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# The role of intratemporal adjustment costs in a multisector economy<sup>1</sup>

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#### Abstract

In this paper we construct a multisector business cycle model which is capable of reproducing the procyclical behavior of cross-sector measures of capital, employment and output. We start by documenting the difficulty that a standard variant of a conventional real business cycle model has in accounting for these facts. We then show how the introduction of intratemporal adjustment costs for investment can significantly enhance the performance of such a model. These costs make it difficult to alter the composition of production of new capital goods. The presence of these costs eliminates many counterfactual observations of the model that would otherwise be present. The dynamic response of variables in the model is different from what one would observe in the standard one-sector model. We also examine the implications of imposing intratemporal adjustment costs for labor. The model can also account for the cross-sector behavior of employment observed in the data. © 1999 Elsevier Science B.V. All rights reserved.

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#### 1. Introduction

Modern dynamic general equilibrium models of the business cycle were developed to better understand and explain the salient features of periodic fluctuations in aggregate economic activity.<sup>2</sup> However, to date there has been relatively little research using these models to account for the cyclical behavior of investment, capital, employment and output across sectors. Much of the focus of the existing business-cycle literature is on the behavior of aggregate hours, as well as the causes and consequences of movements in this variable. It thus seems only natural to explore the cyclical behavior of sectoral movements in factors of production and output. Most of the existing models are reticent on this issue and, as will be shown below, possibly for good reason. This paper confronts the facts about sectoral movements in inputs and outputs by constructing and analyzing a simple model that is capable of speaking to these issues. We show how the standard business-cycle framework can be augmented to produce a multisector model whose behavior is consistent with many features of the post-war US data.

A defining characteristic of business cycles, whether in the traditional sense of Burns and Mitchell (1946) or in the contemporary sense of Lucas (1977), is the comovement in the pace of economic activity in *different sectors of the economy*. Burns and Mitchell (1946) emphasize this comovement in their definition of the business cycle.<sup>3</sup> Lucas (1977) notes that the comovement of economic activity across different sectors of the economy is the most important of the regularities common to all business cycles and that this comovement creates the potential for a single unified theory of business cycles.<sup>4</sup> Thus, it is puzzling that so much of the general equilibrium business-cycle literature has tended to ignore this feature when analyzing whether certain models are consistent with certain business-cycle observations.

As shown below, while the levels of investment and employment in various sectors of the economy do not move perfectly in tandem, almost all of these variables do move in a procyclical manner. Although one might suspect that this is an easily explainable observation, this paper shows that it is not straightforward to produce a variant of a real business-cycle model which is capable of

<sup>&</sup>lt;sup>2</sup> For a good review of the current state of this literature see the recent volume by Cooley (1995).

<sup>&</sup>lt;sup>3</sup> Burns and Mitchell (1946) (p. 3) state the following: "Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time *in many economic activities*, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own." (emphasis added.)

<sup>&</sup>lt;sup>4</sup>Lucas employs a somewhat different definition of the business cycle than do Burns and Mitchell, as consisting of fluctuations about trend.

reproducing the key facts. Furthermore, replicating this feature would seem to be a necessary first step in producing a disaggregated real business-cycle model capable of analyzing the sectoral movements in the factors of production and output.

In fact, one of the motivations for the work of Benhabib, Rogerson, and Wright (1991) is that within a standard business-cycle model (specifically, that of Hansen, 1985), employment in the consumption sector is strongly countercyclical. They argue that this property of the model is at variance with the fact that employment in virtually all sectors appears to be procyclical. However, theirs is not a multisector model in the sense employed here, since their second sector is home production, and obviously some nontrivial fraction of the labor 'employed' in this sector could reasonably be interpreted as leisure. <sup>5</sup> Hence, employment in one of their sectors must necessarily be countercyclical. Furthermore, although their introduction of home production reduces the strong negative correlation between employment in the consumption sector and aggregate output, this correlation is not strongly positive. Additionally, they do not address the issue of the sectoral behavior of investment. Greenwood and Hercowitz (1991) study a model in which there is investment in both home and market production, but again they only have one market sector, and consequently do not speak to the issues addressed here.6

A central assumption in what follows is that it is costly to alter the composition of capital goods produced in an economy. Consider an economy composed of many market sectors which have different technologies for producing various goods and services. Examples of these sectors might be construction, manufacturing, farming, mining, services, and so on. Now consider a technological innovation that occurs in, say, the manufacturing sector which enhances the productivity of the factors employed therein. Other things being equal, this shock will lead to increased investment and employment in this sector and increased output in subsequent periods. It is unlikely, however, that a significant quantity of existing capital would move from the other sectors to employment in the sector that has just experienced the favorable shock. In particular, it is

<sup>&</sup>lt;sup>5</sup>The term multisector used here refers to the fact that there is more than one production technology in the market sector, and these technologies employ nontrivial amounts of capital and labor.

<sup>&</sup>lt;sup>6</sup>Benhabib et al. (1991) employ unobservable shocks to home production which are highly correlated with the market shocks. Greenwood and Hercowitz (1991) assume that the exogenous shocks to market and home production are the same. In contrast, we employ productivity disturbances that are directly measurable. Christiano and Fisher (1995) also study the issue of comovement of activity in a model with two sectors. They find it necessary to have both habit persistence in preferences, and limited labor mobility in order to have employment in both sectors moving procyclically. They do not address the issue of the procyclical nature of investment in different sectors.

doubtful that tractors used in farming, restaurant equipment, or drilling equipment in mines, could be rapidly moved into production in the manufacturing sector to take advantage of favorable production opportunities in that sector. More importantly, it seems equally implausible that the processes that produce these capital goods could be easily changed to produce a different mix of new capital goods in response to the shock. For example, the factors of production used to produce computers cannot be quickly or easily converted into the equipment and skilled labor needed to produce heavy industrial equipment. Within a given period there may be plenty of capital, or even labor, which is sector-specific. Higher demand for, say, capital in one sector does not give rise to the rapid movement of capital from other sectors. What we are proposing is somewhat different, namely a cost of reorienting the production of new capital goods from types destined for one sector to types destined for another. We refer to this cost as an intratemporal adjustment cost. Furthermore, there may also be such costs associated with reallocating labor from one sector to another.

It is worth asking at the outset why it is necessary to construct a multisector model of the business cycle, given that it is possible to understand much of the cyclical nature of market economies within a highly aggregated framework. The answer is that by understanding the cross-sectoral movements in inputs and outputs that take place at business-cycle frequencies, we will enhance our understanding of the causes and effects of cyclical fluctuations. For example, one would be more confident in the predictions of general equilibrium models if they were shown to be consistent with the sectoral movements in employment as observed by, say, Lilien (1982), and studied by Loungani and Rogerson (1989). Ultimately, it would be of interest to know how technological innovations or government policies influence the allocation of the factors of production across sectors. It is likely that distortional government policies influence the amount of capital and labor employed in various sectors. Investment tax credits or capital taxation will have different impacts on the amount of capital employed in various sectors, depending on how important capital is in the production process. It is not obvious how government policies influence the allocation of factors of production in various sectors, and thereby the cyclical behavior of the economy. The older monetary business cycle literature addressed the question of how shocks to monetary policy affect different sectors of the economy (see Kretzmer, 1989). It would be interesting to see if the stylized facts uncovered in that literature could also be accounted for by a dynamic general equilibrium model driven by technological shocks.

<sup>&</sup>lt;sup>7</sup> Murphy et al. (1989) also argue that one-sector models are deficient in that they cannot explain the cross-sectoral movement of outputs and labor inputs. However, they suggest that immobile labor may be of central importance. The model studied below suggests that immobility in labor may not be as important as immobility in capital.

Yet another reason to study a disaggregated model is that some sectoral data are useful for other purposes. For example, the Conference Board's Index of Leading Indicators includes many sector-specific variables such as manufacturers' unfilled orders, permits for housing units, contracts for plant and equipment, manufacturers' new orders, and weekly hours of production of workers in manufacturing. Presumably, a better understanding of the business cycle would be gained by studying models in which these same variables were leading indicators as well. Furthermore, it is possible that simple models which are capable of reproducing similar aggregate business cycles, can have quite different predictions for the cross-sector movements in inputs and outputs. Therefore a close investigation into this topic can serve as a useful tool for discriminating between alternative models which have differing predictions. Finally, it seems likely that it will be necessary to move to a multisectoral framework if unemployment is to be introduced in a meaningful manner into general equilibrium models of the business cycle.<sup>8</sup>

The analysis below complements that of Hornstein and Praschnik (1994), who are also interested in understanding the comovement of output, employment, and investment in a multisector setting. Hornstein and Praschnik study a simple two-sector neoclassical model. The key difference between their analysis and that presented below is in the mechanism used to induce comovement. In their setup, the output of one sector is used as an intermediate input in the other sector. Specifically, they argue that the use of the output of the nondurable goods sector as an intermediate input in the production of durable goods is the key to generating plausible comovement of output, employment, and investment across the two sectors. This is tantamount to assuming that all sectors produce capital goods, with some capital goods being longer-lived than others. In our model, comovement comes about as a result of joint production in one sector and limited possibilities for switching production between the two types of capital. In our model, only one sector produces capital or investment goods for future use in both sectors.

The issue of sectoral comovement was explicitly addressed in one of the earliest papers in the real business-cycle literature, namely Long and Plosser (1983). They develop a simple model aimed at demonstrating the possibility of a sectoral comovement in output similar to that characterizing the business cycle. The output of each sector can be used as an input in the production of all other goods. However, as a result of the particular specification of tastes and technology that they utilize, employment in each sector is independent of current technology shock realizations and is constant over the cycle. We will

<sup>&</sup>lt;sup>8</sup> This argument was made by Black (1987). See recent attempts along this line by Greenwood et al. (1996).

show that a close variant of the Long and Plosser (1983) model is unlikely to give rise to cross-sector business-cycle behavior resembling that observed in the data.<sup>9</sup>

In the next section, some of the cross-sector features of the US data are described to illustrate the procyclical nature of the inputs and outputs of various sectors. In Section 3 we describe the physical environment of the model economy, starting with a counterfactual economy that does not have intratemporal adjustment costs and which is not capable of mimicking many of the observed cross-sector features of the US economy. We then introduce these costs and show how the behavior of the model is much more in accord with that of the US data. Next, we show how various variables move in reaction to technological innovations in the various sector. We also demonstrate that the model is able to reproduce the positive (negative) relationship between unemployment (employment) and the measure of dispersion of employment growth described by Lilien (1982). Section 4 presents our final remarks and conclusions.

# 2. Stylized facts about sectoral economic activity

We begin by documenting some of the facts about the cyclical behavior of the economy at the sectoral level so that we have a benchmark against which to measure the performance of the model economy. Some basic facts about sectoral economic activity are presented in Tables 1 and 2.<sup>10</sup> We might note that a major hindrance to a comprehensive investigation of sectoral business cycles is the paucity of data on sectoral economic activity that are available at a monthly or quarterly frequency for the US economy. At a quarterly frequency, only data on the labor input (measured either in terms of hours or number of employees) is available.

The model studied below will have only two sectors so it is necessary to combine the sectoral data that is available into broader aggregates that correspond in some sense to the sectors in our model. The sectors in the model produce consumption goods and investment goods. The latter are long-lived and are used as capital inputs in each sectors production process, while the former are nonstorable and enter only into consumers utility functions. To allocate a sector to the consumption or investment category, we used the 1987 input—output tables (see Lawson and Teske, 1994) to determine how much of

<sup>&</sup>lt;sup>9</sup> The importance of looking at multisector models is also illustrated by Horvath (1996). He looks at the framework of Long and Plosser (1983) to see how the interaction between sectors influences the degree to which sector-specific shocks can produce aggregate fluctuations.

<sup>&</sup>lt;sup>10</sup> A more extensive set of tables is presented in Huffman and Wynne (1995).

a sector's final output goes to consumption as opposed to investment or intermediate uses. If the bulk of a sector's final output is allocated to final consumption demand it is classified as a consumption sector; if the bulk of a sector's output is allocated to investment or intermediate demand, the sector is classified as an investment sector. Using this criterion, the mining, construction, manufacturing, transportation and public utilities, and wholesale trade sector are grouped into the investment category, and finance, insurance and real estate (FIRE), retail trade and services sector are grouped into the consumption category.

Table 1 presents the correlation of Hodrick-Prescott-filtered hours and employment in the consumption and investment sectors of the US economy with (HP-filtered) aggregate output at various leads and lags. 11 Not surprisingly, the labor input in both sectors is strongly procyclical. This finding also characterizes the cyclical behavior of the labor input at a greater degree of disaggregation (Huffman and Wynne, 1995), with the interesting exception of mining. The labor input in mining tends to be slightly countercyclical: high levels of aggregate activity (relative to trend) tend on average to be associated with low levels of employment in mining (relative to trend). This reflects the fact that the mining category includes the oil drilling activities of the oil sector. The labor input in most sectors tends to lag the cycle slightly, in the sense that the one quarter lead of the labor input tends to have the highest correlation with output. As for the volatility of the labor input across sectors, both hours and employment are more volatile in the investment sector than in the consumption sector. Note that the labor input in the investment sector tends to be more strongly correlated with aggregate activity than the labor input in the consumption sector, in addition to being a lot more volatile than the labor input in the consumption goods sector. This finding also holds at a more disaggregated level: employment in goodsproducing sector is systematically more volatile than employment in serviceproducing sector. Much the same pattern is evident when the pattern of hours is studied: again, hours in mining and construction are five times more volatile than hours in services.

As has already been noted, it is only possible to obtain sectoral data on the labor input at a quarterly frequency: data on output, productivity and capital stocks are only available at an annual frequency. Table 2 presents the correlations of a variety of indicators of sectoral activity with aggregate output at various leads and lags at the annual frequency. Note that the output of the

<sup>&</sup>lt;sup>11</sup> We focus on the eight sectors that make up the nonagricultural business sector. The idiosyncratic nature of the shocks that affect the agricultural and government sectors make them of less interest to our purpose here. We define aggregate output as the sum of the outputs of these sectors, (i.e. excluding the output of agriculture and government) using fixed weights to add the output from the two sectors.

Table 1 Correlations with aggregate output (quarterly data)

	4	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	.5 0.178 55 0.025	55 - 0.014 - 0.002	0.240 0.240 0.129
	3	0.164	0.315	0.185	0.350
	2	0.372	0.442	0.406	0.446
	1	Employment (establishment survey) 0.824 0.760 0.581 0.920 0.857 0.633	Employment (household survey) 0.691 0.687 0.563 0.752 0.725 0.563	nt survey) 0.621 0.697	l survey) 0.539 0.579
$\operatorname{Corr}\left(x_{t-j},y_{t}\right)$	0	ent (establis 0.760 0.857	ment (house 0.687 0.725	Hours (establishment survey) 0.814 0.793 0.621 0.914 0.899 0.697	Hours (household survey) 625 0.644 0.53 711 0.720 0.57
Co	- 1	Employm 0.824 0.920	Employi 0.691 0.752	Hours ( 0.814 0.914	Hours 0.625 0.711
	-2	0.795	0.587	0.742	0.515
	-3	0.720 0.747	0.465 0.545	0.639	0.421
	<u> </u>	0.593	0.351	0.504	0.328
	% Std. dev.	0.97	0.89	0.95	1.43
		Consumption sector Investment sector	Consumption sector Investment sector	Consumption sector Investment sector	Consumption sector Investment sector

Notes: Correlations are between HP filtered series with smoothing parameter set equal to 1,600. Note that the Household survey data are for the sample period 1976-1994; the Establishment survey data are for the sample period 1964-1994.

Table 2 Correlations with aggregate output (annual data)

			Corr(	$(x_{t-j}, y_t)$		_
	% Std. dev.	- 2	- 1	0	1	2
				Output		
Consumption sector	1.24	-0.110	0.317	0.856	0.272	-0.297
Investment sector	3.69	-0.153	0.376	0.991	0.393	-0.097
				Capital		
Consumption sector	1.60	0.269	0.293	-0.023	-0.215	-0.076
Investment sector	1.22	0.369	-0.004	-0.408	-0.324	-0.213
			Labor ii	nput (House	hold data)	
Consumption sector	1.66	0.054	0.638	0.931	0.453	-0.031
Investment sector	3.15	0.215	0.673	0.864	0.148	-0.444
			9	Solow residu	als	
Consumption sector	1.10	-0.253	0.159	0.520	-0.002	-0.494
Investment sector	2.52	-0.156	0.277	0.811	0.756	0.374
			La	bor product	ivitv	
Consumption sector	0.58	-0.101	-0.010	0.105	-0.308	-0.613
Investment sector	1.96	-0.315	-0.065	0.463	0.794	0.690
			Iı	nvestment flo	ows	
Consumption sector	8.59	-0.06	0.10	0.54	0.16	-0.11
Investment sector	7.24	0.07	0.63	0.67	- 0.12	-0.34
			Invest	ment sector	deflator	
Consumption sector	1.71	0.00	0.14	0.09	- 0.18	-0.12
Investment sector	2.34	-0.03	- 0.30	- 0.51	- 0.39	-0.13

*Notes*: Correlations are between HP filtered series with smoothing parameter set equal to 100. The Solow residuals are calculated using the Household survey measure of the labor input.

investment goods sector is more correlated with aggregate output than output of the consumption goods sector, and is also nearly three times more volatile. Note that the capital stock in the consumption sector is more volatile than the capital stock in the investment sector. This pattern seems to reflect primarily differences in the cyclical behavior of the stock of equipment in different sectors. For example, there is a strong contemporaneous movement of the stock of equipment in retailing, and essentially no cyclical movement in the stock of structures in that sector. Table 2 also reports the correlations with aggregate output of the labor input calculated from the Household Survey. Note that as in

Table 1, the labor input in the investment sector is nearly twice as volatile as the labor input in the consumption sector. The labor input is also highly procyclical in both sectors at the annual frequency, although the strength of the correlation is the opposite to what we see in the quarterly data. We used the data on the labor input and the capital stock to calculate Solow residuals for each sector. The Solow residual for each sector is measured as

$$\log(z_{i,t}) = \log(y_{i,t}) - \alpha_i \log(k_{i,t}) - (1 - \alpha_i) \log(n_{i,t}),$$

where labor's share in income,  $(1 - \alpha_i)$ , is defined as the ratio of the sum of compensation of employees plus proprietors income to GDP in the *i*th sector, averaged over the period 1948–1992. Output in each sector,  $y_{i,t}$ , is measured as GDP in 1987 dollars produced in that sector. The labor input in each sector,  $n_{i,t}$  is measured as hours worked from the Household Survey, while the capital employed in the *i*th sector during period t,  $k_{i,t}$ , is measured as the net stock of capital outstanding (in 1987 dollars) in that sector at the end of the previous year. In terms of the relative volatilities of the shocks hitting different sectors, the Solow residuals for the investment goods sector are more than twice as volatile as the residuals for the consumption goods sector. We also examined the behavior of labor productivity in the two sectors. Labor productivity appears to be much more volatile in the investment goods sector than in the consumption goods sector. The last two rows of Table 2 document the cyclical properties of investment flows and investment deflators in the two sectors.

While we are restricted to looking at annual data for sectoral investment flows, we can get additional information about the cyclical behavior of investment by examining quarterly data for aggregate investment. In a more detailed analysis contained in Huffman and Wynne (1995), we note that various measures of quarterly investment are strongly procyclical, with residential investment tending to lead the cycle by about two quarters, while business investment tends to move more in line with aggregate activity. <sup>12</sup> Interestingly, business investment in structures tends to *lag* the cycle by about two quarters. Residential investment is a lot more volatile than investment in other structures or investment in producers durable equipment. Thus the cyclical patterns we observe in the sectoral data at an annual frequency (some tendency of structures investment to lag the cycle while investment in equipment tends to move contemporaneously) is borne out in the aggregate data at a quarterly frequency.

Huffman and Wynne (1995) also document the cyclical characteristics of the real rates of return to capital in these different sectors. Not surprisingly the cyclical behavior of rates of return differs across sectors. For example, the real

<sup>&</sup>lt;sup>12</sup> The lead-lag relationship between residential investment and business investment has been studied by Fisher (1997).

rates of return for the construction sector display substantially more volatility than those for the financial services sector.

Thus an analysis of even relatively aggregated sectoral data reveals patterns of correlation with aggregate output that are not apparent from the aggregates that are the typical focus of business cycle analyses. There is an interesting story to be told about what is going on at the sectoral level. In the next section of the paper a simple two-sector growth model is examined for its ability to account for some of the stylized facts listed above.

## 3. A multisector economy

## 3.1. The physical environment

The economic environment studied is a simple two-sector model. This model could be easily and obviously extended to a more complicated environment. In many ways, the structure is quite similar to that described by Hansen (1985). The first sector produces a perishable consumption good from capital and labor. The production technology for this sector is written as  $c_t = f(k_{1,t}, \lambda_{1,t}n_{1,t})$ . Here  $c_t$  refers to consumption in period t, while  $k_{1,t}$  and  $n_{1,t}$  refer to capital and labor employed in the consumption sector in period t, respectively. Additionally  $\lambda_{1,t}$  is a random productivity disturbance that is assumed to be generated by a stochastic process to be specified below. This technology is assumed to take the specific functional form<sup>13</sup>

$$f(k_{1,t}, \lambda_{1,t}, n_{1,t}) = \left[\alpha_1 k_{1,t}^{-\nu} + (1 - \alpha_1)(\lambda_{1,t}, n_{1,t})^{-\nu}\right]^{-1/\nu},\tag{1}$$

where  $\alpha_1 \in (0,1)$  and v > -1.

The second sector produces a durable investment good from capital,  $k_{2,t}$ , and labor,  $n_{2,t}$ , employed in that sector. There will also be a random technology shock,  $\lambda_{2,t}$ , to this production process. We will assume that the technology for the second sector has the familiar Cobb-Douglas functional form. The resource constraint for the second sector is written as

$$\left[\phi i_{1,t}^{-\rho} + (1-\phi)i_{2,t}^{-\rho}\right]^{-1/\rho} = k_{2,t}^{\alpha_2}(\lambda_{2,t}n_{2,t})^{1-\alpha_2},\tag{2}$$

<sup>&</sup>lt;sup>13</sup> Hansen (1985) uses a Cobb–Douglas version of this technology, which corresponds to having v=0. However, in this context with a logarithmic utility function and a Cobb–Douglas technology, the level of employment in the consumption goods sector is constant. Therefore, values of v close to zero were employed in this paper since Hansens model is a useful benchmark. Alternatively we could have employed a Cobb–Douglas technology in both sectors, and used a utility function of the CRRA type (i.e.  $C^{1-\xi}$  with  $\xi>0$ ). As long as  $\xi$  is not too large, the behavior of the model is not substantially changed from what is presented here.

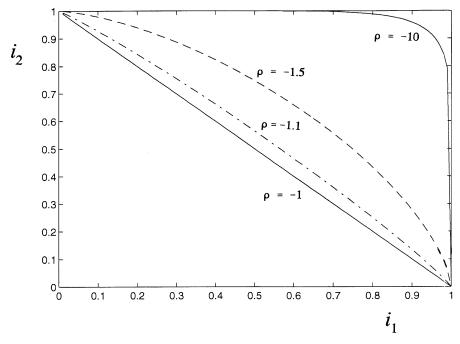


Fig. 1.

where  $i_{1,t}$  and  $i_{2,t}$  represent investment in the two sectors respectively, and  $\alpha_2, \phi \in (0,1)$  are constants. With  $\phi = 0.5$  and  $\rho = -1$ , this results in the standard resource constraint for the capital-goods-producing sector in a two-sector model. However, both these parameters help to determine the relative price of the two investment goods. Even when  $\rho = -1$ , the relative price is determined by  $\phi$ : one unit of  $i_{2,t}$  is equivalent to  $\phi/(1-\phi)$  units of  $i_{1,t}$ . The main focus of this paper is on the implications of having  $\rho < -1$ .

The left side of Eq. (2) may be interpreted as a 'reverse CES' technology since the typical CES technology is restricted to have  $\rho > -1$ . Fig. 1 illustrates this relationship by plotting the graph of the equation  $i_{1,t}^{-\rho} + i_{2,t}^{-\rho} = 1$ . As Fig. 1 illustrates, the restriction  $\rho \le -1$  makes the isoquants concave to the origin, and is necessary for the production possibilities set to be convex. The implications of this specification are as follows. For  $\rho = -1$ , there is an infinite elasticity of substitution between  $i_{1,t}$  and  $i_{2,t}$ . This means that it is very easy to

<sup>&</sup>lt;sup>14</sup> This picture is not totally informative since as  $k_{2,t}$  or  $n_{2,t}$  change (in Eq. (2)), the horizontal and vertical intercepts will change as well.

<sup>&</sup>lt;sup>15</sup> Consequently, the decentralization of the optimal allocations is a straightforward exercise that can be conducted in the usual manner, as illustrated in Prescott and Mehra (1980).

switch from the production of one type of capital good into that of another. Specifically, by cutting back the production of new capital goods for one sector by one unit, it is possible to increase production of new capital goods for the other sector by one unit without any need to increase overall production of new capital goods. It is plausible that an economy can alter its capacity for producing heavy capital equipment for industrial use on the one hand, and alternative capital goods for service sector use on the other. However, in practice it can be costly to do so quickly. Fig. 1 shows that, as the absolute value of  $\rho$  gets bigger, it becomes more difficult to alter the composition of capital goods produced. As  $\rho$  approaches infinity, it becomes impossible to alter the composition of the investment goods that are produced. In other words, there is an infinite cost of doing so, and consequently the two capital stocks and investment quantities will be perfectly correlated.

It is worthwhile to pause to motivate the specification of the intratemporal adjustment costs embedded in the production technology specified in Eq. (2). First, it is trivial to observe that factories cannot immediately be refurbished so as to produce computers instead of pipelines, or trucks instead of cement. It takes time and resources to change the composition of goods produced. These adjustment costs will induce a change in the relative price of the goods that are produced. Furthermore, some types of machinery or equipment used in the production of some goods are entirely useless in the production of other goods. Second, these adjustment costs are likely to be largest for capital goods that are more distinctive. That is, there should be negligible costs for shifting from the production of computers for business to computers for the home. However, there would be substantial cost to changing a factory's production of computers to that of automobiles. To the extent that consumption goods are distinct from investment goods, this separation is an adequate first approximation for this level of aggregation. Third, this approach has the practical appeal of being relatively simple. It also has the added allure of producing an endogenous price for the alternative capital goods, and real rates of return on these capital goods which are not identical.16

The feature shown in Eq. (2) and illustrated in Fig. 1 is what we refer to as intratemporal adjustment costs, since it refers to the decreasing marginal returns

<sup>&</sup>lt;sup>16</sup> Arguably the level of aggregation in our model conceals as much as it reveals. Perhaps we should study a model in which there were many more sectors, possibly which produced both consumption and investment type goods. However, even in an environment with many sectors the sectoral technology specifications would have to be something like that specified in Eq. (2) to prevent capital from freely floating from sector to sector in search of the highest return. In such an environment we could allow for different values of  $\rho$  in the various sectors. For example, we could argue that the equipment used to produce computers for industrial use can easily be switched to producing computers for home use and hence  $\rho = -1$  for the computer sector. But it is probably quite difficult to redirect the equipment and structures used to produce steel to producing airliners

encountered in producing more of one type of investment good while reducing the production of the alternative investment good at a particular moment in time. This specification stands in contrast to the traditional *intertemporal* adjustment costs in which there are decreasing returns to giving up some of the consumption good, which may be perfectly substitutable with existing capital, in one period so as to increase the future capital stock.<sup>17</sup> Another feature of having capital goods produced in a distinct sector is that there is an endogenous price for each type of capital. Greenwood et al. (1988) show how exogenous shocks to the relative price of capital can be an important ingredient in business-cycle fluctuations.

We will assume that capital is not mobile across sectors. The idea here is that capital used in the production of heavy industrial equipment cannot easily be used to produce food or entertainment. This assumption is formalized by specifying separate accumulation equations for the capital stocks in each sector:

$$k_{i,t+1} = (1 - \delta_i)k_{i,t} + i_{i,t}, \text{ for } j = 1,2,$$
 (3)

where  $\delta_j$  denotes the rate of depreciation of capital in sector j. The technology shocks are assumed to obey the following law of motion:

$$\Lambda_t \equiv \begin{bmatrix} \log(\lambda_{1,t}) \\ \log(\lambda_{2,t}) \end{bmatrix} = \Gamma \Lambda_{t-1} + \varepsilon_t.$$

The matrix  $\Gamma$  is described below. Of course, if the off-diagonal elements of  $\Gamma$  are positive, this makes the technological disturbances in the two sectors move together and therefore be more likely to make production in the two sectors move together on average. Here  $\varepsilon_t \equiv [\varepsilon_{1,t}, \varepsilon_{2,t}]'$  is a zero mean two-dimensional vector of normally distributed random variables, with variance-covariance matrix  $\Sigma$ .

The consumers populating the economy have the standard type of timeseparable preferences, which are described by

$$E\left[\sum_{t=1}^{\infty} \beta^{t} U(c_{t}, T - n_{1,t} - n_{2,t})\right], \tag{4}$$

where T is the total time endowment. The utility function is increasing in its two arguments, consumption and leisure. We assume that the point-in-time utility function takes the following form

$$U(c_t, 1 - n_{1,t} - n_{2,t}) = \log(c_t) + (T - n_{1,t} - n_{2,t}).$$
(5)

Preferences are assumed to be linear in leisure for the reasons described in Hansen (1985), although this not a necessary assumption. Note that this

<sup>&</sup>lt;sup>17</sup> Examples of such adjustment costs would be those described by Sargent (1987), chapter 10.

specification of the point-in-time utility function belongs to the class of utility functions identified by King et al. (1988) as consistent with balanced growth.

Before examining the behavior of this model, it should be noted that the presence of multiple sectors gives rise to a subtle measurement issue. The most natural manner to measure aggregate output is to add the amount of the consumption good produced to the amount of the investment good produced, where the latter is measured in consumption units using the (contemporaneous) relative price of capital goods. However, this is not the manner in which aggregate output is constructed in this paper. Instead, a fixed-weight price deflator is employed to add the amount of investment to that of consumption. This method of combining consumption and investment in an aggregate was dictated in part by our use of fixed weight national accounts measures of output. Obviously for the sake of consistency we want to construct aggregates in our model using the same methods employed in the national accounts. Prior to 1995 all national accounts aggregates were constructed using fixed weights, and we used this data rather than the more recent chain-weighted data, to characterize sectoral activity, as the fixed-weight data is more comprehensive in its coverage. 18 In our model economy the fixed weights used to calculate aggregate investment are the relative prices of investment in the nonstochastic steady state. We also aggregate the capital stocks in the two sectors using fixed weights, again following the methodology employed in the national accounts data that we use.

The time period in the model is assumed to be a quarter. As noted in Section 2, there is very little quarterly cross-sector data on employment, output, investment, and technology shocks. Such quarterly data as there is could be time-aggregated in order to scrutinize its annual behavior. However, there is then the danger that some sector aggregates would be procyclical on an annual basis but not at quarterly basis. Since the quarterly data on inputs and outputs presented in the previous section is almost all procyclical, it is important to begin with a model that could potentially explain these observations.<sup>19</sup>

As mentioned in Section 2, we take the structure of the economy seriously in the sense that the investment sector produces investment goods for both market sectors, while the consumption sector produces the consumption good purchased by households. The empirical counterpart of the consumption sector consists of the retail, services and FIRE sectors. The empirical counterpart of the

<sup>&</sup>lt;sup>18</sup> Since 1995 the BEA has adopted chain-weighted Fisher-type indexes to measure prices and quantities the national accounts. The adoption of this approach over the traditional fixed-weight approach was motivated in part by a growing appreciation of the severity of the substitution bias problems associated with the use of fixed weight indexes. However, the BEA has yet to publish sectoral output or capital stock data constructed using the chain weighted approach, so we were forced to use the older 1987 fixed weight series.

<sup>&</sup>lt;sup>19</sup> More extensive documentation of the cyclical behavior of sectoral series is presented in Huffman and Wynne (1995).

investment sector consists of mining, construction, manufacturing, transportation and public utilities, and wholesale trade. Using these classifications, the four key parameters of the production technologies can be calculated using standard assumptions. Specifically, the elasticity of output with respect to the labor input in each sector  $(1-\alpha_j)$  is calculated as the average value over the post-war period of the ratio of the sum of compensation of employees plus proprietors' income to output in each sector. This yields the estimates:  $\alpha_1 = 0.41$  and  $\alpha_2 = 0.34$ . To calculate rates of depreciation in each sector, the ratio of annual depreciation to net capital stock (as reported in US Department of Commerce, 1993a) in each sector is calculated to obtain  $\delta_1 = 0.018$  and  $\delta_2 = 0.020$ . The parameter  $\phi$  is chosen so that the price of each type of capital in each sector, measured in units of the consumption good, is equal in the nonstochastic steady state. Therefore  $\phi = 0.5$  if  $\rho = -1.00$ , and  $\phi = 0.4692$  if  $\rho = -1.10$ . We set  $\beta = 0.99$ , since the unit of time in this is a quarter. We also let v = -0.02 so that the technology in Eq. (1) is close to a Cobb–Douglas technology.

The data on the technological disturbances can be used to derive the parameters of the law of motion for the productivity disturbances as follows<sup>20</sup>

$$\Gamma = \begin{bmatrix} 0.928 & 0.000 \\ 0.000 & 0.786 \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 0.000179 & 0.000332 \\ 0.000332 & 0.000873 \end{bmatrix}.$$

These are the settings for the exogenous stochastic processes used below. As can be seen the technology shocks in the investment sector exhibit more volatility than do those in the consumption sector.

<sup>&</sup>lt;sup>20</sup> Here the period of the model is a quarter, but the sectoral data on capital and output is annual. Therefore, parameters in  $\Gamma$  and  $\Sigma$  were generated under the hypothesis that the actual data was generated by a quarterly process which was temporally aggregated. The off-diagonal elements of  $\Gamma$  are not statistically significant. Nevertheless, none of the results hinge on having these elements be zero. The standard deviations of the shocks present in  $\Sigma$  are not directly comparable to other studies since the technology is assumed to be labor augmenting in both sectors (i.e.  $\lambda_1^{q-1}$  should be interpreted as the Solow residual for the second sector). The corrected (and therefore comparable) standard deviations for the innovations to the shocks to sectors one and two are 0.79% and 1.95%, respectively. The size of the standard deviation of the innovation  $\varepsilon_{1,t}$  is in keeping with the aggregate numbers suggested by Kydland and Prescott (1982) and Hansen (1985). The former use a value of 0.009, while Hansen employs values between 0.007 and 0.01. All of these estimates are less than the value of 0.022 used for the standard deviation of the first difference of the technology shock used by Greenwood and Hercowitz (1991). In the present analysis the size of the standard deviation of the innovation  $\varepsilon_2$ , is larger however, but this should not be too surprising. The more disaggregated an economy becomes, providing the technology shocks are not perfectly correlated across sector, the larger will be the size of the sector-specific shocks that are necessary to reproduce the aggregate behavior. Furthermore, it is not surprising that the technology shocks to sectors such as manufacturing or construction would be larger than those to the service or retail sector. In any event, the goal of the present analysis is not to reproduce some number for the aggregate volatility of output, but to study the cross-sector comovements of inputs and outputs.

Finally, but perhaps most importantly, we need to obtain a value for the parameter that summarizes the adjustment costs,  $\rho$ . It is straightforward to show that the model yields the following relationship between nominal and real flows of investment goods to each sector (see Appendix B):

$$\frac{P_1 i_1}{P_2 i_2} \equiv \frac{I_1}{I_2} = \frac{\phi}{1 - \phi} \left(\frac{i_2}{i_1}\right)^{\rho}. \tag{6}$$

We can use this relationship to retrieve an estimate of the parameter  $\rho$  from time series data on nominal and real investment flows to each sector reported in US Department of Commerce (1993a). However, there is more than one way of going about this. Note, first of all, that we cannot estimate  $\rho$  as a simple average of some quantity except in the special case where  $\phi=0.5$ . Recall that we choose a value for  $\phi$  so as to make the relative price of investment flows to the two sectors equal to unity in the nonstochastic steady state: only when  $\rho=-1$  and there are no costs associated with switching investment from one sector to the other do we want to employ a value of  $\phi=0.5$ . The fact that the value we assign to  $\phi$  depends on the value of  $\rho$  (as well as on the value of every other parameter of the model) precludes the use of least squares estimation of a log-linearized version of Eq. (6). However, note that Eq. (6) implies the following relationship between the deviations from trend of the nominal and real investment flows to the two sectors:

$$\hat{I}_1 - \hat{I}_2 = \rho(\hat{i}_2 - \hat{i}_1),\tag{7}$$

where hats, '^', denote percentage deviations from steady-state values. Using the Hodrick–Prescott filter (with smoothing parameter set equal to 100) to calculate the deviations of the investment flows to the two sectors from trend, we can then estimate  $\rho$  either by applying least squares to Eq. (7) or by calculating the sample average of the ratio  $(\hat{I}_1 - \hat{I}_2)/(\hat{i}_2 - \hat{i}_1)$ . The latter is more in keeping with how we calibrate the other parameters of the model and is our preferred approach. Given the well known and well documented problems with the deflators for equipment investment (see, for example, Gordon, 1990 and the recent study by Greenwood et al., 1997) we focused on the structures investment data to estimate  $\rho$ . The value of  $\rho$  implied by calculating the mean of  $(\hat{I}_1 - \hat{I}_2)/(\hat{i}_2 - \hat{i}_1)$  over the sample period is -1.1. Alternatively, the value of  $\rho$  implied by calculating the mean of  $(\hat{I}_1 - \hat{I}_2)/(\hat{I}_1 - \hat{I}_2)$  is -1.3, so we settled on a moderate value of  $\rho = -1.1$ . Note that choosing even lower values for  $\rho$  only enhances the comovement of the sectoral variables described below.  $^{21,22}$ 

<sup>&</sup>lt;sup>21</sup> Ideally we would use Gordon's (1990) estimates of equipment prices to arrive at a more reliable estimate of real investment flows to the two sectors. However, while Gordon provides a lot of detail for different types of equipment, he does not provide any estimates for deflators of investment in

# 3.2. A specific example

As the focus of this analysis is on how intratemporal adjustment costs can further our understanding of business cycles in multisector environments, much of the remainder of the model will be standard. In particular there is no role for government, externalities, or monetary issues. Therefore, the allocations studied are the optimal ones derived from the solution to the social planning problem. The model is solved in a usual manner by substituting Eqs. (1)–(3) and (5) into the objective function, as given by Eq. (4), and taking a quadratic approximation around the steady state. This is then used to produce (log) linear decision rules for the investment and employment decisions.

Tables 3 and 4 illustrate the behavior of the model without adjustment costs (i.e.  $\rho=-1.0$ ). There are several things to note from these tables, which illustrate the counterfactual behavior of the model. First, consumption is strongly countercyclical. Second, the level of investment in both sectors is much too volatile. This is attributable to the ease with which investment can be switched between the two sectors, as illustrated in Fig. 1. Other flaws in this model are the counter-cyclical behavior of aggregate labor productivity (denoted by  $\pi$ ), investment in the second sector, as well as capital and labor productivity in the first sector (denoted by  $\pi_1$ ). Capital in the second sector is too procyclical and too volatile, and employment in second sector is also too volatile. Lastly, the volatility of aggregate output is more than twice as great as it is in the actual data.

The intuition for why inputs and outputs in sector 2 (the investment sector) are more procyclical than those in sector 1 (the consumption sector) is as

different sectors of the economy. Note also that intuition suggests that if we did have reliable data on real equipment investment flows by sector we would probably obtain a higher estimate of  $\rho$  (it is a lot easier to convert a half-built office building into a hotel than in it is to convert half-built construction equipment into family sedans).

 $<sup>^{22}</sup>$  The regression estimates of Eq. (7) vary depending upon if one considers investment in equipment, structures, or the total. Additionally, the estimates depend upon if one estimates Eq. (7), or the 'reverse' regression. For example, in the case of the estimates for total investment, the estimate range from -1.038 to -1.11, with these estimates having t-statistics on the order of 30.

 $<sup>^{23}</sup>$  In fact with investment so volatile we encounter computational problems. Using the  $\Sigma$  matrix shown in Eq. (7) results in investment in these two sectors being strongly negative in many instances. This creates obvious difficulties for log-linear quadratic approximations, so we multiplied the matrix  $\Sigma$  by 0.0001 to produce the results shown in Tables 3 and 4. This is yet another reason to be disappointed in this model. Because of the volatility in investment in the two sectors, the quadratic approximation, or any such approximation technique, which is useful in studying models that do not display extreme fluctuations, becomes untrustworthy as an investigative tool when such volatility is present. Models with volatile sectors must use other techniques in order to better capture the behavior of variables far away from their steady-state values.

Table 3 Basic model with  $\rho = -1.0$ 

Variable	Standard deviation relative to output	Correlation with output
c	0.364	- 0.584
i	4.441	0.983
$i_1$	47.508	0.212
$i_2$	158.759	-0.096
n	1.299	0.979
$n_1$	0.011	0.963
$n_2$	4.361	0.979
$\pi$	0.381	-0.714
$\pi_1$	0.372	-0.600
$\pi_2$	0.154	0.626
k	0.238	-0.022
$k_1$	0.983	-0.892
$k_2$	3.564	0.931

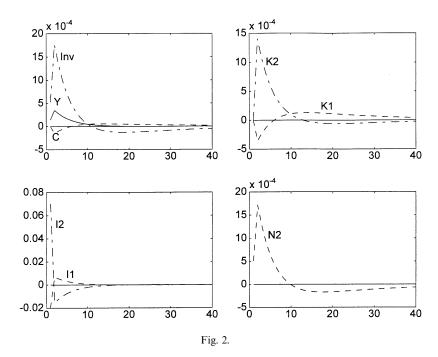
follows. In the event of a favorable productivity shock in the investment sector, labor and capital will be attracted to this sector and thus movements in aggregate activity will be dominated by what is happening in this sector. In the event of a favorable productivity shock to the consumption sector, labor and capital will be attracted to that sector, but this reallocation of factors will be tempered by the fact that the favorable production opportunities can only be propagated by increasing production of capital goods. This becomes more apparent below.

Table 4 also shows many other peculiar features of this baseline model, such as the fact that output, investment, and capital stocks in the two sectors are strongly negatively correlated. Labor productivity in the two sectors is also negatively correlated. Another counterfactual prediction of this model not illustrated in the tables is that the real price of capital across sectors is identical (see Huffman and Wynne, 1995). Consequently, the correlation of the real rate of return to capital in the different sectors is 0.999. One does not have to observe asset markets for very long to realize that this does not appear to reflect the behavior of actual rates of return in different sectors (see, for example, Ferson and Harvey, 1991). In the data the correlation between the quarterly real rates of return in the nondurables and durables sectors is 0.931 for stocks listed on the NYSE from 1949 to 1994. In summation, it is easy to see that many of the variables in this model display behavior that is wildly at odds with the data as described in the previous section.

To better understand the behavior of this model, Fig. 2 describes the behavior of the various variables in response to a one standard deviation shock to  $\varepsilon_2$  at

Table 4 Contemporaneous correlations of key aggregates (model with  $\rho = -1.0$ )

	ý	C	i	$i_1$	$i_2$	и	$n_1$	$n_2$	k	$k_1$	$k_2$	н	$\pi_1$
	-0.584												
	0.983	-0.722											
_	0.212	-0.566	0.309										
2	-0.096	0.493	-0.193	-0.991									
_	0.979	-0.736	1.000	0.309	-0.193								
J	0.963	-0.710	0.980	0.302	-0.189	0.982							
_51	0.979	-0.736	1.000	0.309	-0.193	1.000	0.982						
	-0.022	0.424	-0.111	0.283	-0.305	-0.138	-0.210	-0.137					
, <del>,</del>	-0.892	0.869	0.956	-0.422	0.316	-0.963	-0.965	-0.962	0.305				
,çı	0.931	-0.775	0.968	0.533	-0.429	0.967	0.946	0.967	0.001	-0.952			
L	-0.714	0.982	-0.829	-0.497	0.408	-0.842	-0.824	-0.842	0.423	0.943	-0.853		
Ţ	-0.600	1.000	-0.736	-0.563	0.488	-0.749	-0.724	-0.749	0.421	0.879	-0.786	0.985	
2	0.626	0.002	0.533	0.166	-0.013	0.507	0.443	0.507	0.675	-0.310	0.543	-0.083	-0.011



date  $t=1.^{24}$  Aggregate output and investment output rise immediately and in subsequent periods, while consumption falls for several periods. The reason is that since  $\lambda_2$  has increased,  $i_2$  (and subsequently  $k_2$ ) will increase to take advantage of the increased productivity in the second sector. As  $\lambda_2$  subsequently reverts to its normal level, so too do  $i_2$  and  $k_2$ . Since  $i_2$  has increased so much,  $i_1$  will fall immediately upon the rise in  $\varepsilon_2$ , and consequently consumption falls in the following period. As more capital goods are accumulated,  $k_1$  grows and  $k_2$  falls as agents wish to increase their consumption.

The economics behind this example is as follows. Since new capital goods and labor are substitutable between sectors and perfectly mobile, there is a natural tendency for both factors to move (immediately!) to where their marginal products are highest. If the marginal product of capital in the investment sector rises, there is a strong incentive to reallocate capital to that sector from the consumption sector. However, since capital already in place is immobile, the incentive to reallocate capital will only affect the allocation of new capital goods. Specifically, investment in the investment sector will be increased at the cost of

<sup>&</sup>lt;sup>24</sup> These paths are easily calculated from the linearized system of equations.

reduced investment in the consumption sector. Since there is an infinite elasticity of substitution between the two types of investment goods (see Eq. (2)), this reallocation is feasible. Similarly, there is also an infinite elasticity of substitution between leisure and labor, as well as between labor in the two sectors, so there is even more reason for rapid movement of labor to where its societal marginal product is highest.<sup>25,26</sup> With this rapid movement of inputs across sectors, it is no surprise to find that aggregate output is more volatile than what we see in the data.

It is worth asking whether the addition of *inter*temporal adjustment costs for capital can improve the performance of this baseline model. That is, the left side of Eq. (2) could be changed to:

$$\phi[i_{1,t} + \varphi(k_{1,t+1} - k_{1,t})^2] + (1 - \phi)[i_{2,t} + \varphi(k_{2,t+1} - k_{2,t})^2]$$
  
=  $k_{2,t}^{\alpha_2}(\lambda_{2,t}n_{2,t})^{1-\alpha_2}$ .

This is a straightforward modification of the standard intertemporal adjustment cost specification in which more rapid adjustment of the capital stock over time requires greater inputs of new capital goods. The magnitude of the intertemporal adjustment costs is governed by the parameter  $\varphi$ . It is straightforward to show that moderate values of  $\varphi$  are not sufficient to salvage the baseline model. The reason is that while intertemporal adjustment costs slow the movement of capital from sector to sector, they do not alter the direction of the desired movements. That is, the capital movements are smaller in response to a technological innovation in the investment sector, but there is still a tendency for capital in this sector to be strongly procyclical, and for the reverse to be true for capital in the consumption sector. For higher values of the adjustment cost parameter which produce a sufficiently low variability of aggregate output  $(\varphi \approx 1.0)$ , other things go wrong. Employment and investment in the first sector are strongly countercyclical as is labor productivity in the second sector. Additionally, capital in the first sector is strongly procyclical and investment across sectors is highly negatively correlated. In short, intertemporal adjustment costs of this class do not appear to offer a resolution of the problem.

 $<sup>^{25}</sup>$  The exogenous disturbances of the model are not rigged to produce this result since the off-diagonal elements of  $\Sigma$  are non-negative. Making these elements sufficiently large might enhance the ability of the model to capture comovement, but there will still be some aggregates which would be countercyclical. Nevertheless, highly correlated sector-specific technology shocks seem quite at odds with the data.

<sup>&</sup>lt;sup>26</sup> There are many other types of models in which this perverse behavior arises. In versions of the model of Benhabib et al. (1991), it can be shown analytically that hours in the two sectors will be negatively correlated.

## 3.3. The model with intratemporal adjustment costs for investment

These results motivate the study of an economy in which p < -1. In particular, consider an economy that is identical in every respect to the one presented above, except that now it is assumed that  $\rho = -1.1$ . As Fig. 1 illustrates, this value for  $\rho$  introduces a modest amount of curvature into the tradeoff between the two investment types. However, Tables 5 and 6 show that the behavior of the model is drastically different. Aggregate consumption is now procyclical. The relative volatility of investment in the investment sector is substantially diminished (from 158,759 to 4,269). There is also a dramatic decline in the relative volatility of investment in the consumption sector (from 47.508 to 2.411). This decline is further reflected in declines in the relative volatilities of the capital stocks in the two sectors. Employment in the investment sector is less volatile, while, unfortunately, employment in the consumption sector moves very little (see footnote 12 above). Additionally, with the exception of the capital stock variables, all the aggregates are now procyclical, including labor productivity and investment in the investment sector. The capital stock, both in the aggregate and separately in the two sectors, essentially exhibits acyclic behavior, which is similar to what we observe in the data. In short, when  $\rho = -1.1$ , the behavior of this model is much more similar to the behavior of the data reported in Tables 1 and 2.

Table 6 also illustrates how the cross-sector behavior of labor, capital and labor productivity is much better behaved. The correlation of investment in the two sectors is 0.984, which is perhaps too high – the correlation in the data is 0.55. Nevertheless, this is a significant improvement over the near perfect

Table 5 Model with  $\rho = -1.1$ 

	Standard deviation relative to output	Correlation with output
с	0.433	0.870
i	2.815	0.976
$i_1$	2.411	0.982
$i_2$	4.269	0.954
n	0.810	0.972
$n_1$	0.007	0.913
$n_2$	2.761	0.969
$\pi$	0.284	0.746
$\pi_1$	0.427	0.869
$\pi_2$	0.150	0.483
k	0.147	0.006
$k_1$	0.126	-0.048
$k_2$	0.234	0.115

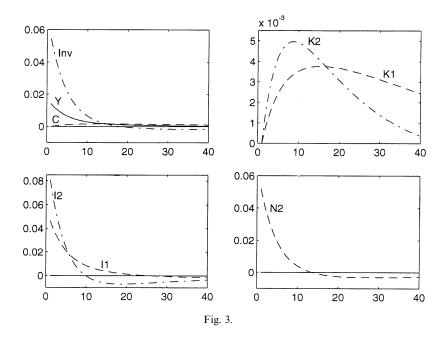
Table 0 Contemporaneous correlations of key aggregates (model with  $\rho=-1.1$ )

	γ	c	i	$i_1$	$i_2$	и	$n_1$	$n_2$	k	$k_1$	$k_2$	π	$\pi_1$
	0.870												
	926.0	0.745											
	0.982	0.767	0.998										
	0.954	0.694	0.993	0.984									
	0.972	0.734	0.999	966.0	966.0								
	0.913	0.956	0.822	0.828	0.801	0.820							
	0.969	0.729	0.999	0.995	0.996	0.999	0.815						
	9000	0.299	-0.122	-0.065	-0.232	-0.154	0.011	-0.155					
	-0.048	0.253	-0.174	-0.118	-0.280	-0.206	-0.039	-0.206	0.995				
	0.115	0.386	-0.012	-0.046	-0.125	-0.044	0.113	-0.045	0.979	0.955			
	0.746	0.972	0.586	0.617	0.519	0.571	0.877	0.563	0.459	0.418	0.529		
	0.869	1.000	0.744	0.766	0.692	0.732	0.955	0.727	0.304	0.257	0.390	0.983	
	0.483	0.565	0.383	0.421	0.296	0.370	0.423	0.337	0.565	0.535	0.610	0.645	0.567
1													

negative correlation (-0.991) of Table 4. We might note that Hornstein and Praschnik's sectoral model, which relies on intermediate goods to generate comovement, is unable to generate a positive correlation between investment in the two sectors (see their Table 4). Another appealing feature of this model is the correlation of aggregate labor productivity and employment, which in this case is 0.571. This correlation is reduced substantially if the correlation in the exogenous shocks is reduced. For example, if the off diagonal elements of  $\Sigma$  are set to zero, then the correlation between aggregate employment and productivity becomes -0.28. Most real business cycle models will have this correlation being something like 0.99, whereas in the data this correlation is closer to zero. Benhabib et al. (1991) stress how the introduction of home production in a standard business cycle model, with both home and market production technologies subject to shocks, can help to reduce this correlation. Christiano and Eichenbaum (1992) employ shocks to government spending to help resolve this puzzle. The present model indicates that a simple two-sector framework is capable of explaining this lack of perfect correlation seen in the data.<sup>27</sup> One benefit of this approach is that there is no need to resort to unmeasurable shocks to home production when some data or information exists on the behavior of cross-sector movements in labor and capital that can be used to discipline the behavior of the model.

Fig. 3 illustrates the behavior of this model in response to a one standard deviation shock to  $\varepsilon_2$ , and is to be compared with Fig. 2. Aggregate consumption, investment, and output, measured in consumption units, appear to all move together. In fact all of the variables in these figures move in a similar manner, and hence are generally procyclical. Investment in both types of capital increases similarly, as does the aggregate capital stock. Agents increase  $n_2$  immediately, because of the increased marginal productivity of labor. Because agents wish to smooth consumption,  $k_1$  and  $i_1$  take a long time to converge back to their steady-state levels. It is interesting that  $n_2$  displays a cycle in the sense that it begins above the steady state level and falls below before converging back to this value. The model with  $\rho = -1.10$  also produces a correlation of the real rates of return of capital across sectors of 0.942 (see Huffman and Wynne, 1995), which is very similar to that observed in the data. In contrast, when  $\rho = -1$  these rates of return are perfectly correlated.

<sup>&</sup>lt;sup>27</sup> This also supports Aiyagari's contention (Aiyagari, 1994) that this correlation would be reduced by having more than one exogenous shock in the model. In fact, the present model can be used to show that it is *not* necessary to have multiple shocks to bring about a *negative* correlation between aggregate employment and labor productivity. It is sufficient to have just one shock (to one sector) but in this case the presence of multiple sectors is also sufficient.



There are some other dimensions along which the model mimics the behavior of the data. For example, employment in the investment sector is more strongly procyclical than is employment in the consumption sector. Labor productivity in the investment sector is much more procyclical than that for the consumption sector. Additionally, the correlation between the level of investment and the price of a unit of capital, measured in units of the consumption good, is negative for the consumption sector, but unfortunately not for the investment sector. The correlations between aggregate output and the real return to capital in the consumption and investment sectors are 0.407 and 0.514, respectively. In the data, these correlations are -0.32 and -0.27. However, as is always the case with financial market variables, it is not clear whether for the purposes of calculating real rates of return the prices in the model should be interpreted as beginning-of-period or end-of-period. Thus it is of interest to know how rates of return are correlated with lags or leads of output. The correlations of the real return to capital in the consumption and investment sectors, with the lagged value of output are -0.246 and -0.274, respectively.

It is also possible to compare the volatility of the price ratio  $(P_{1,t}/P_{2,t})$  from the model with that of the data. In the data, the standard deviation of the log this ratio is between 2–3%, whereas in the model it is 0.60%. Hence, the model does not rely on counterfactually large variability in relative prices to account for the cyclical behavior of sectoral variables.

We also looked at how the model's behavior changed with alternative specifications for the exogenous shocks. Setting the off-diagonal elements of  $\Sigma$  to zero actually enhances the performance of the model. This reduces the correlation of the sector-specific shocks. The result is that aggregate and sector-specific measures of both employment and labor productivity become slightly less procyclical, and therefore more like the behavior in the actual data.

In a multisector model of the sort we are looking at here, there is a large number of variables and moments to scrutinize and compare with the actual data. It comes as no surprise that the model does not match the data along every possible dimension. But then there are also other potential shocks which we are ignoring when conducting this analysis, such as changes in government spending (as in Christiano and Eichenbaum, 1992), shocks to home production (as in Benhabib et al., 1991), or shocks to the relative price of capital goods (Greenwood et al., 1988). What is surprising is that a model as simple as the one outlined here is as successful as it is in characterizing the cyclical behavior of sectoral variables.

## 3.4. Adjustment costs for labor

It is natural to ask whether the addition of adjustment costs for labor instead of investment goods would deliver the same qualitative and quantitative results. The simplest way to introduce such adjustment costs into our framework is to rewrite the constraint on the allocation of time as

$$l_t + \zeta (\psi n_{1,t}^{-\omega} + (1 - \psi) n_{2,t}^{-\omega})^{-1/\omega} \le T,$$

where  $l_t$  denotes leisure at date  $t, \omega \le -1, \zeta > 0$  and  $1 \ge \psi \ge 0$ . This specification of the time allocation constraint captures the idea that it is costly to reallocate labor from one sector to the other. The parameter  $\psi$  is chosen to equate real wages across the two sectors in the nonstochastic steady state. More generally, with  $\omega < -1$  real wages will not be the same in the two sectors. Of course there is an issue as to how to interpret these costs: we ignore this question for now and focus on investigating whether this reduced-form approach has any hope for enhancing the behavior of the model.

Using this constraint to substitute for leisure in the point-in-time utility function we obtain:

$$U(c_t, T - n_{1,t} - n_{2,t}) = \log(c_t) + (T - \zeta(\psi n_{1,t}^{-\omega} + (1 - \psi)n_{2,t}^{-\omega})^{-1/\omega}).$$

Obviously, with  $\zeta = 2$ ,  $\omega = -1$  and  $\psi = 0.5$ , this reduces to the specification of preferences in Eq. (5) above.

We experimented with some different values of the adjustment cost parameter,  $\omega$ , to see how much of a difference it makes to the behavior of our model. Imposing intratemporal adjustment costs only on labor (i.e.  $\omega = -1.1$  and  $\rho = -1.0$ ) does not significantly enhance the performance of the baseline model. Investment and the capital stock in the two sector are highly negatively correlated. Aggregate output and sector-specific investment measures are too volatile. Strangely enough, imposing these costs on labor alone has little significant impact on the behavior of capital or investment, while imposing them on investment alone does improve the performance of the investment and productivity series. Thus intratemporal adjustment costs for labor alone do not solve the problem posed by the benchmark model.

Setting  $\omega=-1.1$  and  $\rho=-1.1$  corresponds in a loose sense to imposing the same degree of difficulty in moving labor and investment. The inclusion of labor adjustment costs along with investment adjustment costs does not significantly enhance the behavior of the model. As we would expect adjustment costs for labor raises the correlation of labor productivity with aggregate output since the adjustment costs make labor less mobile. Labor productivity is more volatile, while employment is less volatile. The cross-sector correlation of both labor productivity and the labor inputs is also higher. A well known feature of most business cycle models driven by technology shocks is that they have difficulty replicating the high degree of correlation between labor productivity and subsequent output. Intratemporal adjustment costs for labor raises this correlation slightly.

## 3.5. Cross-sector behavior of employment

One of the primary achievements of our model is to generate procyclical employment in different sectors which is consistent with what we see in the data. We can also gauge the performance of the model by ascertaining whether the behavior of employment across sectors is similar in other ways to what we see in the data. Thus we asked whether the model is capable of mimicking some of the observations of Lilien (1982). In particular, Lilien observes that the level of unemployment is positively related to a measure of the dispersion of employment growth. This implies that the wider the variation of employment growth across different sectors, the more likely it will be that employment (unemployment) will be low (high), relative to some trend or benchmark level. This is an interesting observation, and perhaps surprisingly, it appears that most recently developed general equilibrium business-cycle models are reticent on this issue. One reason for this is that almost all of these models have only one sector, and hence there is no comparable measure of employment growth across sector.

The model presented above is not as rich in detail as the data presented by Lilien, who analyzed 11 sectors. Nevertheless, it is possible to compare his results to the behavior of employment and the corresponding measure of employment dispersion growth for our model. For the model outlined above, with  $\rho=-1.1$  and  $\omega=-1$ , we obtain a correlation coefficient between employment and Lilien's measure of dispersion of -0.176. Furthermore, the regression coefficient obtained from simply regressing the level of employment on Lilien's measure of dispersion is -0.206. Both of these estimates indicate that our model is capable of mimicking Lilien's finding that there is a negative relationship between aggregate employment and the dispersion of employment growth.<sup>28</sup>

Incidentally, introducing intertemporal adjustment costs on capital, of the type mentioned above, does not help in replicating the observations of Lilien (1982). With intertemporal costs on labor alone there is a positive (negative) correlation between employment (unemployment) growth and the measure of the dispersion of employment growth.

#### 4. Final remarks

Our goal in this paper has been to investigate the dimensions along which a simple real business-cycle model can be extended to account for the sectoral movements in output, employment, and capital observed in the data. We showed that there is a natural reason for some inputs or outputs to tend to move in a countercyclical (and thus counterfactual) manner within a basic multisectoral model. We then showed that by introducing intratemporal adjustment costs for new capital goods (in conjunction with an assumption of complete immobility of existing capital goods) the behavior of the model conforms more closely with what is seen in the data.

Despite the simplicity of the model, it generates a rich array of predictions about what should be expected in the data. For example, it is possible to examine the behavior of the cross-sector returns to capital. The model also generates predictions about variables such as cross-sector employment, capital, labor productivity, and these predictions can also be compared with the data.

<sup>&</sup>lt;sup>28</sup> Abraham and Katz (1986) also present several models in which there is a correlation between the level of unemployment and a measure of the dispersion employment growth. They correctly point out that this does not mean that there is a causal relationship between these variables. Unlike the present approach, their framework is not a general equilibrium model.

The model was shown to be consistent with the following observations from the post-war US data:

- Investment, employment and output in all sectors is procyclical.
- The variability of employment in the investment sector is greater than the variability of employment in consumption sector.
- The capital stock in consumption and investment sectors is acyclic.
- There is a lower correlation between aggregate labor productivity and employment than one finds in many real business-cycle models.
- There is a positive correlation between investment in different sectors.
- The model is consistent with Lilien's (1982) observation that the level of unemployment (employment) should be positively (negatively) related to a measure of the dispersion of employment growth.
- The model developed here can also be used to explore differences in rates of return on capital across sectors. It is straightforward to show that in the context of this model, real rates of return to capital in different sectors are less than perfectly correlated.

# Appendix A.

Annual. The labor compensation series for each sector are from Section 6 of National Income and Product Accounts of the United States: Volume I, 1929–58 (US Department of Commerce, 1993b) and National Income and Product Accounts of the United States: Volume II, 1959–88 (US Department of Commerce, 1992). The output series for each sector are from Yuskavage (1993, 1994) for the period 1977–1992. These data were spliced to the sectoral output series in Section 6 of The National Income and Product Accounts of the United States, 1929–82: Statistical Tables (US Department of Commerce, 1986). The capital stock series for each sector are from Table A1 of Fixed Reproducible Tangible Wealth in the United States, 1925–1989 (US Department of Commerce, 1993a). The annual investment series for each sector are drawn from Table B1 of Fixed Reproducible Tangible Wealth in the United States, 1925–1989 (US Department of Commerce, 1993a).

**Quarterly.** The quarterly data on hours and employment for each sector are from the Bureau of Labor Statistics Household and Establishment Surveys. This data is reported monthly in Tables A-23, B-1 and B-2 of the Bureau of Labor Statistics publication *Employment and Earnings*. Note that when we use the Household Survey data we distribute the self-employed workers and unpaid family workers across sectors on a pro rata basis. We also exclude employees in public administration.

# Appendix B.

It is useful to begin by referring to the right side of Eq. (2) as the level of output from the investment sector. Then, the price of a unit of sector-specific capital is the amount of the consumption good that an agent would be willing to pay for it. To calculate this price, begin by calculating the relative price of a unit of the investment good in units of consumption good. The price of a unit of sector 2 (investment) output in terms of sector 1 (consumption) output (in the extended model which includes intratemporal adjustment costs for labor and capital) is given by

$$\begin{split} P_t &= \left(\frac{1-\psi}{\psi}\right) \!\! \left(\frac{n_{1,t}}{n_{2,t}}\right)^{1+\omega} \\ &\times \! \left[\frac{(1-\alpha_1)\lambda_{1,t}^{-\nu}n_{1,t}^{-\nu-1} \! \left[\alpha_1 k_{1,t}^{-\nu} + (1-\alpha_1)\!(\lambda_{1,t}n_{1,t})^{-\nu}\right]^{-1/\nu-1}}{(1-\alpha_2)\lambda_{2,t}^{2(1-\alpha_2)} (k_{2,t}/n_{2,t})^{\alpha_2}}\right]. \end{split} \tag{B.1}$$

In the absence of adjustment costs associated with reallocating labor between sectors,  $\omega=-1$  and  $\psi=0.5$  and this expression collapses to the standard expression for the relative price of investment goods in a two-sector model. The price of a unit of the sector 1 investment good, measured in units of consumption, is then simply  $P_t$  times the (the inverse of) the increase in sector 1 investment made possible by a unit increase in the output of investment goods times the (inverse of) the increase in sector 1 capital stock next period facilitated by a unit increase in sector 1 investment this period. This can be written as

$$P_{1,t} = P_t \left( \frac{k_{2,t}^{\alpha_2}(\lambda_{2,t} n_{2,t}^{1-\alpha_2})}{i_{1,t}} \right)^{1+\rho} \phi.$$

Similarly, the price of a unit of the sector 2 investment good, measured in units of consumption, is

$$P_{2,t} = P_t \left( \frac{k_{2,t}^{\alpha_2}(\lambda_{2,t}n_{2,t}^{1-\alpha_2})}{i_{2,t}} \right)^{1+\rho} (1-\phi).$$

It is easily seen that if  $\rho = \omega = -1$ , the ratio of these prices is constant, and hence they are perfectly correlated.

It is worth noting that the price of capital in one sector is influenced by the amount of investment undertaken in the other sector. Again setting  $\omega=-1$ , and  $\psi=0.5$  it is straightforward to show that  $P_{1,t}/P_{2,t}=(\phi/(1-\phi))(i_{1,t}/i_{2,t})^{-\rho-1}$ ,

where again  $\rho \leq -1$ . Consider the experiment of increasing  $i_{2,t}$  while simultaneously decreasing  $i_{1,t}$ , and holding employment constant. This makes sector 1 investment goods relatively cheaper to produce, and thereby decreases their value or relative price, and does the opposite to investment goods for sector 2. By substituting Eq. (2) into these pricing relationships it is easy to see that the price of each type of capital good depends on the quantity of capital in both sectors.

The dividend, or marginal product of a unit of capital in sector 1, measured in units of the consumption good is

$$d_{1,t} = \alpha_1 k_{1,t}^{-\nu-1} \left[ \alpha_1 k_{1,t}^{-\nu} + (1 - \alpha_1) (\lambda_{1,t} n_{1,t})^{-\nu} \right]^{-1/\nu-1},$$

while the dividend or marginal product of a unit of capital in sector 2, measured in units of the consumption good is

$$d_{2,t} = P_t \alpha_2 \lambda_{2,t}^{1-\alpha_2} \left( \frac{n_{2,t}}{k_{2,t}} \right)^{1-\alpha_2}.$$

The rates of return to capital are then easily calculated from these formulas. It is possible to show that these rates of return can be written in the form of a factor model, as is frequently done in the finance literature. This provides a link between this latter literature, and that of the dynamic general equilibrium research which is frequently employed in the study of business cycles.

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