

1. IVP (theoreticals)

- (a) Some sample IVPs, e.g. $y' = \lambda y$, Hamiltonian systems, $y' = \frac{y}{1-y^2}$, $y(0) = 0$
- (b) Lipschitz Condition (analyze some examples)
- (c) Well-posedness and its importance
- (d) Theorem: Continuity + Lipschitz condition \Rightarrow well-posedness

2. Euler's Method

- (a) Sample problem $y' = \lambda y$
- (b) Graphical interpretations
- (c) Local truncation error
- (d) $y(t_{n+1}) = y(t_n) + \int_{t_n}^{t_{n+1}} f(s, y(s)) ds$. General methods can be derived by approximating the integral

3. Taylor's Method

- (a) Sample problem $y' = ye^t$, $y(0) = 1$
- (b) Taylor expansion with 2 variables
- (c) Big O notations

4. RK

- (a) Midpoint Method (& it's local truncation error)
- (b) Consistency formula (& higher order formula)
- (c) $y(t_{n+1}) = y(t_n) + \int_{t_n}^{t_{n+1}} f(s, y(s)) ds$; quadrature rules help approximate the integral
- (d) Butcher tableau
- (e) Stability = continuity + Lipschitz condition

5. Multistep Methods

- (a) Proper formulation of multiple Taylor expansions
- (b) Consistency formula (& higher order formula)
- (c) Newton backward-difference formula

$$\begin{aligned} \int_{t_i}^{t_{i+1}} P_m(t) dt &= \int_{t_i}^{t_{i+1}} \sum_{k=0}^m (-1)^k \binom{-s+1}{k} \nabla^k f(t_{i+1}, y(t_{i+1})) dt \\ &= h \sum_{k=0}^m \nabla^k f(t_{i+1}, y(t_{i+1})) (-1)^k \int_0^1 \binom{-s+1}{k} ds \end{aligned}$$

k	0	1	2	3	4	5	...
$(-1)^k \int_0^1 \binom{-s+1}{k} ds$	1	$-\frac{1}{2}$	$-\frac{1}{12}$	$-\frac{1}{24}$	$-\frac{19}{720}$	$-\frac{3}{160}$...

- (d) Stability = root condition (+ continuity + Lipschitz condition)
- (e) Lax equivalence

6. Stiff Equations, BVPs

- (a) Region of absolute stability
- (b) A-stability
- (c) E & U for solutions to BVPs
- (d) Shooting method (linear & nonlinear)
- (e) Finite difference method

7. Nonlinear System Solvers

- (a) Fix-point method
- (b) Newton's method
- (c) (Broydent's method)
- (d) Steepest descent method
- (e) Homotopy method
- (f) (find textbook problems, derive derivatives, and explain the concepts)

8. Eigen Problems (Power Method, QR-Algorithm)

- (a) Diagonalizability conditions
- (b) Norms are all equivalent
- (c) Shifted inverse power method
- (d) Householder transformation and its graphical interpretations and examples
- (e) Why does Gram-Schmidt suck? $v_1 = (1, \epsilon, 0, 0)$, $v_2 = (1, 0, \epsilon, 0)$, $v_3 = (1, 0, 0, \epsilon)$

9. Fast Fourier Transform

- (a) ???