Exercise 3.18 [Boyd & Vandenberghe, 2004]

Adapt the proof of concavity of the log-determinant function in §3.1.5 to show the following.
(a) $f(\mathbf{X}) = \mathbf{tr}(X^{-1})$ is convex on $\mathbf{dom} f = \mathbf{S}^n_{++}$.
(b) $f(\mathbf{X}) = (\det X)^{1/n}$ is concave on $\mathbf{dom} f = \mathbf{S}^n_{++}$.

Ans: Here only does part (a).

Exercise 3.19 [Boyd & Vandenberghe, 2004]

Nonnegative weighted sums and integrals.
(a) Show that $f(x) = \sum_{i=1}^{r} \alpha_i x_{[i]}$ is a convex function of x, where $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_r \geq 0$, and $x_{[i]}$ denotes the ith largest component of x. (You can use the fact that $f(x) = \sum_{i=1}^{k} x_{[i]}$ os convex on $\mathbb{R}^n$.)
(b) Let $T(x, \omega)$ denote the trigonometric polynomial

$$T(x, \omega) = x_1 + x_2 \cos \omega + x_3 \cos 2\omega + \cdots + x_n \cos(n-1)\omega.$$

Show that the function

$$f(x) = -\int_0^{2x} \log T(x, \omega) d\omega$$

is convex on $\{x \in \mathbb{R}^n \mid T(x, \omega) > 0, 0 \leq \omega \leq 2\pi\}$.

Ans: Here only does part (a).

Exercise 3.22 [Boyd & Vandenberghe, 2004]

Composition rules. Show that the following functions are convex.
(a) $f(x) = -\log(-\log(\sum_{i=1}^m e^{a_i^T x+b_i}))$ on $\mathbf{dom} f = \{x \mid \sum_{i=1}^m e^{a_i^T x+b_i} < 1\}$. You can use the fact that $\log(\sum_{i=1}^n e^{y_i})$ is convex.
(b) $f(x,u,v) = -\sqrt{uv - x^T x}$ on $\mathbf{dom} f = \{(x,u,v) \mid uv > x^T x, u, v > 0\}$. Use the fact that $x^T x/u$ is convex in (x,u) for $u > 0$, and that $-\sqrt{x_1 x_2}$ is convex on $\mathbb{R}^2_{++}$.
(c) $f(x,u,v) = -\log(uv - x^T x)$ on $\mathbf{dom} f = \{(x,u,v) \mid uv > x^T x, u, v > 0\}$.
(d) $f(x,t) = -(t^p - \|x\|_p^p)^{1/p}$ where $p > 1$ and $\mathbf{dom} f = \{(x,t) \mid t \geq \|x\|_p\}$. You can use the fact that $\|x\|_p^p/u^{p-1}$ is convex in (x,u) for $u > 0$ (see exercise 3.23), and that $-x^{1/p}y^{1-1/p}$ is convex on $\mathbb{R}^2_+$ (see exercise 3.16).
(e) $f(x,t) = -\log(t^p - \|x\|_p^p)$ where $p > 1$ and $\mathbf{dom} f = \{(x,t) \mid t > \|x\|_p\}$. You can use the fact that $\|x\|_p^p/u^{p-1}$ is convex in (x,u) for $u > 0$ (see exercise 3.23).

Ans: Here only does part (c).

Additional Exercise 2.5 [Boyd & Vandenberghe, 2017]

hi

Ans: Here only does part (a) and (b).

Additional Exercise 2.30 [Boyd & Vandenberghe, 2017]

hi

Ans:

Additional Exercise 2.31 [Boyd & Vandenberghe, 2017]

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Ans:

Additional Exercise 3.17 [Boyd & Vandenberghe, 2017]

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Ans: