Dalton summary

Steps:

1.) Model equilibrium conditions
2.) BGP
3.) Stationarization
4.) Steady state

1.) The model

$$yc_t = bc_t \, h1_t^{1-a-b} \, kc1_t^{a} \, ki1_t^{b} \quad (1)$$

$$yi_t = bi_t \, h2_t^{1-a-b} \, kc2_t^{a} \, ki2_t^{b} \quad (2)$$

$$kc_t = kc1_t + kc2_t \quad (3)$$

$$ki_t = ki1_t + ki2_t \quad (4)$$

$$h_t = h1_t + h2_t \quad (5)$$

$$kc_{t+1} = (1-dc)\,kc_t + ic_t \quad (6)$$

$$ki_{t+1} = (1-di)\,ki_t + it_t \quad (7)$$

$$yc_t = c_t + ic_t \quad (8)$$

$$yi_t = it_t \quad (9)$$

$$p_t = p_t^i / p_t^c \quad (10)$$

bc_t, bi_t evolve according to an AR(1) w/ exog. growth rates γ_c & γ_i. Ass. $\gamma_i > \gamma_c$.

HHs max $\ln c_t - \chi h_t$

s.t. $c_t + ic_t + p_t it_t = w_t h_t + rc_t kc_t + ri_t ki_t$

convert to cons. units

already in units of cons!

$$p_t it_t = p_t ki_{t+1} - (1-di) p_t ki_t$$

So the Lagrangian is

$$\mathcal{L} = \ln c_t - \chi h_t$$
$$+\lambda_t\left[-c_t - kc_{t+1} - p_t ki_{t+1} + w_t h_t + (rc_t + 1 - dc)kc_t + (ri_t + (1-di)p_t)ki_t\right]$$

FOCs:

$$\frac{w_t}{c_t} = \chi$$

$$1 = \beta E_t \frac{c_t}{c_{t+1}}\left(rc_{t+1} + 1 - dc\right)$$

$$1 = \beta E_t \frac{c_t}{c_{t+1}} \frac{p_{t+1}}{p_t}\left(\frac{ri_{t+1}}{p_{t+1}} + 1 - di\right)$$

Firms

GE : $\max\ p^c_t bc_t\, h1_t^{1-a-b}\, kc1_t^{a}\, ki1_t^{b} - w_t h1_t - rc_t kc1_t - ri_t ki1_t$

FOCs

$$w_t = (1-a-b)\, bc_t\, h1_t^{-a-b} kc1_t^{a} ki1_t^{b} \cdot p^c_t$$

$$rc_t = a\ bc_t\, h1_t^{1-a-b}\, kc1_t^{a-1} ki1_t^{b} \cdot p^c_t$$

$$ri_t = b\ bc_t\, h1_t^{1-a-b} kc1_t^{a} ki1_t^{b-1} \cdot p^c_t$$

Google $\max\ p^I_t bi_t\ h2_t^{1-a-b} kc2_t^{a} ki2_t^{b} - w_t h2_t - rc_t kc2_t - ri_t ki1_t$

FOCs

$$w_t = (1-a-b)\, bi_t\, h2_t^{-a-b} kc2_t^{a}\, ki2_t^{b} \cdot p^I_t$$

$$rc_t = a\ bi_t\, h2_t^{1-a-b} kc2_t^{a-1}\ ki2_t^{b}\ p^I_t$$

$$ri_t = b \cdot bi_t\, h2_t^{1-a-b} kc2_t^{a}\ ki2_t^{b-1}\ p^I_t$$

Use $P_t = p^I_t / p^c_t$ and let $p^c_t = 1$

$$w_t = (1-a-b)\, bi_t\, h2_t^{-a-b} kc2_t^{a}\, ki2_t^{b} \cdot P_t$$

$$rc_t = a\ bi_t\, h2_t^{1-a-b} kc2_t^{a-1}\ ki2_t^{b}\ P_t$$

$$ri_t = b \cdot bi_t\, h2_t^{1-a-b} kc2_t^{a}\ ki2_t^{b-1}\ P_t$$

Equilibrium conditions of the model:

$$yc_t = bc_t\, h1_t^{1-a-b}\, kc1_t^{a}\, ki1_t^{b} \tag{1}$$

$$yi_t = bi_t\, h2_t^{1-a-b}\, kc2_t^{a}\, ki2_t^{b} \tag{2}$$

$$kc_t = kc1_t + kc2_t \tag{3}$$

$$ki_t = ki1_t + ki2_t \tag{4}$$

$$h_t = h1_t + h2_t \tag{5}$$

$$kc_{t+1} = (1-dc)\, kc_t + ic_t \tag{6}$$

$$ki_{t+1} = (1-di)\, ki_t + it_t \tag{7}$$

$$yc_t = c_t + ic_t \tag{8}$$

$$yi_t = it_t \tag{9}$$

$$\frac{w_t}{c_t} = \chi \tag{10}$$

$$1 = \beta E_t \frac{c_t}{c_{t+1}} \left(rc_{t+1} + 1 - dc \right) \tag{11}$$

$$1 = \beta E_t \frac{c_t}{c_{t+1}} \frac{p_{t+1}}{p_t} \left(\frac{ri_{t+1}}{p_{t+1}} + 1 - di \right) \tag{12}$$

$$w_t = (1-a-b)\, bc_t\, h1_t^{-a-b}\, kc1_t^{a}\, ki1_t^{b} \tag{13}$$

$$rc_t = a\, bc_t\, h1_t^{1-a-b}\, kc1_t^{a-1}\, ki1_t^{b} \tag{14}$$

$$ri_t = b\, bc_t\, h1_t^{1-a-b}\, kc1_t^{a}\, ki1_t^{b-1} \tag{15}$$

$$w_t = (1-a-b)\, bi_t\, h2_t^{-a-b}\, kc2_t^{a}\, ki2_t^{b} \cdot p_t \tag{16}$$

$$rc_t = a\, bi_t\, h2_t^{1-a-b}\, kc2_t^{a-1}\, ki2_t^{b}\, p_t \tag{17}$$

$$ri_t = b \cdot bi_t\, h2_t^{1-a-b}\, kc2_t^{a}\, ki2_t^{b-1}\, p_t \tag{18}$$

2.) BGP

Denote by "^" growth rates of a variable

- LOMs: $\frac{kc_{t+1}}{kc_t} = (1-\delta c) + \frac{ic_t}{kc_t} \rightarrow \hat{ic} = \hat{kc} =: g$

$$\frac{ki_{t+1}}{ki_t} = (1-\delta i) + \frac{it_t}{ki_t} \rightarrow \hat{it} = \hat{ki} =: gi$$

- GDP: $yc_t = c_t + ic_t \Rightarrow \frac{yc_t}{kc_t} = \frac{c_t}{kc_t} + \frac{ic_t}{ki_t} \Rightarrow \hat{yc} = \hat{c} = \hat{ic} = \hat{kc}$

$$yi_t = it_t \Rightarrow \frac{yi_t}{ki_t} = \frac{it_t}{ki_t} \Rightarrow \hat{yi} = \hat{it} = \hat{ki}$$

- EEs: rc constant

$$\frac{ri_{t+1}}{p_{t+1}} \text{ constant} \rightarrow \hat{ri} = \hat{p} =: gp$$

- h constant
- Frisch: $\frac{w}{c} = \chi \rightarrow \hat{w} = \hat{c}$

So: yc, kc, ic, c, w grow at g

yi, ki, it grow at gi

rc, h constant

ri, p grow (fall, we'll see why) at gp.

Let's map "endog" growth rates to exog. ones $(\gamma c, \gamma i)$.

Differentiate PFs w.r.t time:

$$\hat{yc} = \gamma_c + a \cdot \hat{kc} + b \cdot \hat{ki}$$

i.e. $$g = \gamma_c + a \cdot g + b \cdot gi$$

$$g = \frac{1}{1-a}\gamma_c + \frac{b}{1-a} gi \qquad (i)$$

$$\hat{yi} = \gamma_i + a \cdot \hat{kc} + b \cdot \hat{ki}$$

i.e. $$gi = \gamma_i + a \cdot g + b \cdot gi$$

$$gi = \frac{1}{1-b}\gamma_i + \frac{a}{1-b} g \qquad (ii)$$

In (i):

$$g = \frac{1}{1-a}\gamma_c + \frac{b}{1-a}\frac{1}{1-b}\gamma_i + \frac{b}{1-a}\frac{a}{1-b} g$$

$$\Leftrightarrow \quad g \underbrace{\left(1 - \frac{ab}{(1-a)(1-b)}\right)}_{\frac{(1-a)(1-b)-ab}{(1-a)(1-b)} = \frac{1-a-b}{(1-a)(1-b)}} = \frac{1-b}{(1-a)(1-b)}\gamma_c + \frac{b}{(1-a)(1-b)}\gamma_i$$

$$\rightarrow \quad g = \frac{1-b}{1-a-b}\gamma_c + \frac{b}{1-a-b}\gamma_i \qquad \text{I}$$

Back in (ii)

$$g_i = \frac{1}{1-b}\gamma_i + \frac{a}{1-b}\left[\frac{1-b}{1-a-b}\gamma_c + \frac{b}{1-a-b}\gamma_i\right]$$

$$g_i = \frac{1-a-b}{(1-b)(1-a-b)}\gamma_i + \frac{a(1-b)}{(1-b)(1-a-b)}\gamma_c + \frac{ab}{(1-b)(1-a-b)}\gamma_i$$

$$g_i = \frac{a}{1-a-b}\gamma_c + \frac{(1-a)(1-b)}{(1-b)(1-a-b)}\gamma_i$$

$$g_i = \frac{a}{1-a-b}\gamma_c + \frac{1-a}{1-a-b}\gamma_i \quad \text{III}$$

So I & III say that

- y_c grows at the same rate as $b_c^{\frac{1-b}{1-a-b}} \cdot b_i^{\frac{b}{1-a-b}}$ (I)
- y_i grows at the same rate as $b_c^{\frac{a}{1-a-b}} \cdot b_i^{\frac{1-a}{1-a-b}}$ III

→ so everything that grows by g needs to be stationarized by I

& everything that grows by g_i needs to be stationarized by III

I now just need II, the relationship btwn g_p & (γ_c, γ_i).

For that, PTO:

Form the ratio of value of outputs (following Oulton Annex A.)

$$\frac{P^i \cdot y^i}{P^c \cdot y^c} = \frac{rc \cdot kc2 + ri \cdot ki2 + wh2}{rc \cdot kc1 + ri \cdot ki1 + wh1} \quad \Big| \text{ Diff wrt time.}$$

$$\left. gp + gi - g = \frac{d}{dt}(RHS) \right\} \text{WTS that this} = 0.$$

RHS: $\ln(rc \cdot kc2 + ri \cdot ki2 + w \cdot h2) - \ln(rc \cdot kc1 + ri \cdot ki1 + w \cdot h1)$

$$\frac{d}{dt} \Rightarrow \frac{rc\, kc2\, g + ri\, ki2\,(gp+gi) + w h2 g}{\underbrace{rc \cdot kc2 + ri \cdot ki2 + w \cdot h2}_{yi}} - \frac{\text{the same thing for 1}}{-\,\text{''}\,-}$$

$$\Rightarrow \quad ag + b(gp+gi) + (1-a-b)g - (\text{the same}) = 0 \quad \square$$

So $gp + gi - g = 0$, so:

$gp = g - gi$ (I've called this (IV) in my work.)

Then from PFs we sub in g & gi

$$gp = \frac{1-b}{1-a-b}\gamma c + \frac{b}{1-a-b}\gamma i - \frac{a}{1-a-b}\gamma c - \frac{1-a}{1-a-b}\gamma i$$

$$\Rightarrow gp = \gamma c - \gamma i \qquad \text{(II)}$$

→ so everything that grows by gp grows at the same rate as $\frac{bc}{bi}$ and can thus by stationarized by that!

Summary of stationarization strategy: ($\tilde{\ }$ = stationarized)

1. $$\tilde{x}_t = \frac{x_t}{\underbrace{bc_{t-1}^{\frac{1-b}{1-a-b}} \cdot bi_{t-1}^{\frac{b}{1-a-b}}}_{=: G_{t-1}}}$$ for $x_t = yc_t, c_t, kc_t, ic_t, w_t$

2. $$\tilde{x}_t = \frac{x_t}{\underbrace{bc_{t-1}^{\frac{a}{1-a-b}} \cdot bi_{t-1}^{\frac{1-a}{1-a-b}}}_{=: GI_{t-1}}}$$ for $x_t = yi_t, ki_t, it_t$

3. $$\tilde{x}_t = \frac{x_t}{\frac{bc_{t-1}}{bi_{t-1}}}$$ for $x_t = ri_t, p_t$

4. $$\tilde{x}_t = \frac{x_t}{x_{t-1}} = (1+\gamma_x)$$ for $x_t = bc_t, bi_t$

And why $1+\gamma_x$? B/c the γ_x $(x = c, i)$ are net growth rates.

3.) Stationarization

(1) $yc_t = bc_t \, h1_t^{1-a-b} \, kc1_t^{a} \, ki1_t^{b}$

$$\tilde{yc}_t \cdot G_{t-1} = (1+\gamma_{c,t}) bc_{t-1} h1_t^{1-a-b} \cdot (\tilde{kc1}_t G_{t-1})^a (\tilde{ki1}_t GI_{t-1})^b$$

$$\Leftrightarrow \underline{\underline{\tilde{yc}_t}} = \underbrace{bc_{t-1} G_{t-1}^{a-1} GI_{t-1}^{b}} \cdot \underline{\underline{(1+\gamma_{c,t}) h1_t^{1-a-b} \cdot \tilde{kc1}_t^a \tilde{ki1}_t^b}}$$

$$bc_{t-1}\left(bc_{t-1}^{\frac{1-b}{1-a-b}} \cdot bi_{t-1}^{\frac{b}{1-a-b}}\right)^{a-1} \left(bc_{t-1}^{\frac{a}{1-a-b}} \cdot bi_{t-1}^{\frac{1-a}{1-a-b}}\right)^{b}$$

$$= bc_{t-1}^{\frac{1-a-b-(1-a)(1-b)+ab}{1-a-b}} \cdot bi_{t-1}^{\frac{-(1-a)b+(1-a)b}{1-a-b}}$$

$$= bc_{t-1}^{0} \cdot bi_{t-1}^{0} = 1$$

So $\tilde{yc}_t = (1+\gamma_{c,t}) h1_t^{1-a-b} \tilde{kc1}_t^{a} \tilde{ki1}_t^{b}$ (1)

(2) $yi_t = bi_t \, h2_t^{1-a-b} \, kc2_t^{a} \, ki2_t^{b}$

$$\tilde{yi}_t \cdot GI_{t-1} = (1+\gamma_{i,t}) bi_{t-1} h2_t^{1-a-b} (\tilde{kc2}_t G)^a (\tilde{ki2}_t GI)^b$$

$$\underline{\underline{\tilde{yi}_t}} = \underbrace{bi_{t-1} G_{t-1}^{a} GI_{t-1}^{b-1}} \cdot \underline{\underline{(1+\gamma_{i,t}) h2_t^{1-a-b} \tilde{kc2}_t^a \tilde{ki2}_t^b}}$$

$$bi_{t-1}\left(bc_{t-1}^{\frac{1-b}{1-a-b}} \cdot bi_{t-1}^{\frac{b}{1-a-b}}\right)^{a} \left(bc_{t-1}^{\frac{a}{1-a-b}} \cdot bi_{t-1}^{\frac{1-a}{1-a-b}}\right)^{b-1}$$

$$= bc_{t-1}^{\frac{a(1-b)-a(1-b)}{1-a-b}} \cdot bi_{t-1}^{\frac{1-a-b+ab-(1-a)(1-b)}{1-a-b}}$$

So $\tilde{yi}_t = (1+\gamma_{i,t}) h2_t^{1-a-b} \tilde{kc2}_t^{a} \tilde{ki2}_t^{b}$ (2)

(3), (4), (5) are straightforward b/c LHS grows at the same rate as RHS so:

$\tilde{kc}_t = \tilde{kc1}_t + \tilde{kc2}_t$

$\tilde{ki}_t = \tilde{ki1}_t + \tilde{ki2}_t$

$h_t = h1_t + h2_t$

(6): $kc_{t+1} = (1-dc)kc_t + ic_t$

$$\tilde{kc}_{t+1} G_t = \left[(1-dc)\tilde{kc}_t + \tilde{ic}_t\right] G_{t-1}$$

$$\underline{\underline{\tilde{kc}_{t+1}}} \underbrace{\frac{G_t}{G_{t-1}}} = \underline{\underline{(1-dc)\tilde{kc}_t + \tilde{ic}_t}}$$

$$\frac{bc_t^{\frac{1-b}{1-a-b}} \cdot bi_t^{\frac{b}{1-a-b}}}{bc_{t-1}^{\frac{1-b}{1-a-b}} \cdot bi_{t-1}^{\frac{b}{1-a-b}}} = \left(\frac{bc_t}{bc_{t-1}}\right)^{\frac{1-b}{1-a-b}} \left(\frac{bi_t}{bi_{t-1}}\right)^{\frac{b}{1-a-b}}$$

$$= (1+\gamma_{c,t})^{\frac{1-b}{1-a-b}} (1+\gamma_{i,t})^{\frac{b}{1-a-b}}$$

So $\tilde{kc}_{t+1} (1+\gamma_{c,t})^{\frac{1-b}{1-a-b}} (1+\gamma_{i,t})^{\frac{b}{1-a-b}} = (1-dc)\tilde{kc}_t + \tilde{ic}_t$ (6)

(7) $ki_{t+1} = (1-di)ki_t + it_t$ analogously becomes

$\tilde{ki}_{t+1} \frac{GI_t}{GI_{t-1}} = (1-di)\tilde{ki}_t + \tilde{it}_t$ which is

$\tilde{ki}_{t+1} (1+\gamma_{c,t})^{\frac{a}{1-a-b}} (1+\gamma_{i,t})^{\frac{1-a}{1-a-b}} = (1-di)\tilde{ki}_t + \tilde{it}_t$ (7)

(8), (9), (10) are straightforward b/c LHS grows at the same rate as RHS so:

$\tilde{yc}_t = \tilde{c}_t + \tilde{ic}_t$

$\tilde{yi}_t = \tilde{it}_t$

$\frac{\tilde{w}_t}{\tilde{c}_t} = \chi$

(11): $1 = \beta E_t \frac{c_t}{c_{t+1}} \left(rc_{t+1} + 1 - dc \right)$

$$1 = \beta E_t \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \underbrace{\frac{G_{t-1}}{G_t}} \left(rc_{t+1} + 1 - dc \right)$$

$$\frac{bc_{t-1}^{\frac{1-b}{1-a-b}} \cdot bi_{t-1}^{\frac{b}{1-a-b}}}{bc_t^{\frac{1-b}{1-a-b}} \cdot bi_t^{\frac{b}{1-a-b}}} = \left(\frac{bc_{t-1}}{bc_t} \right)^{\frac{1-b}{1-a-b}} \left(\frac{bi_{t-1}}{bi_t} \right)^{\frac{b}{1-a-b}}$$

$$= \left(\frac{1}{1+\gamma_{c,t}} \right)^{\frac{1-b}{1-a-b}} \left(\frac{1}{1+\gamma_{i,t}} \right)^{\frac{b}{1-a-b}}$$

So

$$1 = \beta E_t \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \left(\frac{1}{1+\gamma_{c,t}} \right)^{\frac{1-b}{1-a-b}} \left(\frac{1}{1+\gamma_{i,t}} \right)^{\frac{b}{1-a-b}} \left(rc_{t+1} + 1 - dc \right) \quad (11)$$

(12) : $1 = \beta E_t \frac{c_t}{c_{t+1}} \frac{p_{t+1}}{p_t} \left(\frac{r_{i,t+1}}{p_{t+1}} + 1 - d_i \right)$

$$1 = \beta E_t \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \underbrace{\frac{G_{t-1}}{G_t}}_{\text{same as in (11)}} \frac{\tilde{p}_{t+1}}{\tilde{p}_t} \underbrace{\frac{\frac{bc_t}{bi_t}}{\frac{bc_{t-1}}{bi_{t-1}}}}_{\frac{bc_t}{bc_{t-1}} \cdot \frac{bi_{t-1}}{bi_t}} \left(\frac{\tilde{r}_{i,t+1} \cancel{\frac{bc_t}{bi_t}}}{\tilde{p}_{t+1} \cancel{\frac{bc_t}{bi_t}}} + 1 - d_i \right)$$

$$= \frac{1+\gamma_{c,t}}{1+\gamma_{i,t}}$$

$\rightarrow$

$$1 = \beta E_t \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \underbrace{\left(\frac{1}{1+\gamma_{c,t}} \right)^{\frac{1-b}{1-a-b}} \left(\frac{1}{1+\gamma_{i,t}} \right)^{\frac{b}{1-a-b}} \cdot \frac{1+\gamma_{i,t}}{1+\gamma_{c,t}}} \left(\frac{\tilde{r}_{i,t+1}}{\tilde{p}_{t+1}} + 1 - d_i \right)$$

$$\left(\frac{1}{1+\gamma_{c,t}} \right)^{\frac{1-b-(1-a-b)}{1-a-b}} \left(\frac{1}{1+\gamma_{i,t}} \right)^{\frac{b+(1-a-b)}{1-a-b}}$$

$$\left(\frac{1}{1+\gamma_{c,t}} \right)^{\frac{a}{1-a-b}} \left(\frac{1}{1+\gamma_{i,t}} \right)^{\frac{1-a}{1-a-b}}$$

So $1 = \beta E_t \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \left(\frac{1}{1+\gamma_{c,t}} \right)^{\frac{a}{1-a-b}} \left(\frac{1}{1+\gamma_{i,t}} \right)^{\frac{1-a}{1-a-b}} \left(\frac{\tilde{r}_{i,t+1}}{\tilde{p}_{t+1}} + 1 - d_i \right)$ (12)

(13): $w_t = (1-a-b)\, bc_t\, hl_t^{-a-b}\, kcl_t^{a}\, kil_t^{b}$

$$\tilde{w}_t \cdot G_{t-1} = (1-a-b)(1+\gamma_{c,t})\, bc_{t-1}\, G_{t-1}^{a}\, Gl_{t-1}^{b}\, hl_t^{-a-b}\, \tilde{kcl}_t^{a}\, \tilde{kil}_t^{b}$$

$$\underline{\underline{\tilde{w}_t}} = \underbrace{G_{t-1}^{a-1}\, Gl_{t-1}^{b}\, bc_{t-1}}\; \underline{\underline{(1-a-b)(1+\gamma_{c,t})\, hl_t^{-a-b}\, \tilde{kcl}_t^{a}\, \tilde{kil}_t^{b}}}$$

$$\left(bc_{t-1}^{\frac{1-b}{1-a-b}} \cdot bi_{t-1}^{\frac{b}{1-a-b}}\right)^{a-1} \left(bc_{t-1}^{\frac{a}{1-a-b}} \cdot bi_{t-1}^{\frac{1-a}{1-a-b}}\right)^{b} bc_{t-1}$$

$$= bc_{t-1}^{\frac{1-a-b-(1-a)(1-b)+ab}{1-a-b}} \cdot bi_{t-1}^{\frac{-(1-a)b+(1-a)b}{1-a-b}}$$

$$= 1$$

so $\tilde{w}_t = (1-a-b)(1+\gamma_{c,t})\, hl_t^{-a-b}\, \tilde{kcl}_t^{a}\, \tilde{kil}_t^{b}$ (13)

(14): $rc_t = a \cdot bc_t\, hl_t^{1-a-b}\, kcl_t^{a-1}\, kil_t^{b}$

$$rc_t = \underbrace{bc_{t-1}\, G_{t-1}^{a-1}\, GI^{b}}_{\text{the same as above}} \quad (1+\gamma_{c,t})\, a\, hl_t^{1-a-b}\, \tilde{kcl}_t^{a-1}\, \tilde{kil}_t^{b}$$

so $rc_t = (1+\gamma_{c,t})\, a\, hl_t^{1-a-b}\, \tilde{kcl}_t^{a-1}\, \tilde{kil}_t^{b}$ (14)

(15) $ri_t = b\; bc_t\, hl_t^{1-a-b}\, kcl_t^{a}\, kil_t^{b-1}$

$$\underline{\underline{\tilde{ri}_t}}\, \frac{\cancel{bc_{t-1}}}{bi_{t-1}} = \underbrace{\cancel{bc_{t-1}}\, G_{t-1}^{a}\, Gl_{t-1}^{b-1}}_{\underbrace{bi_{t-1}\, G_{t-1}^{a}\, Gl_{t-1}^{b-1}}_{\text{same as in (2), so}}} \cdot \underline{\underline{b(1+\gamma_{c,t})\, hl_t^{1-a-b}\, \tilde{kcl}_t^{a}\, \tilde{kil}_t^{b-1}}}$$

$\tilde{ri}_t = b(1+\gamma_{c,t})\, hl_t^{1-a-b}\, \tilde{kcl}_t^{a}\, \tilde{kil}_t^{b-1}$ (15)

(16) $w_t = P_t (1-a-b)\, bi_t\, h2_t^{-a-b}\, kc2_t^{a}\, ki2_t^{b}$

$$\tilde{w}_t G_{t-1} = \tilde{P}_t \underbrace{\frac{bc_{t-1}}{\cancel{bi_{t-1}}} G_{t-1}^{a} Gl_{t-1}^{b}}_{\substack{bc_{t-1} G_{t-1}^{a-1} Gl_{t-1}^{b} \\ \underbrace{\qquad\qquad}_{\text{same as in (14)}}}} \cancel{bi_{t-1}} (1+\gamma_{i,t})(1-a-b) \cdots$$

$$\tilde{w}_t = \tilde{P}_t (1+\gamma_{i,t})(1-a-b)\, h2_t^{-a-b}\, \tilde{kc2}_t^{a}\, \tilde{ki2}_t^{b} \qquad (16)$$

(17) $rc_t = P_t\, a \cdot bi_t\, h2_t^{1-a-b}\, kc2_t^{a-1}\, ki2_t^{b}$

$$= \tilde{P}_t \cdot \underbrace{\frac{bc_{t-1}}{\cancel{bi_{t-1}}} G_{t-1}^{a-1} Gl_{t-1}^{b}}_{\text{same as above, so}} \cancel{bi_{t-1}} (1+\gamma_{i,t})\, a \cdot h2_t^{1-a-b}\, \tilde{kc2}_t^{a-1}\, \tilde{ki2}_t^{b}$$

$$rc_t = \tilde{P}_t (1+\gamma_{i,t})\, a \cdot h2_t^{1-a-b}\, \tilde{kc2}_t^{a-1}\, \tilde{ki2}_t^{b} \qquad (17)$$

(18) $ri_t = P_t\, b\, bi_t\, h2_t^{1-a-b}\, kc2_t^{a}\, ki2_t^{b-1}$

$$\tilde{ri}_t = \tilde{P}_t \underbrace{G_{t-1}^{a} Gl_{t-1}^{b-1} bi_{t-1}}_{\text{same as (15), so}} (1+\gamma_{i,t})\, h2_t^{1-a-b}\, \tilde{kc2}_t^{a}\, \tilde{ki2}_t^{b-1} \cdot b$$

$$\tilde{ri}_t = \tilde{P}_t (1+\gamma_{i,t})\, b \cdot h2_t^{1-a-b}\, \tilde{kc2}_t^{a}\, \tilde{ki2}_t^{b-1} \qquad (18)$$

$\rightarrow$ The stationarized Outton model = model_exog_stat.m

$$\tilde{yc}_t = (1+\gamma_{c,t})\, h1_t^{1-a-b}\, \tilde{kc1}_t^{a}\, \tilde{ki1}_t^{b} \quad (1)$$

$$\tilde{yi}_t = (1+\gamma_{i,t})\, h2_t^{1-a-b}\, \tilde{kc2}_t^{a}\, \tilde{ki2}_t^{b} \quad (2)$$

$$\tilde{kc}_{t+1}\,(1+\gamma_{c,t})^{\frac{1-b}{1-a-b}}\,(1+\gamma_{i,t})^{\frac{b}{1-a-b}} = (1-dc)\,\tilde{kc}_t + \tilde{ic}_t \quad (6)$$

$$\tilde{ki}_{t+1}\,(1+\gamma_{c,t})^{\frac{a}{1-a-b}}\,(1+\gamma_{i,t})^{\frac{1-a}{1-a-b}} = (1-di)\,\tilde{ki}_t + \tilde{it}_t \quad (7)$$

$$\tilde{yc}_t = \tilde{c}_t + \tilde{ic}_t \quad (8)$$

$$\tilde{kc}_t = \tilde{kc1}_t + \tilde{kc2}_t \quad (3)$$

$$\tilde{yi}_t = \tilde{it}_t \quad (9)$$

$$\tilde{ki}_t = \tilde{ki1}_t + \tilde{ki2}_t \quad (4)$$

$$\frac{\tilde{w}_t}{\tilde{c}_t} = \chi \quad (10)$$

$$h_t = h1_t + h2_t \quad (5)$$

$$1 = \beta E_t \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \left(\frac{1}{1+\gamma_{c,t}}\right)^{\frac{1-b}{1-a-b}} \left(\frac{1}{1+\gamma_{i,t}}\right)^{\frac{b}{1-a-b}} \left(rc_{t+1} + 1 - dc\right) \quad (11)$$

$$1 = \beta E_t \frac{\tilde{c}_t}{\tilde{c}_{t+1}} \left(\frac{1}{1+\gamma_{c,t}}\right)^{\frac{a}{1-a-b}} \left(\frac{1}{1+\gamma_{i,t}}\right)^{\frac{1-a}{1-a-b}} \left(\frac{\tilde{ri}_{t+1}}{\tilde{P}_{t+1}} + 1 - di\right) \quad (12)$$

$$\tilde{w}_t = (1-a-b)(1+\gamma_{c,t})\, h1_t^{-a-b}\, \tilde{kc1}_t^{a}\, \tilde{ki1}_t^{b} \quad (13)$$

$$rc_t = (1+\gamma_{c,t})\, a\, h1_t^{1-a-b}\, \tilde{kc1}_t^{a-1}\, \tilde{ki1}_t^{b} \quad (14)$$

$$\tilde{ri}_t = b(1+\gamma_{c,t})\, h1_t^{1-a-b}\, \tilde{kc1}_t^{a}\, \tilde{ki1}_t^{b-1} \quad (15)$$

$$\tilde{w}_t = \tilde{P}_t\,(1+\gamma_{i,t})(1-a-b)\, h2_t^{-a-b}\, \tilde{kc2}_t^{a}\, \tilde{ki2}_t^{b} \quad (16)$$

$$rc_t = \tilde{P}_t\,(1+\gamma_{i,t})\, a \cdot h2_t^{1-a-b}\, \tilde{kc2}_t^{a-1}\, \tilde{ki2}_t^{b} \quad (17)$$

$$\tilde{ri}_t = \tilde{P}_t\,(1+\gamma_{i,t})\, b \cdot h2_t^{1-a-b}\, \tilde{kc2}_t^{a}\, \tilde{ki2}_t^{b-1} \quad (18)$$

4.) Steady state

Procedure:

1. Drop time indexes & rewrite
2. Following Oulton (p.10 & Annex A), set

   $h = h1 = h2$, $kc = kc1 = kc2$ and $ki = ki1 = ki2$

   → need to figure out market clearing for this case!

   Can that be?!

St.st just dropping the time indexes:

$$yc = (1+\gamma_c)\, h1^{1-a-b}\, kc1^{a}\, ki1^{b} \qquad (1)$$

$$yi = (1+\gamma_i)\, h2^{1-a-b}\, kc2^{a}\, ki2^{b} \qquad (2)$$

$$kc\left[(1+\gamma_c)^{\frac{1-b}{1-a-b}}(1+\gamma_i)^{\frac{b}{1-a-b}} - (1-dc)\right] = ic \qquad (6)$$

$$ki\left[(1+\gamma_c)^{\frac{a}{1-a-b}}(1+\gamma_i)^{\frac{1-a}{1-a-b}} - (1-di)\right] = it \qquad (7)$$

$$yc = c + ic \qquad (8)$$

$$kc = kc1 + kc2 \qquad (3)$$

$$yi = it \qquad (9)$$

$$ki = ki1 + ki2 \qquad (4)$$

$$\frac{w}{c} = \chi \qquad (10)$$

$$h = h1 + h2 \qquad (5)$$

$$1 = \beta\left(\frac{1}{1+\gamma_c}\right)^{\frac{1-b}{1-a-b}}\left(\frac{1}{1+\gamma_i}\right)^{\frac{b}{1-a-b}}(rc + 1 - dc) \qquad (11)$$

$$1 = \beta\left(\frac{1}{1+\gamma_c}\right)^{\frac{a}{1-a-b}}\left(\frac{1}{1+\gamma_i}\right)^{\frac{1-a}{1-a-b}}\left(\frac{ri}{p} + 1 - di\right) \qquad (12)$$

$$w = (1-a-b)(1+\gamma_c)\, h1^{-a-b}\, kc1^{a}\, ki1^{b} \qquad (13)$$

$$rc = (1+\gamma_c)\, a\, h1^{1-a-b}\, kc1^{a-1}\, ki1^{b} \qquad (14)$$

$$ri = b(1+\gamma_c)\, h1^{1-a-b}\, kc1^{a}\, ki1^{b-1} \qquad (15)$$

$$w = p\,(1+\gamma_i)(1-a-b)\, h2^{-a-b}\, kc2^{a}\, ki2^{b} \qquad (16)$$

$$rc = p\,(1+\gamma_i)\, a \cdot h2^{1-a-b}\, kc2^{a-1}\, ki2^{b} \qquad (17)$$

$$ri = p\,(1+\gamma_i)\, b \cdot h2^{1-a-b}\, kc2^{a}\, ki2^{b-1} \qquad (18)$$

St. st. imposing $h = h1 = h2$, $kc = kc1 = kc2$, $ki = ki1 + ki2$

$$yc = (1+\gamma_c)\, h^{1-a-b} kc^{a} ki^{b} \qquad (1)$$

$$yi = (1+\gamma_i)\, h^{1-a-b} kc^{a} ki^{b} \qquad (2)$$

$$kc\left[(1+\gamma_c)^{\frac{1-b}{1-a-b}}(1+\gamma_i)^{\frac{b}{1-a-b}} - (1-dc)\right] = ic \qquad (6)$$

$$ki\left[(1+\gamma_c)^{\frac{a}{1-a-b}}(1+\gamma_i)^{\frac{1-a}{1-a-b}} - (1-di)\right] = it \qquad (7)$$

$$yc = c + ic \qquad (8)$$

TROUBLE!

$$kc = kc + kc \qquad (3)$$

$$ki = ki + ki \qquad (4)$$

$$h = h + h \qquad (5)$$

$$yi = it \qquad (9)$$

$$\frac{w}{c} = \chi \qquad (10)$$

$$1 = \beta\left(\frac{1}{1+\gamma_c}\right)^{\frac{1-b}{1-a-b}}\left(\frac{1}{1+\gamma_i}\right)^{\frac{b}{1-a-b}}(rc + 1 - dc) \qquad (11)$$

$$1 = \beta\left(\frac{1}{1+\gamma_c}\right)^{\frac{a}{1-a-b}}\left(\frac{1}{1+\gamma_i}\right)^{\frac{1-a}{1-a-b}}\left(\frac{ri}{p} + 1 - di\right) \qquad (12)$$

$$w = (1-a-b)(1+\gamma_c)\, h^{-a-b} kc^{a} ki^{b} \qquad (13)$$

$$rc = (1+\gamma_c)\, a\, h^{1-a-b} kc^{a-1} ki^{b} \qquad (14)$$

$$ri = b(1+\gamma_c)\, h^{1-a-b} kc^{a} ki^{b-1} \qquad (15)$$

$$w = p\,(1+\gamma_i)(1-a-b)\, h^{-a-b} kc^{a} ki^{b} \qquad (16)$$

$$rc = p\,(1+\gamma_i)\, a \cdot h^{1-a-b} kc^{a-1} ki^{b} \qquad (17)$$

$$ri = p\,(1+\gamma_i)\, b \cdot h^{1-a-b} kc^{a} ki^{b-1} \qquad (18)$$