

Finite Element solution of the fiber/matrix interface crack problem: convergence properties and mode mixity of the Virtual Crack Closure Technique

Luca Di Stasio^{a,b}, Janis Varna^b, Zoubir Ayadi^a

^aUniversité de Lorraine, EEIGM, IJL, 6 Rue Bastien Lepage, F-54010 Nancy, France

^bLuleå University of Technology, University Campus, SE-97187 Luleå, Sweden

Abstract

Priority: 3

Target journal(s): Engineering Fracture Mechanics, Theoretical and Applied Fracture Mechanics, International Journal of Fracture

1. Introduction

Bi-material interfaces represent the basic load transfer mechanism at the heart of Fiber Reinforced Polymer Composite (FRPC) materials. They are present at the macroscale, in the form of adhesive joints; at the mesoscale, as
5 interfaces between layers with different orientations; at the microscale, as fiber-matrix interfaces. Bi-material interfaces have for long attracted the attention of researchers in Fracture Mechanics [1, 2], due to their hidden complexity. The problem was first addressed in the 1950's by Williams [3], who derived through a linear elastic asymptotic analysis the stress distribution around an
10 *open* crack (with crack faces nowhere in contact for any size of the crack) between two infinite half-planes of dissimilar materials and found the existence of a strong oscillatory behavior in the stress singularity at the crack tip of the form

$$r^{-\frac{1}{2}} \sin(\varepsilon \log r) \quad \text{with} \quad \varepsilon = \frac{1}{2\pi} \log \left(\frac{1-\beta}{1+\beta} \right); \quad (1)$$

in which β is one of the two parameters introduced by Dundurs [4] to characterize bi-material interfaces:

$$\beta = \frac{\mu_2(\kappa_1 - 1) - \mu_1(\kappa_2 - 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)} \quad (2)$$

15 where $\kappa = 3 - 4\nu$ in plane strain and $\kappa = \frac{3-4\nu}{1+\nu}$ in plane stress, μ is the shear modulus, ν Poisson's coefficient, and indexes 1,2 refer to the two bulk materials joined at the interface. Defining a as the length of the crack, it was found that the size of the oscillatory region is in the order of $10^{-6}a$ [5]. Given the oscillatory behaviour of the crack tip singularity of the stress field of Eq. 1, 20 the definition of Stress Intensity Factor (SIF) $\lim_{r \rightarrow 0} \sqrt{2\pi r} \sigma$ ceases to be valid as it returns logarithmically infinite terms [1]. Furthermore, it implies that the Mode mixity problem at the crack tip is ill-posed.

It was furthermore observed, always in the context of Linear Elastic Fracture Mechanics (LEFM), that an interpenetration zone exists close to the crack tip [6, 25 7] with a length in the order of 10^{-4} [6]. Following conclusions firstly proposed in [7], the presence of a *contact zone* in the crack tip neighborhood, of a length to be determined from the solution of the elastic problem, was introduced in [8] and shown to provide a physically consistent solution to the straight bi-material interface crack problem.

30 The curved bi-material interface crack, more often referred to as the fiber-matrix interface crack (or debond) due to its relevance in FRPCs, was first treated by England [9] and by Perlman and Sih [10], who provided the analytical solution of stress and displacement fields for a circular inclusion with respectively a single debond and an arbitrary number of debonds. Building on their work, Toya [11] 35 particularized the solution and provided the expression of the Energy Release Rate (ERR) at the crack tip. The same problems exposed previously for the *open* straight bi-material were shown to exist also for the *open* fiber-matrix interface crack: the presence of strong oscillations in the crack tip singularity and crack face interpenetration after a critical initial flaw size.¹

¹For the fiber-matrix interface crack, flaw size is measured in terms of the angle $\Delta\theta$ sub-

40 In order to treat cases more complex than the single partially debonded fiber in
 an infinite matrix of [9, 10, 11], numerical studies followed. In the 1990's, París
 and collaborators [12] developed a Boundary Element Method (BEM) with the
 use of discontinuous singular elements at the crack tip and the Virtual Crack
 Closure Technique (VCCT) [13, 14] for the evaluation of the Energy Release
 45 Rate (ERR). They validated their results [12] with respect to Toya's analytical
 solution [11] and analyzed the effect of BEM interface discretization on the
 stress field in the neighborhood of the crack tip [15]. Following Comninou's
 work on the straight crack [8], they furthermore recognized the importance of
 contact to retrieve a physical solution avoiding interpenetration [12] and studied
 50 the effect of the contact zone on debond ERR [16]. Their algorithm was then
 applied to investigate the fiber-matrix interface crack under different geometrical
 configurations and mechanical loadings [17, 18, 19, 20, 21, 22, 23].
 Recently the Finite Element Method (FEM) was also applied to the solution
 of the fiber-matrix interface crack problem [24, 25, 26], in conjunction with the
 55 VCCT for the evaluation of the ERR at the crack tip. In [24], the authors
 validated their model with respect to the BEM results of [12], but no analysis of
 the effect of the discretization in the crack tip neighborhood comparable to [15]
 was proposed.

tended by half of the arc-crack, i.e. $a = 2\Delta\theta$.

2. Vectorial formulation of the Virtual Crack Closure Technique (VCCT)

60 2.1. *Foundational relations*

2.2. *Formulation of the ERR with respect to FEM variables*

3. Convergence analysis

3.1. *Analytical considerations*

3.2. *Numerical results*

65 4. **Conclusions & Outlook**

Acknowledgements

Luca Di Stasio gratefully acknowledges the support of the European School of Materials (EUSMAT) through the DocMASE Doctoral Programme and the European Commission through the Erasmus Mundus Programme.

70 References

- [1] M. Comninou, An overview of interface cracks, *Engineering Fracture Mechanics* 37 (1) (1990) 197–208. doi:10.1016/0013-7944(90)90343-f.
- [2] D. Hills, J. Barber, Interface cracks, *International Journal of Mechanical Sciences* 35 (1) (1993) 27–37. doi:10.1016/0020-7403(93)90062-y.
- 75 [3] M. L. Williams, The stresses around a fault or crack in dissimilar media, *Bulletin of the Seismological Society of America* 49 (2) (1959) 199.
- [4] J. Dundurs, Discussion: “edge-bonded dissimilar orthogonal elastic wedges under normal and shear loading” (bogy, d. b., 1968, *ASME j. appl. mech.*, 35, pp. 460–466), *Journal of Applied Mechanics* 36 (3) (1969) 650. doi:10.1115/1.3564739.
- 80 [5] F. Erdogan, Stress distribution in a nonhomogeneous elastic plane with cracks, *Journal of Applied Mechanics* 30 (2) (1963) 232. doi:10.1115/1.3636517.

- [6] A. H. England, A crack between dissimilar media, *Journal of Applied Mechanics* 32 (2) (1965) 400. doi:10.1115/1.3625813.
- [7] B. Malyshev, R. Salganik, The strength of adhesive joints using the theory of cracks, *International Journal of Fracture Mechanics* 1-1 (2). doi:10.1007/bf00186749.
URL <https://doi.org/10.1007/bf00186749>
- [8] M. Comninou, The interface crack, *Journal of Applied Mechanics* 44 (4) (1977) 631. doi:10.1115/1.3424148.
URL <https://doi.org/10.1115/1.3424148>
- [9] A. H. England, An arc crack around a circular elastic inclusion, *Journal of Applied Mechanics* 33 (3) (1966) 637. doi:10.1115/1.3625132.
- [10] A. Perlman, G. Sih, Elastostatic problems of curvilinear cracks in bonded dissimilar materials, *International Journal of Engineering Science* 5 (11) (1967) 845–867. doi:10.1016/0020-7225(67)90009-2.
- [11] M. Toya, A crack along the interface of a circular inclusion embedded in an infinite solid, *Journal of the Mechanics and Physics of Solids* 22 (5) (1974) 325–348. doi:10.1016/0022-5096(74)90002-7.
- [12] F. París, J. C. Caño, J. Varna, The fiber-matrix interface crack — a numerical analysis using boundary elements, *International Journal of Fracture* 82 (1) (1996) 11–29. doi:10.1007/bf00017861.
- [13] G. R. Irwin, Fracture, in: *Elasticity and Plasticity / Elastizität und Plastizität*, Springer Berlin Heidelberg, 1958, pp. 551–590. doi:10.1007/978-3-642-45887-3_5.
- [14] R. Krueger, Virtual crack closure technique: History, approach, and applications, *Applied Mechanics Reviews* 57 (2) (2004) 109. doi:10.1115/1.1595677.

- 110 [15] J. C. D. Caño, F. París, On stress singularities induced by the discretization in curved receding contact surfaces: a bem analysis, *International Journal for Numerical Methods in Engineering* 40 (12) (1997) 2301–2320. doi:10.1002/(sici)1097-0207(19970630)40:12<2301::aid-nme166>3.0.co;2-8.
- 115 [16] J. Varna, F. París, J. C. Caño, The effect of crack-face contact on fiber/matrix debonding in transverse tensile loading, *Composites Science and Technology* 57 (5) (1997) 523–532. doi:10.1016/s0266-3538(96)00175-3.
- [17] F. París, E. Correa, V. Mantič, Kinking of transversal interface cracks
120 between fiber and matrix, *Journal of Applied Mechanics* 74 (4) (2007) 703. doi:10.1115/1.2711220.
- [18] E. Correa, E. Gamstedt, F. París, V. Mantič, Effects of the presence of compression in transverse cyclic loading on fibre–matrix debonding in unidirectional composite plies, *Composites Part A: Applied Science and Manufacturing* 38 (11) (2007) 2260–2269. doi:10.1016/j.compositesa.2006.11.002.
125
- [19] E. Correa, V. Mantič, F. París, Effect of thermal residual stresses on matrix failure under transverse tension at micromechanical level: A numerical and experimental analysis, *Composites Science and Technology* 71 (5) (2011) 622–629. doi:10.1016/j.compscitech.2010.12.027.
130
- [20] E. Correa, F. París, V. Mantič, Effect of the presence of a secondary transverse load on the inter-fibre failure under tension, *Engineering Fracture Mechanics* 103 (2013) 174–189. doi:10.1016/j.engfracmech.2013.02.026.
- [21] E. Correa, F. París, V. Mantič, Effect of a secondary transverse load on the inter-fibre failure under compression, *Composites Part B: Engineering* 65 (2014) 57–68. doi:10.1016/j.compositesb.2014.01.005.
135

- [22] C. Sandino, E. Correa, F. París, Numerical analysis of the influence of a nearby fibre on the interface crack growth in composites under transverse tensile load, *Engineering Fracture Mechanics* 168 (2016) 58–75. doi:10.1016/j.engfracmech.2016.01.022.
- [23] C. Sandino, E. Correa, F. París, Interface crack growth under transverse compression: nearby fibre effect, in: *Proceeding of the 18th European Conference on Composite Materials (ECCM-18)*, 2018.
- [24] L. Zhuang, A. Pupurs, J. Varna, R. Talreja, Z. Ayadi, Effects of inter-fiber spacing on fiber-matrix debond crack growth in unidirectional composites under transverse loading, *Composites Part A: Applied Science and Manufacturing* 109 (2018) 463–471. doi:10.1016/j.compositesa.2018.03.031.
- [25] J. Varna, L. Q. Zhuang, A. Pupurs, Z. Ayadi, Growth and interaction of debonds in local clusters of fibers in unidirectional composites during transverse loading, *Key Engineering Materials* 754 (2017) 63–66. doi:10.4028/www.scientific.net/kem.754.63.
- [26] L. Zhuang, R. Talreja, J. Varna, Transverse crack formation in unidirectional composites by linking of fibre/matrix debond cracks, *Composites Part A: Applied Science and Manufacturing* 107 (2018) 294–303. doi:10.1016/j.compositesa.2018.01.013.