# UPDATE 2017-07-17

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# Outline

- Symbols, Models, Equations & Reference Data
- Results

# Symbols, Models, Equations & Reference Data

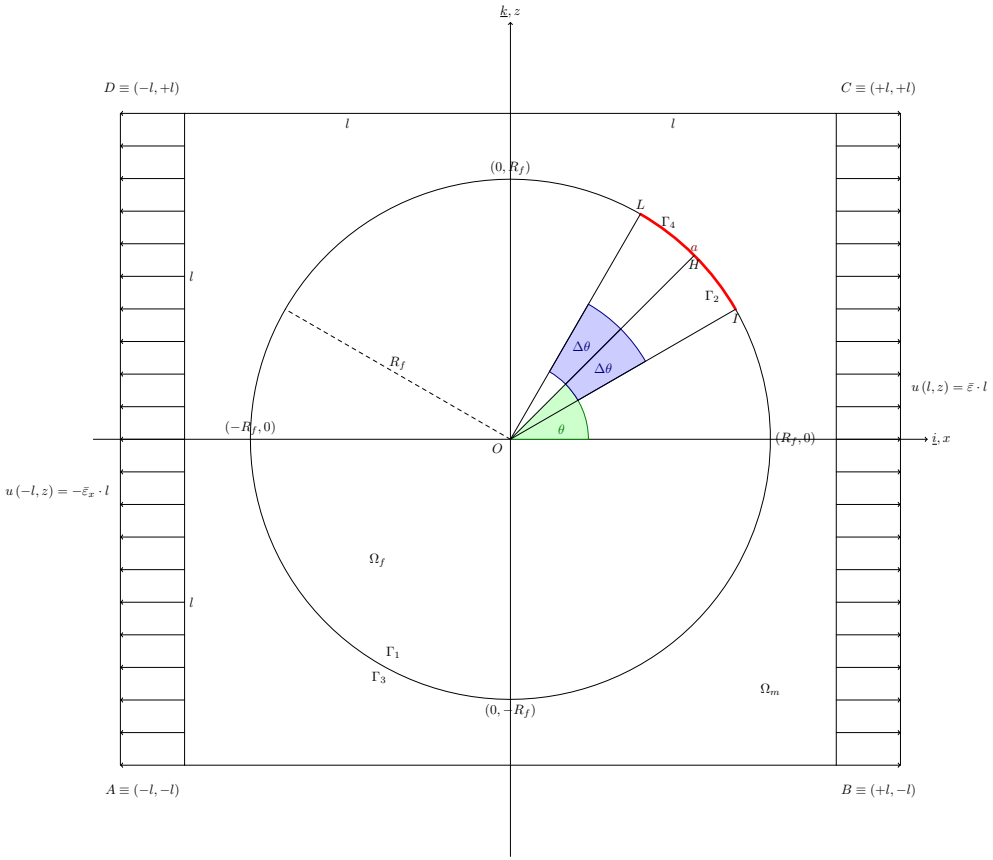
# Symbols

| Symbol | Unit | Description |
|---|---|---|
| $\theta$ | $[^\circ]$ | Debond angular position with respect to the center of the arc defined by the debond itself |
| $\Delta\theta$ | $[^\circ]$ | Debond semi-angular aperture |
| $\delta$ | $[^\circ]$ | Angle subtended by a single element at the fiber/matrix interface |
| $VF_f$ | $[-]$ | Fiber volume fraction |
| $l$ | $[\mu m]$ | Ply's half-length, equal to RVE's half-length (square element) |
| $u$ | $[\mu m]$ | Displacement along x |
| $w$ | $[\mu m]$ | Displacement along z |

# Symbols

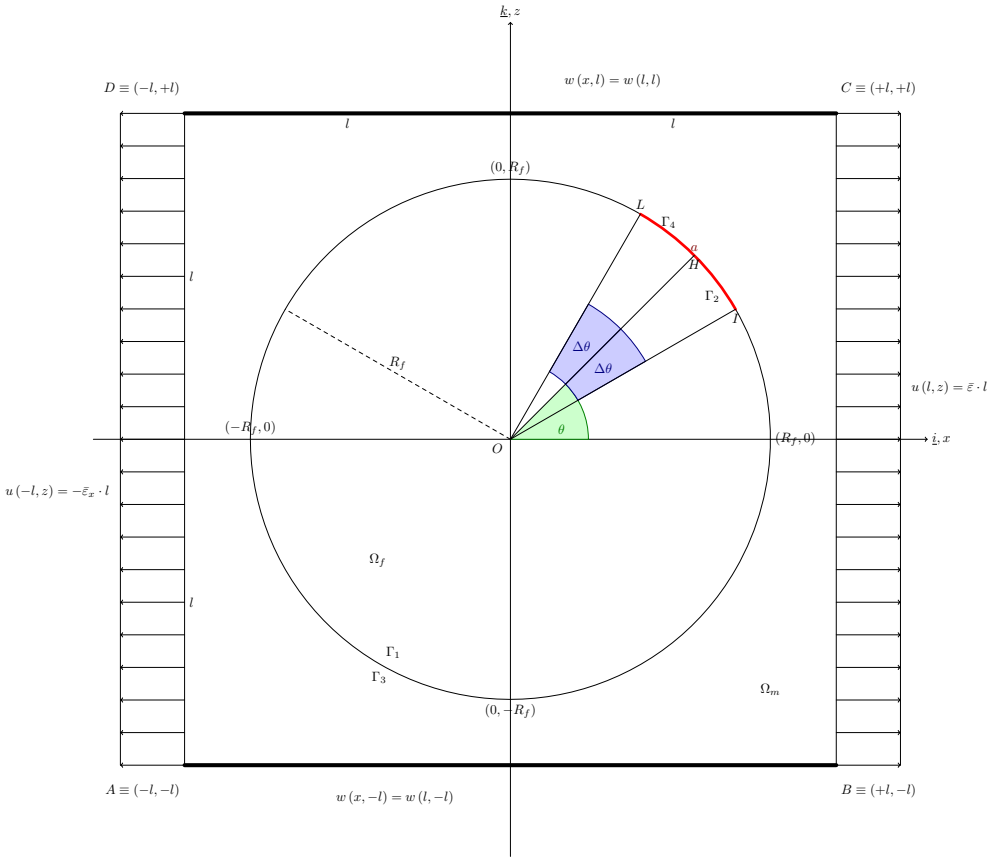
| Symbol | Unit | Description |
|---|---|---|
| $\Gamma_1$ | $[-]$ | Bonded part of fiber surface |
| $\Gamma_2$ | $[-]$ | Free (debonded) part of fiber surface |
| $\Gamma_3$ | $[-]$ | Bonded part of matrix surface |
| $\Gamma_4$ | $[-]$ | Free (debonded) part of matrix surface |

# Reference Models



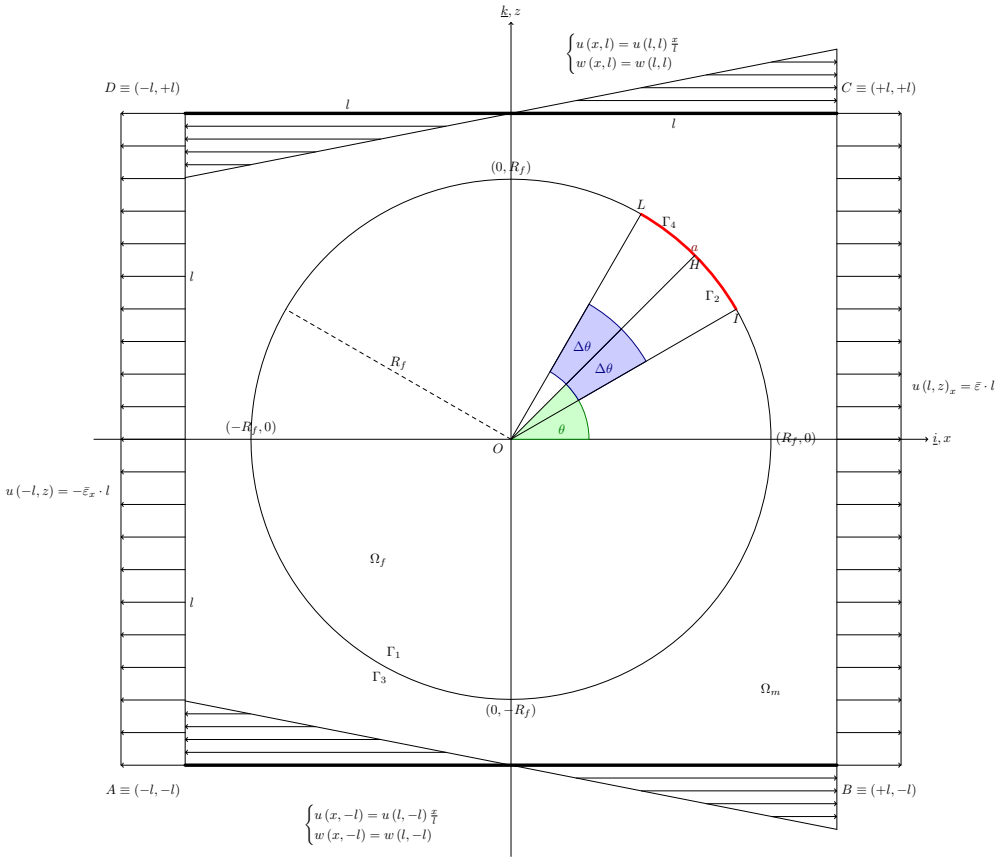

Simple RVE, BC: free.

# Reference Models



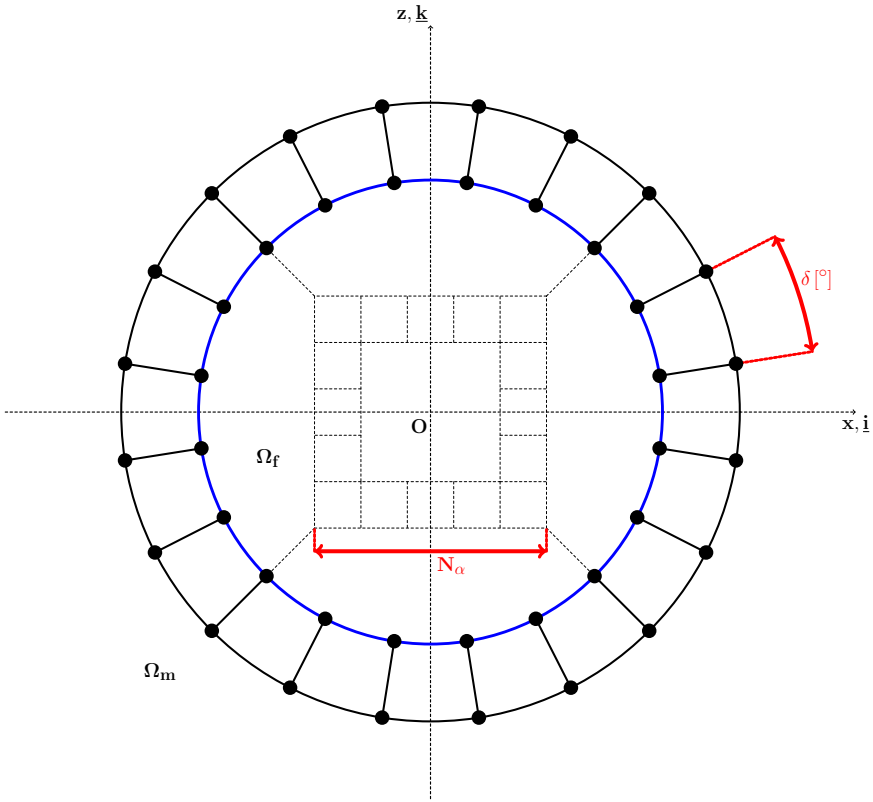

Simple RVE, BC: fixed vertical displacement.

# Reference Models




Simple RVE, BC: fixed vertical and homogeneous horizontal displacement.

# Angular discretization




Angular discretization at fiber/matrix interface: $\delta = \frac{360°}{4N_\alpha}$.

# Material properties

| **Material** | $E\,[GPa]$ | $G\,[GPa]$ | $\nu\,[-]$ |
|---|---|---|---|
| Glass fiber | 70,0 | 29,2 | 0,2 |
| Epoxy | 3,5 | 1,25 | 0,4 |

# Evaluation of $G_0$

$$G_0 = \pi R_f \sigma_0^2 \frac{1 + k_m}{8 G_m} \tag{1}$$

$$k_m = 3 - 4\nu_m \tag{2}$$

$$\sigma_0^{undamaged} = \frac{E_m}{1 - \nu_m^2} \varepsilon_{xx} \tag{3}$$

# VCCT in Forces

$$\Delta u = \left| \Delta u^{matrix}_{\text{1 element before crack tip}} - \Delta u^{fiber}_{\text{1 element before crack tip}} \right| \tag{4}$$

$$\Delta w = \left| \Delta w^{matrix}_{\text{1 element before crack tip}} - \Delta w^{fiber}_{\text{1 element before crack tip}} \right| \tag{5}$$

$$\beta = \arctan\left( \frac{z^{matrix,undef}_{\text{crack tip}}}{x^{matrix,undef}_{\text{crack tip}}} \right) \tag{6}$$

$$\Delta_r = \cos(\beta)\Delta u + \sin(\beta)\Delta w \qquad \Delta_\theta = -\sin(\beta)\Delta u + \cos(\beta)\Delta w \tag{7}$$

$$F_r = \cos(\beta)F_x^{reaction} + \sin(\beta)F_z^{reaction} \qquad F_\theta = -\sin(\beta)F_x^{reaction} + \cos(\beta)F_z^{reaction} \tag{8}$$

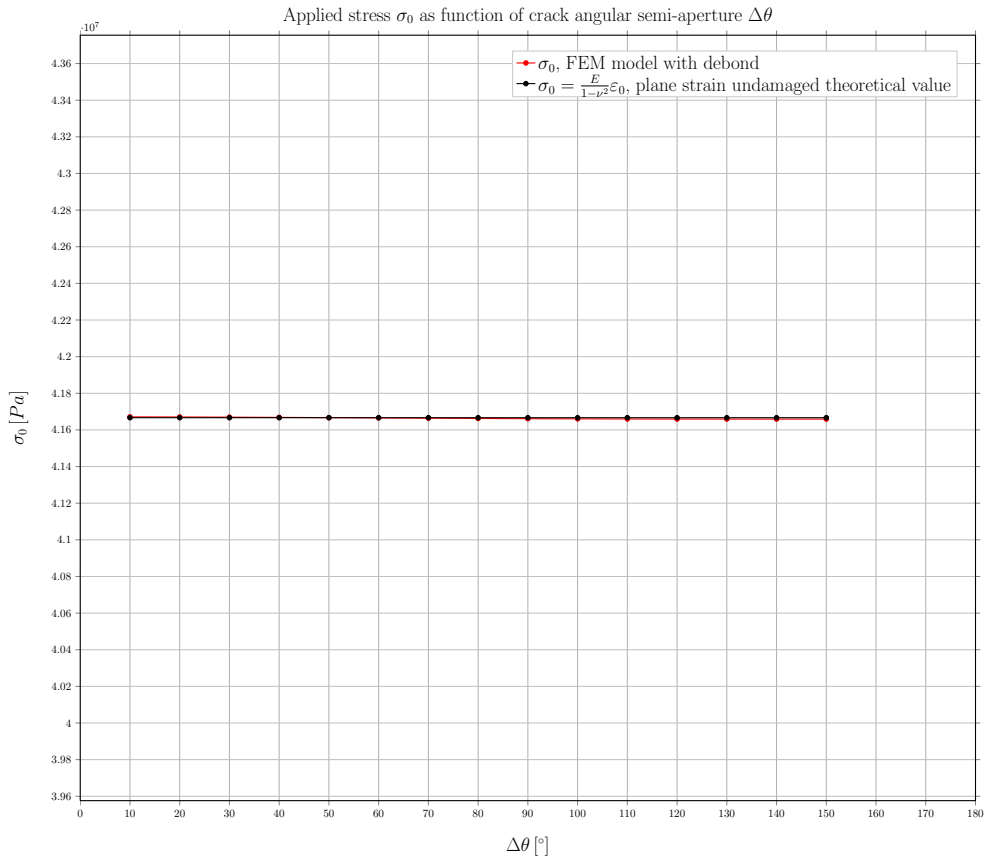
$$G_I = \frac{1}{2}\frac{F_r \Delta_r}{R_f \delta} \qquad G_{II} = \frac{1}{2}\frac{F_\theta \Delta_\theta}{R_f \delta} \qquad b = 1.0 \leftrightarrow \Delta A = b R_f \delta \tag{9}$$

# VCCT in Stresses

# ↘ RESULTS

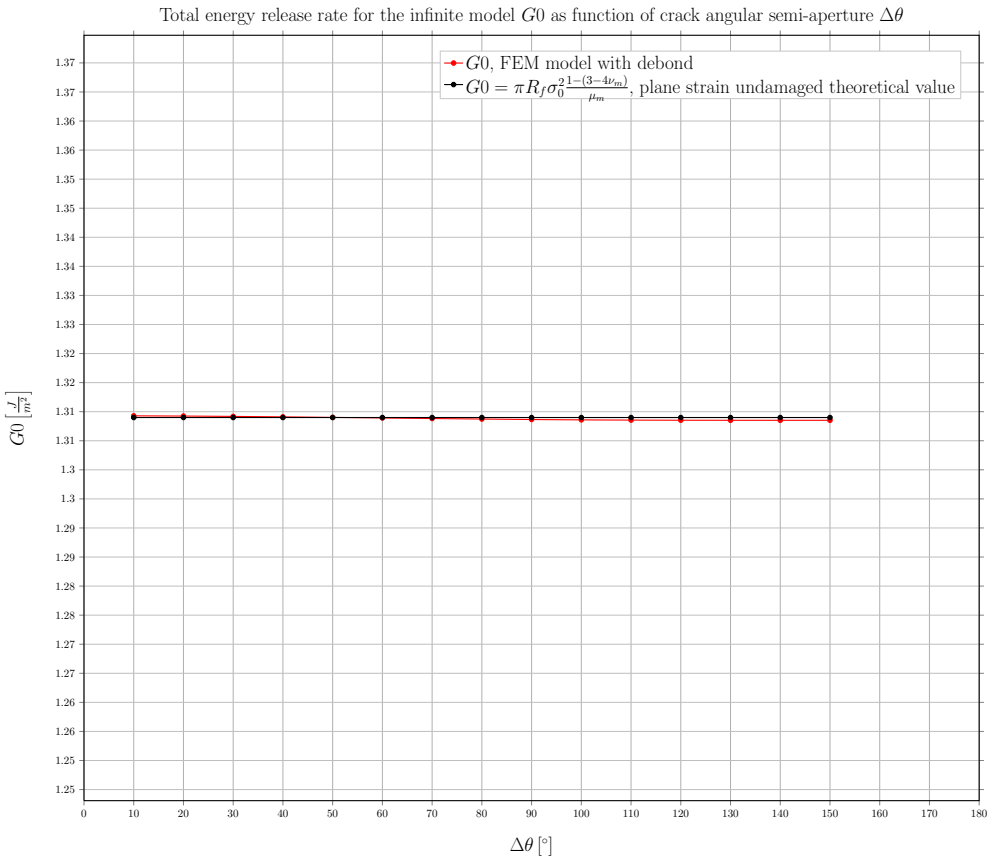
# Model Data

## $\sigma_0$, $\delta = 1.0°$




In red small strain FEM, in black analytical plain strain value.

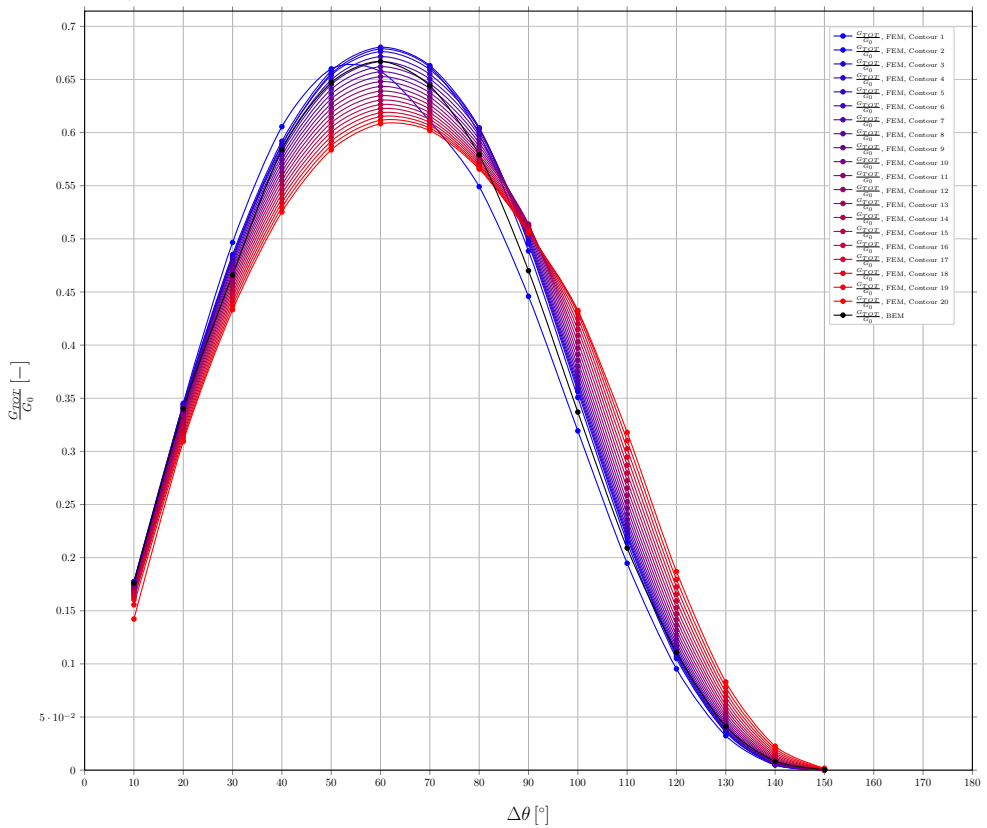
# $G_0$, $\delta = 1.0°$




In red small strain FEM, in black analytical plain strain value.

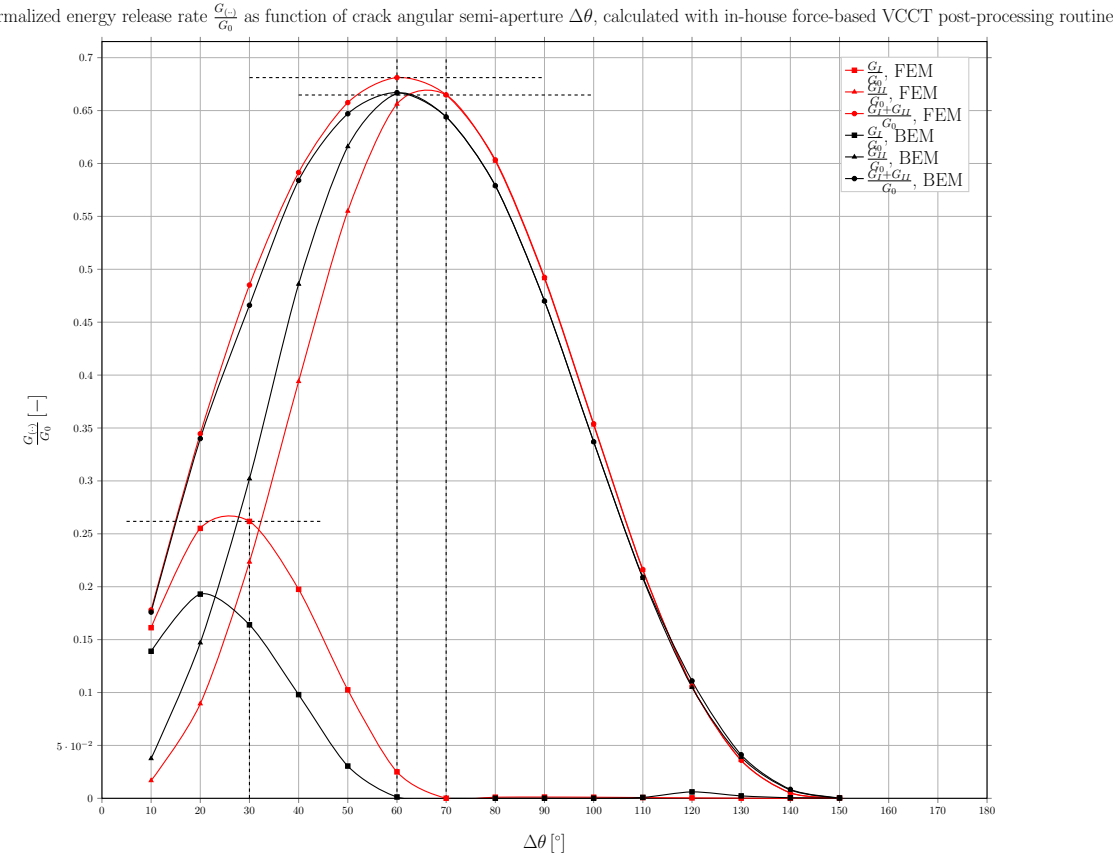
# J-Integral (Abaqus built-in routine), $\delta = 1.0°$

Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)




Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

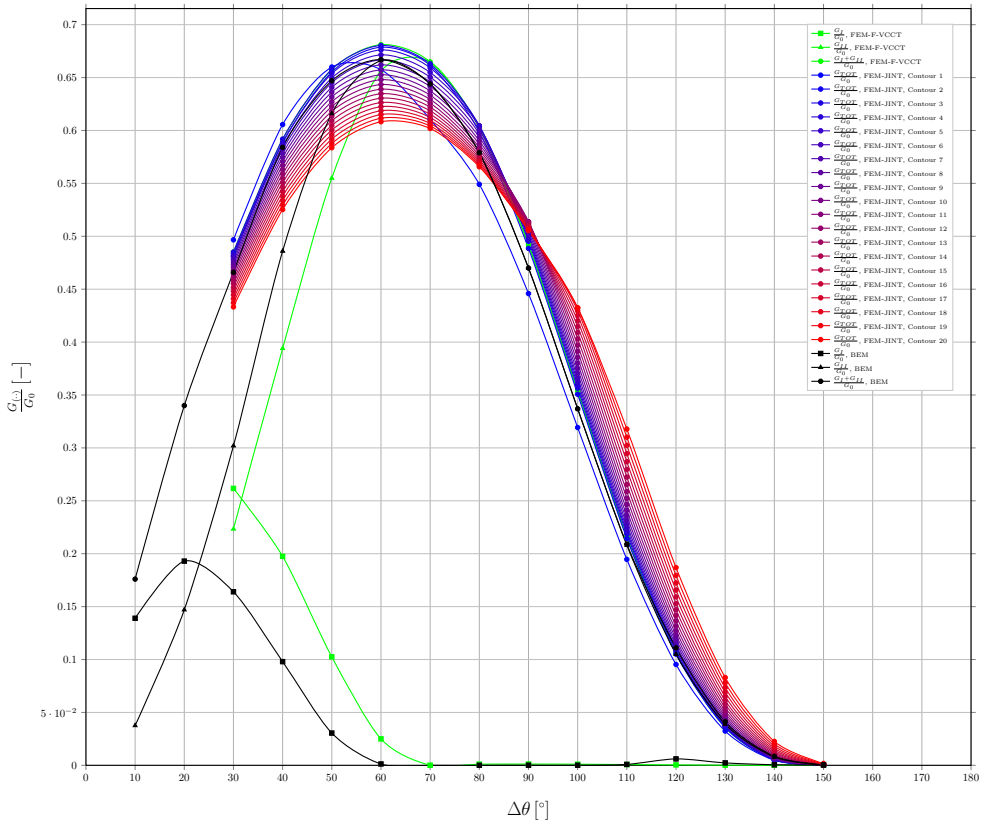
# VCCT in forces (in-house Python routine), $\delta = 1.0°$




In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

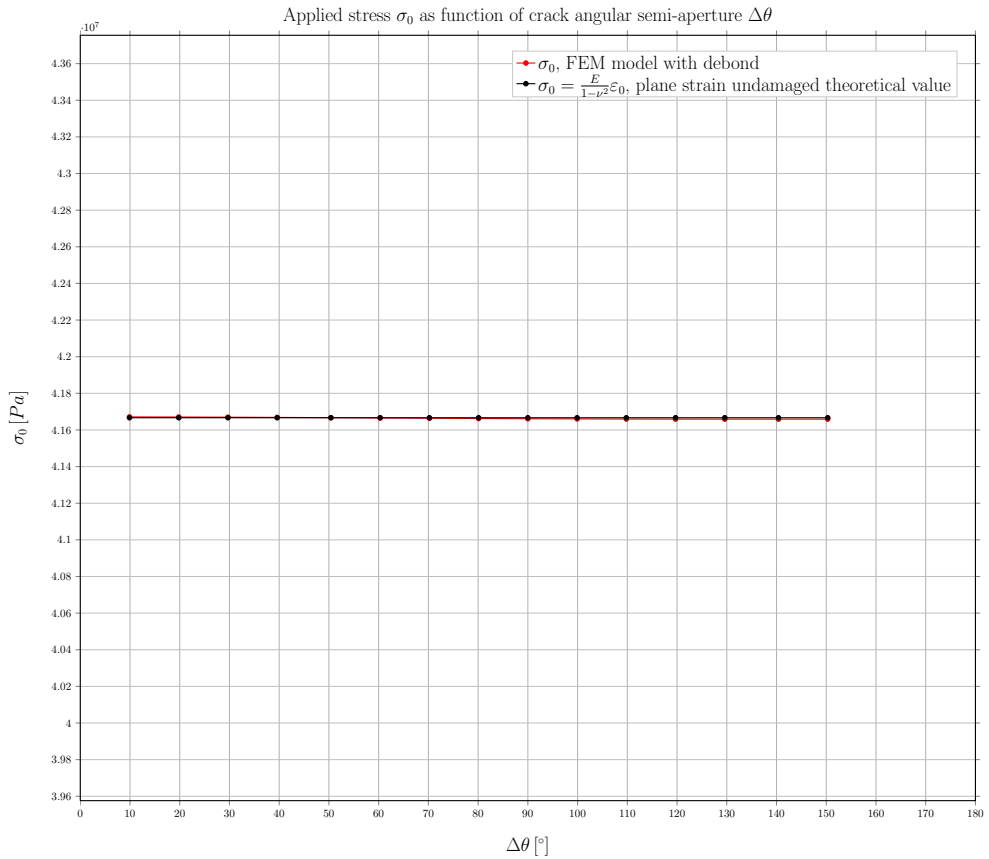
# J-Integral and VCCT in forces, $\delta = 1.0°$

Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines



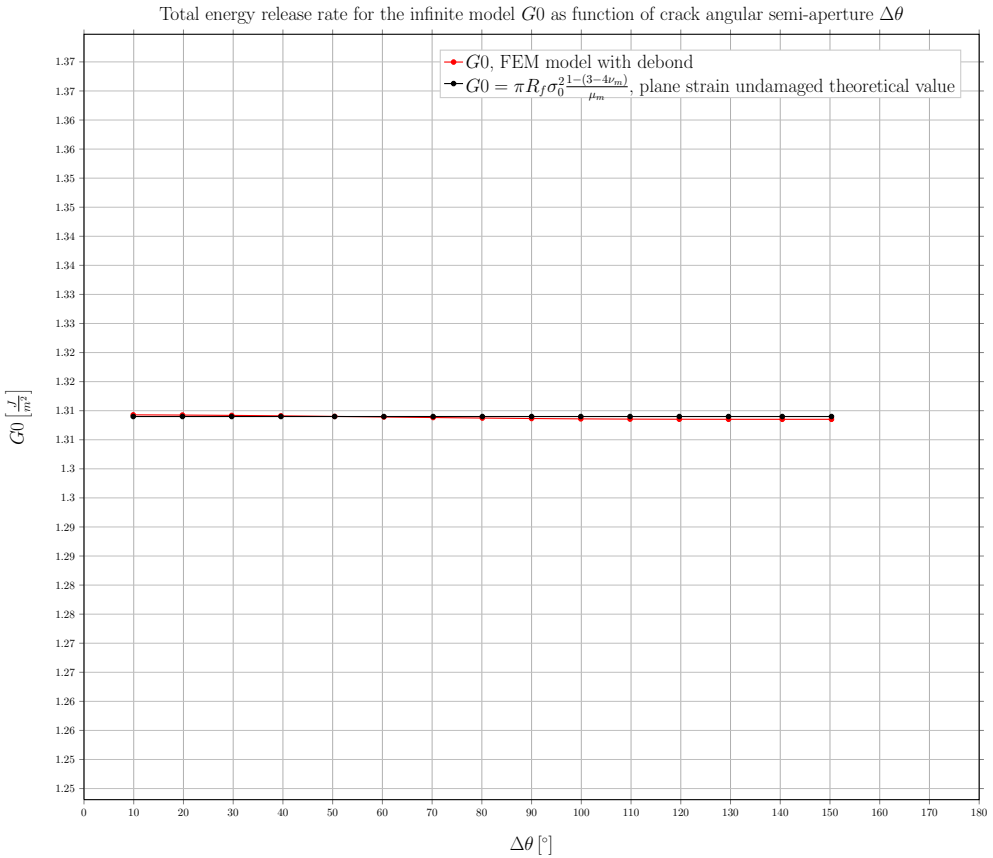

Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.

## $\sigma_0$, $\delta = 0.9°$




In red small strain FEM, in black analytical plain strain value.

# $G_0$, $\delta = 0.9°$

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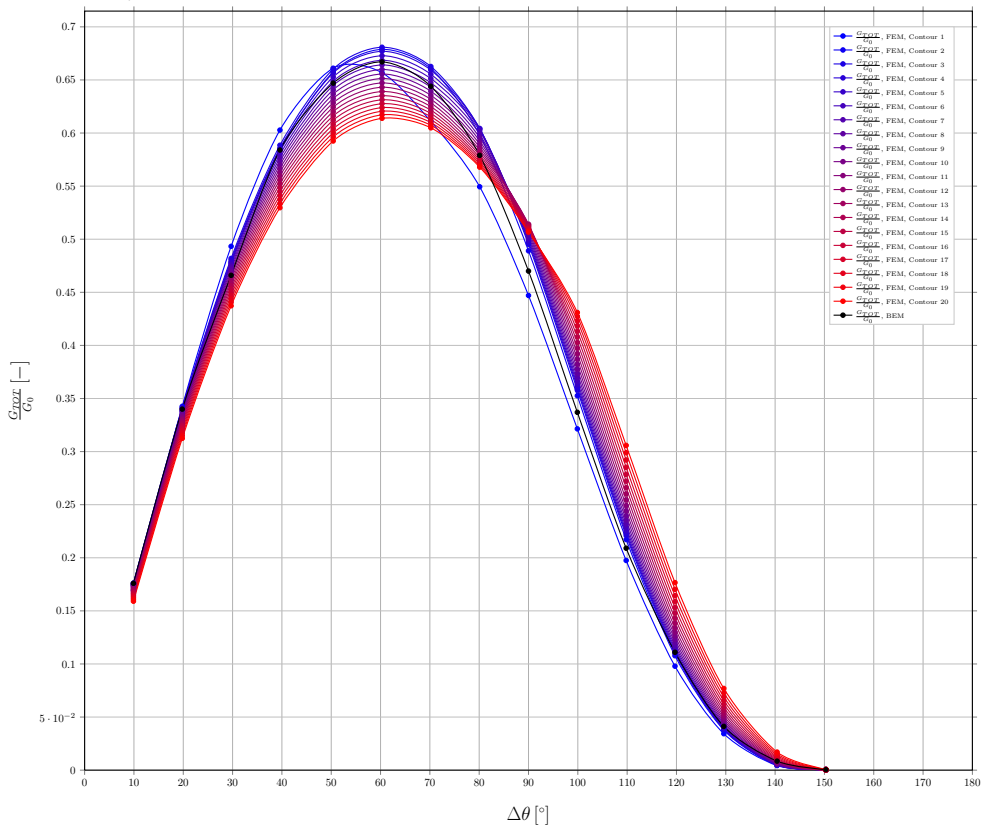
# J-Integral (Abaqus built-in routine), $\delta = 0.9°$

Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)

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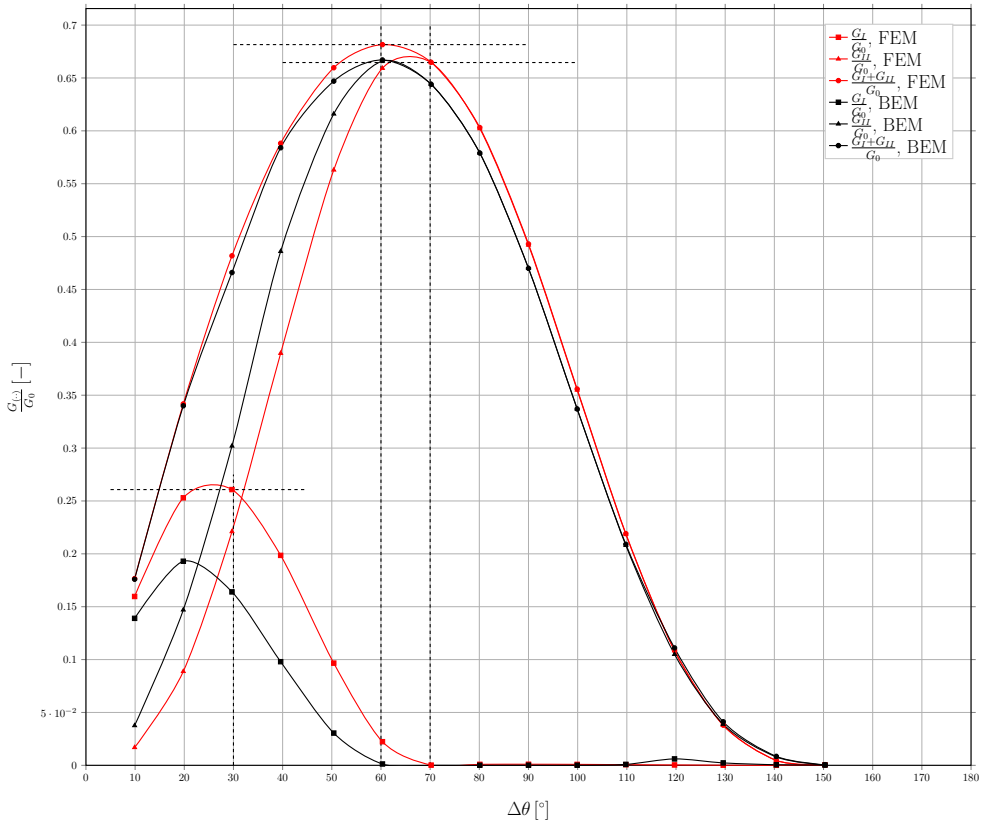
# VCCT in forces (in-house Python routine), $\delta = 0.9°$

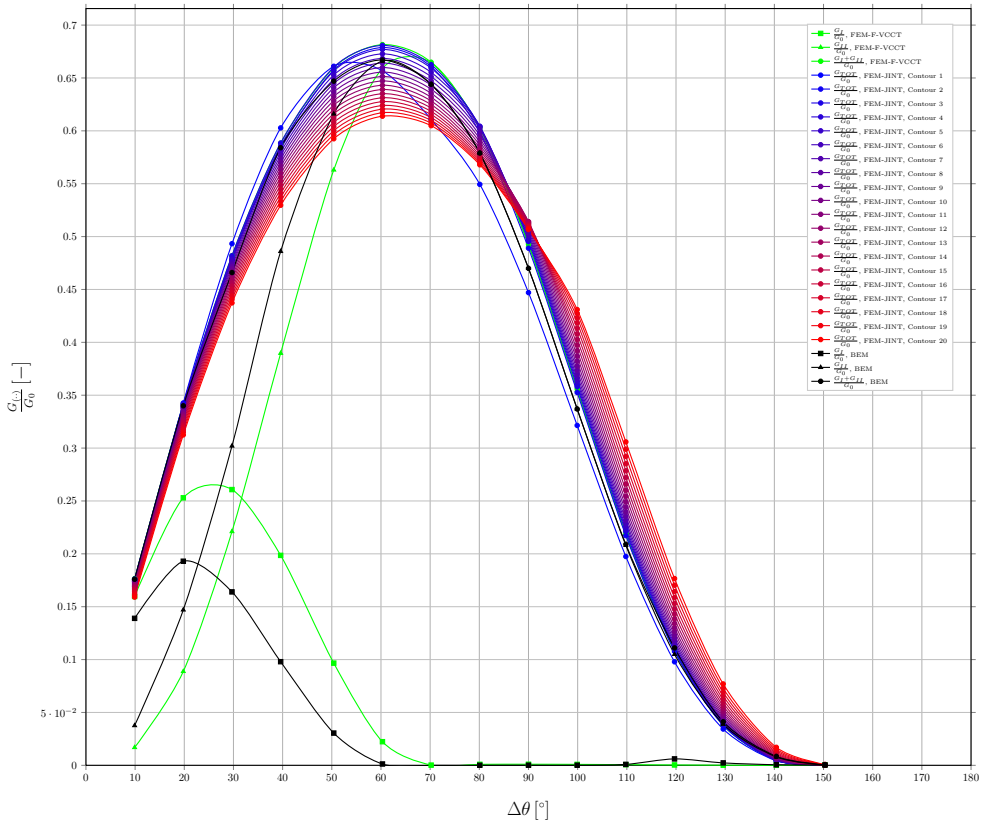
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT post-processing routine




In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

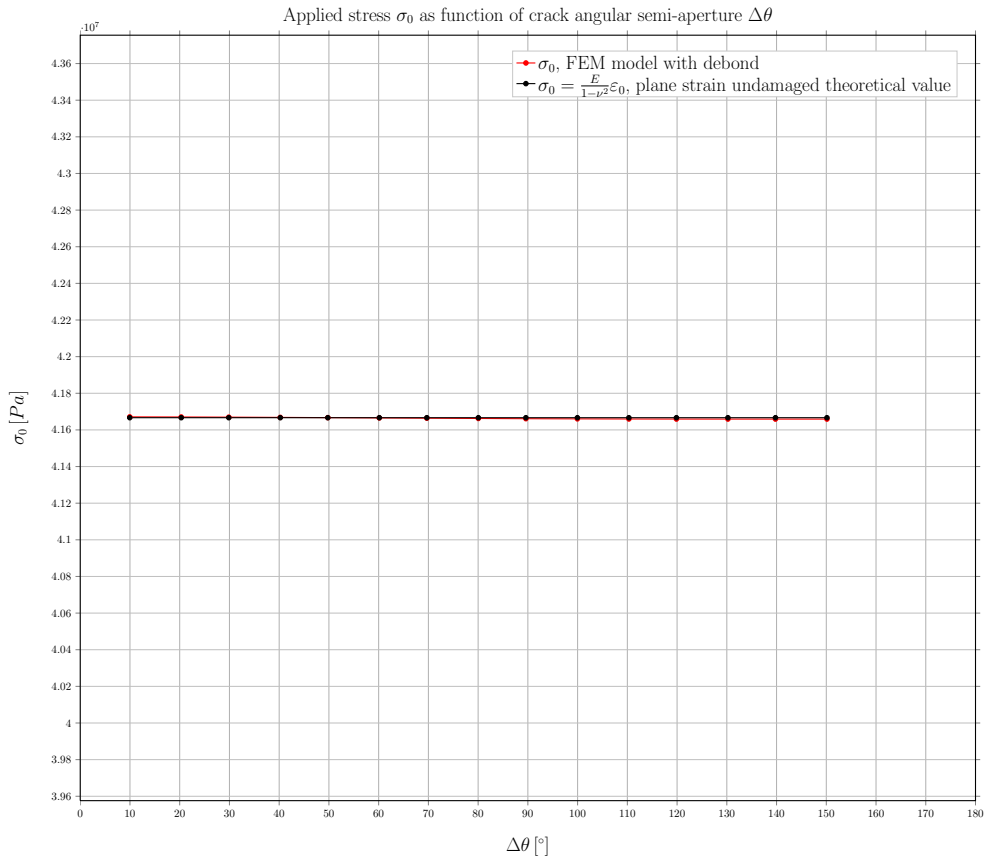
# J-Integral and VCCT in forces, $\delta = 0.9°$

Normalized energy release rate $\frac{G_{(-)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines



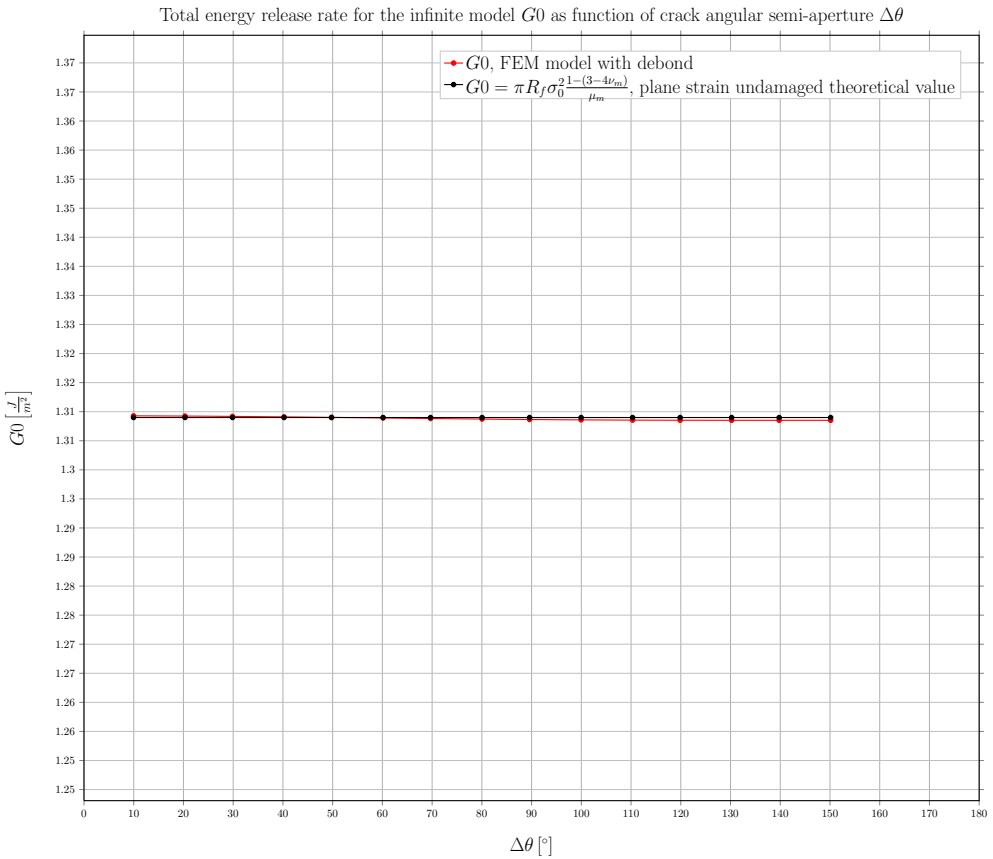

Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.

## $\sigma_0$, $\delta = 0.8°$




In red small strain FEM, in black analytical plain strain value.

## $G_0$, $\delta = 0.8°$

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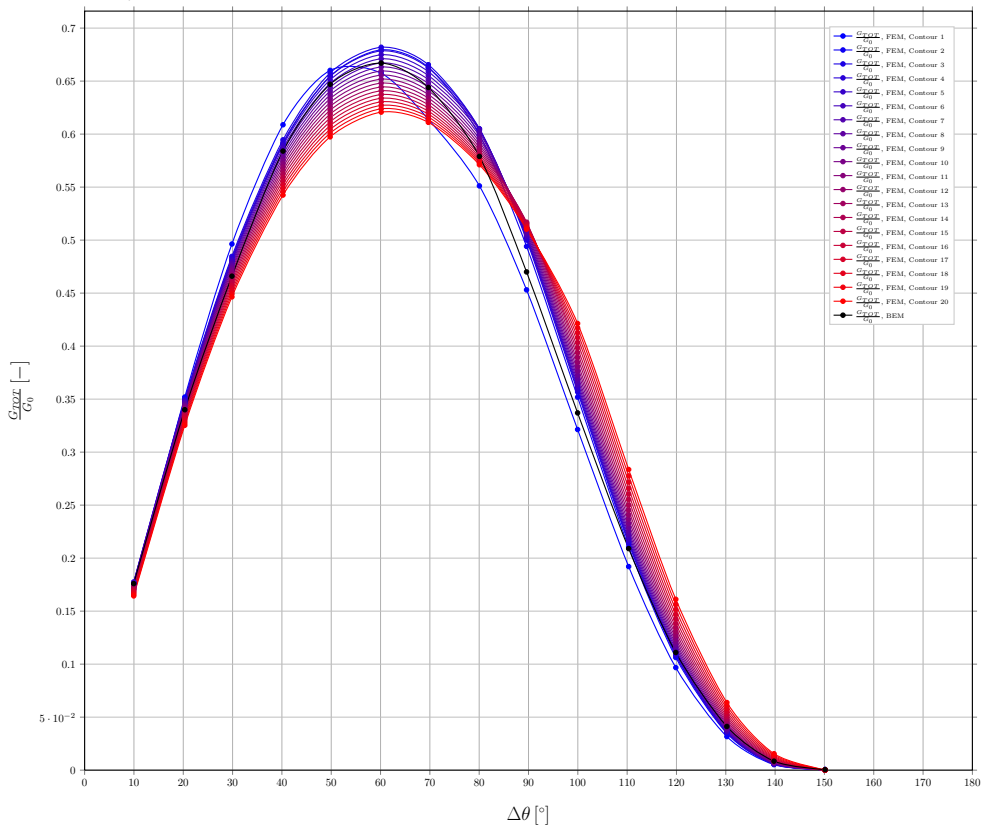
# J-Integral (Abaqus built-in routine), $\delta = 0.8°$

Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)

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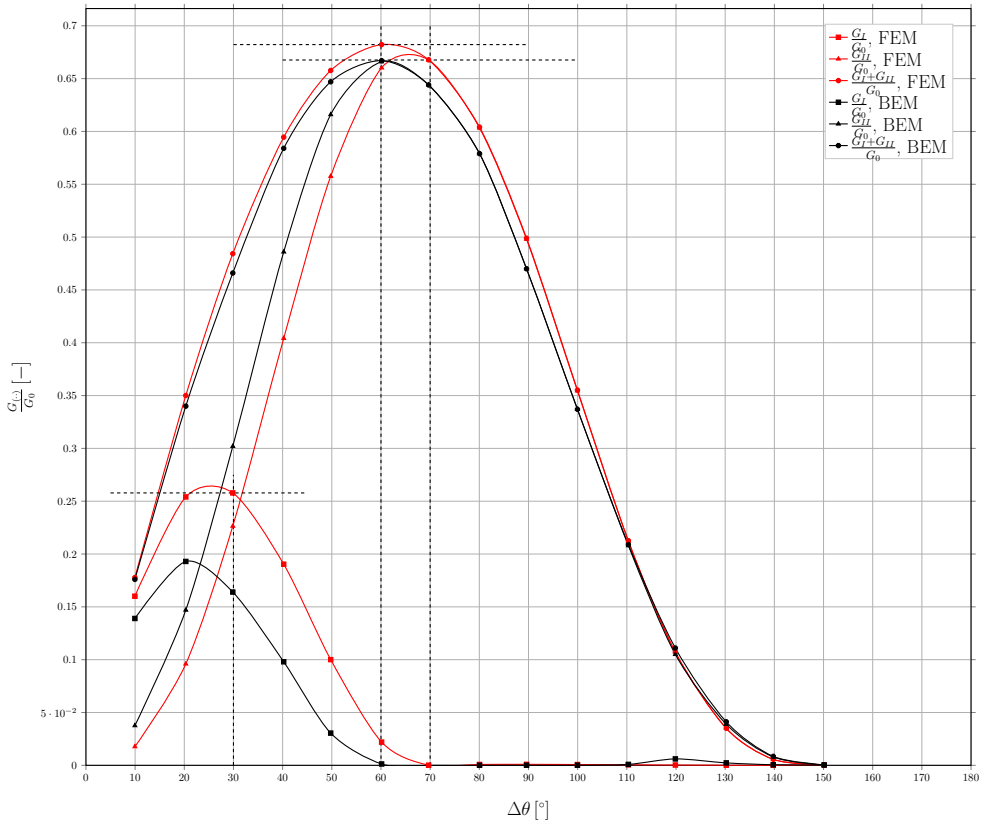
# VCCT in forces (in-house Python routine), $\delta = 0.8°$

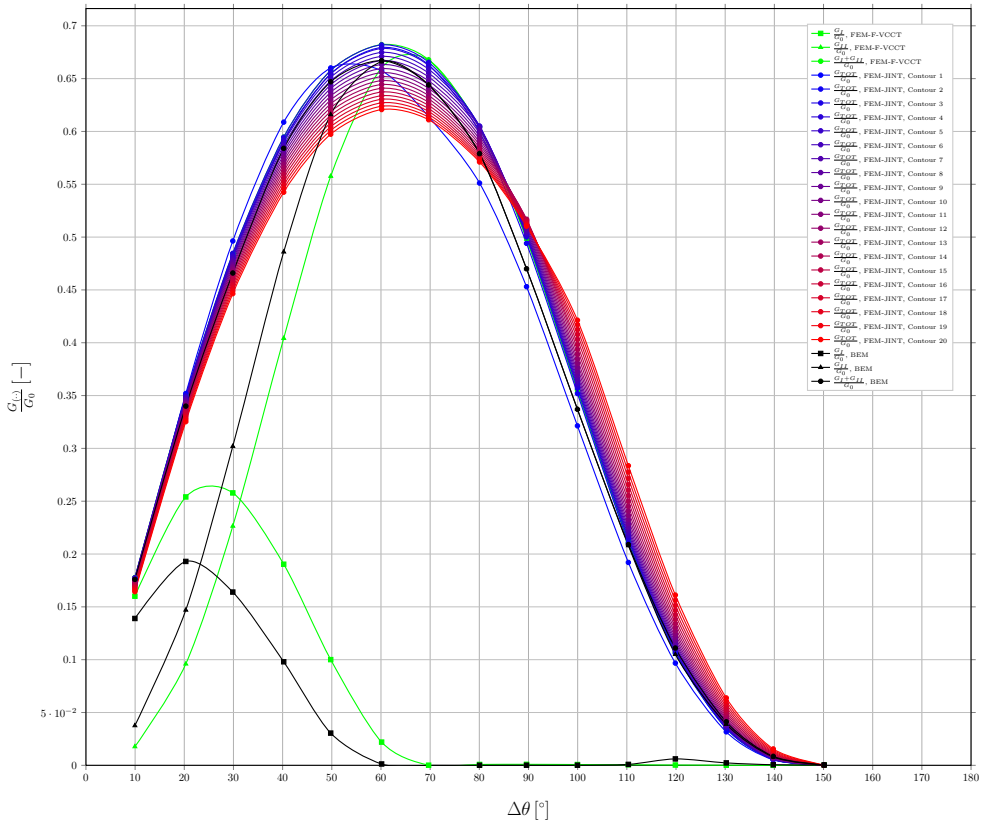
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT post-processing routine




In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

# J-Integral and VCCT in forces, $\delta = 0.8°$

Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines

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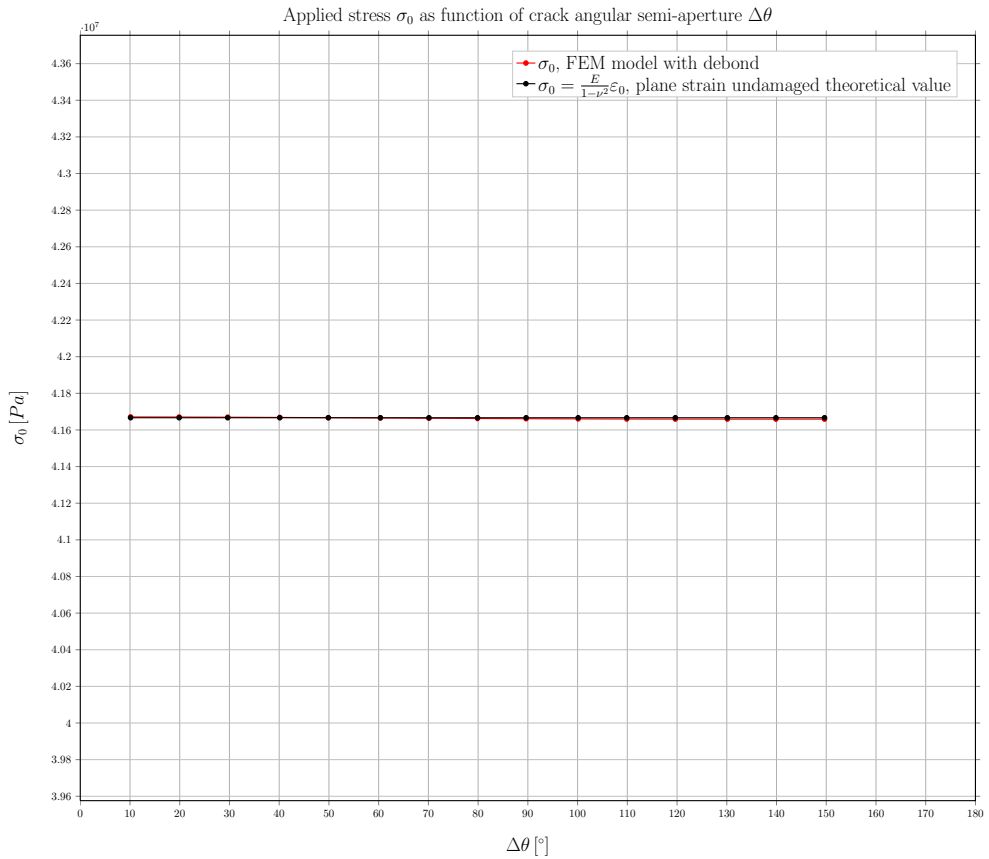
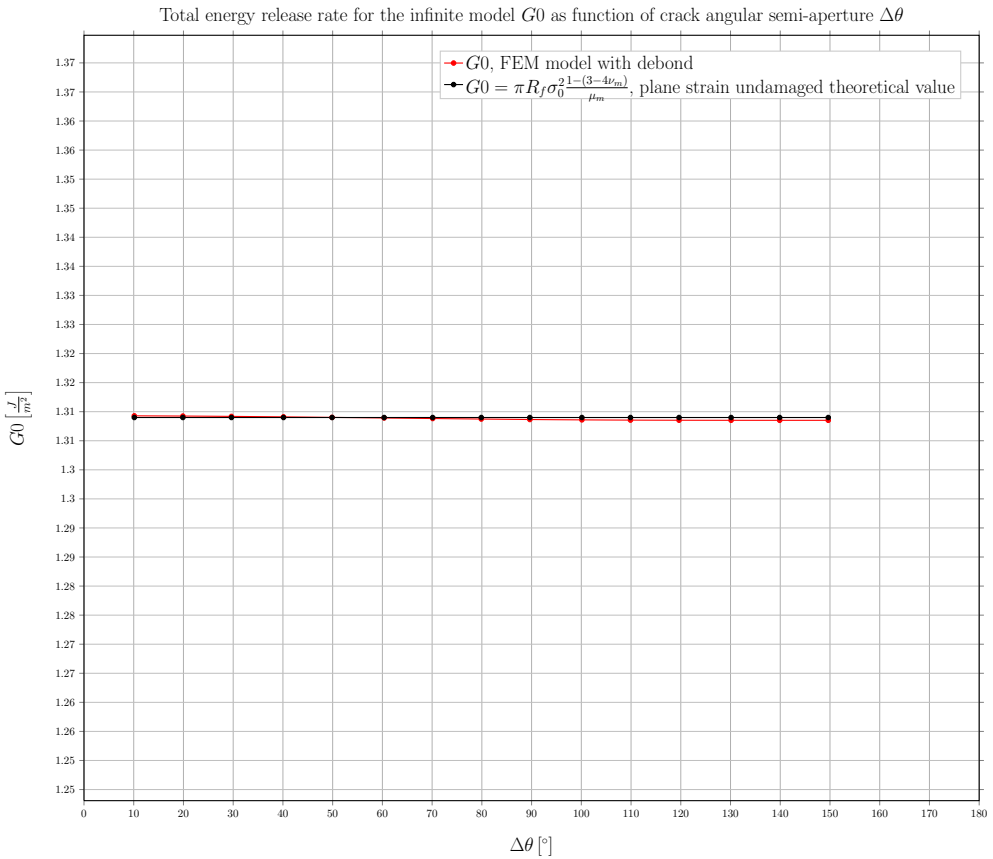
## $\sigma_0$, $\delta = 0.7°$




In red small strain FEM, in black analytical plain strain value.

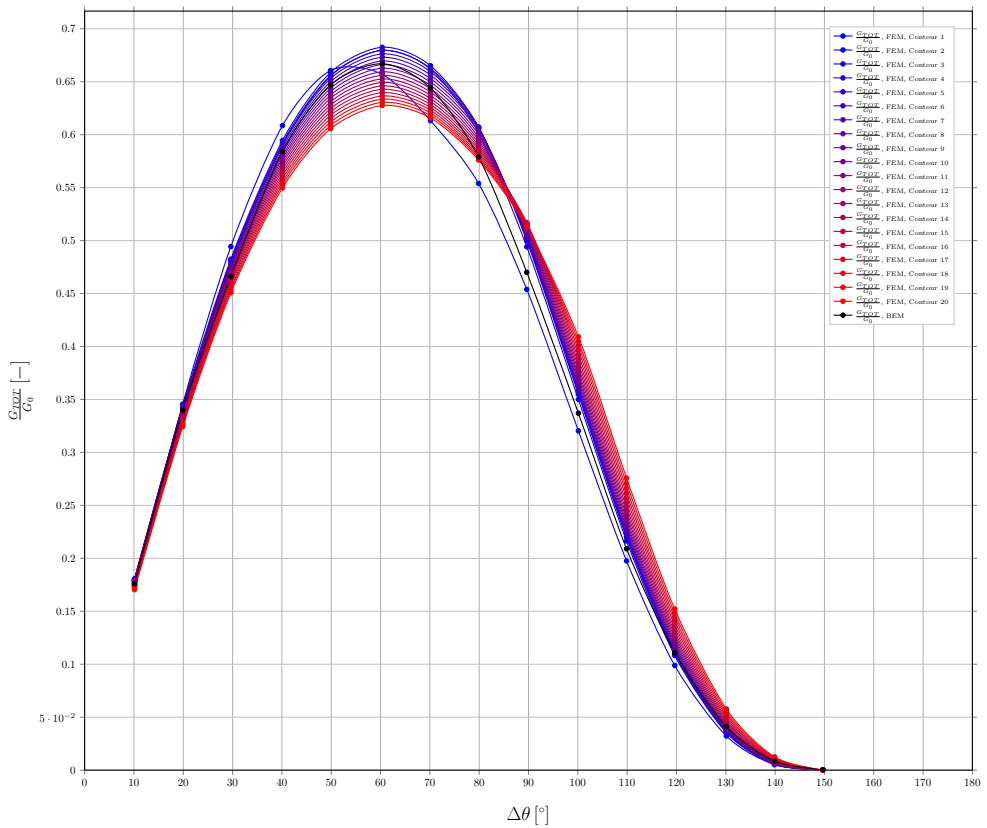
## $G_0$, $\delta = 0.7°$




In red small strain FEM, in black analytical plain strain value.

# J-Integral (Abaqus built-in routine), $\delta = 0.7°$

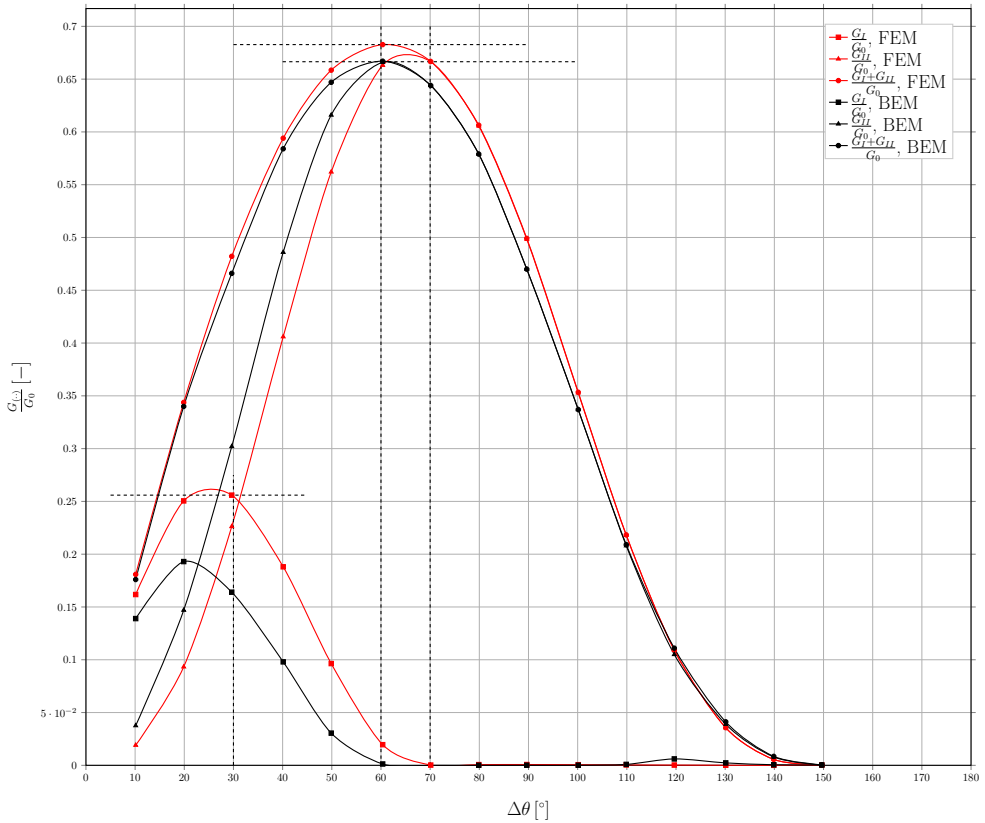
Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)




Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

# VCCT in forces (in-house Python routine), $\delta = 0.7°$

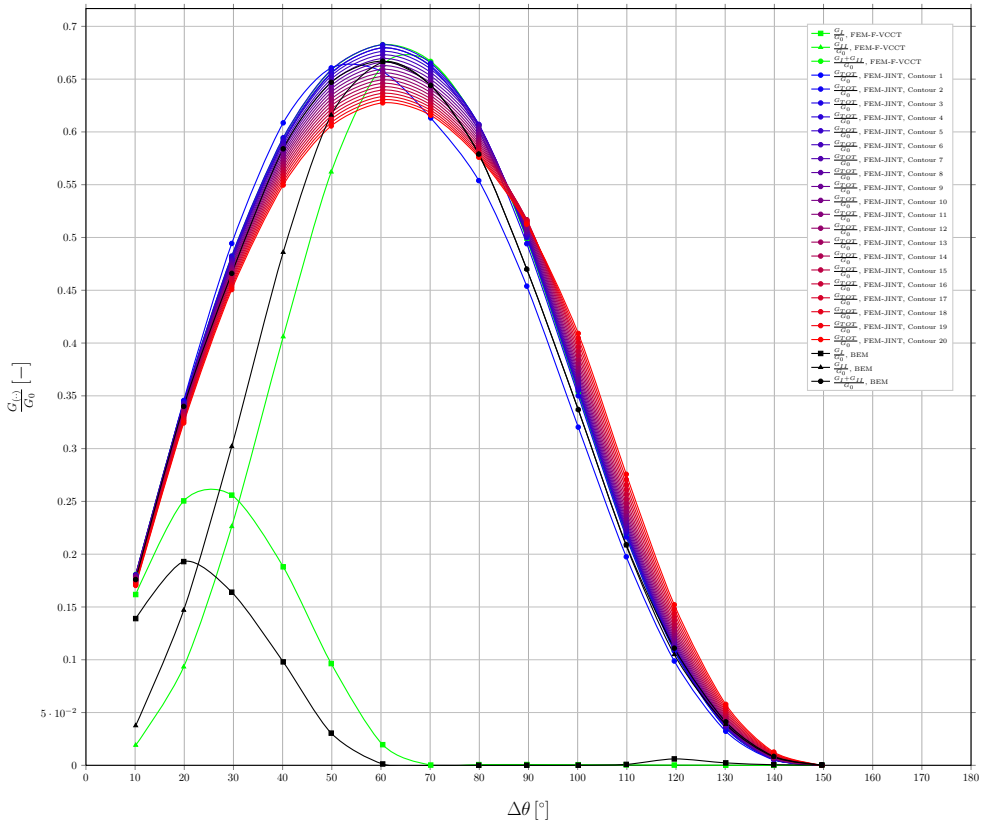
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT post-processing routine




In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

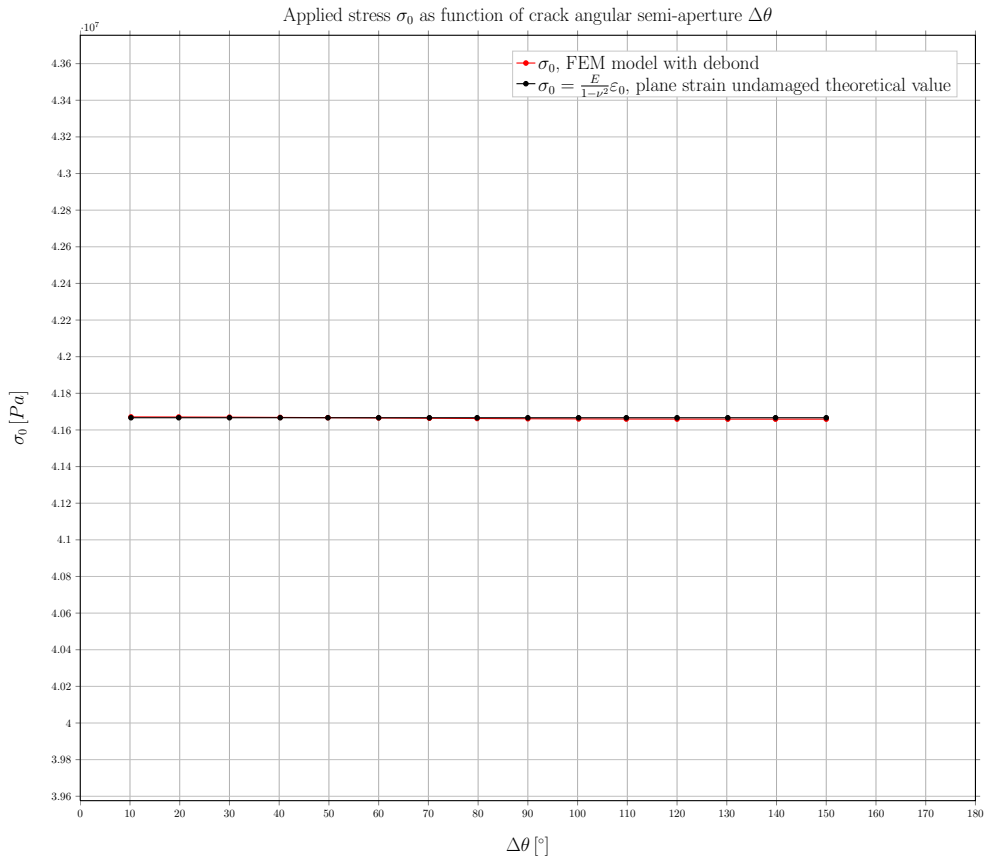
# J-Integral and VCCT in forces, $\delta = 0.7°$

Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines



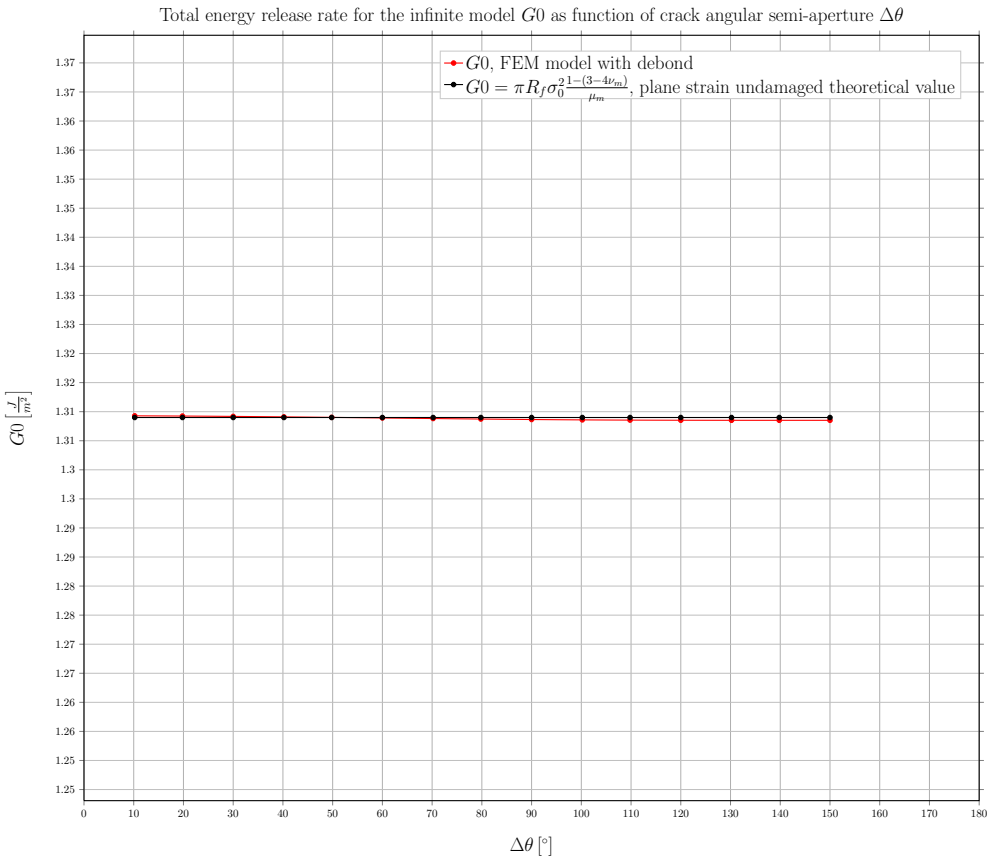

Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.

## $\sigma_0$, $\delta = 0.6°$




In red small strain FEM, in black analytical plain strain value.

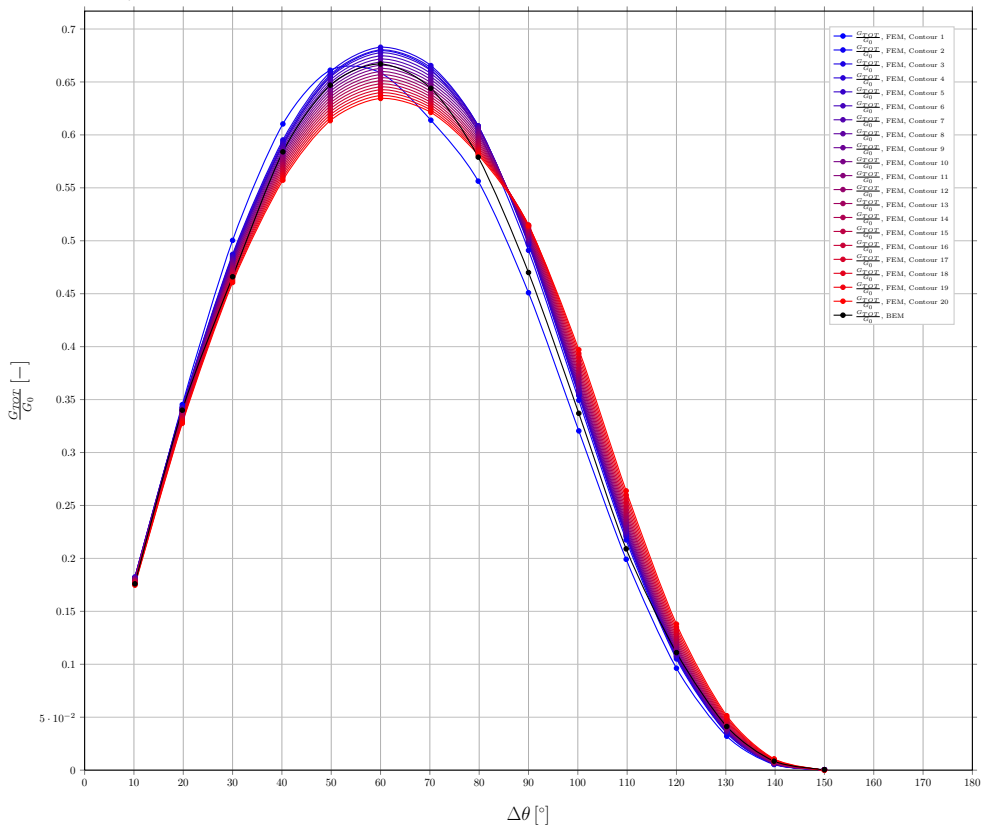
## $G_0$, $\delta = 0.6°$




In red small strain FEM, in black analytical plain strain value.

# J-Integral (Abaqus built-in routine), $\delta = 0.6°$

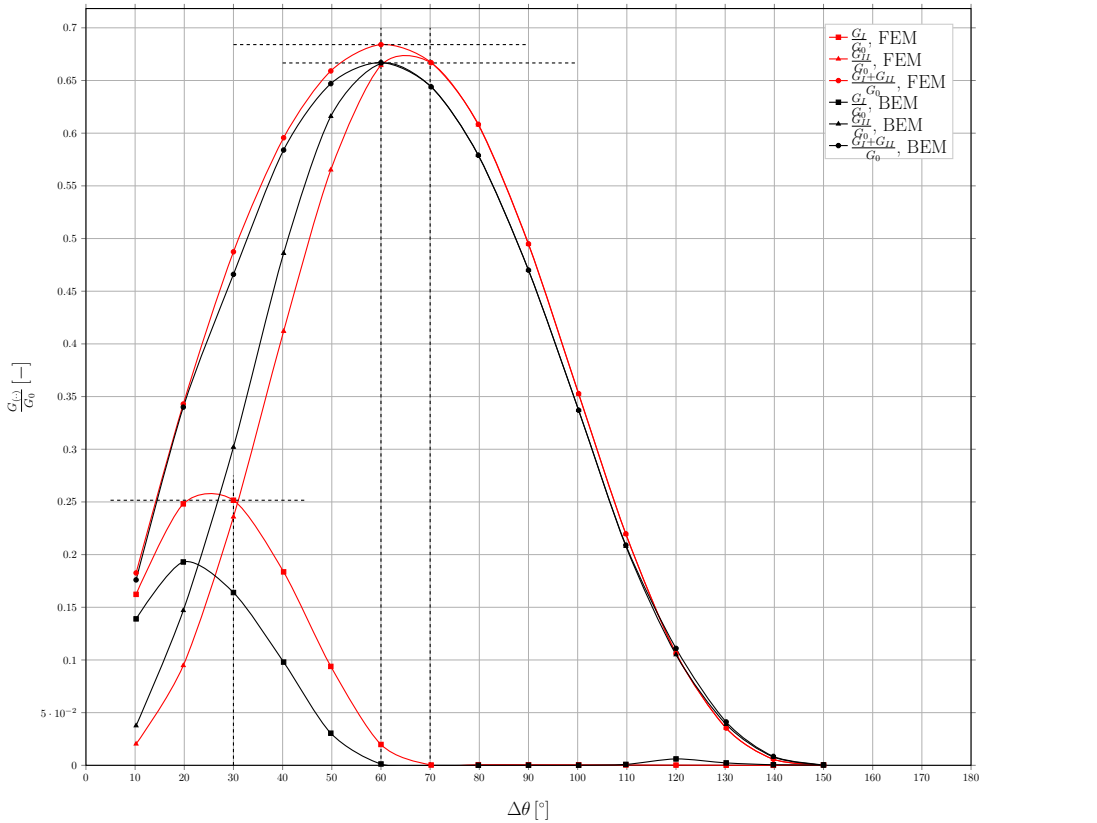
Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)




Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

# VCCT in forces (in-house Python routine), $\delta = 0.6°$

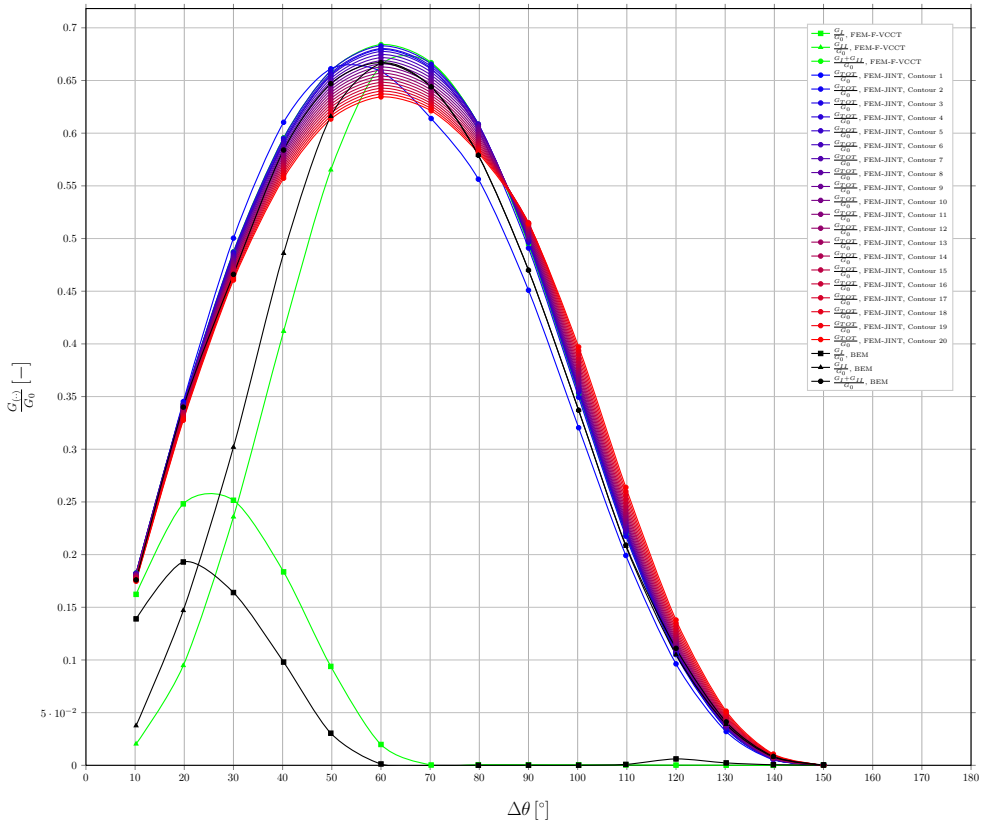
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT post-processing routine




In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

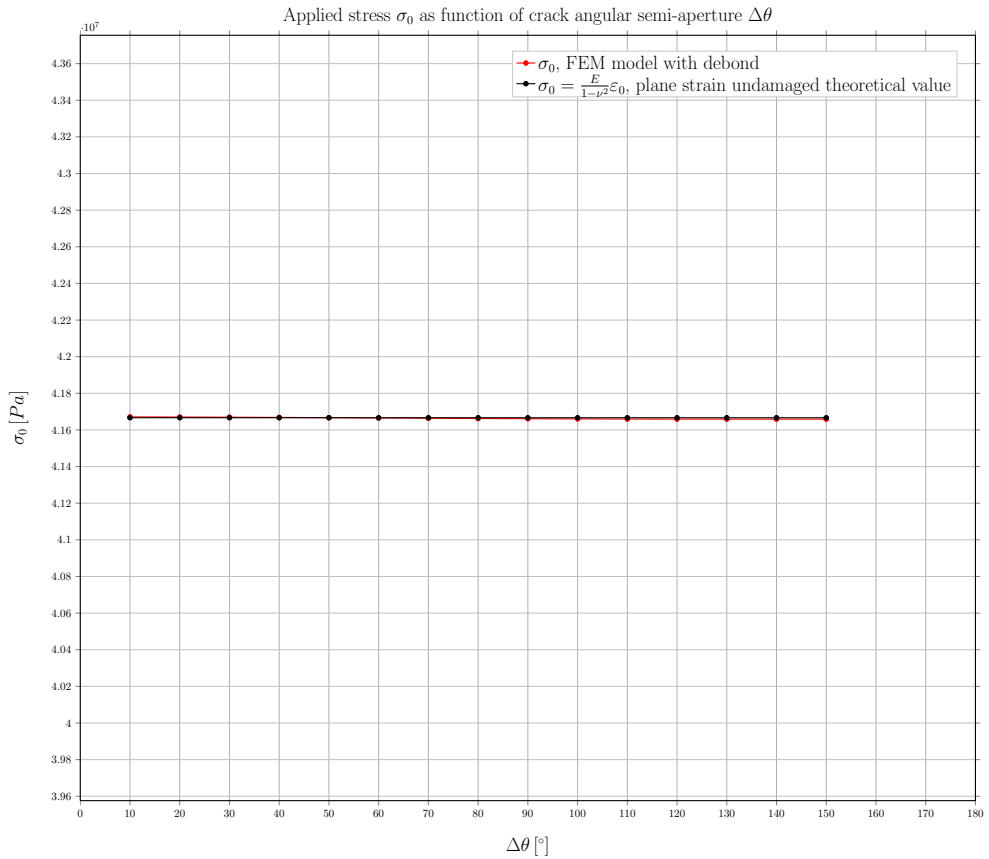
# J-Integral and VCCT in forces, $\delta = 0.6°$

Normalized energy release rate $\frac{G_{(-)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines



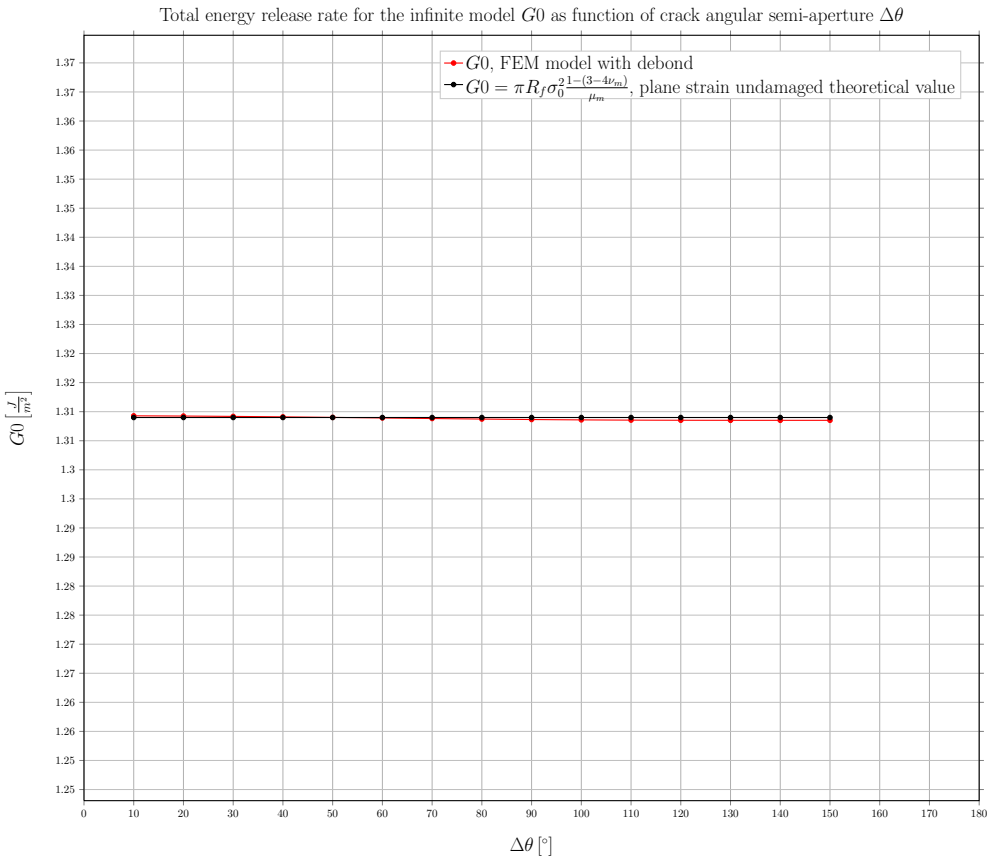

Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.

## $\sigma_0$, $\delta = 0.5°$




In red small strain FEM, in black analytical plain strain value.

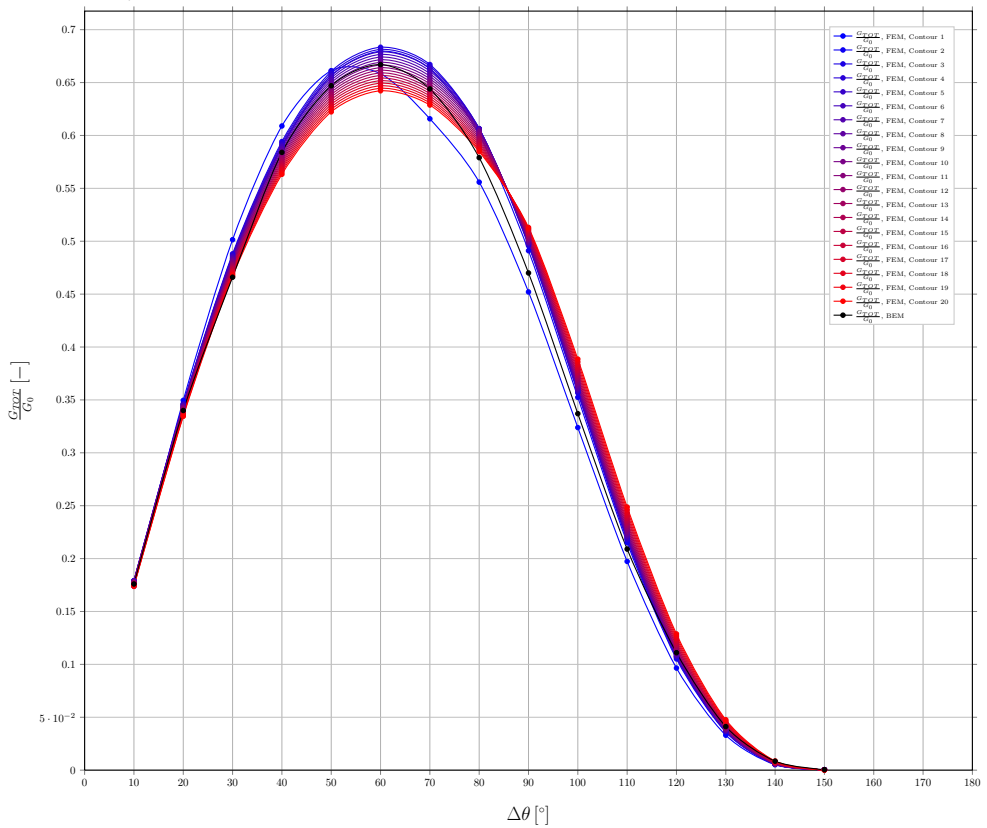
# $G_0$, $\delta = 0.5°$




In red small strain FEM, in black analytical plain strain value.

# J-Integral (Abaqus built-in routine), $\delta = 0.5°$

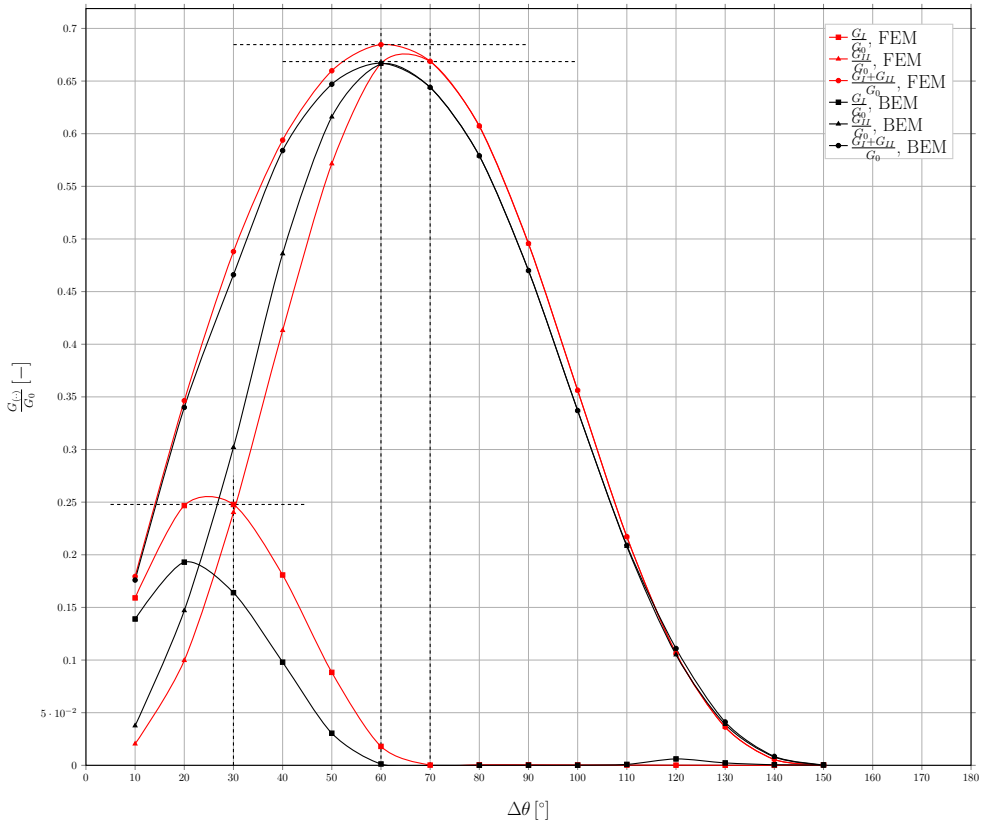
Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)




Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

# VCCT in forces (in-house Python routine), $\delta = 0.5°$

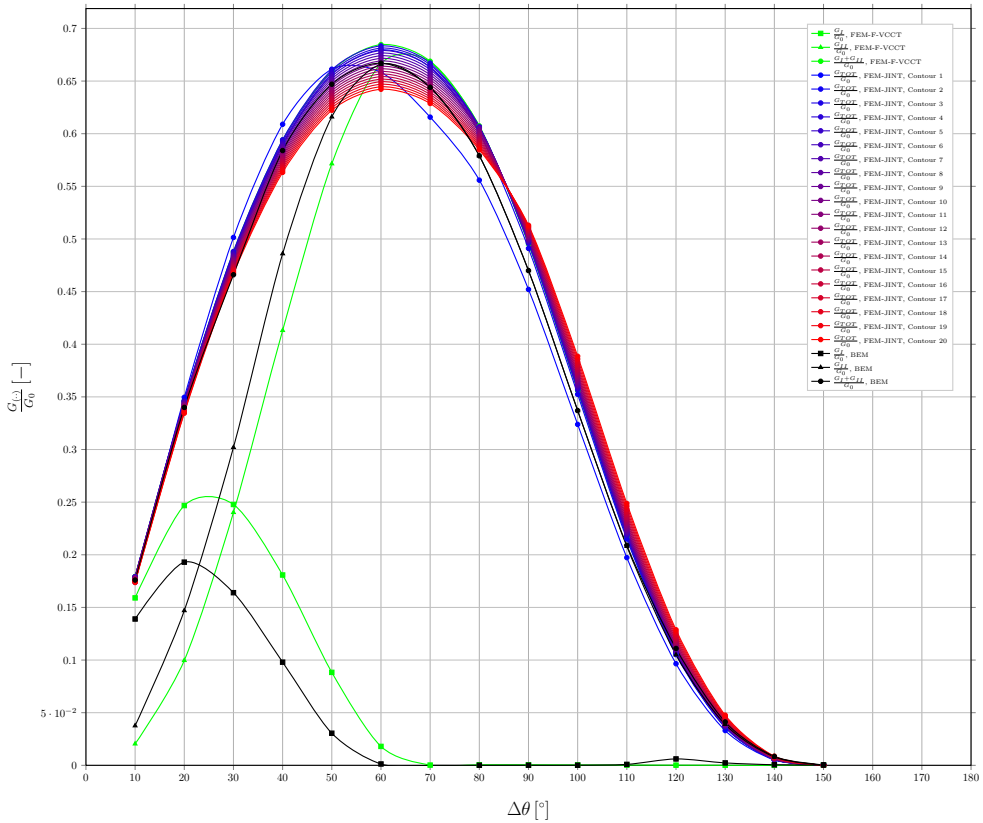
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT post-processing routine




In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

# J-Integral and VCCT in forces, $\delta = 0.5°$

Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines

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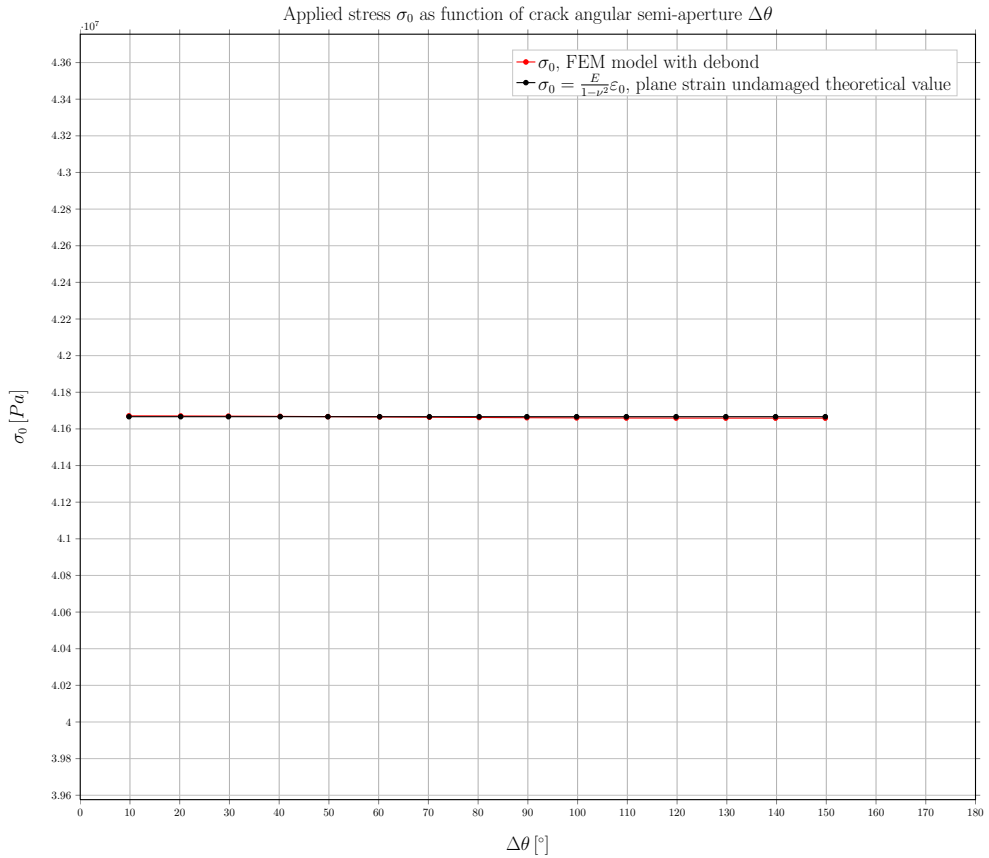
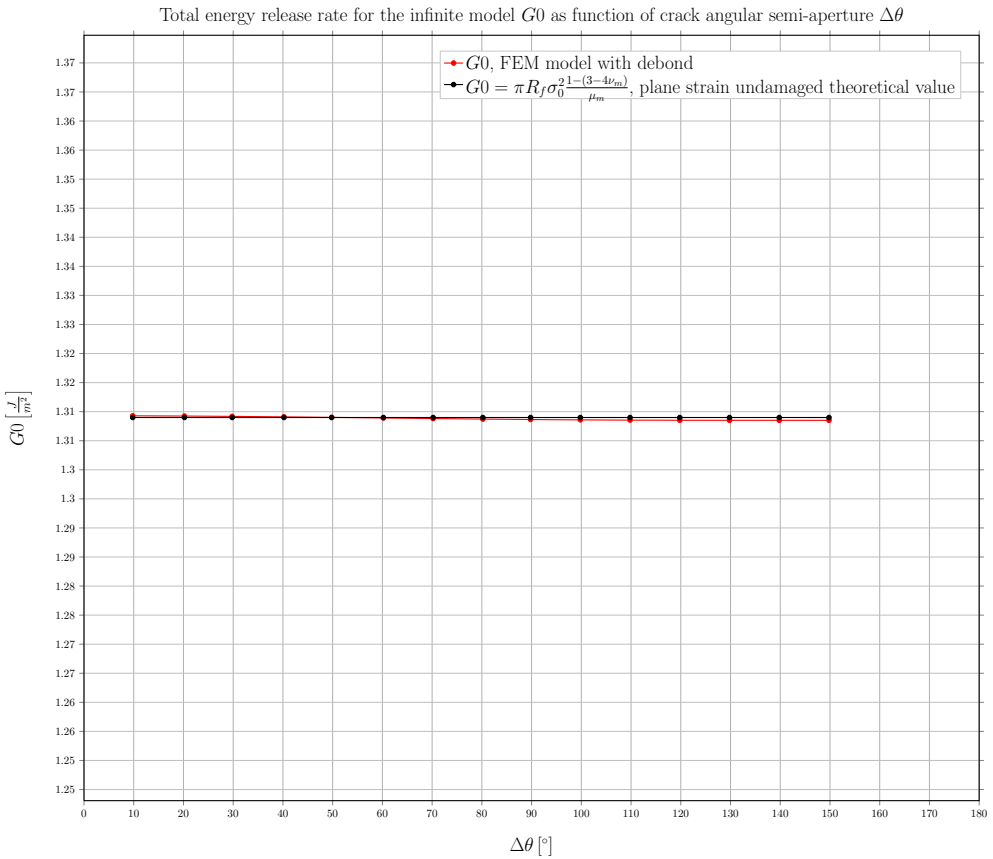
## $\sigma_0$, $\delta = 0.4°$




In red small strain FEM, in black analytical plain strain value.

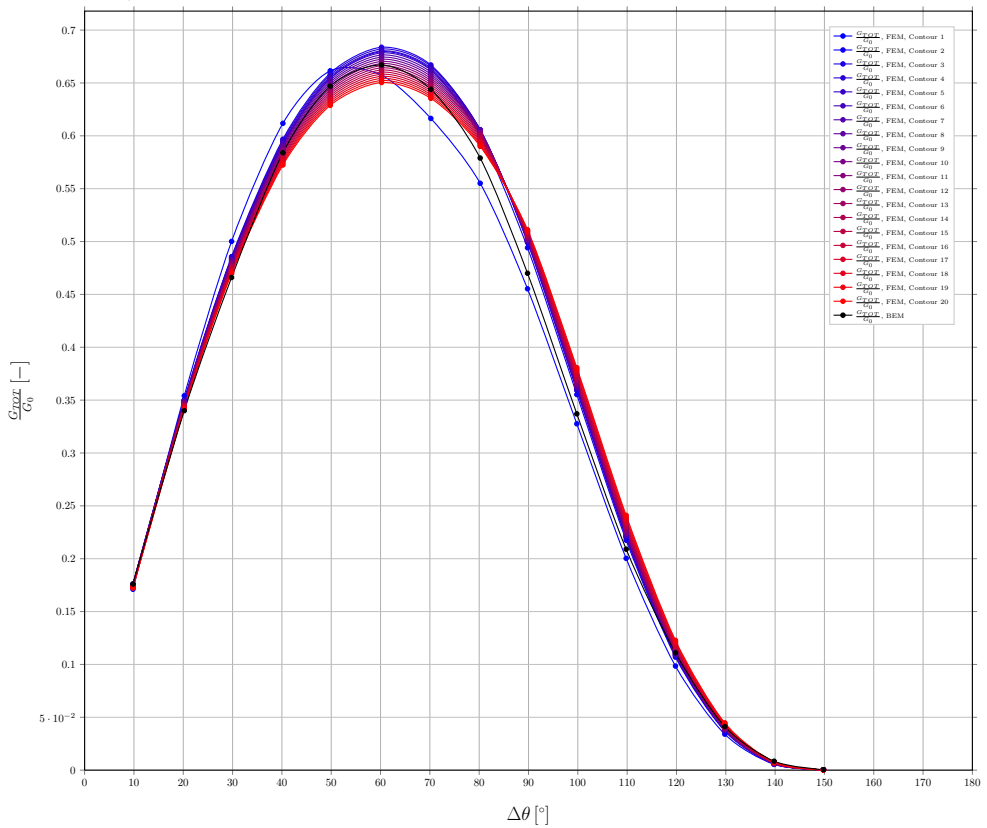
# $G_0$, $\delta = 0.4°$




In red small strain FEM, in black analytical plain strain value.

# J-Integral (Abaqus built-in routine), $\delta = 0.4°$

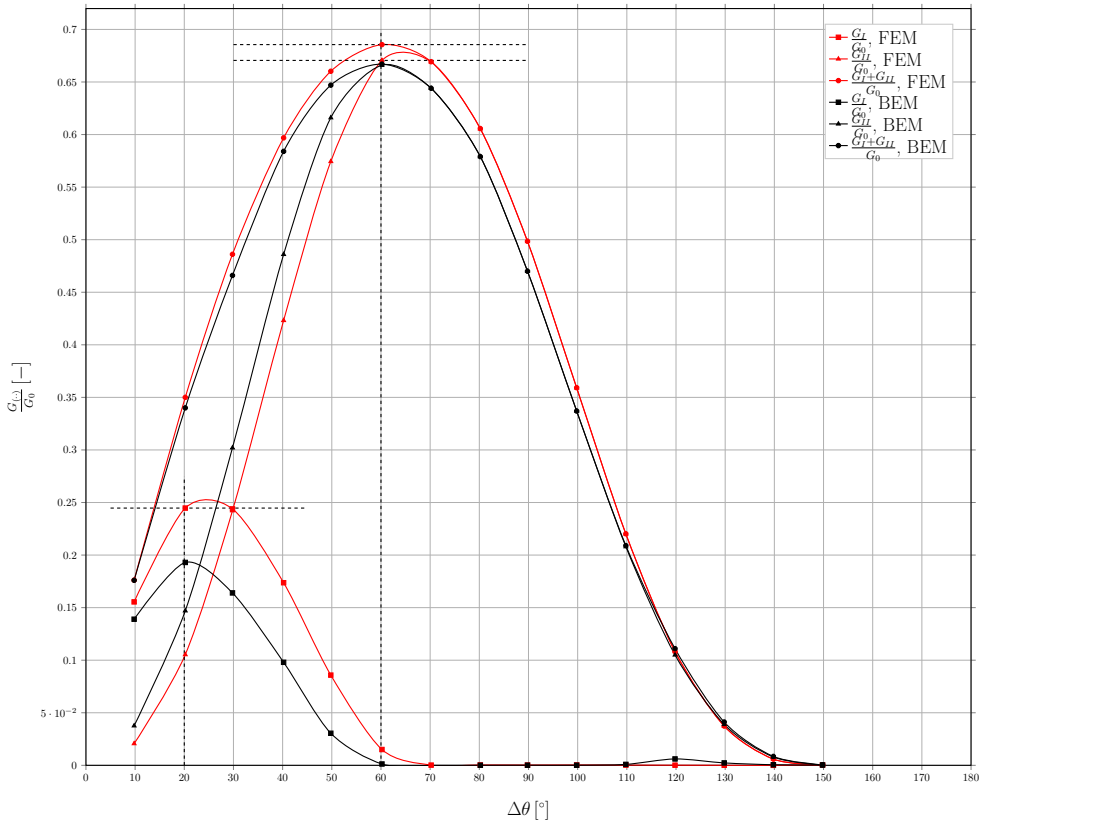
Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)




Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

# VCCT in forces (in-house Python routine), $\delta = 0.4°$

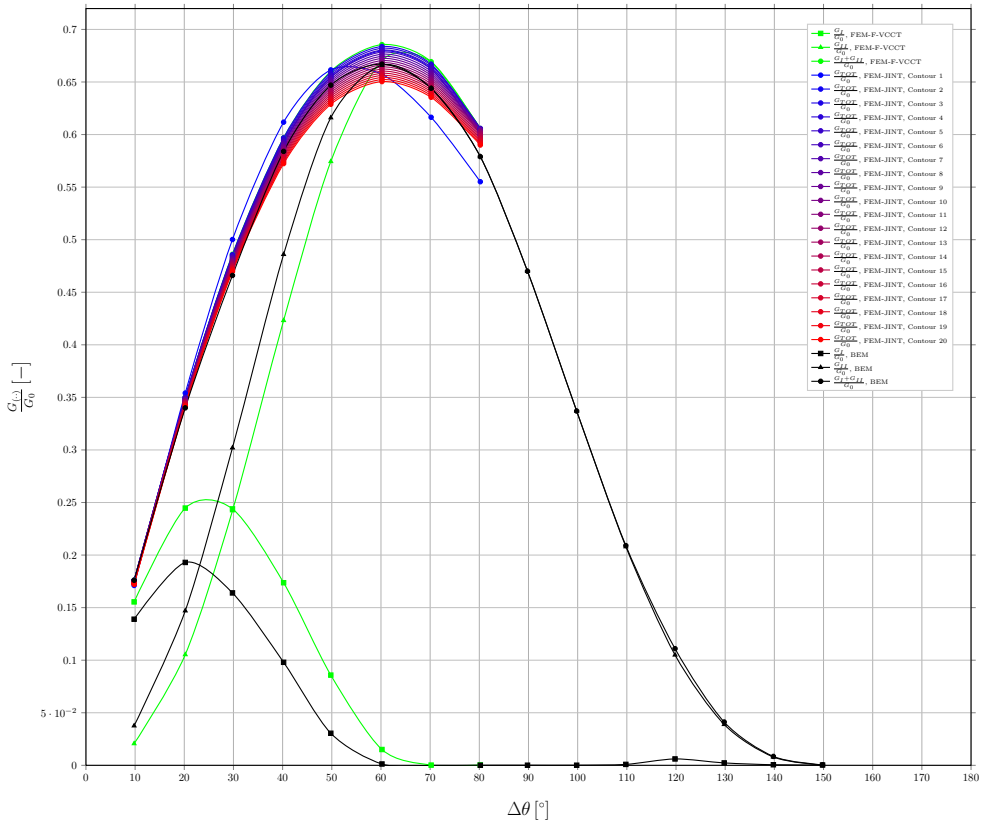
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT post-processing routine




In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

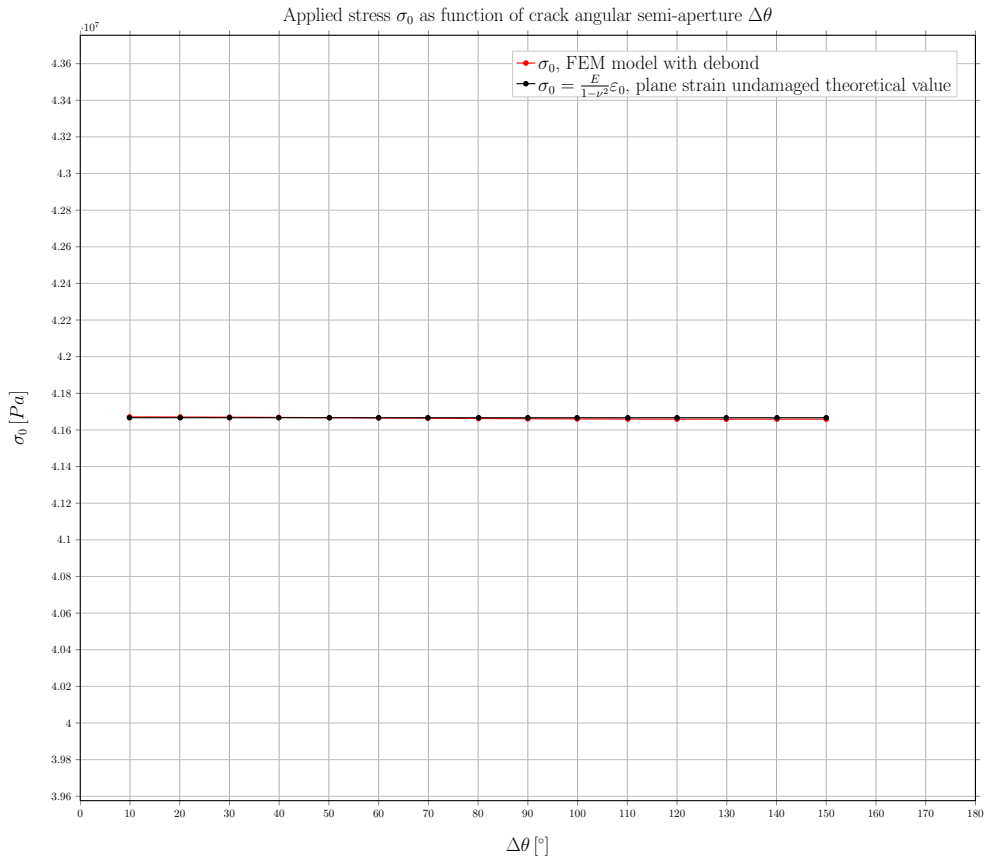
# J-Integral and VCCT in forces, $\delta = 0.4°$

Normalized energy release rate $\frac{G_{(-)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines



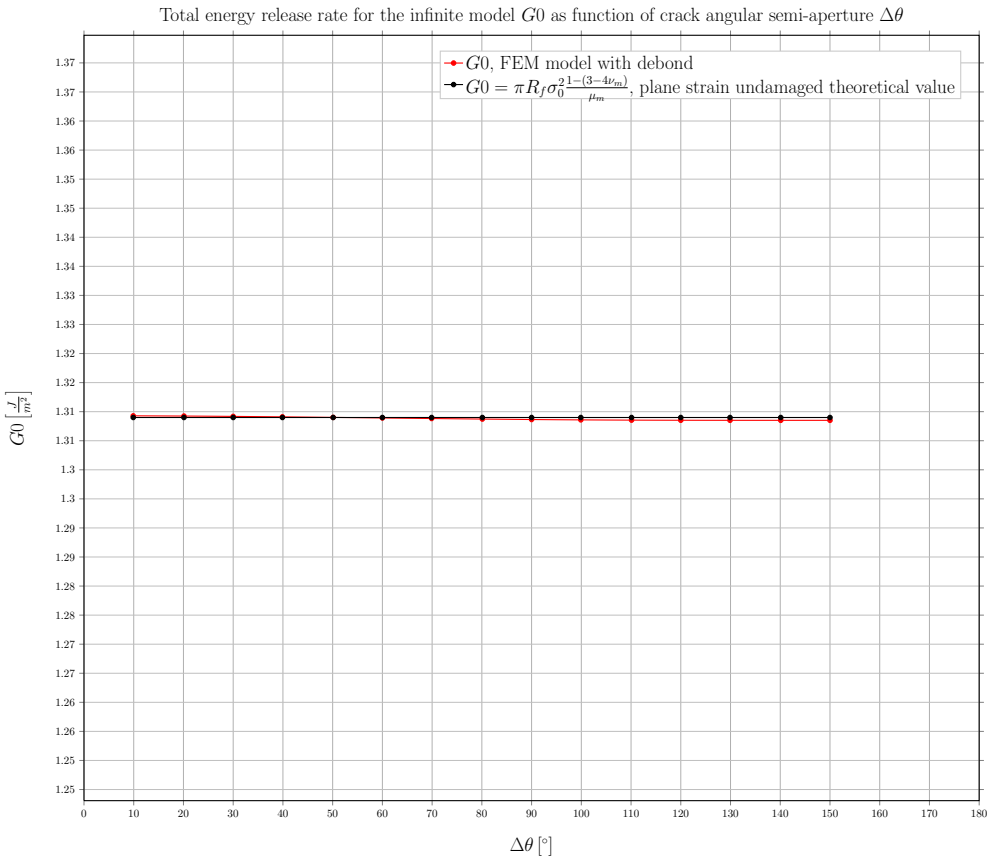

Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.

## $\sigma_0$, $\delta = 0.3°$




In red small strain FEM, in black analytical plain strain value.

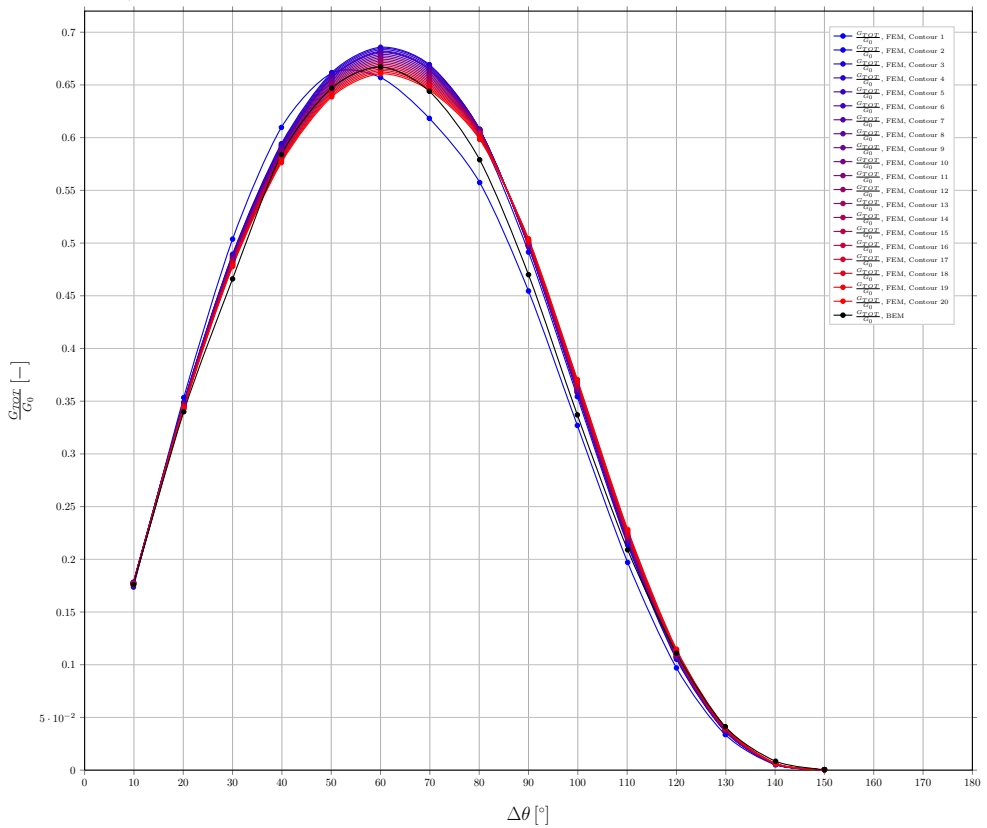
## $G_0$, $\delta = 0.3°$




In red small strain FEM, in black analytical plain strain value.

# J-Integral (Abaqus built-in routine), $\delta = 0.3°$

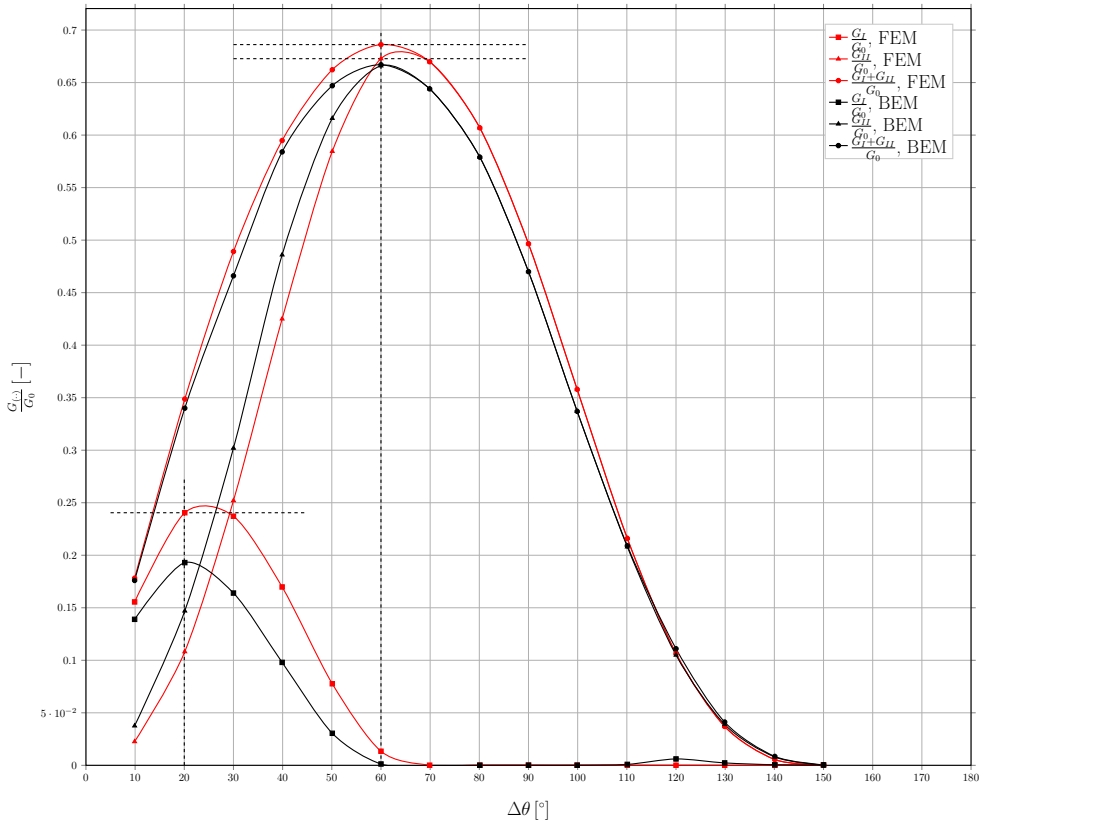
Normalized total energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with Abaqus built-in J-Integral post-processing routine (*CONTOUR INTEGRAL)




Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

# VCCT in forces (in-house Python routine), $\delta = 0.3°$

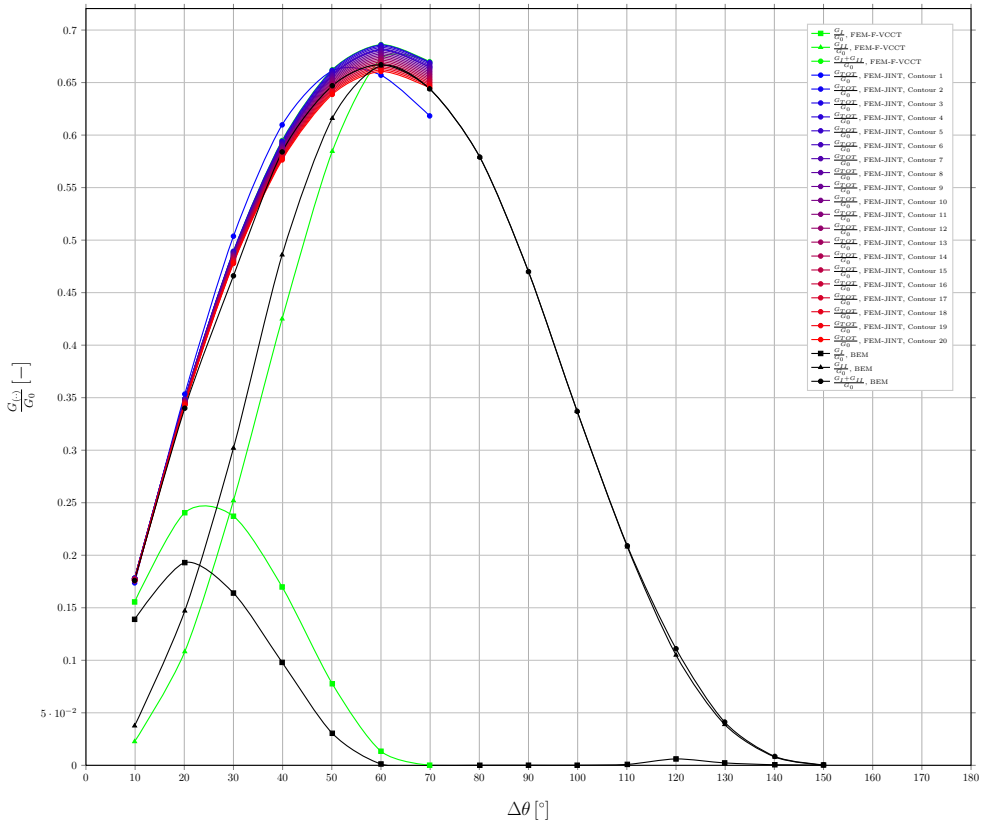
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT post-processing routine




In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

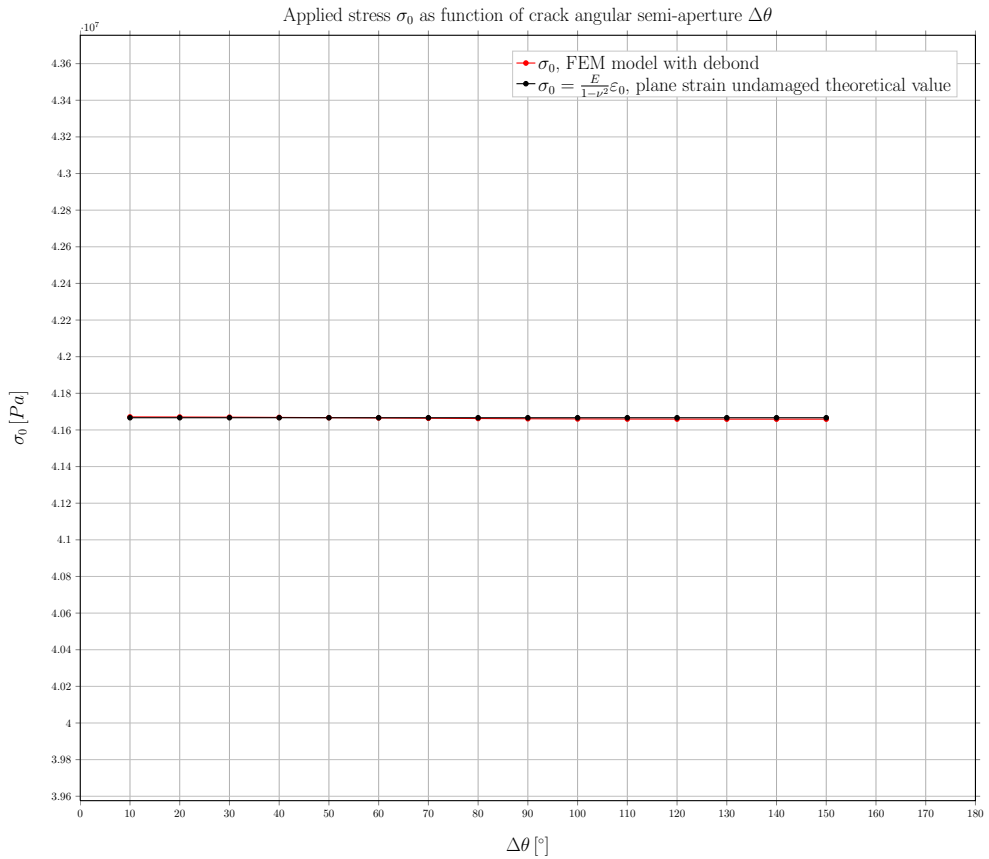
# J-Integral and VCCT in forces, $\delta = 0.3°$

Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines



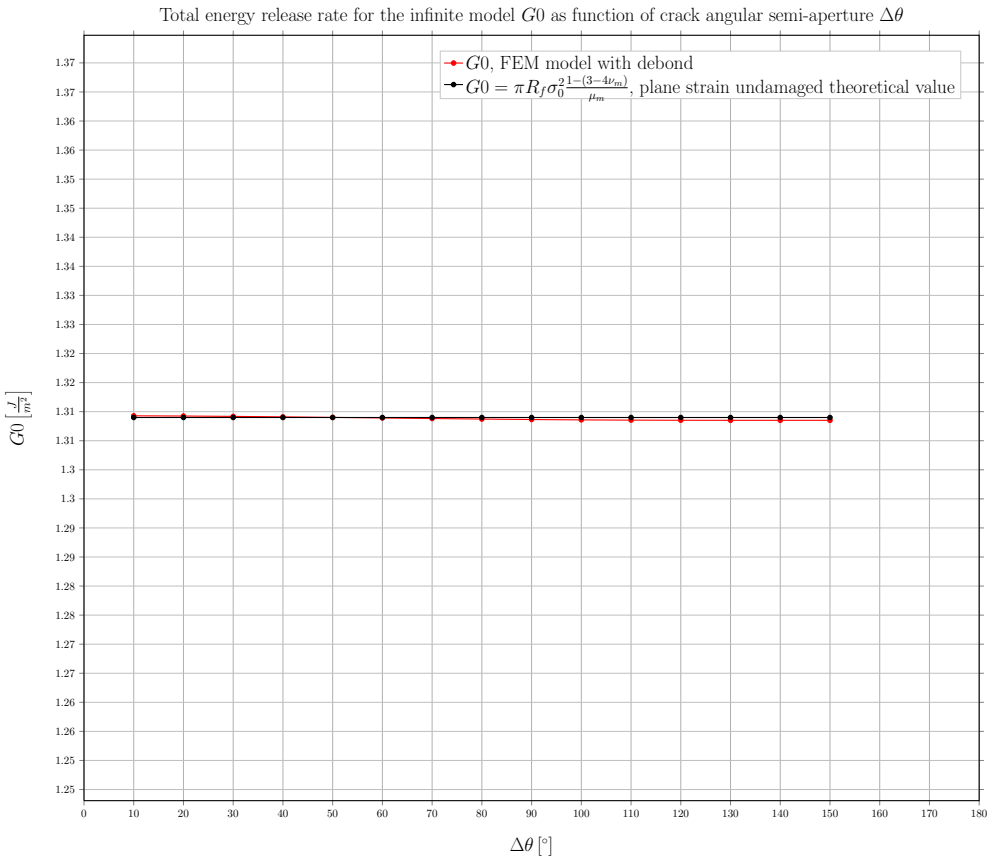

Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.

## $\sigma_0$, $\delta = 0.2°$




In red small strain FEM, in black analytical plain strain value.

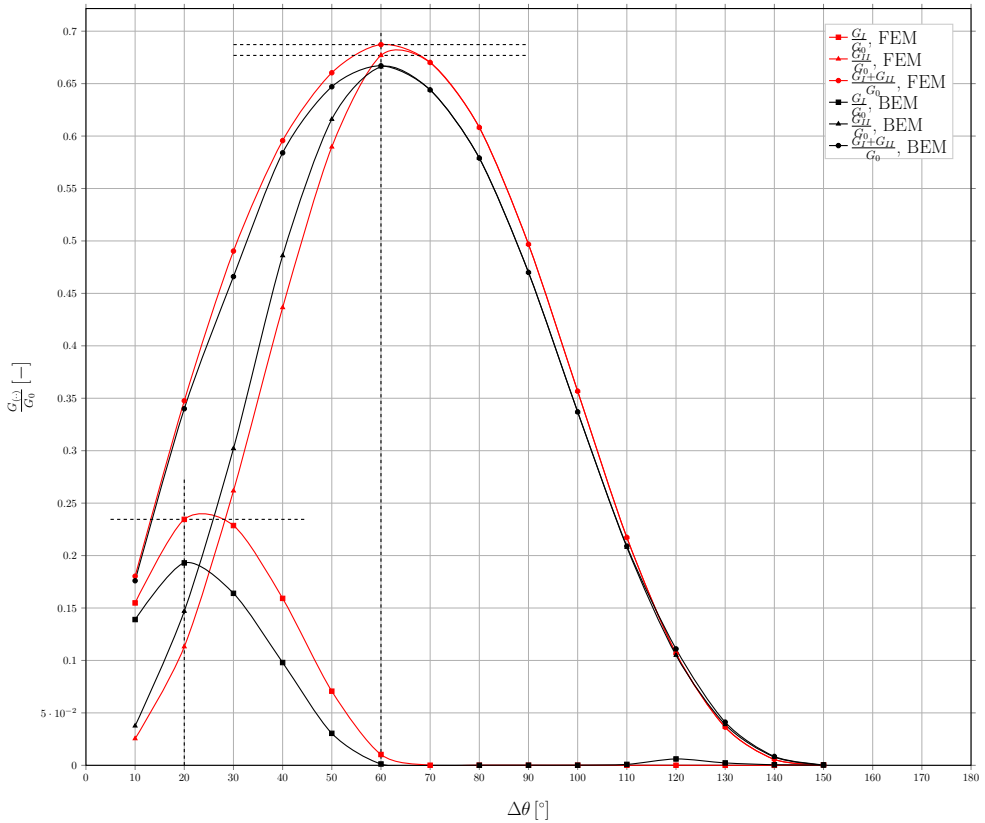
## $G_0$, $\delta = 0.2°$




In red small strain FEM, in black analytical plain strain value.

# VCCT in forces (in-house Python routine), $\delta = 0.2°$

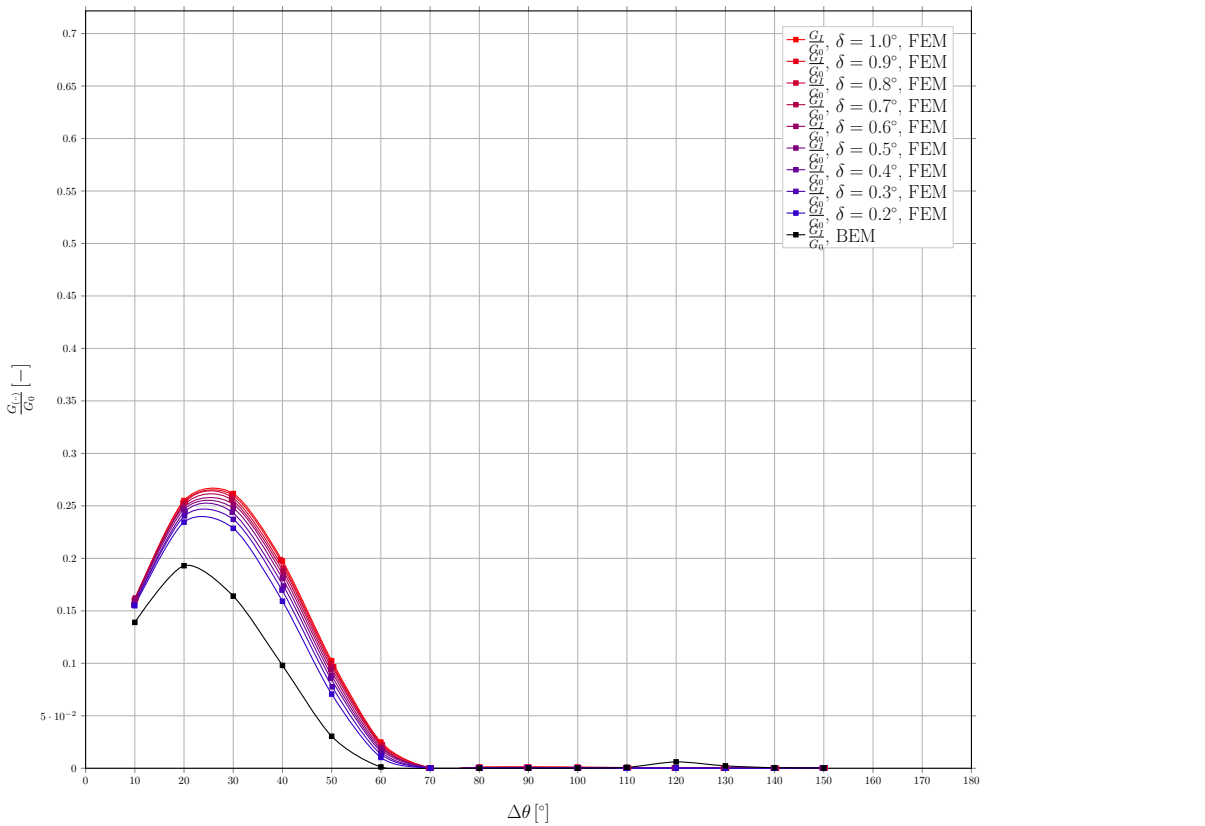
Normalized energy release rate $\frac{G_{(\cdot)}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT post-processing routine




In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

# $G_I$, VCCT in forces

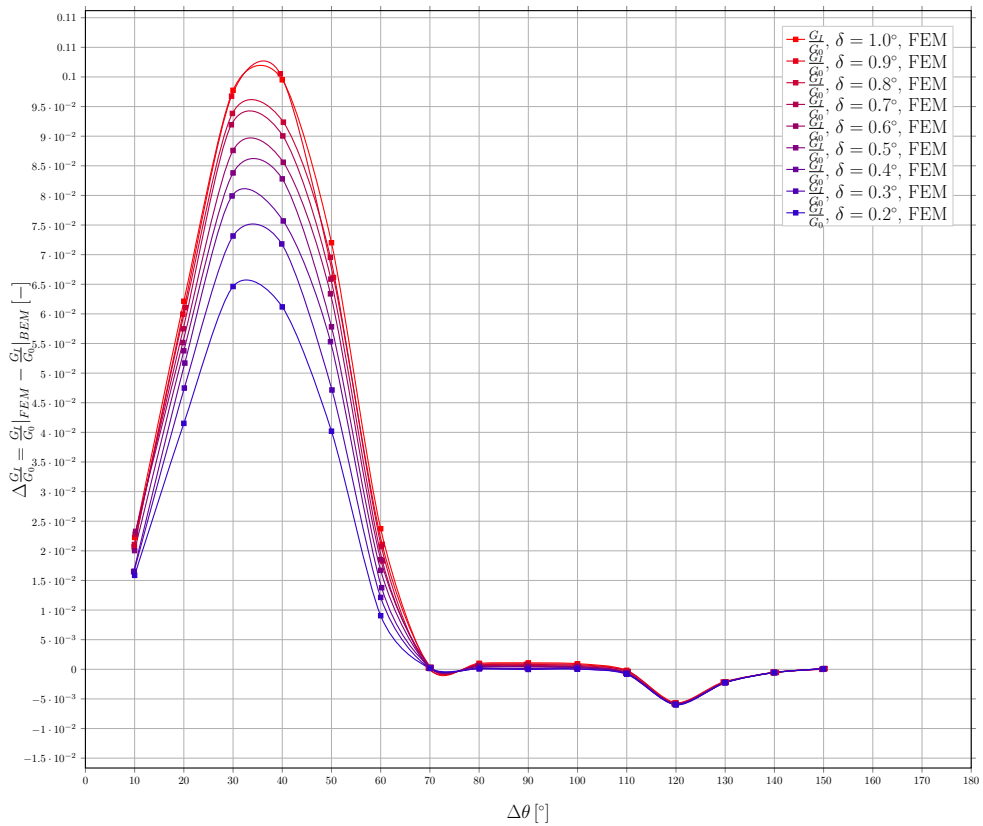
Normalized energy release rate $\frac{G_I}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, $VF_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing routine




Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

# $G_I$ Error with respect to BEM, VCCT in forces

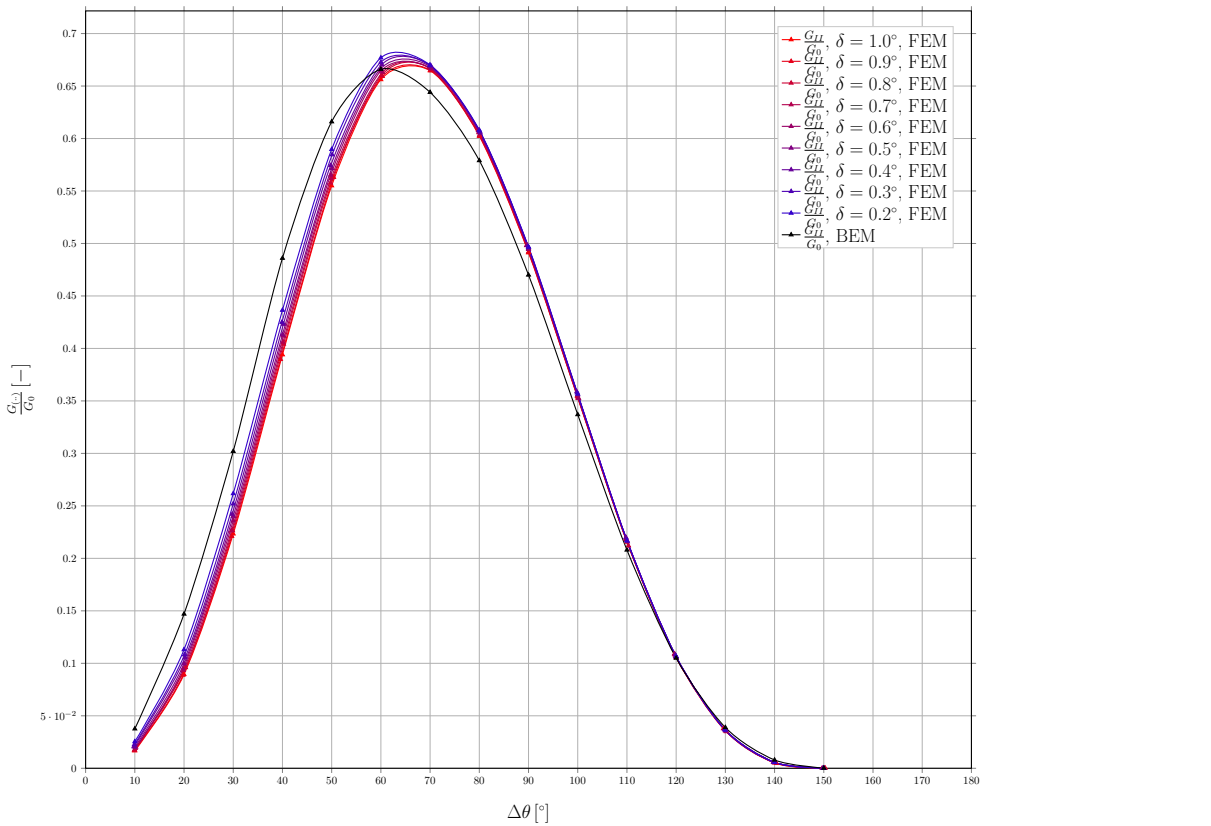
Error of of normalized energy release rate with respect to BEM results $\Delta\frac{G_I}{G_0} = \frac{G_I}{G_0}|_{FEM} - \frac{G_I}{G_0}|_{BEM}$ as function of crack angular semi-aperture $\Delta\theta$, $VF_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing routine




Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results.

# $G_{II}$, VCCT in forces

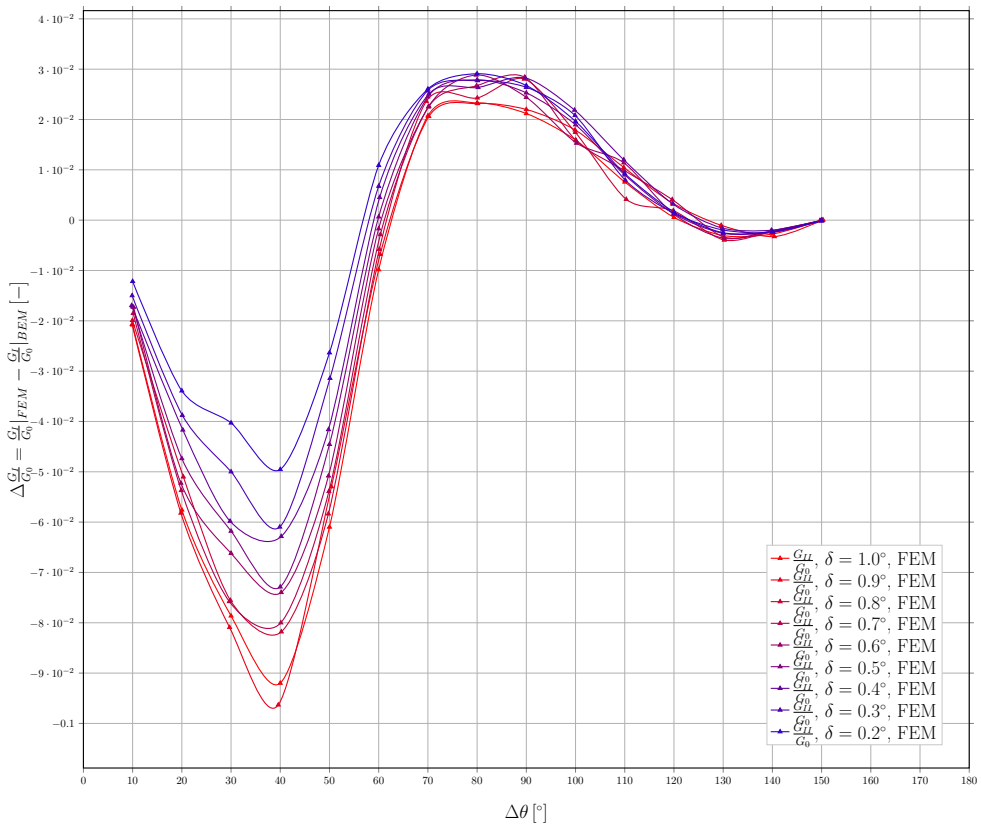
Normalized energy release rate $\frac{G_{II}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, $VF_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing routine




Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

# $G_{II}$ Error with respect to BEM, VCCT in forces

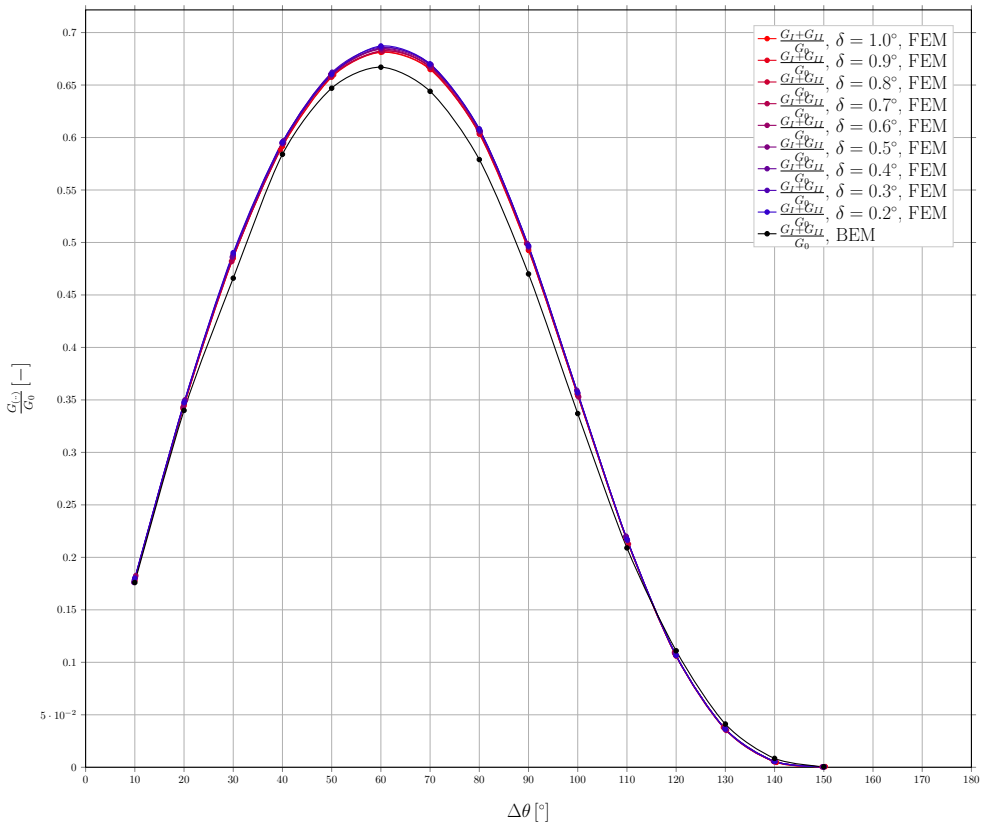
Error of of normalized energy release rate with respect to BEM results $\Delta\frac{G_I}{G_0} = \frac{G_I}{G_0}|_{FEM} - \frac{G_I}{G_0}|_{BEM}$ as function of crack angular semi-aperture $\Delta\theta$, $VF_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing routine




Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results.

# $G_{TOT}$, VCCT in forces

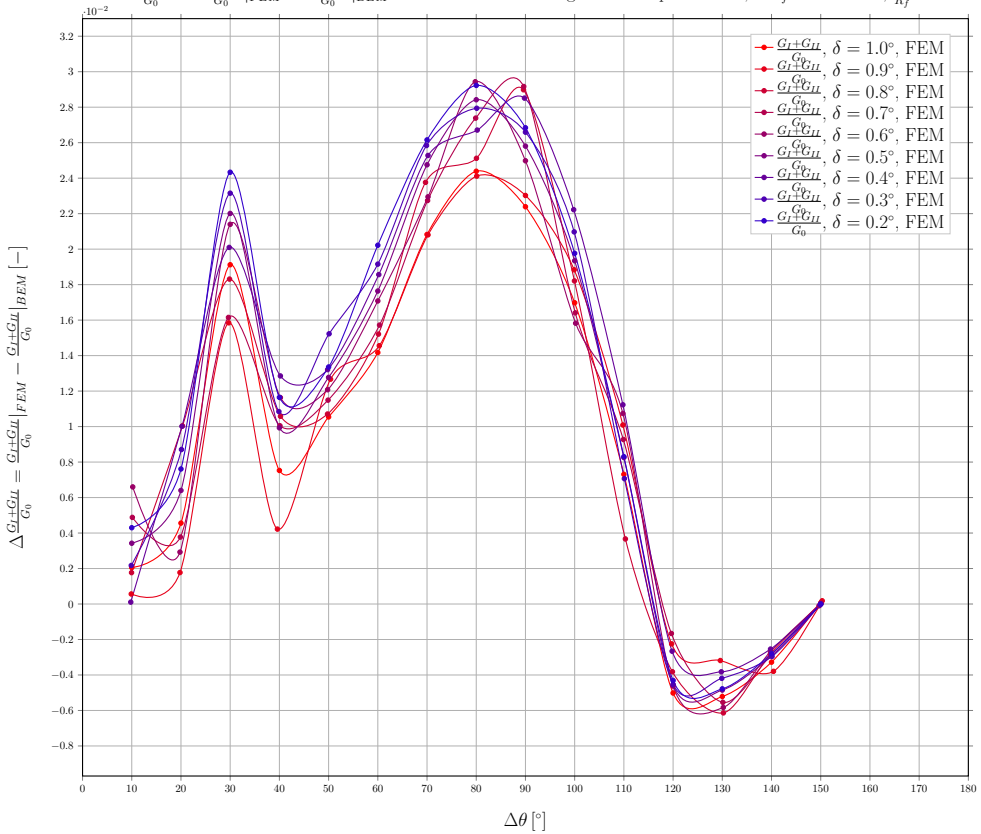
Normalized energy release rate $\frac{G_I+G_{II}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, $VF_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing routine




Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

# $G_{TOT}$ Error with respect to BEM, VCCT in forces

Error of of normalized energy release rate with respect to BEM results $\Delta\frac{G_I+G_{II}}{G_0} = \frac{G_I+G_{II}}{G_0}|_{FEM} - \frac{G_I+G_{II}}{G_0}|_{BEM}$ as function of crack angular semi-aperture $\Delta\theta$, $VF_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing




Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results.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