Stiffness reduction in UD and cross-ply laminates due to fiber/matrix interface cracks

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Abstract

Keywords: Fiber Reinforced Polymer Composite (FRPC), Debonding, Finite element analysis (FEA)

1. Introduction

2. Derivation of constitutive relations

$$\beta_{ij} = \frac{1}{V_k}\int_{S_C}\frac{1}{2}\left(u_i n_j + u_j n_i\right)dS \tag{1}$$

$$\beta_{11} = \frac{1}{V_k}\int_{S_C}\frac{1}{2}\left(u_1 n_1 + u_1 n_1\right)dS = \frac{1}{V_k}\int_{S_C}\cancelto{0}{u_1 n_1}dS = 0 \tag{2}$$

$$\beta_{22} = \frac{1}{V_k}\int_{S_C}\frac{1}{2}\left(u_2 n_2 + u_2 n_2\right)dS = \frac{1}{V_k}\int_{S_C}u_2 n_2 dS \tag{3}$$

$$\begin{aligned}\beta_{22} =& \frac{1}{V_k}\int_{S_C}u_2 n_2 dS = \frac{1}{V_k}\left[\int_{S_C^m}u_2^m n_2^m dS + \int_{S_C^f}u_2^f n_2^f dS\right] = \\ =& \frac{1}{V_k}\left[\int_{S_C^m}u_2^m n_2^m dS + \int_{S_C^f}u_2^f\left(-n_2^m\right)dS\right]\end{aligned} \tag{4}$$

$$S_C^m = S_C^f = R_f\Delta\theta \tag{5}$$

$$
\begin{aligned}
\beta_{22} =& \frac{1}{V_k}\left[n_D R_f w \int_0^{\Delta\theta}\left(u_2^m - u_2^f\right) n_2^m d\theta\right] = \\
=& \frac{1}{Lwt_{90^\circ}}\left[n_D R_f w \int_0^{\Delta\theta}\left(u_2^m - u_2^f\right) n_2^m d\theta\right] = \\
=& \frac{1}{L}\frac{n_D R_f}{t_{90^\circ}}\left[\int_0^{\Delta\theta}\left(u_2^m - u_2^f\right) n_2^m d\theta\right]
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
u_2^m - u_2^f =& \left(u_r^m - u_r^f\right)\cos(\theta) - \left(u_\theta^m - u_\theta^f\right)\sin(\theta) = \\
=& COD(\theta)\cos(\theta) - CSD(\theta)\sin(\theta)
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
u_3^m - u_3^f =& \left(u_r^m - u_r^f\right)\sin(\theta) + \left(u_\theta^m - u_\theta^f\right)\cos(\theta) = \\
=& COD(\theta)\sin(\theta) + CSD(\theta)\cos(\theta)
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
n_2^m &= \cos(\theta) - \sin(\theta) \\
n_3^m &= \sin(\theta) + \cos(\theta)
\end{aligned} \tag{9}
$$

$$
\beta_{22} = \frac{1}{L}\frac{R_f}{t_{90^\circ}}\left[\int_0^{\Delta\theta}\left(COD(\theta)\left(\sin(\theta)\cos(\theta) - \sin^2(\theta)\right) + CSD(\theta)\left(\cos^2(\theta) - \cos(\theta)\sin(\theta)\right)\right) d\theta\right] \tag{10}
$$

$$
\begin{aligned}
COD(\theta) &= COD_{avg} + \delta COD(\theta) = COD_{avg} + COD_{max} f(\theta) \\
CSD(\theta) &= CSD_{avg} + \delta CSD(\theta) = CSD_{avg} + CSD_{max} g(\theta)
\end{aligned} \tag{11}
$$

$$
\int_0^{\Delta\theta} f(\theta) = 0 \qquad \int_0^{\Delta\theta} g(\theta) = 0 \tag{12}
$$

$$
\begin{aligned}
\beta_{22} =& \frac{1}{2}\frac{1}{L}\frac{R_f}{t_{90^\circ}} COD_{avg}\int_0^{\Delta\theta}\left(1 + \sin(2\theta) - \cos(2\theta)\right) d\theta + \\
&+ \frac{1}{2}\frac{1}{L}\frac{R_f}{t_{90^\circ}} COD_{max}\int_0^{\Delta\theta} f(\theta)\left(1 + \sin(2\theta) - \cos(2\theta)\right) d\theta + \\
&- \frac{1}{2}\frac{1}{L}\frac{R_f}{t_{90^\circ}} CSD_{avg}\int_0^{\Delta\theta}\left(1 + \sin(2\theta) - \cos(2\theta)\right) d\theta + \\
&- \frac{1}{2}\frac{1}{L}\frac{R_f}{t_{90^\circ}} CSD_{max}\int_0^{\Delta\theta} g(\theta)\left(1 + \sin(2\theta) - \cos(2\theta)\right) d\theta
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
\int_0^{\Delta\theta} \left(1+\sin\left(2\theta\right)-\cos\left(2\theta\right)\right) d\theta &= \left[\theta - \frac{1}{2}\cos\left(2\theta\right) - \frac{1}{2}\sin\left(2\theta\right)\right]_0^{\Delta\theta} = \\
&= \Delta\theta + \frac{1}{2}\left(1-\cos\left(2\Delta\theta\right)\right) - \frac{1}{2}\sin\left(2\Delta\theta\right) = \\
&= \Delta\theta - \frac{1}{2}\left(\sqrt{2}\sin\left(2\Delta\theta + \frac{\pi}{4}\right) - 1\right)
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&\int_0^{\Delta\theta} f\left(\theta\right)\left(1+\sin\left(2\theta\right)-\cos\left(2\theta\right)\right) d\theta = \\
&= \cancelto{0}{\int_0^{\Delta\theta} f\left(\theta\right) d\theta} - \sqrt{2}\int_0^{\Delta\theta} f\left(\theta\right)\sin\left(\frac{\pi}{4} - 2\Delta\theta\right) d\theta
\end{aligned}
\tag{15}
$$