

3 slides to convince you that it's endogenous TFP

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A two-sector model

Final goods firm (GE)

$$\begin{aligned} \max_{K_t^f(s), L_t^f(s), K_t^{IT}(s)} \quad & Y_t(s) = K_t^f(s)^\alpha L_t^f(s)^\beta K_t^{IT}(s)^{1-\alpha-\beta} A_t \\ & - w_t L_t(s) - R_t K_t^f(s) - P_t^{IT} I_t^{IT}(s) \\ \text{where } & K_t^{IT}(s) = (1 - \delta^{IT}) K_{t-1}^{IT} + I_t^{IT} \\ \text{and } & A_t = S_t \Psi_t(K_t^{IT})^\gamma \end{aligned}$$

IT-producing firm (Google)

$$\max_{K_t^I(s), L_t^I(s)} \quad I_t^{IT}(s) = P_t^{IT} S_t \lambda_t F(K_t^I(s), L_t^I(s)) - w_t L_t^I(s) - R_t K_t^I(s)$$

A news shock is $S_{t+k} \uparrow$ for some positive k .

An IT productivity shock is λ_{t+j} for some positive j .

Ψ is a final-good-specific exogenous technology process.

Proposition 1 - the Solow-residual in this model

$$\begin{aligned}\frac{\dot{A}}{A} &= \frac{\dot{Y}}{Y} - \alpha \frac{\dot{K}^f}{K^f} - \beta \frac{\dot{L}^f}{L^f} - (1 - \alpha - \beta) \frac{\dot{K}^{IT}}{K^{IT}} \\ &= \frac{\dot{S}}{S} + \frac{\dot{\Psi}}{\Psi} + \gamma \frac{\dot{K}^{IT}}{K^{IT}}\end{aligned}$$

TFP consists of an exogenous, common technology process S , a final-good-specific exogenous technology process Ψ and an endogenous component coming as a spillover from the aggregate stock of IT technology.

Proposition 2 - when a news shock leaves relative prices constant

Since the rate of return on the input factors have to equal in the two sectors, we can set the FOCs from GE and Google equal. Let's do this for capital:

$$R_t = \alpha \frac{Y_t(s)}{K_t^f(s)} P_t^C = S_t \lambda_t F_k P_t^{IT}$$
$$\Leftrightarrow \frac{P_t^{IT}}{P_t^C} = \alpha \frac{K_t^f(s)^\alpha L_t^f(s)^\beta K_t^{IT}(s)^{1-\alpha-\beta} S_t \Psi_t(K_t^{IT})^\gamma}{K_t^f(s)} \frac{1}{S_t \lambda_t F_k}$$

A news shock doesn't move relative prices if

- No other exogenous shock hits the economy (Ψ_t and λ_t do not move);
- the news shock doesn't change the relative marginal productivities of the two inputs in the two sectors, so that labor and capital are not reallocated from one sector to another.