

# ESTIMATING THE AVERAGE SIZE OF FIBER/MATRIX INTERFACE CRACKS IN UD AND CROSS-PLY LAMINATES

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Education and Culture

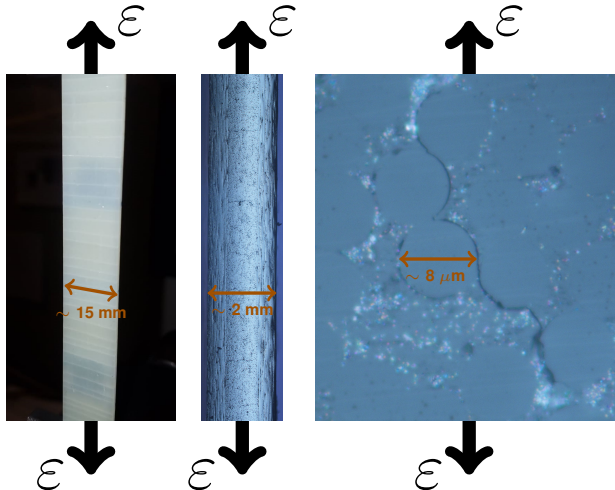
Erasmus Mundus



# Outline

# TRANSVERSE CRACKS INITIATION IN FRPC

## Micromechanics of Initiation: Transverse Tensile Loading



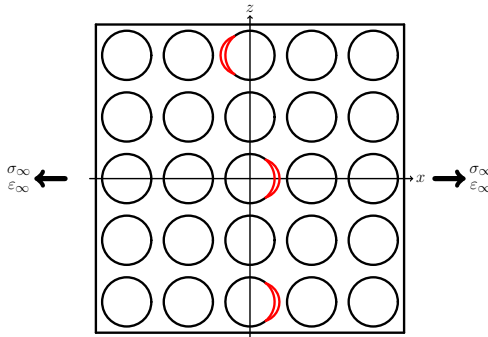
**Left:**  
front view of [0, 90<sub>2</sub>]<sub>S</sub>,  
visual inspection.

**Center:**  
edge view of [0, 90]<sub>S</sub>,  
optical microscope.

**Right:**  
edge view of [0, 90]<sub>S</sub>,  
optical microscope.

## Micromechanics of Initiation: Transverse Tensile Loading

### Stage 1: isolated debonds



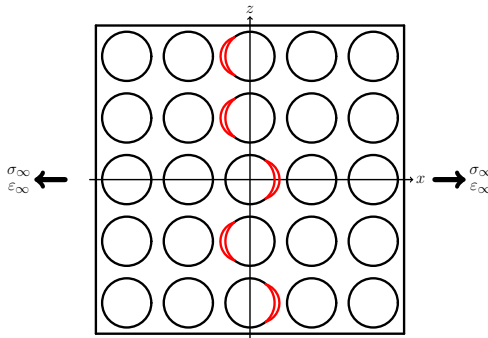
Bailey et al., P. Roy. Soc. A-Math. Phys. **366** (1727), 1979.

Bailey et al., J. Mater. Sci. **16** (3), 1981.

Zhang et al., Compos. Part A-Appl. S. **28** (4), 1997.

## Micromechanics of Initiation: Transverse Tensile Loading

### Stage 2: consecutive debonds



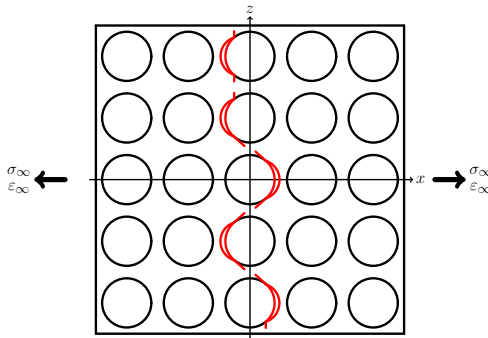
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Zhang et al., Compos. Part A-Appl. S. **28** (4), 1997.

## Micromechanics of Initiation: Transverse Tensile Loading

### Stage 3: kinking



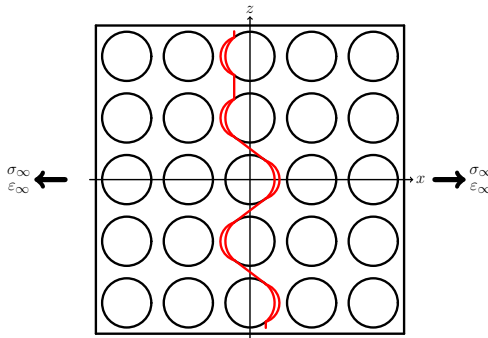
Bailey et al., P. Roy. Soc. A-Math. Phys. **366** (1727), 1979.

Bailey et al., J. Mater. Sci. **16** (3), 1981.

Zhang et al., Compos. Part A-Appl. S. **28** (4), 1997.

## Micromechanics of Initiation: Transverse Tensile Loading

### Stage 4: coalescence



Bailey et al., P. Roy. Soc. A-Math. Phys. **366** (1727), 1979.

Bailey et al., J. Mater. Sci. **16** (3), 1981.

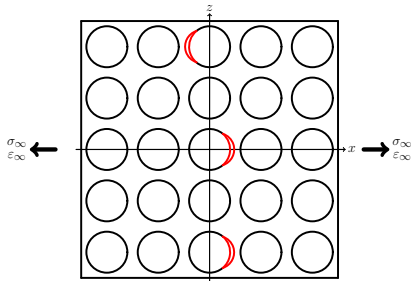
Zhang et al., Compos. Part A-Appl. S. **28** (4), 1997.



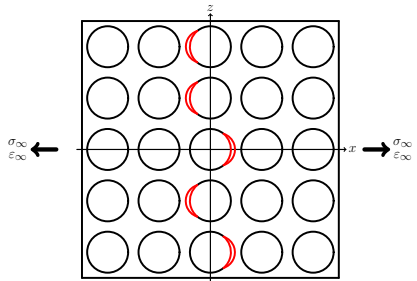
## Objective of the Study

Can we talk about a ply-thickness effect for the fiber-matrix interface crack?

### Stage 1: isolated debonds

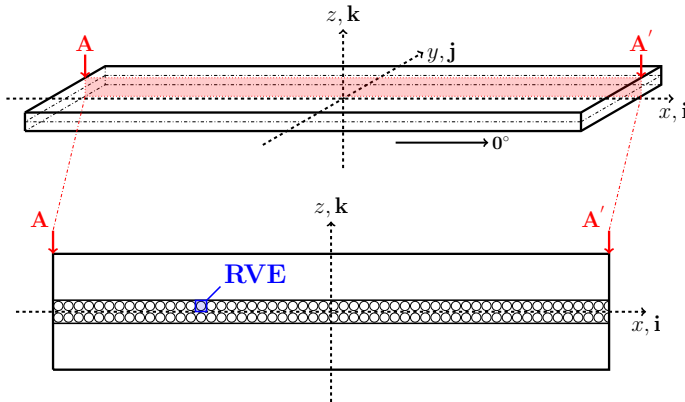


### Stage 2: consecutive debonds



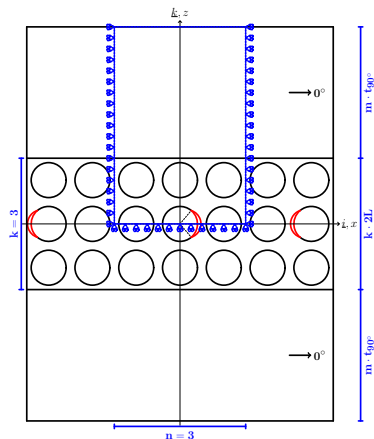
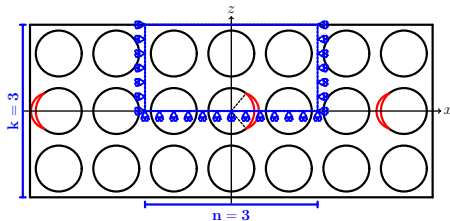
# **MODELING**

## Geometry

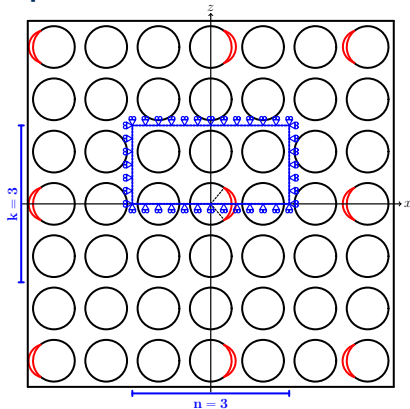


- $L, W \gg t$
- $L, W \rightarrow \infty$
- Square packing
- $L_d \gg \Delta\theta_d$
- 2D RVE

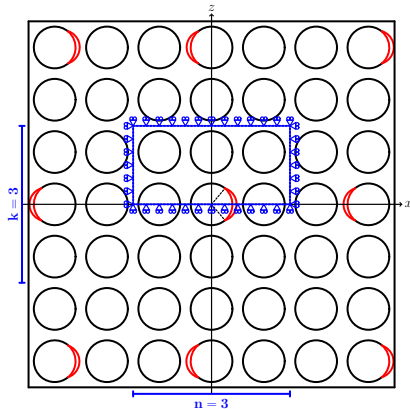
## Representative Volume Elements



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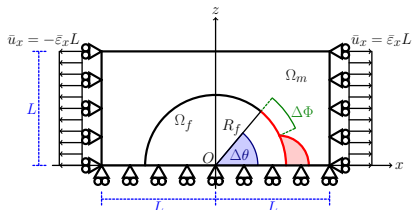


$n \times k - \text{symm}$



$n \times k - \text{asymm}$

## Assumptions

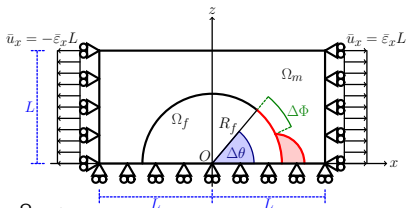


$$R_f = 1 \text{ } [\mu\text{m}] \quad L = \frac{R_f}{2} \sqrt{\frac{\pi}{V_f}}$$

- Linear elastic, homogeneous materials
- Concentric Cylinders Assembly with Self-Consistent Shear Model for UD
- Plane strain
- Frictionless contact interaction
- Symmetric w.r.t. x-axis
- Coupling of x-displacements on left and right side (repeating unit cell)
- Applied uniaxial tensile strain  $\bar{\epsilon}_x = 1\%$
- $V_f = 60\%$

Material	$V_f$ [%]	$E_L$ [GPa]	$E_T$ [GPa]	$\mu_{LT}$ [GPa]	$\nu_{LT}$ [-]	$\nu_{TT}$ [-]
Glass fiber	-	70.0	70.0	29.2	0.2	0.2
Epoxy	-	3.5	3.5	1.25	0.4	0.4
UD	60.0	43.442	13.714	4.315	0.273	0.465

## Solution



in  $\Omega_f, \Omega_m, \Omega_{UD}$  :

$$\frac{\partial^2 \varepsilon_{xx}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial x^2} = \frac{\partial^2 \gamma_{zx}}{\partial x \partial z} \quad \text{for } 0^\circ \leq \alpha \leq \Delta\theta, \Delta\theta \neq 0^\circ :$$

$$\varepsilon_y = \gamma_{xy} = \gamma_{yz} = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = 0$$

$$\sigma_{yy} = \nu (\sigma_{xx} + \sigma_{zz})$$

$$(\vec{u}_m(R_f, \alpha) - \vec{u}_f(R_f, \alpha)) \cdot \vec{n}_\alpha \geq 0$$

$$\text{for } \Delta\theta \leq \alpha \leq 180^\circ :$$

$$\vec{u}_m(R_f, \alpha) - \vec{u}_f(R_f, \alpha) = 0$$

$$\sigma_{ij} = E_{ijkl} \varepsilon_{kl} + BC$$

$$\forall \Delta\theta \neq 0^\circ$$

→ oscillating singularity

$$\sigma \sim r^{-\frac{1}{2}} \sin(\varepsilon \log r), \quad V_f \rightarrow 0$$

$$\varepsilon = \frac{1}{2\pi} \log \left( \frac{1 - \beta}{1 + \beta} \right)$$

$$\beta = \frac{\mu_2 (\kappa_1 - 1) - \mu_1 (\kappa_2 - 1)}{\mu_2 (\kappa_1 + 1) + \mu_1 (\kappa_2 + 1)}$$

→ receding contact

$$\rightarrow \frac{G(R_{f,2})}{G(R_{f,1})} = \frac{R_{f,2}}{R_{f,1}}, \quad \frac{G(\bar{\varepsilon}_{x,2})}{G(\bar{\varepsilon}_{x,1})} = \frac{\bar{\varepsilon}_{x,2}^2}{\bar{\varepsilon}_{x,1}^2}$$

→ FEM + LEFM (VCCT)

→ regular mesh of quadrilaterals at the crack tip:

$$- AR \sim 1, \quad \delta = 0.05^\circ$$

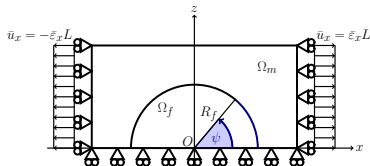
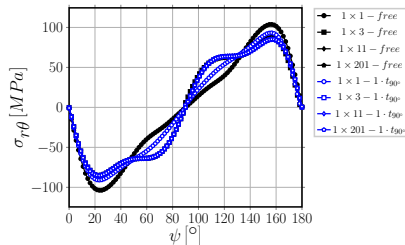
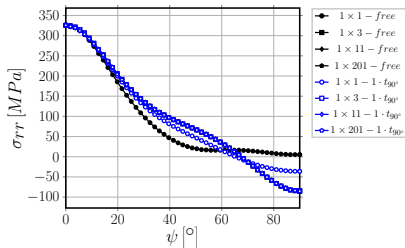
$$\forall \Delta\theta$$

→ 2<sup>nd</sup> order shape functions

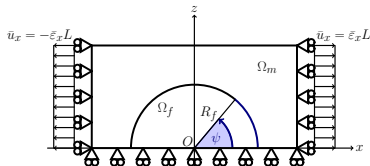
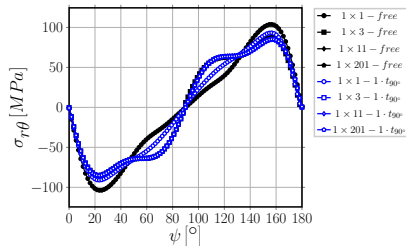
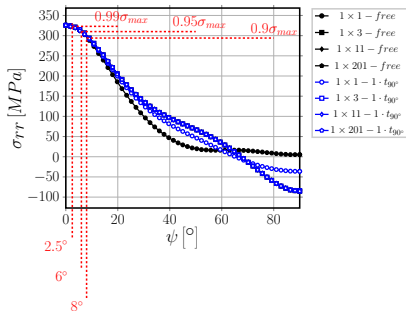
## ➤ DEBOND INITIATION



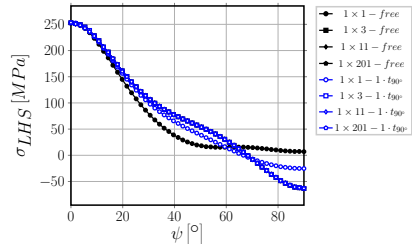
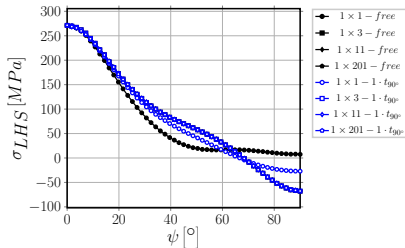
## $\sigma_{rr}$ VS $\tau_{r\theta}$ : radial stress vs tangential shear at the interface



## $\sigma_{rr}$ VS $\tau_{r\theta}$ : radial stress vs tangential shear at the interface

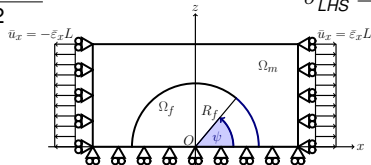


## $\sigma_{LHS}$ : local hydrostatic stress at the interface

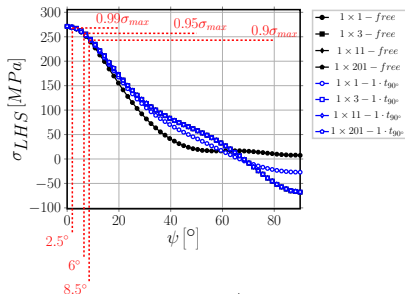


$$\sigma_{LHS}^{2D} = \frac{\sigma_{rr} + \sigma_{\theta\theta}}{2}$$

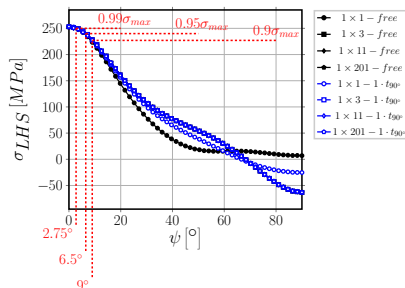
$$\sigma_{LHS}^{3D} = \frac{\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{yy}}{3}$$



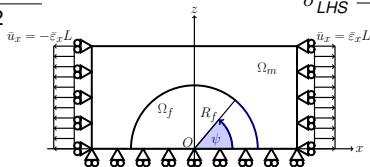
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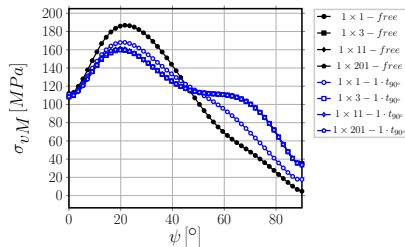
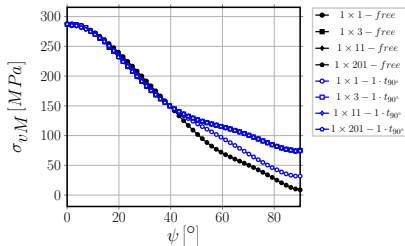
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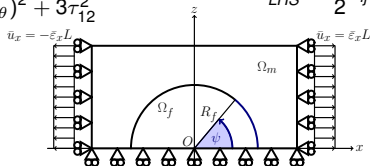


## $\sigma_{vM}$ : von Mises stress at the interface

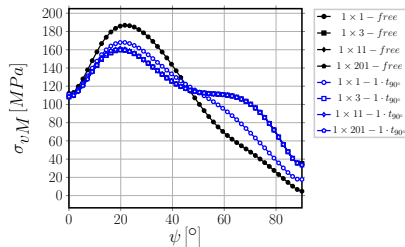
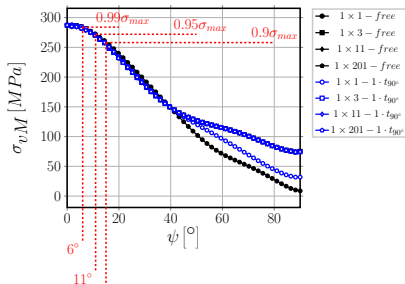


$$\sigma_{vM}^{2D} = \sqrt{(\sigma_{rr} - \sigma_{\theta\theta})^2 + 3\tau_{12}^2}$$

$$\sigma_{LHS}^{3D} = \frac{3}{2} s_{ij} s_{ij} \quad s_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$$

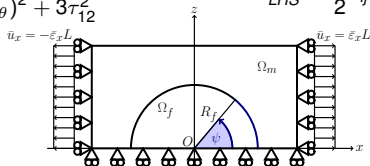


## $\sigma_{vM}$ : von Mises stress at the interface

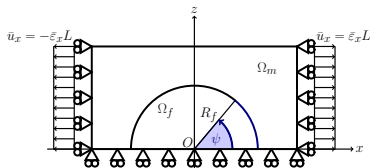
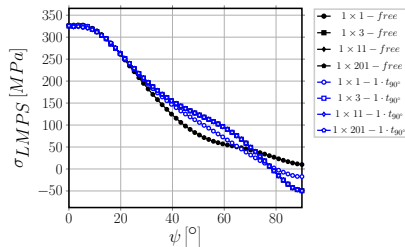
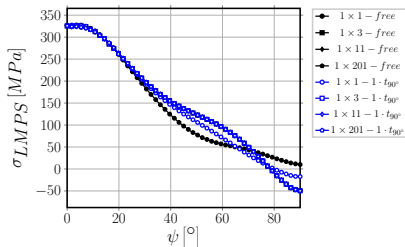


$$\sigma_{vM}^{2D} = \sqrt{(\sigma_{rr} - \sigma_{\theta\theta})^2 + 3\tau_{12}^2}$$

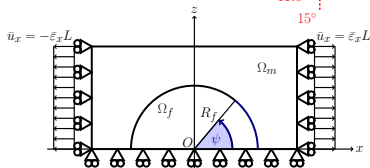
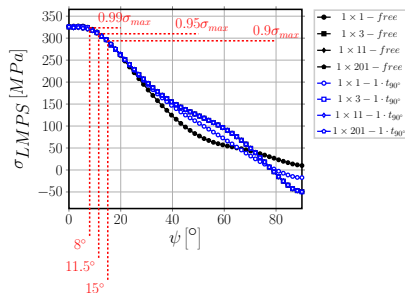
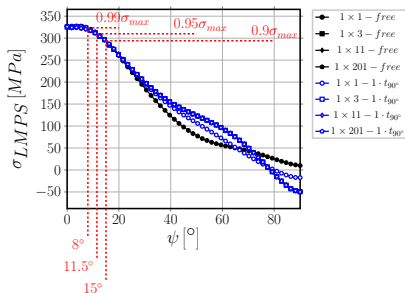
$$\sigma_{LHS}^{3D} = \frac{3}{2} s_{ij} s_{ij} \quad s_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$$



## $\sigma_I$ : maximum principal stress at the interface



## $\sigma_I$ : maximum principal stress at the interface





## Observations & Conclusions

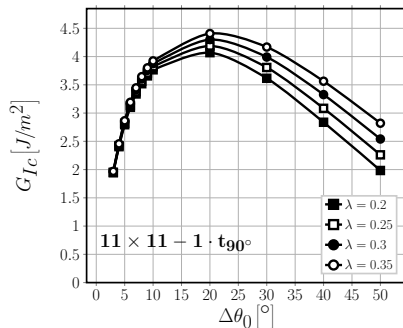
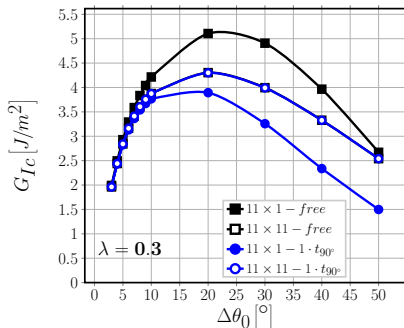
- For all stresses analyzed, no significant difference is present between the different RUCs for  $\psi \leq 10^\circ$ ;
- for all stresses analyzed, no difference can be observed by increasing  $k$  when  $k \geq 3$ ;
- for all stresses analyzed, no difference can be observed between  $1 \times k - free$  and  $1 \times k - 1 \cdot t_{90^\circ}$  for  $k \geq 3$ ;
- $\sigma_{rr}$ ,  $\sigma_{LHS,2D}$ ,  $\sigma_{LHS,3D}$ ,  $\sigma_{VM,2D}$ ,  $\sigma_{LMPS,2D}$  and  $\sigma_{LMPS,3D}$  all reach their peak value at  $0^\circ$  and  $180^\circ$  and decrease to 99% the peak value between  $2^\circ$  and  $8^\circ$ , to 95% the peak value between  $6^\circ$  and  $12^\circ$  and to 90% the peak value between  $8^\circ$  and  $15^\circ$  from the occurrence of the maximum.

### It seems reasonable to conclude that...

...a stress-based criterion would predict, irrespectively of the specific criterion chosen, the onset of an interface crack at  $0^\circ$  or  $180^\circ$  with an initial size at least comprised in the range  $2^\circ - 8^\circ$  (1% margin) and likely in the range  $6^\circ - 12^\circ$  (5% margin).

## DEBOND PROPAGATION

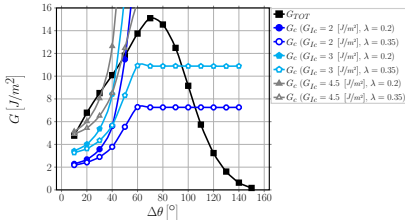
## Estimation of $G_{Ic}$



$$G_{Ic} = \frac{G_c}{1 + \tan^2((1 - \lambda) \Psi_G)} \Big|_{G_c = G_{TOT}(\Delta\theta_0)}, \quad \Psi_G = \tan^{-1} \left( \sqrt{\frac{G_{II}}{G_I}} \right) \Big|_{\Delta\theta_0}$$

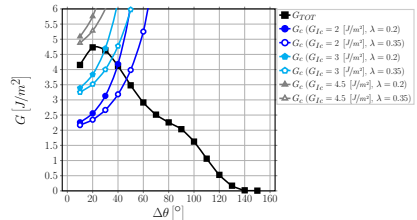
## Estimation of $\Delta\theta_{max}$

$21 \times 1 - free$



$\Delta\theta_{max} \in (30^\circ - 105^\circ)$

$21 \times 1 - 1 \cdot t_{90^\circ}$

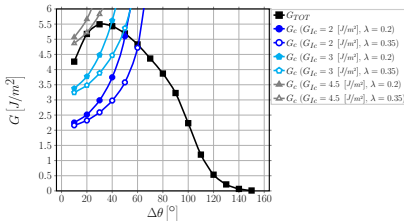


$\Delta\theta_{max} \in (30^\circ - 50^\circ)$

$$G_{TOT}(\Delta\theta) > G_c = G_{Ic} \left( 1 + \tan^2((1 - \lambda) \Psi_G) \right), \quad \Psi_G = \tan^{-1} \left( \sqrt{\frac{G_{II}}{G_I}} \right) \Big|_{\Delta\theta}$$

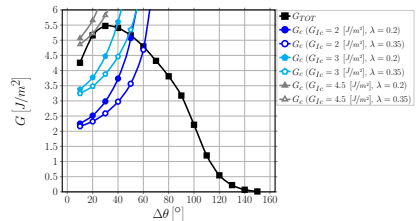
## Estimation of $\Delta\theta_{max}$

$21 \times 3 - free$



$\Delta\theta_{max} \in (40^\circ - 60^\circ)$

$21 \times 3 - 1 \cdot t_{90^\circ}$

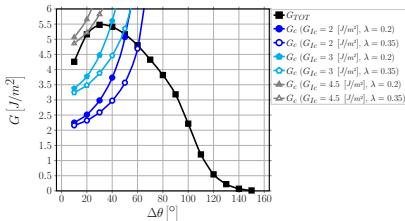


$\Delta\theta_{max} \in (40^\circ - 60^\circ)$

$$G_{TOT}(\Delta\theta) > G_c = G_{Ic} \left( 1 + \tan^2((1 - \lambda) \Psi_G) \right), \quad \Psi_G = \tan^{-1} \left( \sqrt{\frac{G_{II}}{G_I}} \right) \Big|_{\Delta\theta}$$

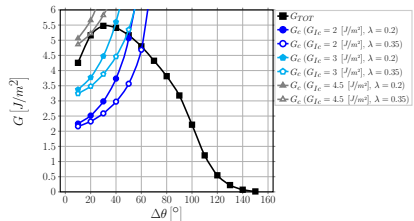
## Estimation of $\Delta\theta_{max}$

$21 \times 21 - free$



$$\Delta\theta_{max} \in (40^\circ - 60^\circ)$$

$21 \times 21 - 1 \cdot t_{90^\circ}$

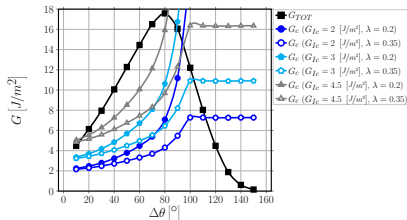


$$\Delta\theta_{max} \in (40^\circ - 60^\circ)$$

$$G_{TOT}(\Delta\theta) > G_c = G_{Ic} \left( 1 + \tan^2((1 - \lambda) \Psi_G) \right), \quad \Psi_G = \tan^{-1} \left( \sqrt{\frac{G_{II}}{G_I}} \right) \Big|_{\Delta\theta}$$

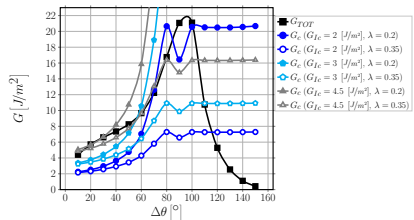
## Estimation of $\Delta\theta_{max}$

21 × 21 – *symm*



$\Delta\theta_{max} \in (80^\circ - 110^\circ)$

21 × 21 – *asymm*



$\Delta\theta_{max} \in (55^\circ - 115^\circ)$

$$G_{TOT}(\Delta\theta) > G_c = G_{Ic} \left( 1 + \tan^2((1 - \lambda) \Psi_G) \right), \quad \Psi_G = \tan^{-1} \left( \sqrt{\frac{G_{II}}{G_I}} \right) \Big|_{\Delta\theta}$$

## Observations & Conclusions

- The effect of the presence of the  $0^\circ$  layer is to reduce the maximum size of debonds;
- however, the estimated debond size is the same for  $n \times k - free$  and  $n \times k - 1 \cdot t_{90^\circ}$  for  $k \geq 3$ ;
- the presence of a debond on the neighboring fiber in the vertical direction ( $21 \times 1 - coupling$  and  $21 \times 1 - asymm$ ) favors instead the growth of larger debonds;
- the largest size is achieved when debonds are on opposite sides of consecutive fibers.

**Estimated debond size range in cross-ply ( $n \times k - 1 \cdot t_{90^\circ}$ )**

$40^\circ - 60^\circ$

**Measured debond size range in cross-ply (Correa et al., Compos. Sci. Technol. 155 (213-220), 2018)**

$21.4^\circ - 89.2^\circ$ , average  $49.3^\circ$ , standard deviation of  $11.7^\circ$   
63% of measurements in  $40^\circ - 60^\circ$  range





# CONCLUSIONS

## Conclusions

- A stress criterion for initiation would likely predict, irrespectively of which criterion from those proposed in the literature is chosen, the onset of a debond at  $0^\circ$  or  $180^\circ$  with a semi-aperture  $\Delta\theta_0$  in the range  $2^\circ - 12^\circ$ , corresponding to a margin of 5% on the satisfaction of the criterion.
- Assuming that debond propagation occurs unstably immediately after debond onset at the same level of global applied strain  $\bar{\epsilon}_x$ , it is possible to evaluate the parameter  $G_{lc}$  in the expression of the critical ERR and with it to estimate the range of expected maximum debond size.
- The prediction for a cross-ply laminate (models  $n \times k - 1 \cdot t_{90^\circ}$ ,  $k \geq 3$ ) agrees well with the debond size distribution in  $[0_3^\circ, 90_3^\circ]_S$  estimated in *Correa et al., Compos. Sci. Technol.* **155** (213-220), 2018 through microscopic observations.

# Thank you for listening today!

