Finite Element solution of the fiber/matrix interface crack problem: convergence properties and mode mixity of the Virtual Crack Closure Technique

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Abstract

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1. Introduction

Bi-material interfaces represent the basic load transfer mechanism at the heart of Fiber Reinforced Polymer Composite (FRPC) materials. They are present at the macroscale, in the form of adhesive joints; at the mesoscale, as interfaces between layers with different orientations; at the microscale, as fiber-matrix interfaces. Bi-material interfaces have for long attracted the attention of researchers in Fracture Mechanics [1, 2], due to their hidden complexity.
The problem was first addressed in the 1950's by Williams [3], who derived through a linear elastic asymptotic analysis the stress distribution around an *open* crack (with crack faces nowhere in contact for any size of the crack) between two infinite half-planes of dissimilar materials and found the existence of a strong oscillatory behavior in the stress singularity at the crack tip of the form

$$r^{-\frac{1}{2}} \sin\left(\varepsilon \log r\right) \quad \text{with} \quad \varepsilon = \frac{1}{2\pi} \log\left(\frac{1-\beta}{1+\beta}\right); \tag{1}$$

in which β is one of the two parameters introduced by Dundurs [4] to characterize bi-material interfaces:

$$\beta = \frac{\mu_2 (\kappa_1 - 1) - \mu_1 (\kappa_2 - 1)}{\mu_2 (\kappa_1 + 1) + \mu_1 (\kappa_2 + 1)} \tag{2}$$

where $\kappa = 3 - 4\nu$ in plane strain and $\kappa = \frac{3-4\nu}{1+\nu}$ in plane stress, μ is the shear modulus, ν Poisson's coefficient, and indexes $1, 2$ refer to the two bulk materials joined at the interface. Defining a as the length of the crack, it was found that the size of the oscillatory region is in the order of $10^{-6}a$ [5]. Given the oscillatory behaviour of the crack tip singularity of the stress field of Eq. 1, the definition of Stress Intensity Factor (SIF) $\lim_{r\to 0} \sqrt{2\pi r}\sigma$ ceases to be valid as it returns logarithmically infinite terms [1]. Furthermore, it implies that the Mode mixity problem at the crack tip is ill-posed.

It was furthermore observed, always in the context of Linear Elastic Fracture Mechanics (LEFM), that an interpenetration zone exists close to the crack tip [6, 7] with a length in the order of 10^{-4} [6]. Following conclusions firstly proposed in[7], the presence of a *contact zone* in the crack tip neighborhood, of a length to be determined from the solution of the elastic problem, was introduced in [8] and shown to provide a physically consistent solution to the straight bi-material interface crack problem.

The curved bi-material interface crack, more often refered to as the fiber-matrix interface crack (or debond) due to its relevance in FRPCs, was first treated by England [9] and by Perlman and Sih [10], who provided the analytical solution of stress and displacement fields for a circular inclusion with respectively a single debond and an arbitrary number of debonds. Building on their work, Toya [11] particularized the solution and provided the expression of the Energy Release Rate (ERR) at the crack tip. The same problems exposed previously for the *open* straight bi-material were shown to exist also for the *open* fiber-matrix interface crack: the presence of strong oscillations in the crack tip singularity and crack face interpenetration after a critical initial flaw size.[1]

[1]For the fiber-matrix interface crack, flaw size is measured in terms of the angle $\Delta\theta$ sub-

In order to treat cases more complex than the single partially debonded fiber in an infinite matrix of [9, 10, 11], numerical studies followed. In the 1990's, París and collaborators [12] developed a Boundary Element Method (BEM) with the use of discontinuous singular elements at the crack tip and the Virtual Crack Closure Technique (VCCT) [13, 14] for the evaluation of the Energy Release Rate (ERR). They validated their results [12] with respect to Toya's analytical solution [11] and analyzed the effect of BEM interface discretization on the stress field in the neighborhood of the crack tip [15]. Following Comninou's work on the straight crack [8], they furthermore recognized the importance of contact to retrieve a physical solution avoiding interpenetration [12] and studied the effect of the contact zone on debond ERR [16]. Their algorithm was then applied to investigate the fiber-matrix interface crack under different geometrical configurations and mechanical loadings [17, 18, 19, 20, 21, 22, 23].

2. Vectorial formulation of the Virtual Crack Closure Technique (VCCT)

2.1. Foundational relations

2.2. Formulation of the ERR with respect to FEM variables

3. Convergence analysis

3.1. Analytical considerations

3.2. Numerical results

4. Conclusions & Outlook

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tended by half of the arc-crack, i.e. $a = 2\Delta\theta$.

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