Bayesian Time Series Methods: Introductory

Computer Tutorial 5: The Unobserved Components Stochastic Volatility Model (UC-SV)

➢ Exercise: The UC-SV Model

- Following Stock and Watson (2007), we write the UC-SV model as

$$\pi_t = \tau_t + \eta_t, \quad \eta_t \sim N(0, \sigma_t^{\eta}),$$

$$\tau_t = \tau_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^{\varepsilon}),$$

$$\log(\sigma_t^{\eta}) = \log(\sigma_{t-1}^{\eta}) + \upsilon_t^{\eta}, \quad \upsilon_t^{\eta} \sim N(0, \gamma_1),$$

$$\log(\sigma_t^{\varepsilon}) = \log(\sigma_{t-1}^{\varepsilon}) + \upsilon_t^{\varepsilon}, \quad \upsilon_t^{\varepsilon} \sim N(0, \gamma_2).$$

- The observable inflation rate π_t is modelled as the sum of the unobserved component τ_t and the disturbance ε_t.

- We are interested in estimating the parameters $\gamma = (\gamma_1, \gamma_2)$ and the state variable τ_t. The parameters γ reflect the smoothness of the evolution of the stochastic volatility process.

Exercise: The UC-SV Model
Rewrite the Model

- Write the model in a more compact way

$$y_t = X_t' B_t + \eta_t,$$

$$X_t' = I_m \otimes (1, y_{t-1}', \ldots, y_{t-k}').$$

- $\otimes$ is the Kronecker product and $t = 1, \ldots, T$.

- All time periods for one variable are grouped together in this setup.

Exercise: The UC-SV Model
Rewrite the Model in MATLAB

```
% Generate lagged Y matrix. This will be part of the X matrix
ylag = mlag2(Y,plag); % Y is [T x m]. ylag is [T x (nk)]

% Form RHS matrix X_t = [1 y_t-1 y_t-2 ... y_t-k] for t=1:T
ylag = ylag(plag+1:t,:);

% m is the number of elements in the state vector
m = 1 + plag; % m is the number of elements in the state vector

% Create X_t matrix
Z = [ones(t-plag,1) ylag];
```

Exercise: The UC-SV Model Priors

- Use OLS with noniformative priors

$$p(B) \sim N(\underline{B}, \underline{V}),$$

$$p(h) \sim G(\underline{s}^{-2}, \underline{v}),$$

```
    %use relatively noninformative values
B_OLS = zeros(m,1);
VB_OLS = eye(m);
h_OLS = ones(m,1);
sigma_OLS = 1;
```

➢ Exercise: The UC-SV Model
Priors

- Set prior means and variances

$$p(B_0) \sim N\left(B_{OLS}, 4 \cdot V_{B_{OLS}}\right),$$

$$p\left(\log(\sigma_0^{\eta})\right) \sim N\left(\log(\sigma_{OLS}^{\eta}), I_n)\right),$$

$$p\left(\log(\sigma_0^{\varepsilon})\right) \sim N\left(\log(\sigma_{OLS}^{\varepsilon}), I_n)\right),$$

```
% Set prior means and variances (_prmean / _prvar)
B_0_prmean = B_OLS;
B_0_prvar = 4*VB_OLS;
h_prmean = h_OLS;
h_prvar = 4*eye(m);
sigma_prmean = sigma_OLS;
sigma_prvar = 4;
```

Exercise: The UC-SV Model
Priors

- Priors for parameters

$$\gamma_1 \sim IW(\kappa_{\gamma_1}^2 \cdot T_0 \cdot V_{B_{OLS}}, T_0),$$

$$\gamma_2 \sim IW(\kappa_{\gamma_2}^2 \cdot T_0 \cdot V_{B_{OLS}}, T_0).$$

- where $\kappa_{\gamma_1} = 0.001, \kappa_{\gamma_2} = 0.001$ and $T_0 = m + 1.$

% Set some hyperparameters

- k_Q = 0.001;
- k_W = 0.001;

% Scale and shape parameters

```
Q_prmean = ((k_Q).^2)*(1+m)*VB_OLS;
Q_prvar = 1 + m;
W_prmean = ((k_W)^2)*2*eye(p);
W_prvar = 2;
```

Exercise: The UC-SV Model
Initial Values

- We assume the prior for the state as

$$p(\tau_t) \sim N(\tau_0, V_0),$$

- Initial values

$$\tau_{0|0} = \tau_{t-1}, \qquad V_{0|0} = 1,$$

$$\gamma_{1,0} = 0.00001 \text{ and } \gamma_{2,0} = 0.0001.$$

Exercise: The UC-SV Model
Gibbs Sampling

- Sample the parameters and the state as follows

1. $p(\sigma_t^{\eta} \mid \sigma_t^{\eta(j-1)}, \sigma_t^{\varepsilon(j-1)}, \gamma_1^{(j-1)}, \gamma_2^{(j-1)}, y_t)$ and $p(\gamma_1 \mid \sigma_t^{\eta(j-1)}, y_t)$.

2. $p(\sigma_t^{\varepsilon} \mid \sigma_t^{\varepsilon(j-1)}, \sigma_t^{\eta(j-1)}, \gamma_1^{(j-1)}, \gamma_2^{(j-1)}, y_t)$ and $p(\gamma_2 \mid \sigma_t^{\varepsilon(j-1)}, y_t)$.

3. $p(\tau_t \mid \gamma_1^{(j-1)}, \gamma_2^{(j-1)}, y_t)$ by using Carter and Kohn (1994).

- Repeat step 1 to 3 until convergence.

References

- Carter, C. K. and Kohn, R. (1994), On Gibbs sampling for state space models, Biometrika, 81, 541–553.
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