Homework 4

Q 1.1

Given two views, we can use fundamental matrix to capture the relationship between the corresponding points. The equation is as follows:

$$x_l^T F x_r = 0$$

Where F is the fundamental matrix with 9 elements, and the f_{33} is 1 when the origin is the intersection of the light ray through the image planar.

$$\mathrm{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$\begin{bmatrix} x_i \\ x_i \\ 1 \end{bmatrix}^T \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = 0$$

One equation for one point correspondence, and we will get a n by 9 matrix:

$$\begin{bmatrix} x'_1 x_1 & x'_1 y_1 & x'_1 & y'_1 x_1 & y'_1 y_1 & y'_1 & x_1 & y_1 & 1 \\ x'_2 x_2 & x'_2 y_2 & x'_2 & y'_2 x_2 & y'_2 y_2 & y'_2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_n x_n & x'_n y_n & x'_n & y'_n x_n & y'_n y_n & y'_n & x_n & y_n & 1 \end{bmatrix} \cdot f = 0$$

Where f is the vectorized fundamental matrix:

$$f = [f_{11} \quad f_{12} \quad f_{13} \quad f_{21} \quad f_{22} \quad f_{23} \quad f_{31} \quad f_{32} \quad f_{33}]^T$$

For correspondence i, we will get one equation:

$$x'_i x_i f_{11} + x'_i y_i f_{12} + x'_i f_{13} + y'_i x_i f_{21} + y'_i y_i f_{22} + y'_i f_{23} + x_i f_{31} + y_i f_{32} + f_{33} = 0$$

Q 1.2

$\mathrm{F} = K^{-T} E K^{-1} = K^{-T} R T K^{-1}$, we can eliminate the rotation matrix R since there is only pure translation along the x axis, so we can get T as:

$$T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -tx \\ 0 & tx & 0 \end{bmatrix}$$

All the x and x' locate in one line, since y and y' are equal. Therefore, the epipole line both parallel to the x axis.

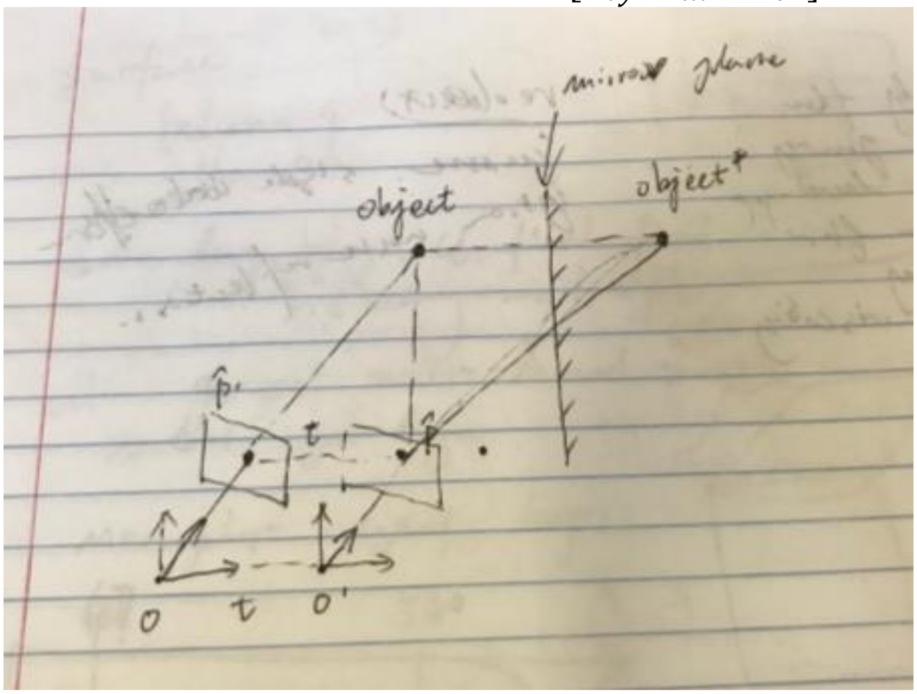
Q 1.3

$$\mathrm{E} = R_{rel} \cdot t_{rel}$$

$$\mathrm{F} = K^{-T} \cdot R_{rel} \cdot t_{rel} \cdot K^{-1}$$

Q 1.4

$$F = K^{-T}EK^{-1} = K^{-T}RTK^{-1} \sim T = \begin{bmatrix} 0 & -tz & ty \\ tz & 0 & -tx \\ -ty & tx & 0 \end{bmatrix}$$



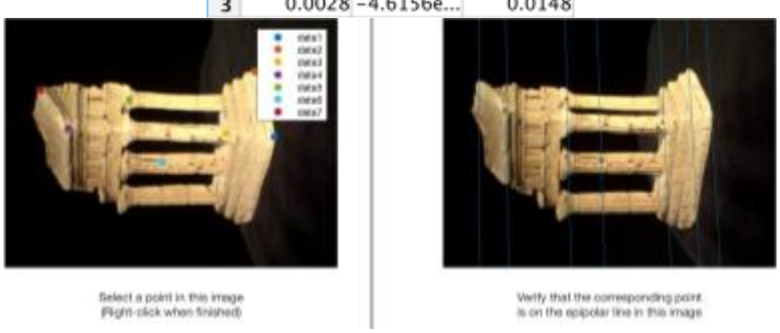
According to the figure, the camera C and C' has a pure translation without rotation, also they have the same intrinsic matrix. Thus, their fundamental matrix can be related to a skew-symmetric matrix.

Q 2.1

F =

0.0000	0.0000	-0.0030
0.0000	-0.0000	-0.0000
0.0028	-0.0000	0.0148

	1	2	3
1	8.5127e-...	1.4383e-...	-0.0030
2	3.5983e-...	-5.1263e...	-1.2604e...
3	0.0028	-4.6156e...	0.0148

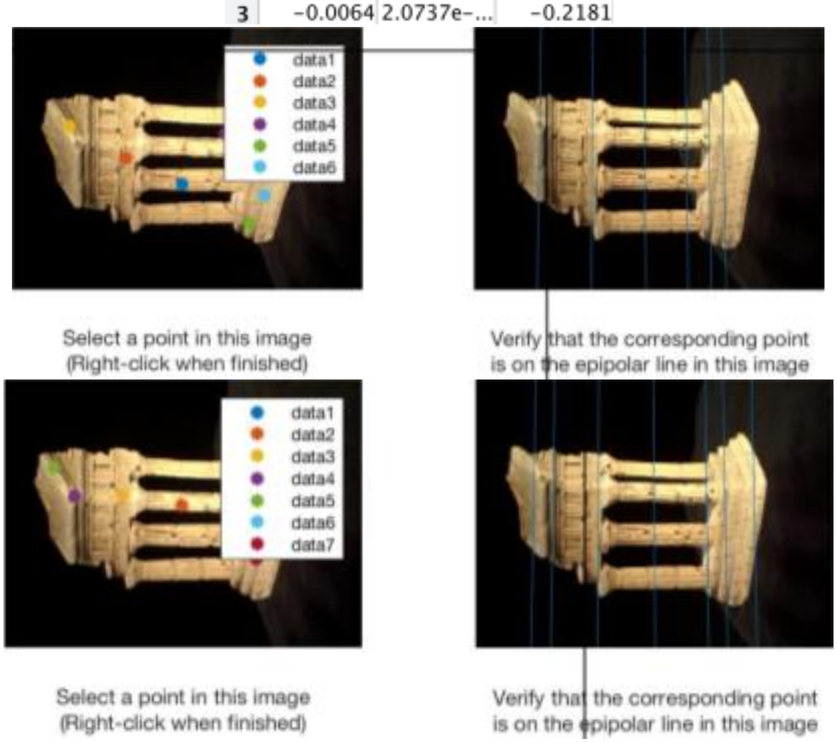


Q 2.2

F =

0.0000	-0.0000	0.0071
-0.0000	0.0000	0.0002
-0.0064	0.0002	-0.2181

	1	2	3
1	2.4291e-...	-4.8019e...	0.0071
2	-6.8335e...	7.7962e-...	1.8656e-...
3	-0.0064	2.0737e-...	-0.2181



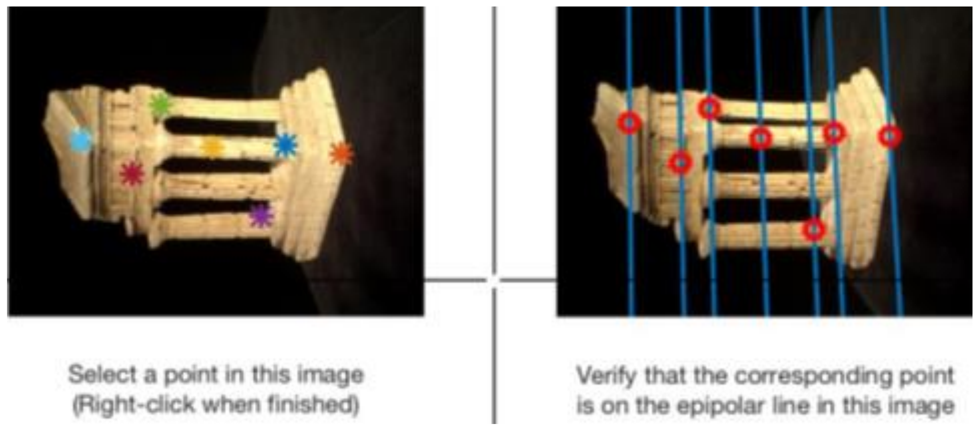
Q 3.2

$$AP = 0$$

$$A = \begin{bmatrix} x_1C_{13} - C_{11} \\ y_1C_{13} - C_{12} \\ x_2C_{23} - C_{21} \\ x_2C_{23} - C_{22} \end{bmatrix}$$

Where (x_i, y_i) is the coordinate of point from camera i; C_{ij} is the projective matrix of camera i; j is the number of row vector in the C_{ij} projective matrix.

Q 4.1


Select a point in this image
(Right-click when finished)
Verify that the corresponding point
is on the epipolar line in this image

Q 4.2