Computer Tutorial 5: The Unobserved Components Stochastic Volatility Model

Data and Matlab code for all questions are available on the course website.

The unobserved components stochastic volatility (UCSV) model of Stock and Watson (2007) "Why Has U.S. Inflation Become Harder to Forecast?," Journal of Money, Credit and Banking is given by their equations (8) - (11) which we replicate here:

$$\begin{aligned}
\pi_t &= \tau_t + \eta_t, \;\; \eta_t \sim N\left(0, \sigma_t^{\eta}\right) \\
\tau_t &= \tau_{t-1} + \varepsilon_t, \;\; \varepsilon_t \sim N\left(0, \sigma_t^{\varepsilon}\right) \\
\log\left(\sigma_t^{\eta}\right) &= \log\left(\sigma_{t-1}^{\eta}\right) + v_t^{\eta}, \;\; v_t^{\eta} \sim N\left(0, \gamma_1\right) \\
\log\left(\sigma_t^{\varepsilon}\right) &= \log\left(\sigma_{t-1}^{\varepsilon}\right) + v_t^{\varepsilon}, \;\; v_t^{\varepsilon} \sim N\left(0, \gamma_2\right)
\end{aligned}$$

This is a state space model which assumes inflation (π_t) is composed of trend inflation (τ_t) which is not directly observed but is assumed to follow a random walk and a transitory component η_t which is uncorrelated over time. The transitory component and the error in the equation for trend inflation are assumed to have stochastic volatility. The data set contains three measures of inflation , CPI inflation, PPI inflation and GDP deflator inflation. Use the code to plot trend inflation and the volatilities σ_t^{η} and σ_t^{ε}. Is there evidence that σ_t^{η} is varying over time? Is there evidence that σ_t^{ε} is varying over time?

Optional things you may wish to explore: Stock and Watson (2007) also estimate a model (see their equations (5) and (6)) where state and measurement equation variances are constant ($\sigma_t^{\eta} = \sigma^{\eta}$ and $\sigma_t^{\varepsilon} = \sigma^{\varepsilon}$). You can also consider models where there is stochastic volatility in one equation but not the other (i.e. $\sigma_t^{\varepsilon} = \sigma^{\varepsilon}$ but σ_t^{η} is time varying or $\sigma_t^{\eta} = \sigma^{\eta}$ but σ_t^{ε} is time varying). Modify the code to estimate these models and compare results to the full model.