In-class Exercise 3.3-3.4

Implementation of Differentiation & Integration, Differentiation of Noisy Data

AMATH 301

University of Washington

Jakob Kotas

1. Returning to the problem from ICE 3.2 #1, we want to approximate $\int_0^4 \sqrt{t}dt$.
   (a) In addition to the previously-seen Midpoint, Trapezoid, and Simpson's Rules, use the built-in `np.trapz` with $N = 1000$ subintervals.
   (b) Approximate using the built-in `scipy.integrate.quad` (no need to specify the subintervalS).
   (c) Try increasing N to achieve the same error as `scipy.integrate.quad`. How large does N have to be to achieve a similar (absolute) error as compared to our Simpson's rule code?

$$\text{absolute error } = \left| \int_{\text{approx}} - \int_{\text{true}} \right|$$

2. Find the volume beneath the surface

$$z = \sin(x) + \sin(y) + 2,$$

above the $z = 0$ plane, and between $-4\pi \le x \le 4\pi$, $-2\pi \le y \le 2\pi$, approximately using `scipy.integrate.dblquad`. Compare to the true value calculated by hand, which is a double integral:

$$\int_{-2\pi}^{2\pi} \int_{-4\pi}^{4\pi} \Big(\sin(x) + \sin(y) + 2 \Big) dxdy = 64\pi^2$$

(Visualization of #2)

3. Consider the data set:

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	4.5	2.5	1	-1	-1	-4	1.5	2.5	3.5

   (a) Find $f'(x)$ at the same x values. Use the fourth-order centered finite difference formula:

$$\frac{df}{dt} = \frac{-f(t+2\Delta t) + 8f(t+\Delta t) - 8f(t-\Delta t) + f(t-2\Delta t)}{12\Delta t} + O((\Delta t)^4)$$

   wherever possible. Where not possible, use the second-order forward/backward finite difference formulas:

$$\frac{df}{dt} = \frac{-3f(t) + 4f(t+\Delta t) - f(t+2\Delta t)}{2\Delta t} + O((\Delta t)^2)$$

$$\frac{df}{dt} = \frac{3f(t) - 4f(t-\Delta t) + f(t-2\Delta t)}{2\Delta t} + O((\Delta t)^2)$$

   (b) Repeat the above to find $f''(x)$ and $f'''(x)$.

4. Smoothing and sharpening.
   (a) Consider the only very slightly "noisy" function:

$$h(x) = x + 0.01\sin(10x)$$

   Compute by hand $h'(x)$, $h''(x)$, and $h'''(x)$ and plot on $-10 \le x \le 10$ in steps of 0.001.

(b) Consider the already quite "noisy" function:

$$j(x) = x + 10\sin(10x)$$

Compute by hand $k(x) = \int_0^x j(y)dy$ and $l(x) = \int_0^x k(y)dy$ and plot on $-10 \leq x \leq 10$ in steps of 0.001.

5. Estimate the total mass M of the cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$ whose density varies with space as $\rho(x, y, z) = xyz$. This is the triple integral:

$$M = \int_0^1 \int_0^1 \int_0^1 xyz\, dxdydz.$$

Use `scipy.integrate.tplquad`. Compare to the true value calculated by hand of 1/8.