IEOR 4150 Homework 6

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Chapter 8

3.
a) 52.5

$$\begin{aligned} t_{63} &= \frac{52.5-50}{20} \\ &= 0.125 \\ &\Rightarrow p-value = 0.9009 \end{aligned}$$

Since $p - value >= 0.05$, so yes.

b) 55.0

$$\begin{aligned} t_{63} &= \frac{55.0-50}{20} \\ &= 0.25 \\ &\Rightarrow p-value = 0.8034 \end{aligned}$$

Since $p - value >= 0.05$, so yes.

c) 57.5

$$\begin{aligned} t_{63} &= \frac{57.5-50}{20} \\ &= 0.375 \\ &\Rightarrow p-value = 0.7089 \end{aligned}$$

Since $p - value >= 0.05$, so yes.

6. The assumptions I am making are: the sample of 20 men are representative for the city's male population.

I used MATLAB to do the t-test, and the p-value is 0.0052. That is, the men in my city are not "average".

8. Since $\mu_1 < \mu_0$, we have

$$\begin{aligned}
\frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}} + z_{\alpha/2} &\geq z_{\alpha/2} \\
\Rightarrow \Phi\left(\frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}} + z_{\alpha/2}\right) &\geq \phi(z_{\alpha/2}) = 1 - \alpha/2 \\
\Rightarrow \Phi\left(\frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}} + z_{\alpha/2}\right) &\geq \phi(z_{\alpha/2}) \approx 1 \\
\Rightarrow \beta \approx 1 - \Phi\left(\frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}} - z_{\alpha/2}\right) \\
\Rightarrow \beta \approx \Phi\left(z_{\alpha/2} - \frac{\mu_0 - \mu_1}{\sigma/\sqrt{n}}\right) \\
\Rightarrow -z_\beta \approx z_{\alpha/2} - (\mu_0 - \mu_1)\frac{\sqrt{n}}{\sigma} \\
\Rightarrow n \approx \frac{(z_{\alpha/2} + z_\beta)^2 \beta^2}{(\mu_1 - \mu_0)^2}
\end{aligned}$$

11.

$$\begin{aligned}
P(\mu < 100) &= P\left(\frac{\mu - 105}{\sigma\sqrt{n}} < \frac{100 - 105}{\sigma\sqrt{n}}\right) \\
&= \Phi(\frac{-5}{\sigma\sqrt{20}})
\end{aligned}$$

a)

$$\begin{aligned}
P_a(\mu < 100) &= P\left(\frac{\mu - 105}{\sigma\sqrt{n}} < \frac{100 - 105}{\sigma\sqrt{n}}\right) \\
&= \Phi(\frac{-5}{5/\sqrt{20}}) \\
&= 0.001
\end{aligned}$$

b)

$$\begin{aligned} P_a(\mu < 100) &= P\left(\frac{\mu - 105}{\sigma\sqrt{n}} < \frac{100 - 105}{\sigma\sqrt{n}}\right) \\ &= \Phi(\frac{-5}{10/\sqrt{20}}) \\ &= 0.03 \end{aligned}$$

c)

$$\begin{aligned} P_a(\mu < 100) &= P\left(\frac{\mu - 105}{\sigma\sqrt{n}} < \frac{100 - 105}{\sigma\sqrt{n}}\right) \\ &= \Phi(\frac{-5}{15/\sqrt{20}}) \\ &= 0.07 \end{aligned}$$

21.

We find that

$$\begin{aligned} \overline{X} &= 237.06 \\ S &= 11.28 \end{aligned}$$

Using t-distribution, we have

$$\begin{aligned} P(\mu < 240) &= P\left(\frac{\mu - \overline{X}}{S/\sqrt{n}} < \frac{240 - \overline{X}}{S/\sqrt{n}}\right) \\ &= P\left(\frac{\mu - 237.06}{11.28/\sqrt{18}} < \frac{240 - 237.06}{11.28/\sqrt{18}}\right) \\ &= P\left(\frac{\mu - 237.06}{11.28/\sqrt{18}} < \frac{240 - 237.06}{11.28/\sqrt{18}}\right) \\ &= P\left(\frac{\mu - 237.06}{11.28/\sqrt{18}} < 1.1508\right) \\ &= 0.2657 \end{aligned}$$

29.

Based on the sample, we have

$$\begin{aligned}\overline{X}_1 &= 137.6667 \\ \overline{X}_2 &= 127.7778\end{aligned}$$

a)

$$\begin{aligned}\overline{X}_1 - \overline{X}_2 = 9.8889 \\ z_{0.95}\sqrt{\frac{\alpha_1}{n_1} + \frac{\alpha_2}{n_2}} = 6.1119 \\ \Rightarrow \overline{X}_1 - \overline{X}_2 > z_{0.95}\sqrt{\frac{\alpha_1}{n_1} + \frac{\alpha_2}{n_2}} \\ \Rightarrow H_0\end{aligned}$$

b)

$$\begin{aligned}\overline{X}_1 - \overline{X}_2 = 9.8889 \\ z_{0.95}\sqrt{\frac{\alpha_1}{n_1} + \frac{\alpha_2}{n_2}} = 7.7310 \\ \Rightarrow \overline{X}_1 - \overline{X}_2 > z_{0.95}\sqrt{\frac{\alpha_1}{n_1} + \frac{\alpha_2}{n_2}} \\ \Rightarrow H_0\end{aligned}$$

c)

$$\begin{aligned}\overline{X}_1 - \overline{X}_2 = 9.8889 \\ z_{0.95}\sqrt{\frac{\alpha_1}{n_1} + \frac{\alpha_2}{n_2}} = 12.2238 \\ \Rightarrow \overline{X}_1 - \overline{X}_2 < z_{0.95}\sqrt{\frac{\alpha_1}{n_1} + \frac{\alpha_2}{n_2}} \\ \Rightarrow H_1\end{aligned}$$

39.
a) Let

$$\begin{aligned}S^2 &= \frac{(n_p - 1)S_p^2 + (n_t - 1)s_t^2}{n_p + n_t - 2} \\ T &= \frac{\overline{X}_t - \overline{X}_p}{\sqrt{S^2(1/n_p + 1/n_t)}}\end{aligned}$$

H_0: The mean lead content of human today is less than it was in the years between 1880 and 1920. Or

$$T < t_{0.01,n_p+n_t-2}$$

H_1: The mean lead content of human today is greater than it was in the years between 1880 and 1920. Or

$$T \geq t_{0.01,n_p+n_t-2}$$

b)

$$\begin{aligned} T &= \frac{\overline{X}_t - \overline{X}_p}{\sqrt{S^2(1/n_p + 1/n_t)}} \\ &= -0.6390 \\ p - value &= 0.524 \end{aligned}$$

57.

Let $p_0 = 0.45$, we can use normal distribution to approximate significance level α test of $H_0 : p \geq p_0$ versus $H_1 : p < p_0$.

$$\begin{aligned} \frac{X - np_0}{\sqrt{np_0(1-p_0)}} &= \frac{70 - 200 \times 0.45}{\sqrt{200 \times 0.45 \times (1 - 0.45)}} \\ &= -2.8427 \end{aligned}$$

a) Since $-2.8427 < z_{0.05} = 1.64$, the claim is not believable.
b) Since $-2.8427 < z_{0.01} = 2.33$, the claim is not believable.

70. Based on the sample, We have

$$\begin{aligned} x_1 &= 256 \\ x_2 &= 119 \\ n &= x_1 + x_2 = 375 \\ P\{Bin(n, 1/(1+1)) \leq x_1\} &= P\{Bin(375, 0.5) \leq 256\} \\ &= P\left\{\frac{X - 375 \times 0.5}{\sqrt{375 \times 0.5(1-0.5)}} \leq \frac{256 - 375 \times 0.5}{\sqrt{375 \times 0.5(1-0.5)}}\right\} \\ &= 7.0746 \geq 0.05/2 \\ &\Rightarrow \lambda_1 \neq \lambda_2 \end{aligned}$$

Chapter 11

1.

$$\begin{aligned} T &= \frac{(141 - 564 \times 0.25)^2}{564 \times 0.25} + \frac{(291 - 564 \times 0.5)^2}{564 \times 0.5} + \frac{(132 - 564 \times 0.25)^2}{564 \times 0.25} \\ &= 0.8617 < \chi^2_{0.05,3-1} = 5.9915 \end{aligned}$$

So the claim is believable.

4.

$$\begin{aligned} T &= \sum_{i=0}^{11} \frac{failure_i - n \times P\{failures = i\}}{n \times P\{failures = i\}} \\ &= 16.3536 < \chi^2_{0.05,12-1} = 19.6751 \end{aligned}$$

So the claim is believable.

16.

$$\begin{aligned} \hat{e}_{11} &= \frac{50 \times 25}{200} = 6 \\ \hat{e}_{12} &= \frac{50 \times 175}{200} = 43 \\ \hat{e}_{21} &= \frac{150 \times 25}{200} = 18 \\ \hat{e}_{22} &= \frac{150 \times 175}{200} = 131 \end{aligned}$$

Then we get

$$\begin{aligned} T &= \frac{(10-6)^2}{6} + \frac{(40-43)^2}{43} + \frac{(15-18)^2}{18} + \frac{(135-131)^2}{131} \\ &= 3.4981 < \chi^2_{0.05,1} = 3.8415 \end{aligned}$$

So two variables are independent.