Factor Models

Introduction

- Macro researchers usually have dozens or hundreds of time series variables to work with
- Especially when forecasting, good to use as much information as possible.
- Want to use lots of variables, but VAR methods suffer proliferation of parameters
- Bayesian large VAR solution: use prior shrinkage on coefficients (e.g. Minnesota prior)
- Another solution: compress the data and then work with parsimonious model
- This is what factor methods do

Static Factor Model

- y_t is $M \times 1$ vector of time series variables
- M is very large
- y_{it} denote a particular variable.
- Simplest static factor model:

$$y_t = \lambda_0 + \lambda f_t + \varepsilon_t$$

- f_t is $q \times 1$ vector of unobserved latent factors (where $q << M$)
- Factors contain information extracted from all the M variables.
- Same f_t occurs in every equation for y_{it} for $i = 1, ..., M$
- But different coefficients (λ is an $M \times q$ matrix of so-called factor loadings).

- Note that restrictions are necessary to identify the model
- Common to say ε_t is i.i.d. $N(0, D)$ where D is diagonal matrix.
- Implication: ε_{it} is pure random shock specific to variable i, co-movements in the different variables in y_t arise only from the factors.
- Note also that $\lambda f_t = \lambda C C^{-1} f_t$ which shows we need identification restriction for factors too.
- Different models arise from different treatment of factors.
- Simplest is: $f_t \sim N(0, I)$
- This can be interpreted as a state equation for "states" f_t
- Factor models are state space models — so Bayesian MCMC methods for state space models can be used

The Dynamic Factor Model (DFM)

- In macroeconomics, usually need to extend static factor model to allow for the dynamic properties which characterize macroeconomic variables.
- A typical DFM:

$$\begin{aligned} y_{it} &= \lambda_{0i} + \lambda_i f_t + \varepsilon_{it} \\ f_t &= \Phi_1 f_{t-1} + .. + \Phi_p f_{t-p} + \varepsilon_t^f \end{aligned}$$

- f_t is as for static model
- λ_i is $1 \times q$ vector of factor loadings.
- Each equation has its own intercept, λ_{0i}.
- ε_{it} is i.i.d. $N\left(0, \sigma_i^2\right)$
- f_t is VAR with ε_t^f being i.i.d. $N\left(0, \Sigma^f\right)$
- Note: usually ε_{it} is autocorrelated (easy extension, omitted here for simplicity)

Replacing Factors by Estimates: Principal Components

- Proper Bayesian analysis of the DFM treats f_t as vector of unobserved latent variables.
- Before doing this, we note a simple approximation.
- The DFM has similar structure to regression model:

$$y_{it} = \lambda_{0i} + \widetilde{\lambda}_{0i} f_t + .. + \widetilde{\lambda}_{pi} f_{t-p} + \widetilde{\varepsilon}_{it}$$

- If f_t were known we could use Bayesian methods for the multivariate Normal regression model to estimate or forecast with the DFM.
- Principal components methods to can be used to approximate f_t.
- Precise details of how principal components is done provided many places

Treating Factors as Unobserved Latent Variables

- DFM is a Normal linear state space model so can use Bayesian methods for state space models
- A bit more detail on MCMC algorithm:
- Conditional on the model's parameters, $\Sigma^f, \Phi_1, .., \Phi_p, \lambda_{0i}, \lambda_i, \sigma_i^2$ for $i = 1, .., M$, use (e.g.) Carter and Kohn algorithm to draw f_t
- Conditional on the factors, measurement equations are just M Normal linear regression models.
- Since ε_{it} is independent of ε_{it} for $i \neq j$, posteriors for $\lambda_{0i}, \lambda_i, \sigma_i^2$ in the M equations are independent over i
- Hence, the parameters for each equation can be drawn one at a time (conditional on factors).
- Finally, conditional on the factors, the state equation is a VAR
- Any of the methods for Bayesian VARs of Lecture 2 can be used.

The Factor Augmented VAR (FAVAR)

- DFMs are good for forecasting (extract all information in huge number of variables)
- VARs are good for macroeconomic policy (e.g. impulse responses).
- Why not combine DFMs and VARs together to get model which can do both?
- FAVAR results
- Bernanke, Boivin and Eliasz (2005, QJE) is pioneering paper

Impulse Response Analysis in DFM

- With VARs impulse responses based on structural VAR:

$$C_0 y_t = c_0 + \sum_{j=1}^{p} C_j y_{t-j} + u_t$$

- u_t is i.i.d. $N(0, I)$ and C_0 chosen to give shocks structural interpretation
- If $C(L) = C_0 - \sum_{j=1}^{p} C_j L^p$ impulse responses obtained from VMA:

$$y_t = C(L)^{-1} u_t$$

- With the DFM, can obtain VMA representation for y_t by substituting in factor equation:

$$\begin{aligned} y_t &= \varepsilon_t + \lambda \Phi(L)^{-1} \varepsilon_t^f \\ &= B(L) \eta_t \end{aligned}$$

- But η_t combines ε_t and ε_t^f — cannot isolate "shock to interest rate equation" as monetary policy shock and do impulse response analysis in standard way.

- FAVAR modifies DFM by adding other explanatory variables:

$$y_{it} = \lambda_{0i} + \lambda_i f_t + \gamma_i r_t + \varepsilon_{it}$$

- r_t is $k_r \times 1$ vector of observed variables of key interest.
- E.g. Bernanke, Boivin and Eliasz (2005) set r_t to be the Fed Funds rate (a monetary policy instrument)
- All other assumptions are same as for the DFM.
- Note: by treating r_t in this way, we can isolate a "monetary policy shock" and calculate impulse responses

- FAVAR state equation extends DFM state equation to include r_t:

$$\begin{pmatrix} f_t \\ r_t \end{pmatrix} = \widetilde{\Phi}_1 \begin{pmatrix} f_{t-1} \\ r_{t-1} \end{pmatrix} + .. + \widetilde{\Phi}_p \begin{pmatrix} f_{t-p} \\ r_{t-p} \end{pmatrix} + \widetilde{\varepsilon}_t^f$$

- where all assumptions are same as DFM with extension that $\widetilde{\varepsilon}_t^f$ is i.i.d. $N\left(0, \widetilde{\Sigma}^f\right)$
- MCMC is very similar to that for the DFM and will not be described here.
- Similar ideas:
- Normal linear state space algorithms can draw f_t
- Measurement equation is series of regressions (conditional on factors)
- The state equation is a VAR (conditional of factors)

Impulse Response Analysis in FAVAR

- FAVAR model can be written:

$$\begin{pmatrix} y_t \\ r_t \end{pmatrix} = \begin{bmatrix} \lambda & \gamma \\ 0 & 1 \end{bmatrix} \begin{pmatrix} f_t \\ r_t \end{pmatrix} + \widetilde{\varepsilon}_t$$
$$\begin{pmatrix} f_t \\ r_t \end{pmatrix} = \widetilde{\Phi}_1 \begin{pmatrix} f_{t-1} \\ r_{t-1} \end{pmatrix} + .. + \widetilde{\Phi}_p \begin{pmatrix} f_{t-p} \\ r_{t-p} \end{pmatrix} + \widetilde{\varepsilon}_t^f$$

- where $\widetilde{\varepsilon}_t = (\varepsilon_t', 0)'$
- VMA obtained by substituting second equation in first and re-arranging

$$\begin{pmatrix} y_t \\ r_t \end{pmatrix} = \begin{bmatrix} \lambda & \gamma \\ 0 & 1 \end{bmatrix} \widetilde{\Phi}(L)^{-1} \widetilde{\varepsilon}_t^f + \widetilde{\varepsilon}_t$$
$$= \widetilde{B}(L)\, \eta_t$$

- Now last k_r elements of η_t are solely associated with original VAR-like equations for r_t and impulse responses with conventional interpretation can be done (e.g. "shock to interest rate equation" can be "monetary policy shock")

The TVP-FAVAR

- With VARs: began with constant parameter model
- then we said it is good to allow the VAR coefficients to vary over time: homoskedastic TVP-VAR
- then we said good to allow for multivariate stochastic volatility: heteroskedastic TVP-VAR
- Recent research is doing the same with FAVARs
- In tomorrow's lecture I will provide an extended example from my recent research
- Note: just as with TVP-VARs, TVP-FAVARs can be over-parameterized and careful incorporation of prior information or the imposing of restrictions (e.g. only allowing some parameters to vary over time) can be important in obtaining sensible results.

- A TVP-FAVAR is just like a FAVAR but with t subscripts on parameters:

$$y_{it} = \lambda_{0it} + \lambda_{it} f_t + \gamma_{it} r_t + \varepsilon_{it},$$

- $$\begin{pmatrix} f_t \\ r_t \end{pmatrix} = \widetilde{\Phi}_{1t} \begin{pmatrix} f_{t-1} \\ r_{t-1} \end{pmatrix} + .. + \widetilde{\Phi}_{pt} \begin{pmatrix} f_{t-p} \\ r_{t-p} \end{pmatrix} + \widetilde{\varepsilon}_t^f$$

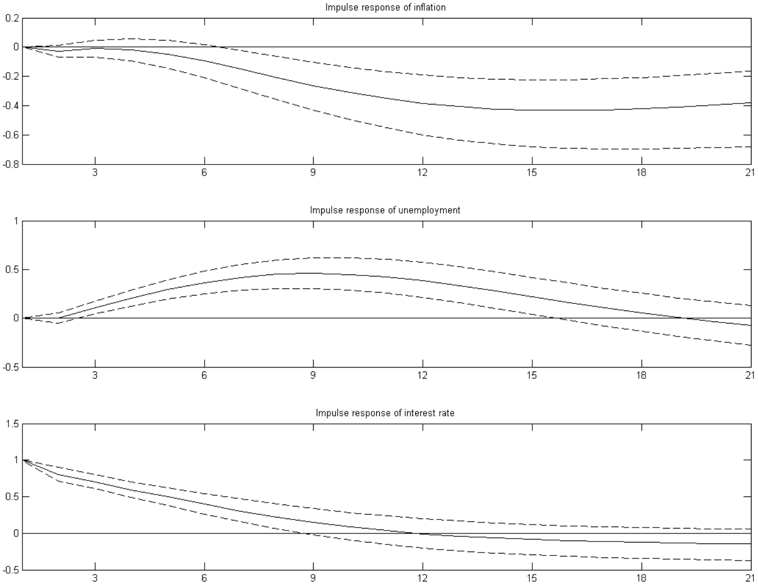
- All each ε_{it} to follow univariate stochastic volatility process
- $var\left(\widetilde{\varepsilon}_t^f\right) = \widetilde{\Sigma}_t^f$ has multivariate stochastic volatility process of the form used in Primiceri (2005).
- Finally, the coefficients (for $i = 1, .., M$) $\lambda_{0it}, \lambda_{it}, \gamma_{it}, \widetilde{\Phi}_{1t}, .., \widetilde{\Phi}_{pt}$ are allowed to evolve according to random walks (i.e. state equations of the same form as in the TVP-VAR complete the model).
- All other assumptions are the same as for the FAVAR.

Bayesian Inference in the TVP-FAVAR

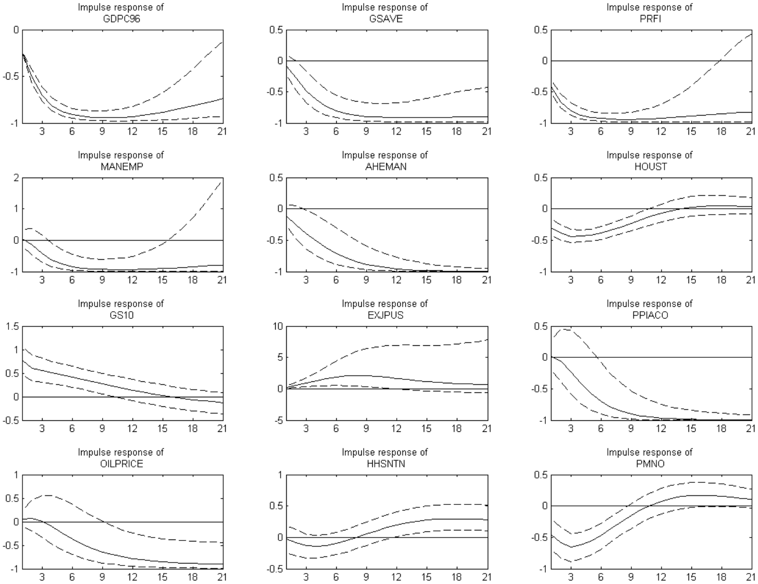
- I will not provide details of MCMC algorithm
- Note only it adds more blocks to the MCMC algorithm for the FAVAR.
- These blocks are all of forms discussed in previous lecture.
- E.g. error variances in measurement equations drawn using the univariate stochastic volatility algorithm of Kim, Shephard and Chib (1998).
- Multivariate stochastic volatility algorithm of Primiceri (2005) can be used to draw $\widetilde{\Sigma}_t^f$.
- The coefficients $\lambda_{0it}, \lambda_{it}, \gamma_{it}, \widetilde{\Phi}_{1t}, .., \widetilde{\Phi}_{pt}$ are all drawn using algorithm for Normal linear state space model

Empirical Illustration of the FAVAR and TVP-FAVAR

- 115 quarterly US macroeconomic variables spanning 1959Q1 though 2006Q3.
- Transform all variables to be stationarity.
- What variables to put in r_t?
- Inflation, unemployment and the interest rate.
- FAVAR is same as VAR from previous illustrations, but augmented with factors, f_t
- We use 2 factors and 2 lags in state equation
- Indentify impulse responses as in our VAR empirical illustration plus Bernanke, Boivin and Eliasz (2005).

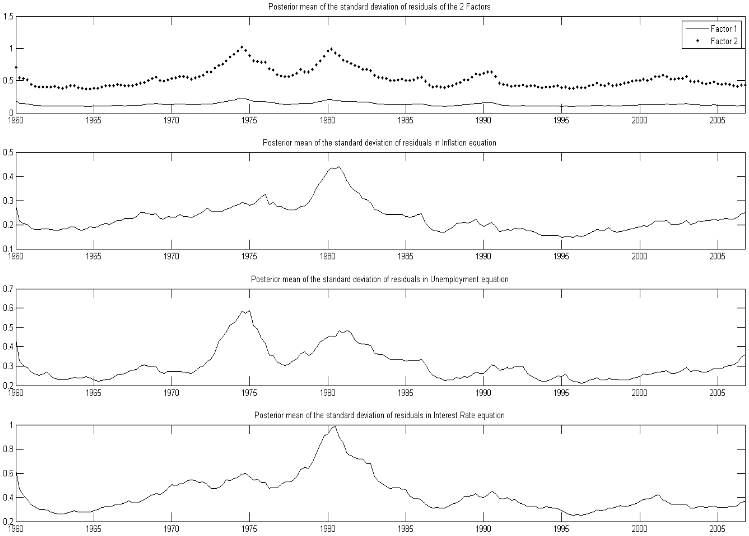


Posterior of impulse responses of main variables to monetary policy shock

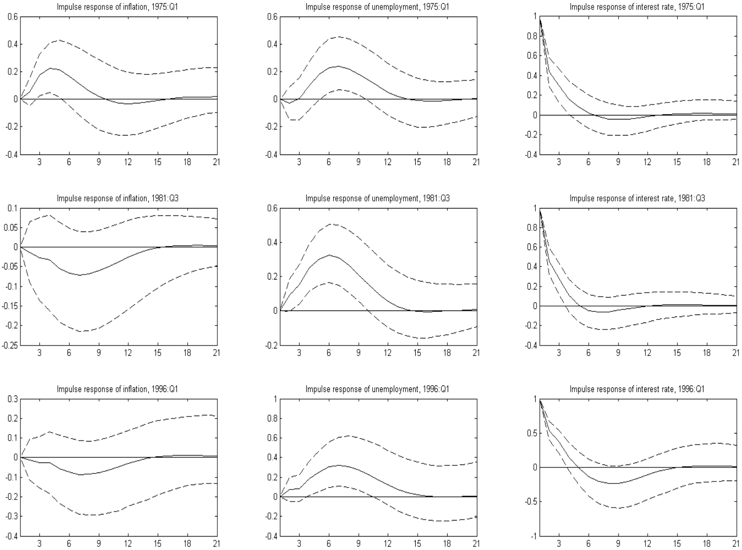


Posterior of impulse responses of selected variables to monetary policy shock

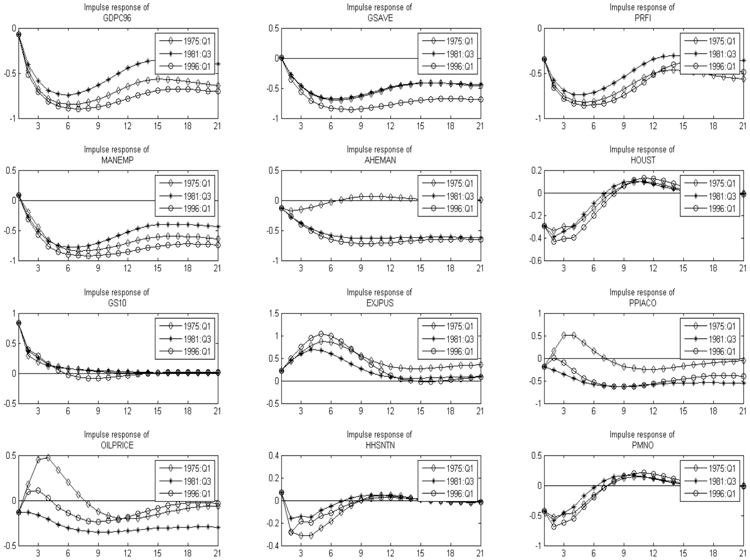
- Now TVP-FAVAR
- Illustrate time varying volatility of equations for r_t and factor equations
- Impulse responses at three different time periods



Time-varying volatilities of errors in five key equations of the TVP-FAVAR



Posterior of impulse responses of main variables to monetary policy shock at different times



Posterior means impulse responses of selected variables to monetary policy shock at different times

Summary

- Factor methods are an attractive way of modelling when the number of variables is large
- DFMs are attractive for forecasting
- FAVARs attractive for macroeconomic policy (e.g. to do impulse response analysis)
- Recently TVP versions of these models have been developed
- Bayesian inference in TVP-FAVAR puts together MCMC algorithm involving blocks from several simple and familiar algorithms.