Bayesian Time Series Methods: Introductory

Computer Tutorial 4: Bayesian Analysis of the TVP-AR model

➢ Exercise: TVP-AR(p) Model and the State Space Form

- Koop and Potter (2001) consider an AR(p) model of the form

$$y_t = \alpha_{0t} + \alpha_{1t} y_{t-1} + \ldots + \alpha_{pt} y_{t-p} + \varepsilon_t,$$

$$\alpha_{it+1} = \alpha_{it} + u_{it}, \text{ for } i = 0, \ldots, p.$$

- where ε_t is assumed to be i.i.d. $N(0, h^{-1})$, u_{it} to be i.i.d. $N(0, \lambda_i h^{-1})$ and $\varepsilon_t, u_{is}, u_{ij}$ are independent of one another.

- The state space form is

- Observation equation: $y_t = X_t \beta + Z_t \alpha_t + \varepsilon_t,$

- State equation: $\alpha_{t+1} = T_t \alpha_t + u_t.$

Exercise: TVP-AR(p) Model and the State Space Form

- By excluding X_t and assuming $T_t = I_{p+1}$, the time series $(y_1, \ldots, y_T)$ can be defined as

$$Z_t = [1, y'_{t-1}, \ldots, y'_{t-p}]',$$

$$\alpha_t = [\alpha_{0,t}, \alpha_{1t}, \ldots, \alpha_{pt}]',$$

$$u_t = [u_{0t}, u_{1t}, \ldots, u_{pt}]',$$

- and

$$H^{-1} = h^{-1} \begin{bmatrix} \lambda & 0 & 0 & \cdot & 0 \\ 0 & \lambda_1 & \cdot & \cdot & \cdot \\ \cdot & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & 0 & \lambda_p \end{bmatrix}.$$

Exercise: The Initial Setup in MATLAB

- Load data for U.K. industrial production from 1701–1992. Set $p = 1$.

```
load dliprod.dat;
t=size(dliprod,1);
p=1;
dliprod=100*dliprod;
y=dliprod(p+1:t,1);
xar=ones(t-p,p+1);
for i=1:p
    xar(:,i+1) = dliprod(p+1-i:t-i,1);
end
t=t-p;
            %k=total number of time varying parameters
k=p+1;
```

Exercise: Bayesian Inference Priors

$$p(h) = f_G(h \mid s^{-2}, \underline{v}),$$

$$p(\lambda_i^{-1} \mid y, \alpha_1, \ldots, \alpha_T) = f_G(\lambda_i^{-1} \mid \overline{\lambda}_i^{-1}, \overline{v}_i),$$

- In the computations, we use an informative prior for the parameters. For h use Gamma prior with $v = 1$ and $s^{-2} = 1$.
- For λ_i^{-1} use Gamma priors with $\underline{v}_i = 1$ and $\underline{\lambda}_i = 1$.
- Priors of the state equation

$$p(\alpha_1, \ldots, \alpha_T \mid H) = p(\alpha_1 \mid H)\, p(\alpha_2 \mid \alpha_1, H) \ldots p(\alpha_T \mid \alpha_{T-1}, H),$$

- For $t = 1, \ldots, T-1$, we obtain

$$p(\alpha_{t+1} \mid \alpha_t, H) = f_N(\alpha_{t+1} \mid T_t \alpha_t, H),$$

- and

$$p(\alpha_1 \mid H) = f_N(\alpha_1 \mid 0, H),$$

- We assume the initial condition for $\alpha_0 = 0$.

Exercise: Writing the Priors in MATLAB

```
        %specify hyperparameters of Gamma prior for h
v0=1;
s02=1;

        %specify hyperparameters for Gamma prior for the vector lambda
vlam0=ones(k,1);
s02lam=1*ones(k,1);
```

Exercise: Bayesian Inference Posteriors

$$\bar{v}_i = T + \underline{v}_i,$$

$$\bar{\lambda}_i = \frac{h\sum_{t=0}^{T-1}(\alpha_{i,t+1} - \alpha_{i,t})(\alpha_{i,t+1} - \alpha_{i,t})' + \underline{v}_i \underline{\lambda}_i}{\bar{v}_i}.$$

- For $i = 0, \ldots, p$ and $t = 1, \ldots, T$.

- For more details, read Koop (2003), pages 194–202.

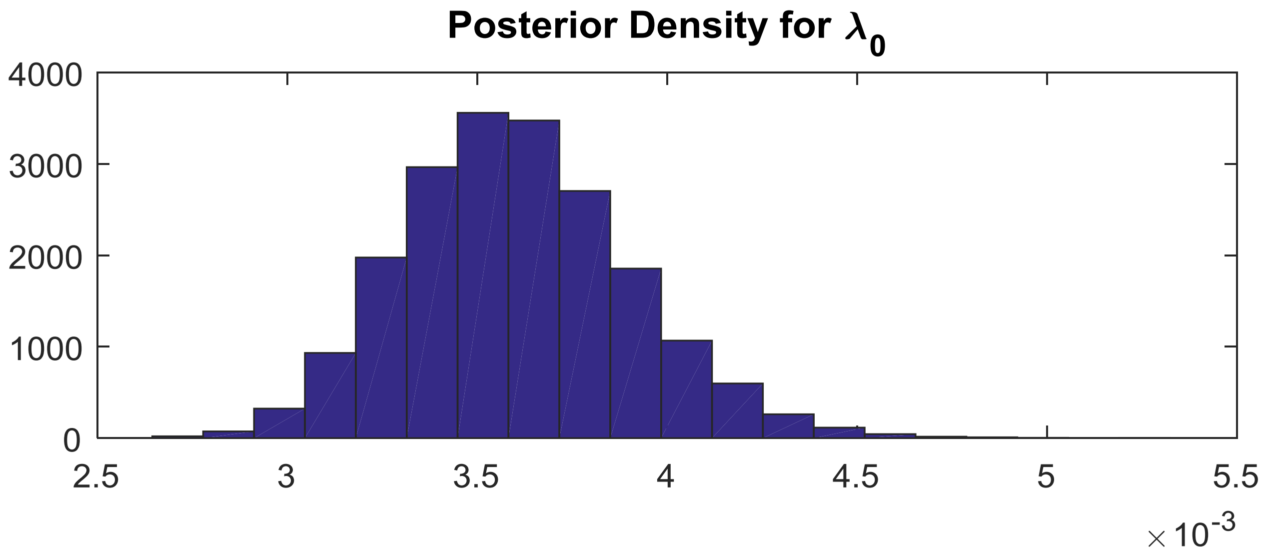
Exercise: Gibbs Sampling algorithm in MATLAB

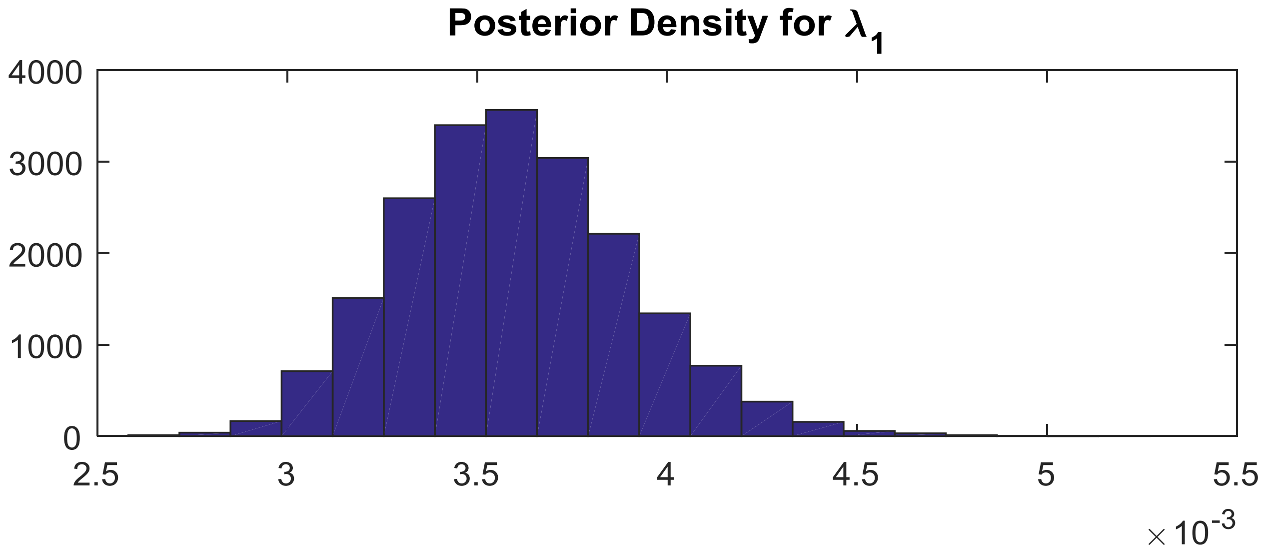
```
        %Specify the number of replications
        %number of burnin replications
s0=100;

        %number of retained replications
s1=1000;
s=s0+s1;

        %choose a starting value for h
hdraw=1;
adraw=zeros(t,k);
lamdraw=zeros(k,1);
```

Exercise: Results





References

- Koop, G. (2003), Bayesian econometrics, Wiley.
- Koop, G., Poirier, D. and Tobias, J. (2007), Bayesian econometric methods. Cambridge: Cambridge University Press.
- Koop, G. and Potter, S. M. (2001), Are apparent findings of nonlinearity due to structural instability in economic time series? Econometrics Journal, 4, 37–55.