Steady state of two-sector models - Questions

Marco Brianti
Laura Gati

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1 A two-sector model à la Oulton (2010)

Why look at this model? Because the one we have laid out maps to this, but here the formulation is simpler because the HH accumulates all kinds of capital. Also, I'm shutting off spillovers to TFP from the IT stock for now.

$$
\begin{aligned}
&yc_t = bc_t \; h1^{1-a-b} kc1_t^a ki1_t^b \\
&yi_t = bi_t \; h2^{1-a-b} kc2_t^a kci1_t^b \\
&kc_t = kc1_t + kc2_t \\
&ki_t = ki1_t + ki2_t \\
&h_t = h1_t + h2_t \\
&kc_{t+1} = (1-\delta_c)kc_t + ic_t \\
&ki_{t+1} = (1-\delta_i)ki_t + it_t \\
&yc_t = c_t + ic_t \\
&yi_t = it_t \\
&p_t = pi_t/pc_t \quad \text{Normalizing } pc = 1 \text{ we have } p \text{ as only price variable left.} \\
&bc_t, bi_t \text{ grow with exogenous growth rates } \gamma_c \text{ and } \gamma_i \\
&w_t/c_t = \chi \\
&1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}}(rc_{t+1} + 1 - \delta_c) \\
&1 = \beta \mathbb{E}_t \frac{c_t}{c_{t+1}}(\frac{1}{p_{t+1}} ri_{t+1} + 1 - \delta_i) \\
&w_t = (1-a-b)bc_t \; h1_t^{-a-b} kc1_t^a ki1_t^b \\
&rc_t = a \; bc_t \; h1_t^{1-a-b} kc1_t^{a-1} ki1_t^b \\
&ri_t = b \; bc_t \; h1_t^{1-a-b} kc1_t^a ki1_t^{b-1} \\
&\frac{1}{p_t} w_t = (1-a-b)bc_t \; h1_t^{-a-b} kc1_t^a ki1_t^b \\
&\frac{1}{p_t} rc_t = a \; bc_t \; h1_t^{1-a-b} kc1_t^{a-1} ki1_t^b \\
&\frac{1}{p_t} ri_t = b \; bc_t \; h1_t^{1-a-b} kc1_t^a ki1_t^{b-1}
\end{aligned}
$$

2 Steady state with growth

BGP: Following Oulton, let's assume that $\gamma_i > \gamma_c$. It can be shown that $g_p = \gamma_c - \gamma_i < 0$, i.e. relative prices are falling. Furthermore it can be shown (from LOMs, GDP equations) that yc, c, ic and kc grow at a shared rate g, while

yi, it and ki grow at the rate g_i which is faster than g. (Something like $g_i = g - g_p$.) From the EEs, we also see that rc is constant while ri is falling at the rate g_p. So at this point I have things growing at 3 different rates (g, g_i and g_p) and how to connect these rates with the exogenous growth rates is tough, especially since I begin to get confused about gross and net growth rates too. But what I find the most challenging is to stationarize. Suppose I just divide the variables by the appropriate variable. For example since I know that yc, c, ic and kc grow at a shared rate g, I can divide all those variables by for example yc_{t-1}. So let's divide the final goods production function by yc_{t-1} and denote by tilde the stationary variables:

$$\tilde{yc}_t = bc_t \; h1^{1-a-b} \tilde{kc}_t^a \; ki_t^b \; yc_{t-1}^{-(1+b)}$$

What makes me confused here is that to get ki to be stationary, we need to divide by additional stuff: the difference between the growth rates of the standard capital sector and the IT sector. But because of the exponents things don't seem to work out. And then we haven't even dealt with bc which is merrily growing at its own rate and I don't see how to connect that with the g, g_i and g_p.

3 Shutting off growth - steady state is still tough

To shut off growth, I let bc, bi be constants for a moment so that nothing grows. Then the steady state is given by the following set of equations (once I've done a couple of substitutions):

$$\frac{w}{\chi} + \delta_c kc1 + \delta_c kc2 = bc \; h1^{1-a-b} \; kc1^a \; ki1^b \tag{1}$$

$$\delta_i ki1 + \delta_i ki2 = bi \; h2^{1-a-b} \; kc2^a \; ki2^b \tag{2}$$

$$rc = 1/\beta - (1 - \delta_c) \tag{3}$$

$$ri = (1/\beta - (1 - \delta_i))p \tag{4}$$

$$w = (1 - a - b) \; bc \; h1^{-a-b} \; kc1^a \; ki1^b \tag{5}$$

$$rc = a \; bc \; h1^{1-a-b} \; kc1^{a-1} \; ki1^b \tag{6}$$

$$ri = b \; bc \; h1^{1-a-b} \; kc1^a \; ki1^{b-1} \tag{7}$$

$$\frac{1}{p} w = (1 - a - b) \; bi \; h2^{-a-b} \; kc2^a \; ki2^b \tag{8}$$

$$\frac{1}{p} rc = a \; bi \; h2^{1-a-b} \; kc2^{a-1} \; ki2^b \tag{9}$$

$$\frac{1}{p} ri = b \; bi \; h2^{1-a-b} \; kc2^a \; ki2^{b-1} \tag{10}$$

Even this one is tricky because so much seems redundant. I've been able to solve it, but it's not a 100% correct.