

# Theoretical Impulse Response Functions

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October 2017

## 1 Zero Balanced-Growth Path

Define  $x_t$  as the value of variable  $x$  at time  $t$ . Define  $\bar{x}$  as the value of variable  $x$  in steady state. Impulse response functions are percentage deviation of  $x$  from its steady state value  $\bar{x}$ . Formally,

$$IRF(x_t|\varepsilon) = \log\left(\frac{x_t}{\bar{x}}\right)$$

where  $\varepsilon$  is a general perturbation from the steady state level of the system. As long as  $\varepsilon$  has a transitory nature, we expect IRFs to converge to zero asymptotically. In other words, we expect that the system will naturally converge to the steady state level once the perturbation ceases.

## 2 Positive Balanced-Growth Path - Temporary Shock

Define  $x_t$  as the value of variable  $x$  at time  $t$ . Define  $\tilde{x}_t$  as the value of  $x$  when divide by the value of an exogenous process which is growing at the same rate of  $x$ . Formally,

$$\tilde{x}_t = \frac{x_t}{G_{t-1}}$$

where  $G_{t-1}$  may represent the value of the exogenous productivity process which is growing at the same rate of  $x$ . Define  $\tilde{x}$  as the value of  $\tilde{x}_t$  when the system is growing at the balanced growth at the BGP without any past or present source of perturbation. In this case, impulse response functions are percentage deviation of  $\tilde{x}_t$  from its steady state value  $\tilde{x}$ . Formally,

$$\begin{aligned} IRF(\tilde{x}_t|\varepsilon) &= \log\left(\frac{\tilde{x}_t}{\tilde{x}}\right) \\ &= \log\left(\frac{\frac{x_t(\varepsilon)}{G_{t-1}}}{\frac{x_t}{G_{t-1}}}\right) \\ &= \log\left(\frac{x_t(\varepsilon)}{x_t}\right) \end{aligned} \tag{1}$$

In other words, we can interpret  $IRF(\tilde{x}_t|\varepsilon)$  as percentage deviation of  $x_t$  from the value it should have had without  $\varepsilon$ . Also in this case we expect IRFs to converge to zero asymptotically. In other words, we expect  $x_t(\varepsilon)$  to converge back to  $x_t$  once the perturbation ceases.

### **3 Positive Balanced-Growth Path - Permanent Shock**