8) elements. A regular mesh of quadrilateral elements with an almost unitary aspect ratio is required at the crack tip, as shown in Fig. 4b. The angular size  $\delta$  of an element in the crack tip region is always

equal to  $0.05^\circ$ . The mode I, mode II and total Energy Release Rates (ERRs) represent the main output of the FEM analysis; they are evaluated using the VCCT technique [2] implemented in a custom Python routine and, for the total ERR, the J-integral [3] by application of the Abaqus built-in functionality. A  
130 glass fiber-epoxy system is considered in every model, and it is assumed that their response lies always in the linear elastic domain. The properties used are listed in Table 1.

#### 2.4. Validation of the model

The model is validated in Fig. 5 against the results reported in [4], obtained  
135 with the Boundary Element Method (BEM) for a single fiber with a symmetric debond placed in an infinite matrix. This situation is modeled using the *free* RVE with  $V_f = 0.0079\%$ , which corresponds to a RUC's length and height of  $\sim 100$ .

To allow for a comparison, the results are normalized following [4] with  
140 respect to a reference Energy Release Rate  $G_0$  defined as

$$G_0 = \frac{1 + k_m}{8\mu_m} \sigma_0^2 \pi R_f \quad (3)$$

where  $\mu$  is the shear modulus,  $k$  is the Kolosov's constant defined as  $3 - 4\nu$  for plane strain conditions,  $R_f$  is the fiber radius and the pedix  $m$  refers to the properties of the matrix.  $\sigma_0$  is the stress at the boundary, computed as the average of the stress extracted at each boundary node along the right side  
145 (arithmetic average as nodes are equispaced by design along both the left and right sides).

### 3. Results & Discussion

#### 3.1. Effect of Fiber Volume Fraction

As shown in Figs. 6 and 7, respectively for mode I and mode II, the fiber  
150 content has a drastic effect on the Energy Release Rate at the crack tip of the fibre/matrix interface crack. The effect of four levels of fiber volume fraction



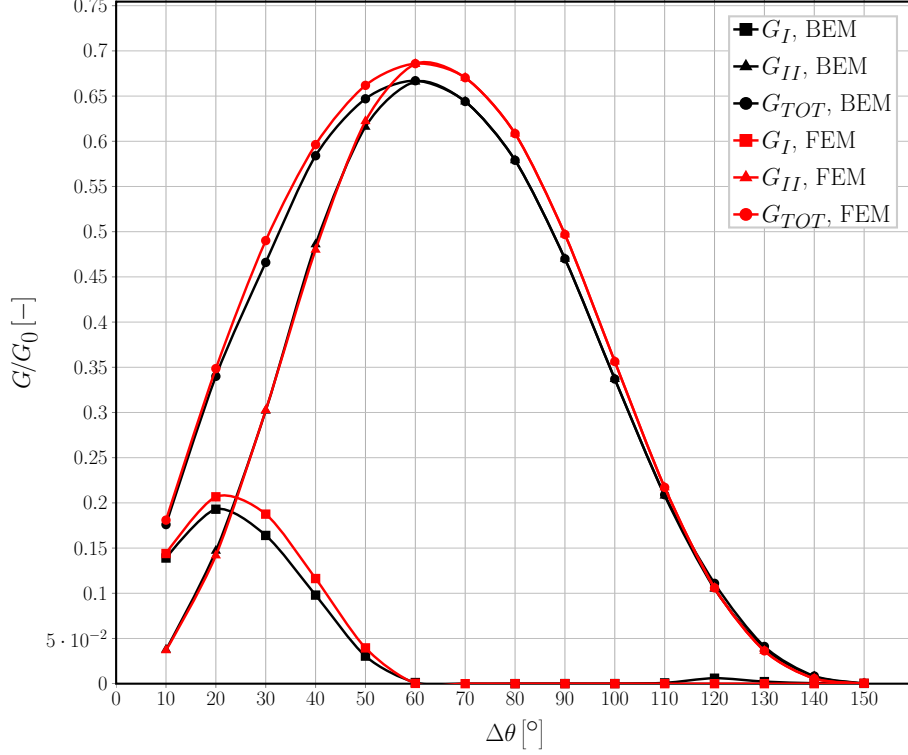


Figure 5: Validation of the single fiber model for the infinite matrix case with respect to the BEM solution in [4].

are compared, 30%, 50%, 60% and 65%, on two microstructural models: a  $11 \times 6 - free$  (a debond every 11<sup>th</sup> fiber in the central layer of an UD with 11 layers of fibers), Figs. 6a and 7a, and a  $21 \times 11 - free$  (a debond every 21<sup>th</sup> fiber in the central layer of an UD with 21 layers of fibers), Figs. 6b and 7b.

Comparison of Fig. 6a with 6b, and of Fig. 7a with 7b, indicates that there exists a specific effect of the fiber content, independent of the microstructure. For mode I, Fig. 6, the maximum value of the ERR is increased by  $\sim 5.2$  times when  $V_f$  changes from 30% to 65% in both models. The debond's size for which the peak value occurs remains unchanged at  $20^\circ$ , but for 60% and 65% the value at  $10^\circ$  and at  $20^\circ$  are almost identical, approximately creating a plateau and thus making the growth of small debonds ( $\leq 20^\circ$ ) in mode I unstable. Furthermore,

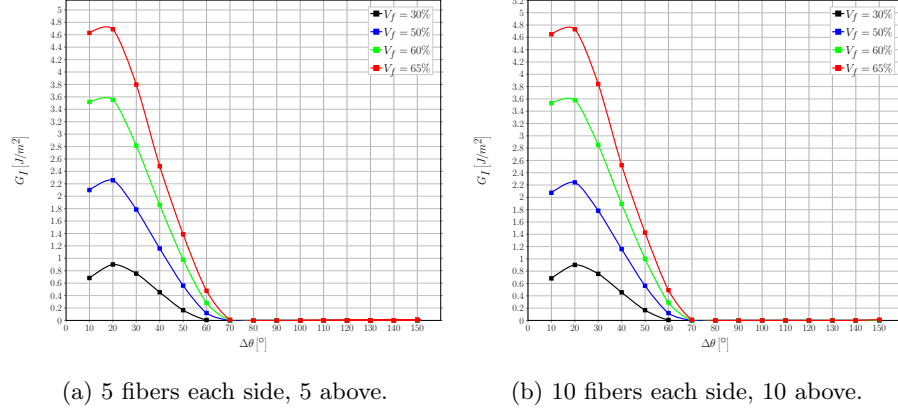


Figure 6: A view of the effect of fiber volume fraction on Mode I ERR in two exemplificative models.

increasing the fiber volume fraction delays the onset of the contact zone, which corresponds in 6 to the first value of  $\Delta\theta$  for which  $G_I$  is equal to zero. For  $V_f = 30\%$ , the contact zone first appears for a debond of  $60^\circ$ , similarly to what happens in the single fiber in infinite matrix model (Fig. 5). For higher fiber contents, the contact zone's onset is delayed to a debond's size equal to  $70^\circ$ .

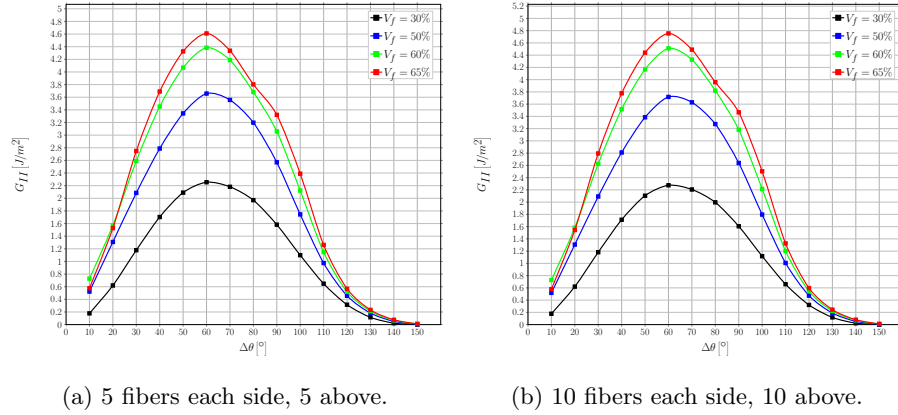


Figure 7: A view of the effect of fiber volume fraction on Mode II ERR in two exemplificative models.

For mode II, Fig. 6, the maximum value of the ERR is increased by  $\sim 2.1$  times when  $V_f$  changes from 30% to 65% in both models. The effect is thus

170 similar to mode I, but with a significantly lower magnitude. As for mode I, the  
 debond's size for which the peak value occurs remains unchanged, at  $60^\circ$  for  
 mode II. The shape of the curve remains instead unchanged, thus no effect on  
 the stability of mode II with respect to debond's size can be observed. It is  
 worthwhile to notice, however, that the ratio of mode II to mode I peak values  
 175 is  $\frac{\max(G_{II})}{\max(G_I)} \sim \frac{2.2}{0.9} \sim 2.4$  for  $V_f = 30\%$ , while it is  $\sim \frac{4.7}{4.7} \sim 1$  for  $V_f = 65\%$  in  
 both models. Given that the peaks occur at different debond's sizes, for which  
 the value of the other ERR is very small or even close to zero, this means that  
 the increase in fiber content creates a long range of very close values of total  
 ERR, and thus has a global destabilizing effect on the debond's growth.

180 The general increasing trends observed in Figs. 6 and 7 are related to the fact  
 that, given that the global and local  $V_f$  are everywhere identical in the models  
 presented, an increase in fiber content corresponds to a decrease in the average  
 distance between fibers. Thus, the relaxation of the stress and strain fields in the  
 matrix domain occurs over smaller lengths causing higher values at the crack tip.

185 The difference in relative magnification between mode I and mode II and the  
 delay in the contact zone's onset are instead due to the interplay between two  
 different mechanisms, both caused by the ordered microstructural arrangement  
 of the model. In the models considered, a fully bonded fiber is always placed  
 along the horizontal direction, aligned with the partially debonded fiber and  
 190 exactly in front of the debond. By increasing  $V_f$ , the former moves closer to the  
 latter and this causes a magnification of the x-strain at the crack tip. For small  
 debonds ( $\leq 20^\circ - 30^\circ$ ), the crack tip is approximately normal to the x-direction  
 and thus an increase in  $\varepsilon_x$  causes an increase in  $G_I$ . On the other hand, for  
 large debonds ( $\geq 70^\circ - 80^\circ$ ) the crack growth is almost aligned with the x-axis,  
 195 thus a magnification in the x-strain translates into an increase of mode II ERR.  
 However, this increasing effect on  $G_{II}$  is counteracted by the presence of a fully  
 bonded fiber along the vertical direction, aligned with the partially debonded  
 one. As fibers are more rigid than the surrounding matrix, the presence of the  
 former will restrain horizontal displacements, thus hampering strong increases in  
 200  $G_{II}$  for large debonds. Furthermore, due to the mismatch in the Poisson's ratios,

the fully bonded fiber placed above generates an upward-directed component of the vertical displacement field in the matrix, which tends to open the debond and causes the delay in the contact zone's onset. The interplay between these mechanisms is governed by the average inter-fiber distance and, in turn, by the fiber volume fraction.

### 3.2. Interaction between debonds in UD laminates with a single layer of fibers

The interaction of debonds appearing at regular intervals in UD composites with a single layer of fibers is studied for mode I (Fig. 8) and mode II (Fig. 9) and fiber content equal to 30% (Figs. 8a and 9a) and 60% (Figs. 8b and 9b). The models treated are  $3 \times 1 - free$ ,  $5 \times 1 - free$ ,  $7 \times 1 - free$ ,  $11 \times 1 - free$ ,  $21 \times 1 - free$ ,  $101 \times 1 - free$  and  $201 \times 1 - free$ , corresponding respectively to a debond every 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, 21<sup>st</sup>, 101<sup>st</sup> and 201<sup>st</sup> fiber (Fig. 1a). Given that the upper surface of the UD is left free, the interaction is stronger than in any other case and the results of this section are thus the most conservative in terms of debond's growth. From both 8 and 9, it can be seen that the presence of a debond decreases the strain magnification effect discussed in Sec. 3.1 and thus reduces the value of the ERR.

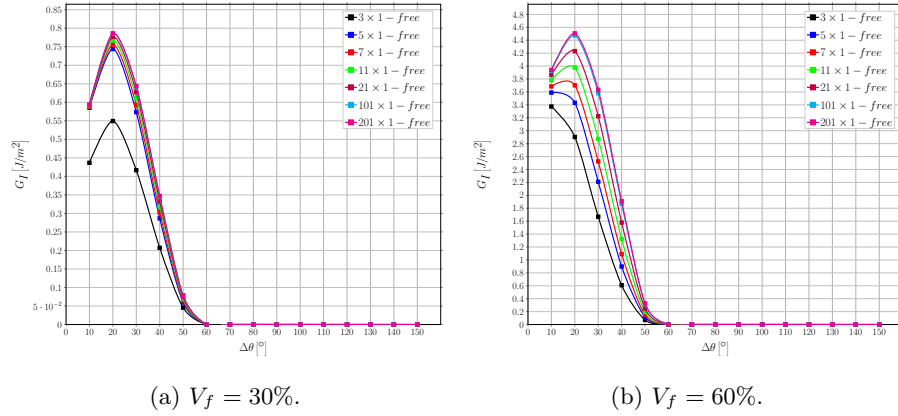


Figure 8: Effect of the interaction between debonds appearing at regular intervals on Mode I ERR in an UD with a single layer of fibers at different levels of fiber volume fraction  $V_f$ .

For mode I, the presence of a free surface, and inversely the absence of

a fully bonded fiber along the vertical direction, implies the absence of the  
 220 counteracting upward-oriented vertical component of the displacement field due  
 to the mismatch in Poisson's ratios. This in turn translates into the constancy  
 of the value of  $\Delta\theta$  corresponding to contact zone's onset, always equal to  $60^\circ$ .  
 For  $V_f = 30\%$ , mode I is reduced going from a debond placed every  $5^{th}$  fiber to  
 every  $3^{th}$  fiber. Larger spacing does not seem to have a sizable effect. Similarly,  
 225 at  $60\%$  no difference can be seen between the case of a debond placed every  $101^{th}$   
 and every  $201^{th}$  fiber. These observations suggest the existence of characteristic  
 distance dependent on the fiber volume fraction which governs the interaction  
 between debonds.

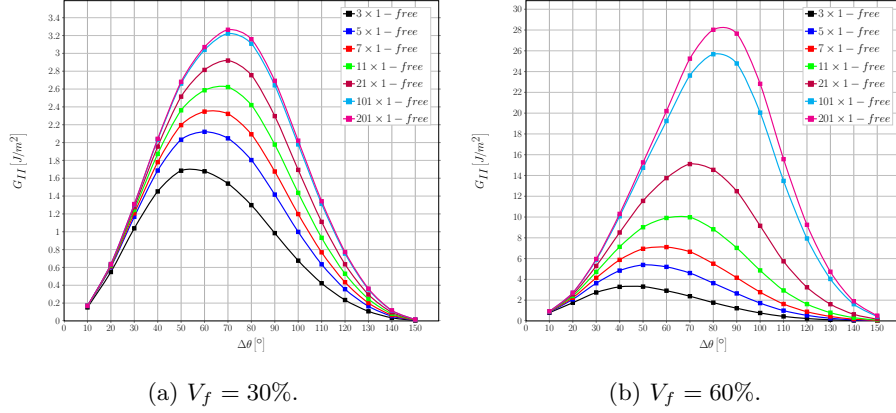


Figure 9: Effect of the interaction between debonds appearing at regular intervals on Mode II ERR in a single-ply laminate with a single layer of fibers at different levels of fiber volume fraction  $V_f$ .

Without constraint on the upper surface, the strain magnification effect cre-  
 230 ates larger displacements in the x-direction, which increase mode II for larger  
 debonds. When debonds are far apart, the series of rigid elements (constituted  
 by fully bonded fibers and their surrounding matrix) creates higher x-strains,  
 which in turn generates higher tangential displacements at the crack tip for  
 larger debonds. Conversely, when debonds are closer, the strain concentration  
 235 is reduced and the tangential component at the crack tip decreases for large  
 $\Delta\theta$ . This is the mechanism behind the change in the value of  $\Delta\theta$  for which the

peak of  $G_{II}$  occurs: from  $70^\circ$  to  $50^\circ$  at 30%, and from  $80^\circ$  to  $40^\circ$  at 60% going from the higher to the smaller spacing of debonds. Differently from mode I, the presence of a characteristic distance is harder to establish. For  $V_f = 30\%$  (Fig. 9a), it seems reasonable to establish it at around 100 fully bonded fibers between each debond. For  $V_f = 60\%$  (Fig. 9b), the difference between models  $101 \times 1 - free$  and  $201 \times 1 - free$  is still sizable, thus preventing the establishment of such characteristic distance. It is possible to observe, however, that the change between  $101 \times 1 - free$  and  $201 \times 1 - free$  is significantly smaller than between  $21 \times 1 - free$  and  $101 \times 1 - free$  ( $2 \left[ \frac{J}{m^2} \right]$  vs  $11 \left[ \frac{J}{m^2} \right]$ ), thus suggesting the existence of the characteristic distance outside the range studied.

### 3.3. Influence of layers of fully bonded fibers on debond's growth in a line of debonded fibers located at mid-thickness

The effect of the presence of layers of fully bonded fibers on debond's growth in a line of partially debonded fibers located at mid-thickness in UD composites is studied for mode I (Fig. 10) and mode II (Fig. 11) and fiber content equal to 30% (Figs. 10a and 11a) and 60% (Figs. 10b and 11b). The models treated are  $1 \times 2 - free$ ,  $1 \times 3 - free$ ,  $1 \times 4 - free$ ,  $1 \times 6 - free$ ,  $1 \times 11 - free$ ,  $1 \times 51 - free$  and  $1 \times 101 - free$ , corresponding to a UD composite with respectively 3, 5, 7, 11, 21, 101 and 201 layers of fibers (Fig. 2a).

The results shown strengthen the considerations made in Sec. 3.1. It can in fact be seen in Fig. 10 that the presence of fully bonded fibers across the thickness delays the onset of the contact zone to a debond of  $70^\circ$  in size, due to the introduction of an additional upward-directed component of the vertical displacement which translates into an opening displacement at the debond's tip.

The results of both mode I and mode II show how the introduction of an increasing number of fully bonded fibers doesn't change the ERR calculated at the crack tip. The effect of the  $V_f$  can be observed at low fiber content (Figs. 10a and 11a), while for high fiber content the smaller model with only fiber above the partially debonded one is already representative.

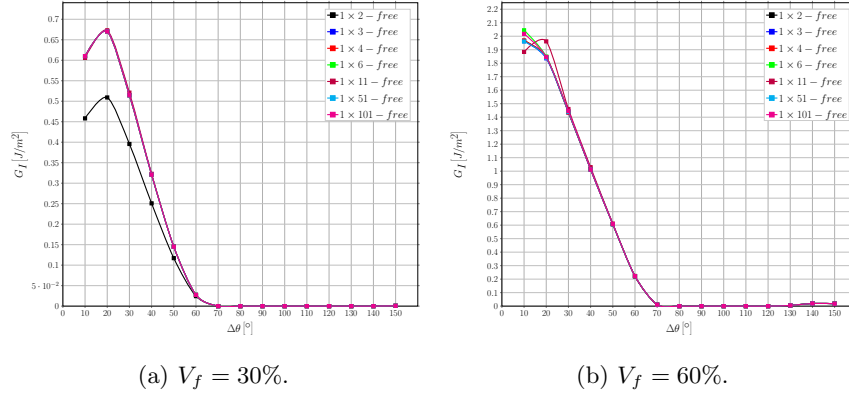


Figure 10: Influence of layers of fully bonded fibers on debond's growth in Mode I ERR in a centrally located line of debonded fibers at different levels of fiber volume fraction  $V_f$ .

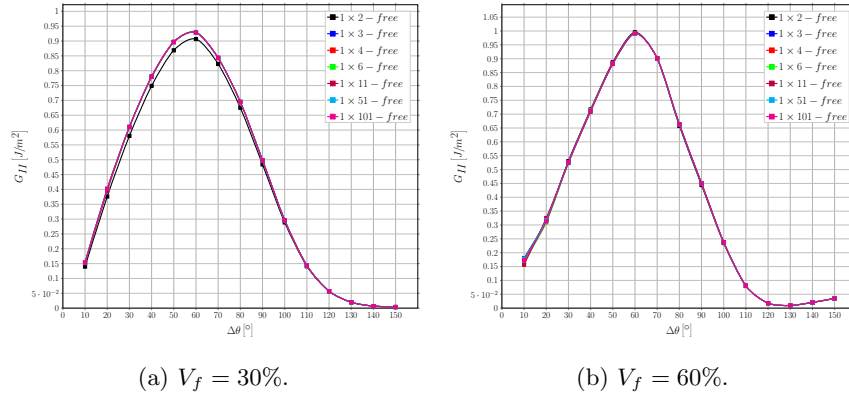


Figure 11: Influence of layers of fully bonded fibers on debond's growth in Mode II ERR in a centrally located line of debonded fibers at different levels of fiber volume fraction  $V_f$ .

### 3.4. Interaction between debonds in UD laminates with multiple layers of fibers

The interaction of debonds appearing at regular intervals in UD composites with multiple layers of fibers is investigated for mode I (Fig. ??) and mode II (Fig. 13) and fiber content equal to 30% (Figs. 12a and 13a) and 60% (Figs. 12b and 13b). The models treated are  $3 \times 2 - \text{free}$ ,  $5 \times 1 - \text{free}$ ,  $5 \times 2 - \text{free}$ ,  $11 \times 1 - \text{free}$ ,  $11 \times 6 - \text{free}$ ,  $21 \times 1 - \text{free}$ ,  $21 \times 11 - \text{free}$ ,  $101 \times 1 - \text{free}$ ,  $101 \times 6 - \text{free}$ ,  $201 \times 1 - \text{free}$  and  $201 \times 6 - \text{free}$  (Fig. 2b).

Comparing with the results in Sec. 3.2, it can be observed how the presence

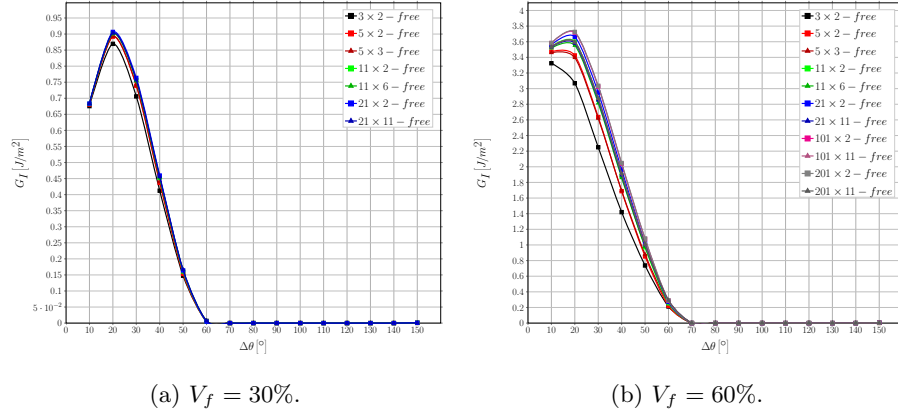


Figure 12: Effect of the interaction between debonds appearing at regular intervals on Mode I ERR in a single-ply laminate with multiple layers of fibers at different levels of fiber volume fraction  $V_f$ .

of fully bonded fibers across the thickness has a restraining effect on the ERR,  
 275 that counteracts the magnification due to an increasing number of fully bonded  
 fibers in the horizontal direction. The interplay is further modulated by the  
 fiber content. For mode I, at low fiber content the contact zone onset starts at  
 60°, while it is delayed to debonds of 70° for  $V_f = 60\%$ . Convergence can also  
 be observed: at 30% fiber volume fraction, the  $5 \times 2 - free$  model can already be  
 280 considered representative of further spaced debonds in arbitrarily thick UD; at  
 60%, the  $21 \times 2 - free$  model can be considered representative of laminates with  
 3 layers of fibers and the  $11 \times 6 - free$  of thicker UD. A less definite situation  
 characterizes instead mode II. An increase in the value of ERR can be observed  
 for any additional fully bonded fiber present in the horizontal direction, while  
 285 a change due to the number of fibers across the thickness can be observed only  
 between 1 and  $> 1$ .

It seems to be apparent that the interaction of debonds is strongly affected  
 by the presence of fully bonded fiber between them: the further apart debonds  
 are, the higher the Energy Release Rate. The presence of layers of fully bonded  
 290 fibers has instead a suppressing effect and there exists a limit value of layers  
 after which no sizeable change is measurable. Such limit value seems however



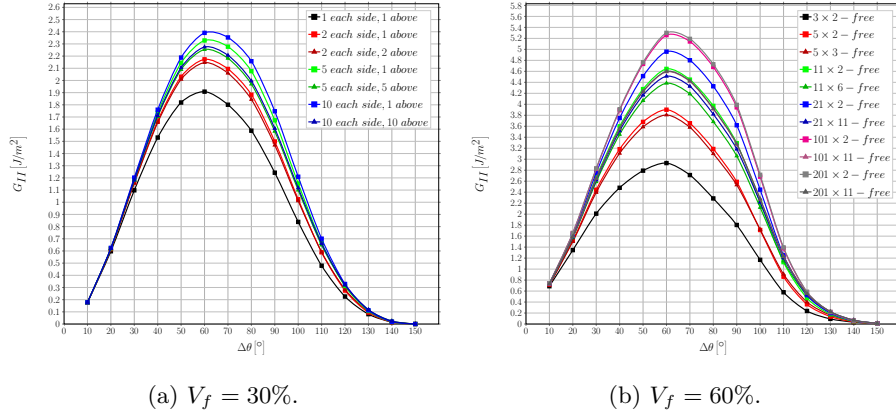


Figure 13: Effect of the interaction between debonds appearing at regular intervals on Mode II ERR in a single-ply laminate with multiple layers of fibers at different levels of fiber volume fraction  $V_f$ .

to depend on the spacing of debonds in the horizontal direction. Increasing the fiber content leads in general to more drastic changes in the ERR.

### 3.5. Comparison with the single fiber model with equivalent boundary conditions

295 The comparison of the single fiber models with the corresponding multi-fiber models (Figs. 14, 15, 14 and 15) show that the former provide in general the lowest estimation of the ERR and correspond to the most damaged state of the laminate, i.e. the state in which the greatest number of debonds is present. The  $1 \times 1 - free$  or simply free model (Figs. 14 and 15), which represents a UD with  
 300 a single layer of partially debonded fibers, agrees with the results of Sec. 3.2 and constitutes the extreme case for UD with a single layer of fibers, i.e. the case in which all the fibers are partially debonded.

The  $1 \times 1 - coupling$  or coupling model (Figs. 16 and 17) underestimates consistently the ERR in mode I and mode II when compared with  $n \times k - free$  models, as it represents an infinitely thick UD with all the fibers partially  
 305 debonded. When compared with the  $1 \times k - free$  model, it shows interestingly a good agreement, especially in mode I (Fig. 16a).

In mode II, it shows a sizeable difference in the range  $50^\circ - 90^\circ$ , while its

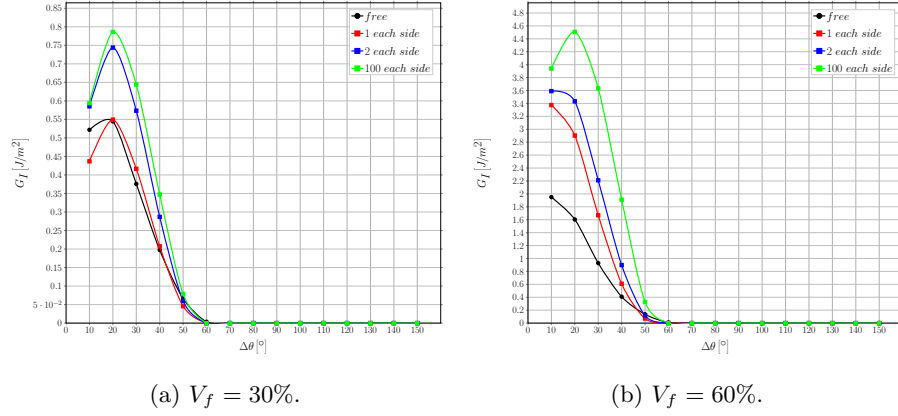


Figure 14: Comparison of Mode I ERR between the single fiber model with free upper boundary and the multiple fibers model with fibers only on the side at different levels of fiber volume fraction  $V_f$ .

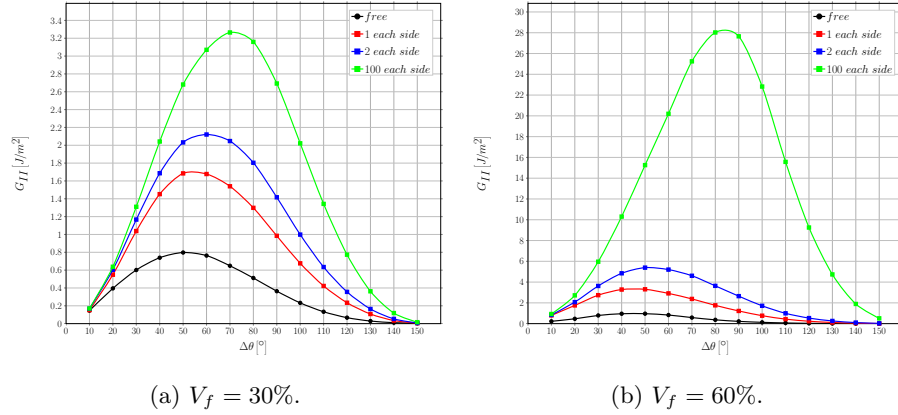


Figure 15: Comparison of Mode II ERR between the single fiber model with free upper boundary and the multiple fibers model with fibers only on the side at different levels of fiber volume fraction  $V_f$ .

310 results coincide with those of the  $1 \times k$ –*free* model for other values of  $\Delta\theta$ . These observations point to the evidence that debonds' interaction has a significant effect in the loading direction and not in the transverse one. The lower estimates of  $G_{II}$  in the range  $50^\circ - 90^\circ$  are due to the presence of a debond of the same size in the fiber just above the central one (modeled by the coupling boundary condition), which leaves the strip of matrix between the two fibers free to deform

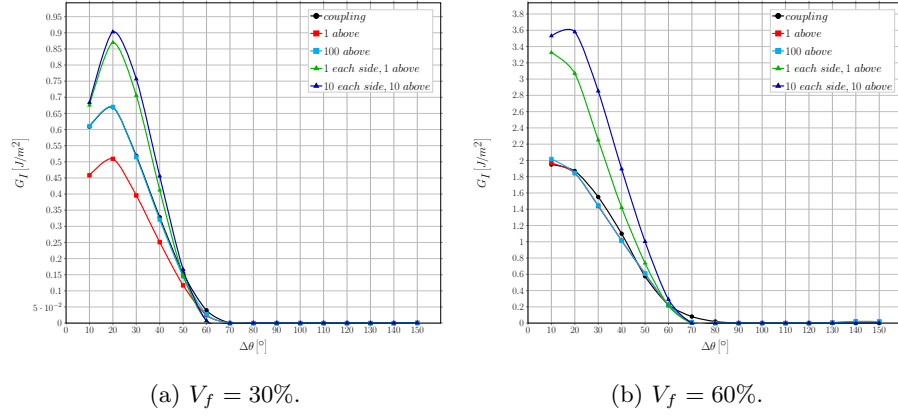


Figure 16: Comparison of Mode I ERR between the single fiber model with coupling conditions along the upper boundary and the multiple fibers model with fibers above and both above and on the side at different levels of fiber volume fraction  $V_f$ .

315 away from both fibers due to Poisson's effect and thus favors mode I and reduces mode II. This translates in the lower estimates in Fig. 17 and to the delay in the appearance of the contact zone, particularly evident in Fig. 16b.

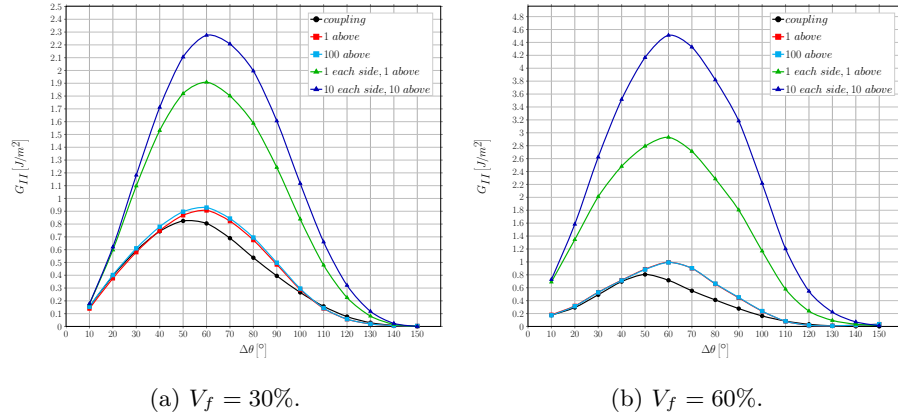


Figure 17: Comparison of Mode II ERR between the single fiber model with coupling conditions along the upper boundary and the multiple fibers model with fibers above and both above and on the side at different levels of fiber volume fraction  $V_f$ .

## 4. Conclusions & Outlook

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### References

- [1] Simulia, Providence, RI, USA, ABAQUS/Standard User's Manual, Version  
325 6.12 (2012).
- [2] R. Krueger, Virtual crack closure technique: History, approach, and appli-  
cations, *Applied Mechanics Reviews* 57 (2) (2004) 109. doi:10.1115/1.  
1595677.  
URL <https://doi.org/10.1115/1.1595677>
- 330 [3] J. R. Rice, A path independent integral and the approximate analysis of  
strain concentration by notches and cracks, *Journal of Applied Mechanics*  
35 (2) (1968) 379. doi:10.1115/1.3601206.  
URL <https://doi.org/10.1115/1.3601206>
- [4] C. Sandino, E. Correa, F. París, Numerical analysis of the influence of a  
335 nearby fibre on the interface crack growth in composites under transverse  
tensile load, *Engineering Fracture Mechanics* 168 (2016) 58–75. doi:10.  
1016/j.engfracmech.2016.01.022.  
URL <https://doi.org/10.1016/j.engfracmech.2016.01.022>