Bayesian Time Series Methods: Introductory

Computer Tutorial 1: MATLAB Basics

➢ What is MATLAB?

- MATLAB is an interpreted matrix programming language. MATLAB’s interpreter translates statements into machine code one by one.

- We can work in MATLAB in three ways. From the command prompt, writing scripts and writing functions.

- By using M-files we can create a script, which can take no arguments. It is a series of statements that will be executed sequentially. M-files can also be used to create a function that can take and return arguments.

➢ Exercise 1 (a): Basic Matrix Commands
Creating a Matrix

- How to type the following 3-by-3 x matrix into MATLAB?

$$x = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

- Write a square bracket, separate elements in a row by space or comma, use semicolon to separate rows, and end the matrix with another square bracket.

```
>> x = [1 2 3; 4 5 6; 7 8 9];
```

```
>> disp x; >> disp(x);
```

- The former, prints x in the command window and the latter, prints only the value of x on command window.

➢ Exercise 1 (a cont.): Basic Matrix Commands
Matrix Arithmetic Operators

Operator	**Example**	**Meaning**
+	$x + y$	Matrix addition is valid if x and y are of the same size.
-	$x - y$	Likewise, matrix subtraction is valid if x and y are of the same size.
*	$x*y$	Matrix multiplication is valid if the number of columns of x is the same as number of rows of y.

Exercise 1 (a cont.): Basic Matrix Commands Matrix Arithmetic Operators

Command	Meaning
a = [x, y]	Concatenates the 3-by-3 matrix x by adding the 3-by-3 matrix y as an additional column.
b = [x; y]	Concatenates the 3-by-3 matrix x by adding the 3-by-3 matrix y as an additional row.
c = x(:, 2)	Returns second column of x.
d = y(:, 1)	Accesses first column in y.
e = x(2, 3)	We can access specific elements in each argument. Here the command returns second element (row) in argument (column) 3 of x.

Exercise 1 (b, c, d): Basic Matrix Commands
Matrix Arithmetic Operators

Command	Meaning
$f = x'$	The transpose operator is defined by a single quote. It flips a matrix around its main diagonal and it returns a row vector into a column vector.
eye(m)	Returns an m-by-m identity matrix.
eye(m, n)	Returns an m-by-n matrix.
i = eye(size(x))	Defines an identity matrix of size x.
g = plus(i,x)	Returns the identity matrix i plus x.
ginv = inv(g)	Returns inverse of g.

Exercise 2: OLS Estimation Using Artificial Data

- We are interested in the regression model of the form

$$y_i = \beta_1 + \beta_2 x_{i2} + \ldots + \beta_k x_{ik} + \varepsilon_i, \text{ for } i = 1, \ldots, n.$$

- Let x_i and ε_i be random draws from the $U(0, 1)$ and $N(0, 1)$ distributions, respectively. The above model in matrix notation is

$$\underset{(n\times 1)}{y} = \underset{(n\times k)}{X}\;\underset{(k\times 1)}{\beta} + \underset{(n\times 1)}{\varepsilon}.$$

- where $X = [\iota, x_2]$, ι being an $n \times 1$ vector of ones.

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}, \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}.$$

Exercise 2 (a): OLS Estimation Using Artificial Data

- We compute the OLS estimates β from $b = (X'X)^{-1}X'y$.

- Recall that the OLS estimate of the error variance is $s^2 = e'e/(n-k)$, where $e = y - Xb$, and

$$
\begin{aligned}
e'e = (y - Xb)'(y - Xb) &= y'y - 2b'X'y + b'X'Xb \\
&= y'y - 2b'X'y + b'X'X(X'X)^{-1}X'y \\
&= y'y - b'X'y.
\end{aligned}
$$

Type:

```
>> bhat = inv(x'*x)*x'*y;
>> resids = y - x*bhat;
>> s2 = resids'*resids/(n-2);
```

Exercise 2 (b): OLS Estimation Using Artificial Data

- How well does the estimated model fit the data? Let's attempt to measure this in terms of the proportion of the variation in y explained by the model. We calculate R^2.

$$R^2 = 1 - \frac{\sum e_i^2}{\sum (y_i - \bar{y})^2} = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2},$$

$$= 1 - \frac{RSS}{TSS} \frac{\leftarrow \text{Residual sum of squares}}{\leftarrow \text{Total sum of squares}}.$$

- where $\bar{y} = \sum y_i / n$ is the sample mean and $\sum$ denotes $\sum_{i=1}^{n}$.

Type:

```
>> yhat = x*bhat;
>> Rsq = 1 - sum((y - yhat).^2)/sum((y - mean(y)).^2);
>> disp 'The R-squared of the regression is';
>> disp(Rsq);
```

Exercise 2 (c): OLS Estimation Using Artificial Data

- Recall that $b = \beta + (X'X)^{-1}X'\varepsilon$. The conditional covariance matrix of b is

$$\begin{aligned}
\operatorname{var}(b \mid X) &= E[(b-\beta)(b-\beta)' \mid X] \\
&= E[(X'X)^{-1}X'\varepsilon\varepsilon'X(X'X)^{-1} \mid X] \\
&= (X'X)^{-1}X'E(\varepsilon\varepsilon' \mid X)X(X'X)^{-1} \\
&= \sigma^2(X'X)^{-1}X'X(X'X)^{-1} \text{ since } E(\varepsilon\varepsilon' \mid X) = \sigma^2 I_n \\
&= \sigma^2(X'X)^{-1}.
\end{aligned}$$

Type:

```
varb = s2*inv(x'*x);
disp 'The covariance matrix of the OLS estimator is';
disp(varb);
```

➢ Exercise 3 (a): For *loops* and *if* statements

Type: >> x=[1;2;7;5;9;3;6;9;1;11;1];

- This command returns an 11-by-1 column vector of *x*.

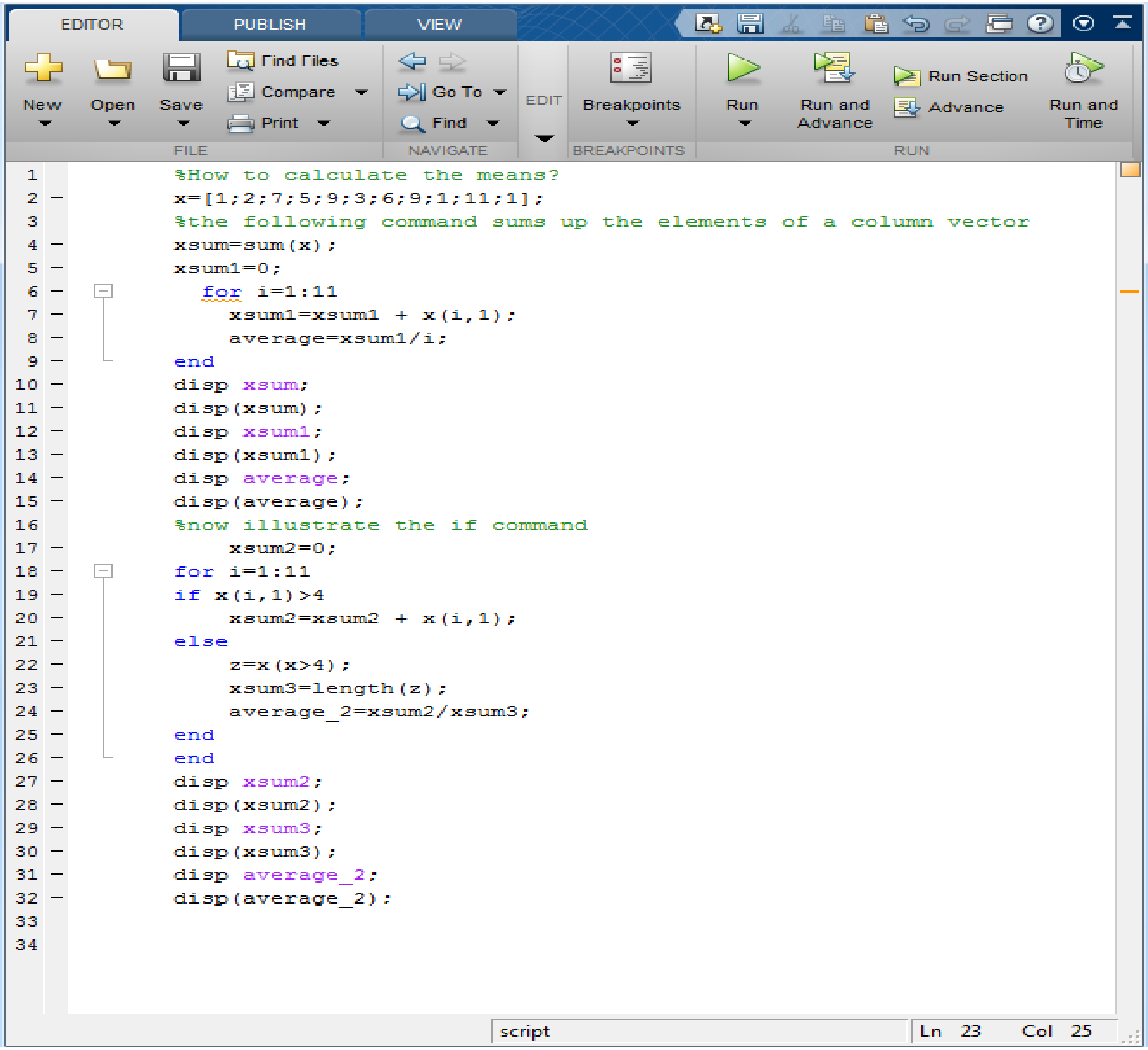
>> xsum=sum(x); xsum1=0;

- First command sums up the elements of the vector *x* and the second command returns a zero element.

- The *for* command, iterates over procedure “xsum1 = xsum1 + x(i,1)”; incrementing *i* from 1 to 11 by 1. In other words, it computes the sum of all numbers from 1 to 11 in the procedure.

- The *If* command proceeds as follows, *if* criteria “x(i,1) > 4” is true do “xsum2=xsum2 + x(i,1)”.

➢ Exercise 3 (b): For *loops* and *if* statements



➢ Exercise 4: Drawing from Standard Distributions Uniform Distribution

- To generate a uniform random number, use the MATLAB command “rand”. (type ‘help rand’ in console)

Type: >> tempp = rand(num_draws, 1);

- “rand(*m*, *n*)” command returns *m*-by-*n* matrix of independent uniformly distributed random entries. In the example, it will generate a 10-by-1 column vector of uniformly distributed random numbers.

>> unifans = [mean(tempp) std(tempp)];

- “tempp” is an 10-by-1 column vector, “mean(tempp)” returns a row vector of size 1 whose *j*th entry is the mean of the *j*th column of “num_draws”. “std(tempp)” returns the standard deviation of “tempp”.

➢ Exercise 4 (cont.): Drawing from Standard Distributions Normal Distribution

- Obtaining draws from normal random number generator in MATLAB can be done by the function "randn" and its functionality is similar to the function "rand".

Type: >> tempp = randn(num_draws, 1);

- Generates an 10-by-1 vector of normally distributed random numbers.

>> normans = [mean(tempp) std(tempp)];

- "tempp" is an 10-by-1 column vector, "mean(tempp)" returns a row vector of size 1 whose *j*th entry is the mean of the *j*th column of "num_draws". "std(tempp)" returns the standard deviation of "tempp".

Exercise 4 (cont.): Drawing from Standard Distributions
Student's-*t* Distribution

Type: >> tempp = trnd(3, num_draws, 1)

- "trnd" command generates a 10-by-1 column vector of random numbers from Student's *t* distribution with degrees of freedom set to $v = 3$.

Exercise 4 (cont.): Drawing from Standard Distributions
Beta Distribution

- Generate random variables from the Beta distribution with parameters using the command "betarnd(α_1, α_2)" by setting $\alpha_1 = 3$ and $\alpha_2 = 2$.

Type >> tempp = betarnd(3, 2, num_draws, 1);

- The command returns a 10-by-1 column vector of random numbers generated from the Beta distribution.

➢ Exercise 4 (cont.): Drawing from Standard Distributions Exponential Distribution

Type: >> tempp = exprnd(5, num_draws, 1);

- This command returns random numbers from exponential distribution with mean parameter set to μ = 5 and number of draws set to a column vector with 10 rows.

Exercise 4 (cont.): Drawing from Standard Distributions
Chi-Square Distribution

- A Chi-square distribution with *v* degrees of freedom calls for the use of "chi2rnd2" command. It generates random numbers from the Chi square distribution.

Type: >> tempp = chi2rnd(3, num_draws, 1);

- The command returns a 10-by-1 column vector of random numbers.

➢ Exercise 4 (cont.): Drawing from Standard Distributions
Gamma Distribution

- We can generate random numbers from the Gamma distribution with mean μ and degrees of freedom v as follows

```
>> tempp = gamrnd(4, 2, num_draws,1);
```

- The command returns a 10-by-1 column vector of random numbers generated from the Gamma distribution with $\mu = 4$ and $v = 2$.

➢ Exercise 5: Monte Carlo Integration

- Suppose that the posterior density

$$p(\theta \mid y),$$

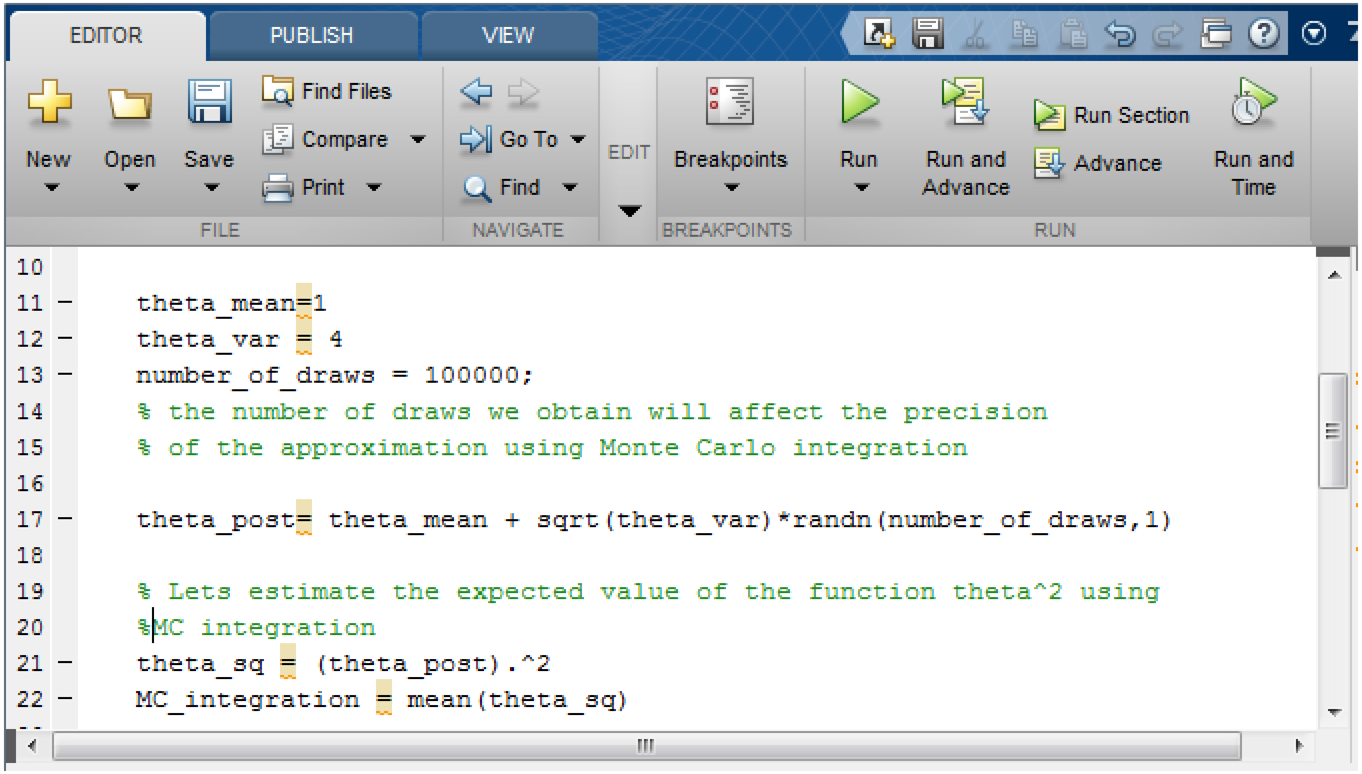
- has a known form. Thus, we can generate R i.i.d. random draws of the parameters, $\theta^{(r)}$, for $r = 1, \ldots, R$.
- But we are interested in the parameters with a functional form $g(\theta)$.
- To estimate the $E(g(\theta) \mid y)$, sampling methods use a sample average as

$$E(g(\theta) \mid y) = \int g(\theta) P(\theta \mid y) d\theta \approx \frac{\sum_{r=1}^{R} g\left(\theta^{(r)}\right)}{R}.$$

➢ Exercise 5: Monte Carlo Integration

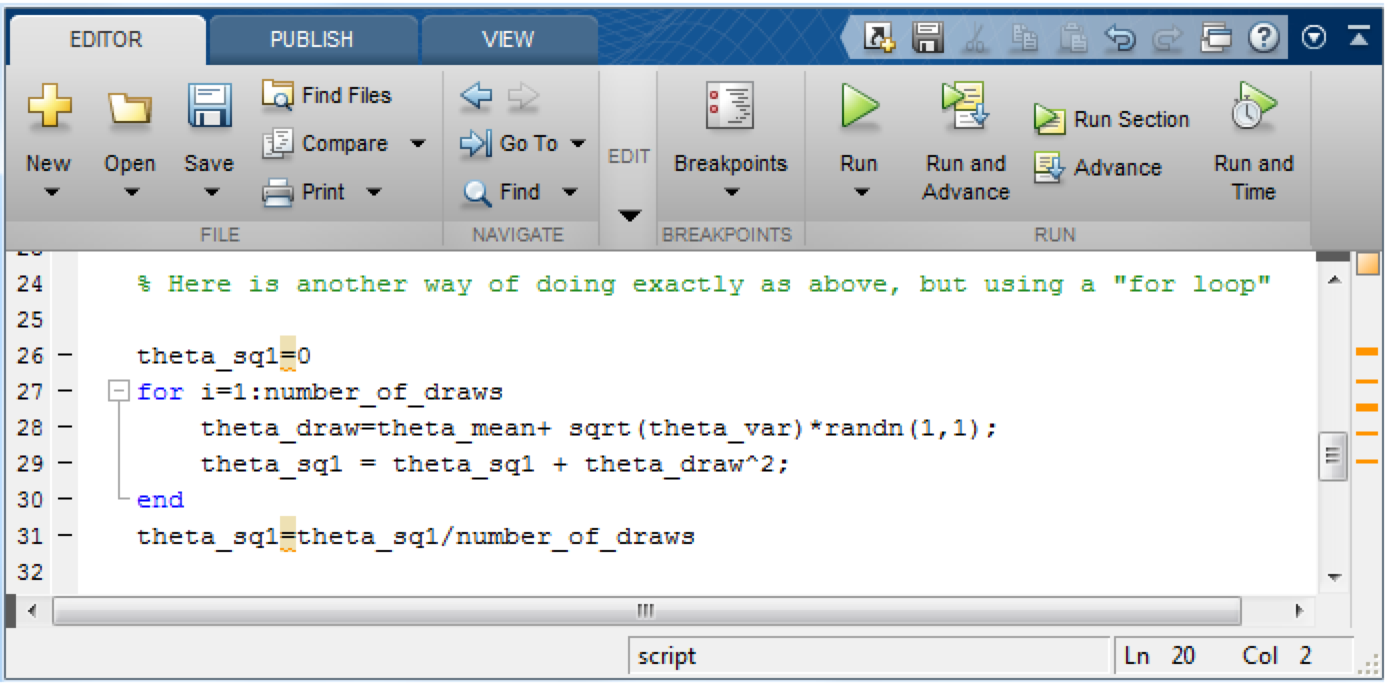
- Lets generate the parameter of interest from the function $g(\theta) = \theta^2$ from a Normal density

$$p(\theta \mid y) \sim N(1,4).$$



➤ Exercise 5 (cont.): Monte Carlo Integration

- Alternative method using control flow



References

- Koop, G. (2003), Bayesian econometrics, Wiley.

- Koop, G., Poirier, D. and Tobias, J. (2007), Bayesian econometric methods. Cambridge: Cambridge University Press.