

On the meaning of the normalisation parameter G_0 and the normalised Energy Release Rates in the fiber/matrix interface problem

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Abstract.

List of acronyms

BEM	Boundary Element Method
ERR	Energy Release Rate (here used as synonym of SERR)
FEM	Finite Element Method
SERR	Strain Energy Release Rate
VCCT	Virtual Crack Closure Technique

List of symbols

1. Introduction

The numerical analysis of the fiber/matrix interface has focused on the determination of mode mixity through the calculation of mode I and mode II energy release rates. In early papers, where the problem was solved in the complex domain by means of the Airy function and conformal transformations, it was shown that the stress field was due to oscillate in a very small region close to the crack tip. Thus, the evaluation of stresses and stress intensity factors represents an arduous obstacle when to the discrete counterpart of the problem.

Due to their definition as rates of energy change, the calculation of ERRs fits better into the frameworks of discrete procedures, whether the Finite Element Method (FEM) or the Boundary Element Method (BEM). Different variations of the same principle has been derived over the years, namely the Crack Closure Technique, the Crack Closure Integral, the Virtual Crack Closure Technique and the Virtual Crack Closure Integral. The idea at the core of these methods is that, given that the crack is propagating in a linear elastic medium, the energy released by the creation of a unit area of crack's surfaces is equal to the work needed to close the new created surfaces back together.

Energy Release Rates have been so far reported in a normalized form, where a reference energy release rate G_0 is used as normalization parameter. In [?], the authors claim that the use of

$$G_0 = \frac{1 + k_m}{8\mu_m} \sigma_0^2 R_f \pi \quad (1)$$

would make the results comparable between different material systems. In equation 1, k_m is the Kolosov constant for the matrix, which is equal to $3 - 4\nu$ for plane strain and $\frac{3-\nu}{1+\nu}$ for plane stress conditions, μ_m is the shear modulus of the matrix, σ_0 is the applied stress at the boundary and R_f is the radius of the inclusion.

In [?], the same normalization parameter is used to analyze the effect of a neighbouring fiber on the Energy Release Rates. A similar use can be found in [?] It seems that the first appearance of G_0 can be retrieved in [?], soon followed by [?] where a parametric study of Toya's analytical results is performed and tabulated. However, in Toya the normalization is performed for the maximum? stress at the crack tip, and its actual formulation is *formula from toya*.

The question thus arises: what is the meaning of G_0 ? And consequently, what is the meaning of the normalized Energy Release Rates? How does the selection of this peculiar normalization value make results comparable across different material systems and ply geometries? In this brief note we will try to answer these questions.

References

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