Finite Element solution of the fiber/matrix interface crack problem: convergence properties and mode mixity of the Virtual Crack Closure Technique

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Abstract

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1. Introduction

Bi-material interfaces represent the basic load transfer mechanism at the heart of Fiber Reinforced Polymer Composite (FRPC) materials. They are present at the macroscale, in the form of adhesive joints; at the mesoscale, as interfaces between layers with different orientations; at the microscale, as fiber-matrix interfaces. Bi-material interfaces have for long attracted the attention of researchers in Fracture Mechanics [1, 2], due to their hidden complexity.

The problem was first addressed in the 1950's by Williams [3], who derived through a linear elastic asymptotic analysis the stress distribution around an *open* crack (with crack faces nowhere in contact for any size of the crack) between two infinite half-planes of dissimilar materials and found the existence of a strong oscillatory behavior in the stress singularity at the crack tip of the form

$$r^{-\frac{1}{2}} \sin\left(\varepsilon \log r\right) \quad \text{with} \quad \varepsilon = \frac{1}{2\pi} \log\left(\frac{1-\beta}{1+\beta}\right); \tag{1}$$

in which β is one of the two parameters introduced by Dundurs [4] to characterize bi-material interfaces:

$$\beta = \frac{\mu_2 (\kappa_1 - 1) - \mu_1 (\kappa_2 - 1)}{\mu_2 (\kappa_1 + 1) + \mu_1 (\kappa_2 + 1)} \tag{2}$$

where $\kappa = 3 - 4\nu$ in plane strain and $\kappa = \frac{3-4\nu}{1+\nu}$ in plane stress, μ is the shear modulus, ν Poisson's coefficient, and indexes $1, 2$ refer to the two bulk materials joined at the interface. Defining a as the length of the crack, it was found that the size of the oscillatory region is in the order of $10^{-6}a$ [5]. Given the oscillatory behaviour of the crack tip singularity of the stress field of Eq. 1, the definition of Stress Intensity Factor (SIF) $\lim_{r\to 0}\sqrt{2\pi r}\sigma$ ceases to be valid as it returns logarithmically infinite terms [1]. Furthermore, it implies that the Mode mixity problem at the crack tip is ill-posed.

It was furthermore observed, always in the context of Linear Elastic Fracture Mechanics (LEFM), that an interpenetration zone exists close to the crack tip [6, 7] with a length in the order of 10^{-4} [6]. Following conclusions firstly proposed in[7], the presence of a *contact zone* in the crack tip neighborhood, of a length to be determined from the solution of the elastic problem, was introduced in [8] and shown to provide a physically consistent solution to the straight bi-material interface crack problem.

The curved bi-material interface crack, more often refered to as the fiber-matrix interface due to its relevance in FRPCs, was first treated by Perlman and Sih [9], who provided the analytical solution of stress and displacement fields for a circular inclusion with an arbitrary number of cracks. Building on their work, Toya [10] particularized the solution for an inclusion with a single interface crack (or debond) and provided the expression of the Energy Release Rate (ERR) at the crack tip.

2. Vectorial formulation of the Virtual Crack Closure Technique (VCCT)

3. Formulation of the ERR with respect to the FEM solution's variables

4. Convergence analysis

4.1. Analytical considerations

4.2. Numerical results

5. Conclusions & Outlook

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