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Abstract

Updates for June 2018. Main topics covered:

- comment on recent FEM results with multiple fibers with focus on effect of debond interaction, see Excel workbooks;
- some thoughts (with derivations) on relationship between contact zone onset angle and local ratio of matrix strains;
- a modified proposal about paper writing

1. FEM results with multiple fibers

Geometries studied.

- n fibers on each side: n fibers are placed to the right and to left of the debonded fiber.
- 5 • n fibers on top: n fibers are placed on top of the debonded one, aligned with it.
- n fibers on each side and m fibers on top: n fibers are placed to the right and to the left, then the layer of $2n + 1$ fibers is repeated m times on top; only the central fiber is debonded.
- 10 • The model has symmetry applied to the bottom boundary, i.e. the first layer from the bottom is always of half fibers.
- The top boundary is always free.

Mode I ERRT.

- Effect of debonds on the side: reducing drastically the value of mode I ERR, but shape, debond size of maximum and debond size when $G_I = 0$ remain the same.
- Effect of debonds on top: retards the occurrence of $G_I = 0$.

Mode II ERRT.

- Effect of debonds on the side: reducing drastically the value of mode II ERR, but shape and debond size of maximum remain the same.
- Effect of bonded fiber on top: maximum is shifted towards the right by $\approx 20^\circ$ and there's a plateau

Contact zone.

- Effect of debonds on the side: onset unchanged, size decreased by $\approx 4 - 5^\circ$
- Effect of bonded fiber on top: onset retarded by $\approx 20^\circ$, size reduced by $\approx 15^\circ$

Some thoughts on the mechanics.

- Effect of debonds on the side: it reduces the strain concentration between fibers (strain magnification) and thus the displacement of crack faces. There seems to be a constant (given uncertainty) ratio between values with fibers on the side vs free single fiber for G_I and crack displacements, but it's more elusive for G_{II} because the standard deviation is quite high.
- Effect of debonds on top: it provides the matrix with a free surface on top, which makes it shrink due to Poisson effect toward the center line between the two free surface, i.e. debonds. Thus it shrinks away from the fiber interface, favoring mode I and retarding contact zone onset. But the effect is small compared with the effect of the bounding material which acts in the opposite way.

2. On a relationship between contact zone onset angle and local ratio of matrix strains

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Reference case: square element of matrix in plane strain under applied axial strain. At any given circle in the element, we can compute the strains in a circumferential reference system as:

$$\begin{aligned}\varepsilon_{rr} &= \varepsilon_{xx} \cos^2 \theta + \varepsilon_{zz} \sin^2 \theta + 2\gamma_{xz} \cos \theta \sin \theta \\ \varepsilon_{zz} &= \varepsilon_{xx} \sin^2 \theta + \varepsilon_{zz} \cos^2 \theta - 2\gamma_{xz} \cos \theta \sin \theta \\ \gamma_{rz} &= (\varepsilon_{zz} - \varepsilon_{xx}) \cos \theta \sin \theta + \gamma_{xz} (\cos^2 \theta - \sin^2 \theta)\end{aligned}\tag{1}$$

We can express the radial strain as a function of the axial one through Poisson's ratio and assuming $\gamma_{xz} = 0$

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$$\varepsilon_{rr} = \varepsilon_{xx} \cos^2 \theta - \nu \varepsilon_{xx} \sin^2 \theta = (\cos^2 \theta - \nu \sin^2 \theta) \varepsilon_{xx}\tag{2}$$

From which we can derive the condition for $\varepsilon_{rr} = 0$

$$\cos^2 \theta - \nu \sin^2 \theta = 1 - (1 + \nu) \sin^2 \theta = 0\tag{3}$$

the solution of which is

$$\theta_0 = \sin^{-1} \left(\pm \sqrt{\frac{1}{1 + \nu}} \right)\tag{4}$$

Given an applied traction, i.e. $\varepsilon_{xx} > 0$, the radial strain is positive for

$$\sin^{-1} \left(-\sqrt{\frac{1}{1 + \nu}} \right) < \theta < \sin^{-1} \left(\sqrt{\frac{1}{1 + \nu}} \right)\tag{5}$$

or, remembering $\sin x$ is antisymmetric,

$$-\theta_0 < \theta < \theta_0\tag{6}$$

50 For an epoxy with $\nu = 0.4$, the angle for which the radial strain is zero and then becomes compressive is

$$\theta_0 = 57.69^\circ\tag{7}$$

which is strikingly close to the 60° value for the contact zone onset calculated for the single fiber in infinite matrix (matrix dominated case).

Contact zone onset as a proxy for local strain ratio. The presence of fiber, debond, boundaries and neighbouring material has a perturbative effect on the elastic solution in the matrix. We have seen from the ERR's results that the ratio of axial to transverse strain is one of the key factor, we can use the angle of contact zone onset to gauge the change in their ratio with respect to the undisturbed matrix case using one of the two forms:

$$\begin{aligned}\theta_0 &= \sin^{-1} \left(\pm \sqrt{\frac{1}{1+SR}} \right) \\ SR &= \frac{1}{\sin^2 \theta} - 1\end{aligned}\tag{8}$$

where $SR = -\frac{\varepsilon_{zz}}{\varepsilon_{xx}}$ stands for strain ratio.

Example: single fiber with debond in infinite matrix. For the single fiber with debond in infinite matrix, $\theta_0 \approx 60^\circ$, slightly greater than the 57.69° , which means SR is a bit lower than the pure matrix case. The presence of the fiber and of the debond will locally constrain more the vertical than the horizontal strain, but the effect is small due to the size of the matrix.

3. Proposal about paper writing

I think the new results show that there actually is an interaction effect between debonds. Thus, gathering the old and new results and studying some new cases, I think it might be better to write 2 papers instead of 1.

- *Paper 1: effect of debond interaction in thin and thick plies and its modeling.* We compare the results of: the free single fiber, the single fiber with vertical displacement coupling, the debonded fiber with fibers on the side and free upper surface, the debonded fiber with fibers on top, the debonded fiber with fibers on the side and on top. We might add another

75 case: fibers on the side with vertical displacement coupling on top. We
 focus on the effect of debond interaction in the different directions (longi-
 tudinal, x, and transverse, z) and how good is the single fiber model with
 equivalent boundary conditions. We find (especially in the longitudinal
 case) the minimum number of fibers required for the debonds' interaction
 80 effect to die off.

- *Paper 2: effect of bounding 0° ply in thick- and thin-ply cross-ply lami-
 nates.* We compare results for: free single fiber (as reference), single fiber
 with vertical displacement coupling and linear horizontal displacement on
 top, single fiber with 0° ply on top, debonded fiber with fibers on the side
 85 and vertical displacement coupling and linear horizontal displacement on
 top, debonded fiber with fibers on the side with 0° ply on top. At least 2
 ratios of 0° vs 90° ply thickness are studied: 1 for thick-ply laminate and
 10 for thin-ply laminate. Here again we find the minimum number (this
 time only in the longitudinal direction) of fiber for the debonds' interac-
 90 tion to vanish. Does it change with respect to the free thin and thick ply?
 And between thin- and thick- ply laminates?

- All models are studied at least for $V_f = [30\%, 50\%, 60\%, 65\%]$. The 65%
 case is used because it's the fastest to solve and thus it is useful both
 for debugging and model checking as well as for feedback on the results.
 95 However, it could be of interest also as an extreme case of a very thin,
 very dense ply.