

INVESTIGATION OF SCALING LAWS OF THE FIBER/MATRIX INTERFACE CRACK IN POLYMER COMPOSITES THROUGH FINITE ELEMENT-BASED MICROMECHANICAL MODELING

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Education and Culture

Erasmus Mundus

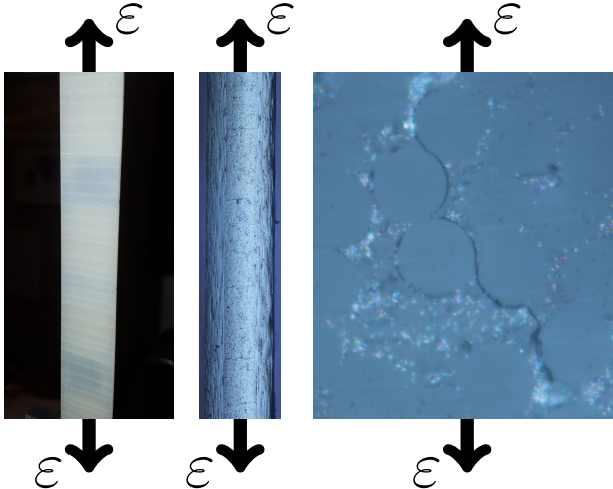


Outline

- Initiation of Transverse Cracking in Fiber Reinforced Polymer Composites (FRPCs): Microscopic Observations & Modeling
- The Fiber-Matrix Interface Crack Problem
- Investigation of Scaling Laws of the Fiber/Matrix Interface Crack

TRANSVERSE CRACKING IN FRPCs

Observations: From Macro to Micro



Left:
front view of $[0, 90_2]_S$,
visual inspection.

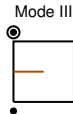
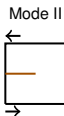
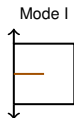
Center:
edge view of $[0, 90]_S$,
optical microscope.

Right:
edge view of $[0, 90]_S$,
optical microscope.

Mathematical Modeling of Fracture: Linear Elastic Fracture Mechanics (LEFM)

Fracture Mode

I, II, III, I/II, I/III, II/III



→ Energy Release Rate: $G \left[\frac{J}{m^2} = \frac{N}{m} \right]$

$$G = \frac{\partial W}{\partial A} - \left(\frac{\partial U}{\partial A} + \frac{\partial E_k}{\partial A} \right)$$

→ Stress Intensity Factor: $K \left[Pa\sqrt{m} \right]$

$$K_{I/II/III} = \lim_{r \rightarrow 0} \sqrt{2\pi r} \cdot \sigma_{I/II/III}(r, 0)$$

Variables

geometry
materials
boundary conditions
loading mode
scale

→ J-Integral: $J \left[\frac{J}{m^2} = \frac{N}{m} \right]$

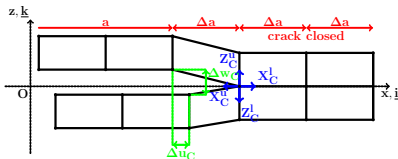
$$J = \lim_{\delta \rightarrow 0} \int_{\Gamma_\delta} \left(W - n_j \sigma_{jk} \frac{\partial u_k}{\partial x_j} \right) d\Gamma$$

→ Average Crack Opening & Shear Displacement:
 $COD, CSD_{II/III} [m]$

$$\left\{ \begin{array}{c} COD \\ CSD_{II} \\ CSD_{III} \end{array} \right\} = \frac{1}{S_C} \int_{S_C} \overrightarrow{\Delta u_C} \cdot \left\{ \begin{array}{c} \vec{n}_I \\ \vec{n}_{II} \\ \vec{n}_{III} \end{array} \right\} dS$$

Numerical Characterization of Fracture: VCCT & J-Integral

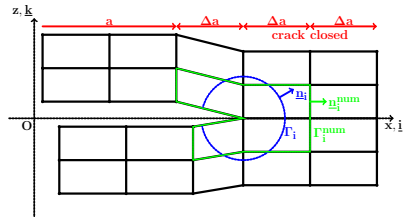
Virtual Crack Closure Technique (VCCT)



$$G_I = \frac{Z_C \Delta w_C}{2B \Delta a} \quad G_{II} = \frac{X_C \Delta u_C}{2B \Delta a}$$

Krueger R.; *Virtual crack closure technique: History, approach, and applications. Appl. Mech. Rev.* **57** (2) 109–143, 2004.

J-Integral

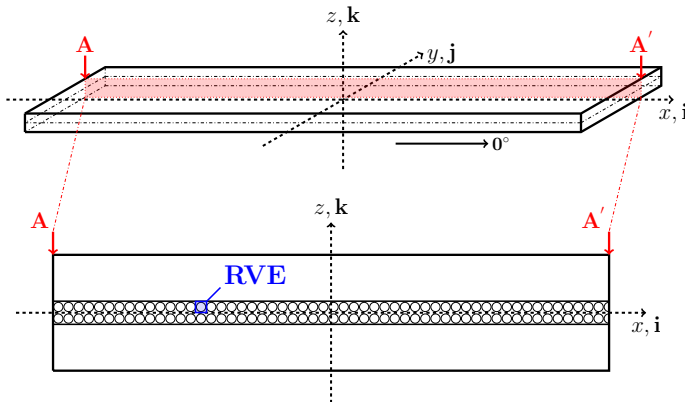


$$J_i = \sum_{k=1}^{n_{\text{segments}}} \sum_{j=1}^{n_{\text{nodes}}} \left[w_j \left(W - n_j \sigma_{jk} \frac{\partial u_k}{\partial x_i} \right) \right]_{(x_{kj}, y_{kj})}$$

Rice J. R.; *A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks. J. Appl. Mech.* **35** (2) 379–386, 1968.

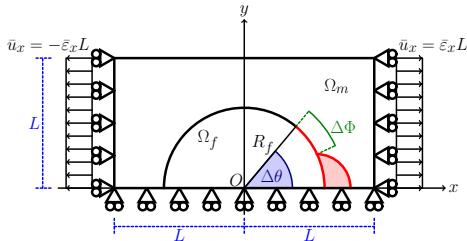
THE FIBER-MATRIX INTERFACE CRACK PROBLEM

The Fiber-Matrix Interface Crack Problem: Geometry



- $L, W \gg t$
- $L, W \rightarrow \infty$
- 2D RVE

The Fiber-Matrix Interface Crack Problem: Assumptions

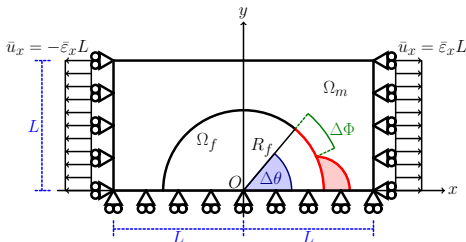


$$R_f = 1 \text{ } [\mu\text{m}] \quad L = \frac{R_f}{2} \sqrt{\frac{\pi}{V_f}}$$

| Material | E_1 | ν_{12} |
|-------------|-------|------------|
| glass fiber | 70.0 | 0.2 |
| epoxy | 3.5 | 0.4 |

- Linear elastic, homogeneous and isotropic materials
- Plane strain
- Frictionless contact interaction
- Symmetric w.r.t. x-axis
- Coupling of x-displacements on left and right side (repeating unit cell)
- Applied uniaxial tensile strain $\bar{\epsilon}_x = 1\%$

The Fiber-Matrix Interface Crack Problem: Solution



in Ω_f, Ω_m :

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\varepsilon_z = \gamma_{zx} = \gamma_{yz} = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

$$\sigma_{zz} = \nu (\sigma_{xx} + \sigma_{yy})$$

for $0^\circ \leq \alpha \leq \Delta\theta$:

$$(\vec{u}_m(R_f, \alpha) - \vec{u}_f(R_f, \alpha)) \cdot \vec{n}_\alpha \geq 0$$

for $\Delta\theta \leq \alpha \leq 180^\circ$:

$$\vec{u}_m(R_f, \alpha) - \vec{u}_f(R_f, \alpha) = 0$$

$$\sigma_{ij} = E_{ijkl} \varepsilon_{kl}$$

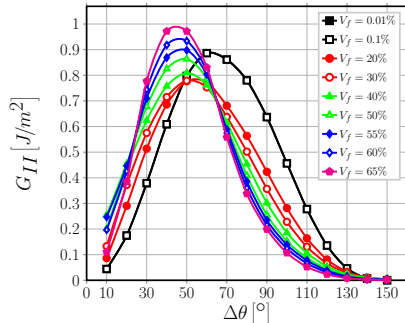
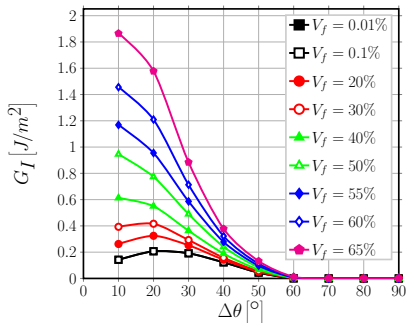
$$+ BC$$

- Finite Element Method (FEM) in AbaqusTM
- 2nd order shape functions
- 6-nodes triangles & 8-nodes quadrilaterals
- regular mesh of quadrilaterals at the crack tip:

$$- AR \sim 1$$

$$- \delta = 0.05^\circ$$

The Fiber-Matrix Interface Crack Problem: Normalization & Scaling



$$(\?) \quad G_I = G_0 (V_f, E_f, \nu_f, E_m, \nu_m, \bar{\epsilon}_x, \Delta\theta) g_I (\Delta\theta, BC, \text{microstructure, damage})$$

$$(\?) \quad G_{II} = G_0 (V_f, E_f, \nu_f, E_m, \nu_m, \bar{\epsilon}_x, \Delta\theta) g_{II} (\Delta\theta, BC, \text{microstructure, damage})$$

INVESTIGATION OF SCALING LAWS

Dimensional Analysis

→ From the definition of Energy Release Rate

$$G = \frac{\partial W}{\partial A} - \left(\frac{\partial U}{\partial A} + \frac{\partial E_k}{\partial A} \right) \quad \left[\frac{J}{m^2} \right]$$

$$\left[\frac{J}{m^2} \right] \longleftrightarrow \frac{E}{L^2} = \frac{F \cdot L}{L^2} = \frac{F}{L^2} \frac{L}{L} \cdot L = \sigma \varepsilon L$$

$$G_0 \sim \sigma_{\infty} \varepsilon_{\infty} L_c$$

→ From the assumption of linear elasticity and uniaxial loading

$$\sigma_{\infty} = E_{eq} \varepsilon_{\infty} \quad \varepsilon_{\infty} = \frac{\sigma_{\infty}}{E_{eq}}$$

$$G_0 \sim E_{eq} \varepsilon_{\infty}^2 L_c \quad G_0 \sim \frac{\sigma_{\infty}^2}{E_{eq}} L_c$$

→ From crack geometry

$$L_c \sim a = R_f \Delta \theta \longrightarrow L_c \sim R_f f(\Delta \theta)$$

$$G_0 = A \cdot E_{eq} \varepsilon_{\infty}^2 R_f f(\Delta \theta)$$

Homogenization of Material Properties: Concentric Cylinders Assembly (CCA)

$$E_L = V_f E_f + (1 - V_f) E_m + 2\lambda_1 (\nu_m - \nu_f)^2 V_f (1 - V_f)$$

$$\nu_{LT} = V_f \nu_f + (1 - V_f) \nu_m + \frac{\lambda_1}{2} (\nu_m - \nu_f) \left(\frac{1}{k_{fT}} - \frac{1}{k_{mT}} \right) V_f (1 - V_f)$$

$$G_{TT} = \frac{E_m}{2(1 + \nu_m)} + \frac{V_f}{\frac{1}{\frac{E_f}{2(1 + \nu_f)} - \frac{E_m}{2(1 + \nu_m)}} + \frac{k_{mT} + \frac{E_m}{1 + \nu_m}}{\frac{E_m}{1 + \nu_m} \left(k_{mT} + \frac{E_m}{2(1 + \nu_m)} \right)}} (1 - V_f)$$

$$K_{TT} = \frac{k_{mT} \left(k_{fT} + \frac{E_m}{2(1 + \nu_m)} \right) (1 - V_f) + k_{fT} \left(k_{mT} + \frac{E_m}{2(1 + \nu_m)} \right) V_f}{\left(k_{fT} + \frac{E_m}{2(1 + \nu_m)} \right) (1 - V_f) + \left(k_{mT} + \frac{E_m}{2(1 + \nu_m)} \right) V_f}$$

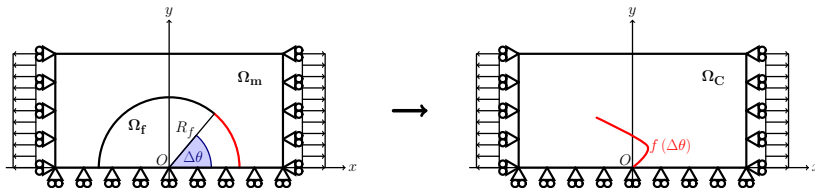
$$E_T = \frac{4G_{TT}}{1 + \frac{\left(1 + \frac{4K_{TT}\nu_{LT}^2}{E_L} \right) G_{TT}}{K_{23}}} \quad \nu_{TT} = \frac{E_T}{2G_{TT}} - 1$$

$$k_{fT} = \frac{E_f}{2(1 - \nu_f - 2\nu_f^2)} \quad k_{mT} = \frac{E_m}{2(1 - \nu_m - 2\nu_m^2)} \quad \lambda_1 = 2 \left(\frac{2(1 + \nu_m)}{E_m} + \frac{V_f}{k_{mT}} + \frac{1 - V_f}{k_{fT}} \right)^{-1}$$

Homogenization of Material Properties: Plane Strain Conditions

$$E_{\text{plane strain}} = \frac{E_T}{1 - \frac{E_T}{E_L} \nu_{LT}^2}$$

$$E_{eq} = \frac{E_T}{1 - \frac{E_T}{E_L} \nu_{LT}^2}$$



References



Kawabe K., Tomoda S. and Matsuo T.; *A pneumatic process for spreading reinforcing fiber tow Proc. 42nd Int. SAMPE USA (Anaheim, CA, USA) 6576, 1997.*



Kawabe K., Tomoda S.; *Method of producing a spread multi-filament bundle and an apparatus used in the same.* Japan: Fukui Prefectural Government; 2003. JP 2003-193895, 2003.



Kawabe K.; *New Spreading Technology for Carbon Fiber Tow and Its Application to Composite Materials Sen'i Gakkaishi* **64** (8) 262–267, 2008 [in Japanese].

References



Sasayama H. and Tomoda S.; *New Carbon Fiber Tow-Spread Technology and Applications to Advanced Composite Materials S.A.M.P.E. journal* **45** (2) 6–17, 2009.



Meijer A.; *NTPT makes worlds thinnest prepreg even thinner [Internet] [cited 30 April 2017]* North Thin Ply Technology (NTPT) press release 2015. Available from http://www.thinplytechnology.com/mesimages/Press_Release_NTPT_16JUN2015.pdf.



oXeon TECHNOLOGIES 2014 [Internet] [cited 30 April 2017] Available from <http://oxeon.se/technologies/>.

References



Donald L. Flaggs, Murat H. Kural; *Experimental Determination of the In Situ Transverse Lamina Strength in Graphite/Epoxy Laminates*. *J. Comp. Mat.* **16** 2, 1982.



Krueger R.; *Virtual crack closure technique: History, approach, and applications* *Appl. Mech. Rev.* **57** (2) 109–143, 2004.



Rice J. R.; *A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks* *J. Appl. Mech.* **35** 379–386, 1968.

References



Toya M.; *A Crack Along the Interface of a Circular Inclusion Embedded in an Infinite Solid* *J. Mech. Phys.* **22** 325–348, 1975.



París F., Caño J. C., Varna J.; *The fiber-matrix interface crack - A numerical analysis using Boundary Elements* *Int. J. Fract.* **82** 1 11–29, 1996.

