Finite Element solution of the fiber/matrix interface crack problem: convergence properties and mode mixity of the Virtual Crack Closure Technique

Luca Di Stasio[a,b], Janis Varna[b], Zoubir Ayadi[a]

[a] *Université de Lorraine, EEIGM, IJL, 6 Rue Bastien Lepage, F-54010 Nancy, France*
[b] *Luleå University of Technology, University Campus, SE-97187 Luleå, Sweden*

Abstract

Priority: 3

Target journal(s): Engineering Fracture Mechanics, Theoretical and Applied Fracture Mechanics, International Journal of Fracture

---

1. Introduction

Bi-material interfaces represent the basic load transfer mechanism at the heart of Fiber Reinforced Polymer Composite (FRPC) materials. They are present at the macroscale, in the form of adhesive joints; at the mesoscale, as interfaces between layers with different orientations; at the microscale, as fiber-matrix interfaces. Bi-material interfaces have for long attracted the attention of researchers in Fracture Mechanics [1, 2], due to their hidden complexity.

The problem was first addressed in the 1950's by Williams [3], who derived through a linear elastic asymptotic analysis the stress distribution around an *open* crack (with crack faces nowhere in contact for any size of the crack) between two infinite half-planes of dissimilar materials and found the existence of a strong oscillatory behavior in the stress singularity at the crack tip of the form

$$r^{-\frac{1}{2}}\sin\left(\varepsilon\log r\right) \quad \text{with} \quad \varepsilon=\frac{1}{2\pi}\log\left(\frac{1-\beta}{1+\beta}\right); \tag{1}$$

in which β is one of the two parameters introduced by Dundurs [4] to characterize bi-material interfaces:

$$\beta = \frac{\mu_2\left(\kappa_1 - 1\right) - \mu_1\left(\kappa_2 - 1\right)}{\mu_2\left(\kappa_1 + 1\right) + \mu_1\left(\kappa_2 + 1\right)} \tag{2}$$

where $\kappa = 3 - 4\nu$ in plane strain and $\kappa = \frac{3-4\nu}{1+\nu}$ in plane stress, μ is the shear modulus, ν Poisson's coefficient, and indexes $1, 2$ refer to the two bulk materials joined at the interface. Defining a as the length of the crack, it was found that the size of the oscillatory region is in the order of $10^{-6}a$ [5]. Given the oscillatory behaviour of the crack tip singularity of the stress field of Eq. 1, the definition of Stress Intensity Factor (SIF) $\lim_{r\rightarrow 0}\sqrt{2\pi r}\sigma$ ceases to be valid as it returns logarithmically infinite terms [1]. Furthermore, it implies that the Mode mixity problem at the crack tip is ill-posed.

It was furthermore observed, always in the context of Linear Elastic Fracture Mechanics (LEFM), that an interpenetration zone exists close to the crack tip [6, 7] with a length in the order of 10^{-4} [6]. Following conclusions firstly proposed in[7], the presence of a *contact zone* in the crack tip neighborhood, of a length to be determined from the solution of the elastic problem, was introduced in [8] and shown to provide a physically consistent solution to the straight bi-material interface crack problem.

The curved bi-material interface crack, more often refered to as the fiber-matrix interface crack (or debond) due to its relevance in FRPCs, was first treated by England [9] and by Perlman and Sih [10], who provided the analytical solution of stress and displacement fields for a circular inclusion with respectively a single debond and an arbitrary number of debonds. Building on their work, Toya [11] particularized the solution and provided the expression of the Energy Release Rate (ERR) at the crack tip. The same problems exposed previously for the *open* straight bi-material were shown to exist also for the *open* fiber-matrix interface crack: the presence of strong oscillations in the crack tip singularity and crack face interpenetration after a critical initial flaw size.[1]

[1]For the fiber-matrix interface crack, flaw size is measured in terms of the angle $\Delta\theta$ sub-

In order to treat cases more complex than the single partially debonded fiber in an infinite matrix of [9, 10, 11], numerical studies followed. In the 1990's, París and collaborators [12] developed a Boundary Element Method (BEM) with the use of discontinuous singular elements at the crack tip and the Virtual Crack Closure Integral (VCCI) [13] for the evaluation of the Energy Release Rate (ERR). They validated their results [12] with respect to Toya's analytical solution [11] and analyzed the effect of BEM interface discretization on the stress field in the neighborhood of the crack tip [14]. Following Comninou's work on the straight crack [8], they furthermore recognized the importance of contact to retrieve a physical solution avoiding interpenetration [12] and studied the effect of the contact zone on debond ERR [15]. Their algorithm was then applied to investigate the fiber-matrix interface crack under different geometrical configurations and mechanical loadings [16, 17, 18, 19, 20, 21, 22].

Recently the Finite Element Method (FEM) was also applied to the solution of the fiber-matrix interface crack problem [23, 24, 25], in conjunction with the Virtual Crack Closure Technique (VCCT) [26, 27] for the evaluation of the ERR at the crack tip. In [23], the authors validated their model with respect to the BEM results of [12], but no analysis of the effect of the discretization in the crack tip neighborhood comparable to [14] was proposed. Thanks to the interest in evaluating the ERR of interlaminar delamination, different studies exist in the literature on the effect of mesh discretization on Mode I and Mode II ERR of the bi-material interface crack when evaluated with the VCCT in the context of the FEM [28, 29, 30]. However, no comparable analysis can be found in the literature on the application of the VCCT to the fiber-matrix interface crack (circular bi-material interface crack) problem in the context of a linear elastic FEM solution. It is this gap that the present work aims to address. We first present the FEM formulation of the problem, together with the main geometrical characteristics, material properties, boundary conditions and loading. We then propose a vectorial formulation of the VCCT and express

tended by half of the arc-crack, i.e. $a = 2\Delta\theta$.

the Mode I and Mode II ERR in terms of the FEM natural variables. With this tool, we derive an analytical estimate of the ERR convergence and compare it with numerical results.

2. FEM formulation of the fiber-matrix interface crack problem

3. Vectorial formulation of the Virtual Crack Closure Technique (VCCT)

3.1. Foundational relations

3.2. Formulation of the ERR with respect to FEM variables

4. Convergence analysis

4.1. Analytical considerations

4.2. Numerical results

5. Conclusions & Outlook

Acknowledgements

Luca Di Stasio gratefully acknowledges the support of the European School of Materials (EUSMAT) through the DocMASE Doctoral Programme and the European Commission through the Erasmus Mundus Programme.

References

[1] M. Comninou, An overview of interface cracks, Engineering Fracture Mechanics 37 (1) (1990) 197–208. `doi:10.1016/0013-7944(90)90343-f`.

[2] D. Hills, J. Barber, Interface cracks, International Journal of Mechanical Sciences 35 (1) (1993) 27–37. `doi:10.1016/0020-7403(93)90062-y`.

[3] M. L. Williams, The stresses around a fault or crack in dissimilar media, Bulletin of the Seismological Society of America 49 (2) (1959) 199.

[4] J. Dundurs, Discussion: “edge-bonded dissimilar orthogonal elastic wedges under normal and shear loading” (bogy, d. b., 1968, ASME j. appl. mech., 35, pp. 460–466), Journal of Applied Mechanics 36 (3) (1969) 650. doi:10.1115/1.3564739.

[5] F. Erdogan, Stress distribution in a nonhomogeneous elastic plane with cracks, Journal of Applied Mechanics 30 (2) (1963) 232. doi:10.1115/1.3636517.

[6] A. H. England, A crack between dissimilar media, Journal of Applied Mechanics 32 (2) (1965) 400. doi:10.1115/1.3625813.

[7] B. Malyshev, R. Salganik, The strength of adhesive joints using the theory of cracks, International Journal of Fracture Mechanics 1-1 (2). doi:10.1007/bf00186749.
URL https://doi.org/10.1007/bf00186749

[8] M. Comninou, The interface crack, Journal of Applied Mechanics 44 (4) (1977) 631. doi:10.1115/1.3424148.
URL https://doi.org/10.1115/1.3424148

[9] A. H. England, An arc crack around a circular elastic inclusion, Journal of Applied Mechanics 33 (3) (1966) 637. doi:10.1115/1.3625132.

[10] A. Perlman, G. Sih, Elastostatic problems of curvilinear cracks in bonded dissimilar materials, International Journal of Engineering Science 5 (11) (1967) 845–867. doi:10.1016/0020-7225(67)90009-2.

[11] M. Toya, A crack along the interface of a circular inclusion embedded in an infinite solid, Journal of the Mechanics and Physics of Solids 22 (5) (1974) 325–348. doi:10.1016/0022-5096(74)90002-7.

[12] F. París, J. C. Caño, J. Varna, The fiber-matrix interface crack — a numerical analysis using boundary elements, International Journal of Fracture 82 (1) (1996) 11–29. doi:10.1007/bf00017861.

[13] G. R. Irwin, Fracture, in: Elasticity and Plasticity / Elastizität und Plastizität, Springer Berlin Heidelberg, 1958, pp. 551–590. doi:10.1007/978-3-642-45887-3_5.

[14] J. C. D. Caño, F. París, On stress singularities induced by the discretization in curved receding contact surfaces: a bem analysis, International Journal for Numerical Methods in Engineering 40 (12) (1997) 2301–2320. doi:10.1002/(sici)1097-0207(19970630)40:12<2301::aid-nme166>3.0.co;2-8.

[15] J. Varna, F. París, J. C.Caño, The effect of crack-face contact on fiber/matrix debonding in transverse tensile loading, Composites Science and Technology 57 (5) (1997) 523–532. doi:10.1016/s0266-3538(96)00175-3.

[16] F. París, E. Correa, V. Mantič, Kinking of transversal interface cracks between fiber and matrix, Journal of Applied Mechanics 74 (4) (2007) 703. doi:10.1115/1.2711220.

[17] E. Correa, E. Gamstedt, F. París, V. Mantič, Effects of the presence of compression in transverse cyclic loading on fibre–matrix debonding in unidirectional composite plies, Composites Part A: Applied Science and Manufacturing 38 (11) (2007) 2260–2269. doi:10.1016/j.compositesa.2006.11.002.

[18] E. Correa, V. Mantič, F. París, Effect of thermal residual stresses on matrix failure under transverse tension at micromechanical level: A numerical and experimental analysis, Composites Science and Technology 71 (5) (2011) 622–629. doi:10.1016/j.compscitech.2010.12.027.

[19] E. Correa, F. París, V. Mantič, Effect of the presence of a secondary transverse load on the inter-fibre failure under tension, Engineering Fracture Mechanics 103 (2013) 174–189. doi:10.1016/j.engfracmech.2013.02.026.

[20] E. Correa, F. París, V. Mantič, Effect of a secondary transverse load on the inter-fibre failure under compression, Composites Part B: Engineering 65 (2014) 57–68. doi:10.1016/j.compositesb.2014.01.005.

[21] C. Sandino, E. Correa, F. París, Numerical analysis of the influence of a nearby fibre on the interface crack growth in composites under transverse tensile load, Engineering Fracture Mechanics 168 (2016) 58–75. doi:10.1016/j.engfracmech.2016.01.022.

[22] C. Sandino, E. Correa, F. París, Interface crack growth under transverse compression: nearby fibre effect, in: Proceeding of the 18^{th} European Conference on Composite Materials (ECCM-18), 2018.

[23] L. Zhuang, A. Pupurs, J. Varna, R. Talreja, Z. Ayadi, Effects of inter-fiber spacing on fiber-matrix debond crack growth in unidirectional composites under transverse loading, Composites Part A: Applied Science and Manufacturing 109 (2018) 463–471. doi:10.1016/j.compositesa.2018.03.031.

[24] J. Varna, L. Q. Zhuang, A. Pupurs, Z. Ayadi, Growth and interaction of debonds in local clusters of fibers in unidirectional composites during transverse loading, Key Engineering Materials 754 (2017) 63–66. doi:10.4028/www.scientific.net/kem.754.63.

[25] L. Zhuang, R. Talreja, J. Varna, Transverse crack formation in unidirectional composites by linking of fibre/matrix debond cracks, Composites Part A: Applied Science and Manufacturing 107 (2018) 294–303. doi:10.1016/j.compositesa.2018.01.013.

[26] E. Rybicki, M. Kanninen, A finite element calculation of stress intensity factors by a modified crack closure integral, Engineering Fracture Mechanics 9 (4) (1977) 931–938. doi:10.1016/0013-7944(77)90013-3.

[27] R. Krueger, Virtual crack closure technique: History, approach, and appli-

cations, Applied Mechanics Reviews 57 (2) (2004) 109. doi:10.1115/1.1595677.

[28] C. Sun, C. Jih, On strain energy release rates for interfacial cracks in bi-material media, Engineering Fracture Mechanics 28 (1) (1987) 13–20. doi:10.1016/0013-7944(87)90115-9.
URL https://doi.org/10.1016/0013-7944(87)90115-9

[29] M. Manoharan, C. Sun, Strain energy release rates of an interfacial crack between two anisotropic solids under uniform axial strain, Composites Science and Technology 39 (2) (1990) 99–116. doi:10.1016/0266-3538(90)90049-b.
URL https://doi.org/10.1016/0266-3538(90)90049-b

[30] C. Sun, W. Qian, The use of finite extension strain energy release rates in fracture of interfacial cracks, International Journal of Solids and Structures 34 (20) (1997) 2595–2609. doi:10.1016/s0020-7683(96)00157-6.
URL https://doi.org/10.1016/s0020-7683(96)00157-6