

INVESTIGATION OF SCALING LAWS OF THE FIBER/MATRIX INTERFACE CRACK IN POLYMER COMPOSITES THROUGH FINITE ELEMENT-BASED MICROMECHANICAL MODELING

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Education and Culture

Erasmus Mundus

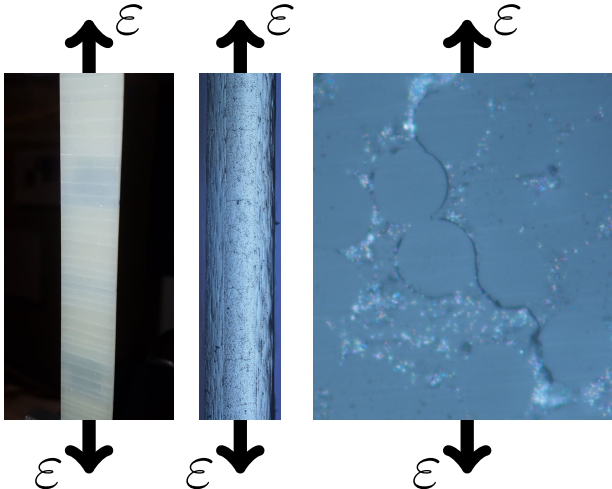


Outline

- The Fiber-Matrix Interface Crack Problem
- Investigation of Scaling Laws of the Fiber/Matrix Interface Crack
- Conclusions

➤ THE FIBER-MATRIX INTERFACE CRACK PROBLEM

Initiation of Transverse Cracking in FRPCs: Microscopic Observations

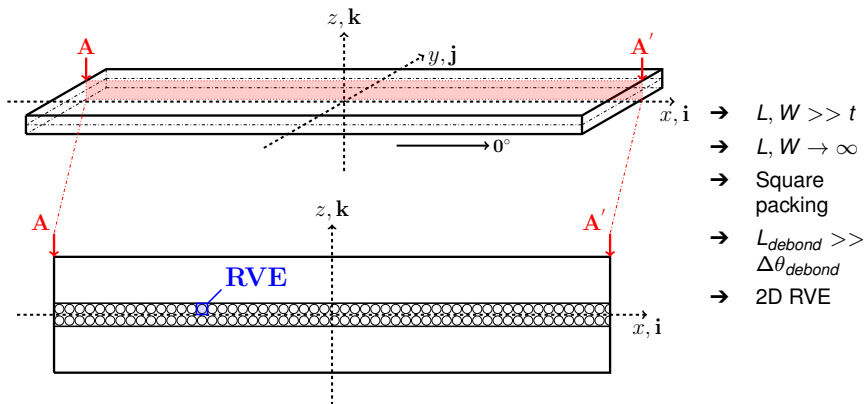


Left:
front view of $[0, 90]_S$,
visual inspection.

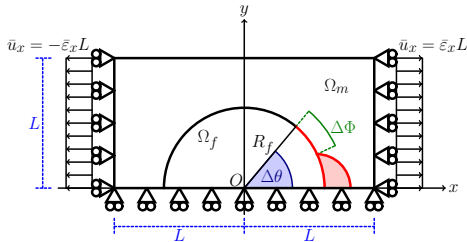
Center:
edge view of $[0, 90]_S$,
optical microscope.

Right:
edge view of $[0, 90]_S$,
optical microscope.

The Fiber-Matrix Interface Crack Problem: Geometry



The Fiber-Matrix Interface Crack Problem: Assumptions

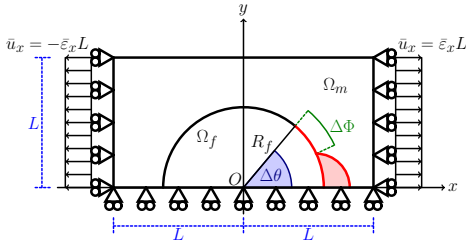


$$R_f = 1 \text{ } [\mu\text{m}] \quad L = \frac{R_f}{2} \sqrt{\frac{\pi}{V_f}}$$

Material	E_1	ν_{12}
glass fiber	70.0	0.2
epoxy	3.5	0.4

- Linear elastic, homogeneous and isotropic materials
- Plane strain
- Frictionless contact interaction
- Symmetric w.r.t. x-axis
- Coupling of x-displacements on left and right side (repeating unit cell)
- Applied uniaxial tensile strain $\bar{\epsilon}_x = 1\%$

The Fiber-Matrix Interface Crack Problem: Solution



in Ω_f, Ω_m :

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\varepsilon_z = \gamma_{zx} = \gamma_{yz} = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

$$\sigma_{zz} = \nu (\sigma_{xx} + \sigma_{yy})$$

for $0^\circ \leq \alpha \leq \Delta\theta$:

$$(\vec{u}_m(R_f, \alpha) - \vec{u}_f(R_f, \alpha)) \cdot \vec{n}_\alpha \geq 0$$

for $\Delta\theta \leq \alpha \leq 180^\circ$:

$$\vec{u}_m(R_f, \alpha) - \vec{u}_f(R_f, \alpha) = 0$$

$$\sigma_{ij} = E_{ijkl} \varepsilon_{kl} + BC$$

→ Oscillating singularity

$$\sigma \sim r^{-\frac{1}{2}} \sin(\varepsilon \log r), \quad V_f \rightarrow 0$$

$$\varepsilon = \frac{1}{2\pi} \log \left(\frac{1 - \beta}{1 + \beta} \right)$$

$$\beta = \frac{\mu_2 (\kappa_1 - 1) - \mu_1 (\kappa_2 - 1)}{\mu_2 (\kappa_1 + 1) + \mu_1 (\kappa_2 + 1)}$$

→ Finite Element Method (FEM) in AbaqusTM

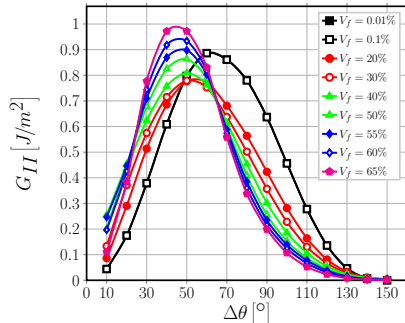
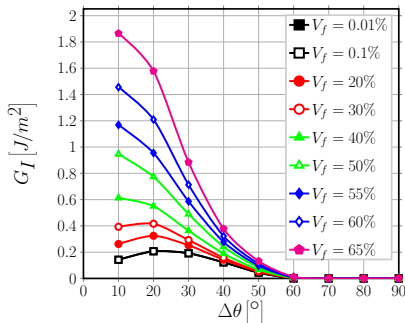
→ 2nd order shape functions

→ 6-nodes triangles & 8-nodes quadrilaterals

→ regular mesh of quadrilaterals at the crack tip:

- $AR \sim 1$
- $\delta = 0.05^\circ$

The Fiber-Matrix Interface Crack Problem: Normalization & Scaling



$$(\?) \quad G_I = G_0(V_f, E_f, \nu_f, E_m, \nu_m, \bar{\epsilon}_x, \Delta\theta) g_I(\Delta\theta, BC, \text{microstructure, damage})$$

$$(\?) \quad G_{II} = G_0(V_f, E_f, \nu_f, E_m, \nu_m, \bar{\epsilon}_x, \Delta\theta) g_{II}(\Delta\theta, BC, \text{microstructure, damage})$$

INVESTIGATION OF SCALING LAWS

Dimensional Analysis

→ From the definition of Energy Release Rate

$$G = \frac{\partial W}{\partial A} - \left(\frac{\partial U}{\partial A} + \frac{\partial E_k}{\partial A} \right) \quad \left[\frac{J}{m^2} \right]$$

$$\left[\frac{J}{m^2} \right] \longleftrightarrow \frac{E}{L^2} = \frac{F \cdot L}{L^2} = \frac{F}{L^2} \frac{L}{L} \cdot L = \sigma \varepsilon L$$

$$G_0 \sim \sigma_\infty \varepsilon_\infty L_c$$

→ From the assumption of linear elasticity and uniaxial loading

$$\sigma_\infty = E_{eq} \varepsilon_\infty \quad \varepsilon_\infty = \frac{\sigma_\infty}{E_{eq}}$$

$$G_0 \sim E_{eq} \varepsilon_\infty^2 L_c \quad G_0 \sim \frac{\sigma_\infty^2}{E_{eq}} L_c$$

→ From crack geometry

$$L_c \sim a = R_f \Delta \theta \longrightarrow L_c \sim R_f f(\Delta \theta)$$

$$G_0 = A \cdot E_{eq} \varepsilon_\infty^2 R_f f(\Delta \theta)$$

Homogenization of Material Properties: Concentric Cylinders Assembly (CCA)

$$E_L = V_f E_f + (1 - V_f) E_m + 2\lambda_1 (\nu_m - \nu_f)^2 V_f (1 - V_f)$$

$$\nu_{LT} = V_f \nu_f + (1 - V_f) \nu_m + \frac{\lambda_1}{2} (\nu_m - \nu_f) \left(\frac{1}{k_{fT}} - \frac{1}{k_{mT}} \right) V_f (1 - V_f)$$

$$G_{TT} = \frac{E_m}{2(1 + \nu_m)} + \frac{V_f}{\frac{1}{\frac{E_f}{2(1 + \nu_f)} - \frac{E_m}{2(1 + \nu_m)}} + \frac{k_{mT} + \frac{E_m}{1 + \nu_m}}{\frac{E_m}{1 + \nu_m} \left(k_{mT} + \frac{E_m}{2(1 + \nu_m)} \right)}} (1 - V_f)$$

$$K_{TT} = \frac{k_{mT} \left(k_{fT} + \frac{E_m}{2(1 + \nu_m)} \right) (1 - V_f) + k_{fT} \left(k_{mT} + \frac{E_m}{2(1 + \nu_m)} \right) V_f}{\left(k_{fT} + \frac{E_m}{2(1 + \nu_m)} \right) (1 - V_f) + \left(k_{mT} + \frac{E_m}{2(1 + \nu_m)} \right) V_f}$$

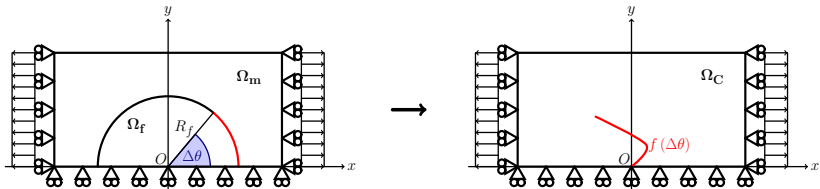
$$E_T = \frac{4G_{TT}}{1 + \frac{\left(1 + \frac{4K_{TT}\nu_{LT}^2}{E_L} \right) G_{TT}}{K_{23}}} \quad \nu_{TT} = \frac{E_T}{2G_{TT}} - 1$$

$$k_{fT} = \frac{E_f}{2(1 - \nu_f - 2\nu_f^2)} \quad k_{mT} = \frac{E_m}{2(1 - \nu_m - 2\nu_m^2)} \quad \lambda_1 = 2 \left(\frac{2(1 + \nu_m)}{E_m} + \frac{V_f}{k_{mT}} + \frac{1 - V_f}{k_{fT}} \right)^{-1}$$

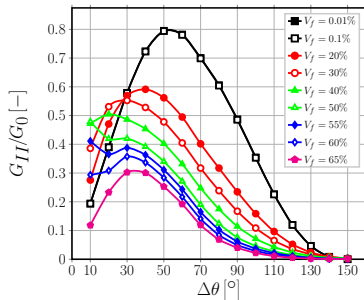
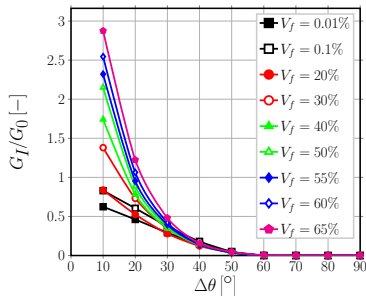
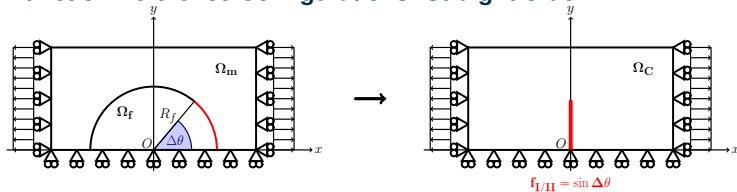
Homogenization of Material Properties: Plane Strain Conditions

$$E_{\text{plane strain}} = \frac{E_T}{1 - \frac{E_T}{E_L} \nu_{LT}^2}$$

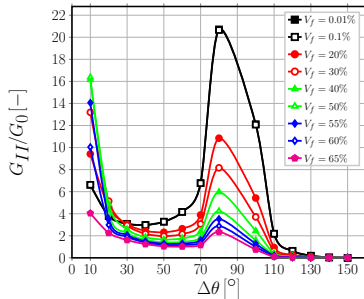
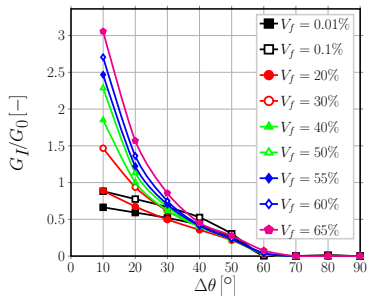
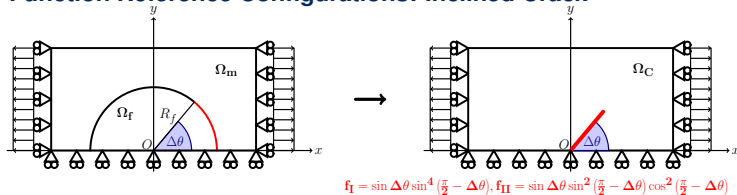
$$E_{eq} = \frac{E_T}{1 - \frac{E_T}{E_L} \nu_{LT}^2}$$



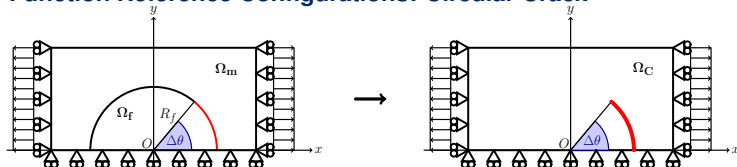
Shape Function Reference Configurations: Straight Crack



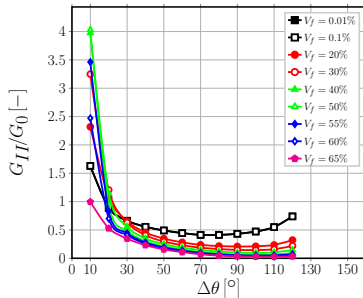
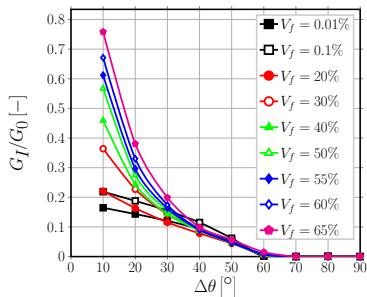
Shape Function Reference Configurations: Inclined Crack



Shape Function Reference Configurations: Circular Crack



$$f_I = \frac{\sin \Delta\theta}{\Delta\theta} \left(\frac{1 - \sin^2 \left(\frac{\Delta\theta}{2} \right) \cos^2 \left(\frac{\Delta\theta}{2} \right)}{1 + \sin^2 \left(\frac{\Delta\theta}{2} \right)} \cos \left(\frac{\Delta\theta}{2} \right) + \cos \left(\frac{3}{2} \Delta\theta \right) \right)^2, f_{II} = -\frac{\sin \Delta\theta}{\Delta\theta} \left(\frac{1 - \sin^2 \left(\frac{\Delta\theta}{2} \right) \cos^2 \left(\frac{\Delta\theta}{2} \right)}{1 + \sin^2 \left(\frac{\Delta\theta}{2} \right)} \sin \left(\frac{\Delta\theta}{2} \right) + \sin \left(\frac{3}{2} \Delta\theta \right) \right)^2$$



CONCLUSIONS

Conclusions

→ $f_{\text{straight crack}}(\Delta\theta)$: ✓ G_I , ✗ G_{II}

$f_{\text{inclined crack}}(\Delta\theta)$: ✓ G_I , ✓ G_{II} , ✗ $\nexists f_{\text{inclined crack}}(\Delta\theta = \frac{\pi}{2})$

$f_{\text{curved crack}}(\Delta\theta)$: ✓ G_I , ✓ G_{II}

→ scaling breaks for $\Delta\theta \leq 20^\circ \rightarrow$ microstructure is important for small debonds!

