

Assignment 6

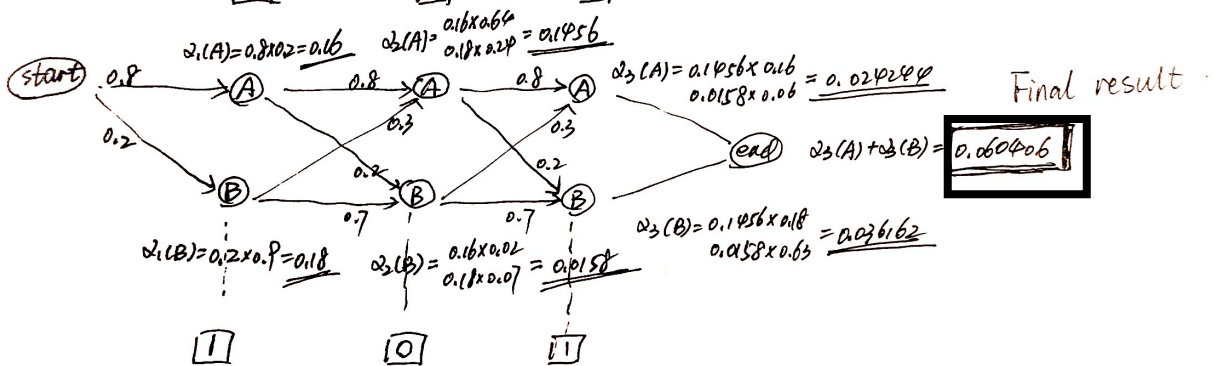
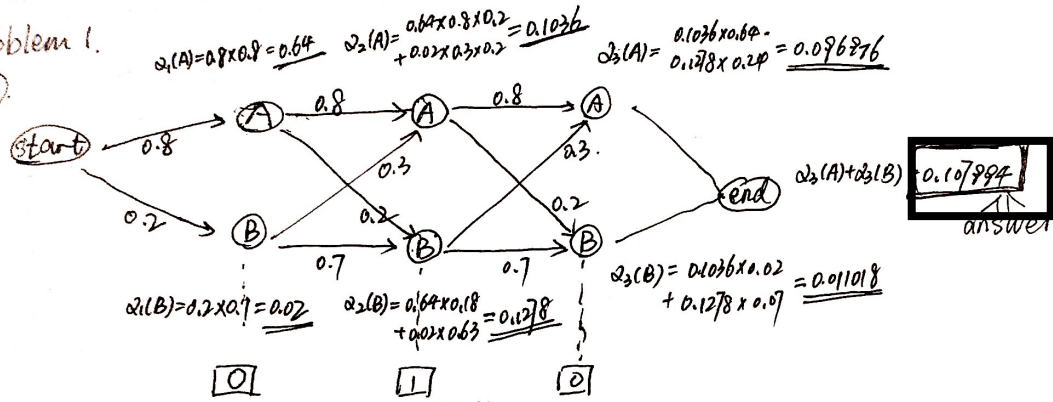
Jiazhao Li

Probl:

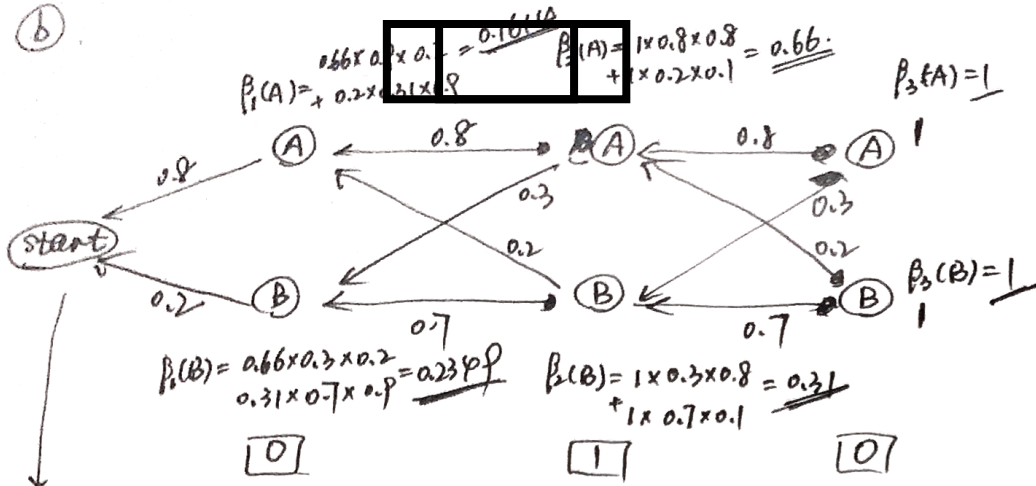
Forward algorithm:

Problem 1.

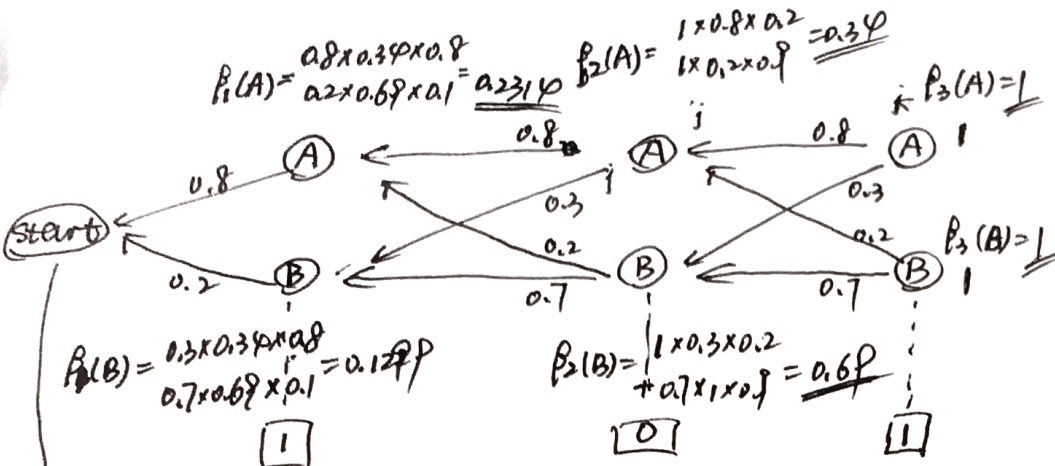
a.



⑥

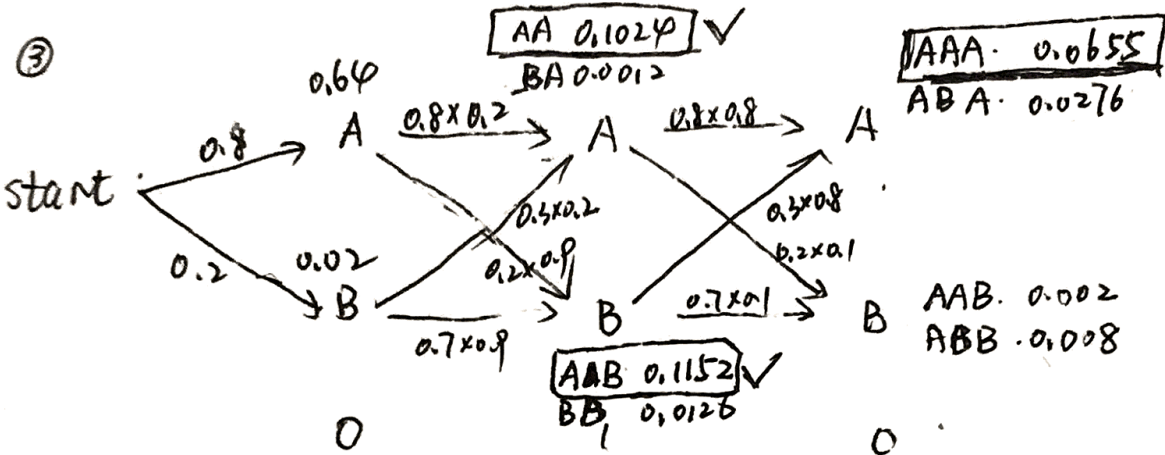


$$0.1614 \times 0.8 \times 0.8 + 0.2349 \times 0.2 \times 0.1 = 0.107894$$



$$0.2314 \times 0.8 \times 0.2 + 0.1279 \times 0.2 \times 0.9 = 0.060406$$

③



So the most likely path should be AAA

Prob2:

a:

Calculation is in code

K = 2

Transition prob matrix A:

```
[0.12044337, 0.87955663]
[0.9910059 , 0.0089941]
```

Emission Prob matrix B:

```
[5.9981e-01, 2.0020e-01, 1.99940e-01, 3.180954e-05]
[4.3345e-06, 6.6658e-01, 2.07313e-16, 3.334080e-01]
```

K = 4

Transition prob matrix A:

```
[5.14751e-05, 7.19341e-01, 2.69370e-05, 2.80580e-01]
[4.40058e-02, 3.45646e-04, 4.52532e-01, 5.03116e-01]
[2.09043e-04, 9.99703e-01, 2.79228e-10, 8.72000e-05]
[9.82158e-01, 1.57920e-02, 5.65622e-04, 1.48419e-03]
```

Emission Prob matrix B:

```
[8.69938e-01, 1.3001e-01, 2.32347e-08, 4.76056e-05]
[2.49142e-05, 9.89941e-01, 3.1605e-18, 1.00334e-02]
[3.05193e-01, 1.55962e-01, 5.3884e-01, 2.29273e-10],
[3.49642e-01, 2.32387e-03, 3.6455e-06, 6.48029e-01]
```

b:

K = 2

The most possible state sequence should be **S0-S1-S0-S1**.

For the first state, we use Viterbi, for the rest, we use transition matrix to get the most likely state.

K = 4

The most possible state sequence should be **S0-S1-S3-S0**.

For the first state, we use Viterbi, for the rest, we use transition matrix to get the most likely state.

Prob 3:

Problem 3

$$a) P(D, I, G, L, S) = P(D)P(I)P(G|D, I)P(L|G)P(S|I).$$

$$b) P(L=L') = \sum_D \sum_I \sum_G \sum_S P(D)P(I)P(G|D, I)P(L=L'|G) \\ = 0.6 \times 0.7 \times (0.3 \times 0.9 + 0.4 \times 0.6 + 0.3 \times 0.01) + 0.7 \times 0.4 \times (0.05 \times 0.9 + 0.25 \times 0.6 + 0.7 \times 0.01) \\ + 0.6 \times 0.3 \times (0.9 \times 0.9 + 0.08 \times 0.6 + 0.02 \times 0.01) + 0.4 \times 0.3 \times (0.5 \times 0.9 + 0.3 \times 0.6 + 0.2 \times 0.01) \\ = 0.502336$$

$$c) P(L=L' | I=i') = \frac{P(L=L', I=i')}{P(I=i')} = \frac{\sum_D \sum_G \sum_S P(D)P(G|D, I=i')P(L=L'|G)P(I=i')}{P(I=i')} \\ = 0.6 \times 0.8582 + 0.4 \times 0.632 = 0.76672$$

$$d) P(G=g' | S=s') = \frac{P(G=g', S=s')}{P(S=s')}. \quad P(S=s') = \sum_i P(S=s', I=i) = 0.275 \\ P(G=g' | S=s') = \sum_D \sum_I P(G, S, D, I) = \sum_D \sum_I P(G|D, I)P(D)P(I)P(S|I) \\ = 0.1846.$$

$$P(G=g' | S=s') = 0.6712727$$

$$e) P(D=d' | i', s', l') = \frac{P(D=d', i', s', l')}{P(i', s', l')} = \frac{P(d')P(i')P(l'|d', i')P(s'|i')}{P(s'|i')P(l'|i')P(i')} = \frac{0.060672}{0.184258} = 0.3283$$

Prob4 :

first two principal
direction is:

```
[[-0.24959319 -0.31318631]
 [ 0.25652131 -0.32130825]
 [-0.3468611   0.11181554]
 [-0.005099    0.45672596]
 [-0.34297566  0.21985693]
 [ 0.18943673  0.15387677]
 [-0.31385097  0.31174761]
 [ 0.32173451 -0.34918069]
 [-0.31981745 -0.2703984 ]
 [-0.33853899 -0.23885931]
 [-0.20502118 -0.30870354]
 [ 0.20273245  0.23495727]
 [-0.30984085 -0.07598235]]
```

