

# Effect of uniform distributions of bonded and debonded fibers on the growth of the fiber/matrix interface crack in thin UD laminates with different fiber contents under transverse loading

Luca Di Stasio<sup>a,b</sup>, Janis Varna<sup>b</sup>, Zoubir Ayadi<sup>a</sup>

<sup>a</sup>*Université de Lorraine, EEIGM, IJL, 6 Rue Bastien Lepage, F-54010 Nancy, France*

<sup>b</sup>*Luleå University of Technology, University Campus, SE-97187 Luleå, Sweden*

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## Abstract

The effects of crack shielding, finite thickness and fiber content on fiber/matrix debond growth in thin unidirectional composites are investigated with Representative Volume Elements (RVEs) of different ordered microstructures. Debond growth is characterized by the estimation of the Energy Release Rates (ERRs) using the Virtual Crack Closure Technique (VCCT) and the J-integral. It is found that the fiber content, the distance between debonds in the loading direction and the presence of a free surface close to the debond have all a strong effect on the ERR. The presence of fully bonded fibers in the direction transverse to the load has instead a restraining effect, and it is shown to be very localized. An explanation of these observations is proposed based on mechanical considerations.

*Keywords:* Polymer-matrix Composites (PMCs), Thin-ply, Transverse Failure, Debonding, Finite Element Analysis (FEA)

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## 1. Introduction

Stimulated by the ever more stringent requirements in terms of weight and mechanical performances of the aerospace industry, the composite community has turned its attention in recent years to *thin-ply* laminates and alternative design approaches are now considered based on this non-conventional laminate in

applications ranging from cryogenic pressure vessels [1], to airplanes' wings [2], and even reusable space launchers [3].

*Thin-ply* laminates are the result of a technological innovation, the *spread tow technology*, which consists in opening or spreading the tows in which fibers (carbon, glass, aramid, basalt among others) are usually shipped in into very thin tapes then used for laminate production. Ply thicknesses of less than  $50\ \mu m$  can nowadays be mass-produced, and record thicknesses of around  $20 - 25\ \mu m$ , or  $\sim 4 - 5$  times the average fiber's diameter, have been achieved. The technique in its current form, sometimes referred to as "FUKUI method" from the name of the Japanese prefecture it originated in, was firstly proposed towards the end of the 1990s [4] and perfected in the subsequent decade [5, 6].

Several experimental investigations on *thin ply* laminates have highlighted their main properties [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]: increased fiber content; more regular packing of fibers; delay and even suppression of transverse cracking (also matrix- or micro-cracking) and delamination. A very insightful work documenting how these mechanisms are affected by the morphology of *thin ply* laminates is the microscopic study of Saito & al. [20], which focuses on the effect of ply thickness on the onset and propagation of transverse cracking. In their investigation, they performed tensile tests on a carbon fiber/epoxy  $[0_2, 90_n, 0_2]$  laminate for  $n = 1, 2, 4$  and measured the crack density at several level of applied tensile strain in the range 0% and 1.5%. Furthermore, they performed microscopic observations on the specimen's edge at each level of strain. They observed the onset of fiber/matrix interface cracks (referred to as debonds in the following) at lower levels of strain in thinner plies, while at the same time coalescence of debonds and through-the-thickness propagation of transverse cracks were delayed and even suppressed as ply thickness decreases. In fact, they reported the first onset of debonds at 0.4% for  $n = 1, 2$  and 0.7% for  $n = 4$ ; for  $n = 1$ , however, at  $\varepsilon = 1.5\%$  coalescence had started to take place but the crack had not completely propagated through the thickness, while for  $n = 2$  and  $n = 4$  the latter already happened at a value of strain respectively of 1.3% and 1%.

Early studies on the effect of ply thickness on the onset and propagation of transverse cracks were conducted on glass fiber/epoxy cross-ply laminates by Bailey, Parvizi and collaborators [21, 22, 23], who firstly observed the beneficial effect of thickness reduction on the delay of transverse cracking. They furthermore pointed the attention to the appearance of debonds at the fiber/matrix interface and their subsequent coalescence as the mechanism at the origin of transverse cracks [24]. Moreover, they identified the main mechanical driver of the damage process in the mismatch of elastic properties, and particularly of Poisson's ratios, between fibers and matrix [25]. A full understanding of damage onset and propagation in *thin-ply* laminates thus requires the comprehension of the mechanisms governing its very first stage, i.e. the fiber/matrix interface crack. First results on its behavior were obtained through analytical models in the case of a single fiber in an infinite matrix under transverse tension by England [26] and Perlman & Sih [27], who obtained the stresses at the interface and calculated the stress intensity factors at the crack tip, and by Taya [28], who evaluated the Energy Release Rate (ERR). Drawing upon the results for the straight bi-material interface crack by Comninou [29], the effect of crack face contact in fiber-matrix debonding was investigated in [30, 31]. In [32], it was showed in terms of ERR why the case of a single asymmetric debond is more likely to be observed under remote transverse tension than two symmetric debonds on the same fiber. The effect of different types and combinations of loads on debonding have been studied for the single fiber model: compression [33], residual thermal stresses [34], biaxial tension-tension and tension-compression [35], biaxial compression-compression and compression-tension [36]. The effect of the presence of nearby fibers has also been studied for a Representative Volume Element (RVE) under uniaxial transverse tension [37], biaxial tension [38] and uniaxial transverse compression [39]. The effect of inter-fiber distance on debond growth has been studied for a partially debonded fiber at the center of an hexagonal cluster (the center partilly debonded fiber being the 7<sup>th</sup>) inside an homogenized UD composite in the case of fully bonded neighbouring fibers [40] and of two partially debonded fibers out of the surrounding six [41]. An understanding of crack

shielding and finite thickness effects on debond growth in non-homogenized microstructural models of UD composites seems thus to be lacking: this is the problem that  
70 we want to address in the present work.

## 2. RVE models & FE discretization

### 2.1. Introduction & Nomenclature

In this paper, we analyze debond development in unidirectional (UD) composites subjected to in-plane transverse tensile loading. The interaction between  
75 debonds in UD composites is studied developing models of different Repeating Unit Cells (RUC) of laminates where only the central fiber in the cell has a damage in the form of a fiber/matrix interface crack (debond). The composite RUC may be repeating in the transverse direction only (representing an ultra-thin composite) or repeating also in the composite thickness direction,  
80 representing an infinite composite in a limiting case. Thus, the conditions at the UD composite's upper and lower boundaries are one of the parameters for the investigation. The used RUCs allow for the consideration of the composite with debonds as a sequence of damaged and undamaged rows, each row with only one fiber in the thickness direction. Since all of these RUCs feature regular  
85 microstructures with fibers placed according to a square-packing tiling, they are Representative Volume Elements (RVE) of composites with a certain distribution of debonds. Introducing in-plane coordinates  $x$  and  $y$ , where  $x$  is in the transverse direction of the UD composite under consideration, the strain in the  $y$ -direction due to a load in the  $x$ -direction is small, due to the very small  
90 minor Poisson's ratio of the UD composite. Additionally, debonds are considered to be significantly longer in the fiber direction than in the arc direction. Therefore, we use 2D models under the assumption of plane strain, defined in the  $x - z$  section of the composite. Thus, the analysis presented applies to long debonds, with a focus on understanding the mechanisms of growth along its arc  
95 direction. The composites are subjected to transverse tensile strain, applied as a constant displacement in the  $x$ -direction along the vertical boundary of the

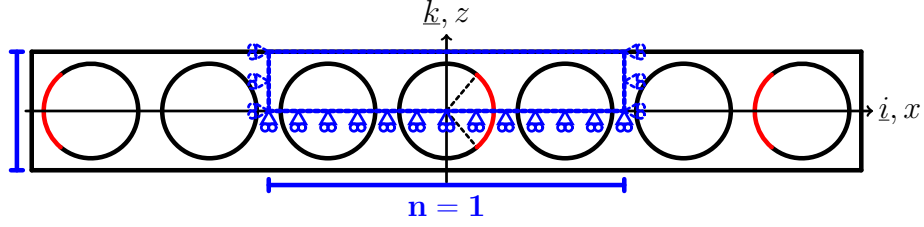
RUC as shown in Figure 1 to 4. As the models are differentiated by the number of layers of fibers and by the spacing between debonds along the vertical and horizontal directions, the corresponding RUCs can be distinguished from each other based on the number  $n$  of fibers in the horizontal direction and  $k$  in the vertical direction. Furthermore, the horizontal surfaces can be either free or vertical displacement coupling can be applied. We thus introduce the common notation  $n \times k - free$  and  $n \times k - coupling$  to denote a RUC with  $n \times k$  fibers and, respectively, a free upper surface or kinematic coupling applied to it. The specific combinations of particular choices of  $n$ ,  $k$ , and boundary conditions are detailed in Section 2.2, together with the corresponding models of damaged composite they are representing.

## 2.2. Models of Representative Volume Element (RVE)

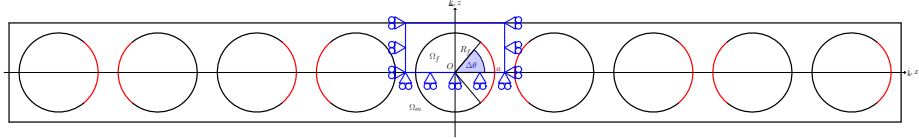
The first two models feature, as shown in Fig. 1, an ultra-thin UD laminate with only one row of fibers across its thickness,  $k = 1$ . This is quite an extreme model from the microstructural point of view; however, it allows to focus the analysis on the interaction between debonded fibers placed along the x-direction. Furthermore, as the horizontal surfaces are considered free, the interaction is stronger in this case than in any other, making the predictions of this model rather conservative. In retrospective, if only 20 years ago such a model would have been considered too abstracted from the physical reality, the recent advancements in the spread tow technology make this approach appealing also as a limiting case for practical considerations.

In the first sub-model, Fig. 1a, every  $n^{th}$  fiber in the composite is partially debonded on alternating sides of the fiber. The symmetries of the model allow the use of the upper part of the RUC. It is highlighted by blue lines in Fig. 1 to 3. Following the notation introduced in Section 2.1, we will refer to this model as  $n \times 1 - free$ . In the second sub-model  $n = 1$ , Fig. 1b, and a debond appears on each fiber on alternating sides and the corresponding RUC contains only one fiber. We will refer to this model as  $1 \times 1 - free$ .

The second set of models in Fig. 2 and Fig. 3 considers laminates with



(a) Single row of fibers with a debond appearing every  $m$  fibers.

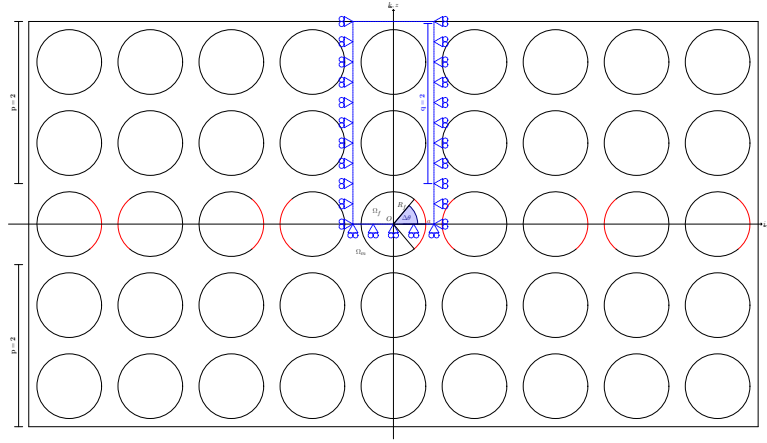


(b) Single row of fibers with debonds appearing on each fiber.

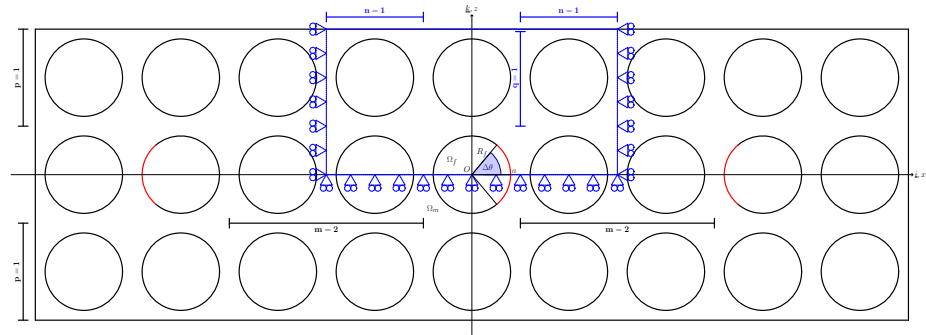
Figure 1: Models of ultra-thin UD composites with a single “row” of fibers and debonds repeating at different distances. The corresponding repeating element (RUC) is highlighted in blue, while debonds are represented in red.

multiple rows of fibers across the thickness: a finite number of rows in the first two sub-models in Fig. 2; an infinite number in the model of Fig. 3. In Fig. 2a, the RUC contains  $n = 1$  fiber in the x-direction,  $k$  fibers across the thickness and the central fiber is debonded. This model will be referred to in the following as  $1 \times k - free$ . Thinking in terms of rows, in this model we have a central row where each fiber is debonded. This row is surrounded from each side by  $(k-1)/2$  rows with perfectly bonded fibers. In the sub-model in Fig. 2b, each  $n^{th}$  fiber in the central row is debonded and this row is surrounded by  $(k-1)/2$  rows of undamaged fibers from each side. We will refer to this model as  $n \times k - free$  (because the horizontal boundary of the RUC is free of any constraint).

Finally, the model in Fig. 3 considers an UD composite with an infinite number of rows; all of them with partially debonded fibers. As all fibers have debonds, the corresponding RUC is made of a single partially debonded fiber with kinematic coupling conditions applied to the upper boundary to assure periodicity. This model is referred to as  $1 \times 1 - coupling$ .



(a) Multiple rows of fibers with debonds appearing on each fiber belonging to the central row.



(b) Mutiple rows of fibers with a debond appearing every  $m$  fibers within the central row.

Figure 2: Models of UD composites with different “rows” of fibers and debonds repeating at different distances. The corresponding repeating element (RUC) is highlighted in blue, while debonds are represented in red.

### 2.3. Finite Element (FE) discretization

Each RUC is discretized using the Finite Element Method (FEM) within the Abaqus environment, a commercial FEM package [42]. The length  $l$  and height  $h$  of the model are determined by number of fibers  $n$  in the horizontal direction and  $k$  across the thickness (see 2.2) according to Eq. 1:

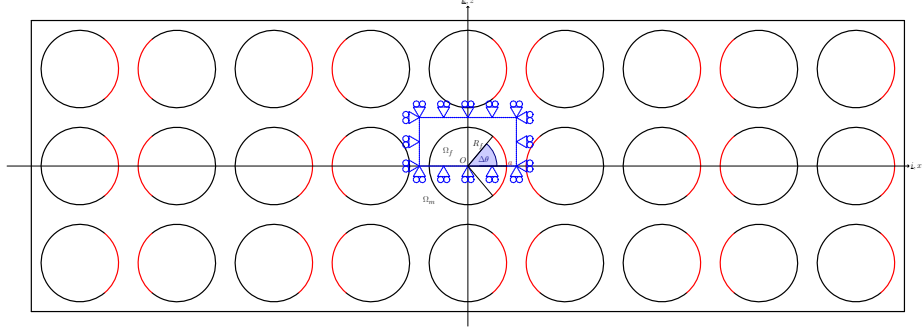


Figure 3: Model of UD composites with an infinite number of “rows” of fibers and debonds appearing on each fiber. The corresponding repeating element (RUC) is highlighted in blue, while debonds are represented in red.

$$l = 2nL \quad h = 2kL; \quad (1)$$

where the reference length  $L$ , see Fig. 4a, is defined as a function of the fiber volume fraction  $V_f$  and the fibers’ radius according to

$$L = \frac{R_f}{2} \sqrt{\frac{\pi}{V_f}}. \quad (2)$$

The fibers’ radius  $R_f$  is assumed to be the same for each fiber present in the model and equal to  $1 \mu m$ . The latter value is not physical and it has been chosen for simplicity. It is worth to note at this point that, in a linear elastic solution as the one presented here, the ERR is proportional to the geometrical dimensions and recalculation of the ERR for fibers of any size thus requires a simple multiplication. Furthermore, notice that the relationships in Eqs. 1 and 2 ensure that the local and global  $V_f$  are everywhere equal.

The debond is placed symmetrically with respect to the  $x$  axis (in red in 4a) and has an angular size of  $\Delta\theta$  (the full debond’s size is thus  $2\Delta\theta$ ). For large debond’s sizes ( $\geq 60^\circ - 80^\circ$ ), a region of variable size  $\Delta\Phi$  appears at the crack tip in which the crack’s faces are in contact and slide on each other. Due to its appearance, frictionless contact is considered between the two crack’s faces to allow free sliding and avoid interpenetration. Symmetry with respect to



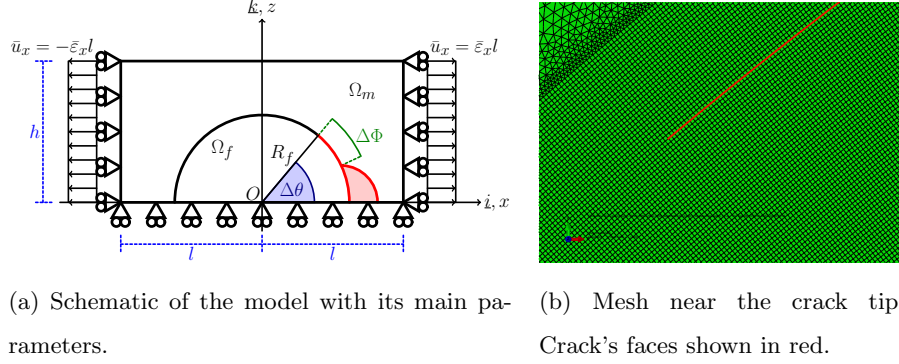


Figure 4: Details and main parameters of the Finite Element model.

the  $x$  axis is applied on the lower boundary and kinematic coupling on the  $x$ -displacement along the left and right sides. The upper boundary is in general free, except for the model  $1 \times 1 - coupling$  (Fig. 3) which requires kinematic coupling of vertical displacements also on the upper side. Constant transverse strain  $\bar{\varepsilon}$  equal to 1% is applied to the right and left sides by means of an imposed  $x$ -displacement of, respectively,  $\pm \bar{\varepsilon}l$ .

Table 1: Summary of the mechanical properties of fiber and matrix.

Material	$E$ [GPa]	$G$ [GPa]	$\nu$ [-]
Glass fiber	70.0	29.2	0.2
Epoxy	3.5	1.25	0.4

The model is meshed using second order, 2D, plane strain triangular (CPE6) and rectangular (CPE8) elements. A regular mesh of quadrilateral elements with an almost unitary aspect ratio is required at the crack tip, as shown in Fig. 4b. The angular size  $\delta$  of an element in the crack tip region is always equal to  $0.05^\circ$ . The crack faces are modeled as element-based surfaces and a small-sliding contact pair interaction with no friction is established between them. The Mode I, Mode II and total Energy Release Rates (ERRs) (respectively referred to as  $G_I$ ,  $G_{II}$  and  $G_{TOT}$ ) represent the main output of the FEM analysis; they are evaluated using the VCCT technique [43] implemented in a custom Python

routine and, for the total ERR, the J-integral [44] by application of the Abaqus built-in functionality. A glass fiber-epoxy system is considered in every model, and it is assumed that their response lies always in the linear elastic domain.

180 The properties used are listed in Table 1.

#### 2.4. Validation of the model

The model is validated in Fig. 5 against the results reported in [45, 37], obtained with the Boundary Element Method (BEM) for a single fiber with a symmetric debond placed in an infinite matrix. This situation is modeled using the *free* RVE with  $V_f = 0.0079\%$ , which corresponds to a RUC's length and height of  $\sim 100$ .

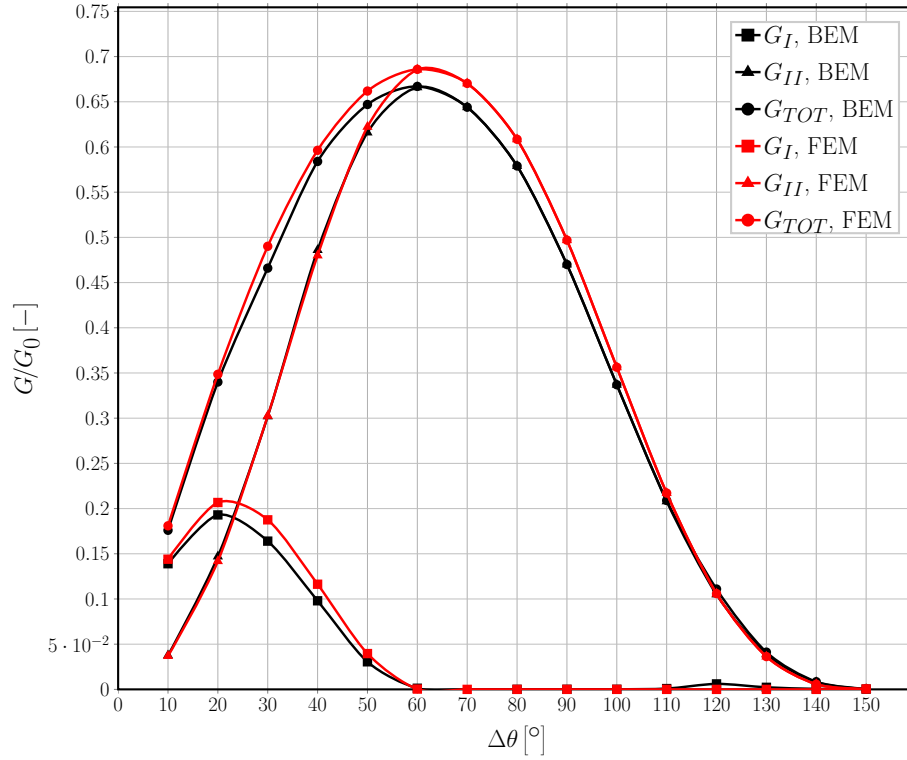


Figure 5: Validation of the single fiber model for the infinite matrix case with respect to the BEM solution in [37].

To allow for a comparison, the results are normalized following [37] with

respect to a reference Energy Release Rate  $G_0$  defined as

$$G_0 = \frac{1 + k_m}{8\mu_m} \sigma_0^2 \pi R_f \quad (3)$$

where  $\mu$  is the shear modulus,  $k$  is the Kolosov's constant defined as  $3 - 4\nu$  for plane strain conditions,  $R_f$  is the fiber radius and the index  $m$  refers to the properties of the matrix.  $\sigma_0$  is the stress at the boundary, computed as the average of the stress extracted at each boundary node along the right side (arithmetic average as nodes are equispaced by design along both the left and right sides). The agreement is good: the difference between the BEM solution, which is considered more accurate, and the FEM solution does not exceed 5%. The ERRs' maxima are in the same positions and the size of the contact zone is the same. Nevertheless, an analysis of phenomena leading to less than 5% differences in ERR would not be reliable and, therefore, it is not recommended.

### 3. Results & Discussion

#### 3.1. Effect of Fiber Volume Fraction

As shown in Figs. 6 and 7, respectively for Mode I and Mode II, the fiber content has a drastic effect on the Energy Release Rate at the tip of the fibre/matrix interface crack. The effect of four levels of fiber volume fraction are compared, 30%, 50%, 60% and 65%, on two microstructural models: a  $11 \times 11$  – *free* (every 11<sup>th</sup> fiber in the central fiber row is partially debonded and, on the top of this row, we have 5 undamaged fiber rows), Figs. 6a and 7a, and a  $21 \times 21$  – *free* (every 21<sup>th</sup> fiber in the central fiber row is partially debonded and, on the top of this row, we have 10 undamaged fiber rows), Figs. 6b and 7b.

Comparing Fig. 6a with 6b, and Fig. 7a with 7b, we can observe that the ERRs' values are very similar for RUCs with  $11 \times 11$  and  $21 \times 21$  fibers, though they are slightly higher for the larger RUC where the next debonded fiber and the free surface are further away from the debonded fiber. From these results we conclude that both RUCs are large enough to represent a single debonded fiber in an infinite array of bonded fibers. Obviously, there exists a specific effect of

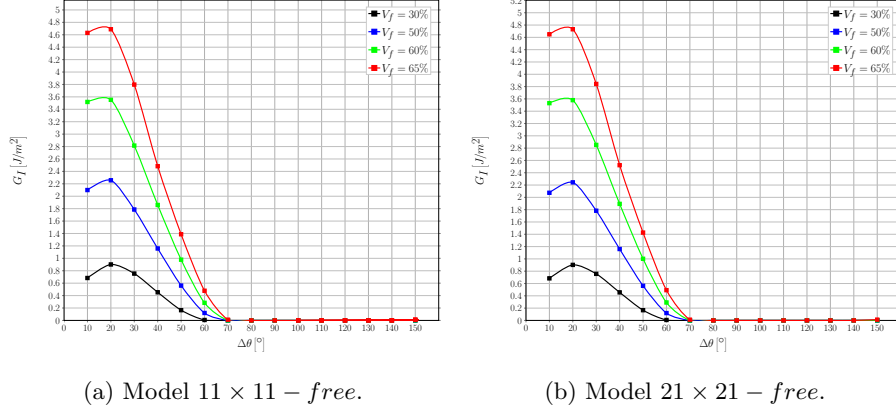


Figure 6: A view of the effect of fiber volume fraction on Mode I ERR in two exemplificative models, subject to an applied transverse strain  $\varepsilon_x$  of 1%.

the fiber content. For Mode I, Fig. 6, the maximum value of the ERR increases by  $\sim 5.2$  times when  $V_f$  changes from 30% to 65%. The debond's angular size for which the peak value occurs remains unchanged at  $20^\circ$ , but for  $V_f = 60\%$  and  $65\%$  the Mode I ERR at  $10^\circ$  and at  $20^\circ$  are rather similar, approximately creating a plateau. Furthermore, increasing the fiber volume fraction delays the onset of the contact zone, which corresponds in Fig. 6 to the first value of  $\Delta\theta$  for which  $G_I$  is equal to zero. For  $V_f = 30\%$ , the contact zone first appears for a debond of  $60^\circ$ , similarly to what happens in the single fiber in infinite matrix model (Fig. 5). For higher fiber contents, the contact zone's onset is delayed to a debond's size approximately equal to  $70^\circ$ .

For Mode II, Fig. 7, the maximum value of the ERR is increases by  $\sim 2.1$  times when  $V_f$  changes from 30% to 65%. The effect is thus similar to Mode I, but with a significantly lower magnitude. Similar to Mode I, the debond's size for which the peak value of Mode II occurs remains unchanged, at  $60^\circ$  for Mode II. There is a distinct maximum in the curve and its shape does not depend on the fiber content. It is worthwhile to notice that the ratio of Mode II to Mode I peak values is  $\frac{\max(G_{II})}{\max(G_I)} \sim \frac{2.2}{0.9} \sim 2.4$  for  $V_f = 30\%$ , while it is  $\sim \frac{4.7}{4.7} \sim 1$  for  $V_f = 65\%$ . Given that the peaks occur at different debond's sizes, for which the value of the other ERR is very small or even close to zero, this means that the

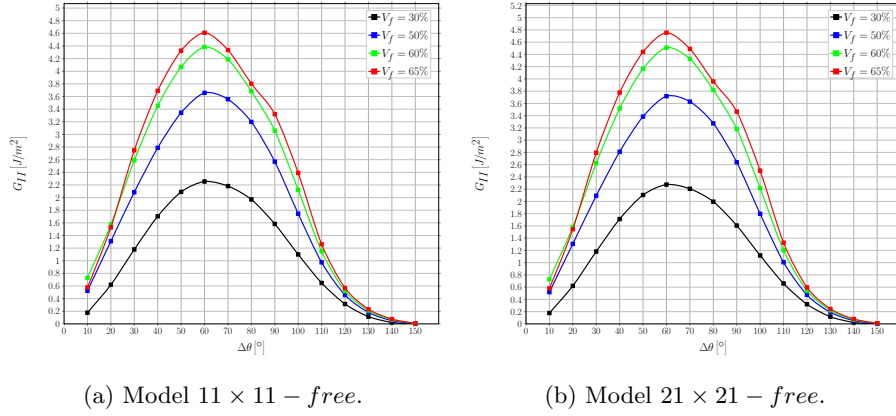


Figure 7: A view of the effect of fiber volume fraction on Mode II ERR in two exemplificative models, subject to an applied transverse strain  $\varepsilon_x$  of 1%.

increase in fiber content creates a long range of very close values of the total  
 235 ERR, that may have a global destabilizing effect on the debond's growth.

The general increasing trends observed in Figs. 6 and 7 are related to the fact  
 that, given that the global and local  $V_f$  are everywhere identical in the models  
 presented, an increase in fiber content corresponds to a decrease in the average  
 distance between fibers. Thus, the decay of the local stress and strain fields  
 240 in the matrix domain occurs over smaller lengths causing higher values at the  
 crack tip. The difference in relative magnification between Mode I and Mode  
 II and the delay in the contact zone's onset are instead due to the interplay  
 between two different mechanisms, both caused by the ordered microstructural  
 arrangement of the model. In the models considered, a fully bonded fiber is al-  
 245 ways placed along the horizontal direction, aligned with the partially debonded  
 fiber and exactly in front of the debond. By increasing  $V_f$ , the former moves  
 closer to the latter and for small debonds this causes a magnification of the  
 x-strain at the crack tip. For small debonds ( $\leq 20^\circ - 30^\circ$ ) in fact, the crack tip  
 is approximately normal to the  $x$ -direction and thus an increase in  $\varepsilon_x$  causes an  
 250 increase in  $G_I$ . On the other hand, for large debonds ( $\geq 70^\circ - 80^\circ$ ) the crack  
 growth direction is almost aligned with the  $x$ -axis, thus a magnification in the  
 $x$ -strain translates into an increase of Mode II ERR. However, this increasing

effect on  $G_{II}$  is partially counteracted by the presence of a fully bonded fiber on  
 top of the debonded fiber and aligned with it. As fibers are more rigid than the  
 255 surrounding matrix, the presence of the former will restrain horizontal displace-  
 ments, thus hampering strong increases in  $G_{II}$  for large debonds. Furthermore,  
 due to the mismatch in the Poisson's ratios, the fully bonded fiber placed above  
 generates an upward-directed component of the vertical displacement field in  
 the matrix, which tends to open the debond and causes the delay in the con-  
 260 tact zone's onset. The interplay between these mechanisms is governed by the  
 average inter-fiber distance and, in turn, by the fiber volume fraction.

These observations are in strong agreement with the results reported in [37],  
 where the effect of two fully bonded nearby fibers, placed symmetrically with  
 respect to the loading direction, is studied for different angular positions and  
 265 radial distances in a model with an effectively infinite matrix. They reported an  
 increase in both Mode I and Mode II ERR with respect to the single fiber case  
 when the second fiber was placed at an angle of  $25^\circ, 30^\circ, 140^\circ, 150^\circ, 155^\circ$  with  
 respect to the loading direction, with the partially debonded fiber placed in the  
 center. In the range  $40^\circ - 130^\circ$ , the presence of the second fiber resulted in a re-  
 270 duction of the ERR at the crack tip. Furthermore, in the range  $80^\circ - 120^\circ$  results  
 were very close and almost identical in the case of Mode I; the minimum values  
 in both Mode I and Mode II were found when the fiber was placed at  $50^\circ - 60^\circ$ .  
 They furthermore observed that the further the fully bonded fibers were placed  
 from the central one, the lower was their effect on the ERR, in agreement with  
 275 our observations on the effect of fiber content. Analogous results can be found  
 in [40], where the authors consider a centrally-placed partially debonded fiber  
 surrounded by an hexagonal cluster inside an homogenized UD composite. They  
 observed a reduction in the ERR when the spacing between fibers was reduced,  
 i.e. when the local fiber volume fraction was increased.

280 Comparing the results presented in this paper with those in [37, 40], an hypothe-  
 sis on the robustness of our results with respect to deviations in fibers' positions  
 can be formulated: it is reasonable to assume that the present results will still  
 be valid for deviations of max.  $\pm 30^\circ$  with respect to the loading direction and

of max.  $\pm 20^\circ$  with respect to the through-the-thickness direction.

### 285 3.2. Interaction between debonds in UD laminates with a single layer of fibers

The interaction of debonds appearing at regular intervals in an ultra-thin UD composite with a single row of fibers is studied for Mode I (Fig. 8) and Mode II (Fig. 9) and fiber content equal to 30% (Figs. 8a and 9a) and 60% (Figs. 8b and 9b). The models treated are  $3 \times 1 - free$ ,  $5 \times 1 - free$ ,  $7 \times 1 - free$ ,  
 290  $11 \times 1 - free$ ,  $21 \times 1 - free$ ,  $101 \times 1 - free$  and  $201 \times 1 - free$ , corresponding respectively to a debond every  $3^{rd}$ ,  $5^{th}$ ,  $7^{th}$ ,  $11^{th}$ ,  $21^{st}$ ,  $101^{st}$  and  $201^{st}$  fiber (Fig. 1a). Given that the upper surface of the UD row is left free, the interaction with the next RUC is stronger than in any other case and the results of this section are thus the most conservative in terms of debond's growth: the ERRs  
 295 should be the largest. The effect is enhanced in composites with high  $V_f$  and especially for  $G_{II}$ : at  $V_f = 60\%$  the highest  $G_{II}$  value for the  $201 \times 1 - free$  composite in Fig. 9b is more than 3 times higher than the  $G_{II}$  value for the  $21 \times 21 - free$  composite in Fig. 7b. Even the maximum is shifted to larger angles. The  $G_I$  value is only 30% higher.

300 From both Fig. 8 and Fig. 9, it can be seen that the presence of a debond decreases the strain magnification effect discussed in Sec. 3.1 and thus reduces the value of the ERR. This phenomenon is called “crack shielding” [32].

For Mode I, the presence of a free surface, and inversely the absence of a fully bonded fiber along the vertical direction, implies the absence of the  
 305 counteracting upward-oriented vertical component of the displacement field due to the mismatch in Poisson's ratios. This in turn translates into the constancy of the value of  $\Delta\theta$  corresponding to contact zone's onset, always equal to  $60^\circ$ . For  $V_f = 30\%$ , Mode I is reduced when the spacing between debonds (in terms of fully bonded fibers in our models) decreases, but the magnitude of change is  
 310 significant only when the spacing is reduced from a debond every  $5^{th}$  fiber to one every  $3^{rd}$ . For comparison, the difference of peak  $G_I$  values for  $V_f = 30\%$  between  $5 \times 1 - free$  and  $3 \times 1 - free$  is  $\sim 0.2 \frac{J}{m^2}$  (around 30% of the lower value), while between  $201 \times 1 - free$  and  $5 \times 1 - free$  is  $\sim 0.05 \frac{J}{m^2}$  (around 7%

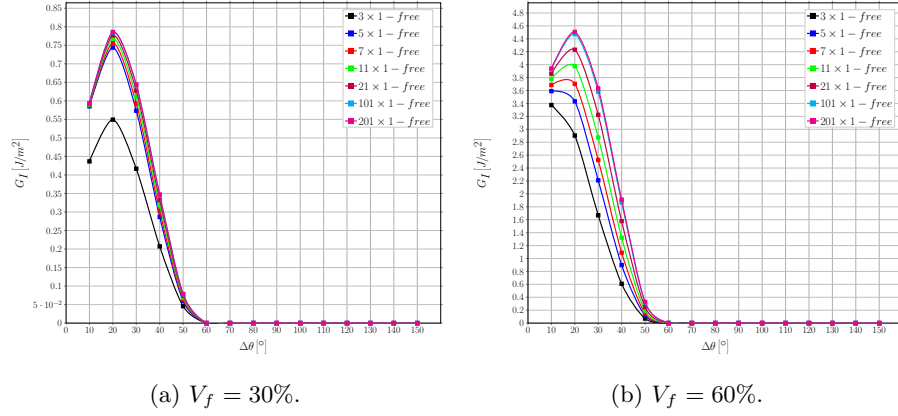


Figure 8: Effect of the interaction between debonds appearing at regular intervals on Mode I ERR in an UD with a single layer of fibers at different levels of fiber volume fraction  $V_f$ , subject to an applied transverse strain  $\varepsilon_x$  of 1%.

of the lower value). A similar observation can be made for  $V_f = 60\%$ , but for  
 315 larger spacings: no difference can be seen between the case of a debond placed  
 every  $101^{th}$  and every  $201^{th}$  fiber. These observations suggest the existence of  
 characteristic distance dependent on the fiber volume fraction which governs the  
 interaction between debonds: in low  $V_f$  composites ( $V_f = 30\%$ ) the convergence  
 to a non-interactive solution is faster (less interaction between debonded fibers  
 320 in neighboring RUCs).

Without constraint on the upper surface, the strain magnification effect cre-  
 ates a larger displacement gap in the  $x$ -direction, which increases Mode II for  
 larger debonds. When debonds are far apart, the series of rigid elements in the  
 ultra-thin composite row (constituted by fully bonded fibers and their surround-  
 325 ing matrix) creates higher  $x$ -strains in the element with the debonded fiber,  
 which in turn generates higher tangential displacements at the crack tip for  
 larger debonds. Conversely, when debonds are closer, the strain concentration  
 in the debonded element is more similar to the applied strain (the magnifica-  
 tion is reduced) and the tangential displacement component at the crack tip  
 330 decreases for large  $\Delta\theta$ . This is the mechanism behind the change in the value  
 of  $\Delta\theta$  for which the peak of  $G_{II}$  occurs: from  $70^\circ$  to  $50^\circ$  at 30%, and from



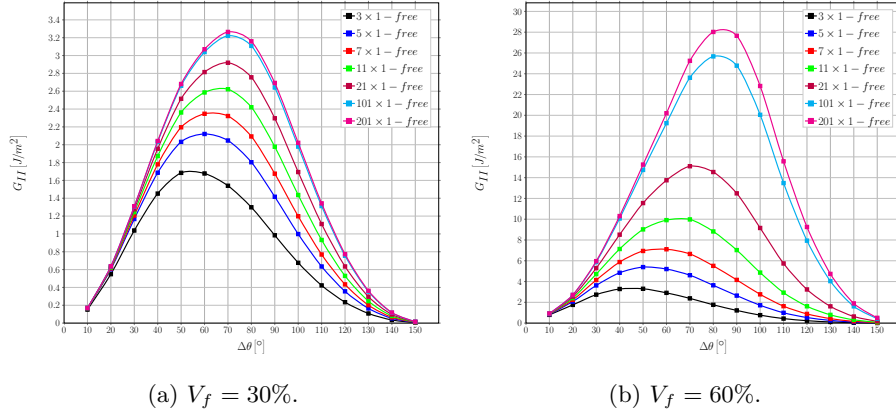


Figure 9: Effect of the interaction between debonds appearing at regular intervals on Mode II ERR in a single-ply laminate with a single layer of fibers at different levels of fiber volume fraction  $V_f$ , subject to an applied transverse strain  $\varepsilon_x$  of 1%.

80° to 40° at 60% going from the higher to the smaller spacing of debonds. Differently from Mode I, the presence of a characteristic distance is harder to establish. For  $V_f = 30\%$  (Fig. 9a), it seems reasonable to establish it at around 100 fully bonded fibers between each debond. For  $V_f = 60\%$  (Fig. 9b), the difference between models  $101 \times 1 - free$  and  $201 \times 1 - free$  is still sizable, thus preventing the establishment of such characteristic distance. It is possible to observe, however, that the change between  $101 \times 1 - free$  and  $201 \times 1 - free$  is significantly smaller than between  $21 \times 1 - free$  and  $101 \times 1 - free$  ( $2 \left[ \frac{J}{m^2} \right]$  vs  $11 \left[ \frac{J}{m^2} \right]$ ), thus suggesting the existence of the characteristic distance outside the range studied. Nevertheless, one should question whether the single row composite with free surface is an appropriate RUC for defining the upper bound for  $G_{II}$ :  $G_{II}$  may be more affected by the free surface than by the effect of the interaction between debonds in the row.

### 3.3. Influence of rows of fully bonded fibers on debond's growth in RUCs with debonds in the central row

The effect of the presence of layers of fully bonded fibers on debond's growth in a line of partially debonded fibers located at mid-thickness in UD composites

is studied for Mode I (Fig. 10) and Mode II (Fig. 11) and fiber content equal to 30% (Figs. 10a and 11a) and 60% (Figs. 10b and 11b). The models treated are  $1 \times 3 - free$ ,  $1 \times 5 - free$ ,  $1 \times 7 - free$ ,  $1 \times 11 - free$ ,  $1 \times 21 - free$ ,  $1 \times 101 - free$  and  $1 \times 201 - free$ , corresponding to a UD composite with respectively 3, 5, 7, 11, 21, 101 and 201 rows of fibers (Fig. 2a) and each fiber debonded in the central row.

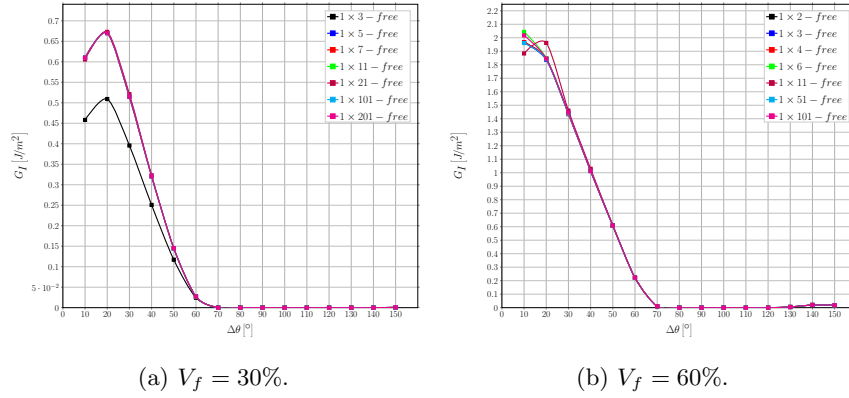


Figure 10: Influence of layers of fully bonded fibers on debond's growth in Mode I ERR in a centrally located line of debonded fibers at different levels of fiber volume fraction  $V_f$ , subject to an applied transverse strain  $\varepsilon_x$  of 1%.

The results shown strengthen the considerations made in Sec. 3.1. It can in fact be seen in Fig. 10 that an increasing number of bonded fibers' rows across the thickness delays the onset of the contact zone to a debond of  $70^\circ$  in size, due to the introduction of an additional positive component of the vertical displacement which translates into an opening displacement at the debond's tip. Comparing Fig. 9b with Fig. 11b, we observe that the presence of bonded fibers' rows significantly reduce the  $G_{II}$  and its maximum is shifted back to  $60^\circ$ , thus confirming the hypothesis in Section 3.2 that the absence of  $G_{II}$  convergence with the increasing distance in a single-row composite is caused more by the free surface than by the interaction between debonds.

The results of both Mode I and Mode II show that the introduction of an increasing number of fully bonded fibers's rows doesn't change the ERR

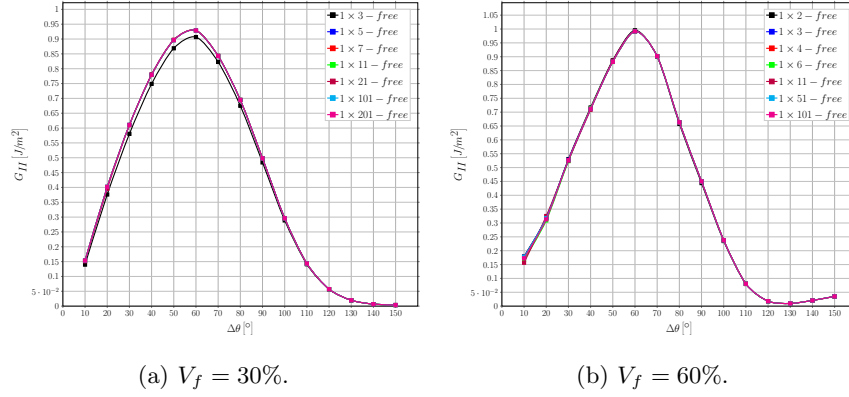


Figure 11: Influence of layers of fully bonded fibers on debond's growth in Mode II ERR in a centrally located line of debonded fibers at different levels of fiber volume fraction  $V_f$ , subject to an applied transverse strain  $\varepsilon_x$  of 1%.

calculated at the crack tip (the convergence is very fast). Some effect of the  $V_f$  (mostly on Mode I) can be observed at low fiber content (Figs. 10a and 11a), while for high fiber content the smaller model with only one fiber row above the partially debonded one is already representative.

### 3.4. Interaction between debonds in UD composites with multiple rows of fibers

The interaction of debonds appearing at regular intervals in UD composites with multiple rows of fibers is investigated using different combinations of horizontal debonds' spacing and number of rows of fibers across the thickness, corresponding to the models:  $3 \times 3 - free$ ,  $5 \times 3 - free$ ,  $5 \times 5 - free$ ,  $7 \times 3 - free$ ,  $7 \times 5 - free$ ,  $7 \times 7 - free$ ,  $11 \times 3 - free$ ,  $11 \times 5 - free$ ,  $11 \times 7 - free$ ,  $11 \times 11 - free$ ,  $21 \times 3 - free$ ,  $21 \times 5 - free$ ,  $21 \times 7 - free$ ,  $21 \times 11 - free$ ,  $21 \times 21 - free$ ,  $101 \times 3 - free$ ,  $101 \times 5 - free$ ,  $101 \times 7 - free$ ,  $101 \times 11 - free$ ,  $201 \times 3 - free$ ,  $201 \times 5 - free$ ,  $201 \times 7 - free$ ,  $201 \times 11 - free$  (Fig. 2b).

The results shown in Fig. 12 confirm the observations discussed in Sec. 3.2: the presence of fully bonded fibers across the thickness has a restraining effect on the ERR, that counteracts the magnification due to an increasing number of fully bonded fibers in the horizontal direction. The interplay is further modulated by the fiber content. For Mode I, at high fiber content the contact zone onset

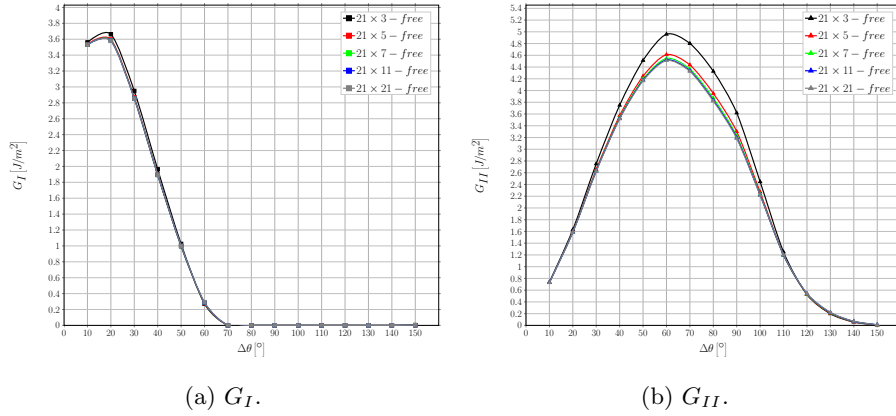
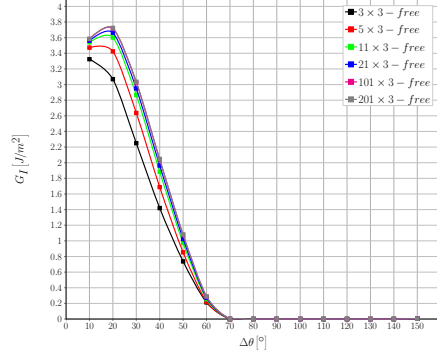


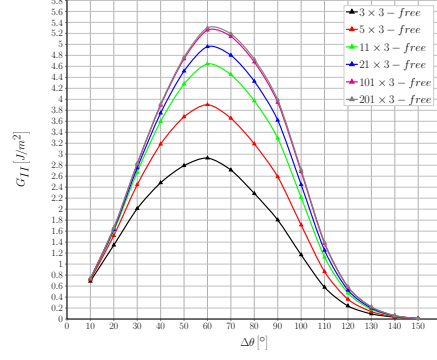
Figure 12: Effect on Mode I and Mode II ERR of the presence of an increasing number of rows of fully bonded fibers in UD composites with debonds appearing every  $10^{th}$  fiber (model  $21 \times k - free$ ).  $V_f = 60\%$  and  $\varepsilon_x = 1\%$ .

starts at  $70^\circ$  for  $V_f = 60\%$ , delayed with respect to the low fiber content case of  $60^\circ$ . Comparing Fig. 12 with Fig. 10b and Fig. 11b, it is furthermore possible to observe that the number of fully bonded fibers' rows necessary to reach convergence to a non-interacting solution in the vertical direction depends on the spacing of debonds in the central row. In Figures 10b and 11b the results for the  $1 \times 3 - free$  model (1 row below and above) are already representative of all the other cases; in Fig. 12 the solution doesn't change anymore once at least 3 rows below and above the central one are present, when convergence in both  $G_I$  and  $G_{II}$  is required.

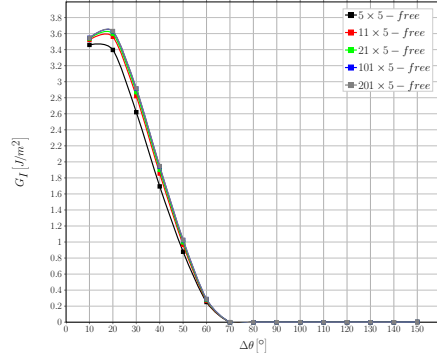
The results in Fig. 13 show that the converse is as well as true: the characteristic distance (in terms of fully bonded fibers) between debonds for which a non-interactive solution is attained changes in relation to the thickness of the UD composite (defined by the number of rows in the vertical direction). Mode I appears to be far less sensitive than  $G_{II}$  to the spacing of debonds in the horizontal direction when rows of fully bonded fibers are present above and below: in Fig. 13a the increase in the peak value of  $G_I$  is  $\sim 8\%$  going from model  $5 \times 3 - free$  to  $201 \times 3 - free$ , while  $< 5\%$  for larger spacings. In UDs of increased thickness, Figures 13c and 13e, the variation is further



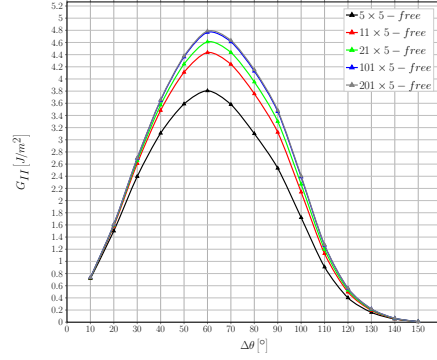
(a)  $k = 3$ ,  $G_I$ .



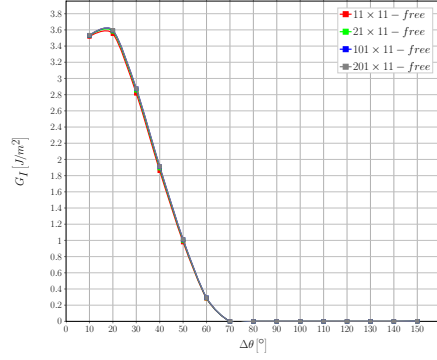
(b)  $k = 3$ ,  $G_{II}$ .



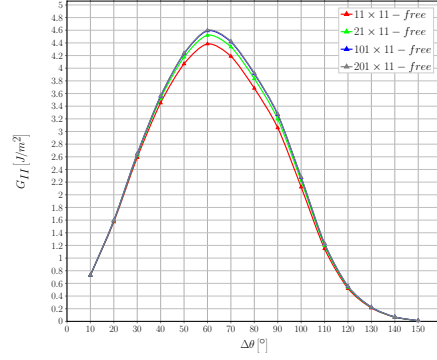
(c)  $k = 5$ ,  $G_I$ .



(d)  $k = 5$ ,  $G_{II}$ .



(e)  $k = 11$ ,  $G_I$ .



(f)  $k = 11$ ,  $G_{II}$ .

Figure 13: Effect on Mode I and Mode II ERR of increasing the spacing between debonds appearing in the central row of fibers in a UD composite with a fixed number of rows across the thickness.  $V_f = 60\%$  and  $\varepsilon_x = 1\%$ .

reduced. For Mode II, convergence to a non-interactive solution is reached with a spacing of 100 fully bonded fibers for a UD with 3 rows of fibers across the thickness ( $\frac{G_{II}^{201 \times 3}(60^\circ) - G_{II}^{101 \times 3}(60^\circ)}{G_{II}^{101 \times 3}(60^\circ)} \sim 0.7\%$ ), of 20 fibers in a UD with 5 rows ( $\frac{G_{II}^{101 \times 5}(60^\circ) - G_{II}^{21 \times 5}(60^\circ)}{G_{II}^{21 \times 5}(60^\circ)} \sim 4.3\%$ ) and of 10 fibers in a UD with 11 rows ( $\frac{G_{II}^{21 \times 11}(60^\circ) - G_{II}^{11 \times 11}(60^\circ)}{G_{II}^{11 \times 11}(60^\circ)} \sim 3.4\%$ ).

### 3.5. Comparison with the single fiber model with equivalent boundary conditions

The single fiber RUC ( $1 \times 1 - \text{free}$  or  $1 \times 1 - \text{coupling}$ ) corresponds to the most damaged state of the composite, i.e. the state in which all fibers have debonds. The  $1 \times 1 - \text{free}$  model represents an ultra-thin UD composite with a single row of partially debonded fibers. The  $1 \times 1 - \text{coupling}$  model, where the displacement coupling is used to enforce periodic boundary conditions, represents an infinite composite.

The comparison of the  $1 \times 1 - \text{free}$  model with one row multi-fiber models  $n \times 1 - \text{free}$  in Figure 14 show that the former provide in general the lowest value of the ERR (the highest crack shielding case) which is consistent with the trends observed in Section 3.2.

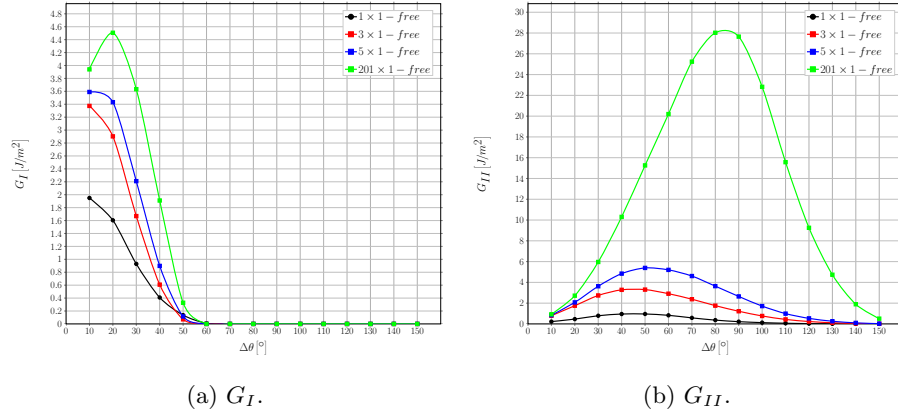


Figure 14: Comparison of the ERR between the single fiber model with free upper boundary and the multiple fibers model with fibers only on the side.  $V_f = 60\%$  and  $\varepsilon_x = 1\%$ .

The  $1 \times 1 - \text{coupling}$  model is compared with  $1 \times 3 - \text{free}$  and  $1 \times 201 - \text{free}$  models in Fig. 15. In all three models the distance between debonds in the

$x$ -direction is the same and the difference is in the vertical direction. The  $1 \times 1 - \text{coupling}$  model describes the interaction between debonds in different rows of debonded fibers whereas the  $1 \times k - \text{free}$  models describe the effect of the proximity of the composite's free surface. The Mode I ERR in the  $1 \times 3 - \text{free}$  model and in the  $1 \times 1 - \text{coupling}$  model is very similar, which leads to a rather surprising conclusion. In both models we have, on the top of the central one, a large amount of fibers (bonded in one case and debonded in the other case). It appears that the effect of bonded and debonded fibers on the central debond is the same. This implies that the interaction between debonded fibers in elements placed on top of each other is small. The volume fraction effect is much smaller in high fiber content composites of this type.

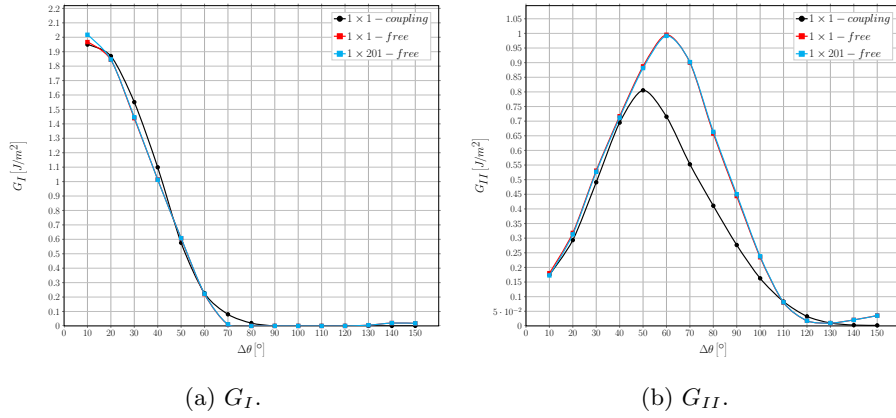


Figure 15: Comparison of the ERR between the single fiber model with coupling conditions along the upper boundary and the multiple fibers model with fibers above.  $V_f = 60\%$  and  $\varepsilon_x = 1\%$ .

The same comparison for Mode II shows a sizeable difference in the range  $50^\circ - 90^\circ$ , while the results almost coincide for smaller values of  $\Delta\theta$ . These observations point to the evidence that debond interaction is more significant in the loading direction than in the transverse one. The lower values of  $G_{II}$  of the  $1 \times 1 - \text{coupling}$  model in the range  $50^\circ - 90^\circ$  are due to the shielding effect of a debond of the same size in the fiber just above the central one (modeled by the coupling boundary condition), which leaves the strip of matrix between

the two fibers free to deform away from both of them due to the Poissons effect  
 440 and thus favors Mode I and reduces Mode II. This translates into the lower  
 estimates in Fig. 15b and into the delay in the appearance of the contact zone,  
 particularly evident in Fig. 15a.

#### 4. Conclusions & Outlook

Several models of Repeating Unit Cell, representative of different microstruc-  
 445 tural arrangements, have been studied in order to investigate the effect on in-  
 terface crack growth of the presence of debonds and/or fully bonded fibers in  
 the loading and through-the-thickness direction in UD composites. Regular  
 microstructures based on square-packing of fibers have been considered, with  
 debonds appearing at regular intervals measured in terms of fully bonded fibers  
 450 between them. Local and global fiber volume fractions are everywhere equal to  
 each other by design, which establishes a direct relationship between fiber con-  
 tent and inter-fiber distance. The main conclusions of this work are summarized  
 here in the following.

1. The presence of a free surface close to the debond causes the effect of the  
 455 presence of fully bonded fibers along the loading direction to be strongly  
 amplified, leading to higher Mode I and Mode II ERRs and a displacement  
 of the peak  $G$  values to larger debonds.
2. The presence of fully bonded fibers in the loading directions causes an  
 increase in ERR, proportional to the number of fully fibers present be-  
 460 fore the appearance of the closest aligned debond. It seems to exist a  
 characteristic distance between debonds which defines the transition to a  
 non-interactive solution; however, it has not been proved for Mode II in  
 the range of parameters studied.
3. The presence of fibers (fully or partially bonded) in the through-the-  
 465 thickness direction appears to have a restraining effect on both  $G_I$  and  
 $G_{II}$ , which opposes the magnifying effect of fully bonded fibers placed  
 along the loading direction. Transition to a non-interactive solution is



fast, and no change in the solution can be observed by adding more than 2 fully bonded fibers below and above the central partially debonded one.

- 470 4. The presence of a debond in the fiber above the central partially debonded one only delays the appearance of the contact zone, while no significant effect on the ERR can be observed.
5. Increasing the fiber content, which corresponds to a decrease in the inter-fiber distance, magnifies in general the effects described in the previous  
475 points.
6. The results and conclusions presented agree strongly with previous observations reported in the literature ([37, 40]). A mechanical explanation has been presented based on the mismatch in elastic properties, and particularly Poisson's ratios, and the positions of fibers and debonds with  
480 respect to the loading direction.

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## 485 References

- [1] D. A. McCarville, J. C. Guzman, A. K. Dillon, J. R. Jackson, J. O. Birkland, 3.5 Design, Manufacture and Test of Cryotank Components, Elsevier, 2018, pp. 153–179. doi:10.1016/b978-0-12-803581-8.09958-6.  
URL <https://doi.org/10.1016/b978-0-12-803581-8.09958-6>
- 490 [2] Y. H. N. Kim, S. Ko, W.-S. Lay, J. Tian, P. Chang, S. U. Thielk, H.-J. Bang, J. Yang, Effects of shallow biangle, thin-ply laminates on structural performance of composite wings, AIAA Journal 55 (6) (2017) 2086–2092. doi:10.2514/1.j055465.  
URL <https://doi.org/10.2514/1.j055465>

- 495 [3] A. Kopp, S. Stappert, D. Mattsson, K. Olofsson, E. Marklund, G. Kurth,  
E. Mooij, E. Roorda, The aurora space launcher concept, CEAS Space  
Journal 10 (2) (2017) 167–187. doi:10.1007/s12567-017-0184-2.  
URL <https://doi.org/10.1007/s12567-017-0184-2>
- [4] K. Kawabe, S. Tomoda, T. Matsuo, A pneumatic process for spreading  
500 reinforcing fiber tow, in: Proceedings of the 42<sup>nd</sup> International SAMPE  
Symposium and Exhibition, SAMPE, pp. 65–76.
- [5] K. Kawabe, New spreading technology for carbon fiber tow and its ap-  
plication to composite materials, Sen'i Gakkaishi 64 (8) (2008) 262–267.  
doi:10.2115/fiber.64.p\_262.  
505 URL [https://doi.org/10.2115/fiber.64.p\\_262](https://doi.org/10.2115/fiber.64.p_262)
- [6] K. Kawabe, H. Sasayama, S. Tomoda, New carbon fiber tow-spread  
technology and applications to advanced composite materials, SAMPE  
Journal 45 (2) (2008) 6–17.  
URL [https://researchmap.jp/?action=cv\\_download\\_main&upload\\_](https://researchmap.jp/?action=cv_download_main&upload_id=161885)  
510 [id=161885](https://researchmap.jp/?action=cv_download_main&upload_id=161885)
- [7] H. Sasayama, K. Kawabe, S. Tomoda, I. Ohsawa, K. Kageyama, N. Ogata,  
Effect of lamina thickness on first ply failure in multidirectionally lami-  
nated composites, in: Proceedings of the 8<sup>th</sup> Japan SAMPE Symposium,  
SAMPE, 2003.
- 515 [8] S. Tsai, S. Sih, R. Kim, Thin ply composites, in: Proceedings of 46<sup>th</sup>  
AIAA/ASME/AHS/ASC Structures, Structural Dynamics & Materials  
Conference, 2005.
- [9] K. Yamaguchi, H. Hahn, The improved ply cracking resistance of thin-  
ply laminates, in: Proceedings of the 15<sup>th</sup> International Conference on  
520 Composite Materials (ICCM-15), SAMPE, 2005.
- [10] S. SIHN, R. KIM, K. KAWABE, S. TSAI, Experimental studies of thin-ply  
laminated composites, Composites Science and Technology 67 (6) (2007)

996–1008. doi:10.1016/j.compscitech.2006.06.008.

URL <https://doi.org/10.1016/j.compscitech.2006.06.008>

- 525 [11] T. Yokozeki, Y. Aoki, T. Ogasawara, Experimental characterization of strength and damage resistance properties of thin-ply carbon fiber/toughened epoxy laminates, *Composite Structures* 82 (3) (2008) 382–389. doi:10.1016/j.compstruct.2007.01.015.

URL <https://doi.org/10.1016/j.compstruct.2007.01.015>

- 530 [12] T. Yokozeki, A. Kuroda, A. Yoshimura, T. Ogasawara, T. Aoki, Damage characterization in thin-ply composite laminates under out-of-plane transverse loadings, *Composite Structures* 93 (1) (2010) 49–57. doi:10.1016/j.compstruct.2010.06.016.

URL <https://doi.org/10.1016/j.compstruct.2010.06.016>

- 535 [13] J.-B. Moon, M.-G. Kim, C.-G. Kim, S. Bhowmik, Improvement of tensile properties of CFRP composites under LEO space environment by applying MWNTs and thin-ply, *Composites Part A: Applied Science and Manufacturing* 42 (6) (2011) 694–701. doi:10.1016/j.compositesa.2011.02.011.

URL <https://doi.org/10.1016/j.compositesa.2011.02.011>

- 540 [14] A. Arteiro, G. Catalanotti, J. Xavier, P. Camanho, Notched response of non-crimp fabric thin-ply laminates, *Composites Science and Technology* 79 (2013) 97–114. doi:10.1016/j.compscitech.2013.02.001.

URL <https://doi.org/10.1016/j.compscitech.2013.02.001>

- [15] A. Arteiro, G. Catalanotti, J. Xavier, P. Camanho, Large damage capability of non-crimp fabric thin-ply laminates, *Composites Part A: Applied Science and Manufacturing* 63 (2014) 110–122. doi:10.1016/j.compositesa.2014.04.002.

545 URL <https://doi.org/10.1016/j.compositesa.2014.04.002>

- [16] R. Amacher, J. Cugnoni, J. Botsis, L. Sorensen, W. Smith, C. Dransfeld, 550 Thin ply composites: Experimental characterization and modeling of size-effects, *Composites Science and Technology* 101 (2014) 121–132. doi:10.

1016/j.compscitech.2014.06.027.

URL <https://doi.org/10.1016/j.compscitech.2014.06.027>

- 555 [17] G. Guillaumet, A. Turon, J. Costa, J. Renart, P. Linde, J. Mayugo, Damage occurrence at edges of non-crimp-fabric thin-ply laminates under off-axis uniaxial loading, *Composites Science and Technology* 98 (2014) 44–50. doi:10.1016/j.compscitech.2014.04.014.

URL <https://doi.org/10.1016/j.compscitech.2014.04.014>

- 560 [18] C. Huang, S. Ju, M. He, Q. Zheng, Y. He, J. Xiao, J. Zhang, D. Jiang, Identification of failure modes of composite thin-ply laminates containing circular hole under tension by acoustic emission signals, *Composite Structures* 206 (2018) 70–79. doi:10.1016/j.compstruct.2018.08.019.

URL <https://doi.org/10.1016/j.compstruct.2018.08.019>

- 565 [19] J. Cugnoni, R. Amacher, S. Kohler, J. Brunner, E. Kramer, C. Dransfeld, W. Smith, K. Scobbie, L. Sorensen, J. Botsis, Towards aerospace grade thin-ply composites: Effect of ply thickness, fibre, matrix and interlayer toughening on strength and damage tolerance, *Composites Science and Technology* 168 (2018) 467–477. doi:10.1016/j.compscitech.2018.08.037.

570 URL <https://doi.org/10.1016/j.compscitech.2018.08.037>

- [20] H. Saito, H. Takeuchi, I. Kimpara, Experimental evaluation of the damage growth restraining in 90 layer of thin-ply cfrp cross-ply laminates, *Advanced Composite Materials* 21 (1) (2012) 57–66. doi:10.1163/156855112X629522.

- 575 [21] K. W. Garrett, J. E. Bailey, Multiple transverse fracture in 90° cross-ply laminates of a glass fibre-reinforced polyester, *Journal of Materials Science* 12 (1) (1977) 157–168. doi:10.1007/bf00738481.

URL <https://doi.org/10.1007/bf00738481>

- 580 [22] A. Parvizi, J. E. Bailey, On multiple transverse cracking in glass fibre epoxy cross-ply laminates, *Journal of Materials Science* 13 (10) (1978) 2131–2136.

doi:10.1007/bf00541666.

URL <https://doi.org/10.1007/bf00541666>

- [23] A. Parvizi, K. W. Garrett, J. E. Bailey, Constrained cracking in glass fibre-reinforced epoxy cross-ply laminates, *Journal of Materials Science* 13 (1) (1978) 195–201. doi:10.1007/bf00739291.

URL <https://doi.org/10.1007/bf00739291>

- [24] J. E. Bailey, A. Parvizi, On fibre debonding effects and the mechanism of transverse-ply failure in cross-ply laminates of glass fibre/thermoset composites, *Journal of Materials Science* 16 (3) (1981) 649–659. doi:10.1007/bf02402782.

URL <https://doi.org/10.1007/bf02402782>

- [25] J. E. Bailey, P. T. Curtis, A. Parvizi, On the transverse cracking and longitudinal splitting behaviour of glass and carbon fibre reinforced epoxy cross ply laminates and the effect of poisson and thermally generated strain, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 366 (1727) (1979) 599–623. doi:10.1098/rspa.1979.0071.

URL <https://doi.org/10.1098/rspa.1979.0071>

- [26] A. H. England, An arc crack around a circular elastic inclusion, *Journal of Applied Mechanics* 33 (3) (1966) 637. doi:10.1115/1.3625132.

URL <https://doi.org/10.1115/1.3625132>

- [27] A. Perlman, G. Sih, Elastostatic problems of curvilinear cracks in bonded dissimilar materials, *International Journal of Engineering Science* 5 (11) (1967) 845–867. doi:10.1016/0020-7225(67)90009-2.

URL [https://doi.org/10.1016/0020-7225\(67\)90009-2](https://doi.org/10.1016/0020-7225(67)90009-2)

- [28] M. Toya, A crack along the interface of a circular inclusion embedded in an infinite solid, *Journal of the Mechanics and Physics of Solids* 22 (5) (1974) 325–348. doi:10.1016/0022-5096(74)90002-7.

URL [https://doi.org/10.1016/0022-5096\(74\)90002-7](https://doi.org/10.1016/0022-5096(74)90002-7)

- [29] M. Comninou, The interface crack, *Journal of Applied Mechanics* 44 (4) (1977) 631. doi:10.1115/1.3424148.  
 URL <https://doi.org/10.1115/1.3424148>
- [30] F. París, J. C. Caño, J. Varna, The fiber-matrix interface crack — a numerical analysis using boundary elements, *International Journal of Fracture* 82 (1) (1996) 11–29. doi:10.1007/bf00017861.  
 URL <https://doi.org/10.1007/bf00017861>
- [31] J. Varna, F. París, J. C. Caño, The effect of crack-face contact on fiber/matrix debonding in transverse tensile loading, *Composites Science and Technology* 57 (5) (1997) 523–532. doi:10.1016/s0266-3538(96)00175-3.  
 URL [https://doi.org/10.1016/s0266-3538\(96\)00175-3](https://doi.org/10.1016/s0266-3538(96)00175-3)
- [32] I. García, V. Mantič, E. Graciani, Debonding at the fibre–matrix interface under remote transverse tension. one debond or two symmetric debonds?, *European Journal of Mechanics - A/Solids* 53 (2015) 75–88. doi:10.1016/j.euromechsol.2015.02.007.  
 URL <https://doi.org/10.1016/j.euromechsol.2015.02.007>
- [33] E. Correa, E. Gamstedt, F. París, V. Mantič, Effects of the presence of compression in transverse cyclic loading on fibre–matrix debonding in unidirectional composite plies, *Composites Part A: Applied Science and Manufacturing* 38 (11) (2007) 2260–2269. doi:10.1016/j.compositesa.2006.11.002.  
 URL <https://doi.org/10.1016/j.compositesa.2006.11.002>
- [34] E. Correa, V. Mantič, F. París, Effect of thermal residual stresses on matrix failure under transverse tension at micromechanical level: A numerical and experimental analysis, *Composites Science and Technology* 71 (5) (2011) 622–629. doi:10.1016/j.compscitech.2010.12.027.  
 URL <https://doi.org/10.1016/j.compscitech.2010.12.027>

- [35] E. Correa, F. París, V. Mantič, Effect of the presence of a secondary transverse load on the inter-fibre failure under tension, *Engineering Fracture Mechanics* 103 (2013) 174–189. doi:10.1016/j.engfracmech.2013.02.026.  
 640 URL <https://doi.org/10.1016/j.engfracmech.2013.02.026>
- [36] E. Correa, F. París, V. Mantič, Effect of a secondary transverse load on the inter-fibre failure under compression, *Composites Part B: Engineering* 65 (2014) 57–68. doi:10.1016/j.compositesb.2014.01.005.  
 URL <https://doi.org/10.1016/j.compositesb.2014.01.005>
- 645 [37] C. Sandino, E. Correa, F. París, Numerical analysis of the influence of a nearby fibre on the interface crack growth in composites under transverse tensile load, *Engineering Fracture Mechanics* 168 (2016) 58–75. doi:10.1016/j.engfracmech.2016.01.022.  
 URL <https://doi.org/10.1016/j.engfracmech.2016.01.022>
- 650 [38] C. Sandino, E. Correa, F. París, Composite materials under transverse biaxial loads: Study of the influence of a nearby fibre on the interface crack growth under tension, in: *Proceeding of the 17<sup>th</sup> European Conference on Composite Materials (ECCM-17)*, 2016.
- [39] C. Sandino, E. Correa, F. París, Interface crack growth under transverse  
 655 compression: nearby fibre effect, in: *Proceeding of the 18<sup>th</sup> European Conference on Composite Materials (ECCM-18)*, 2018.
- [40] L. Zhuang, A. Pupurs, J. Varna, R. Talreja, Z. Ayadi, Effects of inter-fiber spacing on fiber-matrix debond crack growth in unidirectional composites under transverse loading, *Composites Part A: Applied Science and Manufacturing* 109 (2018) 463–471. doi:10.1016/j.compositesa.2018.03.031.  
 660 URL <https://doi.org/10.1016/j.compositesa.2018.03.031>
- [41] J. Varna, L. Q. Zhuang, A. Pupurs, Z. Ayadi, Growth and interaction of debonds in local clusters of fibers in unidirectional composites during

- 665       transverse loading, Key Engineering Materials 754 (2017) 63–66. doi:  
10.4028/www.scientific.net/kem.754.63.  
URL <https://doi.org/10.4028/www.scientific.net/kem.754.63>
- [42] Simulia, Providence, RI, USA, ABAQUS/Standard User’s Manual, Version  
6.12 (2012).
- 670 [43] R. Krueger, Virtual crack closure technique: History, approach, and appli-  
cations, Applied Mechanics Reviews 57 (2) (2004) 109. doi:10.1115/1.  
1595677.  
URL <https://doi.org/10.1115/1.1595677>
- [44] J. R. Rice, A path independent integral and the approximate analysis of  
675 strain concentration by notches and cracks, Journal of Applied Mechanics  
35 (2) (1968) 379. doi:10.1115/1.3601206.  
URL <https://doi.org/10.1115/1.3601206>
- [45] F. París, E. Correa, V. Mantič, Kinking of transversal interface cracks  
between fiber and matrix, Journal of Applied Mechanics 74 (4) (2007) 703.  
680 doi:10.1115/1.2711220.  
URL <https://doi.org/10.1115/1.2711220>