

# Stiffness reduction in UD and cross-ply laminates due to fiber/matrix interface cracks

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## Abstract

**Keywords:** Fiber Reinforced Polymer Composite (FRPC), Debonding, Finite element analysis (FEA)

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## 1. Introduction

## 2. Derivation of constitutive relations

$$\beta_{ij} = \frac{1}{V_k} \int_{S_C} \frac{1}{2} (u_i n_j + u_j n_i) dS \quad (1)$$

$$\beta_{11} = \frac{1}{V_k} \int_{S_C} \frac{1}{2} (u_1 n_1 + u_1 n_1) dS = \frac{1}{V_k} \int_{S_C} u_1 n_1 dS = 0 \quad (2)$$

$$\beta_{22} = \frac{1}{V_k} \int_{S_C} \frac{1}{2} (u_2 n_2 + u_2 n_2) dS = \frac{1}{V_k} \int_{S_C} u_2 n_2 dS \quad (3)$$

$$\begin{aligned} \beta_{22} &= \frac{1}{V_k} \int_{S_C} u_2 n_2 dS = \frac{1}{V_k} \left[ \int_{S_C^m} u_2^m n_2^m dS + \int_{S_C^f} u_2^f n_2^f dS \right] = \\ &= \frac{1}{V_k} \left[ \int_{S_C^m} u_2^m n_2^m dS + \int_{S_C^f} u_2^f (-n_2^m) dS \right] \end{aligned} \quad (4)$$

$$S_C^m = S_C^f = R_f \Delta \theta \quad (5)$$

$$\begin{aligned}
\beta_{22} &= \frac{1}{V_k} \left[ R_f w \int_0^{\Delta\theta} (u_2^m - u_2^f) n_2^m d\theta \right] = \\
&= \frac{1}{L w t_{90^\circ}} \left[ R_f w \int_0^{\Delta\theta} (u_2^m - u_2^f) n_2^m d\theta \right] = \\
&= \frac{1}{L} \frac{R_f}{t_{90^\circ}} \left[ \int_0^{\Delta\theta} (u_2^m - u_2^f) n_2^m d\theta \right]
\end{aligned} \tag{6}$$

$$\begin{aligned}
u_2^m - u_2^f &= (u_r^m - u_r^f) \cos(\theta) - (u_\theta^m - u_\theta^f) \sin(\theta) = \\
&= COD(\theta) \cos(\theta) - CSD(\theta) \sin(\theta)
\end{aligned} \tag{7}$$

$$\begin{aligned}
u_3^m - u_3^f &= (u_r^m - u_r^f) \sin(\theta) + (u_\theta^m - u_\theta^f) \cos(\theta) = \\
&= COD(\theta) \sin(\theta) + CSD(\theta) \cos(\theta)
\end{aligned} \tag{8}$$

$$\begin{aligned}
n_2^m &= \cos(\theta) - \sin(\theta) \\
n_3^m &= \sin(\theta) + \cos(\theta)
\end{aligned} \tag{9}$$

$$\beta_{22} = \frac{1}{L} \frac{R_f}{t_{90^\circ}} \left[ \int_0^{\Delta\theta} (COD(\theta) (\sin(\theta) \cos(\theta) - \sin^2(\theta)) + CSD(\theta) (\cos^2(\theta) - \cos(\theta) \sin(\theta))) d\theta \right] \tag{10}$$

$$\begin{aligned}
COD(\theta) &= COD_{avg} + \delta COD(\theta) = COD_{avg} + COD_{max} f(\theta) \\
CSD(\theta) &= CSD_{avg} + \delta CSD(\theta) = CSD_{avg} + CSD_{max} g(\theta)
\end{aligned} \tag{11}$$

$$\int_0^{\Delta\theta} f(\theta) d\theta = 0 \quad \int_0^{\Delta\theta} g(\theta) d\theta = 0 \tag{12}$$