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Outline

- Symbols, Models, Equations & Reference Data
- Normal stress distribution at the loaded boundary
- σ_0 and G_0
- Finite strain and small strain formulations
- Elements's aspect ratio
- Next steps

Symbols, Models, Equations & Reference Data

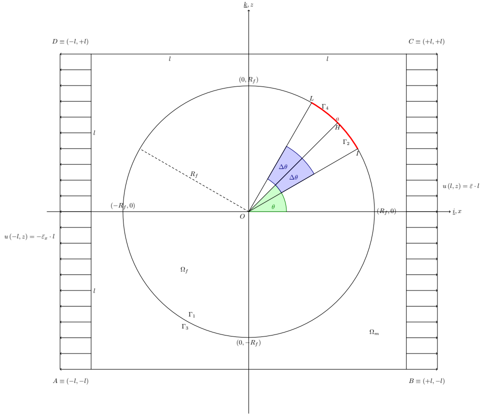
Symbols

Symbol	Unit	Description
θ	[°]	Debond angular position with respect to the center of the arc defined by the debond itself
$\Delta\theta$	[°]	Debond semi-angular aperture
δ	[°]	Angle subtended by a single element at the fiber/matrix interface
VF_f	[−]	Fiber volume fraction
l	[μm]	Ply's half-length, equal to RVE's half-length (square element)
u	[μm]	Displacement along x
w	[μm]	Displacement along z

Symbols

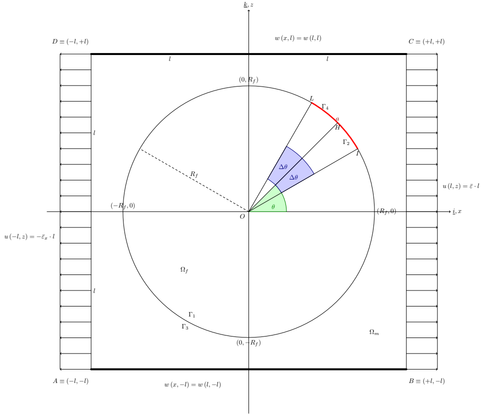
Symbol	Unit	Description
Γ_1	$[-]$	Bonded part of fiber surface
Γ_2	$[-]$	Free (debonded) part of fiber surface
Γ_3	$[-]$	Bonded part of matrix surface
Γ_4	$[-]$	Free (debonded) part of matrix surface

Reference Models



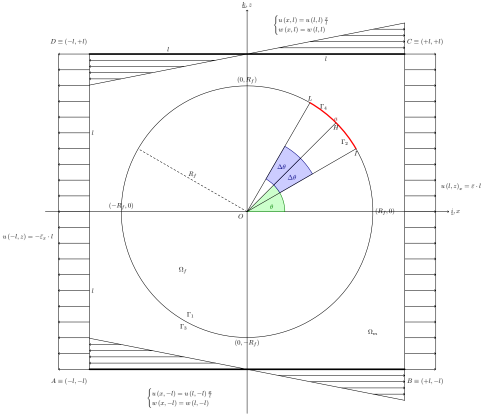
Simple RVE, BC: free.

Reference Models



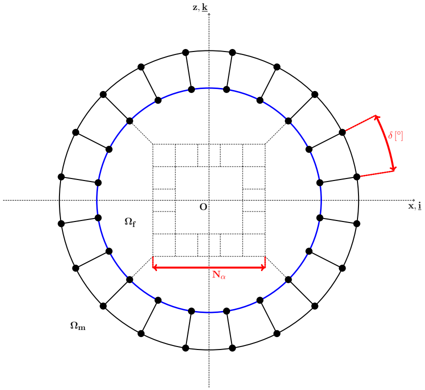
Simple RVE, BC: fixed vertical displacement.

Reference Models



Simple RVE, BC: fixed vertical and homogeneous horizontal displacement.

Angular discretization



Angular discretization at fiber/matrix interface: $\delta = \frac{360^\circ}{4N_\alpha}$.

Material properties

Material	$E\,[GPa]$	$G\,[GPa]$	$\nu\,[-]$
Glass fiber	70,0	29,2	0,2
Epoxy	3,5	1,25	0,4

Evaluation of G_0

$$G_0 = \pi R_f \sigma_0^2 \frac{1 + k_m}{8G_m} \tag{1}$$

$$k_m = 3 - 4\nu_m \tag{2}$$

$$\sigma_0^{undamaged} = \frac{E_m}{1 - \nu_m^2} \varepsilon_{xx} \tag{3}$$

VCCT in Forces

$$\Delta u = \left(x^{fiber,def}_{\text{1 element before crack tip}} - x^{fiber,undef}_{\text{1 element before crack tip}}\right) - \left(x^{matrix,def}_{\text{1 element before crack tip}} - x^{matrix,undef}_{\text{1 element before crack tip}}\right) \tag{4}$$

$$\Delta w = \left(z^{fiber,def}_{\text{1 element before crack tip}} - z^{fiber,undef}_{\text{1 element before crack tip}}\right) - \left(z^{matrix,def}_{\text{1 element before crack tip}} - z^{matrix,undef}_{\text{1 element before crack tip}}\right) \tag{5}$$

$$\beta = \arctan\left(\frac{z^{matrix,def}_{\text{crack tip}}}{x^{matrix,def}_{\text{crack tip}}}\right) \tag{6}$$

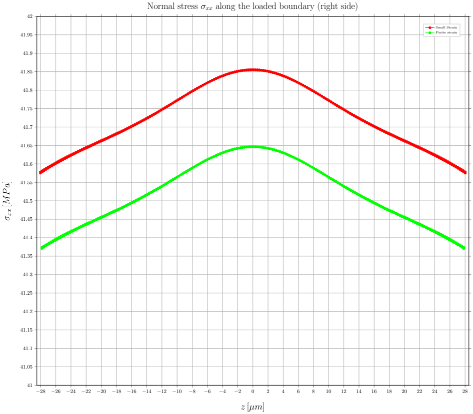
$$\Delta_r = \cos(\beta)\Delta u + \sin(\beta)\Delta w \qquad \Delta_\theta = -\sin(\beta)\Delta u + \cos(\beta)\Delta w \tag{7}$$

$$F_r = \cos(\beta)F^{reaction}_x + \sin(\beta)F^{reaction}_z \qquad F_\theta = -\sin(\beta)F^{reaction}_x + \cos(\beta)F^{reaction}_z \tag{8}$$

$$G_I = \frac{1}{2}\frac{F_r\Delta_r}{R_f\delta} \qquad G_{II} = \frac{1}{2}\frac{F_\theta\Delta_\theta}{R_f\delta} \qquad b = 1.0 \leftrightarrow \Delta A = bR_f\delta \tag{9}$$

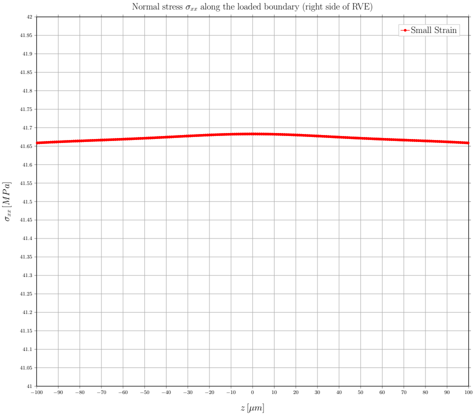
↘ $\sigma_{xx}(x = L, z)$

$\sigma_{xx}\left(x=L,z\right)$ for $Vf_f=0.001$, $\frac{L}{R_f}\sim 28$ and $\delta=0.4°$



In red small strain FEM, in green finite strain FEM.

$\sigma_{xx}\,(x=L,z)$ for $Vf_f = 0.000079$, $\frac{L}{R_f} \sim 100$ and $\delta = 0.4°$



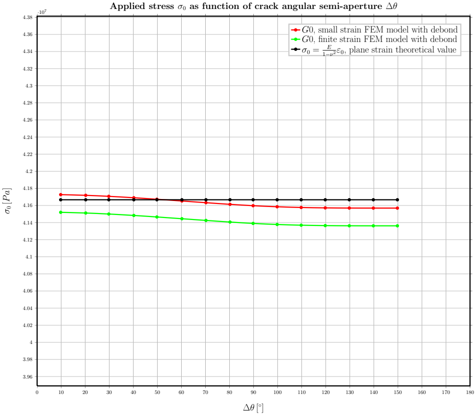
In red small strain FEM.

Conclusions

- ➔ Maximum and minimum are equal due to symmetry
- ➔ For $\frac{L}{R_f} \sim 28$ in small strain, the relative difference between maximum/minimum and mean value is 0.34%
- ➔ For $\frac{L}{R_f} \sim 28$ in finite strain, the relative difference between maximum/minimum and mean value is 0.33%
- ➔ For $\frac{L}{R_f} \sim 100$ in small strain, the relative difference between maximum/minimum and mean value is 0.03%
- ➔ The stress at the boundary can thus be effectively approximated as constant and equal to the mean value

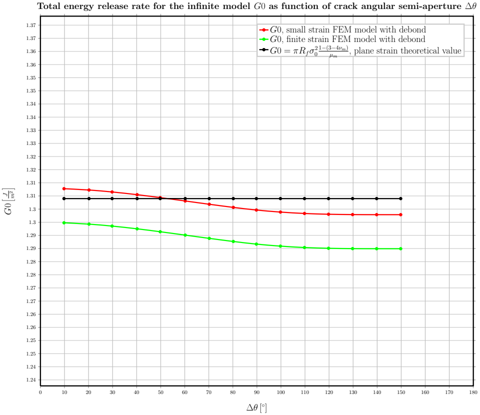
σ_0 AND G_0

σ_0 for $Vf_f = 0.001$, $\frac{L}{R_f} \sim 28$ and $\delta = 0.4°$



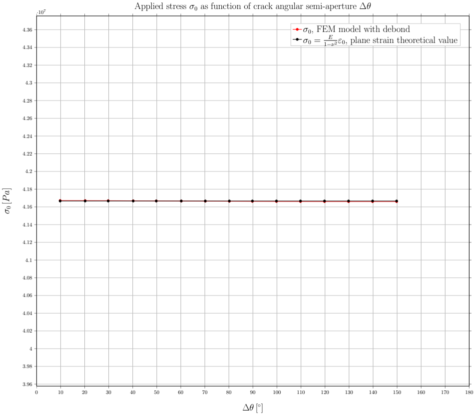
In red small strain FEM, in green finite strain FEM, in black $\sigma_0 = \frac{E}{1-\nu^2}\varepsilon$.

G_0 for $Vf_f = 0.001$, $\frac{L}{R_f} \sim 28$ and $\delta = 0.4°$



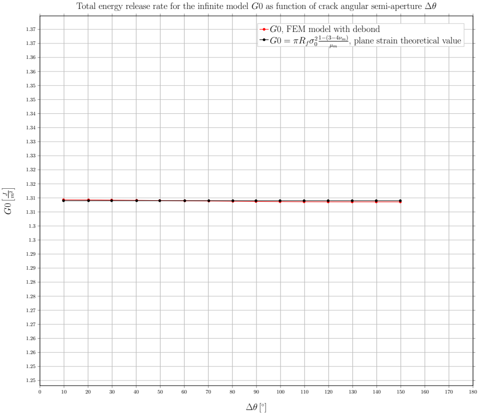
In red small strain FEM, in green finite strain FEM, in black G_0 calculated assuming $\sigma_0 = \frac{E}{1-\nu^2}\varepsilon$.

σ_0 for $Vf_f = 0.000079$, $\frac{L}{R_f} \sim 100$ and $\delta = 0.4°$



In red small strain FEM, in black $\sigma_0 = \frac{E}{1-\nu^2}\varepsilon$.

G_0 for $Vf_f = 0.000079$, $\frac{L}{R_f} \sim 100$ and $\delta = 0.4°$



In red small strain FEM, in black G_0 calculated assuming $\sigma_0 = \frac{E}{1-\nu^2}\varepsilon$.

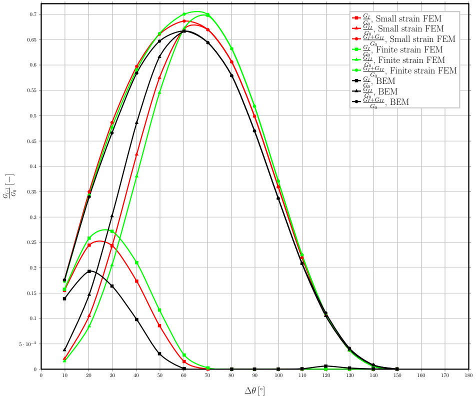
Conclusions

- σ_0 and G_0 depend on $\Delta\theta$ for finite sizes of the RVE
- As the RVE size $\rightarrow \infty$, i.e. $\frac{L}{R_f} \rightarrow \infty$ ($\sim$ 100), σ_0 and G_0 tend to the theoretical undamaged value given by $\sigma_0 = \frac{E_m}{1-\nu_m^2}\varepsilon_0$
- σ_0 and G_0 might be taken as a good measure of "infinetess" for strain-/displacement-controlled simulations
- By selecting $\Delta\theta = 10^\circ$ and running a parametric study with a comparatevely coarse mesh the minimum ratio $\frac{L}{R_f}$ or equivalently maximum Vf_f volume to have an infinite RVE could be found

↘ FS & SS

$\frac{G_{(\cdot\cdot)}}{G_0}$ for $V_f = 0.001$, $\frac{L}{R_f} \sim 28$ and $\delta = 0.4°$

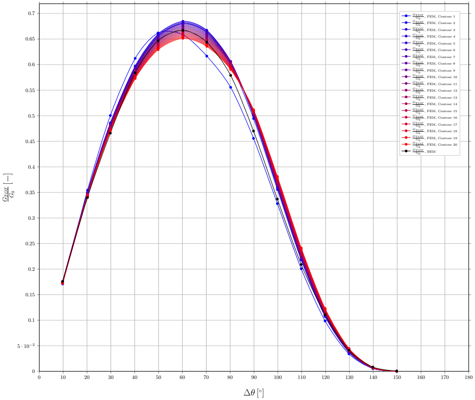
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house VCCT post-processing routine



In red small strain FEM, in green finite strain FEM, in black BEM results.

$\frac{G_{(\cdot\cdot)}}{G_0}$ for $V_f = 0.001$, $\frac{L}{R_f} \sim 28$ and $\delta = 0.4°$, small strain formulation

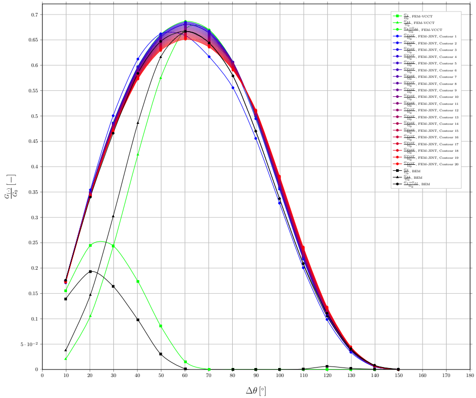
Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)



Fading from blue to red J-Integrals evaluated at contours at increasing distance from the crack tip, in black BEM results.

$\frac{G_{(\cdot\cdot)}}{G_0}$ for $V_f = 0.001$, $\frac{L}{R_f} \sim 28$ and $\delta = 0.4°$, small strain formulation

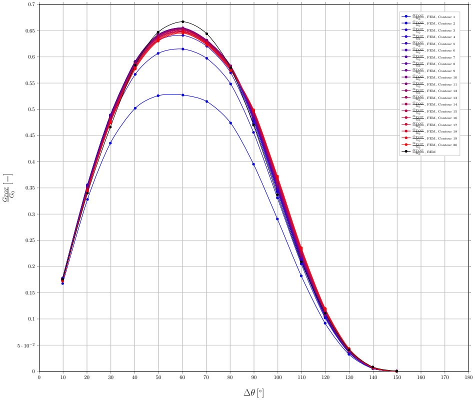
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines



Fading from blue to red J-Integrals evaluated at contours at increasing distance from the crack tip, in green evaluation with in-house VCCT routine, in black BEM results.

$\frac{G_{(\cdot\cdot)}}{G_0}$ for $V_f = 0.001$, $\frac{L}{R_f} \sim 28$ and $\delta = 0.4°$, finite strain formulation

Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)



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$\frac{G_{(\cdot\cdot)}}{G_0}$ for $V_f = 0.001$, $\frac{L}{R_f} \sim 28$ and $\delta = 0.4°$, finite strain formulation

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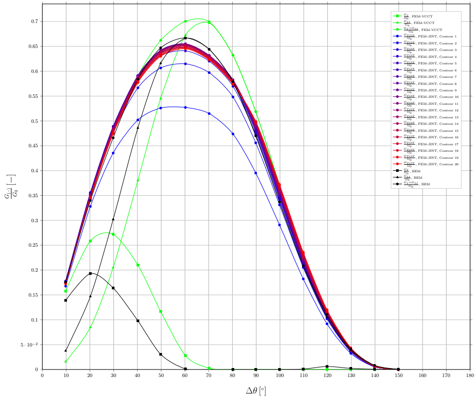


Fading from blue to red J-Integrals evaluated at contours at increasing distance from the crack tip, in green evaluation with in-house VCCT routine, in black BEM results.

Conclusions

- For both small and finite strain formulations, J-integrals are already in good agreement with $\frac{G_{TOT}}{G_0}$ from BEM, i.e. no sizeable finite size effect already at $\frac{L}{R_f} \sim 28$
- For both small and finite strain formulations, J-integrals correctly measure the peak value of $\frac{G_{TOT}}{G_0}$ at 60°
- J-Integrals in small strain slightly overestimate the BEM result
- J-integrals in small strain shows poor convergence in the range $50° - 80°$
- J-Integrals in finite strain slightly underestimate the BEM result
- J-integrals in finite strain shows very good convergence in all the range $10° - 150°$

Conclusions

- ➔ $\frac{G_{TOT}}{G_0}$ is correctly calculated by the VCCT in small strain, in good agreement with BEM results
- ➔ The peak value of $\frac{G_{TOT}}{G_0}$ is correctly calculated by the VCCT in small strain, at 60°
- ➔ $\frac{G_{TOT}}{G_0}$ is wrongly calculated by the VCCT in finite strain, with a peak at $65^\circ - 70^\circ$
- ➔ Small strain VCCT shows better results than finite strain VCCT
- ➔ Mode ratio is still not correct, i.e. probably finite size effect

Observations & Questions

- ➔ J-Integral is a far-field technique, using stresses, strain and displacements far from the crack tip; convergence is in fact far from crack tip (at least 10 contours, i.e. 10 ring of elements)
- ➔ VCCT is a local technique, using forces and displacements at the crack tip
- ➔ The difference between small and finite strain results rests mainly in the displacements
- ➔ Previously, we observed that changing the formulation of the bonded interface, all other parameters equal, the result doesn't change
- ➔ All the convergence problem reduces to the correct evaluation of displacements of debonded surfaces close to the crack tip
- ➔ Displacements of debonded surfaces close to the crack tip are influenced by RVE size

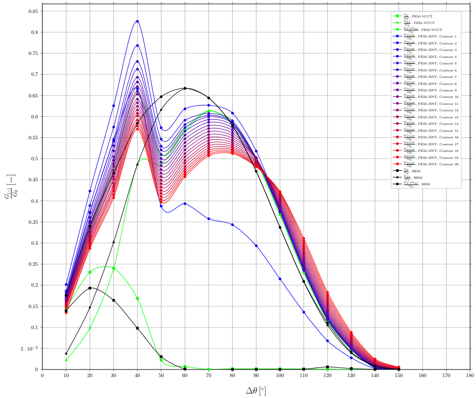
Observations & Questions

- ➔ Small strain shows (correctly) better results than finite strain formulation with respect to infinite reference values
- ➔ However, Abaqus documentation suggests that, if contact between surfaces is present in the model, finite strain formulation (nonlinear geometry) should be used
- ➔ For finite sizes of RVE, which formulation should be chosen?

ELEMENTS'S ASPECT RATIO

$\frac{G_{(\cdot)}}{G_0}$ for $Vf_f = 0.000079$, $\frac{L}{R_f} \sim 100$ and $\delta = 1.0°$, small strain formulation

Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines



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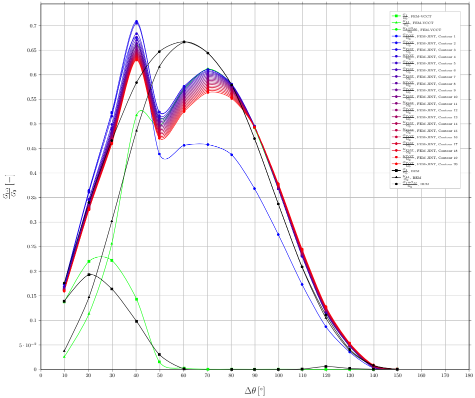
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines



Fading from blue to red J-Integrals evaluated at contours at increasing distance from the crack tip, in green evaluation with in-house VCCT routine, in black BEM results.

Conclusions

- Elements' aspect ratio (maximum side length/minimum side length) was very high in the exterior part of the matrix in this set of simulations
- Spurious stresses adn deformations were created at $45°, 135°, 225°, 315°$
- Results are badly affected by this in the range $40° - 70°$ with a marked oscillation between $40° - 50°$
- Elements' aspect ratio in the matrix is more important than the elements' size at the fiber/matrix interface
- Program has already been changed to receive aspect ratios as input instead of number of elements
- Results from previous sections were calculated with meshes with controlled aspect ratios

↘ Next steps

Next steps

- Simulations for $Vf_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ for different δ (mesh size) for both finite and small strain: already running, results during next week
- Simulations over Vf_f for fixed $\Delta\theta$ and δ to find the value of Vf_f for which the model can be considered infinite by measuring σ_0 and G_0: starting beginning next week (∼Monday)
- Simulations over elements' aspect ratio for fixed size and Vf_f to measure its effect on the solution: starting mid next week (∼Wednesday)

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