

# INVESTIGATION OF SCALING LAWS OF THE FIBER/MATRIX INTERFACE CRACK IN POLYMER COMPOSITES THROUGH FINITE ELEMENT-BASED MICROMECHANICAL MODELING

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# Outline

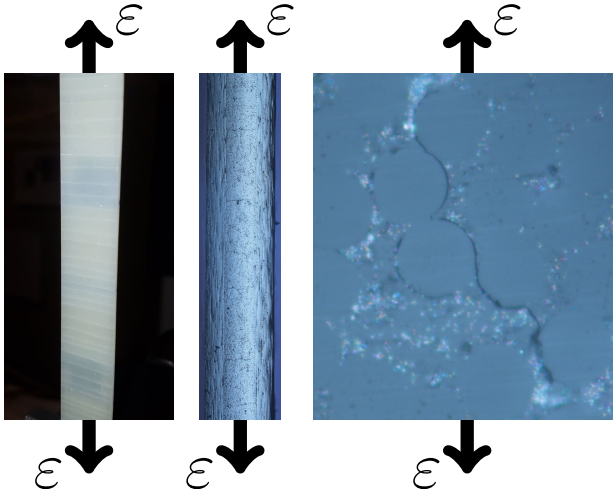
- Initiation of Transverse Cracking in Fiber Reinforced Polymer Composites (FRPCs): Microscopic Observations & Modeling
- The Fiber-Matrix Interface Crack Problem

Transverse Cracking in FRPCs The Fiber-Matrix Interface Crack Problem

Observations: From Macro to Micro Mathematical Modeling of Fracture Numerical Characterization of Fracture

# TRANSVERSE CRACKING IN FRPCs

## Observations: From Macro to Micro



**Left:**  
front view of  $[0, 90_2]_S$ ,  
visual inspection.

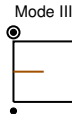
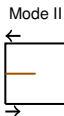
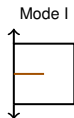
**Center:**  
edge view of  $[0, 90]_S$ ,  
optical microscope.

**Right:**  
edge view of  $[0, 90]_S$ ,  
optical microscope.

## Mathematical Modeling of Fracture: Linear Elastic Fracture Mechanics (LEFM)

### Fracture Mode

*I, II, III, I/II, I/III, II/III*



→ Energy Release Rate:  $G \left[ \frac{J}{m^2} = \frac{N}{m} \right]$

$$G = \frac{\partial W}{\partial A} - \left( \frac{\partial U}{\partial A} + \frac{\partial E_k}{\partial A} \right)$$

→ Stress Intensity Factor:  $K \left[ Pa\sqrt{m} \right]$

$$K_{I/II/III} = \lim_{r \rightarrow 0} \sqrt{2\pi r} \cdot \sigma_{I/II/III}(r, 0)$$

### Variables

geometry

materials

boundary conditions

loading mode

scale

→ J-Integral:  $J \left[ \frac{J}{m^2} = \frac{N}{m} \right]$

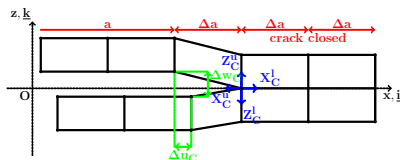
$$J = \lim_{\delta \rightarrow 0} \int_{\Gamma_\delta} \left( W - n_j \sigma_{jk} \frac{\partial u_k}{\partial x_j} \right) d\Gamma$$

→ Average Crack Opening & Shear Displacement:  
 $COD, CSD_{II/III} [m]$

$$\left\{ \begin{matrix} COD \\ CSD_{II} \\ CSD_{III} \end{matrix} \right\} = \frac{1}{S_C} \int_{S_C} \overrightarrow{\Delta u_C} \cdot \left\{ \begin{matrix} \vec{n}_I \\ \vec{n}_{II} \\ \vec{n}_{III} \end{matrix} \right\} dS$$

## Numerical Characterization of Fracture: VCCT & J-Integral

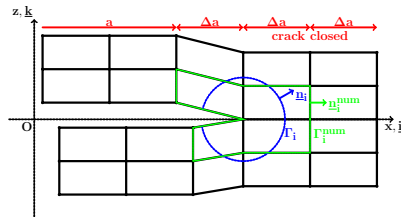
### Virtual Crack Closure Technique (VCCT)



$$G_I = \frac{Z_C \Delta w_C}{2B \Delta a} \quad G_{II} = \frac{X_C \Delta u_C}{2B \Delta a}$$

Krueger R.; *Virtual crack closure technique: History, approach, and applications. Appl. Mech. Rev.* **57** (2) 109–143, 2004.

### J-Integral



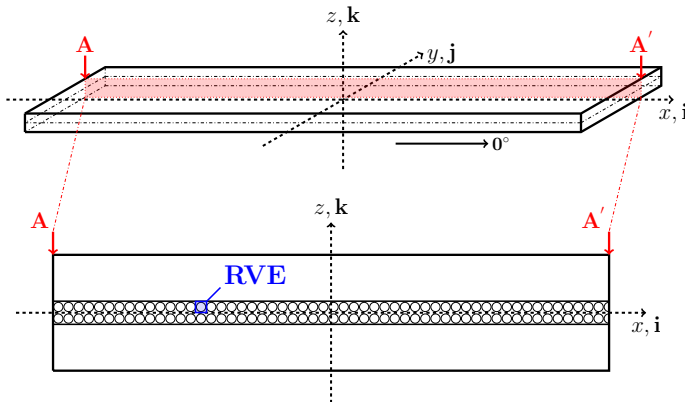
$$J_i = \sum_{k=1}^{n_{segments}} \sum_{j=1}^{n_{nodes}} \left[ w_j \left( W - n_j \sigma_{jk} \frac{\partial u_k}{\partial x_i} \right) \right]_{(x_{kj}, y_{kj})}$$

Rice J. R.; *A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks. J. Appl. Mech.* **35** (2) 379–386, 1968.



# THE FIBER-MATRIX INTERFACE CRACK PROBLEM

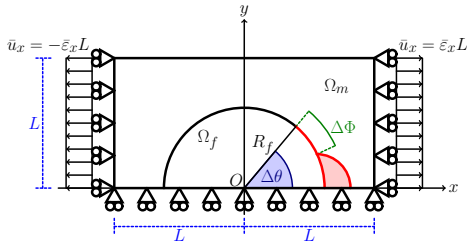
## The Fiber-Matrix Interface Crack Problem: Geometry



- $L, W \gg t$
- $L, W \rightarrow \infty$
- 2D RVE



## The Fiber-Matrix Interface Crack Problem: Assumptions

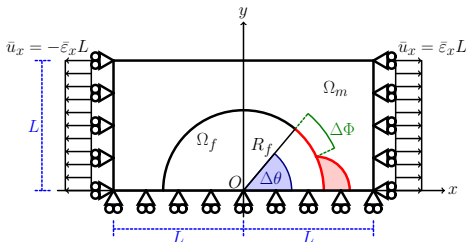


$$R_f = 1 \text{ } [\mu\text{m}] \quad L = \frac{R_f}{2} \sqrt{\frac{\pi}{V_f}}$$

Material	$E_1$	$\nu_{12}$
glass fiber	70.0	0.2
epoxy	3.5	0.4

- Linear elastic, homogeneous and isotropic materials
- Plane strain
- Frictionless contact interaction
- Symmetric w.r.t. x-axis
- Coupling of x-displacements on left and right side (repeating unit cell)
- Applied uniaxial tensile strain  $\bar{\epsilon}_x = 1\%$

## The Fiber-Matrix Interface Crack Problem: Solution



in  $\Omega_f, \Omega_m$  :

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\varepsilon_z = \gamma_{zx} = \gamma_{yz} = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

$$\sigma_{zz} = \nu (\sigma_{xx} + \sigma_{yy})$$

for  $0^\circ \leq \alpha \leq \Delta\theta$  :

$$(\vec{u}_m(R_f, \alpha) - \vec{u}_f(R_f, \alpha)) \cdot \vec{n}_\alpha \geq 0$$

for  $\Delta\theta \leq \alpha \leq 180^\circ$  :

$$\vec{u}_m(R_f, \alpha) - \vec{u}_f(R_f, \alpha) = 0$$

$$\sigma_{ij} = E_{ijkl} \varepsilon_{kl}$$

$$+ BC$$

- Finite Element Method (FEM) in Abaqus<sup>TM</sup>
- 2<sup>nd</sup> order shape functions
- 6-nodes triangles & 8-nodes quadrilaterals
- regular mesh of quadrilaterals at the crack tip:

$$- AR \sim 1$$

$$- \delta = 0.05^\circ$$

## The Fiber-Matrix Interface Crack Problem: Normalization & Scaling

## References



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