



## UPDATE 2017-07-31

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## **Outline**

- Symbols, Models, Equations & Reference Data
- **Nesults**
- Summary & Conclusion









Symbols Reference Models Angular discretization Material properties Evaluation of Gn VCCT VCC

# SYMBOLS, MODELS, EQUATIONS & REFERENCE DATA









Description

Symbols Reference Models Angular discretization Material properties Evaluation of G<sub>0</sub> VCCT VC

### **Symbols**

**Symbol** 

Unit

$\theta$	[°]	Debond angular position with respect to the center of the arc defined by the debond itself
$\Delta \theta$	[°]	Debond semi-angular aperture
δ	[°]	Angle subtended by a single element at the fiber/matrix interface
$VF_f$	[-]	Fiber volume fraction
I	[ <i>µm</i> ]	Ply's half-length, equal to RVE's half-length (square element)
и	$[\mu m]$	Displacement along x
W	$[\mu m]$	Displacement along z









Symbols, Models, Equations & Reference Data Results Summary & Conclusion

Symbols Reference Models Angular discretization Material properties Evaluation of  $G_0$  VCCT

#### **Symbols**

Symbol	Unit	Description
Γ <sub>1</sub>	[-]	Bonded part of fiber surface
$\Gamma_2$	[-]	Free (debonded) part of fiber surface
$\Gamma_3$	[-]	Bonded part of matrix surface
$\Gamma_4$	[-]	Free (debonded) part of matrix surface



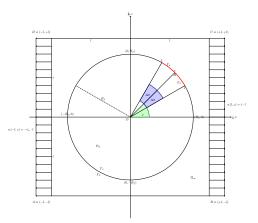






Reference Models Angular discretization Material properties Evaluation of G<sub>0</sub> VCCT VCCI

#### **Reference Models**



Simple RVE, BC: free.



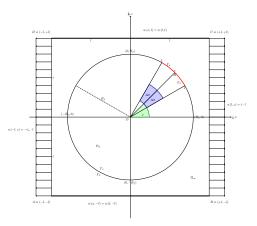






Symbols Reference Models Angular discretization Material properties Evaluation of G<sub>0</sub> VCCT VCCI

#### **Reference Models**



Simple RVE, BC: fixed vertical displacement.



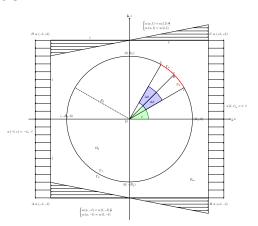






Symbols Reference Models Angular discretization Material properties Evaluation of G<sub>0</sub> VCCT VCC

#### **Reference Models**



Simple RVE, BC: fixed vertical and homogeneous horizontal displacement.



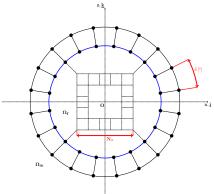






Symbols Reference Models Angular discretization Material properties Evaluation of G<sub>0</sub> VCCT VCC

#### **Angular discretization**



Angular discretization at fiber/matrix interface:  $\delta = \frac{360^{\circ}}{4N_{\odot}}$ .









Symbols Reference Models Angular discretization Material properties Evaluation of  $G_0$  VCCT VCC

### **Material properties**

Material	E [GPa]	G [GPa]	$\nu\left[- ight]$
Glass fiber	70,0	29,2	0,2
Ероху	3,5	1,25	0,4









ymbols Reference Models Angular discretization Material properties **Evaluation of**  $G_0$  VCCT VCC

#### Evaluation of $G_0$

$$G_0 = \pi R_f \sigma_0^2 \frac{1 + k_m}{8G_m} \tag{1}$$

$$k_m = 3 - 4\nu_m \tag{2}$$

$$\sigma_0^{undamaged} = \frac{E_m}{1 - \nu_m^2} \varepsilon_{xx} \tag{3}$$









Symbols, Models, Equations & Reference Data Results

Symbols Reference Models Angular discretization Material properties Evaluation of Go VCCT VCCI

# Virtual Crack Closure Technique (Nodal Forces at Crack Tip)

$$\Delta u = \left| \Delta u_{1}^{\text{matrix}} \right|_{\text{element before crack tip}} - \Delta u_{1}^{\text{fiber}}$$
(4)

$$\Delta w = \left| \Delta w_1^{matrix} - \Delta w_1^{fiber} \right|$$
 (5)

$$\beta = \arctan \begin{pmatrix} z_{\text{crack tip}}^{\text{matrix}, \text{undef}} \\ z_{\text{crack tio}}^{\text{matrix}, \text{undef}} \end{pmatrix}$$
 (6)

$$\Delta_{r} = \cos(\beta)\Delta u + \sin(\beta)\Delta w \qquad \Delta_{\theta} = -\sin(\beta)\Delta u + \cos(\beta)\Delta w \tag{7}$$

$$F_r = \cos(\beta)F_\chi^{reaction} + \sin(\beta)F_Z^{reaction}$$
  $F_\theta = -\sin(\beta)F_\chi^{reaction} + \cos(\beta)F_Z^{reaction}$  (8)

$$G_{I} = \frac{1}{2} \frac{F_{r} \Delta_{r}}{R_{r} \delta}$$
  $G_{II} = \frac{1}{2} \frac{F_{\theta} \Delta_{\theta}}{R_{r} \delta}$   $b = 1.0 \leftrightarrow \Delta A = bR_{f} \delta$  (9)









Reference Models Angular discretization Material properties Evaluation of G<sub>0</sub> VC

# Virtual Crack Closure Integral (Stress at Surface Nodes)

$$G_{I} = \frac{1}{2\Delta A} \int_{0}^{\Delta c} \sigma_{II}(s) \, \delta u_{II}(s - \Delta c) \, ds \quad G_{II} = \frac{1}{2\Delta A} \int_{0}^{\Delta c} \tau_{SII}(s) \, \delta u_{SI}(s - \Delta c) \, ds \tag{10}$$

with the reference frame centered on the crack tip and rotated according to the orientation of the crack tip.

$$\beta = \arctan \left( \frac{z_{\text{crack tip}}^{\textit{matrix}}, \textit{undef}}{x_{\text{matrix}}^{\textit{matrix}}, \textit{undef}} \right) \tag{11}$$

$$\Delta u^i = \left| \Delta u^{\textit{matrix}}_{i \text{ elements before crack tip}} - \Delta u^{\textit{fiber}}_{i \text{ elements before crack tip}} \right| \tag{12}$$

$$\Delta w^{i} = \left| \Delta w_{i}^{matrix} - \Delta w_{i}^{fiber} \right|$$
 (13)

$$\Delta_f^i = \cos(\beta)\Delta u^i + \sin(\beta)\Delta w^i \qquad \Delta_\theta^i = -\sin(\beta)\Delta u^i + \cos(\beta)\Delta w^i$$
 (14)









Symbols, Models, Equations & Reference Data Results

Reference Models Angular discretization Material properties Evaluation of Go

# Virtual Crack Closure Integral (Stress at Surface Nodes)

$$\sigma_{II}^{m,i} = \sigma_{XX}^{m,i}$$
 elements after c.t.  $\cos^2 \beta + \sigma_{ZZ}^{m,i}$  elements after c.t.  $\sin^2 \beta + 2\tau_{XZ}^{m,i}$  elements after c.t.  $\sin \beta \cos \beta$  (15)

$$\tau_{\ell\theta}^{m,i} = \left(\sigma_{ZZ}^{m,i \text{ elements after c:ack tip}} - \sigma_{\chi\chi}^{m,i \text{ elements after c:t.}}\right) \sin\beta\cos\beta + \tau_{\chi Z}^{m,i \text{ elements after c:t.}} \left(\cos^2\beta - \sin^2\beta\right)$$
(16)

where m stands for material, i.e. stresses can be extracted either on the fiber or the matrix surface.

$$G_{I}^{m} = \frac{1}{2R_{f}\delta b} \sum_{i=1}^{N \ln t} \frac{1}{2} R_{f}\delta \left( \sigma_{rr}^{m,i} \Delta_{r}^{i} + \sigma_{rr}^{m,i-1} \Delta_{r}^{i-1} \right) \qquad G_{II}^{m} = \frac{1}{2R_{f}\delta b} \sum_{i=1}^{N \ln t} \frac{1}{2} R_{f}\delta \left( \tau_{r\theta}^{m,i} \Delta_{\theta}^{i} + \tau_{r\theta}^{m,i-1} \Delta_{\theta}^{i-1} \right)$$
(17)

remembering b = 1, i.e. unit depth in the out-of-plane direction, they simplify to

$$G_{I}^{m} = \frac{1}{4} \left( \sigma_{rr}^{m,0} \Delta_{r}^{0} + \sum_{i=1}^{N \text{ Int } EI-1} \left( 2\sigma_{rr}^{m,i} \Delta_{r}^{i} \right) + \sigma_{rr}^{m,N \text{ Int } EI} \Delta_{r}^{N \text{ Int } EI} \right)$$
(18)

$$G_{II}^{m} = \frac{1}{4} \left( \tau_{r\theta}^{m,0} \Delta_{\theta}^{0} + \sum_{i=1}^{N \text{ Int } EI - 1} \left( 2\tau_{r\theta}^{m,i} \Delta_{\theta}^{i} \right) + \tau_{r\theta}^{m,N \text{ Int } EI} \Delta_{\theta}^{N \text{ Int } EI} \right)$$

$$\tag{19}$$









Model Data  $\delta=1.0^\circ$   $\delta=0.9^\circ$   $\delta=0.8^\circ$   $\delta=0.7^\circ$   $\delta=0.6^\circ$   $\delta=0.5^\circ$   $\delta=0.4^\circ$   $\delta=0.3^\circ$   $\delta=0.2^\circ$  Summary











Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta=0.5^{\circ}$ 

 $0.4^{\circ}$   $\delta=0.3^{\circ}$   $\delta=0.2^{\circ}$  Summ

# **Model Data**

Quantity	Value
$\theta$ [ $^{\circ}$ ]	0
$\Delta  heta\left[^{\circ} ight]$	$\in [10,150]$
$\delta$ [ $^{\circ}$ ]	$\in [1,0.2]$
$VF_f[-]$	$7.9\cdot 10^{-5}$
$\frac{L}{R_t}[-]$	$\sim 100$
$R_f[\mu m]$	1



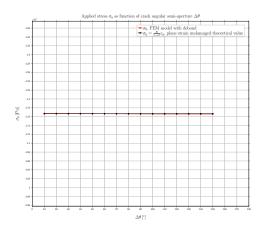






 $\label{eq:delta-$ 

 $\sigma_0$ ,  $\delta=1.0^\circ$ 



In red small strain FEM, in black analytical plain strain value.



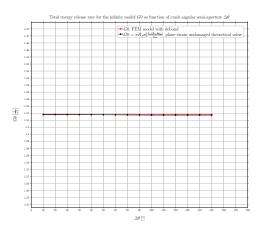






 $\label{eq:def-Model Data} \ \ \delta = \textbf{1.0}^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary and Model}$ 

 $G_0, \delta = 1.0^{\circ}$ 



In red small strain FEM, in black analytical plain strain value.



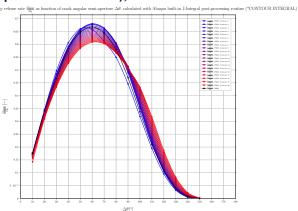






Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

#### J-Integral (Abaqus built-in routine), $\delta=1.0^\circ$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.



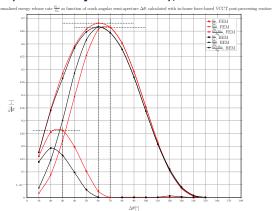






Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

#### VCCT in forces (in-house Python routine), $\delta = 1.0^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.



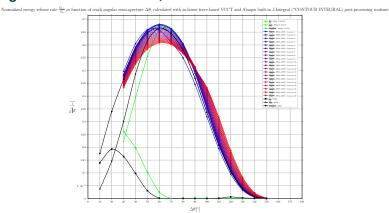






Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

#### J-Integral and VCCT in forces, $\delta = 1.0^{\circ}$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.





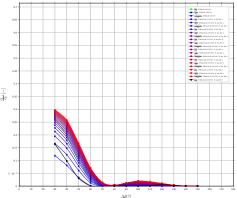




Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

### $G_l$ from VCCI, stresses extracted on fiber surface, $\delta = 1.0^{\circ}$

Normalized energy release rate  $\frac{G_{i-1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black





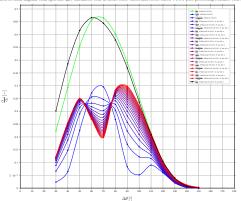




Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

#### $G_{\parallel}$ from VCCI, stresses extracted on fiber surface, $\delta=1.0^{\circ}$

Normalized energy release rate  $\frac{G_{s,t}}{st}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black





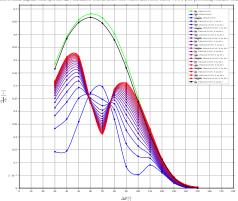




Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

### $G_{TOT}$ from VCCI, stresses extracted on fiber surface, $\delta=1.0^{\circ}$

Normalized energy release rate  $\frac{G_{i+1}}{G_{i+1}}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black

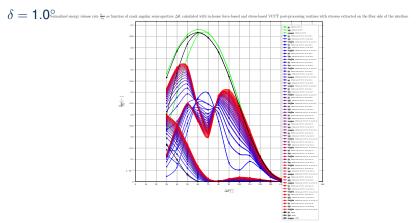








## Summary of $G_{(...)}$ from VCCI, stresses extracted on liber surface,



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black BEM results.





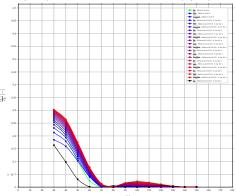




Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

#### $G_l$ from VCCl, stresses extracted on matrix surface, $\delta = 1.0^{\circ}$

Normalized energy release rate  $\frac{G_{i+1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black





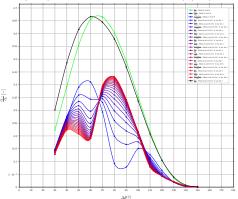




Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

#### $G_{II}$ from VCCI, stresses extracted on matrix surface, $\delta=1.0^{\circ}$

Normalized energy release rate  $\frac{G_{ij}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black





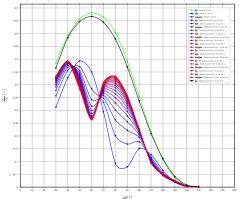




Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

#### $G_{TOT}$ from VCCI, stresses extracted on matrix surface, $\delta = 1.0^{\circ}$

Normalized energy release rate  $\frac{G_{1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



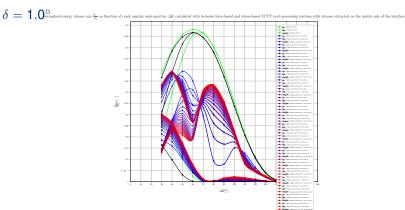






Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

## Summary or $G_{(...)}$ from VCCI, stresses extracted on matrix surface,



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black BEM results.



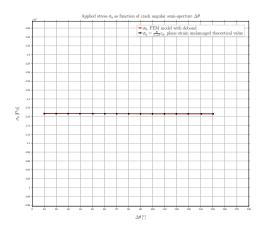


Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary





 $\sigma_{0}$ ,  $\delta=0.9^{\circ}$ 



In red small strain FEM, in black analytical plain strain value.

30



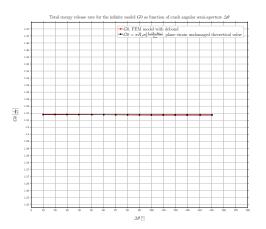






Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

$$G_0$$
,  $\delta = 0.9^\circ$ 



In red small strain FEM, in black analytical plain strain value.



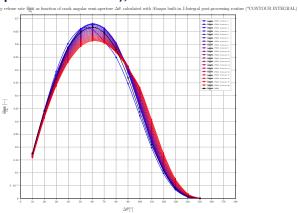






Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

#### J-Integral (Abaqus built-in routine), $\delta = 0.9^{\circ}$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.





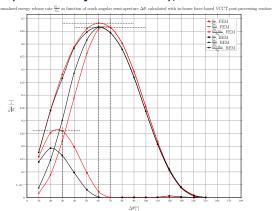




Symbols, Models, Equations & Reference Data Results Summary & Conclusion

Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

### VCCT in forces (in-house Python routine), $\delta=0.9^\circ$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.



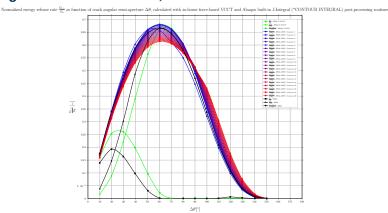






Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

## J-Integral and VCCT in forces, $\delta = 0.9^{\circ}$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.







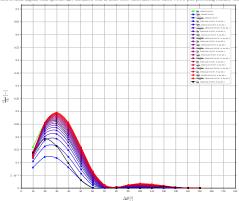


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta=0.7^{\circ}$ 

 $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta =$ 

 $G_l$  from VCCI, stresses extracted on fiber surface,  $\delta = 9.0^{\circ}$ 

Normalized energy release rate  $\frac{G_{s,t}}{st}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black





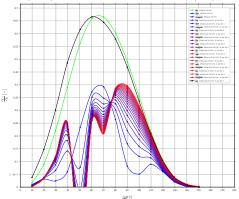




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#### $G_{\parallel}$ from VCCI, stresses extracted on fiber surface, $\delta=9.0^{\circ}$

Normalized energy release rate  $\frac{a_{ij}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



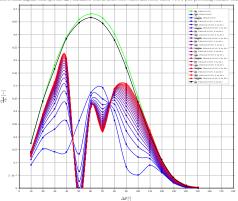






# $G_{TOT}$ from VCCI, stresses extracted on fiber surface, $\delta = 9.0^{\circ}$

Normalized energy release rate  $\frac{G_{i-1}}{d_i}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface











Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta=0.4^{\circ}$   $\delta=0.3^{\circ}$   $\delta=0.2^{\circ}$  Summary Summary of  $G_{(...)}$  from VGGI, Stresses extracted on Tiber Surface,

8 = 9.0° Committed energy robous rate  $\frac{G_{ij}}{G_{ij}}$  as function of crack angular semi-sporture  $\Delta \theta_i$ , calculated with in locus brew-based and stross-based AVCCT post-processing rootines with strosses extracted on the liber side of the interface







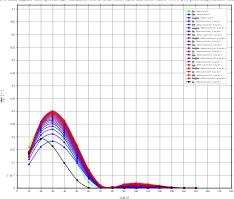


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$ 

 $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

### $G_l$ from VCCI, stresses extracted on matrix surface, $\delta = 9.0^{\circ}$

Normalized energy release rate  $\frac{G_{i-1}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





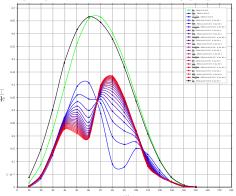






# $G_{II}$ from VCCI, stresses extracted on matrix surface, $\delta = 9.0^{\circ}$

Normalized energy release rate  $\frac{G_{i+1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





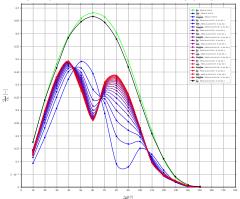






#### $G_{TOT}$ from VCCI, stresses extracted on matrix surface, $\delta = 9.0^{\circ}$

Normalized energy release rate  $\frac{G_{cr}}{2\pi^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





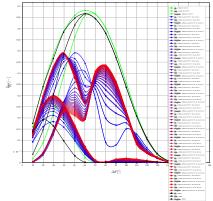






Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta=0.4^{\circ}$   $\delta=0.3^{\circ}$   $\delta=0.2^{\circ}$  Summary Summary of  $G_{(...)}$  from VGG, Stresses extracted on matrix surface,

 $\delta=9.0^{\circ}_{
m combined energy release rate rac{\alpha_{
m co}}{m_{
m co}}}$  as function of crack angular semi-operture  $\Delta\theta$ , calculated with in-loose force-based and stross-based VCCT post-processing restricts with strosses extracted on the matrix side of the interface





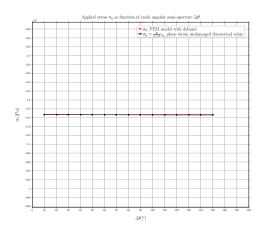






 $\label{eq:model_delta_$ 

$$\sigma_0$$
,  $\delta = 0.8^\circ$ 



In red small strain FEM, in black analytical plain strain value.



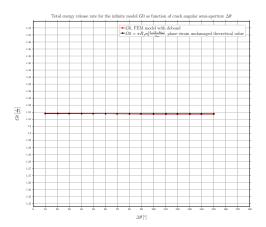






 $\label{eq:delta-$ 

 $G_0$ ,  $\delta = 0.8^\circ$ 



In red small strain FEM, in black analytical plain strain value.

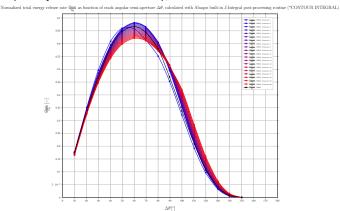








#### J-Integral (Abaqus built-in routine), $\delta = 0.8^{\circ}$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

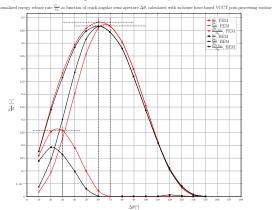








## VCCT in forces (in-house Python routine), $\delta = 0.8^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

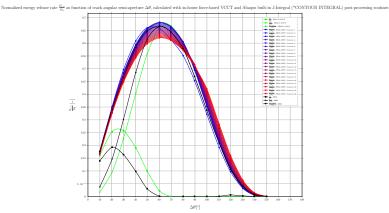








#### J-Integral and VCCT in forces, $\delta = 0.8^{\circ}$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



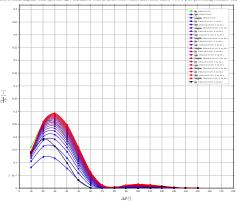






### $G_l$ from VCCl, stresses extracted on fiber surface, $\delta = 8.0^{\circ}$

Normalized energy release rate  $\frac{G_{12}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





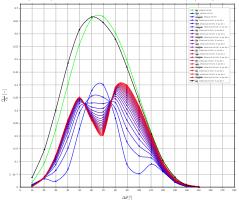






#### $G_{II}$ from VCCI, stresses extracted on fiber surface, $\delta = 8.0^{\circ}$

Normalized energy release rate  $\frac{a_{ij}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface









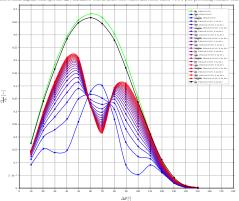


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta=0.5^{\circ}$ 

 $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta=0.4^{\circ}$   $\delta=0.3^{\circ}$   $\delta=0.2^{\circ}$  Summary

# $G_{TOT}$ from VCCI, stresses extracted on fiber surface, $\delta=8.0^\circ$

Normalized energy release rate  $\frac{d_{i+1}}{d_i}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





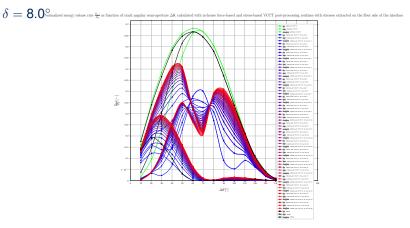






 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = \textbf{0.8}^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary } \delta = 0.2^{\circ} \quad \delta = 0.2^{\circ$ 

Summary of  $G_{(\cdot\cdot)}$  from VCCI, stresses extracted on liber surface,









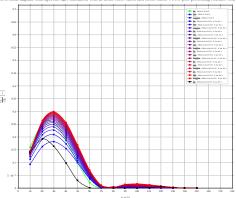


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$ 

 $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta=0.4^{\circ}$   $\delta=0.3^{\circ}$   $\delta=0.2^{\circ}$  Summary

#### $G_l$ from VCCI, stresses extracted on matrix surface, $\delta = 8.0^{\circ}$

Normalized energy release rate  $\frac{G_{i-1}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





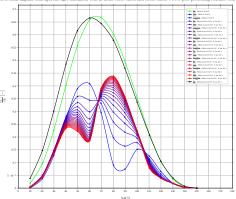






## $G_{II}$ from VCCI, stresses extracted on matrix surface, $\delta=8.0^{\circ}$

Normalized energy release rate  $\frac{G_{ij}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





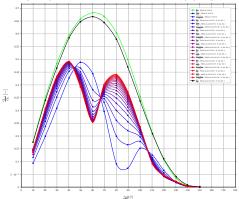






#### $G_{TOT}$ from VCCI, stresses extracted on matrix surface, $\delta = 8.0^{\circ}$

Normalized energy release rate  $\frac{G_{12}}{G_{12}}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



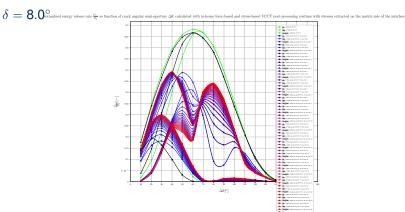








## Summary or $G_{(...)}$ from VCCI, stresses extracted on matrix surface,



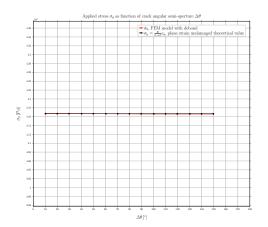






Model Data  $\delta=1.0^\circ$   $\delta=0.9^\circ$   $\delta=0.8^\circ$   $\delta=0.7^\circ$   $\delta=0.6^\circ$   $\delta=0.5^\circ$   $\delta=0.4^\circ$   $\delta=0.3^\circ$   $\delta=0.2^\circ$  Summary

$$\sigma_0$$
,  $\delta=0.7^\circ$ 



In red small strain FEM, in black analytical plain strain value.



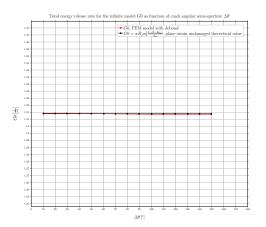






 $\label{eq:decomposition} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary and } \mbox{Summary$ 

 $G_0, \delta = 0.7^{\circ}$ 



In red small strain FEM, in black analytical plain strain value.

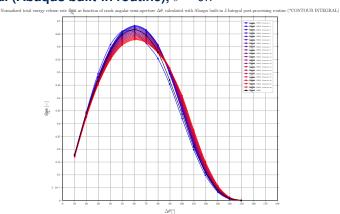








#### J-Integral (Abagus built-in routine), $\delta = 0.7^{\circ}$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

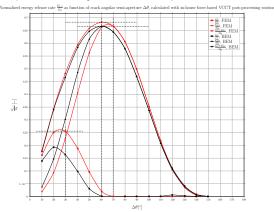








### VCCT in forces (in-house Python routine), $\delta=0.7^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.



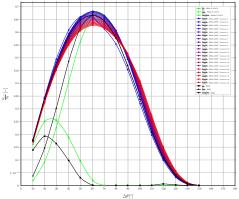






#### J-Integral and VCCT in forces, $\delta = 0.7^{\circ}$

Normalized energy release rate  $\frac{a_{i+}}{a_i}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based VCCT and Abaqus built-in J-Integral (\*CONTOUR INTEGRAL) post-processing routines



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



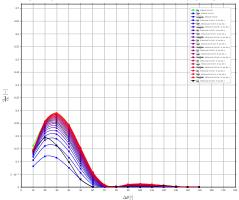






## $G_l$ from VCCl, stresses extracted on fiber surface, $\delta = 7.0^{\circ}$

Normalized energy release rate  $\frac{G_{+}}{dr}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





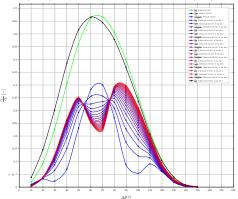






#### $G_{II}$ from VCCI, stresses extracted on fiber surface, $\delta = 7.0^{\circ}$

Normalized energy release rate  $\frac{G_{12}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





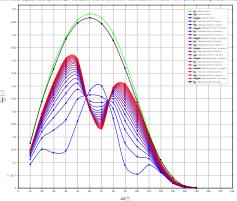






### $G_{TOT}$ from VCCI, stresses extracted on fiber surface, $\delta = 7.0^{\circ}$

Normalized energy release rate  $\frac{d_{i+1}}{d_i}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





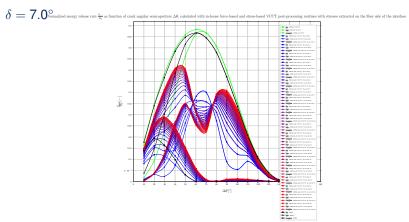






 $\label{eq:decomposition} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary Summary Polynomials} \quad \delta = 0.00^{\circ} \quad \delta = 0.$ 

#### Summary of $G_{(\cdot,\cdot)}$ from VCCI, stresses extracted on liber surface,









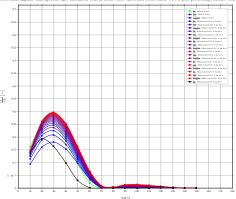


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta$ 

 $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

# $G_l$ from VCCI, stresses extracted on matrix surface, $\delta = 7.0^{\circ}$

Normalized energy release rate  $\frac{G_{i,j}}{G}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





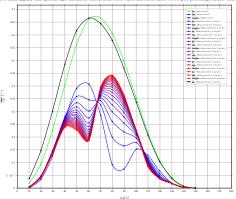






#### $G_{II}$ from VCCI, stresses extracted on matrix surface, $\delta = 7.0^{\circ}$

Normalized energy release rate  $\frac{G_{ij}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





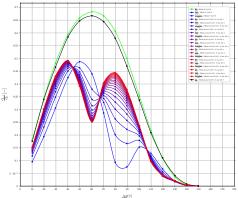






#### $G_{TOT}$ from VCCI, stresses extracted on matrix surface, $\delta = 7.0^{\circ}$

Normalized energy release rate  $\frac{G_{12}}{G_{12}}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



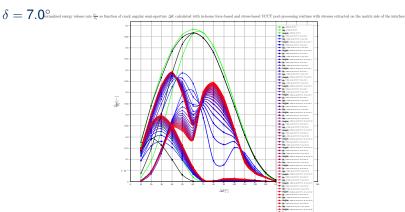








### Summary of $G_{(\cdot\cdot)}$ from VCCI, stresses extracted on matrix surface,





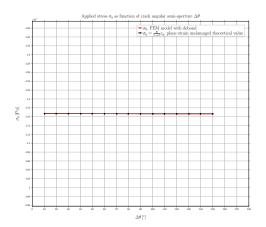






 $\mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary Summary Polynomials} \quad \delta = 0.00^{\circ} \quad \delta$ 

$$\sigma_0$$
,  $\delta = 0.6^\circ$ 



In red small strain FEM, in black analytical plain strain value.



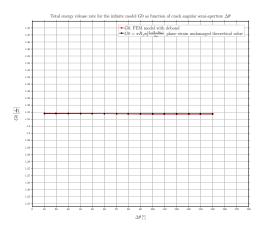






Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

 $G_{0}$ ,  $\delta = 0.6^{\circ}$ 



In red small strain FEM, in black analytical plain strain value.

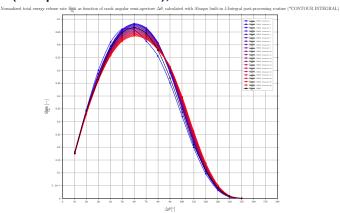








#### J-Integral (Abaqus built-in routine), $\delta = 0.6^{\circ}$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

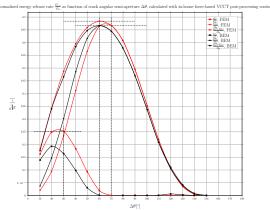








## VCCT in forces (in-house Python routine), $\delta=0.6^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

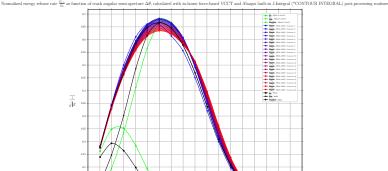








### J-Integral and VCCT in forces, $\delta=0.6^\circ$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



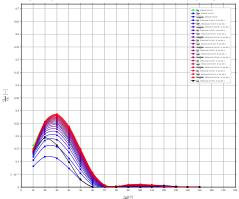






### $G_l$ from VCCI, stresses extracted on fiber surface, $\delta = 6.0^{\circ}$

Normalized energy release rate  $\frac{a_{ij}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





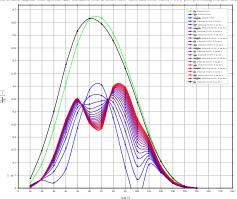






#### $G_{II}$ from VCCI, stresses extracted on fiber surface, $\delta = 6.0^{\circ}$

Normalized energy release rate  $\frac{a_{ij}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





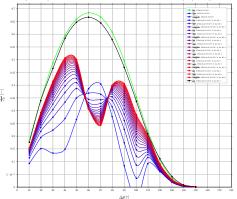






### $G_{TOT}$ from VCCI, stresses extracted on fiber surface, $\delta = 6.0^{\circ}$

Normalized energy release rate  $\frac{G_{i-1}}{d_i}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





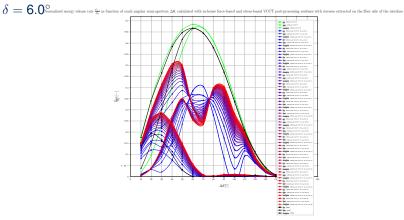






 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = \textbf{0.6}^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary } \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \delta = 0.2^{\circ$ 

Summary of  $G_{(...)}$  from VCCI, stresses extracted on liber surface,









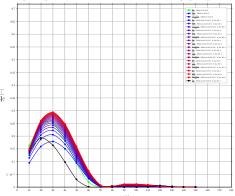


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$ 

 $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

#### $G_l$ from VCCl, stresses extracted on matrix surface, $\delta = 6.0^{\circ}$

Normalized energy release rate  $\frac{G_{i+1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



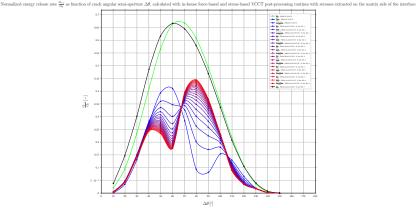








 $G_{II}$  from VCCI, stresses extracted on matrix surface,  $\delta=6.0^{\circ}$ 





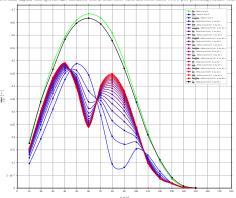






### $G_{TOT}$ from VCCI, stresses extracted on matrix surface, $\delta = 6.0^{\circ}$

Normalized energy release rate  $\frac{G_{c,b}}{dc}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





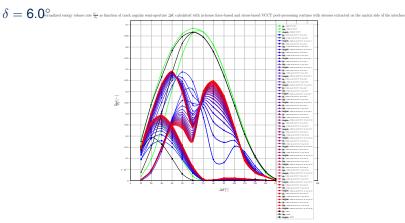






 $\label{eq:delta-$ 

# Summary of $G_{(\cdot,\cdot)}$ from VCCI, stresses extracted on matrix surface,





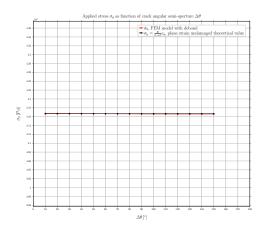






Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta=0.4^{\circ}$   $\delta=0.3^{\circ}$   $\delta=0.2^{\circ}$  Summary

$$\sigma_0$$
,  $\delta=0.5^\circ$ 



In red small strain FEM, in black analytical plain strain value.



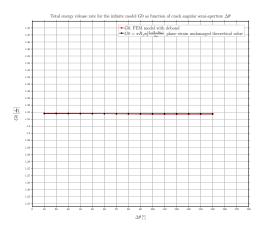






Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

$$G_0$$
,  $\delta = 0.5^\circ$ 



In red small strain FEM, in black analytical plain strain value.

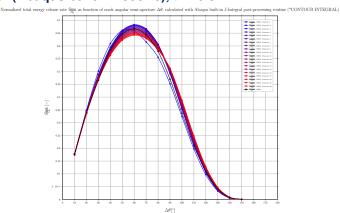








#### J-Integral (Abagus built-in routine), $\delta=0.5^{\circ}$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

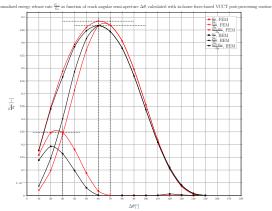








## VCCT in forces (in-house Python routine), $\delta=0.5^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

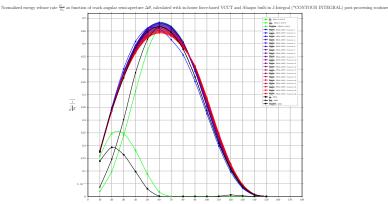








### J-Integral and VCCT in forces, $\delta=0.5^\circ$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



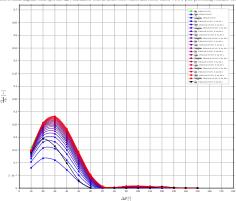






#### $G_l$ from VCCI, stresses extracted on fiber surface, $\delta = 5.0^{\circ}$

Normalized energy release rate  $\frac{G_{12}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





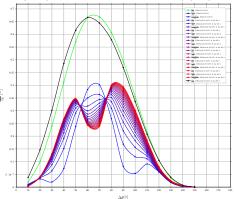






#### $G_{\parallel}$ from VCCI, stresses extracted on fiber surface, $\delta = 5.0^{\circ}$

Normalized energy release rate  $\frac{G_{i-1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface







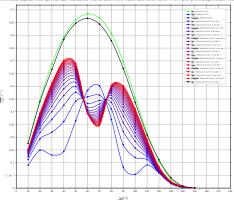




Symbols, Models, Equations & Reference Data Results Summary & Conclusion  $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$ 

# $G_{TOT}$ from VCCI, stresses extracted on fiber surface, $\delta = 5.0^{\circ}$

Normalized energy release rate  $\frac{G_{i-1}}{d_i}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





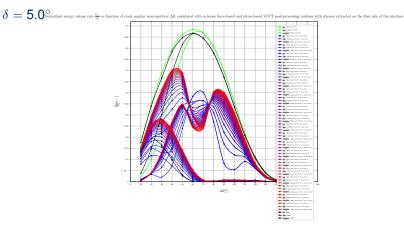






 $\label{eq:delta-$ 

## Summary or G(...) from VCCI, stresses extracted on liber surface,









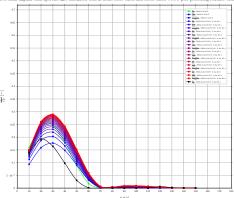


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$ 

 $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

### $G_l$ from VCCI, stresses extracted on matrix surface, $\delta = 5.0^{\circ}$

Normalized energy release rate  $\frac{G_{i,j}}{G_{i,j}}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





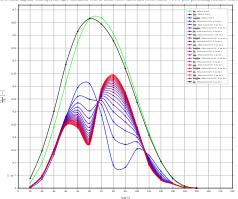






### $G_{II}$ from VCCI, stresses extracted on matrix surface, $\delta = 5.0^{\circ}$

Normalized energy release rate  $\frac{G_{ij}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





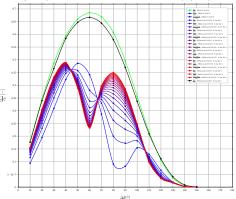






### $G_{TOT}$ from VCCI, stresses extracted on matrix surface, $\delta = 5.0^{\circ}$

Normalized energy release rate  $\frac{G_{i,i}}{G_{i,j}}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





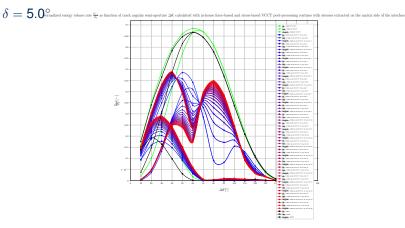






 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary Summary Sum$ 

# Summary of $G_{(\cdot\cdot)}$ from VGGI, stresses extracted on matrix surface,





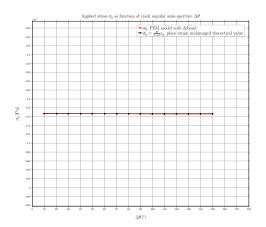






 $\label{eq:model_delta_$ 

 $\sigma_0$ ,  $\delta = 0.4^\circ$ 



In red small strain FEM, in black analytical plain strain value.

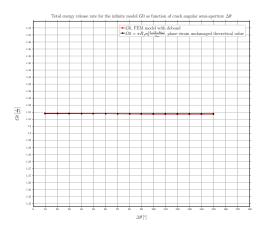






 $\label{eq:decomposition} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary Summary Polynomials} \quad \delta = 0.00^{\circ} \quad \delta = 0.$ 

 $G_0$ ,  $\delta = 0.4^\circ$ 



In red small strain FEM, in black analytical plain strain value.

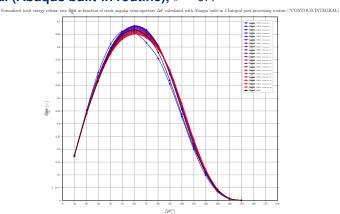








### J-Integral (Abaqus built-in routine), $\delta = 0.4^{\circ}$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

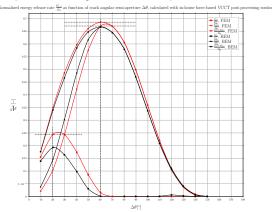








### VCCT in forces (in-house Python routine), $\delta = 0.4^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.









Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$ 

 $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

## VCCT, percentual error on BEM, $\delta=0.4^\circ$

$\Delta \theta$	[°]	$\frac{\frac{G_I}{G_0} _{FEM} - \frac{G_I}{G_0} _{BEM}}{\frac{G_I}{G_0} _{BEM}}$	$\frac{\frac{G_{II}}{G_0} _{\textit{FEM}} - \frac{G_{II}}{G_0} _{\textit{BEM}}}{\frac{G_{II}}{G_0} _{\textit{BEM}}}$	$\frac{\frac{G_{TOT}}{G_0} _{FEM} - \frac{G_{TOT}}{G_0} _{BEM}}{\frac{G_{TOT}}{G_0} _{BEM}}$
10	0	11.84%	-45.09%	0.06%
20	0	26.79%	-28.36%	2.95%
30	0	48.73%	-19.81%	4.31%
40	0	77.24%	-12.93%	2.20%
50	0	181.34%	-6.75%	2.04%
60	0	1084.50%	0.68%	2.78%
70	0		3.99%	3.93%
80	0		4.79%	4.61%
90	0		5.62%	6.07%
10	00		6.18%	6.59%
11	0		3.83%	5.37%
12	20		1.31%	-2.40%
13	30		-4.97%	-9.28%
14	10		-30.42%	-29.99%
15	50		-61.36%	-14.84%

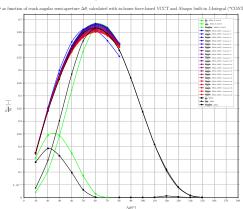








#### J-Integral and VCCT in forces, $\delta=0.4^\circ$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



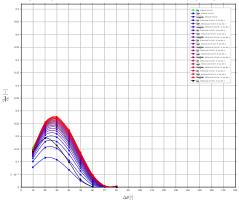






#### $G_l$ from VCCI, stresses extracted on fiber surface, $\delta = 4.0^{\circ}$

Normalized energy release rate  $\frac{G_{s,t}}{st}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





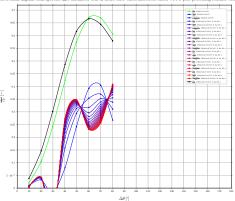






#### $G_{II}$ from VCCI, stresses extracted on fiber surface, $\delta = 4.0^{\circ}$

Normalized energy release rate  $\frac{G_{+}}{d^{2}}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





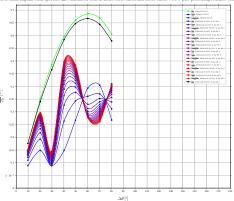






### $G_{TOT}$ from VCCI, stresses extracted on fiber surface, $\delta = 4.0^{\circ}$

Normalized energy release rate  $\frac{G_{c}^{2}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



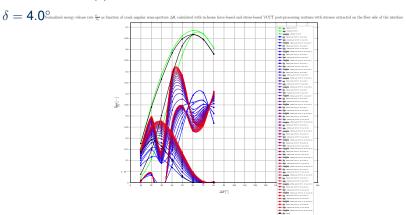








# Summary or $G_{(...)}$ from VCCI, stresses extracted on liber surface,





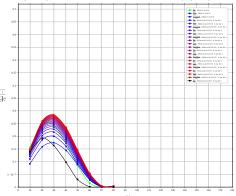






#### $G_l$ from VCCl, stresses extracted on matrix surface, $\delta = 4.0^{\circ}$

Normalized energy release rate  $\frac{G_{i+1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





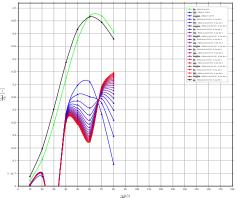






### $G_{II}$ from VCCI, stresses extracted on matrix surface, $\delta=4.0^{\circ}$

Normalized energy release rate  $\frac{G_{i+1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





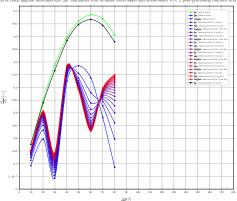






## $G_{TOT}$ from VCCI, stresses extracted on matrix surface, $\delta = 4.0^{\circ}$

Normalized energy release rate  $\frac{G_{1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





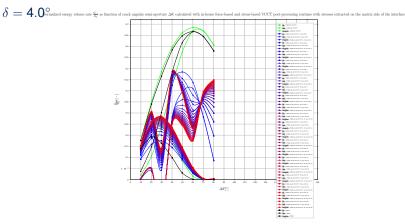






 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = \textbf{0.4}^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary } \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \delta = 0.2^{\circ$ 

# Summary of $G_{(\cdot\cdot)}$ from VCCI, stresses extracted on matrix surface,









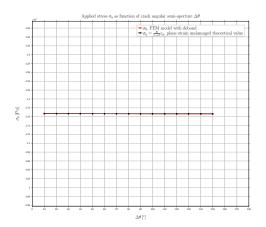


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Symbols, Models, Equations & Reference Data Results Summary & Conclusion

 $\mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = \textbf{0.3}^{\circ} \quad \delta = \textbf{0.2}^{\circ} \quad \mbox{Summary Summary Properties of the properties of th$ 

 $\sigma_0$  ,  $\delta=0.3^\circ$ 



In red small strain FEM, in black analytical plain strain value.





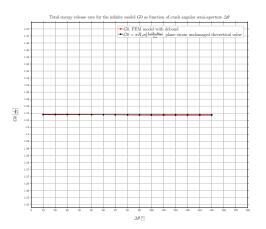




Symbols, Models, Equations & Reference Data Results Summary & Conclusion

 $\mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = \textbf{0.3}^{\circ} \quad \delta = \textbf{0.2}^{\circ} \quad \mbox{Summary Summary Polynomials} \quad \delta = 0.8^{\circ} \quad \delta = 0.8$ 

$$G_0$$
,  $\delta=0.3^\circ$ 



In red small strain FEM, in black analytical plain strain value.

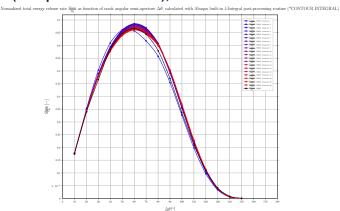








#### J-Integral (Abaqus built-in routine), $\delta=0.3^\circ$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

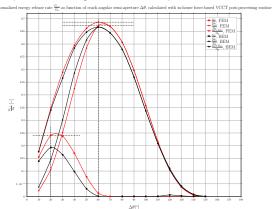








## VCCT in forces (in-house Python routine), $\delta=0.3^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.









Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta=1.0^{\circ}$   $\delta=0.9^{\circ}$   $\delta=0.8^{\circ}$   $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$ 

 $\delta=0.7^{\circ}$   $\delta=0.6^{\circ}$   $\delta=0.5^{\circ}$   $\delta=0.4^{\circ}$   $\delta=0.3^{\circ}$   $\delta=0.2^{\circ}$  Summary

## VCCT, percentual error on BEM, $\delta=0.3^\circ$

$\Delta \theta$ [°	$\frac{G_I}{G_0} _{FEM} - \frac{G_I}{G_0} _{BEM}$	$\frac{\frac{G_{II}}{G_0}\mid_{\textit{FEM}} - \frac{G_{II}}{G_0}\mid_{\textit{BEM}}}{\frac{G_{II}}{G_0}\mid_{\textit{BEM}}}$	$\frac{\frac{G_{TOT}}{G_0}\mid_{\textit{FEM}} - \frac{G_{TOT}}{G_0}\mid_{\textit{BEM}}}{\frac{G_{TOT}}{G_0}\mid_{\textit{BEM}}}$
10	11.91%	-39.86%	1.23%
20	24.60%	-26.37%	2.56%
30	44.60%	-16.55%	4.97%
40	73.27%	-12.54%	1.86%
50	154.60%	-5.10%	2.35%
60	955.70%	1.01%	2.87%
70		3.99%	4.01%
80		4.79%	4.83%
90		5.62%	5.66%
100		6.18%	6.22%
110		3.83%	3.38%
120		1.31%	-4.08%
130		-4.97%	-10.17%
140		-30.42%	-34.90%
150		-61.36%	-0.52%

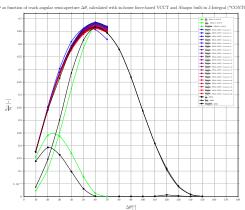








#### J-Integral and VCCT in forces, $\delta=0.3^\circ$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



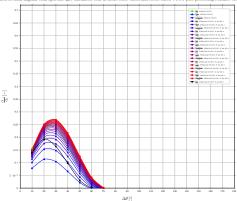






#### $G_l$ from VCCI, stresses extracted on fiber surface, $\delta = 3.0^{\circ}$

Normalized energy release rate  $\frac{G_{+}}{dr}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



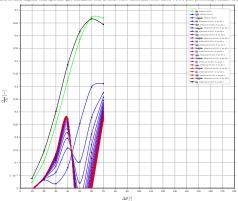






#### $G_{II}$ from VCCI, stresses extracted on fiber surface, $\delta=3.0^{\circ}$

Normalized energy release rate  $\frac{a_{ij}}{L^2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



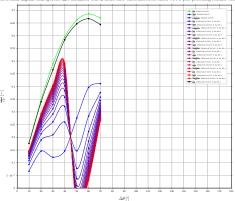






# $G_{TOT}$ from VCCI, stresses extracted on fiber surface, $\delta=3.0^\circ$

Normalized energy release rate  $\frac{G_{+}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black





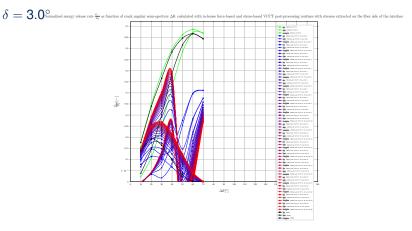




Symbols, Models, Equations & Reference Data Results Summary & Conclusion

 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = \textbf{0.3}^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary } \delta = 0.8^{\circ} \quad \delta = 0.8^{\circ$ 

# Summary or G<sub>(...)</sub> from VCCI, stresses extracted on liber surface,



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black BEM results.



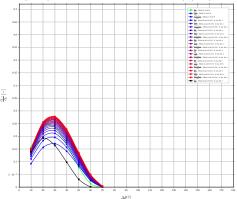






#### $G_l$ from VCCl, stresses extracted on matrix surface, $\delta = 3.0^{\circ}$

Normalized energy release rate  $\frac{G_{i+1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



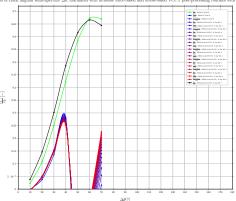






# $G_{II}$ from VCCI, stresses extracted on matrix surface, $\delta=3.0^{\circ}$

Normalized energy release rate  $\frac{G_{i+1}}{2}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



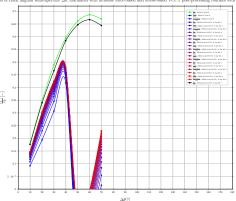






#### $G_{TOT}$ from VCCI, stresses extracted on matrix surface, $\delta=3.0^{\circ}$

Normalized energy release rate  $\frac{G_{i,i}}{G_{i,j}}$  as function of crack angular semi-aperture  $\Delta\theta$ , calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black

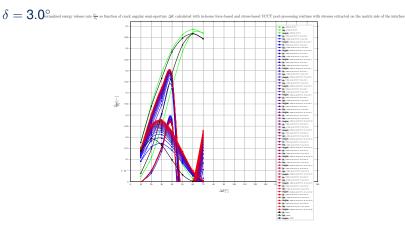








## Summary of Gan from VCCI, stresses extracted on matrix surface,



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black BEM results.





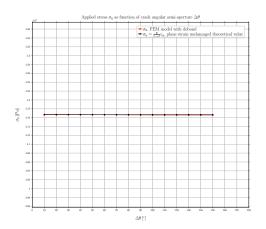




Symbols, Models, Equations & Reference Data Results Summary & Conclusion

Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$   $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

$$\sigma_0$$
,  $\delta=0.2^\circ$ 



In red small strain FEM, in black analytical plain strain value.





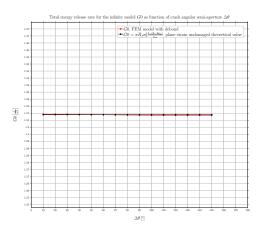




Symbols, Models, Equations & Reference Data Results Summary & Conclusion

 $\label{eq:model_delta} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = \textbf{0.2}^{\circ} \quad \mbox{Summary and the model}$ 

 $G_0$ ,  $\delta = 0.2^\circ$ 



In red small strain FEM, in black analytical plain strain value.

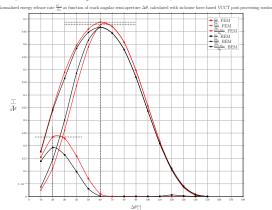








## VCCT in forces (in-house Python routine), $\delta = 0.2^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.









Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$ 

 $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary

## VCCT, percentual error on BEM, $\delta=0.2^{\circ}$

$\Delta  heta\left[^{\circ} ight]$	$\frac{\frac{G_I}{G_0}\mid_{\textit{FEM}} - \frac{G_I}{G_0}\mid_{\textit{BEM}}}{\frac{G_I}{G_0}\mid_{\textit{BEM}}}$	$\frac{\frac{G_{II}}{G_0}\mid_{\textit{FEM}} - \frac{G_{II}}{G_0}\mid_{\textit{BEM}}}{\frac{G_{II}}{G_0}\mid_{\textit{BEM}}}$	$\frac{\frac{G_{TOT}}{G_0}\mid_{FEM} - \frac{G_{TOT}}{G_0}\mid_{BEM}}{\frac{G_{TOT}}{G_0}\mid_{BEM}}$
10	11.41%	-32.33%	2.44%
20	21.51%	-23.07%	2.24%
30	39.40%	-13.34%	5.22%
40	62.43%	-10.19%	1.99%
50	131.80%	-4.28%	2.06%
60	712.56%	1.64%	3.03%
70		4.05%	4.06%
80		5.02%	5.05%
90		5.69%	5.71%
100		5.83%	5.86%
110		4.45%	3.97%
120		1.55%	-3.88%
130		-6.42%	-11.60%
140		-28.46%	-33.11%
150		-67.88%	-0.52%

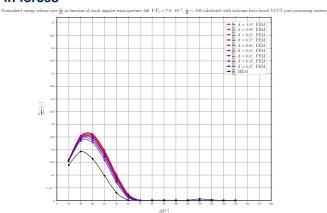








#### $G_l$ , VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

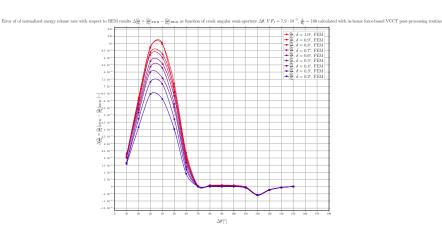








#### G<sub>1</sub> Error with respect to BEM, VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from

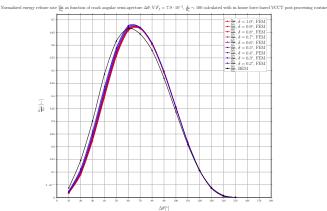








#### $G_{\parallel}$ , VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.



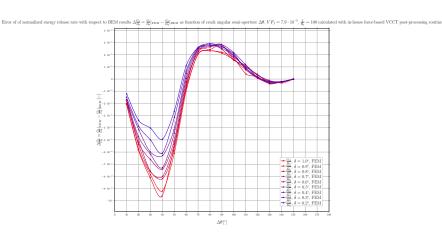






Symbols, Models, Equations & Reference Data Results Summary & Conclusion  $\delta = 0.7^{\circ}$   $\delta = 0.6^{\circ}$   $\delta = 0.5^{\circ}$   $\delta = 0.4^{\circ}$   $\delta = 0.3^{\circ}$   $\delta = 0.2^{\circ}$  Summary Model Data  $\delta = 1.0^{\circ}$   $\delta = 0.9^{\circ}$   $\delta = 0.8^{\circ}$ 

# $G_{\parallel}$ Error with respect to BEM, VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from

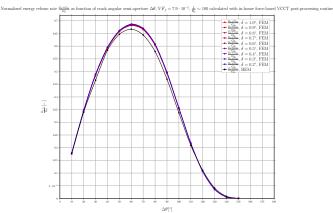








#### $G_{TOT}$ , VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

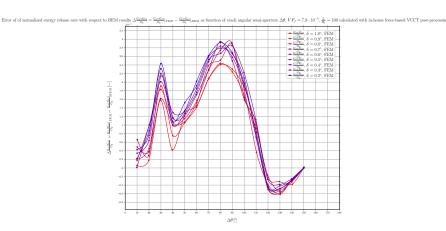








# $G_{TOT}$ Error with respect to BEM, VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from









Symbols, Models, Equations & Reference Data Results Summary & Conclusion

# **≥** SUMMARY & CONCLUSION









mbols, Models, Equations & Reference Data Results Summary & Conc

#### **Summary**

- Implemented Virtual Crack Closure Integral (VCCI) method for the calculation of  $G_l$  and  $G_{ll}$
- ✓ Analysis of free infinite RVE ( $\frac{L}{R_f}$  ~ 100) for several mesh refinements  $\delta \in [1.0^{\circ}, 0.2^{\circ}]$
- $\checkmark$   $G_I, G_{II}$  and  $G_{TOT}$  calculated using Abaqus built-in J-Integral routine, in-house implemented VCCT and VCCI routines









mbols Models Equations & Reference Data Results Summary &

#### Conclusion

- ✓ Good agreement of J-Integral results with  $G_{TOT}$  from BEM
- J-Integral convergence improves refining the mesh
- For  $\delta = 0.4^{\circ}, 0.3^{\circ}, 0.2^{\circ}$  maxima are at the right angle (20° for  $G_{II}$ , 60° for  $G_{II}$  and  $G_{TOT}$ ) with in-house VCCT
- $\checkmark$   $G_{TOT}$  relative errors of VCCT over BEM are small ( $\sim$  5% or less) for every  $\Delta\theta$
- $\checkmark$   $G_{II}$  relative errors of VCCT over BEM are small ( $\sim 5\%$  or less) for  $\Delta \theta > 40^\circ$
- Results tend to converge to BEM values as the mesh is refined





mbols, Models, Equations & Reference Data Results Summary & Conclusion

#### Conclusion

- $\times$   $G_l$  relative errors of VCCT over BEM are high (> 10%)
- $ightharpoonup G_{II}$  relative errors of VCCT over BEM are high (> 10%) for  $\Delta heta \leq 40^\circ$
- $\times$   $G_l$  of VCCI has correct functional form but values are overestimated (except for very small integration lengths)
- $\times$   $G_{II}$  (and consequently  $G_{TOT}$ ) of VCCI provides strange results







Symbols, Models, Equations & Reference Data Results Summary & Conclusion

# **Next steps**

New analysis with  $\delta = 0.05^{\circ}$  to see if a better agreement of  $G_l$  values is attained



