In-class Exercise 2.6
Nonlinear systems of equations
AMATH 301
University of Washington
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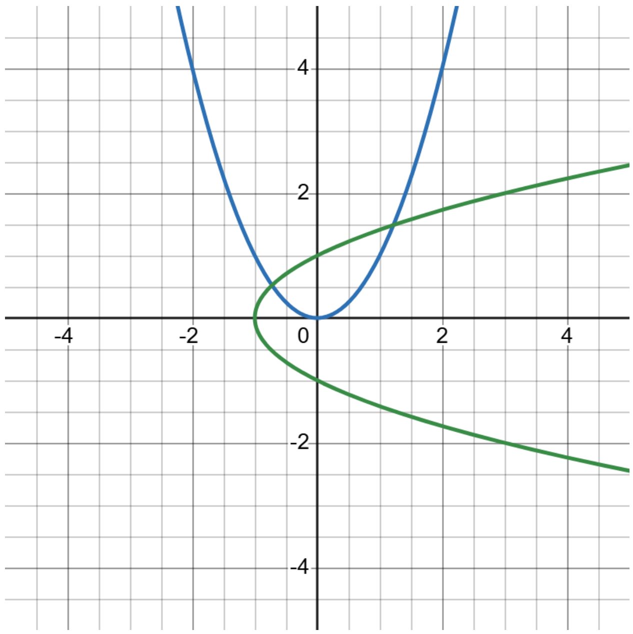
1. We wish to find the points of intersection of the two parabolas:

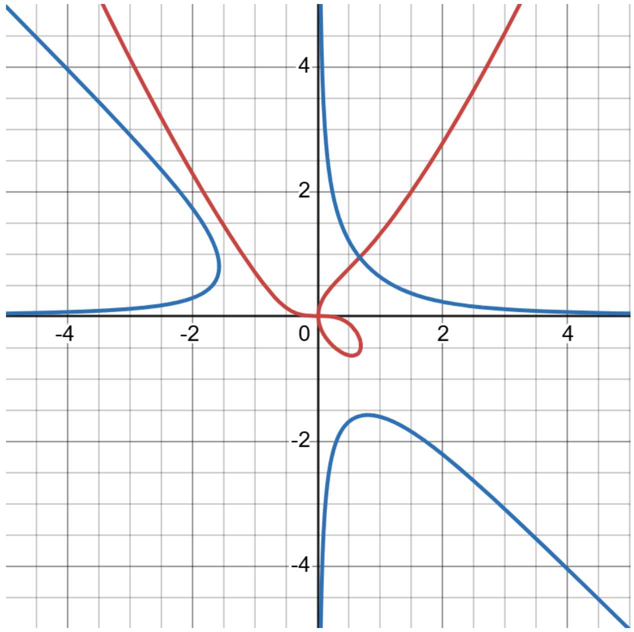
$$\begin{cases} y &= x^2 \\ x &= y^2 - 1 \end{cases}.$$



Rewrite the nonlinear system of equations as a single nonlinear equation. Use the Newton-Raphson method in 1D. Start at different starting points to find both intersections. Stop when either the step size from one iteration to the next has a 2-norm of less than 10^{-6}, or after 100 iterations, whichever comes first.

2. We wish to find the point of intersection of the two implicitly-defined functions:

$$\begin{cases} xy + x^4 &= y^3 \\ x^2y + y^2x &= 1 \end{cases}.$$



(a) Use the Newton-Raphson method. Start at $[\ x \quad y\ ]^T = [\ 1 \quad 0\ ]^T$. Use `np.linalg.solve` to solve the linear system of equations. Stop when either the step size from one iteration to the next has a 2-norm of less than 10^{-6}, or after 100 iterations, whichever comes first.

(b) Use the built in Python command `fsolve`. You will need to import the library:

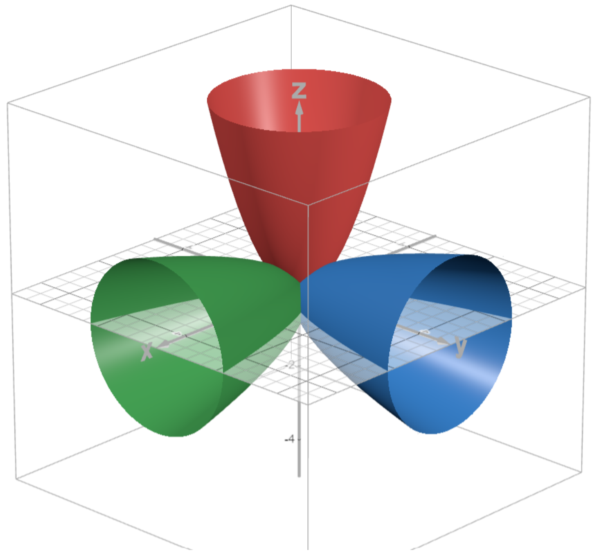
```
from scipy.optimize import fsolve
```

(c) As with the single-variable Newton-Raphson method that we saw before, convergence only happens if the initial guess is good enough. Try Newton-Raphson again, with different initial guesses, to see the resulting behavior.

3. The three paraboloids:

$$\begin{cases} z &= x^2 + y^2 \\ y &= x^2 + z^2 \\ x &= y^2 + z^2 \end{cases}$$

intersect at $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and one other point.



Use the Newton-Raphson method to find the non-origin intersection point. Start at $\begin{bmatrix} x & y & z \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$. Use `np.linalg.solve` to solve the linear system of equations. Stop when either the step size from one iteration to the next has a 2-norm of less than 10^{-6}, or after 100 iterations, whichever comes first.