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Outline

- Symbols, Models, Equations & Reference Data
- **Nesults**
- Summary & Conclusion









Symbols Reference Models Angular discretization Material properties Evaluation of G_0 VCCT VCC

SYMBOLS, MODELS, EQUATIONS & REFERENCE DATA









Description

Symbols Reference Models Angular discretization Material properties Evaluation of G₀ VCCT

Symbols

Symbol

Unit

θ	[°]	Debond angular position with respect to the center of the arc defined by the debond itself
$\Delta \theta$	[°]	Debond semi-angular aperture
δ	[°]	Angle subtended by a single element at the fiber/matrix interface
VF_f	[-]	Fiber volume fraction
1	[<i>µ</i> m]	Ply's half-length, equal to RVE's half-length (square element)
и	$[\mu m]$	Displacement along x
W	$[\mu m]$	Displacement along z









Symbols Reference Models Angular discretization Material properties Evaluation of G₀

Symbols

Symbol	Unit	Description
Γ ₁	[-]	Bonded part of fiber surface
Γ_2	[-]	Free (debonded) part of fiber surface
Γ_3	[-]	Bonded part of matrix surface
Γ₄	[-]	Free (debonded) part of matrix surface



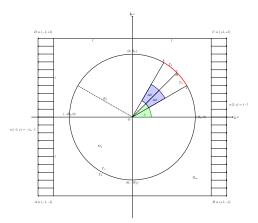






Symbols Reference Models Angular discretization Material properties Evaluation of G_0 VCCT VCCI

Reference Models



Simple RVE, BC: free.



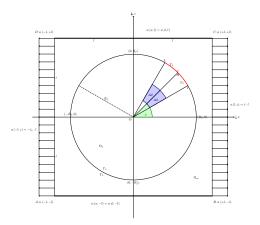






Symbols Reference Models Angular discretization Material properties Evaluation of G₀ VCCT VCCI

Reference Models



Simple RVE, BC: fixed vertical displacement.



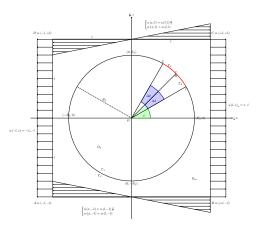






Symbols Reference Models Angular discretization Material properties Evaluation of G₀ VCCT VCC

Reference Models



Simple RVE, BC: fixed vertical and homogeneous horizontal displacement.

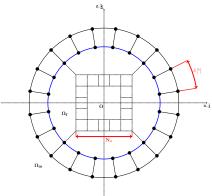






Symbols Reference Models Angular discretization Material properties Evaluation of G_0 VCCT VCC

Angular discretization



Angular discretization at fiber/matrix interface: $\delta = \frac{360^{\circ}}{4N_{\odot}}$.









Symbols Reference Models Angular discretization Material properties Evaluation of G_0 VCCT VCC

Material properties

Material	E [GPa]	G [GPa]	ν [-]
Glass fiber	70,0	29,2	0,2
Ероху	3,5	1,25	0,4









Reference Models Angular discretization Material properties **Evaluation of** G₀ VCCT VCC

Evaluation of G_0

$$G_0 = \pi R_f \sigma_0^2 \frac{1 + k_m}{8G_m} \tag{1}$$

$$k_m = 3 - 4\nu_m \tag{2}$$

$$\sigma_0^{undamaged} = \frac{E_m}{1 - \nu_m^2} \varepsilon_{xx} \tag{3}$$









Symbols, Models, Equations & Reference Data Results

Symbols Reference Models Angular discretization Material properties Evaluation of Go VCCT VCCI

Virtual Crack Closure Technique (Nodal Forces at Crack Tip)

$$\Delta u = \left| \Delta u_1^{\text{matrix}} \right| = \left| \Delta u_1^{\text{matrix}} \right| = \left| \Delta u_1^{\text{fiber}} \right|$$
 (4)

$$\Delta w = \left| \Delta w_1^{matrix} \right| = \left| \Delta w_1^{matrix} \right| = \left| \Delta w_1^{fiber} \right|$$
 (5)

$$\beta = \arctan \begin{pmatrix} z_{\text{crack tip}}^{\text{matrix}, \text{undef}} \\ z_{\text{crack tio}}^{\text{matrix}, \text{undef}} \end{pmatrix}$$
 (6)

$$\Delta_r = \cos(\beta)\Delta u + \sin(\beta)\Delta w \qquad \Delta_\theta = -\sin(\beta)\Delta u + \cos(\beta)\Delta w \tag{7}$$

$$F_r = \cos(\beta)F_\chi^{reaction} + \sin(\beta)F_Z^{reaction}$$
 $F_\theta = -\sin(\beta)F_\chi^{reaction} + \cos(\beta)F_Z^{reaction}$ (8)

$$G_{I} = \frac{1}{2} \frac{F_{r} \Delta_{r}}{R_{r} \delta}$$
 $G_{II} = \frac{1}{2} \frac{F_{\theta} \Delta_{\theta}}{R_{r} \delta}$ $b = 1.0 \leftrightarrow \Delta A = bR_{f} \delta$ (9)









Reference Models Angular discretization Material properties Evaluation of G₀ V

Virtual Crack Closure Integral (Stress at Surface Nodes)

$$G_{IJ} = \frac{1}{2\Delta A} \int_{0}^{\Delta c} \sigma_{II}(s) \, \delta u_{II}(s - \Delta c) \, ds \quad G_{IJ} = \frac{1}{2\Delta A} \int_{0}^{\Delta c} \tau_{SII}(s) \, \delta u_{S}(s - \Delta c) \, ds \quad (10)$$

with the reference frame centered on the crack tip and rotated according to the orientation of the crack tip.

$$\beta = \arctan \left(\frac{z_{\text{crack tip}}^{\text{matrix}, \, undef}}{x_{\text{matrix}}^{\text{matrix}, \, undef}} \right) \tag{11}$$

$$\Delta u^i = \left| \Delta u^{\textit{matrix}}_{i \text{ elements before crack tip}} - \Delta u^{\textit{fiber}}_{i \text{ elements before crack tip}} \right| \tag{12}$$

$$\Delta w^{i} = \left| \Delta w_{i}^{matrix} - \Delta w_{i}^{fiber} \right|$$
 (13)

$$\Delta_f^i = \cos(\beta)\Delta u^i + \sin(\beta)\Delta w^i \qquad \Delta_\theta^i = -\sin(\beta)\Delta u^i + \cos(\beta)\Delta w^i$$
 (14)









Symbols, Models, Equations & Reference Data Results

Reference Models Angular discretization

Material properties Evaluation of Go

Virtual Crack Closure Integral (Stress at Surface Nodes)

$$\sigma_{II}^{m,i} = \sigma_{XX}^{m,i}$$
 elements after c.t. $\cos^2 \beta + \sigma_{ZZ}^{m,i}$ elements after c.t. $\sin^2 \beta + 2\tau_{XZ}^{m,i}$ elements after c.t. $\sin \beta \cos \beta$ (15)

$$\tau_{\ell\theta}^{m,i} = \left(\sigma_{ZZ}^{m,i\text{ elements after crack tip}} - \sigma_{\chi\chi}^{m,i\text{ elements after c.t.}}\right) \sin\beta\cos\beta + \tau_{\chi Z}^{m,i\text{ elements after c.t.}} \left(\cos^2\beta - \sin^2\beta\right) \tag{16}$$

where m stands for material, i.e. stresses can be extracted either on the fiber or the matrix surface.

$$G_{I}^{m} = \frac{1}{2R_{f}\delta b} \sum_{i=1}^{N \ln t} \frac{1}{2} R_{f}\delta \left(\sigma_{rr}^{m,i} \Delta_{r}^{i} + \sigma_{rr}^{m,i-1} \Delta_{r}^{i-1} \right) \qquad G_{II}^{m} = \frac{1}{2R_{f}\delta b} \sum_{i=1}^{N \ln t} \frac{1}{2} R_{f}\delta \left(\tau_{r\theta}^{m,i} \Delta_{\theta}^{i} + \tau_{r\theta}^{m,i-1} \Delta_{\theta}^{i-1} \right)$$
(17)

remembering b = 1, i.e. unit depth in the out-of-plane direction, they simplify to

$$G_{I}^{m} = \frac{1}{4} \left(\sigma_{rr}^{m,0} \Delta_{r}^{0} + \sum_{i=1}^{N \text{ Int } EI-1} \left(2\sigma_{rr}^{m,i} \Delta_{r}^{i} \right) + \sigma_{rr}^{m,N \text{ Int } EI} \Delta_{r}^{N \text{ Int } EI} \right)$$
(18)

$$G_{II}^{m} = \frac{1}{4} \left(\tau_{r\theta}^{m,0} \Delta_{\theta}^{0} + \sum_{i=1}^{N \text{ Int } EI - 1} \left(2\tau_{r\theta}^{m,i} \Delta_{\theta}^{i} \right) + \tau_{r\theta}^{m,N \text{ Int } EI} \Delta_{\theta}^{N \text{ Int } EI} \right)$$

$$\tag{19}$$









Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$ $\delta=0.4^{\circ}$ $\delta=0.3^{\circ}$ $\delta=0.2^{\circ}$ Summary











Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$

 $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summa

Model Data

Quantity	Value
θ [$^{\circ}$]	0
$\Delta heta\left[^{\circ} ight]$	$\in [10,150]$
δ [$^{\circ}$]	$\in [1,0.2]$
$VF_f[-]$	$7.9 \cdot 10^{-5}$
$\frac{L}{R_t}[-]$	~ 100
$R_f^{'}[\mu m]$	1



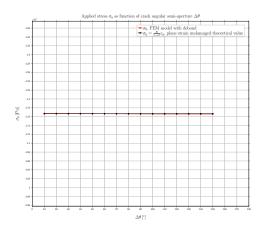






Model Data $\delta = 1.0^{\circ}$ $\delta = 0.9^{\circ}$ $\delta = 0.8^{\circ}$ $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

 σ_0 , $\delta=1.0^\circ$



In red small strain FEM, in black analytical plain strain value.

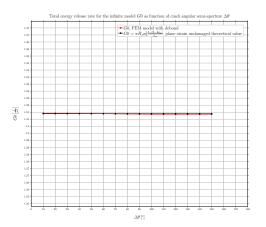






Model Data $\delta=1.0^\circ$ $\delta=0.9^\circ$ $\delta=0.8^\circ$ $\delta=0.7^\circ$ $\delta=0.6^\circ$ $\delta=0.5^\circ$ $\delta=0.4^\circ$ $\delta=0.3^\circ$ $\delta=0.2^\circ$ Summary

 $G_0, \delta = 1.0^{\circ}$



In red small strain FEM, in black analytical plain strain value.

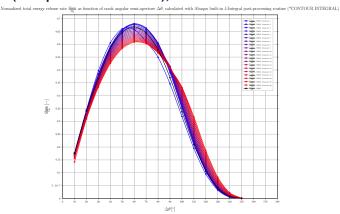








J-Integral (Abaqus built-in routine), $\delta=1.0^\circ$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

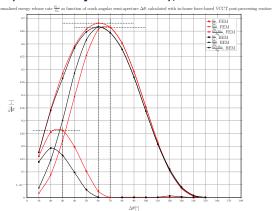








VCCT in forces (in-house Python routine), $\delta = 1.0^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

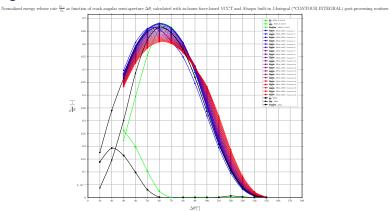








J-Integral and VCCT in forces, $\delta = 1.0^{\circ}$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



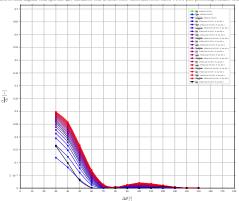






G_l from VCCI, stresses extracted on fiber surface, $\delta = 1.0^{\circ}$

Normalized energy release rate $\frac{G_{+}}{c^{2}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



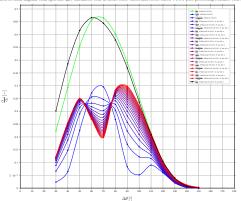






G_{\parallel} from VCCI, stresses extracted on fiber surface, $\delta=1.0^{\circ}$

Normalized energy release rate $\frac{G_{+}}{c^{2}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



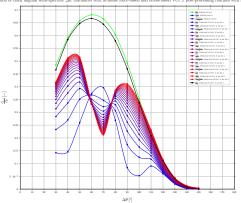






G_{TOT} from VCCI, stresses extracted on fiber surface, $\delta=1.0^\circ$

Normalized energy release rate $\frac{G_{r}^{2}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black

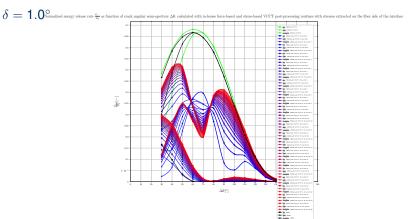








Summary of $G_{(\cdot,\cdot)}$ from VCCI, stresses extracted on liber surface,



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black BEM results.



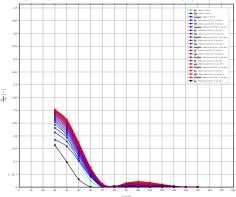






G_l from VCCI, stresses extracted on matrix surface, $\delta = 1.0^{\circ}$

Normalized energy release rate $\frac{G_{cb}}{dc^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



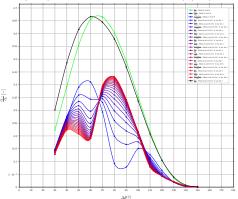






G_{II} from VCCI, stresses extracted on matrix surface, $\delta=1.0^{\circ}$

Normalized energy release rate $\frac{G_{22}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



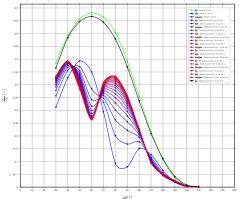






G_{TOT} from VCCI, stresses extracted on matrix surface, $\delta = 1.0^{\circ}$

Normalized energy release rate $\frac{G_{cr}}{2\pi^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black

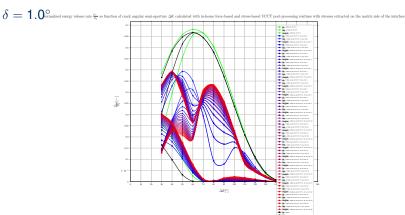








Summary or G(...) from VCCI, stresses extracted on matrix surface,



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black BEM results.



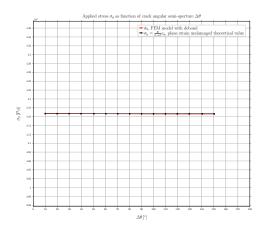






 $\label{eq:decomposition} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = \mbox{0.9}^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary and the sum of the sum of$

$$\sigma_0$$
, $\delta=0.9^\circ$



In red small strain FEM, in black analytical plain strain value.



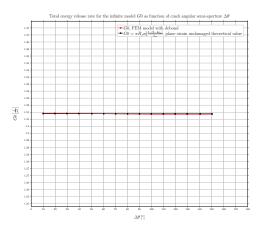






 $\label{eq:model_delta_$

 G_{0} , $\delta = 0.9^{\circ}$



In red small strain FEM, in black analytical plain strain value.

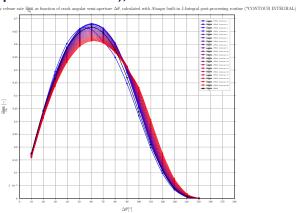








J-Integral (Abaqus built-in routine), $\delta=0.9^\circ$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

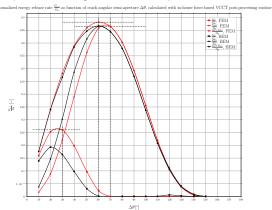








VCCT in forces (in-house Python routine), $\delta = 0.9^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.





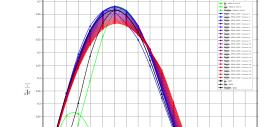
Normalized energy release rate $\frac{G_{cr}}{dc}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based VCCT and Abaqus built-in J-Integral (*CONTOUR INTEGRAL) post-processing routines





Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta = 1.0^{\circ}$ $\delta = 0.9^{\circ}$ $\delta = 0.8^{\circ}$ $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

J-Integral and VCCT in forces, $\delta=0.9^\circ$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.





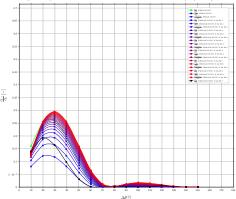




Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta = 1.0^{\circ}$ $\delta = 0.9^{\circ}$ $\delta = 0.8^{\circ}$ $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary & Conclusion Model Data $\delta = 0.00^{\circ}$ $\delta = 0.00^{\circ}$ $\delta = 0.00^{\circ}$ $\delta = 0.00^{\circ}$ $\delta = 0.00^{\circ}$ Summary & Conclusion

G_l from VCCI, stresses extracted on fiber surface, $\delta = 9.0^{\circ}$

Normalized energy release rate $\frac{G_{22}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



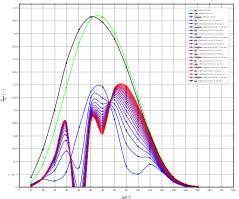






G_{II} from VCCI, stresses extracted on fiber surface, $\delta = 9.0^{\circ}$

Normalized energy release rate $\frac{a_{ij}}{L^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



Fading from blue to red for increasing number of integration elements, Virtual Crack Closure Integral (VCCI) from FEM results; in green VCCT from FEM results; in black



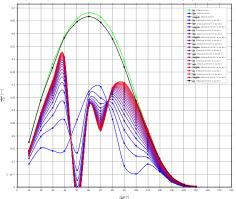






G_{TOT} from VCCI, stresses extracted on fiber surface, $\delta = 9.0^{\circ}$

Normalized energy release rate $\frac{G_{+}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





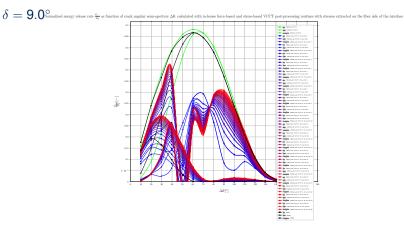






 $\label{eq:decomposition} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary and } \mbox{Summary$

Summary of $G_{(\cdot\cdot)}$ from VCCI, stresses extracted on liber surface,









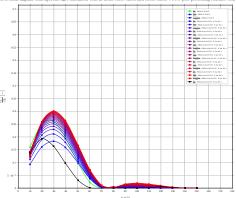


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$

 $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

G_l from VCCI, stresses extracted on matrix surface, $\delta = 9.0^{\circ}$

Normalized energy release rate $\frac{G_{i,i}}{G_{i,j}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





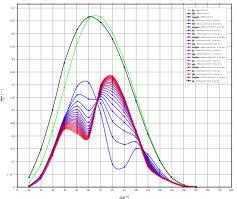






G_{II} from VCCI, stresses extracted on matrix surface, $\delta=9.0^\circ$

Normalized energy release rate $\frac{\sigma_{re}^2}{2\pi^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





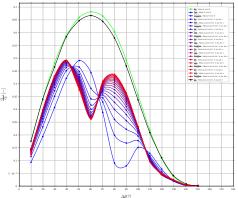






G_{TOT} from VCCI, stresses extracted on matrix surface, $\delta = 9.0^{\circ}$

Normalized energy release rate $\frac{G_{12}}{G_{12}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





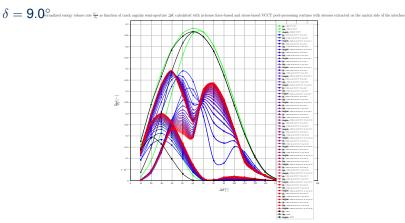






 $\label{eq:definition} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary and the property of the sum of the property of the pro$

Summary of $G_{(\cdot,\cdot)}$ from VCCI, stresses extracted on matrix surface,





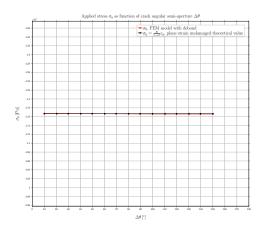






 $\label{eq:delta-$

$$\sigma_0$$
, $\delta=0.8^\circ$



In red small strain FEM, in black analytical plain strain value.

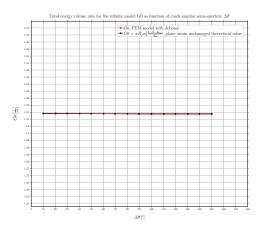






 $\label{eq:decomposition} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary Summary Polynomials} \quad \delta = 0.00^{\circ} \quad \delta = 0.$

 G_0 , $\delta = 0.8^\circ$



In red small strain FEM, in black analytical plain strain value.

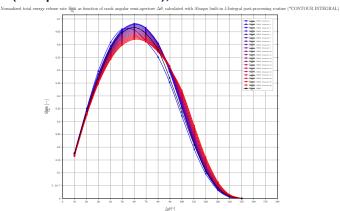








J-Integral (Abaqus built-in routine), $\delta = 0.8^{\circ}$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

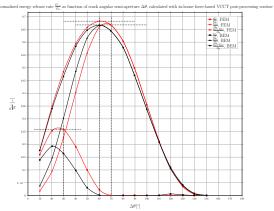








VCCT in forces (in-house Python routine), $\delta=0.8^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

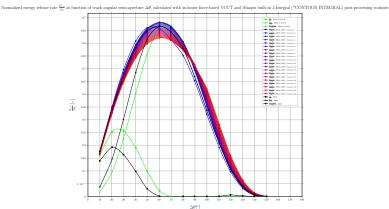








J-Integral and VCCT in forces, $\delta=0.8^\circ$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



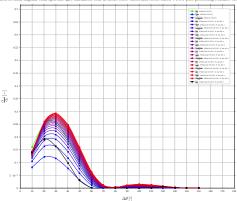






G_l from VCCl, stresses extracted on fiber surface, $\delta = 8.0^{\circ}$

Normalized energy release rate $\frac{G_{12}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





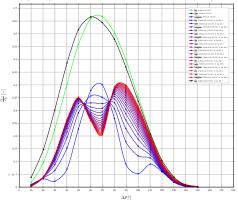






G_{II} from VCCI, stresses extracted on fiber surface, $\delta = 8.0^{\circ}$

Normalized energy release rate $\frac{a_{ij}}{L^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





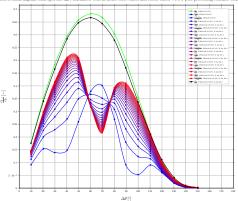






G_{TOT} from VCCI, stresses extracted on fiber surface, $\delta = 8.0^{\circ}$

Normalized energy release rate $\frac{d_{i+1}}{d_i}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



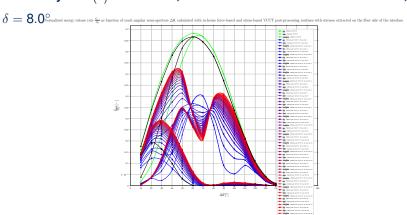








Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$ $\delta=0.4^{\circ}$ $\delta=0.3^{\circ}$ $\delta=0.2^{\circ}$ Summary Summary of $G_{(...)}$ from VGG, stresses extracted on liber surface,









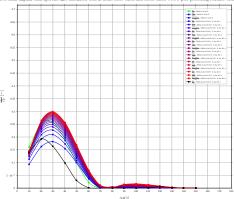


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$

 $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$ $\delta=0.4^{\circ}$ $\delta=0.3^{\circ}$ $\delta=0.2^{\circ}$ Summary

G_l from VCCI, stresses extracted on matrix surface, $\delta = 8.0^{\circ}$

Normalized energy release rate $\frac{G_{i-1}}{L^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





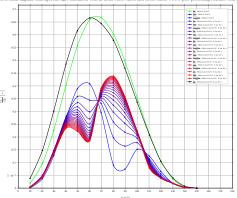






G_{II} from VCCI, stresses extracted on matrix surface, $\delta=8.0^{\circ}$

Normalized energy release rate $\frac{G_{i-1}}{L^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





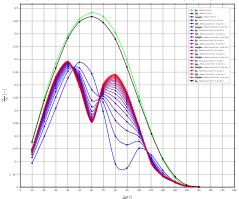






G_{TOT} from VCCI, stresses extracted on matrix surface, $\delta = 8.0^{\circ}$

Normalized energy release rate $\frac{G_{i-1}}{\omega}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





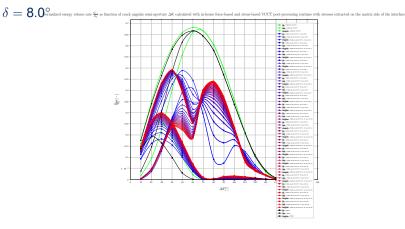






Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$ $\delta=0.4^{\circ}$ $\delta=0.3^{\circ}$ $\delta=0.2^{\circ}$ Summary

Summary of $G_{(\cdot\cdot)}$ from VCCI, stresses extracted on matrix surface,





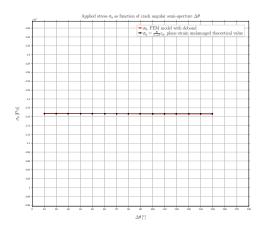






Model Data $~\delta=1.0^\circ~\delta=0.9^\circ~\delta=0.8^\circ~\delta=0.7^\circ~\delta=0.6^\circ~\delta=0.5^\circ~\delta=0.4^\circ~\delta=0.3^\circ~\delta=0.2^\circ~$ Summary

$$\sigma_0$$
, $\delta=0.7^\circ$



In red small strain FEM, in black analytical plain strain value.



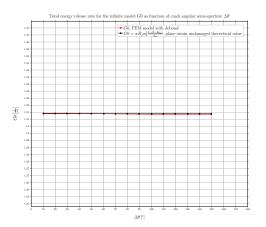






 $\label{eq:model_delta_$

 $G_0, \delta = 0.7^{\circ}$



In red small strain FEM, in black analytical plain strain value.

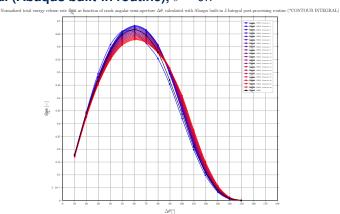








J-Integral (Abaqus built-in routine), $\delta=0.7^\circ$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

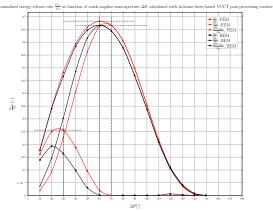








VCCT in forces (in-house Python routine), $\delta=0.7^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

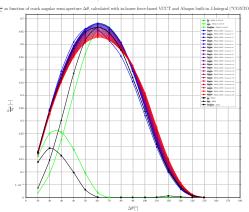








J-Integral and VCCT in forces, $\delta = 0.7^{\circ}$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



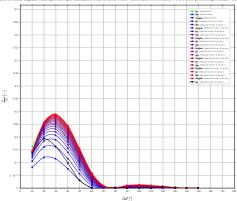






G_l from VCCI, stresses extracted on fiber surface, $\delta = 7.0^{\circ}$

Normalized energy release rate $\frac{G_{tr}}{2\pi}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





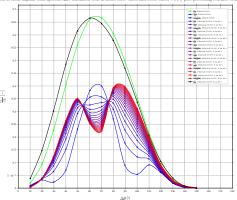






G_{II} from VCCI, stresses extracted on fiber surface, $\delta = 7.0^{\circ}$

Normalized energy release rate $\frac{G_{2}}{c^{2}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





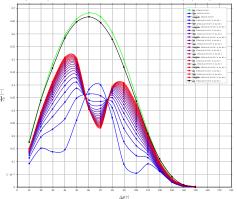






G_{TOT} from VCCI, stresses extracted on fiber surface, $\delta = 7.0^{\circ}$

Normalized energy release rate $\frac{G_{+}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



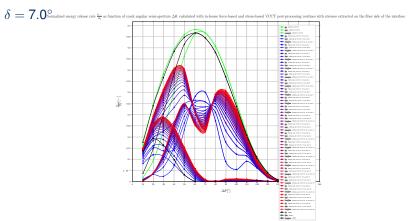








Summary of $G_{(\cdot,\cdot)}$ from VCCI, stresses extracted on liber surface,





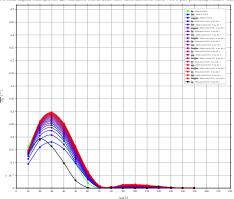






G_l from VCCl, stresses extracted on matrix surface, $\delta = 7.0^{\circ}$

Normalized energy release rate $\frac{G_{i+1}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





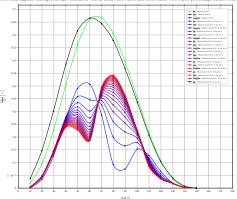






G_{II} from VCCI, stresses extracted on matrix surface, $\delta = 7.0^{\circ}$

Normalized energy release rate $\frac{G_{ij}}{L^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





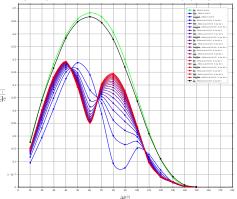






G_{TOT} from VCCI, stresses extracted on matrix surface, $\delta = 7.0^{\circ}$

Normalized energy release rate $\frac{G_{i,j}}{G_{i,j}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface



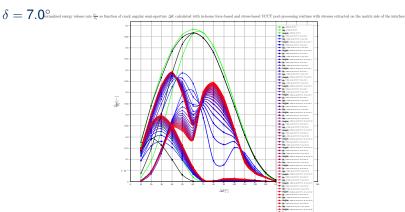








Summary of $G_{(\cdot,\cdot)}$ from VCCI, stresses extracted on matrix surface,





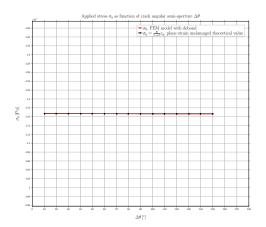






 $\label{eq:decomposition} \text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary and Summary and Summary and Summary Advanced Part of the Summary Adva$

 σ_0 , $\delta=0.6^\circ$



In red small strain FEM, in black analytical plain strain value.

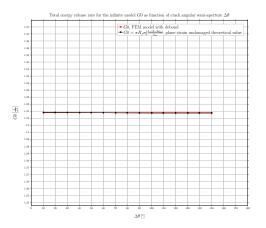






 $\label{eq:decomposition} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary and } \mbox{Summary$

 G_0 , $\delta = 0.6^\circ$



In red small strain FEM, in black analytical plain strain value.

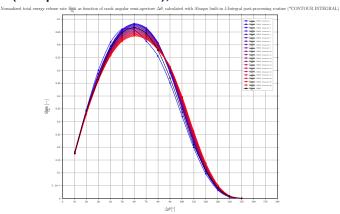








J-Integral (Abaqus built-in routine), $\delta = 0.6^{\circ}$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

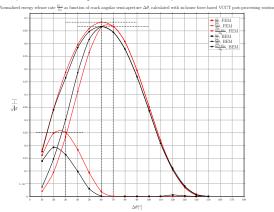








VCCT in forces (in-house Python routine), $\delta = 0.6^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

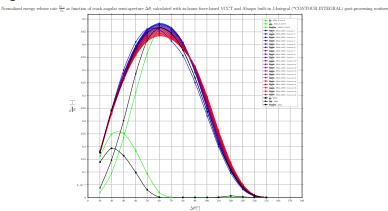








J-Integral and VCCT in forces, $\delta=0.6^\circ$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



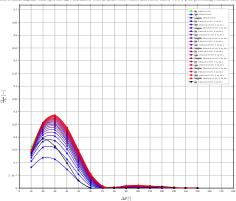






G_l from VCCI, stresses extracted on fiber surface, $\delta = 6.0^{\circ}$

Normalized energy release rate $\frac{G_{tr}}{2\pi}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





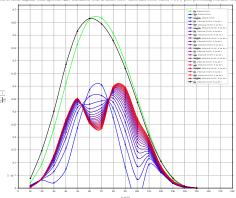






G_{II} from VCCI, stresses extracted on fiber surface, $\delta=6.0^{\circ}$

Normalized energy release rate $\frac{a_{ij}}{L^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





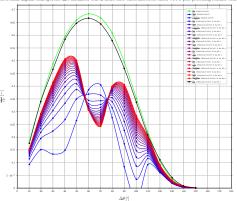






G_{TOT} from VCCI, stresses extracted on fiber surface, $\delta=6.0^\circ$

Normalized energy release rate $\frac{G_{ij}}{L^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





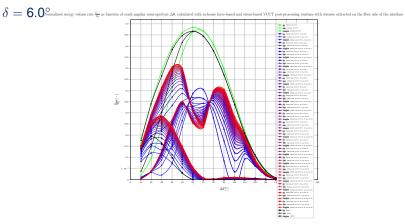






Model Data $\delta = 1.0^{\circ}$ $\delta = 0.9^{\circ}$ $\delta = 0.8^{\circ}$ $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

Summary of $G_{(\cdot\cdot)}$ from VCCI, stresses extracted on liber surface,









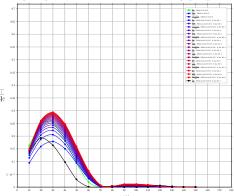


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta = 1.0^{\circ}$ $\delta = 0.9^{\circ}$ $\delta = 0.8^{\circ}$ $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$

 $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

G_l from VCCI, stresses extracted on matrix surface, $\delta = 6.0^{\circ}$

Normalized energy release rate $\frac{G_{i,j}}{G_{i,j}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





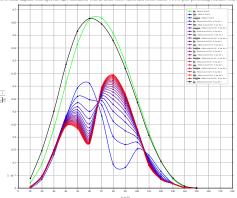






G_{\parallel} from VCCI, stresses extracted on matrix surface, $\delta = 6.0^{\circ}$

Normalized energy release rate $\frac{G_{1}}{k^{2}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





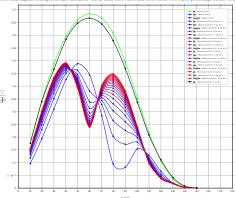






G_{TOT} from VCCI, stresses extracted on matrix surface, $\delta = 6.0^{\circ}$

Normalized energy release rate $\frac{G_{c,b}}{dc}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





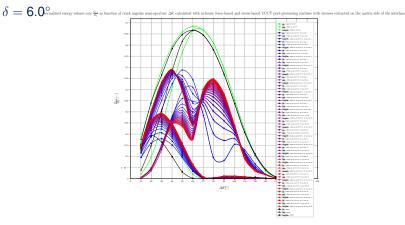






 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = \textbf{0.6}^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary Summary Summary$

Summary of $G_{(\cdot\cdot)}$ from VCCI, stresses extracted on matrix surface,





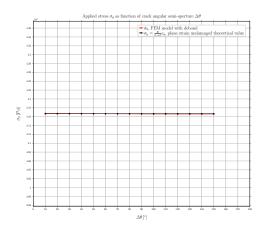






 $\text{Model Data} \ \ \delta = 1.0^{\circ} \ \ \delta = 0.9^{\circ} \ \ \delta = 0.8^{\circ} \ \ \delta = 0.7^{\circ} \ \ \delta = 0.6^{\circ} \ \ \delta = 0.5^{\circ} \ \ \delta = 0.4^{\circ} \ \ \delta = 0.3^{\circ} \ \ \delta = 0.2^{\circ} \ \ \text{Summary Boundary Boundary$

 σ_0 , $\delta=0.5^\circ$



In red small strain FEM, in black analytical plain strain value.



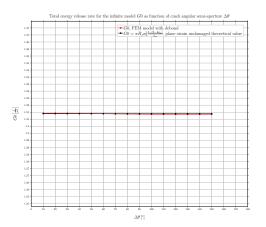






 $\label{eq:decomposition} \text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary summary sum$

 G_0 , $\delta = 0.5^\circ$



In red small strain FEM, in black analytical plain strain value.

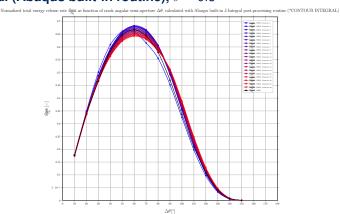








J-Integral (Abaqus built-in routine), $\delta=0.5^\circ$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

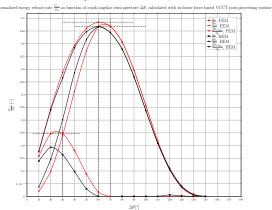








VCCT in forces (in-house Python routine), $\delta=0.5^\circ$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.

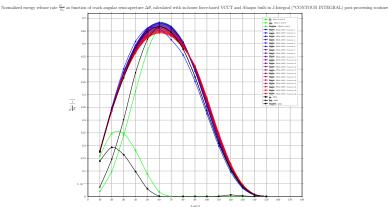








J-Integral and VCCT in forces, $\delta = 0.5^{\circ}$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.







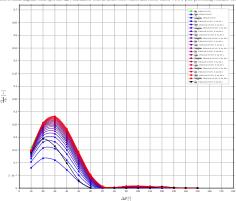


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$

 $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

G_l from VCCI, stresses extracted on fiber surface, $\delta = 5.0^{\circ}$

Normalized energy release rate $\frac{G_{22}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





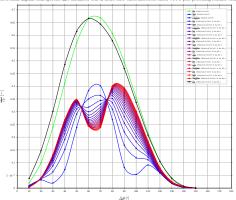






G_{\parallel} from VCCI, stresses extracted on fiber surface, $\delta = 5.0^{\circ}$

Normalized energy release rate $\frac{G_{i,j}}{d_i}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





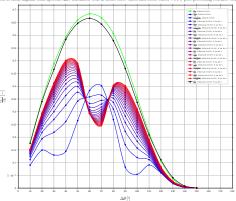






G_{TOT} from VCCI, stresses extracted on fiber surface, $\delta = 5.0^{\circ}$

Normalized energy release rate $\frac{G_{i+1}}{G_{i+1}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





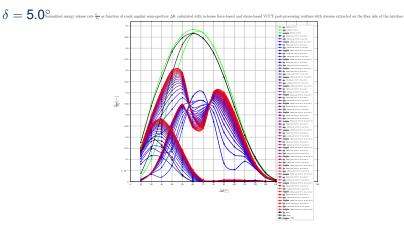






 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary Summary Sum$

Summary of $G_{(...)}$ from VCCI, stresses extracted on liber surface,









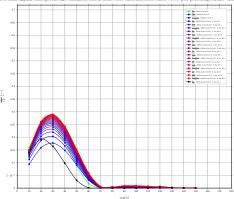


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$

 $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

$\textit{G}_{\textit{I}}$ from VCCI, stresses extracted on matrix surface, $\delta = 5.0^{\circ}$

Normalized energy release rate $\frac{G_{i-1}}{L^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface







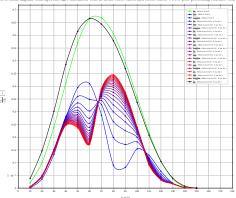




 $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$ $\delta=0.4^{\circ}$ $\delta=0.3^{\circ}$ $\delta=0.2^{\circ}$ Summary

G_{II} from VCCI, stresses extracted on matrix surface, $\delta=5.0^\circ$

Normalized energy release rate $\frac{G_{i-1}}{L^2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





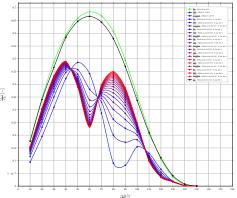






G_{TOT} from VCCI, stresses extracted on matrix surface, $\delta = 5.0^{\circ}$

Normalized energy release rate $\frac{G_{++}}{G_{-}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





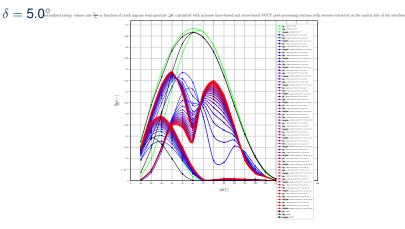






 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary } \delta = 0.2^{\circ} \quad \delta = 0.2^{\circ} \quad$

Summary of $G_{(\cdot\cdot)}$ from VCCI, stresses extracted on matrix surface,





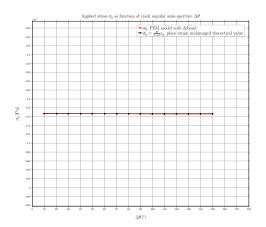






 $\label{eq:decomposition} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary summary sum$

$$\sigma_0$$
, $\delta = 0.4^\circ$



In red small strain FEM, in black analytical plain strain value.



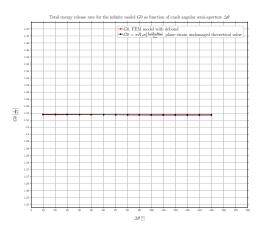






Model Data $\delta = 1.0^{\circ}$ $\delta = 0.9^{\circ}$ $\delta = 0.8^{\circ}$ $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

 G_{0} , $\delta = 0.4^{\circ}$



In red small strain FEM, in black analytical plain strain value.

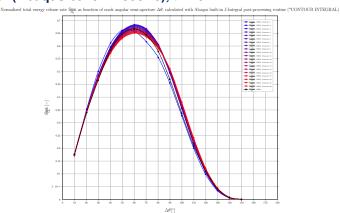








J-Integral (Abaqus built-in routine), $\delta=0.4^\circ$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

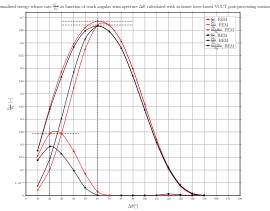








VCCT in forces (in-house Python routine), $\delta=0.4^\circ$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.









Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$

 $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

VCCT, percentual error on BEM, $\delta=0.4^\circ$

$\Delta heta\left[^{\circ} ight]$	$\frac{\frac{G_I}{G_0} _{FEM} - \frac{G_I}{G_0} _{BEM}}{\frac{G_I}{G_0} _{BEM}}$	$\frac{\frac{G_{II}}{G_0} _{FEM} - \frac{G_{II}}{G_0} _{BEM}}{\frac{G_{II}}{G_0} _{BEM}}$	$\frac{\frac{G_{TOT}}{G_0}\left _{FEM} - \frac{G_{TOT}}{G_0}\right _{BEM}}{\frac{G_{TOT}}{G_0}\left _{BEM}}$
10	11.84%	-45.09%	0.06%
20	26.79%	-28.36%	2.95%
30	48.73%	-19.81%	4.31%
40	77.24%	-12.93%	2.20%
50	181.34%	-6.75%	2.04%
60	1084.50%	0.68%	2.78%
70		3.99%	3.93%
80		4.79%	4.61%
90		5.62%	6.07%
100		6.18%	6.59%
110		3.83%	5.37%
120		1.31%	-2.40%
130		-4.97%	-9.28%
140		-30.42%	-29.99%
150		-61.36%	-14.84%

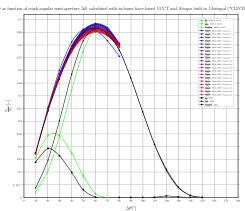








J-Integral and VCCT in forces, $\delta=0.4^\circ$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



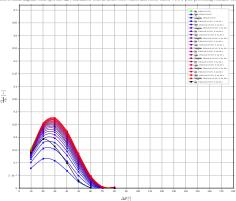






G_l from VCCI, stresses extracted on fiber surface, $\delta = 4.0^{\circ}$

Normalized energy release rate $\frac{G_{i-1}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





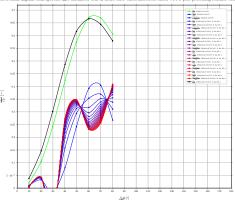






G_{II} from VCCI, stresses extracted on fiber surface, $\delta = 4.0^{\circ}$

Normalized energy release rate $\frac{G_{s,t}}{st}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





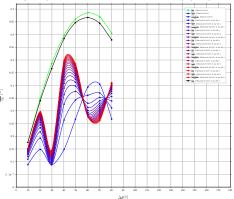






G_{TOT} from VCCI, stresses extracted on fiber surface, $\delta = 4.0^{\circ}$

Normalized energy release rate $\frac{G_{i-1}}{d_i}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





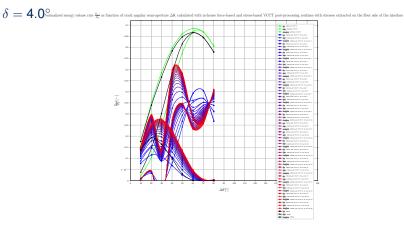






 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = \textbf{0.4}^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \text{Summary } \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \delta = 0.2^{\circ$

Summary of $G_{(\cdot\cdot)}$ from VCCI, stresses extracted on liber surface,









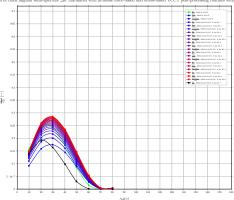


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$ $\delta=0.8^{\circ}$

 $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

G_l from VCCI, stresses extracted on matrix surface, $\delta = 4.0^{\circ}$

Normalized energy release rate $\frac{G_{ch}}{L_0}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





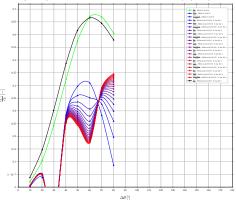






G_{II} from VCCI, stresses extracted on matrix surface, $\delta=4.0^{\circ}$

Normalized energy release rate $\frac{G_{i+1}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





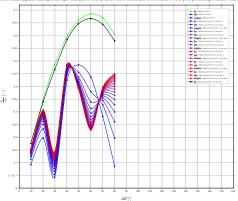






G_{TOT} from VCCI, stresses extracted on matrix surface, $\delta = 4.0^{\circ}$

Normalized energy release rate $\frac{G_{+}}{c^{2}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





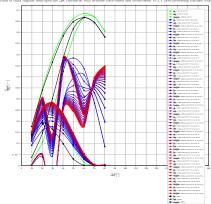






Summary of G(...) from VCGI, stresses extracted on matrix surface,

δ = 4.0 cmailized energy release rate $\frac{c_0}{c_0}$ as function of crack angular semi-spectrum Δθ, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





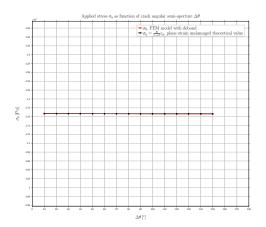






Model Data $\delta = 1.0^{\circ}$ $\delta = 0.9^{\circ}$ $\delta = 0.8^{\circ}$ $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

 σ_0 , $\delta=0.3^\circ$



In red small strain FEM, in black analytical plain strain value.



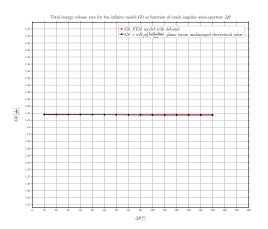






Model Data $\delta = 1.0^{\circ}$ $\delta = 0.9^{\circ}$ $\delta = 0.8^{\circ}$ $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

 G_0 , $\delta = 0.3^{\circ}$



In red small strain FEM, in black analytical plain strain value.

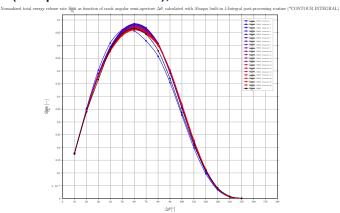








J-Integral (Abaqus built-in routine), $\delta=0.3^\circ$



Fading from blue to red for contours further from the crack tip, FEM results; in black BEM results.

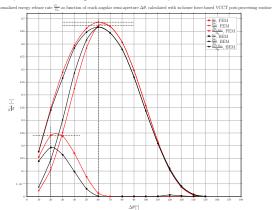








VCCT in forces (in-house Python routine), $\delta=0.3^\circ$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.









Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$

 $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

VCCT, percentual error on BEM, $\delta=0.3^\circ$

$\Delta heta\left[^{\circ} ight]$	$\frac{\frac{G_I}{G_0}\mid_{\textit{FEM}} - \frac{G_I}{G_0}\mid_{\textit{BEM}}}{\frac{G_I}{G_0}\mid_{\textit{BEM}}}$	$\frac{\frac{G_{II}}{G_0} _{FEM} - \frac{G_{II}}{G_0} _{BEM}}{\frac{G_{II}}{G_0} _{BEM}}$	$\frac{\frac{G_{TOT}}{G_0} _{FEM} - \frac{G_{TOT}}{G_0} _{BEM}}{\frac{G_{TOT}}{G_0} _{BEM}}$
10	11.91%	-39.86%	1.23%
20	24.60%	-26.37%	2.56%
30	44.60%	-16.55%	4.97%
40	73.27%	-12.54%	1.86%
50	154.60%	-5.10%	2.35%
60	955.70%	1.01%	2.87%
70		3.99%	4.01%
80		4.79%	4.83%
90		5.62%	5.66%
100		6.18%	6.22%
110		3.83%	3.38%
120		1.31%	-4.08%
130		-4.97%	-10.17%
140		-30.42%	-34.90%
150		-61.36%	-0.52%

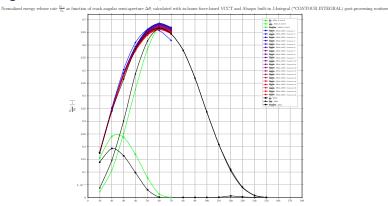








J-Integral and VCCT in forces, $\delta=0.3^\circ$



Fading from blue to red for contours further from the crack tip, J-Integral from FEM results; in green VCCT from FEM results; in black BEM results.



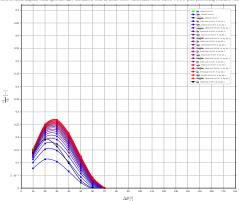






G_l from VCCI, stresses extracted on fiber surface, $\delta = 3.0^{\circ}$

Normalized energy release rate $\frac{G_{22}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface





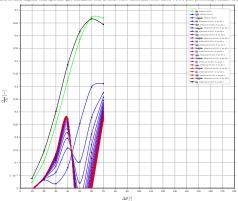






G_{II} from VCCI, stresses extracted on fiber surface, $\delta=3.0^{\circ}$

Normalized energy release rate $\frac{G_{+}}{d^{2}}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface







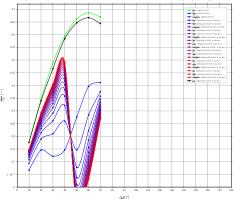




The Data 0 = 1.0 - 0.9 - 0.9 = 0.0 - 0 = 0.7 - 0 = 0.0 - 0 = 0.3 - 0 = 0.4 - 0 = 0.3 - 0 = 0.2 Sufficiently

G_{TOT} from VCCI, stresses extracted on fiber surface, $\delta=3.0^\circ$

Normalized energy release rate $\frac{G_{i,j}}{d_i}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the fiber side of the interface



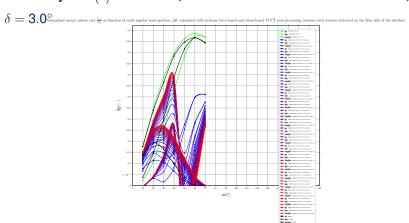








Model Data $\delta=1.0^{\circ}$ $\delta=0.9^{\circ}$ $\delta=0.8^{\circ}$ $\delta=0.7^{\circ}$ $\delta=0.6^{\circ}$ $\delta=0.5^{\circ}$ $\delta=0.4^{\circ}$ $\delta=0.3^{\circ}$ $\delta=0.2^{\circ}$ Summary Summary of $G_{(...)}$ from VGG, stresses extracted on liber surface,









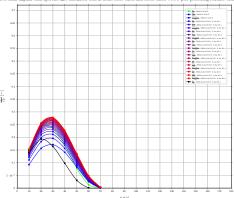


Symbols, Models, Equations & Reference Data Results Summary & Conclusion Model Data $\delta = 1.0^{\circ}$ $\delta = 0.9^{\circ}$ $\delta = 0.8^{\circ}$

 $\delta = 0.7^{\circ}$ $\delta = 0.6^{\circ}$ $\delta = 0.5^{\circ}$ $\delta = 0.4^{\circ}$ $\delta = 0.3^{\circ}$ $\delta = 0.2^{\circ}$ Summary

G_l from VCCl, stresses extracted on matrix surface, $\delta = 3.0^{\circ}$

Normalized energy release rate $\frac{G_{i+1}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





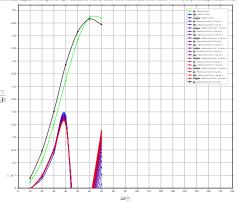






$G_{\rm II}$ from VCCI, stresses extracted on matrix surface, $\delta=3.0^{\circ}$

Normalized energy release rate $\frac{G_{22}}{2}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





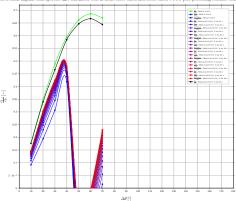






G_{TOT} from VCCI, stresses extracted on matrix surface, $\delta = 3.0^{\circ}$

Normalized energy release rate $\frac{G_{i-1}}{\omega}$ as function of crack angular semi-aperture $\Delta\theta$, calculated with in-house force-based and stress-based VCCT post-processing routines with stresses extracted on the matrix side of the interface





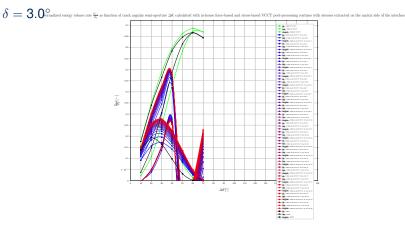






 $\text{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = \textbf{0.3}^{\circ} \quad \delta = \textbf{0.2}^{\circ} \quad \text{Summary } \delta = 0.8^{\circ} \quad \delta = 0.8$

Summary of $G_{(\cdot,\cdot)}$ from VCCI, stresses extracted on matrix surface,





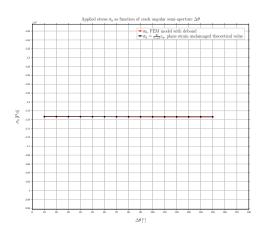






 $\label{eq:model_delta} \mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary and the model}$

$$\sigma_0$$
, $\delta=0.2^\circ$



In red small strain FEM, in black analytical plain strain value.



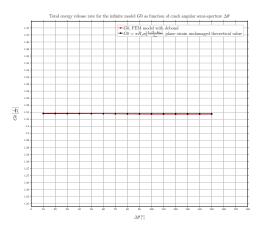






 $\mbox{Model Data} \quad \delta = 1.0^{\circ} \quad \delta = 0.9^{\circ} \quad \delta = 0.8^{\circ} \quad \delta = 0.7^{\circ} \quad \delta = 0.6^{\circ} \quad \delta = 0.5^{\circ} \quad \delta = 0.4^{\circ} \quad \delta = 0.3^{\circ} \quad \delta = 0.2^{\circ} \quad \mbox{Summary Summary Model Data} \quad \delta = 0.00^{\circ} \quad \delta$

$$G_0$$
, $\delta=0.2^\circ$



In red small strain FEM, in black analytical plain strain value.

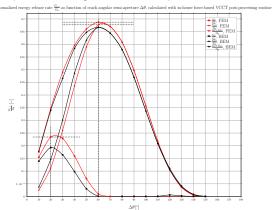








VCCT in forces (in-house Python routine), $\delta = 0.2^{\circ}$



In green VCCT from FEM results, in black BEM results; positions of maxima highlighted by dashed lines.









VCCT, percentual error on BEM, $\delta=0.2^\circ$

$\Delta heta\left[^{\circ} ight]$	$\frac{\frac{G_I}{G_0}\mid_{\textit{FEM}} - \frac{G_I}{G_0}\mid_{\textit{BEM}}}{\frac{G_I}{G_0}\mid_{\textit{BEM}}}$	$\frac{\frac{G_{II}}{G_0} _{FEM} - \frac{G_{II}}{G_0} _{BEM}}{\frac{G_{II}}{G_0} _{BEM}}$	$\frac{\frac{G_{TOT}}{G_0} _{FEM} - \frac{G_{TOT}}{G_0} _{BEM}}{\frac{G_{TOT}}{G_0} _{BEM}}$
10	11.41%	-32.33%	2.44%
20	21.51%	-23.07%	2.24%
30	39.40%	-13.34%	5.22%
40	62.43%	-10.19%	1.99%
50	131.80%	-4.28%	2.06%
60	712.56%	1.64%	3.03%
70		4.05%	4.06%
80		5.02%	5.05%
90		5.69%	5.71%
100		5.83%	5.86%
110		4.45%	3.97%
120		1.55%	-3.88%
130		-6.42%	-11.60%
140		-28.46%	-33.11%
150		-67.88%	-0.52%

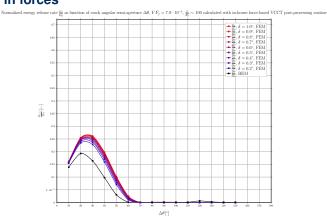








G_l , VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

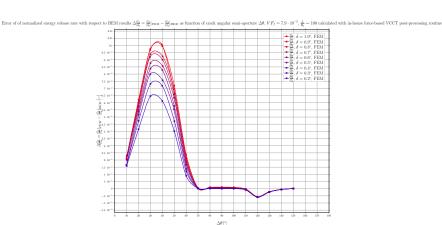








G_l Error with respect to BEM, VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from

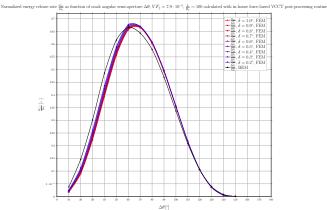








G_{\parallel} , VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

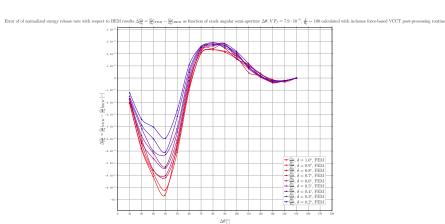








G_{II} Error with respect to BEM, VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from

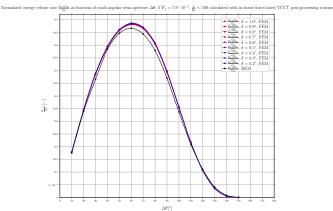








G_{TOT} , VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

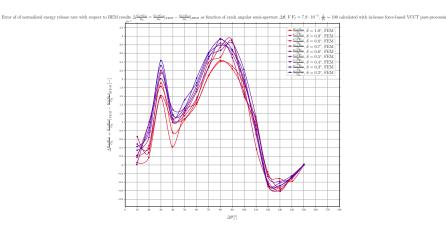








G_{TOT} Error with respect to BEM, VCCT in forces



Fading from red to blue for decreasing size of elements at the interface, VCCT from









≥ SUMMARY & CONCLUSION







mbols, Models, Equations & Reference Data Results Summar

Summary & Conclusion

Summary

- Implemented Virtual Crack Closure Integral (VCCI) method for the calculation of G_l and G_{ll}
- ✓ Analysis of free infinite RVE ($\frac{L}{R_f}$ ~ 100) for several mesh refinements $\delta \in [1.0^{\circ}, 0.2^{\circ}]$
- \checkmark G_I, G_{II} and G_{TOT} calculated using Abaqus built-in J-Integral routine, in-house implemented VCCT and VCCI routines









mbols Models Equations & Reference Data Results Sur

Summary & Conclusion

Conclusion

- ✓ Good agreement of J-Integral results with G_{TOT} from BEM
- √ J-Integral convergence improves refining the mesh.
- For $\delta = 0.4^{\circ}, 0.3^{\circ}, 0.2^{\circ}$ maxima are at the right angle (20° for G_{I} , 60° for G_{II} and G_{TOT}) with in-house VCCT
- \checkmark G_{TOT} relative errors of VCCT over BEM are small (\sim 5% or less) for every $\Delta\theta$
- \checkmark G_{II} relative errors of VCCT over BEM are small ($\sim 5\%$ or less) for $\Delta \theta > 40^\circ$
- √ Results tend to converge to BEM values as the mesh is refined.







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Summary & Conclusion

Conclusion

- \times G_I relative errors of VCCT over BEM are high (> 10%)
- $ightharpoonup G_{II}$ relative errors of VCCT over BEM are high (> 10%) for $\Delta heta \leq 40^\circ$
- G_I of VCCI has correct functional form but values are overestimated (except for very small integration lengths)
- \times G_{II} (and consequently G_{TOT}) of VCCI provides strange results







Symbols, Models, Equations & Reference Data Results

Summary & Conclusion

Next steps

New analysis with $\delta = 0.05^{\circ}$ to see if a better agreement of G_{I} values is attained



