

FINITE ELEMENTS SOLUTION OF THE FIBER-MATRIX INTERFACE CRACK: EFFECTS OF MESH REFINEMENT AND DOMAIN SIZE

L. Di Stasio^{1,2}, Z. Ayadi¹, J. Varna²

¹EEIGM, Université de Lorraine, Nancy, France

²Division of Materials Science, Luleå University of Technology, Luleå, Sweden

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Education and Culture

Erasmus Mundus

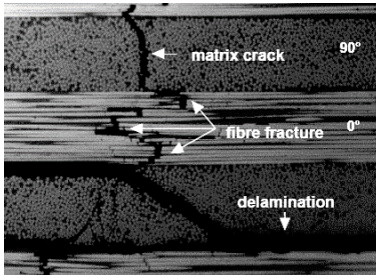


Outline

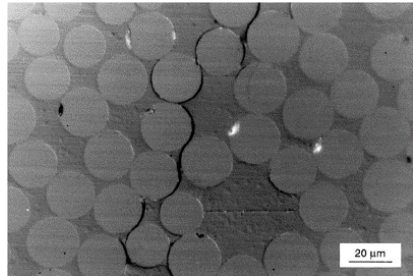
- Characterization of Fracture in FRPC Laminates
- The Fiber-Matrix Interface Problem in Fiber Reinforced Polymer Laminates
- Effects of Mesh Refinement & Domain Size
- Conclusions & Outlook

CHARACTERIZATION OF FRACTURE IN FRPC

Damage Onset and Propagation in FRPC Laminates



(a) By Dr. R. Olsson, Swerea, SE.



(b) By Prof. Dr. E. K. Gamstedt, KTH, SE.

A visual definition of intralaminar transverse cracking.

Characterization of the Fracture Process

→ Energy Release Rate

$$G_m = G_m(p_1, \dots, p_i, \dots, p_n) \quad \text{where} \quad G = \frac{\partial W}{\partial A} - \left(\frac{\partial U}{\partial A} + \frac{\partial E_k}{\partial A} \right)$$

→ Stress Intensity Factor

$$K_m = K_m(p_1, \dots, p_i, \dots, p_n) \quad \text{where} \quad \sigma_m \sim K_m \frac{\alpha}{(x-a)^\beta} \quad \alpha, \beta > 0$$

→ J-Integral

$$J = J(p_1, \dots, p_i, \dots, p_n) \quad \text{where} \quad J = \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} \left(W(\Gamma) n_i - n_j \sigma_{jk} \frac{\partial u_k(\Gamma, x_i)}{\partial x_i} \right) d\Gamma$$

→ Crack Opening & Shear Displacement

$$COD = COD(p_1, \dots, p_i, \dots, p_n) \quad \text{and} \quad CSD = CSD(p_1, \dots, p_i, \dots, p_n)$$

$$p_i \in \{\text{geometry, materials, boundary conditions, loading mode, scale}\}$$

$$m \in \{I, II, III, I/II, I/III, II/III\}$$

Evaluation of Fracture Parameters

→ Analytical

- ✓ Closed form
- ✓ Every material scale can be studied
- ✗ Available only for particular configurations

→ Experimental

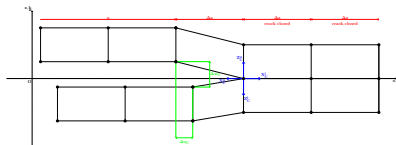
- ✓ Complex geometries can be studied
- ✗ Not every material scale is accessible

→ Numerical

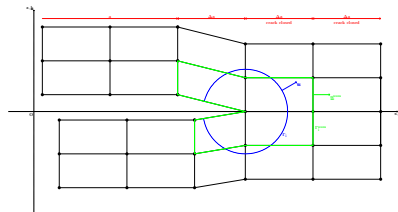
- ✓ Complex geometries can be studied
- ✓ Every material can be studied
- ✗ Discretization
- ✗ Finite domains

Numerical Estimation of Energy Release Rates

→ Virtual Crack Closure Technique (VCCT) → J-Integral



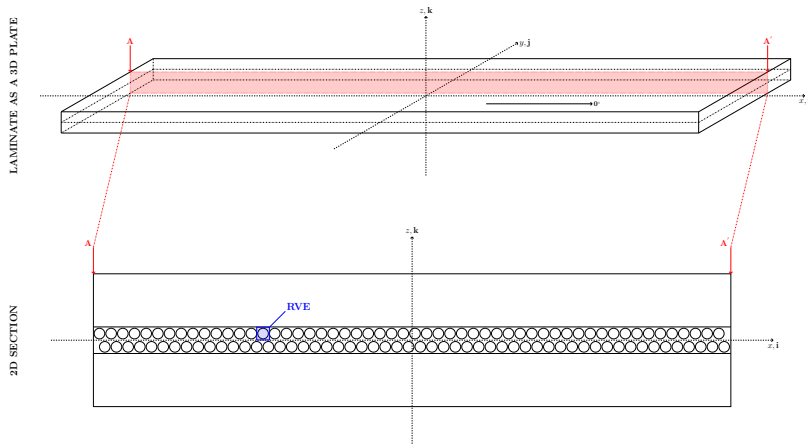
$$G_I = \frac{Z_C \Delta w_C}{2B \Delta a} \quad G_{II} = \frac{X_C \Delta u_C}{2B \Delta a}$$



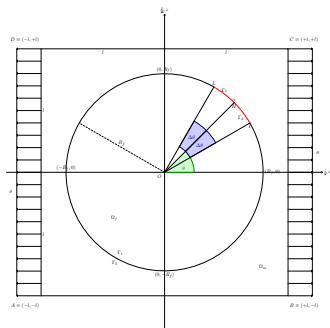
$$J_i = \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} \left(W(\Gamma) n_i - n_j \sigma_{jk} \frac{\partial u_k(\Gamma, x_i)}{\partial x_i} \right) d\Gamma$$

THE FIBER-MATRIX INTERFACE PROBLEM IN FRPC

From macro to micro

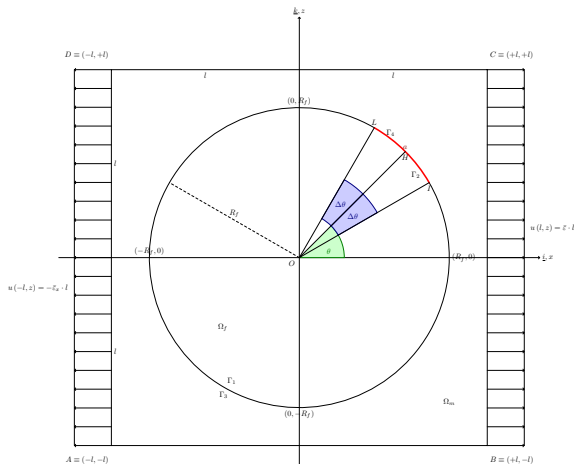


The Fiber-Matrix Interface Crack Problem



	Analytical	Numerical
<i>Method</i>	Analytical (complex) functions	FEM
<i>Domain Type</i>	Continuous	Discrete
<i>Domain Size</i>	Infinite	Finite
<i>Natural variable</i>	Stress (stress function)	Displacement field
<i>Conjugate variable</i>	Displacement	Stress
<i>Dirichlet BC</i>	Stress	Displacement
<i>Loading process</i>	Force-controlled	Displacement- controlled

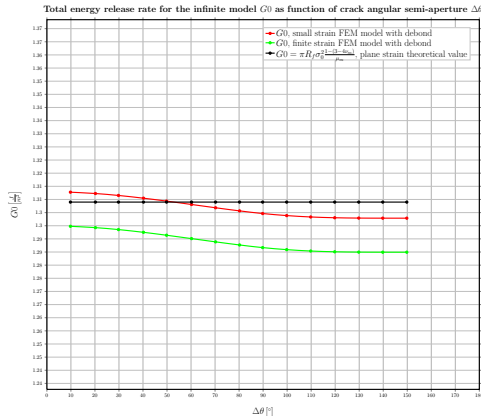
FEM Model of the Fiber-Matrix Interface Crack



- 2D space
- Linear elastic materials
- Displacement-controlled
- Dirichlet-type BC
- LEFM
- Contact interaction
- Bi-linear quadrilateral elements

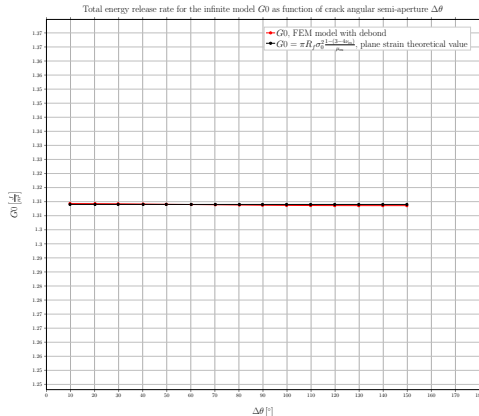
➤ MESH & DOMAIN SIZE

G_0 for $Vf_f = 0.001$, $\frac{L}{R_f} \sim 28$, $\delta = 0.4^\circ$



In red small strain FEM, in green finite strain FEM, in black G_0 calculated assuming $\sigma_0 = \frac{E}{1-\nu^2} \varepsilon$.

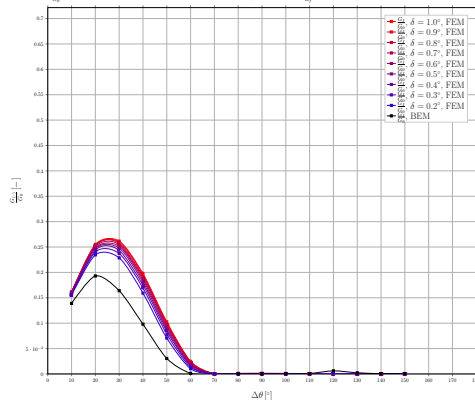
G_0 for $Vf_f = 0.000079$, $\frac{L}{R_f} \sim 100$, $\delta = 0.4^\circ$



In red small strain FEM, in green finite strain FEM, in black G_0 calculated assuming $\sigma_0 = \frac{E}{1-\nu^2} \varepsilon$.

$$G_I, \text{ VCCT}, Vf_f = 0.000079, \frac{L}{R_f} \sim 100$$

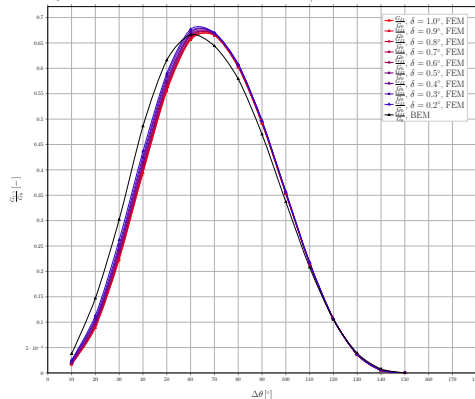
Normalized energy release rate $\frac{G_0}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, $Vf_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing routine



Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

$$G_{II}, \text{VCCT}, V_f = 0.000079, \frac{L}{R_f} \sim 100$$

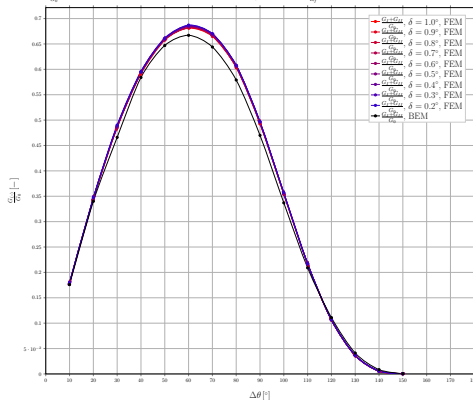
Normalized energy release rate $\frac{G_{II}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, $V_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing routine



Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

$$G_{TOT}, VCCT, Vf_f = 0.000079, \frac{L}{R_f} \sim 100$$

Normalized energy release rate $\frac{G_{TOT}}{G_0}$ as function of crack angular semi-aperture $\Delta\theta$, $Vf_f = 7.9 \cdot 10^{-5}$, $\frac{L}{R_f} \sim 100$ calculated with in-house force-based VCCT post-processing routine



Fading from red to blue for decreasing size of elements at the interface, VCCT from FEM results; in black BEM results.

➤ CONCLUSIONS

Conclusions & Outlook

Conclusions

- Domain size is a fundamental parameter in determining the RVE behaviour between finite and effectively infinite size
- Mesh refinement affects directly mode ratio, increasing mode I with respect to mode II

Outlook

- Analyze the dependence on δ for $\frac{L}{R_f} = 200, 300, \dots$
- Analyze the dependence on $\frac{L}{R_f}$ for constant δ
- Study finite size effects

