

# Convergence properties of the ERR in the FEM solution to the fiber/matrix interface crack problem evaluated with the VCCT

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## Abstract

*Priority: 3*

*Target journal(s): Engineering Fracture Mechanics, Theoretical and Applied Fracture Mechanics, International Journal of Fracture*

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## 1. Normalization function

$$G_0 = G_0(\varepsilon_0, V_f, E_{1f}, E_{2f}, E_m, \nu_{12f}, \nu_{23f}, \nu_m, G_{12f}, G_{23f}) \quad (1)$$

Given the elastic properties of the transversely isotropic UD ply  $E_1, E_2, \nu_{12}, \nu_{23}$ , for a  $90^\circ$  ply under transverse tension the cross section along the direction of the load coincides with the plane of transversal isotropy. It is thus possible, for  
5 a system in plane strain, to define equivalent isotropic Young's modulus and Poisson's ratio as follows. The effective Young's modulus and Poisson's ratio in plane strain in the plane of isotropy are defined as

$$E^* = \frac{E_2}{1 - \nu_{21}\nu_{12}} \quad \nu^* = \frac{\nu_{23} + \nu_{21}\nu_{12}}{1 + \nu_{23}} \quad (2)$$

$$G_0 = \frac{\sigma_0^2}{E^*} \pi R_f \quad \text{for a stress or force controlled test} \quad (3)$$

$$G_0 = E^* \varepsilon_0^2 \pi R_f \quad \text{for a strain or displacement controlled test}$$

## 2. Boundary conditions

The ratio of maximum radial and tangential crack displacements with respect  
 10 to the free case (single repeating element or single fiber layer ply?) can be  
 considered as proxies for the effect of boundary conditions

$$\frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \quad (4)$$

## 3. Initiation of fiber-matrix debonds

Following Asp,

$$U_{\nu,m} = \frac{1-2\nu}{6E} (\sigma_{1,m} + \sigma_{2,m} + \sigma_{3,m}) \quad (5)$$

$$U_{\nu,m} \geq U_{\nu,m}^{cr} \quad (6)$$

$$\theta_0 = \max_{\theta} U_{\nu,m}, \quad U_{\nu,m} \geq U_{\nu,m}^{cr} \quad (7)$$

→ Measurable with hybrid laminate  $[[90_2^{\circ}, 0^{\circ}]_S, epoxy, [90_2^{\circ}, 0^{\circ}]_S]$  as in Paper

15 III Asp's thesis; from which we can derive  $p(U_{\nu,m}^{cr})$

## 4. Propagation of fiber-matrix debonds

$$\frac{G_I}{G_0} = \begin{cases} A_{\delta} (V_f) \log(\delta) + A_{\Delta\theta} \left( V_f, \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) \sin(B_{\Delta\theta} \Delta\theta + C_{\Delta\theta}) + D \\ B_{\Delta\theta} \Delta\theta_{max} \left( V_f, \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) + C_{\Delta\theta} = \frac{\pi}{2} \\ B_{\Delta\theta} \Delta\theta_{CZ} \left( \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) + C_{\Delta\theta} = \pi \\ for \ \Delta\theta < \Delta\theta_{CZ} \left( \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) \\ 0 & otherwise \end{cases} \quad (8)$$

$$\frac{G_{II}}{G_0} = \begin{cases} E_\delta (V_f) \log (\delta) & +F_{\Delta\theta} \left( V_f, \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) \sin (G_{\Delta\theta} \Delta\theta + H_{\Delta\theta}) + \\ & +I_{\Delta\theta} \left( V_f, \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) \sin (2G_{\Delta\theta} \Delta\theta + H_{\Delta\theta}) + L \\ & G_{\Delta\theta} \Delta\theta_{max} \left( V_f, \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) + H_{\Delta\theta} = \frac{\pi}{2} \\ & G_{\Delta\theta} \Delta\theta_{CZ} \left( \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) + H_{\Delta\theta} = \pi \\ & \text{for } \Delta\theta < \Delta\theta_{CZ} \left( \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) \\ F_{\Delta\theta} \left( V_f, \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) & \sin (G_{\Delta\theta} \Delta\theta + H_{\Delta\theta}) + \\ & +I_{\Delta\theta} \left( V_f, \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) \sin (2G_{\Delta\theta} \Delta\theta + H_{\Delta\theta}) + L \\ & G_{\Delta\theta} \Delta\theta_{max} \left( V_f, \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) + H_{\Delta\theta} = \frac{\pi}{2} \\ & G_{\Delta\theta} \Delta\theta_{CZ} \left( \frac{u_{r,max}^{BC}}{u_{r,max}^{free}}, \frac{u_{\theta,max}^{BC}}{u_{\theta,max}^{free}} \right) + H_{\Delta\theta} = \pi \\ & \text{otherwise} \end{cases} \quad (9)$$

$$\frac{\Delta\Phi}{\Delta\theta} = \begin{cases} A_{\Delta\Phi} (\Delta\theta - \Delta\theta_{CZ}) & \text{for } \Delta\theta \geq \Delta\theta_{CZ} \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

$$\Delta\Phi = \begin{cases} A_{\Delta\Phi} (\Delta\theta - \Delta\theta_{CZ})^2 & \text{for } \Delta\theta \geq \Delta\theta_{CZ} \\ 0 & \text{otherwise} \end{cases} \quad (11)$$

## 5. Fracture toughness

$$G_c = G_{Ic} (1 + \tan^2 ((1 - \lambda) \psi)) \quad \psi = \tan^{-1} \left( \sqrt{\frac{G_{II}}{G_I}} \right) \quad (12)$$

*Hypothesis*

$$p(\Delta\theta) = p(\Delta\theta|\varepsilon) \sim \frac{1}{\sqrt{2\pi}\sigma_{\Delta\theta}(\varepsilon)} e^{\left( \frac{\Delta\theta - \overline{\Delta\theta}}{\sigma_{\Delta\theta}}(\varepsilon) \right)} \quad (13)$$

→ Verified by measuring debond's size at different strain levels (see preliminary experimental work)

$$\begin{cases} G_{TOT}(\Delta\theta) = G_{Ic} (1 + \tan^2((1-\lambda)\psi)) \\ \psi = \tan^{-1} \left( \sqrt{\frac{G_{II}(\Delta\theta)}{G_I(\Delta\theta)}} \right) \end{cases} \quad \forall \Delta\theta : p(\Delta\theta) \neq 0 \quad (14)$$

$$p(G_c|\psi) = p(G_{Ic}|\psi) p(\lambda|\psi) \quad (15)$$

## 6. Transition to collective mesoscopic behavior

$$\begin{cases} \frac{G_{TOT}}{G_0}|_{\text{debond}} > \frac{G_{TOT}}{G_0}|_{\text{transverse crack}} \\ \rightarrow \text{Propagation of debonds at fiber/matrix interface level will occur, discrete events, "debonds' regime"} \\ \frac{G_{TOT}}{G_0}|_{\text{debond}} < \frac{G_{TOT}}{G_0}|_{\text{transverse crack}} \\ \rightarrow \text{Propagation of transverse cracks will occur, collective behavior of debonds} \\ \text{inter-fiber matrix cracks propagating and coalescing, "transverse cracks' regime"} \end{cases} \quad (16)$$

## 7. Global propagation function

Hypothesis

$$\frac{G_{TOT}}{G_0} \left( a, \frac{t_{0^\circ}}{t_{90^\circ}} \right) = -A \cdot \left( \frac{t_{0^\circ}}{t_{90^\circ}} - \frac{t_{0^\circ}}{t_{90^\circ}}|_{ref} \right)^{2n+1} \sqrt{a} + \frac{G_{TOT}}{G_0}|_0 \quad (17)$$