INVESTIGATION OF SCALING LAWS OF THE FIBER/MATRIX INTERFACE CRACK IN POLYMER COMPOSITES THROUGH FINITE ELEMENT-BASED MICROMECHANICAL MODELING

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Education and Culture

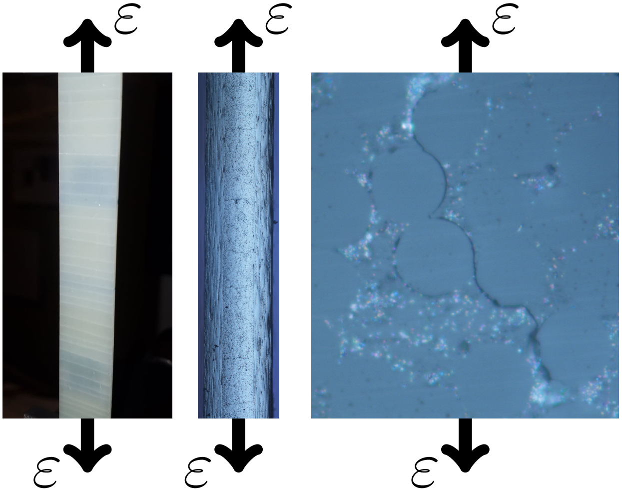
Erasmus Mundus

Outline

- Initiation of Transverse Cracking in Fiber Reinforced Polymer Composites (FRPCs): Microscopic Observations & Modeling
- The Fiber-Matrix Interface Crack Problem
- Investigation of Scaling Laws of the Fiber/Matrix Interface Crack

TRANSVERSE CRACKING IN FRPCs

Observations: From Macro to Micro



Left:
front view of $[0, 90_2]_S$, visual inspection.

Center:
edge view of $[0, 90]_S$, optical microscope.

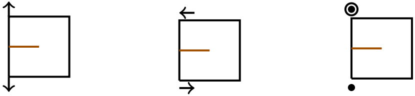
Right:
edge view of $[0, 90]_S$, optical microscope.

Mathematical Modeling of Fracture: Linear Elastic Fracture Mechanics (LEFM)

Fracture Mode

$I, II, III, I/II, I/III, II/III$

Mode I Mode II Mode III



➔ Energy Release Rate: $G\ \left[\frac{J}{m^2}=\frac{N}{m}\right]$

$$G=\frac{\partial W}{\partial A}-\left(\frac{\partial U}{\partial A}+\frac{\partial E_k}{\partial A}\right)$$

➔ Stress Intensity Factor: $K\ \left[Pa\sqrt{m}\right]$

$$K_{I/II/III}=\lim_{r\to 0}\sqrt{2\pi r}\cdot\sigma_{I/II/III}(r,0)$$

Variables

geometry
materials
boundary conditions
loading mode
scale

➔ J-Integral: $J\ \left[\frac{J}{m^2}=\frac{N}{m}\right]$

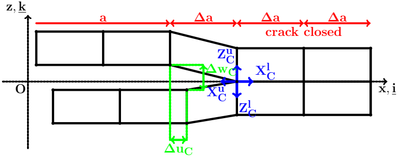
$$J=\lim_{\delta\to 0}\int_{\Gamma_\delta}\left(W-n_j\sigma_{jk}\frac{\partial u_k}{\partial x_i}\right)d\Gamma$$

➔ Average Crack Opening & Shear Displacement: COD, $CSD_{II/III}$ $[m]$

$$\begin{Bmatrix}COD\\CSD_{II}\\CSD_{III}\end{Bmatrix}=\frac{1}{S_C}\int_{S_C}\overrightarrow{\Delta u_C}\cdot\begin{Bmatrix}\overrightarrow{n_I}\\\overrightarrow{n_{II}}\\\overrightarrow{n_{III}}\end{Bmatrix}dS$$

Numerical Characterization of Fracture: VCCT & J-Integral

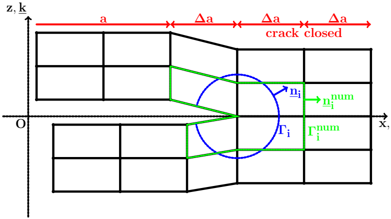
Virtual Crack Closure Technique (VCCT)



$$G_I = \frac{Z_C \Delta w_C}{2B\Delta a} \quad G_{II} = \frac{X_C \Delta u_C}{2B\Delta a}$$

Krueger R.; *Virtual crack closure technique: History, approach, and applications. Appl. Mech. Rev.* **57** (2) 109–143, 2004.

J-Integral

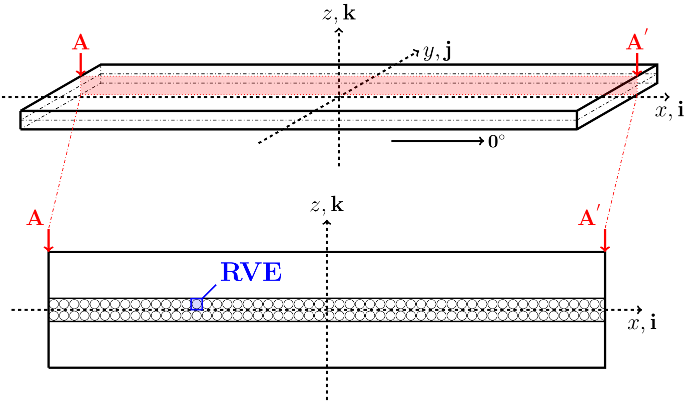


$$J_i = \sum_{k=1}^{n_{segments}} \sum_{j=1}^{n_{nodes}} \left[w_j \left(W - n_j \sigma_{jk} \frac{\partial u_k}{\partial x_i} \right) \Bigg|_{(x_{kj}, y_{kj})} \right]$$

Rice J. R.; *A Path Independent Integral and the Approximate Analysis of Strain Concentration by Notches and Cracks. J. Appl. Mech.* **35** (2) 379–386, 1968.

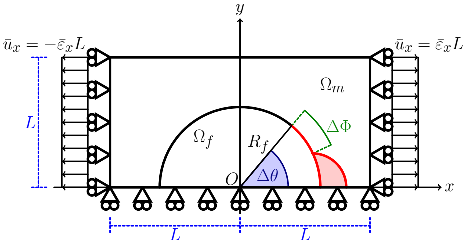
The Fiber-Matrix Interface Crack Problem

The Fiber-Matrix Interface Crack Problem: Geometry



- ➔ $L, W >> t$
- ➔ $L, W \rightarrow \infty$
- ➔ 2D RVE

The Fiber-Matrix Interface Crack Problem: Assumptions

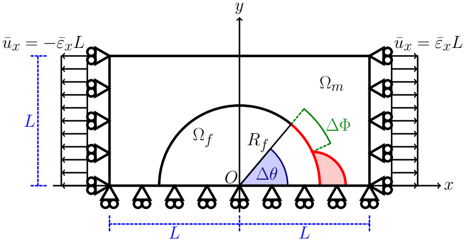


$$R_f = 1 \ [\mu m] \qquad L = \frac{R_f}{2}\sqrt{\frac{\pi}{V_f}}$$

Material	E_1	ν_{12}
glass fiber	70.0	0.2
epoxy	3.5	0.4

- ➔ Linear elastic, homogeneous and isotropic materials
- ➔ Plane strain
- ➔ Frictionless contact interaction
- ➔ Symmetric w.r.t. x-axis
- ➔ Coupling of x-displacements on left and right side (repeating unit cell)
- ➔ Applied uniaxial tensile strain $\bar{\varepsilon}_x = 1\%$

The Fiber-Matrix Interface Crack Problem: Solution



in Ω_f, Ω_m :

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\varepsilon_z = \gamma_{zx} = \gamma_{yz} = 0$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0$$

$$\sigma_{zz} = \nu \left(\sigma_{xx} + \sigma_{yy}\right)$$

for $0^\circ \leq \alpha \leq \Delta\theta$:

$$\left(\vec{u}_m\left(R_f, \alpha\right) - \vec{u}_f\left(R_f, \alpha\right)\right) \cdot \vec{n}_\alpha \geq 0$$

for $\Delta\theta \leq \alpha \leq 180^\circ$:

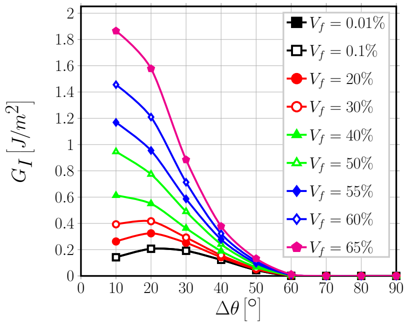
$$\vec{u}_m\left(R_f, \alpha\right) - \vec{u}_f\left(R_f, \alpha\right) = 0$$

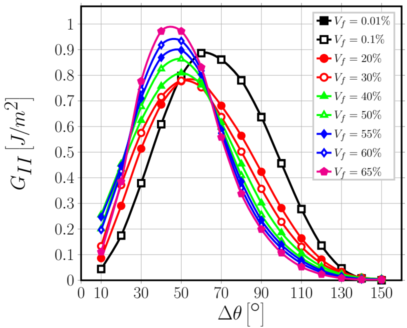
$$\sigma_{ij} = E_{ijkl}\varepsilon_{kl}$$

$$+ BC$$

- ➔ Finite Element Method (FEM) in Abaqus™
- ➔ 2^{nd} order shape functions
- ➔ 6-nodes triangles & 8-nodes quadrilaterals
- ➔ regular mesh of quadrilaterals at the crack tip:
  - $AR \sim 1$
  - $\delta = 0.05^\circ$

The Fiber-Matrix Interface Crack Problem: Normalization & Scaling





(?) $\quad G_I = G_0\left(V_f, E_f, \nu_f, E_m, \nu_m, \bar{\varepsilon}_x, \Delta\theta\right) g_I\left(\Delta\theta, BC, microstructure, damage\right)$

(?) $\quad G_{II} = G_0\left(V_f, E_f, \nu_f, E_m, \nu_m, \bar{\varepsilon}_x, \Delta\theta\right) g_{II}\left(\Delta\theta, BC, microstructure, damage\right)$

INVESTIGATION OF SCALING LAWS

Dimensional Analysis

➔ From the definition of Energy Release Rate

$$G = \frac{\partial W}{\partial A} - \left(\frac{\partial U}{\partial A} + \frac{\partial E_k}{\partial A} \right) \quad \left[\frac{J}{m^2} \right]$$

$$\left[\frac{J}{m^2} \right] \longleftrightarrow \frac{E}{L^2} = \frac{F \cdot L}{L^2} = \frac{F}{L^2} \frac{L}{L} \cdot L = \sigma \varepsilon L$$

$$G_0 \sim \sigma_\infty \varepsilon_\infty L_c$$

➔ From the assumption of linear elasticity and uniaxial loading

$$\sigma_\infty = E_{eq} \varepsilon_\infty \qquad \varepsilon_\infty = \frac{\sigma_\infty}{E_{eq}}$$

$$G_0 \sim E_{eq} \varepsilon_\infty^2 L_c \qquad G_0 \sim \frac{\sigma_\infty^2}{E_{eq}} L_c$$

➔ From crack geometry

$$L_c \sim a = R_f \Delta\theta \longrightarrow L_c \sim R_f f(\Delta\theta)$$

$$G_0 = A \cdot E_{eq} \varepsilon_\infty^2 R_f f(\Delta\theta)$$

Homogenization of Material Properties: Concentric Cylinders Assembly (CCA)

$$E_L = V_f E_f + (1 - V_f) E_m + 2\lambda_1 (\nu_m - \nu_f)^2 V_f (1 - V_f)$$

$$\nu_{LT} = V_f \nu_f + (1 - V_f) \nu_m + \frac{\lambda_1}{2} (\nu_m - \nu_f) \left(\frac{1}{k_{fT}} - \frac{1}{k_{mT}} \right) V_f (1 - V_f)$$

$$G_{TT} = \frac{E_m}{2(1 + \nu_m)} + \frac{V_f}{\frac{1}{\frac{E_f}{2(1+\nu_f)} - \frac{E_m}{2(1+\nu_m)}} + \frac{k_{mT} + \frac{E_m}{1+\nu_m}}{\frac{E_m}{1+\nu_m}\left(k_{mT} + \frac{E_m}{2(1+\nu_m)}\right)}(1 - V_f)}$$

$$K_{TT} = \frac{k_{mT}\left(k_{fT} + \frac{E_m}{2(1+\nu_m)}\right)(1 - V_f) + k_{fT}\left(k_{mT} + \frac{E_m}{2(1+\nu_m)}\right)V_f}{\left(k_{fT} + \frac{E_m}{2(1+\nu_m)}\right)(1 - V_f) + \left(k_{mT} + \frac{E_m}{2(1+\nu_m)}\right)V_f}$$

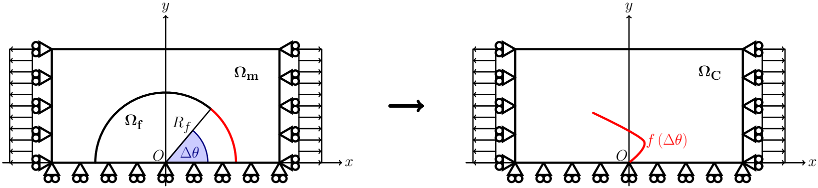
$$E_T = \frac{4G_{TT}}{1 + \frac{\left(1 + \frac{4K_{TT}\nu_{LT}^2}{E_L}\right)G_{TT}}{K_{23}}} \qquad \nu_{TT} = \frac{E_T}{2G_{TT}} - 1$$

$$k_{fT} = \frac{E_f}{2\left(1 - \nu_f - 2\nu_f^2\right)} \quad k_{mT} = \frac{E_m}{2\left(1 - \nu_m - 2\nu_m^2\right)} \quad \lambda_1 = 2\left(\frac{2(1 + \nu_m)}{E_m} + \frac{V_f}{k_{mT}} + \frac{1 - V_f}{k_{fT}}\right)^{-1}$$

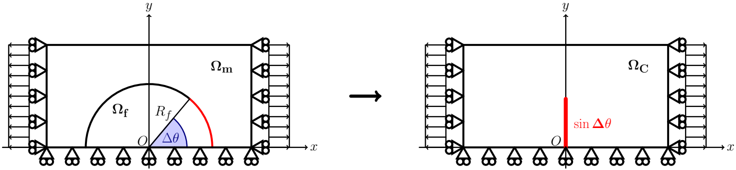
Homogenization of Material Properties: Plane Strain Conditions

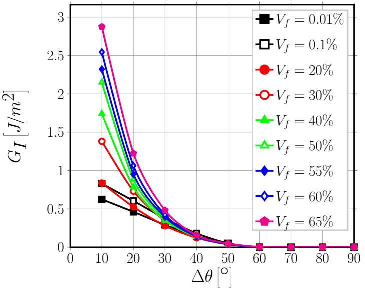
$$E_{\text{plane strain}} = \frac{E_T}{1 - \frac{E_T}{E_L}\nu_{LT}^2}$$

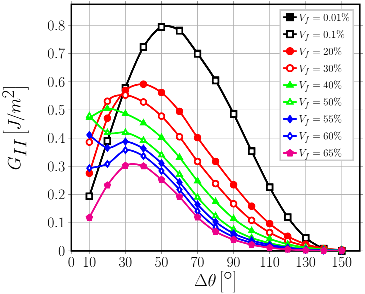
$$E_{eq} = \frac{E_T}{1 - \frac{E_T}{E_L}\nu_{LT}^2}$$



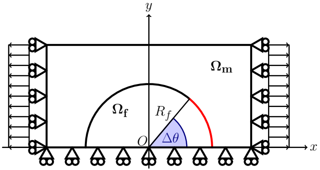
Shape Function Reference Configurations: Straight Crack

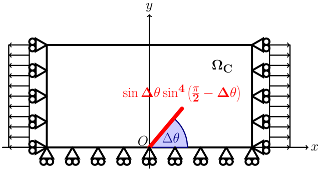


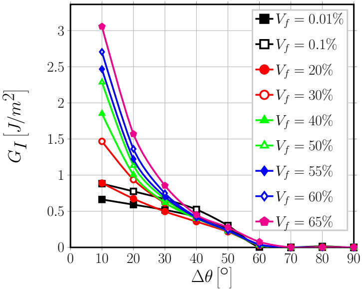


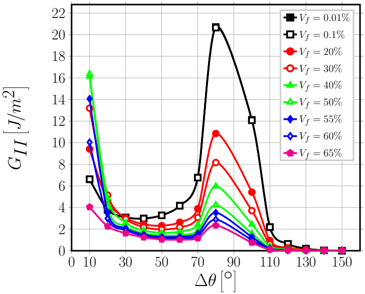


Shape Function Reference Configurations: Inclined Crack

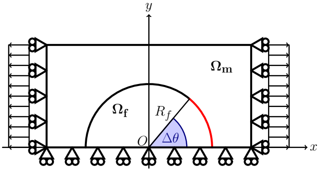


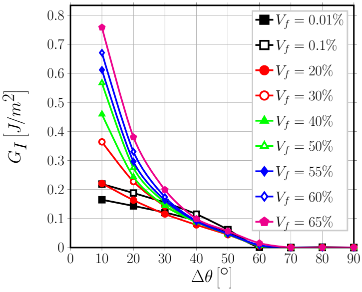


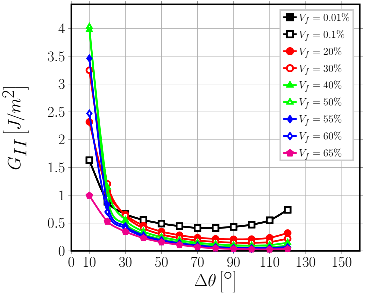




Shape Function Reference Configurations: Circular Crack







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