

Energy release rate of the fiber/matrix interface crack in UD composites under transverse loading: debond-debond and debond-free boundary interactions

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Abstract

The effects of crack shielding, finite thickness of the composite and fiber content on fiber/matrix debond growth in thin unidirectional composites are investigated analyzing Representative Volume Elements (RVEs) of different ordered microstructures. Debond growth is characterized by estimation of the Energy Release Rates (ERRs) in Mode I and Mode II using the Virtual Crack Closure Technique (VCCT) and the J-integral. It is found that increasing fiber content, larger distance between debonds in the loading direction and the presence of a free surface close to the debond have all a strong enhancing effect on the ERR. The presence of fully bonded fibers in the composite thickness direction has instead a constraining effect, and it is shown to be very localized. An explanation of these observations is proposed based on mechanical considerations.

Keywords: Polymer-matrix Composites (PMCs), Thin-ply, Transverse Failure, Debonding, Finite Element Analysis (FEA)

1. Introduction

Stimulated by the ever more stringent requirements in terms of weight and mechanical performances of the aerospace industry, in recent years the composite community has returned its attention to the mechanisms of intralaminar crack initiation in general and to initiation of multiple cracking in thin-ply laminates in particular. Alternative design approaches are now considered based on

this non-conventional laminate in applications ranging from cryogenic pressure vessels [1], to airplanes' wings [2], and even reusable space launchers [3].

Thin-ply laminates are the result of a technological innovation, the *spread tow*
10 *technology*, which consists in opening or spreading the tows in which fibers (carbon, glass, aramid, basalt among others) are usually shipped in into very thin tapes then used for laminate production. Ply thicknesses of less than $50\ \mu m$ can nowadays be mass-produced, and record thicknesses of around $20 - 25\ \mu m$, or $\sim 4 - 5$ times the average fiber's diameter, have been achieved. The technique
15 in its current form, sometimes referred to as "FUKUI method" from the name of the Japanese prefecture it originated in, was firstly proposed towards the end of the 1990s [4] and perfected in the subsequent decade [5, 6].

Several experimental investigations on *thin ply* laminates have highlighted their main properties [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]: increased fiber content; more uniform packing of fibers; delay and even suppression of intralaminar
20 cracking (called also transverse-, matrix- or micro-cracking) and delamination. A very insightful work documenting how these mechanisms are affected by the morphology of *thin ply* laminates is the microscopic study of Saito & al. [20], which focuses on the effect of ply thickness on the onset and propagation of
25 intralaminar cracking. In their investigation, tensile tests were performed on carbon fiber/epoxy $[0_2, 90_n, 0_2]$ *thin-ply* laminates for $n = 1, 2, 4$ and the crack density was measured at several levels of applied tensile strain in the range between 0% and 1.5%. Furthermore, they performed microscopic observations on the specimen's edge at each level of strain. They observed the onset of
30 fiber/matrix interface cracks (referred to as debonds in the following) at lower levels of strain in thinner plies, while at the same time coalescence of debonds and through-the-thickness propagation of transverse cracks in thin plies were delayed and even suppressed as ply thickness decreased. In particular, they reported the first onset of debonds at 0.4% for $n = 1, 2$ and 0.7% for $n = 4$. For
35 $n = 1$, however, at $\varepsilon = 1.5\%$ coalescence of debonds had started to take place but the crack had not completely propagated through the thickness, while for $n = 2$ and $n = 4$ the latter already happened at a value of strain respectively

of 1.3% and 1%. Our inability to explain these observations with the currently accumulated knowledge demonstrates the necessity of further investigation of
40 interactions between debonds and studies of the constraining (or accelerating) effect of presence of bonded fibers, free and constrained boundaries in the vicinity of a partially debonded fiber.

Early studies on the effect of ply thickness on the onset and propagation of transverse cracks were conducted on glass fiber/epoxy cross-ply laminates by Bailey,
45 Parvizi and collaborators [21, 22, 23], who firstly observed the beneficial effect of thickness reduction on the delay of transverse cracking. They furthermore pointed the attention to the appearance of debonds at the fiber/matrix interface and their subsequent coalescence as the mechanism at the origin of transverse cracks [24]. Moreover, they identified the main mechanical driver of the damage
50 process in the mismatch of elastic properties, and particularly of Poisson's ratios, between fibers and matrix [25]. A full understanding of damage onset and propagation in *thin-ply* laminates thus requires the comprehension of the mechanisms governing its very first stage, i.e. the fiber/matrix interface crack. First results were obtained through analytical models in the case of a single fiber with
55 an arc crack (debond) in an infinite matrix under transverse tension by England [26] and Perlman & Sih [27], who obtained the stresses at the interface and calculated the stress intensity factors at the crack tip, and by Toya [28], who evaluated the Energy Release Rate (ERR). Drawing upon the results for the straight bi-material interface crack by Comninou [29], the effect of crack face contact in
60 fiber-matrix debonding was investigated in [30, 31]. In [32], it was showed in terms of ERR why the case of a single asymmetric debond is more likely to be observed under remote transverse tension than two symmetric debonds on the same fiber. The effect of different types and combinations of loads on debonding have been studied for the single fiber model: compression [33], residual
65 thermal stresses [34], biaxial tension-tension and tension-compression [35], biaxial compression-compression and compression-tension [36]. The effect of the presence of nearby bonded fibers on the debonding of a fiber embedded in an infinite matrix has been studied under uniaxial transverse tension [37], biaxial

tension [38] and uniaxial transverse compression [39]. The effect of inter-fiber
70 distance on debond growth has been studied for a partially debonded fiber at the
center of a hexagonal cluster inside a homogenized UD composite in the case
of fully bonded neighbouring fibers [40] and of two partially debonded fibers
out of the surrounding six [41]. An understanding of crack shielding and finite
thickness effects on debond growth in non-homogenized microstructural models
75 of UD composites seems thus to be lacking: this is the problem that we want to address
in the present work. Mode I and Mode II energy release rates will be analyzed
using stress fields calculated with the FEM for a variety of Repeating Unit Cell
(RUC) of the composite with square packing of fibers under transverse ten-
sile loading. These RUCs represent composites with different distance between
80 partially debonded fibers in the transverse direction which allows to study the
effect of crack shielding on the ERR. In the ply thickness direction, the varying
number of perfectly bonded fiber rows exposes the effect of the free boundary of
the composite on debond growth. Finally, using coupling of thickness direction
displacements on horizontal boundaries of the RUC, the accelerating effect of
85 the interaction between debonds of fibers located on the same vertical line is
studied.

2. RVE models & FE discretization

2.1. Introduction & Nomenclature

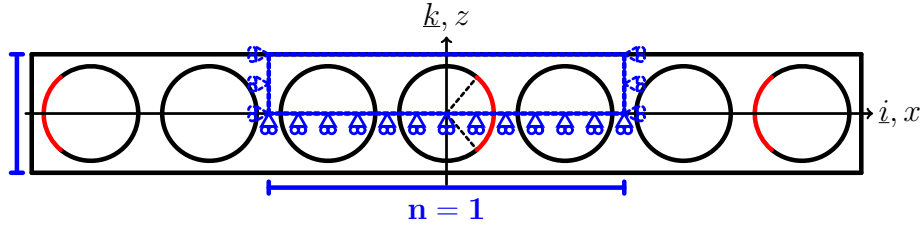
In this paper, we analyze debond development in unidirectional (UD) com-
90 posites subjected to in-plane transverse tensile loading. The interaction between
debonds in UD composites is studied developing models of different Repeating
Unit Cells (RUC) of laminates where only the central fiber in the cell has a
damage in the form of a fiber/matrix interface crack (debond). The composite
RUC may be repeating in the in-plane transverse direction only (representing
95 an ultra-thin composite) or repeating also in the composite thickness direction,
representing an infinite composite in a limiting case. Thus, the conditions at
the UD composite's upper and lower boundaries are one of the parameters for

the investigation. The used RUCs allow for considering the composite with debonds as a sequence of stacked damaged and undamaged rows, each row with only one fiber in the thickness direction. Since all of these RUCs feature regular microstructures with fibers placed according to a square-packing configuration, they are Representative Volume Elements (RVE) of composites with a certain distribution of debonds. Introducing in-plane coordinates x and y , where x is in the transverse direction of the UD composite under consideration, the strain in the y -direction due to a load in the x -direction is small, caused in turn by the very small minor Poisson's ratio of the UD composite. Additionally, debonds are considered to be significantly longer in the fiber direction than in the arc direction. Therefore, we use 2D models under the assumption of plane strain, defined in the $x - z$ section of the composite. Thus, the analysis presented applies to long debonds, with a focus on understanding the mechanisms of growth along their arc direction. The composites are subjected to transverse tensile strain, applied as a constant displacement in the x -direction along the vertical boundary of the RUC as shown in Figure 1 to 4. As the models are differentiated by the number of rows of fibers and by the spacing between debonds along the vertical and horizontal directions, the corresponding RUCs can be distinguished from each other based on the number n of fibers in the horizontal direction and k in the vertical direction. Furthermore, the horizontal surfaces can be either free or vertical displacement coupling can be applied. We thus introduce the common notation $n \times k - free$ and $n \times k - coupling$ to denote a RUC with $n \times k$ fibers and, respectively, a free upper surface or kinematic coupling applied to it. The specific combinations of particular choices of n , k , and boundary conditions are detailed in Section 2.2, together with the description of the corresponding models of damaged composite they are representing.

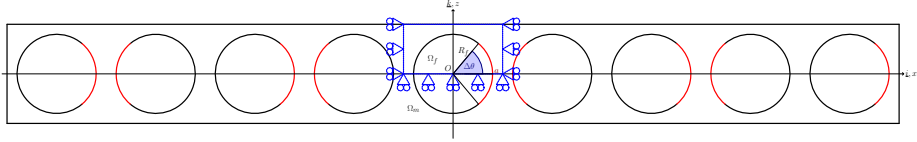
2.2. Models of Representative Volume Element (RVE)

The first two models feature, as shown in Fig. 1, an ultra-thin UD laminate with only one row of fibers across its thickness, $k = 1$. This is quite an extreme model from the microstructural point of view; however, it allows to

focus the analysis on the interaction between debonded fibers placed along the x-direction. Furthermore, as the horizontal surfaces are considered free, the interaction is stronger in this case than in any other, making the trends very clear and the predictions of this model rather conservative. In retrospective, if only 20 years ago such a model would have been considered too abstracted from the physical reality, the recent advancements in the spread tow technology make this approach appealing also as a limiting case for practical considerations.



(a) Single row of fibers with a debond appearing every n fibers: model $n \times 1 - free$ ($n = 3$ in the figure).

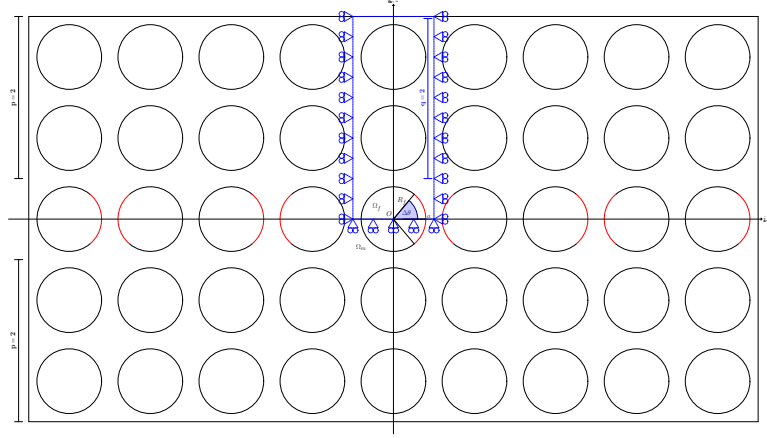


(b) Single row of fibers with debonds appearing on each fiber: model $1 \times 1 - free$.

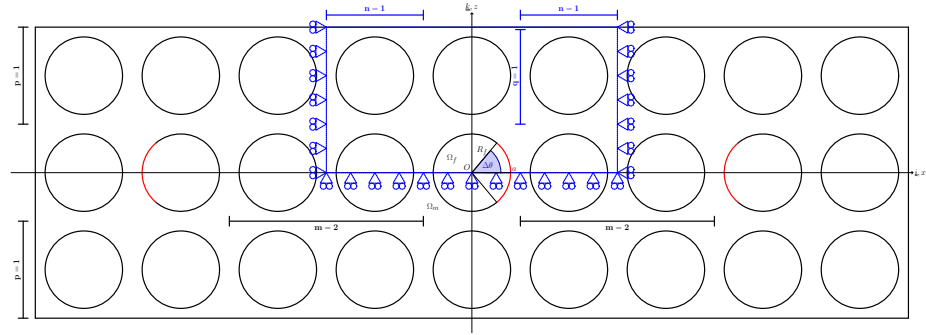
Figure 1: Models of ultra-thin UD composites with a single “row” of fibers and debonds repeating at different distances. The corresponding repeating element (RUC) is highlighted in blue, while debonds are represented in red.

In the sub-model of Fig. 1a, every n^{th} fiber in the composite is partially debonded on alternating sides of the fiber. The symmetries of the model allow the use of the upper part of the RUC. It is highlighted by blue lines in Fig. 1 to 3. Following the notation introduced in Section 2.1, we will refer to this model as $n \times 1 - free$. In the sub-model $n = 1$, Fig. 1b, a debond appears on each fiber on alternating sides and the corresponding RUC contains only one fiber. We will refer to this model as $1 \times 1 - free$.

The second set of models in Fig. 2 and Fig. 3 considers laminates with



(a) Multiple rows of fibers with debonds appearing on each fiber belonging to the central row: model $1 \times k - free$ ($k = 3$ in the figure).



(b) Multiple rows of fibers with a debond appearing every n fibers within the central row: model $n \times k - free$ ($n = 3$ and $k = 3$ in the figure).

Figure 2: Models of UD composites with different “rows” of fibers and debonds repeating at different distances. The corresponding repeating element (RUC) is highlighted in blue, while debonds are represented in red.

multiple rows of fibers across the thickness: a finite number of rows in the first two sub-models in Fig. 2; an infinite number in the model of Fig. 3. In Fig. 2a, the RUC contains $n = 1$ fiber in the x-direction, k fibers across the thickness and the central fiber is debonded. This model will be referred to in the following as $1 \times k - free$. Thinking in terms of rows, in this model we have a central row where each fiber is debonded. This row is surrounded from each side by $(k-1)/2$

rows with perfectly bonded fibers. In the sub-model in Fig. 2b, each n^{th} fiber
 150 in the central row is debonded and this row is surrounded by $(k-1)/2$ rows of
 undamaged fibers from each side. We will refer to this model as $n \times k - free$
 (because the horizontal boundary of the RUC is free of any constraint).

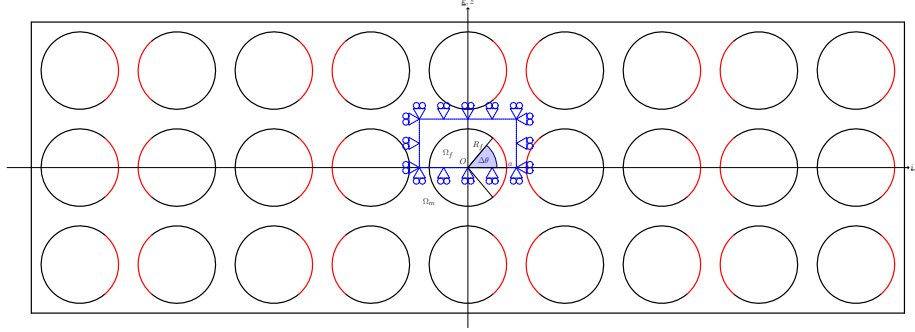


Figure 3: Model of UD composites with an infinite number of “rows” of fibers and debonds
 appearing on each fiber: model $1 \times 1 - coupling$. The corresponding repeating element (RUC)
 is highlighted in blue, while debonds are represented in red.

Finally, the model in Fig. 3 represents an UD composite with an infinite
 number of rows; all of them with partially debonded fibers. As all fibers have
 155 debonds, the corresponding RUC is made of a single partially debonded fiber
 with kinematic coupling conditions applied to the upper boundary to assure
 periodicity. This model is referred to as $1 \times 1 - coupling$.

2.3. Finite Element (FE) discretization

Each RUC is discretized using the Finite Element Method (FEM) within
 160 the Abaqus environment, a commercial FEM package [42]. The length l and
 height h of the model are determined by the number of fibers n in the horizontal
 direction and k across the thickness (see 2.2) according to Eq. 1:

$$l = 2nL \quad h = 2kL; \quad (1)$$

where L is the length of a one-fiber unit, see Fig. 4a, defined as a function
 of the fiber volume fraction V_f and the fiber radius according to

$$L = \frac{R_f}{2} \sqrt{\frac{\pi}{V_f}}. \quad (2)$$

165 The fiber radius R_f is assumed to be the same for each fiber in the model and equal to $1 \mu m$. The latter value is not physical and it has been chosen for simplicity. It is worth to note at this point that, in a linear elastic solution as the one presented here, the ERR is proportional to the geometrical dimensions and recalculation of the ERR for fibers of any size, thus, requires a simple
170 multiplication. Furthermore, notice that the relationships in Eqs. 1 and 2 ensure that the local and global V_f are everywhere equal.

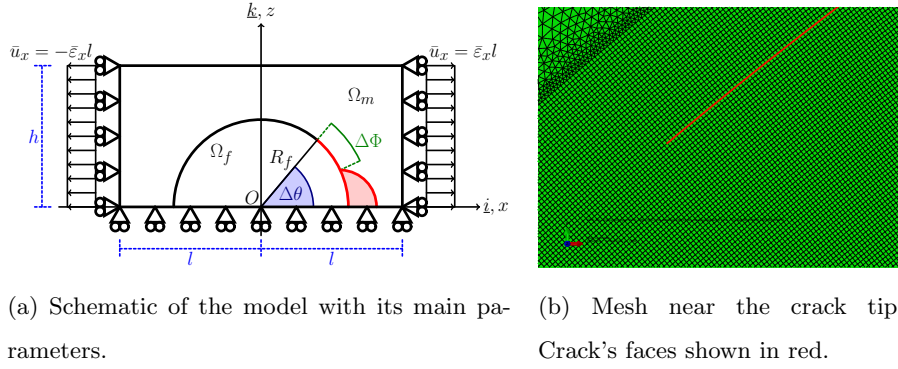


Figure 4: Details and main parameters of the Finite Element model.

The debond is placed symmetrically with respect to the x axis (in red in 4a) and has an angular size of $\Delta\theta$ (the full debond's size is thus $2\Delta\theta$). For large debond's sizes ($\geq 60^\circ - 80^\circ$), a region of variable size $\Delta\Phi$ appears at the crack
175 tip in which the crack faces are in contact and slide on each other. Due to its appearance, frictionless contact is considered between the two crack's faces to allow free sliding and avoid interpenetration. Symmetry with respect to the x axis is applied on the lower boundary and kinematic coupling on the x -displacement along the left and right sides. The upper boundary is in general free, except
180 for the model $1 \times 1 - coupling$ (Fig. 3) which requires kinematic coupling of vertical displacements also on the upper side. Constant x -displacement $\pm\bar{\varepsilon}l$, corresponding to transverse strain $\bar{\varepsilon}$ equal to 1% is applied to the right and left

boundaries.

Table 1: Summary of the mechanical properties of fiber and matrix.

Material	E [GPa]	G [GPa]	ν [-]
Glass fiber	70.0	29.2	0.2
Epoxy	3.5	1.25	0.4

The model is meshed using second order, 2D, plane strain triangular (CPE6) and rectangular (CPE8) elements. A regular mesh of quadrilateral elements with an almost unitary aspect ratio is required at the crack tip, as shown in Fig. 4b. The angular size δ of an element in the crack tip region is always equal to 0.05° . The crack faces are modeled as element-based surfaces and a small-sliding contact pair interaction with no friction is established between them. The Mode I, Mode II and total Energy Release Rates (ERRs) (respectively referred to as G_I , G_{II} and G_{TOT}) represent the main output of the FEM analysis; they are evaluated using the VCCT technique [43] implemented in a custom Python routine and, for the total ERR, the J-integral [44] by application of the Abaqus built-in functionality. A glass fiber-epoxy system is considered in every model, and it is assumed that their response lies always in the linear elastic domain. The properties used are listed in Table 1.

2.4. Validation of the model

The model is validated in Fig. 5 against the results reported in [45, 37], obtained with the Boundary Element Method (BEM) for a single fiber with a symmetric debond placed in an infinite matrix. This situation is modeled using the *free* RVE with $V_f = 0.0079\%$, which corresponds to a RUC's length and height of ~ 100 .

To allow for a comparison, the results are normalized following [37] with respect to a reference Energy Release Rate G_0 defined as

$$G_0 = \frac{1 + k_m}{8\mu_m} \sigma_0^2 \pi R_f \quad (3)$$

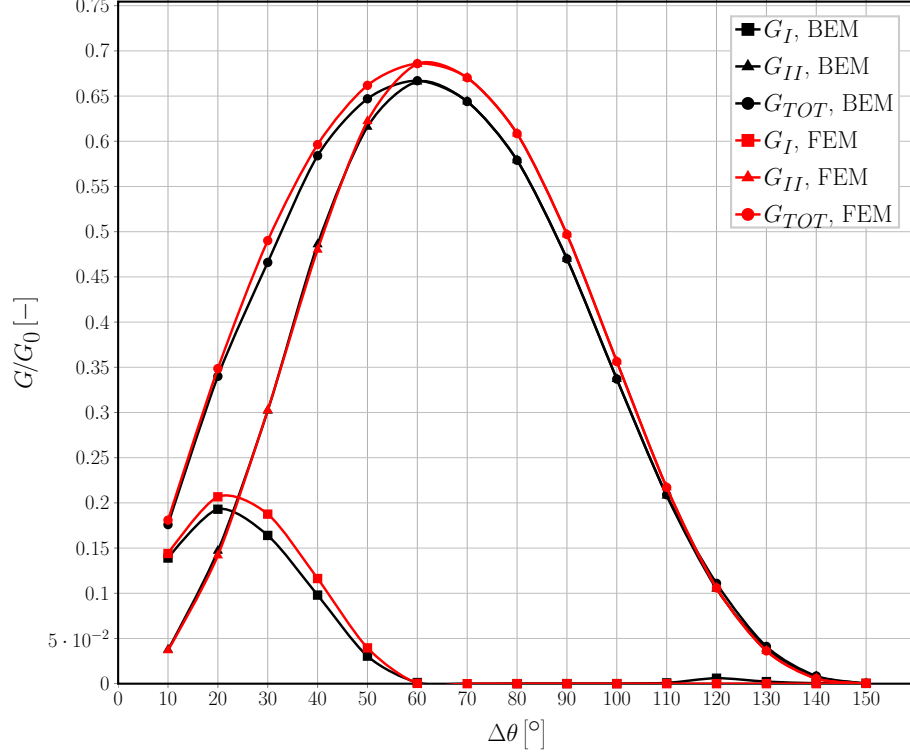


Figure 5: Validation of the single fiber model for the infinite matrix case with respect to the BEM solution in [37].

where μ is the shear modulus, k is the Kolosov's constant defined as $3 - 4\nu$ for plane strain conditions, R_f is the fiber radius and the index m refers to the properties of the matrix. σ_0 is the stress at the boundary, computed as the average of the stress extracted at each boundary node along the right side (arithmetic average as nodes are equispaced by design along both the left and right sides). The agreement is good: the difference between the BEM solution, which is considered more accurate, and the FEM solution does not exceed 5%. The ERRs' maxima are in the same positions and the size of the contact zone is the same. Nevertheless, an analysis of phenomena leading to less than 5% differences in ERR would not be reliable and, therefore, it is not recommended.

215 3. Results & Discussion

3.1. Effect of Fiber Volume Fraction

As shown in Figs. 6 and 7, respectively for Mode I and Mode II, the fiber content has a drastic effect on the Energy Release Rate at the tip of the fiber/matrix interface crack. The effect of four levels of fiber volume fraction are compared, 30%, 50%, 60% and 65%, on two microstructural models: a 11×11 – *free* (every 11th fiber in the central fiber row is partially debonded and, on the top of this row, we have 5 undamaged fiber rows), Figs. 6a and 7a, and a 21×21 – *free* (every 21th fiber in the central fiber row is partially debonded and, on the top of this row, we have 10 undamaged fiber rows), Figs. 6b and 7b.

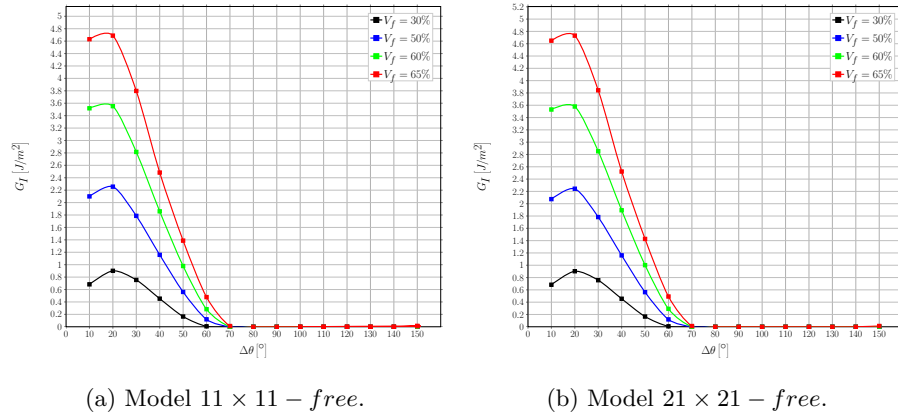


Figure 6: A view of the effect of fiber volume fraction on Mode I ERR in two exemplificative models, subject to an applied transverse strain ε_x of 1%.

Comparing Fig. 6a with 6b, and Fig. 7a with 7b, we can observe that the ERRs' values are very similar for RUCs with 11×11 and 21×21 fibers, though they are slightly higher for the larger RUC where the next debonded fiber and the free surface are further away from the debonded fiber. From these results we conclude that both RUCs are large enough to represent a single debonded fiber in an infinite array of bonded fibers. Obviously, there exists a specific effect of the fiber content. For Mode I, Fig. 6, the maximum value of the ERR increases by ~ 5.2 times when V_f changes from 30% to 65%. The debond's angular size

for which the peak value occurs remains unchanged at 20° , but for $V_f = 60\%$ and 65% the Mode I ERR at 10° and at 20° are rather similar, approximately creating a plateau. Furthermore, increasing the fiber volume fraction delays the onset of the contact zone, which corresponds in Fig. 6 to the first value of $\Delta\theta$ for which G_I is equal to zero. For $V_f = 30\%$, the contact zone first appears for a debond of 60° , similarly to what happens in the single fiber in infinite matrix model (Fig. 5). For higher fiber contents, the contact zone's onset is delayed to a debond's size approximately equal to 70° .

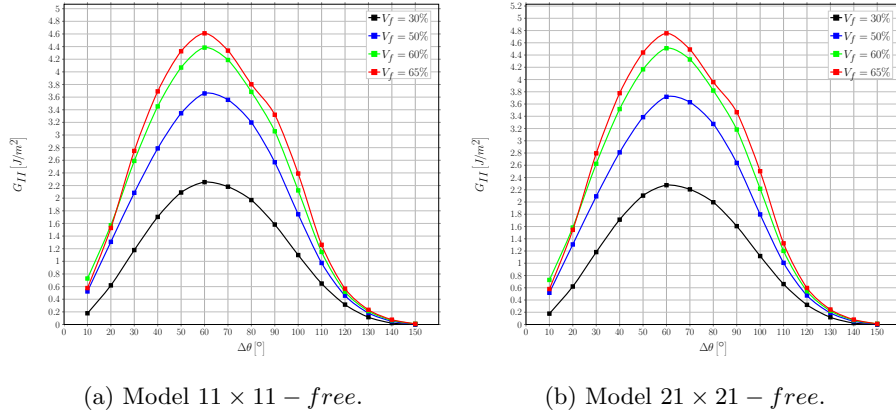


Figure 7: A view of the effect of fiber volume fraction on Mode II ERR in two exemplificative models, subject to an applied transverse strain ε_x of 1%.

For Mode II, Fig. 7, the maximum value of the ERR increases by ~ 2.1 times when V_f changes from 30% to 65%. The effect is thus similar to Mode I, but with a significantly lower magnitude. Similar to Mode I, the debond's size for which the peak value of Mode II occurs remains unchanged, at 60° . There is a distinct maximum in the curve and its shape does not depend on the fiber content. It is worthwhile to notice that the ratio of Mode II to Mode I peak values is $\frac{\max(G_{II})}{\max(G_I)} \sim \frac{2.2}{0.9} \sim 2.4$ for $V_f = 30\%$, while it is $\sim \frac{4.7}{4.7} \sim 1$ for $V_f = 65\%$. Given that the peaks occur at different debond's sizes, for which the value of the other ERR is very small or even close to zero, this means that the increase in fiber content creates a long range of very close values of the total ERR, that may have a global destabilizing effect on the debond's growth.

The general increasing trends observed in Figs. 6 and 7 are related to the fact that, given that the global and local V_f are everywhere identical in the models presented, an increase in fiber content corresponds to a decrease in the average distance between fibers. Thus, the decay of the local stress and strain fields in the matrix domain occurs over smaller lengths causing higher values at the crack tip. The difference in relative magnitude between Mode I and Mode II and the delay in the contact zone's onset are instead due to the interplay between two different mechanisms, both caused by the ordered microstructural arrangement of the model. In the models considered, a fully bonded fiber is always placed along the horizontal direction, aligned with the partially debonded fiber and exactly in front of the debond. By increasing V_f , the former moves closer to the latter and for small debonds this causes a magnification of the x -strain at the crack tip. For small debonds ($\leq 20^\circ - 30^\circ$) in fact, the crack tip is approximately normal to the x -direction and thus an increase in ε_x causes an increase in G_I . On the other hand, for large debonds ($\geq 70^\circ - 80^\circ$) the crack growth direction is almost aligned with the x -axis, thus a magnification in the x -strain translates into an increase of Mode II ERR. However, this increasing effect on G_{II} is partially counteracted by the presence of a fully bonded fiber on top of the debonded fiber and aligned with it. As fibers are more rigid than the surrounding matrix, the presence of the former will restrain horizontal displacements, thus hampering strong increases in G_{II} for large debonds. Furthermore, due to the mismatch in the Poisson's ratios, the fully bonded fiber placed above generates an upward-directed component of the vertical displacement field in the matrix, which tends to open the debond and causes the delay in the contact zone's onset. The interplay between these mechanisms is governed by the average inter-fiber distance and, in turn, by the fiber volume fraction.

These observations are in strong agreement with the results reported in [37], where the effect on the ERR of a partially debonded fiber of two fully bonded nearby fibers, placed symmetrically with respect to the loading direction, is studied for different angular positions (denoted as θ_2) and radial distances in a model with an effectively infinite matrix ($V_f \sim 0.09\%$). The effect of the former

is studied for a constant value of the radial distance between the debonded and bonded fibers, which corresponds to a local V_f^{local} of $\sim 62\%$ assuming hexagonal packing. They report an increase in both Mode I and Mode II ERR with respect to the single fiber case when the two fibers are placed at an angle of respectively $\pm 25^\circ, \pm 30^\circ, \pm 140^\circ, \pm 150^\circ, \pm 155^\circ$, i.e. closest to the loading direction. Notice that for $\pm 25^\circ$ and $\pm 155^\circ$ the two fully bonded fibers are almost in contact, with an inter-fiber distance of ~ 0.04 times their radius. This result confirms the considerations made in the previous paragraph about the x -strain magnification caused by the presence of fully bonded fibers along the loading direction. The effect is further analyzed and discussed in Sec. 3.2 and Sec. 3.4. In the range $\pm 40^\circ - \pm 130^\circ$ instead, the presence of the other fibers causes a reduction of the ERR and, particularly in the range $80^\circ - 120^\circ$, results are very close and almost insensitive to variations in θ_2 , which supports the previous conclusion about the effect of a fully bonded fiber on top the partially debonded one. This effect is treated in more detail in Sec. 3.3.

Comparing the results from [37] with those presented in this paper, an hypothesis can be furthermore formulated about the robustness of the results of the present article with respect to deviations in fiber position: it seems reasonable to assume a tolerance to deviations of max. $\pm 30^\circ$ with respect to the loading direction and of max. $\pm 20^\circ$ with respect to the through-the-thickness direction. The effect of the local fiber content is also investigated in [37], by changing the radial distance between the partially debonded fiber and the fully bonded ones. They observe that the further the fully bonded fibers are placed from the central one, i.e. the lower the local V_f , the lower is their effect on the ERR. The magnitude of the effect is however small: the maximum increase of the total ERR is of ~ 1.15 times for $\theta_2 = 30^\circ$ and 150° when increasing V_f^{local} from 28% to 62%; of ~ 1.25 times for $\theta_2 = 60^\circ$, ~ 1.4 times for $\theta_2 = 90^\circ$ and ~ 1.75 times for $\theta_2 = 120^\circ$ when decreasing V_f^{local} from 62% to 28%. Analogous results can be found in [40], where the authors consider a centrally-placed partially debonded fiber surrounded by an hexagonal cluster inside an homogenized UD composite. They observe a reduction in the ERR when the local fiber volume fraction is

increased, i.e. when the spacing between fibers is reduced. The strongest change
 315 is reported for Mode II, which registers a maximum increase of ~ 1.36 times
 when the local fiber volume fraction is decreased from 78% to 66%. Thus, the
 trends presented in [37, 40] agree with the results on the effect of V_f and sup-
 port the considerations made so far. The stark difference in magnitude however
 highlights the contrast between the effect of the local V_f of a cluster of fibers
 320 inside an infinite medium and of the global V_f of long-range microstructural
 arrangements, such the ones considered in this article. In the former, the fiber
 volume fraction controls the size of a localized perturbation to a constant elas-
 tic solution; in the latter, V_f determines the characteristic lengths of a global
 periodic solution.

3.2. Interaction between debonds in UD composites with a single row of fibers

The interaction of debonds appearing at regular intervals in an ultra-thin
 UD composite with a single row of fibers is studied for Mode I (Fig. 8) and Mode
 II (Fig. 9) and fiber content equal to 30% (Figs. 8a and 9a) and 60% (Figs. 8b
 and 9b). The models treated are $3 \times 1 - free$, $5 \times 1 - free$, $7 \times 1 - free$,
 330 $11 \times 1 - free$, $21 \times 1 - free$, $101 \times 1 - free$ and $201 \times 1 - free$, corresponding
 respectively to a debond every 3^{rd} , 5^{th} , 7^{th} , 11^{th} , 21^{st} , 101^{st} and 201^{st} fiber
 (Fig. 1a). Given that the upper surface of the UD row is left free, the interaction
 with the debonded fiber in the next RUC is stronger than in any other case and
 the results of this section are thus the most conservative in terms of debond's
 335 growth: the ERRs should be the largest. The effect is enhanced in composites
 with high V_f and especially for G_{II} : at $V_f = 60\%$ the highest G_{II} value for the
 $201 \times 1 - free$ composite in Fig. 9b is more than 3 times higher than the G_{II}
 value value for the $21 \times 21 - free$ composite in Fig. 7b. Even the maximum is
 shifted to larger angles. The G_I value is only 30% higher.

340 From both Fig. 8 and Fig. 9, it can be seen that the presence of a debond close
 to the analyzed debond decreases the strain magnification effect discussed in
 Sec. 3.1 and thus reduces the value of the ERR. This phenomenon is called
 “crack shielding” [32].

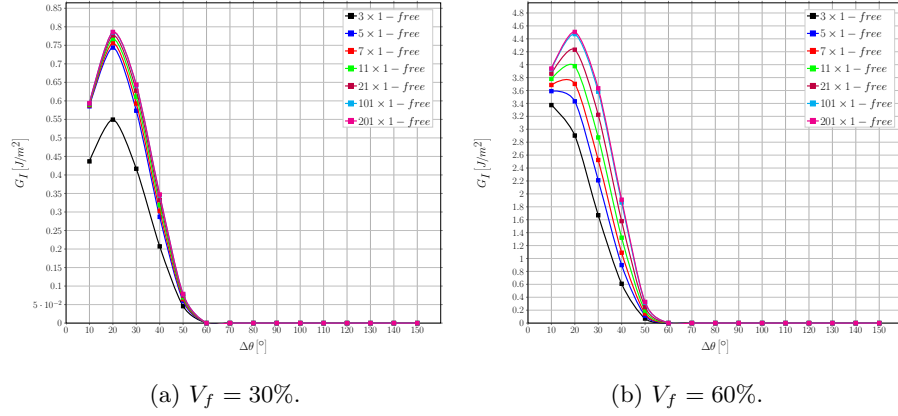


Figure 8: Effect of the interaction between debonds appearing at regular intervals on Mode I ERR in an UD with a single layer of fibers at different levels of fiber volume fraction V_f , subject to an applied transverse strain ε_x of 1%.

For Mode I, the presence of a free surface, and inversely the absence of

345 a fully bonded fiber along the vertical direction, implies the absence of the counteracting upward-oriented vertical component of the displacement field due to the mismatch in Poisson's ratios. This in turn translates into the constancy of the value of $\Delta\theta$ corresponding to contact zone's onset, always equal to 60° . For $V_f = 30\%$, Mode I is reduced when the spacing between debonds (in terms

350 of number of fully bonded fibers between them in our models) decreases, but the magnitude of change is significant only when the spacing is reduced from a debond every 5^{th} fiber to one every 3^{rd} . For comparison, the difference of peak G_I values for $V_f = 30\%$ between $5 \times 1 - free$ and $3 \times 1 - free$ is $\sim 0.2 \frac{J}{m^2}$ (around 30% of the lower value), while between $201 \times 1 - free$ and

355 $5 \times 1 - free$ is $\sim 0.05 \frac{J}{m^2}$ (around 7% of the lower value). A similar observation can be made for $V_f = 60\%$, but for larger spacings: no difference can be seen between the case of a debond placed every 101^{th} and every 201^{th} fiber. These observations suggest the existence of characteristic distance dependent on the fiber volume fraction which governs the interaction between debonds: in low V_f

360 composites ($V_f = 30\%$) the convergence to a non-interactive solution is faster (less interaction between debonded fibers in neighboring RUCs).

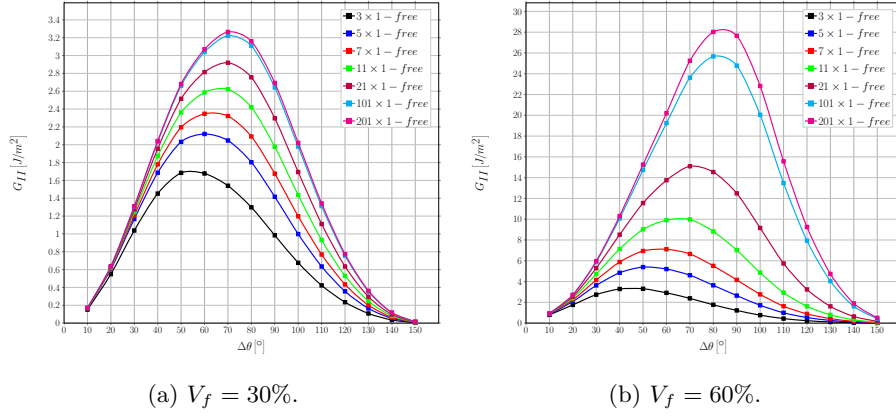


Figure 9: Effect of the interaction between debonds appearing at regular intervals on Mode II ERR in a single-ply laminate with a single layer of fibers at different levels of fiber volume fraction V_f , subject to an applied transverse strain ε_x of 1%.

Without constraint on the upper surface, the strain magnification effect creates a larger displacement gap in the x -direction, which increases Mode II for larger debonds. When debonds are far apart, the series of rigid elements in the ultra-thin composite row (constituted by fully bonded fibers and their surrounding matrix) creates higher x -strains than in average in the element with the debonded fiber, which in turn generates higher tangential displacements at the crack tip for larger debonds. Conversely, when debonds are closer (smaller number of rigid elements between them), the strain concentration in the debonded element is more similar to the applied strain (the magnification is reduced) and the tangential displacement component at the crack tip decreases for large $\Delta\theta$. This is the mechanism behind the change in the value of $\Delta\theta$ for which the peak of G_{II} occurs: from 70° to 50° at 30%, and from 80° to 40° at 60% going from the higher to the smaller spacing of debonds. Differently from Mode I, the presence of a characteristic distance is harder to establish. For $V_f = 30\%$ (Fig. 9a), it seems reasonable to establish it at around 100 fully bonded fibers between each debond. For $V_f = 60\%$ (Fig. 9b), the difference between models $101 \times 1 - free$ and $201 \times 1 - free$ is still sizable, thus preventing the establishment of such characteristic distance. It is possible to observe, however, that

the change between $101 \times 1 - free$ and $201 \times 1 - free$ is significantly smaller than between $21 \times 1 - free$ and $101 \times 1 - free$ ($2 [\frac{J}{m^2}]$ vs $11 [\frac{J}{m^2}]$), thus suggesting the existence of the characteristic distance outside the range studied. Nevertheless, one should question whether the single row composite with free surface is an appropriate RUC for defining the upper bound for G_{II} : G_{II} may be more affected by the free surface than by the effect of the interaction between debonds in the row.

3.3. Influence of rows of fully bonded fibers on debond's ERR in the middle row

The effect of the presence of rows of fully bonded fibers on debond's growth in the central row with all fibers partially debonded is studied for Mode I (Fig. 10) and Mode II (Fig. 11) and fiber content equal to 30% (Figs. 10a and 11a) and 60% (Figs. 10b and 11b). The models treated are $1 \times 3 - free$, $1 \times 5 - free$, $1 \times 7 - free$, $1 \times 11 - free$, $1 \times 21 - free$, $1 \times 101 - free$ and $1 \times 201 - free$, corresponding to a UD composite with respectively 3, 5, 7, 11, 21, 101 and 201 rows of fibers (Fig. 2a).

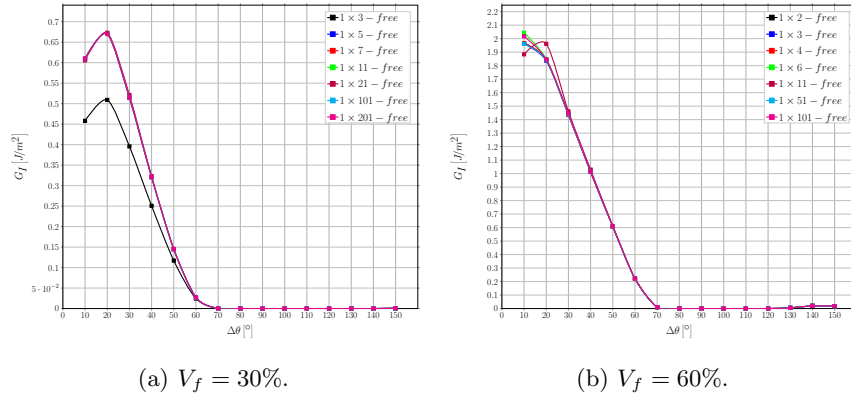


Figure 10: Influence of layers of fully bonded fibers on debond's growth in Mode I ERR in a centrally located line of debonded fibers at different levels of fiber volume fraction V_f , subject to an applied transverse strain ε_x of 1%.

The results shown strengthen the arguments made in Sec. 3.1 and Sec. 3.2. It can, in fact, be seen in Fig. 10 that an increasing number of bonded fiber rows

across the thickness delays the onset of the contact zone to a debond of 70° in size, due to the introduction of an additional positive component of the vertical displacement which translates into an opening displacement at the debond's tip.

400 Comparing Fig. 9b with Fig. 11b, we observe that the presence of bonded fiber rows significantly reduce the G_{II} and its maximum is shifted back to 60° , thus confirming the hypothesis in Section 3.2 that the absence of G_{II} convergence with the increasing distance in a single-row composite is caused more by the free surface than by the interaction between debonds.

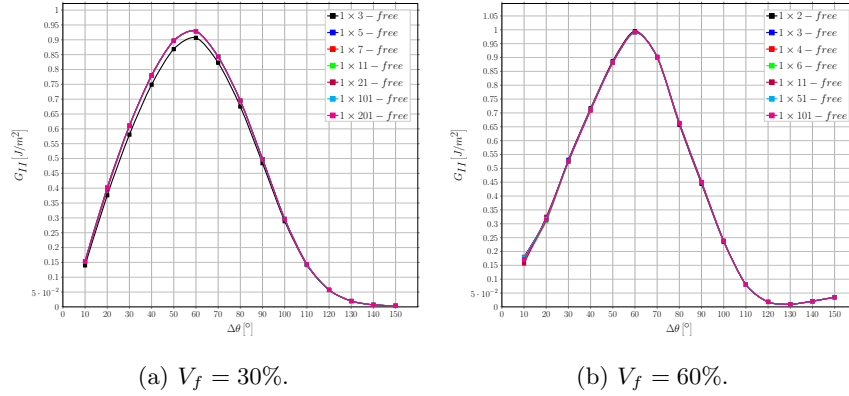


Figure 11: Influence of layers of fully bonded fibers on debond's growth in Mode II ERR in a centrally located line of debonded fibers at different levels of fiber volume fraction V_f , subject to an applied transverse strain ε_x of 1%.

405 The results of both Mode I and Mode II show that the introduction of an increasing number of fully bonded fiber rows doesn't change the ERR calculated at the crack tip (the convergence is very fast). A small effect, mostly on Mode I, of the number of bonded fiber rows can be observed at low fiber content (Figs. 10a and 11a), while for high fiber content the smaller model with only
 410 one fiber row above the partially debonded one is already representative.

3.4. Interaction between debonds in central row of a UD composite with multiple rows of bonded fibers

The interaction of debonds appearing at regular intervals in the central row of fibers in UD composites with multiple rows of fibers is investigated using

different combinations of horizontal debond spacing and the number of rows of bonded fibers across the thickness, corresponding to the models: $3 \times 3 - free$, $5 \times 3 - free$, $5 \times 5 - free$, $7 \times 3 - free$, $7 \times 5 - free$, $7 \times 7 - free$, $11 \times 3 - free$, $11 \times 5 - free$, $11 \times 7 - free$, $11 \times 11 - free$, $21 \times 3 - free$, $21 \times 5 - free$, $21 \times 7 - free$, $21 \times 11 - free$, $21 \times 21 - free$, $101 \times 3 - free$, $101 \times 5 - free$, $101 \times 7 - free$, $101 \times 11 - free$, $201 \times 3 - free$, $201 \times 5 - free$, $201 \times 7 - free$, $201 \times 11 - free$ (Fig. 2b).

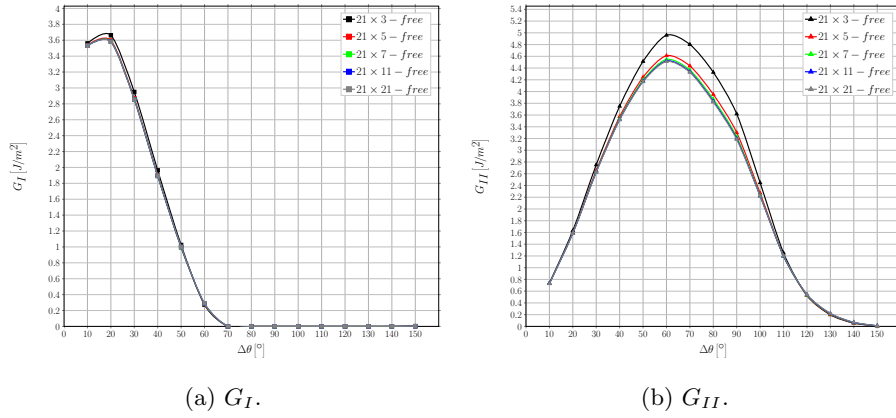


Figure 12: Effect on Mode I and Mode II ERR of the presence of an increasing number of rows of fully bonded fibers in UD composites with debonds appearing every 10^{th} fiber (model $21 \times k - free$). $V_f = 60\%$ and $\varepsilon_x = 1\%$.

The results shown in Fig. 12 confirm the observations discussed in Sec. 3.2: the presence of fully bonded fibers across the thickness has a restraining effect on the ERR, that counteracts the magnification due to an increasing number of fully bonded fibers in the horizontal direction. The interplay is further modulated by the fiber content. For Mode I, at high fiber content the contact zone onset starts at 70° for $V_f = 60\%$, delayed with respect to the low fiber content case of 60° . Comparing Fig. 12 with Fig. 10b and Fig. 11b, it is furthermore possible to observe that the number of fully bonded fibers' rows necessary to reach convergence to a non-interacting solution in the vertical direction depends on the spacing of debonds in the central row. In Figures 10b and 11b the results for the $1 \times 3 - free$ model (1 row below and above) are already representative

of all the other cases; in Fig. 12 the solution doesn't change anymore once at least 3 rows below and above the central one are present, when convergence in both G_I and G_{II} is required.

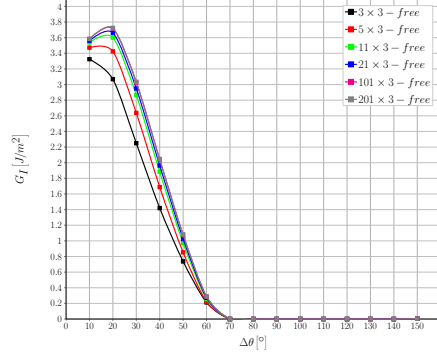
The results in Fig. 13 show that the converse is as well as true: the characteristic distance (in terms of fully bonded fibers) between debonds for which a non-interactive solution is attained changes in relation to the thickness of the UD composite (defined by the number of rows in the vertical direction). Mode I appears to be far less sensitive than G_{II} to the spacing of debonds in the horizontal direction when rows of fully bonded fibers are present above and below: in Fig. 13a the increase in the peak value of G_I is $\sim 8\%$ going from model $5 \times 3 - free$ to $201 \times 3 - free$, while $< 5\%$ for larger spacings. In UDs of increased thickness, Figures 13c and 13e, the variation is further reduced. For Mode II, convergence to a non-interactive solution is reached with a spacing of 100 fully bonded fibers for a UD with 3 rows of fibers across the thickness ($\frac{G_{II}^{201 \times 3}(60^\circ) - G_{II}^{101 \times 3}(60^\circ)}{G_{II}^{101 \times 3}(60^\circ)} \sim 0.7\%$), of 20 fibers in a UD with 5 rows ($\frac{G_{II}^{101 \times 5}(60^\circ) - G_{II}^{21 \times 5}(60^\circ)}{G_{II}^{21 \times 5}(60^\circ)} \sim 4.3\%$) and of 10 fibers in a UD with 11 rows ($\frac{G_{II}^{21 \times 11}(60^\circ) - G_{II}^{11 \times 11}(60^\circ)}{G_{II}^{11 \times 11}(60^\circ)} \sim 3.4\%$).

3.5. Comparison with the single fiber model with equivalent boundary conditions

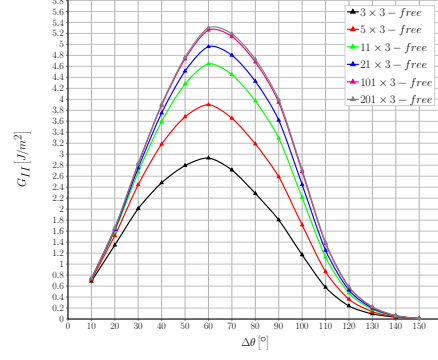
The single fiber RUC ($1 \times 1 - free$ or $1 \times 1 - coupling$) corresponds to the most damaged state of the composite, i.e. the state in which all fibers have debonds. The $1 \times 1 - free$ model represents an ultra-thin UD composite with a single row of partially debonded fibers. The $1 \times 1 - coupling$ model, where the displacement coupling is used to enforce periodic boundary conditions, represents an infinite composite.

The comparison of the $1 \times 1 - free$ model with one row multi-fiber models $n \times 1 - free$ in Figure 14 show that the former provides in general the lowest value of the ERR (the highest crack shielding case) which is consistent with the trends observed in Section 3.2.

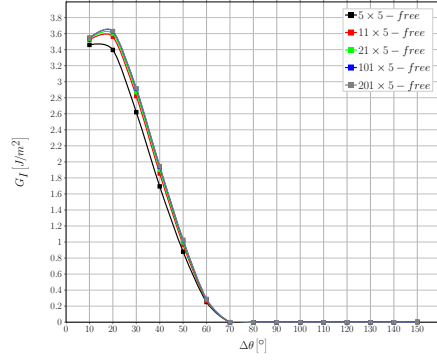
The $1 \times 1 - coupling$ model is compared with $1 \times 3 - free$ and $1 \times 201 - free$ models in Fig. 15. In all three models the distance between debonds in the



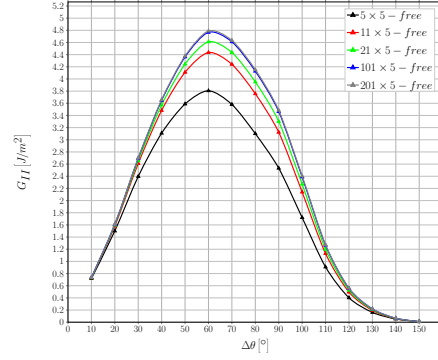
(a) $k = 3$, G_I .



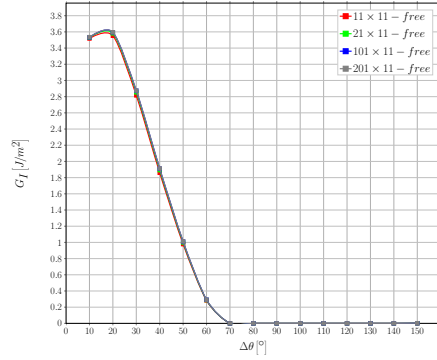
(b) $k = 3$, G_{II} .



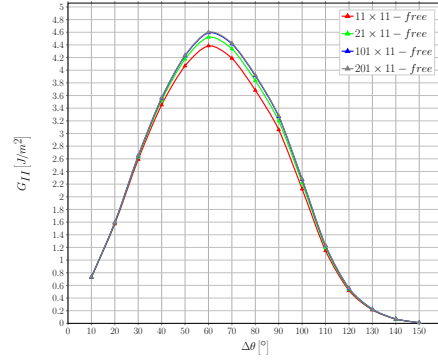
(c) $k = 5$, G_I .



(d) $k = 5$, G_{II} .



(e) $k = 11$, G_I .



(f) $k = 11$, G_{II} .

Figure 13: Effect on Mode I and Mode II ERR of increasing the spacing between debonds appearing in the central row of fibers in a UD composite with a fixed number of rows across the thickness. $V_f = 60\%$ and $\varepsilon_x = 1\%$.

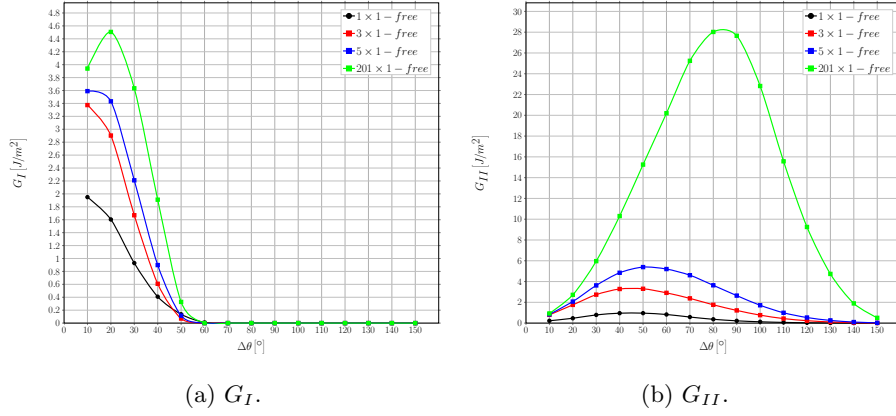


Figure 14: Comparison of the ERR between the single fiber model with free upper boundary and the multiple fibers model with fibers only on the side. $V_f = 60\%$ and $\varepsilon_x = 1\%$.

x -direction is the same and the difference is in the vertical direction. The $1 \times 1 - \text{coupling}$ model describes the interaction between debonds in different rows of debonded fibers whereas the $1 \times k - \text{free}$ models describe the effect of the proximity of the composite's free surface. The Mode I ERR in the $1 \times 3 - \text{free}$ model and in the $1 \times 1 - \text{coupling}$ model is very similar, which leads to a rather surprising conclusion. In both models we have, on the top of the central one, a large amount of fibers (bonded in one case and debonded in the other case). It appears that the effect of bonded and debonded fibers on the central debond is the same. This implies that the interaction between debonded fibers in elements placed on top of each other is small. The volume fraction effect is much smaller in high fiber content composites of this type.

The same comparison for Mode II shows a sizeable difference in the range $50^\circ - 90^\circ$, while the results almost coincide for smaller values of $\Delta\theta$. These observations point to the evidence that debond interaction is more significant in the loading direction than in the transverse one. The lower values of G_{II} of the $1 \times 1 - \text{coupling}$ model in the range $50^\circ - 90^\circ$ are due to the shielding effect of a debond of the same size in the fiber just above the central one (modeled by the coupling boundary condition), which leaves the strip of matrix between the two fibers free to deform away from both of them due to the Poissons effect

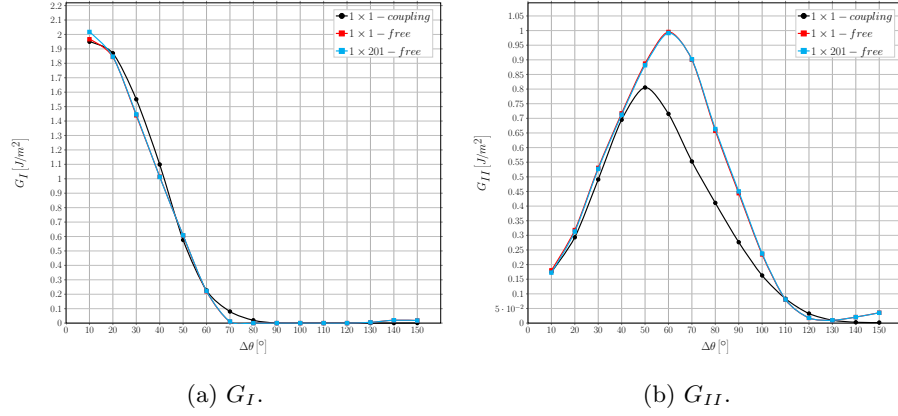


Figure 15: Comparison of the ERR between the single fiber model with coupling conditions along the upper boundary and the multiple fibers model with fibers above. $V_f = 60\%$ and $\varepsilon_x = 1\%$.

and thus favors Mode I and reduces Mode II. This translates into the lower estimates in Fig. 15b and into the delay in the appearance of the contact zone, particularly evident in Fig. 15a.

4. Conclusions & Outlook

Several models of Repeating Unit Cell, representative of different microstructural arrangements of a unidirectional (UD) composite, have been studied in order to investigate the effect on fiber/matrix interface crack growth of the presence of partially debonded and/or fully bonded fibers. Regular microstructures based on square-packing arrangements of fibers have been loaded in transverse tension, with debonds appearing in the central row of fibers at regular intervals measured in terms of number of fully bonded fibers between them. This central row is embedded in-between a varying number of rows with perfectly bonded fibers. The surface of the composite is either traction-free or with imposed vertical displacement constraint imitating a periodic structure in the composite thickness direction.

Local and global fiber volume fractions are everywhere equal to each other by

design, which establishes a direct relationship between fiber content and inter-fiber distance. The main conclusions of this work are summarized here in the following.

1. With a decreasing number of fully bonded fibers between two partially debonded fibers in the central row, the ERR decreases. It seems to exist a characteristic distance between debonds which defines the transition to a non-interactive solution. However, this distance depends on the number of perfectly bonded fiber rows surrounding the central row.
2. The presence of a free surface close to the debond leads to higher Mode I and Mode II ERRs and a shift of the peak G values to larger debonds.
3. The presence of fibers (fully or partially bonded) in the composite thickness direction, along the same vertical line as the analyzed central fiber, appears to have a restraining effect on both G_I and G_{II} . The free composite surface effect on the ERR decays very fast: adding more than 2 fully bonded fibers below and above the central row leads to stable constant values of ERR.
4. The presence of a debond in the fiber above the central partially debonded one only delays the appearance of the contact zone, while no significant effect on the ERR has been observed.
5. Increasing the fiber content (decreasing the inter-fiber distance), magnifies in general the effects described in the previous points.
6. The results and conclusions presented agree well with previous observations reported in the literature [37, 40]. A mechanical explanation of the observed trends has been presented based on the mismatch in elastic properties, and particularly Poisson's ratios, and the positions of fibers and debonds with respect to the loading direction.

Acknowledgements

Luca Di Stasio gratefully acknowledges the support of the European School of Materials (EUSMAT) through the DocMASE Doctoral Programme and the

European Commission through the Erasmus Mundus Programme.

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