In-class Exercise 2.2
Iterative Solution Methods for Ax=b
AMATH 301
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1. Solve the systems of equations from ICE 2.1 using Jacobi iteration. Stop when the 2-norm of the step size from one iteration to the next, $||x_{j+1} - x_j||_2 < 10^{-5}$. Remember that for $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$,

$$||x||_2 = \sqrt{x_1^2 + x_2^2 + x_3^2}.$$

   (a) This system was solvable with Naive Gaussian elimination.

$$\underbrace{\begin{bmatrix} 5 & 1 & 1 \\ 0 & 3 & 4 \\ 4 & 0 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -14 \\ 25 \\ 5 \end{bmatrix}}_{b}$$

   (b) This system was not solvable with Naive Gaussian elimination, but it was solvable with Full Gaussian elimination with partial pivoting.

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 1 & 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -8 \\ -19 \\ 0 \end{bmatrix}}_{b}$$

   (c) This system was not solvable with Full Gaussian elimination because the A matrix is singular.

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 6 \\ 13 \\ 19 \end{bmatrix}}_{b}$$

2. Solve the same system from #1a using Gauss-Seidel iteration. Again stop when the 2-norm of the step size from one iteration to the next, $||x_{j+1} - x_j||_2 < 10^{-5}$.