Computer Tutorial 4: Bayesian Analysis of the Time-varying Parameter AR model

Data and Matlab code for all questions are available on the course website. The data set consists of the annual percentage change in UK industrial production from 1701-1992. In order to investigate whether the dynamic structure of this time series model is changing over time, use an AR(p) model with time varying coefficients:

$$y_t = \alpha_{0t} + \alpha_{1t} y_{t-1} + ... + \alpha_{pt} y_{t-p} + \varepsilon_t,$$

where for $i = 0, .., p$

$$\alpha_{it+1} = \alpha_{i,t} + u_{it}.$$

Assume ε_t to be i.i.d. $N\left(0, h^{-1}\right)$ and u_{it} to i.i.d. $N\left(0, \lambda_i h^{-1}\right)$ where ε_t, u_{is} and u_{jr} are independent of one another for all s, t, r, i and j. Use an informative prior for the parameters h and λ_i for $i = 0, .., p$. For h use a Gamma prior with $\underline{\nu} = 1$ and $\underline{s}^{-2} = 1$. For λ_i^{-1} use Gamma priors with $\underline{\nu}_i = 1$ and $\underline{\lambda}_i = 1$. With this prior, the conditional posteriors have the form:

$$p\left(\lambda_i^{-1} | y, \alpha_1, .., \alpha_T\right) = f_G\left(\lambda_i^{-1} | \overline{\lambda}_i^{-1}, \overline{\nu}_i\right),$$

for $i = 0, .., p$, where

$$\overline{\nu}_i = T + \underline{\nu}_i$$

and

$$\overline{\lambda}_i = \frac{h \sum_{t=0}^{T-1} \left(\alpha_{i,t+1} - \alpha_{it}\right)\left(\alpha_{i,t+1} - \alpha_t\right)' + \underline{\nu}_i \underline{\lambda}_i}{\overline{\nu}_i}.$$

You are asked to write code which implements the Gibbs sampling algorithm above. I have provided code which does this and produces some basic parameter estimates for the $p = 1$ case. You may wish to extend my code to produce other features of interest. For instance, you may wish to produce graphs which plot posterior means of α_{0t} and α_{1t} along with +/- 2 posterior standard deviation interval. These can be used to provide evidence as to whether there is parameter change in this model.