INVESTIGATION OF SCALING LAWS OF THE FIBER/MATRIX INTERFACE CRACK IN POLYMER COMPOSITES THROUGH FINITE ELEMENT-BASED MICROMECHANICAL MODELING

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10th EEIGM International Conference on Advanced Materials Research

Moscow (RU), April 25-26, 2019

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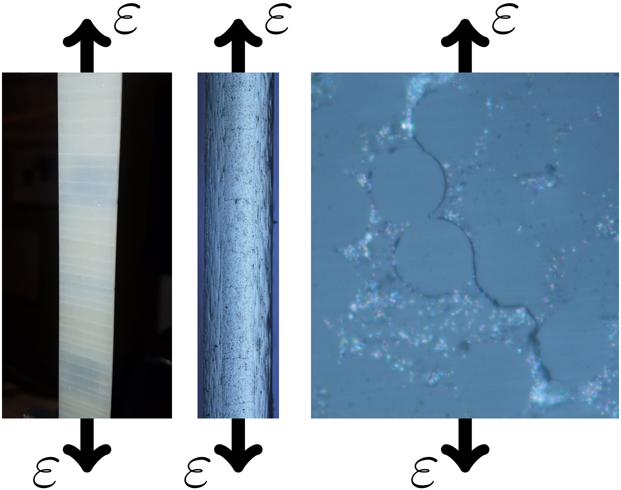
Outline

- The Fiber-Matrix Interface Crack Problem
- Investigation of Scaling Laws of the Fiber/Matrix Interface Crack
- Conclusions

THE FIBER-MATRIX INTERFACE CRACK PROBLEM

Initiation of Transverse Cracking in FRPCs: Microscopic Observations

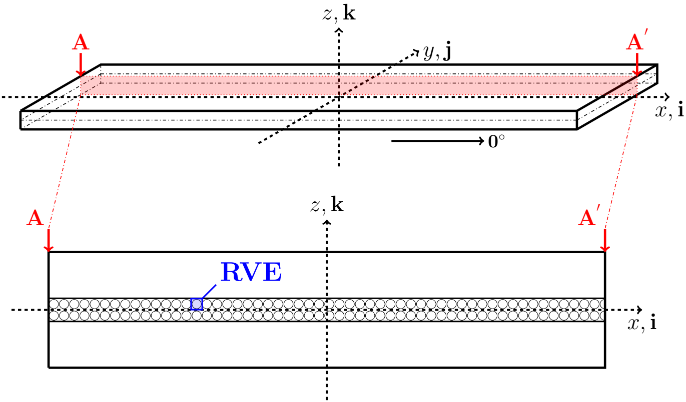


Left:
front view of $[0, 90_2]_S$,
visual inspection.

Center:
edge view of $[0, 90]_S$,
optical microscope.

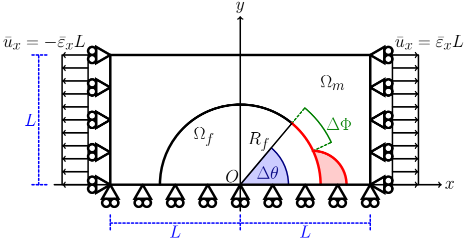
Right:
edge view of $[0, 90]_S$,
optical microscope.

The Fiber-Matrix Interface Crack Problem: Geometry



- ➔ $L, W >> t$
- ➔ $L, W \to \infty$
- ➔ 2D RVE

The Fiber-Matrix Interface Crack Problem: Assumptions

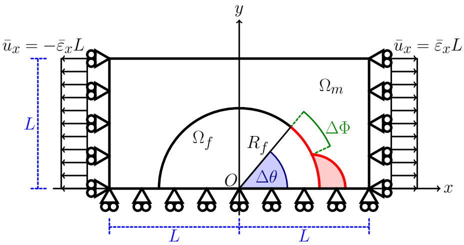


$$R_f = 1 \;\; [\mu m] \qquad L = \frac{R_f}{2}\sqrt{\frac{\pi}{V_f}}$$

Material	E_1	ν_{12}
glass fiber	70.0	0.2
epoxy	3.5	0.4

- ➔ Linear elastic, homogeneous and isotropic materials
- ➔ Plane strain
- ➔ Frictionless contact interaction
- ➔ Symmetric w.r.t. x-axis
- ➔ Coupling of x-displacements on left and right side (repeating unit cell)
- ➔ Applied uniaxial tensile strain $\bar{\varepsilon}_x = 1\%$

The Fiber-Matrix Interface Crack Problem: Solution



in Ω_f, Ω_m :

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\varepsilon_z = \gamma_{zx} = \gamma_{yz} = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

$$\sigma_{zz} = \nu \left(\sigma_{xx} + \sigma_{yy} \right)$$

for $0^\circ \leq \alpha \leq \Delta\theta$:

$$\left(\vec{u}_m (R_f, \alpha) - \vec{u}_f (R_f, \alpha) \right) \cdot \vec{n}_\alpha \geq 0$$

for $\Delta\theta \leq \alpha \leq 180^\circ$:

$$\vec{u}_m (R_f, \alpha) - \vec{u}_f (R_f, \alpha) = 0$$

$$\sigma_{ij} = E_{ijkl} \varepsilon_{kl}$$

$$+ BC$$

- ➔ Oscillating singularity

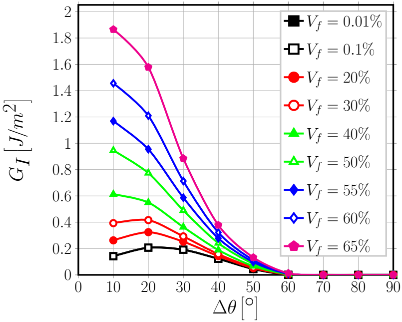
$$\sigma \sim r^{-\frac{1}{2}} \sin\left(\varepsilon \log r\right), \quad V_f \to 0$$

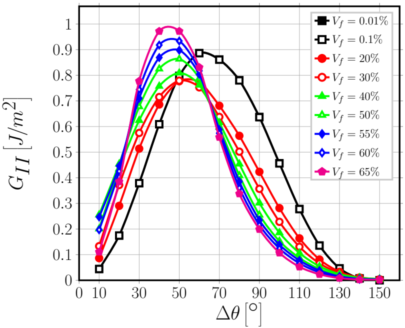
$$\varepsilon = \frac{1}{2\pi} \log\left(\frac{1-\beta}{1+\beta} \right)$$

$$\beta = \frac{\mu_2 (\kappa_1 - 1) - \mu_1 (\kappa_2 - 1)}{\mu_2 (\kappa_1 + 1) + \mu_1 (\kappa_2 + 1)}$$

- ➔ Finite Element Method (FEM) in Abaqus™
- ➔ 2^{nd} order shape functions
- ➔ 6-nodes triangles & 8-nodes quadrilaterals
- ➔ regular mesh of quadrilaterals at the crack tip:
  - $AR \sim 1$
  - $\delta = 0.05^\circ$

The Fiber-Matrix Interface Crack Problem: Normalization & Scaling





$$(?)\qquad G_I = G_0\left(V_f, E_f, \nu_f, E_m, \nu_m, \bar{\varepsilon}_x, \Delta\theta\right) g_I\left(\Delta\theta, BC, microstructure, damage\right)$$

$$(?)\qquad G_{II} = G_0\left(V_f, E_f, \nu_f, E_m, \nu_m, \bar{\varepsilon}_x, \Delta\theta\right) g_{II}\left(\Delta\theta, BC, microstructure, damage\right)$$

↘ INVESTIGATION OF SCALING LAWS

Dimensional Analysis

- From the definition of Energy Release Rate

$$G = \frac{\partial W}{\partial A} - \left(\frac{\partial U}{\partial A} + \frac{\partial E_k}{\partial A} \right) \quad \left[\frac{J}{m^2} \right]$$

$$\left[\frac{J}{m^2} \right] \longleftrightarrow \frac{E}{L^2} = \frac{F \cdot L}{L^2} = \frac{F}{L^2} \frac{L}{L} \cdot L = \sigma \varepsilon L$$

$$G_0 \sim \sigma_\infty \varepsilon_\infty L_c$$

- From the assumption of linear elasticity and uniaxial loading

$$\sigma_\infty = E_{eq} \varepsilon_\infty \qquad \varepsilon_\infty = \frac{\sigma_\infty}{E_{eq}}$$

$$G_0 \sim E_{eq} \varepsilon_\infty^2 L_c \qquad G_0 \sim \frac{\sigma_\infty^2}{E_{eq}} L_c$$

- From crack geometry

$$L_c \sim a = R_f \Delta\theta \longrightarrow L_c \sim R_f f(\Delta\theta)$$

$$G_0 = A \cdot E_{eq} \varepsilon_\infty^2 R_f f(\Delta\theta)$$

Homogenization of Material Properties: Concentric Cylinders Assembly (CCA)

$$E_L = V_f E_f + (1 - V_f)\, E_m + 2\lambda_1 \left(\nu_m - \nu_f\right)^2 V_f \left(1 - V_f\right)$$

$$\nu_{LT} = V_f \nu_f + (1 - V_f)\, \nu_m + \frac{\lambda_1}{2} \left(\nu_m - \nu_f\right) \left(\frac{1}{k_{fT}} - \frac{1}{k_{mT}}\right) V_f \left(1 - V_f\right)$$

$$G_{TT} = \frac{E_m}{2\left(1 + \nu_m\right)} + \frac{V_f}{\frac{1}{\frac{E_f}{2(1+\nu_f)} - \frac{E_m}{2(1+\nu_m)}} + \frac{k_{mT} + \frac{E_m}{1+\nu_m}}{\frac{E_m}{1+\nu_m}\left(k_{mT} + \frac{E_m}{2(1+\nu_m)}\right)} \left(1 - V_f\right)}$$

$$K_{TT} = \frac{k_{mT}\left(k_{fT} + \frac{E_m}{2(1+\nu_m)}\right)(1 - V_f) + k_{fT}\left(k_{mT} + \frac{E_m}{2(1+\nu_m)}\right) V_f}{\left(k_{fT} + \frac{E_m}{2(1+\nu_m)}\right)(1 - V_f) + \left(k_{mT} + \frac{E_m}{2(1+\nu_m)}\right) V_f}$$

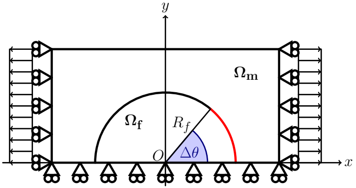
$$E_T = \frac{4G_{TT}}{1 + \frac{\left(1 + \frac{4K_{TT}\nu_{LT}^2}{E_L}\right) G_{TT}}{K_{23}}} \qquad \nu_{TT} = \frac{E_T}{2G_{TT}} - 1$$

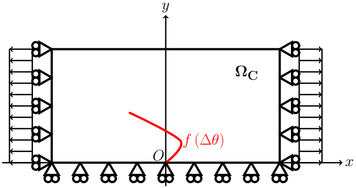
$$k_{fT} = \frac{E_f}{2\left(1 - \nu_f - 2\nu_f^2\right)} \quad k_{mT} = \frac{E_m}{2\left(1 - \nu_m - 2\nu_m^2\right)} \quad \lambda_1 = 2\left(\frac{2\left(1 + \nu_m\right)}{E_m} + \frac{V_f}{k_{mT}} + \frac{1 - V_f}{k_{fT}}\right)^{-1}$$

Homogenization of Material Properties: Plane Strain Conditions

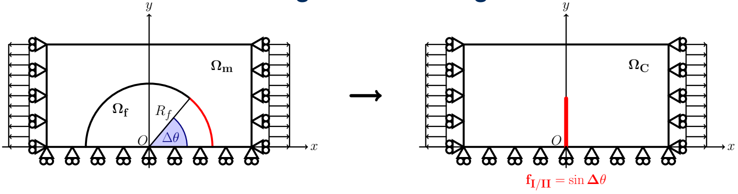
$$E_{\text{plane strain}} = \frac{E_T}{1 - \frac{E_T}{E_L}\nu_{LT}^2}$$

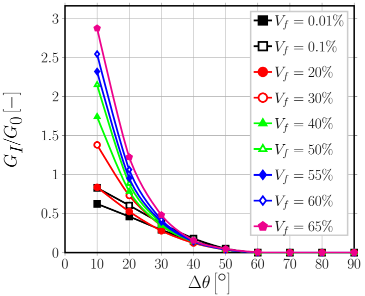
$$E_{eq} = \frac{E_T}{1 - \frac{E_T}{E_L}\nu_{LT}^2}$$

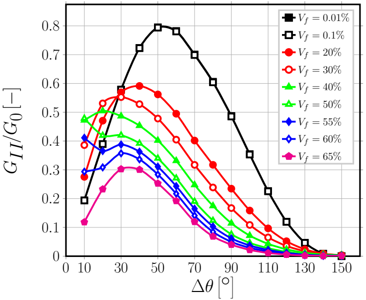




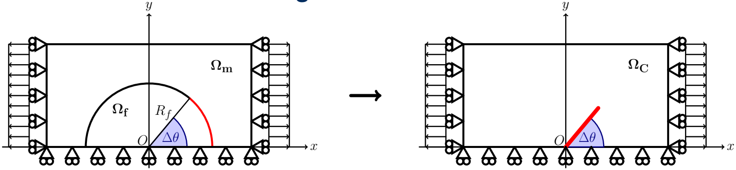
Shape Function Reference Configurations: Straight Crack



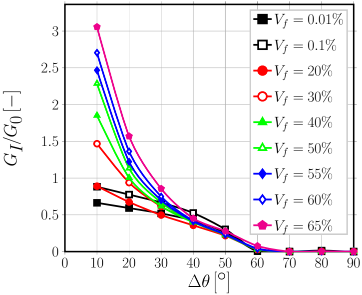


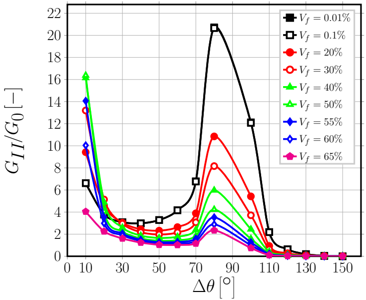


Shape Function Reference Configurations: Inclined Crack

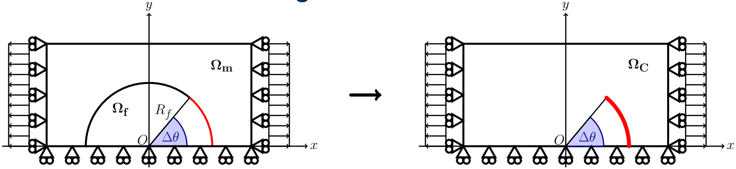


$\mathbf{f_I} = \sin\Delta\theta \sin^4\left(\frac{\pi}{2} - \Delta\theta\right), \mathbf{f_{II}} = \sin\Delta\theta \sin^2\left(\frac{\pi}{2} - \Delta\theta\right) \cos^2\left(\frac{\pi}{2} - \Delta\theta\right)$

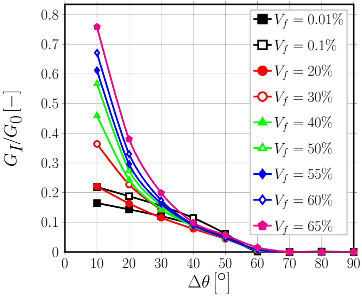


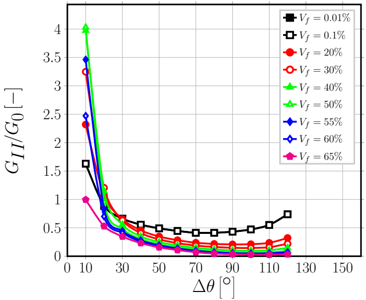


Shape Function Reference Configurations: Circular Crack



$$\mathbf{f_I} = \frac{\sin\Delta\theta}{\Delta\theta}\left(\frac{1-\sin^2\left(\frac{\Delta\theta}{2}\right)\cos^2\left(\frac{\Delta\theta}{2}\right)}{1+\sin^2\left(\frac{\Delta\theta}{2}\right)}\cos\left(\frac{\Delta\theta}{2}\right)+\cos\left(\frac{3}{2}\Delta\theta\right)\right)^2,\ \mathbf{f_{II}} = -\frac{\sin\Delta\theta}{\Delta\theta}\left(\frac{1-\sin^2\left(\frac{\Delta\theta}{2}\right)\cos^2\left(\frac{\Delta\theta}{2}\right)}{1+\sin^2\left(\frac{\Delta\theta}{2}\right)}\sin\left(\frac{\Delta\theta}{2}\right)+\sin\left(\frac{3}{2}\Delta\theta\right)\right)^2$$





↘ CONCLUSIONS

Conclusions

- ➔ $f_{\text{straight crack}}\left(\Delta\theta\right)$: $\checkmark G_{I}$, $\times$ G_{II}

  $f_{\text{inclined crack}}\left(\Delta\theta\right)$: $\checkmark G_{I}$, $\checkmark G_{II}$, $\times$ $\nexists$ $f_{\text{inclined crack}}\left(\Delta\theta=\frac{\pi}{2}\right)$

  $f_{\text{curved crack}}\left(\Delta\theta\right)$: $\checkmark G_{I}$, $\checkmark G_{II}$

- ➔ scaling breaks for $\Delta\theta\leq 20^{\circ}\rightarrow$ microstructure is important for small debonds!

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