Bayesian evaluation of DSGE models with financial frictions - Appendix

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A LOG-LINEARIZED EQUATIONS

This section lays out the systems of log-linearized equations which make up the NK baseline, as well as the EFP and CC extensions. Variables without time subscripts refer to the steady state, while hats denote log deviations from the steady state. Lower case letters are the real counterparts of nominal variables. For partial derivatives of functions (ψ, χ^{EFP}, ϱ), we use subscripts to indicate with respect to which variable the derivative is taken. Otherwise, the notation is as in the main text.

A.1 NK model

The log-linearized NK model consists of the following equations.

Marginal utility

$$\hat{\lambda}_t = \hat{\Gamma}_t - \frac{\sigma_c}{1-\xi}(\hat{c}_t - \xi \hat{c}_{t-1}) \tag{A.1}$$

Euler equation

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \hat{R}_t - E_t \hat{\pi}_{t+1} \tag{A.2}$$

Capital accumulation

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta(\hat{i}_t + \hat{\Psi}_t) \tag{A.3}$$

Investment demand

$$\hat{i}_t = \frac{\beta}{1+\beta}E_t\hat{i}_{t+1} + \frac{1}{1+\beta}\hat{i}_{t-1} + \frac{1}{\kappa(1+\beta)}(\hat{q}_t + \hat{\Psi}_t) \tag{A.4}$$

Real price of capital

$$\hat{q}_t = E_t\lambda_{t+1} - \lambda_t + \beta r_k \hat{r}_{k,t+1} + \beta(1-\delta)\hat{q}_{t+1} \tag{A.5}$$

Real wage

$$\begin{aligned} \hat{w}_t &= \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\sigma_n\frac{\phi_w}{\phi_w-1})(1+\beta)}(\sigma_n\hat{n}_t - \hat{\lambda}_t - \hat{w}_t + \hat{\phi}_{w,t}) \\ &+ \frac{\beta}{1+\beta}(E_t\hat{w}_{t+1} + E_t\hat{\pi}_{t+1}) - \frac{1+\beta\zeta_w}{1+\beta}\hat{\pi}_t + \frac{1}{1+\beta}(\hat{w}_{t-1} + \zeta_w\hat{\pi}_{t-1}) \end{aligned} \tag{A.6}$$

Inflation

$$\hat{\pi}_t = \frac{(1-\beta\theta)(1-\theta)}{\theta(1+\beta\zeta)}(\alpha\hat{r}_{k,t} + (1-\alpha)\hat{w}_t - \hat{A}_t + \hat{\phi}_t) + \frac{\beta}{1+\beta\zeta}E_t\hat{\pi}_{t+1} + \frac{\zeta}{1+\beta\zeta}\hat{\pi}_{t-1} \tag{A.7}$$

Production function

$$\hat{y}_t = \hat{A}_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t \tag{A.8}$$

Optimal factor input

$$\hat{r}_{k,t} + \hat{k}_{t-1} = \hat{w}_t + \hat{n}_t \tag{A.9}$$

Aggregate resource constraint

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{g}{y}\hat{g}_t \tag{A.10}$$

Taylor rule

$$\hat{R}_t = \gamma_R \hat{R}_{t-1} + (1-\gamma_R)(\gamma_\pi \hat{\pi}_t + \gamma_y \hat{y}_t) + \hat{\varphi}_t \tag{A.11}$$

The model is driven by six autoregressive shocks ($\hat{A}_t$, $\hat{\Gamma}_t$, $\hat{\Psi}_t$, $\hat{\phi}_t$, $\hat{\phi}_{w,t}$ and $\hat{g}_t$), and one iid shock ($\hat{\varphi}_t$).

A.2 EFP model

This model includes all equations showing up in the NK benchmark, except for (A.5) and (A.10). The additional equations are as follows.

Entrepreneurial loans

$$\hat{l}_t = \frac{k}{l}(\hat{k}_t + \hat{q}_t) - \frac{v}{l}\hat{v}_t \tag{A.12}$$

Rate of return on capital

$$\hat{R}_{E,t} = \frac{1}{r_k + 1 - \delta}(r_k \hat{r}_{k,t} + (1-\delta)\hat{q}_t) - \hat{q}_{t-1} + \hat{\pi}_t \tag{A.13}$$

Banking sector's zero profit condition

$$\hat{R}_{E,t} + \hat{q}_{t-1} + \hat{k}_{t-1} + \hat{\psi}_t = \hat{l}_{t-1} + \hat{R}_{t-1} \tag{A.14}$$

Optimal financial contract

$$E_t \hat{R}_{E,t+1} = \hat{R}_t + E_t \hat{\chi}^{EFP}_{t+1} \tag{A.15}$$

Non-default lending rate

$$\hat{R}_{L,t} = \tilde{a}_E^{-1}\hat{\tilde{a}}_{E,t} + \hat{R}_{E,t} + \hat{q}_{t-1} + \hat{k}_{t-1} - \hat{l}_{t-1} \tag{A.16}$$

Net worth accumulation

$$\begin{aligned} \hat{v}_t &= \frac{R_E k(1-\mu\varrho)}{v\pi\varepsilon_\nu^{-1}}(\hat{R}_{E,t} + \hat{q}_{t-1} + \hat{k}_{t-1}) - \frac{Rl}{v\pi\varepsilon_\nu^{-1}}(\hat{R}_{t-1} + \hat{l}_{t-1}) \\ &- \frac{\mu\varrho R_E k}{v\pi\varepsilon_\nu^{-1}}\hat{\varrho}_t + (\frac{t_E}{v} - 1)(\hat{\pi}_t - \hat{\nu}_t) \end{aligned} \tag{A.17}$$

Aggregate resource constraint

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{g}{y}\hat{g}_t + \frac{\mu\varrho R_E k}{y}(\hat{\varrho}_t + \hat{R}_{E,t} + \hat{q}_{t-1} + \hat{k}_{t-1}) \tag{A.18}$$

Auxiliary functions

$$\hat{\psi}_t = \frac{\psi_1}{\psi}E_t\hat{\tilde{a}}_{E,t} + \frac{\psi_2}{\psi}E_t\hat{\sigma}_{E,t} \tag{A.19}$$

$$\hat{\chi}_t^{EFP} = \chi_1^{EFP}E_t\hat{\tilde{a}}_{E,t} + \chi_2^{EFP}E_t\hat{\sigma}_{E,t} \tag{A.20}$$

$$\hat{\varrho}_t = \frac{\varrho_1}{\varrho}E_t\hat{\tilde{a}}_{E,t} + \frac{\varrho_2}{\varrho}E_t\hat{\sigma}_{E,t} \tag{A.21}$$

The model includes two additional shocks, i.e. $\hat{\nu}_t$ and $\hat{\sigma}_{E,t}$, both assumed to follow an autoregressive process.

A.3 CC model

This model consists of all equations showing up in the NK model but (A.5) and (A.10), plus the following equilibrium conditions.

Marginal utility of entrepreneurs

$$\hat{\lambda}_{E,t} = -\frac{\sigma_c}{1-\xi}(\hat{c}_{E,t} - \xi\hat{c}_{E,t-1}) \tag{A.22}$$

Euler equation for entrepreneurs

$$\begin{aligned}\hat{\lambda}_{E,t} &= -\hat{q}_t + \beta_E r_k E_t\{\hat{\lambda}_{E,t+1} + \hat{r}_{k,t+1}\} + \beta_E (1-\delta) E_t\{\hat{\lambda}_{E,t+1} + \hat{q}_{t+1}\} \\ &+ \frac{\pi}{R_L} m (1-\delta) E_t\{\hat{\lambda}_{E,t} + \hat{\pi}_{t+1} - \hat{R}_{L,t} + \hat{m}_t + \hat{q}_{t+1}\} \\ &- \beta_E m (1-\delta) E_t(\hat{\lambda}_{E,t+1} + \hat{m}_t + \hat{q}_{t+1})\end{aligned} \tag{A.23}$$

Entrepreneurs' budget constraint

$$\frac{c^E}{y}\hat{c}_t^E = r_k \frac{k}{y}(\hat{r}_{k,t} + \hat{k}_{t-1}) + \frac{k}{y}(1-\delta)(\hat{q}_t + \hat{k}_{t-1}) + \frac{l}{y}\hat{l}_t - \frac{k}{y}(\hat{q}_t + \hat{k}_t) - \frac{R_L}{\pi}\frac{l}{y}(\hat{R}_{L,t-1} + \hat{l}_{t-1} - \hat{\pi}_t) \tag{A.24}$$

Collateral constraint

$$\hat{R}_{L,t} + \hat{l}_t - E_t\hat{\pi}_{t+1} = \hat{m}_t + \hat{k}_t + E_t\hat{q}_{t+1} \tag{A.25}$$

Lending rate

$$\hat{R}_{L,t} = \hat{\phi}_{L,t} + \hat{R}_t \tag{A.26}$$

Aggregate resource constraint

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{i}{y}\hat{i}_t + \frac{g}{y}\hat{g}_t + \frac{c_E}{y}\hat{c}_{E,t} \tag{A.27}$$

The two additional shocks are $\hat{m}_t$ and $\hat{\phi}_{L,t}$. Both are assumed to follow an autoregressive process.

B DATA

Output is measured as gross domestic product, consumption is personal consumption expenditures and investment is defined as gross fixed private domestic investment. All this data are expressed in real terms and come from the Bureau of Economic Analysis. Inflation is measured as log change in the CPI index, published by the Bureau of Labor Statistics. The interest rate is defined as the federal funds rate and taken from the Federal Reserve Board.

Labor market variables are calculated using the Bureau of Labor Statistics data. Hours worked are defined as average weekly hours in the non-farm business sector, multiplied with the civilian employment (16 years and over), and divided by the population level (16 years and over). Real wages is nominal compensation of employees in the non-farm business sector, deflated by the implicit GDP deflator (taken from the Bureau of Economic Analysis).

Loans to firms are defined as credit market instruments liabilities of nonfarm nonfinancial businesses, taken from the Flow of Funds Account of the Federal Reserve Board and deflated with the GDP deflator. Spread on loans to firms is the difference between the industrial BBB corporate bond yield, backcasted using BAA corporate bond yields (both series taken from Bloomberg), and the federal funds rate.

C ROBUSTNESS CHECKS

Table C.1 below shows the prior assumptions for financial sector parameters that we use in the Robustness checks section of the main paper.

Table C.1: Additional prior assumptions

Parameter	Dist. type	Mean	Std.
EFP			
μ	beta	0.10	0.025
ν	beta	0.977	0.002
σ_E	gamma	0.29	0.05
CC			
β_E	beta	0.985	0.002
ϕ_L	normal	1.002	0.002
m	beta	0.52	0.05

D PRIOR AND POSTERIOR DISTRIBUTIONS

The following figures plot the marginal prior (grey lines) and posterior (black lines) distributions for all estimated model parameters. The vertical dashed lines indicate the posterior mode obtained from posterior maximization.

Figure D.1: Priors and posteriors - NK

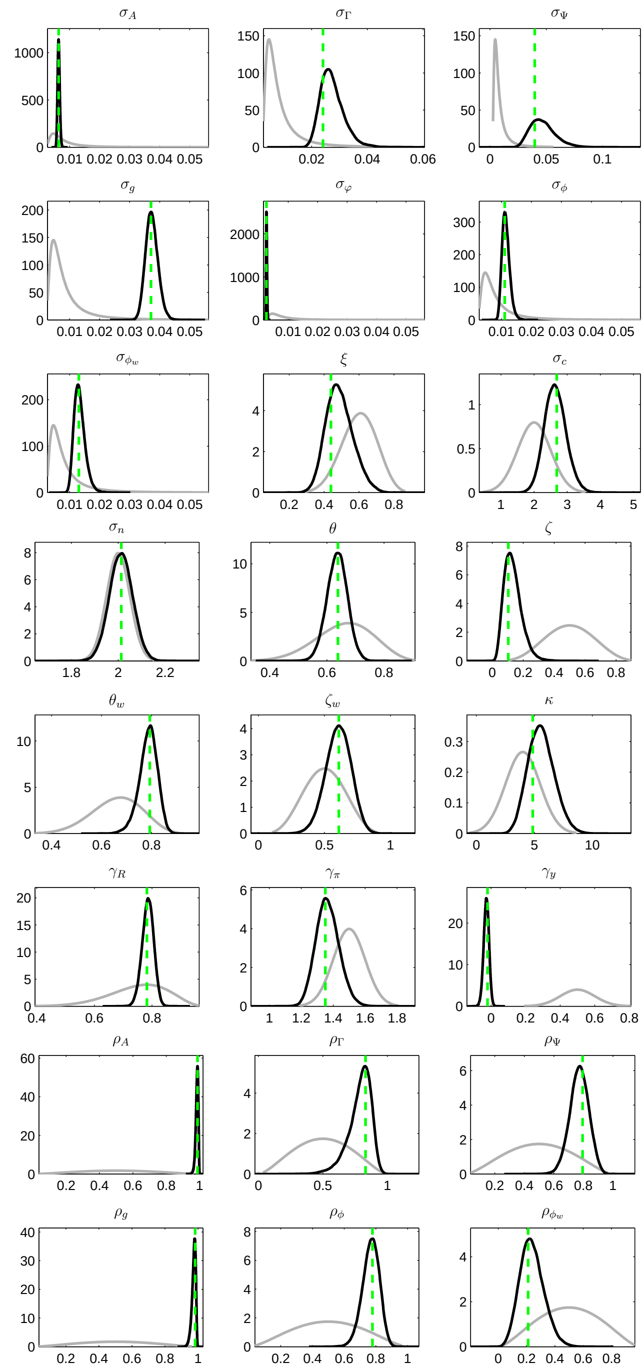


Figure D.2: Priors and posteriors - EFP

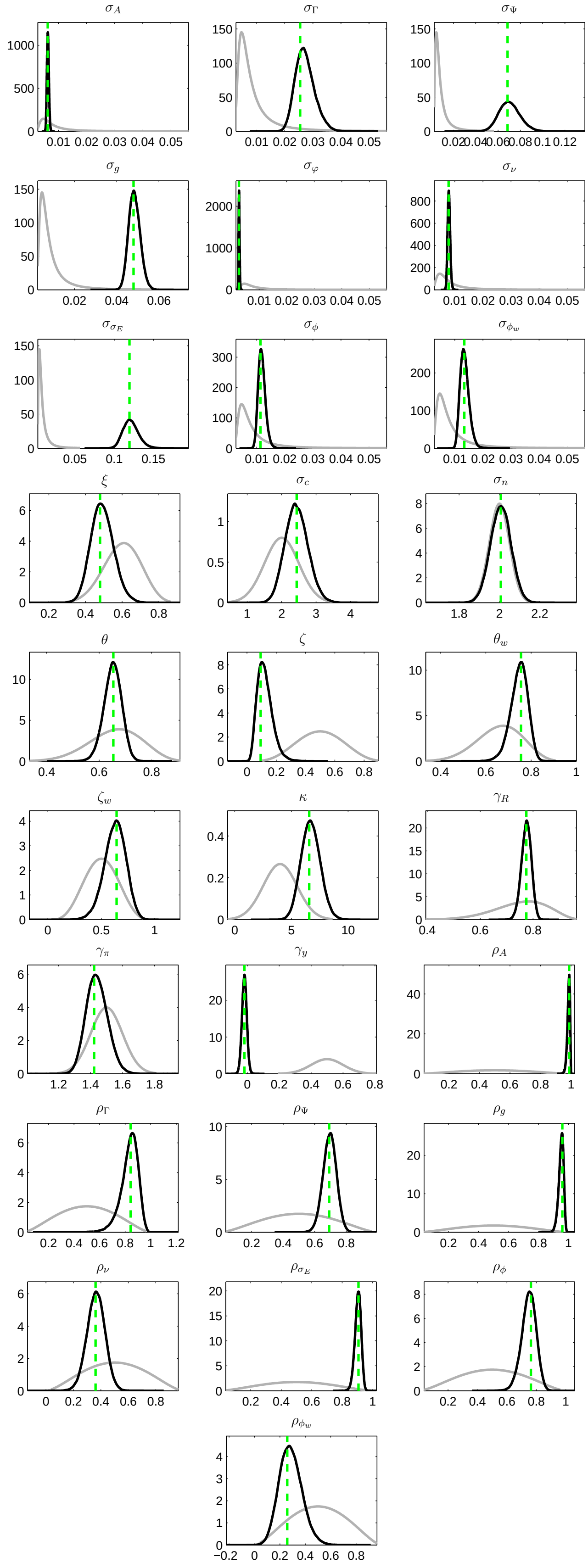
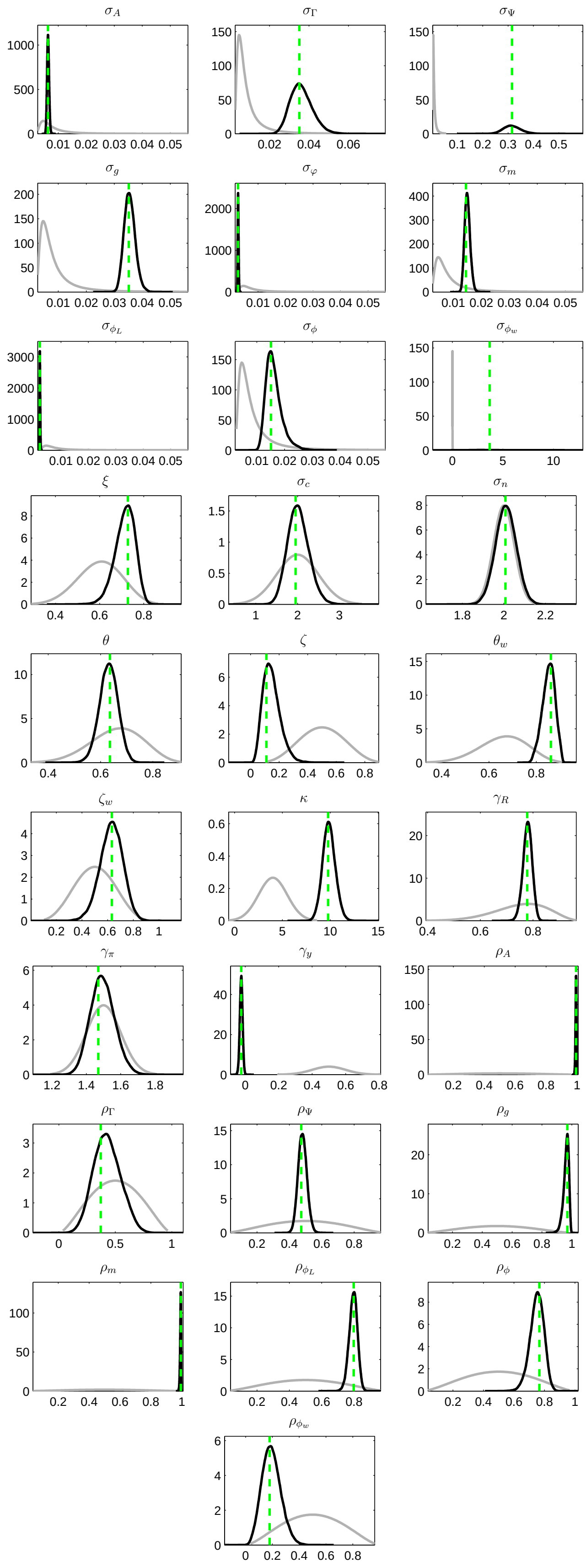


Figure D.3: Priors and posteriors - CC



E CONVERGENCE DIAGNOSTICS

The figures below illustrate the convergence diagnostic test suggested by Brooks and Gelman (1998). The lines plot the recursive mean width of the 80% confidence intervals obtained by pooling draws from all chains (blue), or by averaging across the chains (red).

Figure E.1: Convergence diagnostics - NK

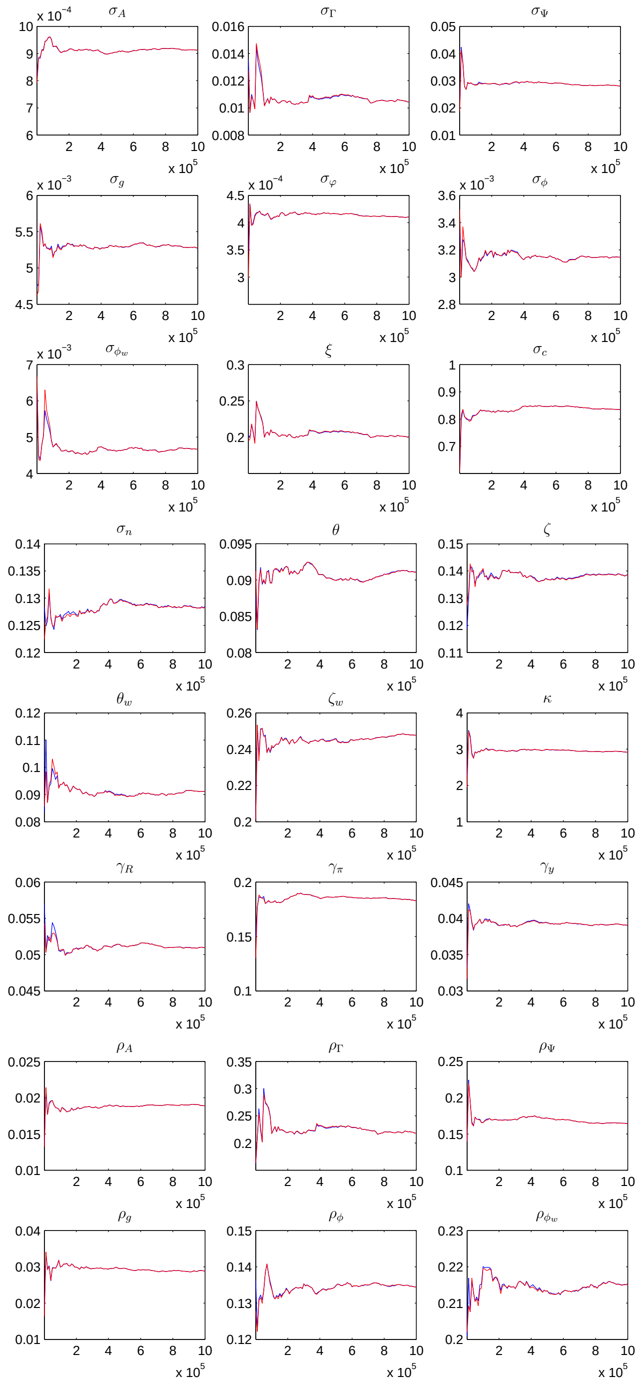


Figure E.2: Convergence diagnostics - EFP

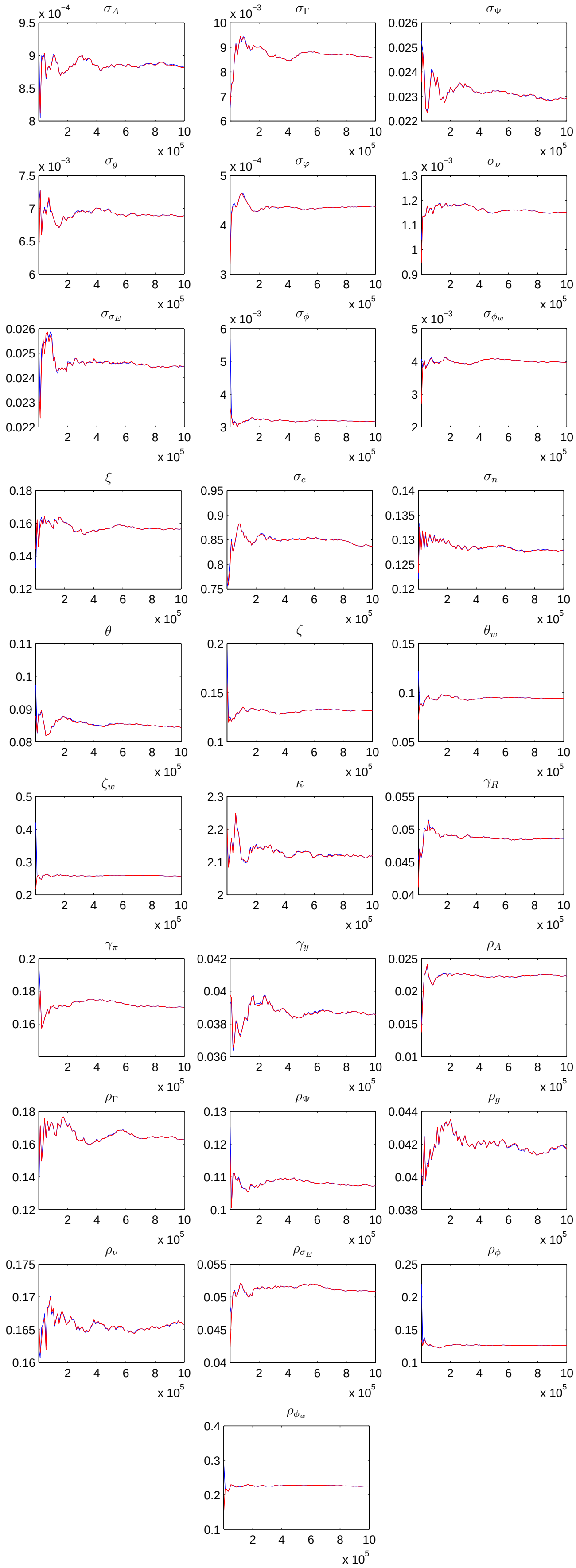


Figure E.3: Convergence diagnostics - CC