

# HDDA Tutorial: MatrixBasics : Solutions

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*Tutorial 7*

## Vectors

Consider the vectors

$$\mathbf{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

Work out the following (without using R). All multiplication is matrix multiplication.

1.  $\mathbf{a} + \mathbf{b}$
2.  $\mathbf{a}'\mathbf{a}$
3.  $\mathbf{a}'\mathbf{b}$
4.  $\mathbf{a}'\mathbf{c}$
5.  $\mathbf{ab}'$

*Solutions provided in section using R*

## Matrices

Consider the matrices

Work out the following (without using R). All multiplication is matrix multiplication.

$$\mathbf{X} = \begin{pmatrix} 1 & 2 \\ 1 & 4 \\ 0 & -1 \end{pmatrix} \quad \mathbf{Y} = \begin{pmatrix} 2 & -1 \\ 3 & 0 \\ 3 & -1 \end{pmatrix}$$

1.  $\mathbf{X} + \mathbf{Y}$
2.  $\mathbf{XY}$
3.  $\mathbf{X}'\mathbf{Y}$

*Solutions provided in section using R*

## Vectors and Matrices in R

Repeat all above questions using R. Useful functions are `c` for setting a vector, `matrix` for setting a matrix and `t` for the transpose. Also note that `*` does NOT do matrix multiplication. Instead use `%*`.

```
#Vectors
```

```
a<-c(2,1)
b<-c(1,0)
c<-c(-2,1)
```

```
a+b
```

```
## [1] 3 1
```

```
t(a)%*%a
```

```
##      [,1]  
## [1,]    5
```

```
t(a)%*%b
```

```
##      [,1]  
## [1,]    2
```

```
t(a)%*%c
```

```
##      [,1]  
## [1,]   -3
```

```
a%*%t(b)
```

```
##      [,1] [,2]  
## [1,]    2    0  
## [2,]    1    0
```

```
#Matrices
```

```
#Make sure you set byrow=T
```

```
X<-matrix(c(1,2,  
            1,4,  
            0,-1),3,2,byrow = T)
```

```
X
```

```
##      [,1] [,2]  
## [1,]    1    2  
## [2,]    1    4  
## [3,]    0   -1
```

```
Y<-matrix(c(2,-1,  
            3,0,  
            3,-1),3,2,byrow = T)
```

```
Y
```

```
##      [,1] [,2]  
## [1,]    2   -1  
## [2,]    3    0  
## [3,]    3   -1
```

```
X+Y
```

```
##      [,1] [,2]  
## [1,]    3    1  
## [2,]    4    4  
## [3,]    3   -2
```

```
# X%*%Y This is non-conformable
```

```
t(X)%*%Y
```

```
##      [,1] [,2]  
## [1,]    5   -1  
## [2,]   13   -1
```

## Data matrix

Consider the data matrix  $\mathbf{Y}$

$$\mathbf{Y} = \begin{pmatrix} y_{11} & y_{12} & \dots & y_{1p} \\ y_{21} & y_{22} & \dots & y_{2p} \\ \vdots & \vdots & \dots & \vdots \\ y_{n1} & y_{n2} & \dots & y_{np} \end{pmatrix}$$

where  $y_{ij}$  is the value of variable  $j$  for observation  $i$

1. How many rows are there in  $\mathbf{Y}$ ?

*There are  $n$  rows, i.e. each observation is a row*

2. How many columns are there in  $\mathbf{Y}$ ?

*There are  $p$  columns, i.e. each variable is a column*

3. What are the dimensions of  $\mathbf{Y}$ ?

*The matrix is an  $n \times p$  matrix*

4. Find an expression for the first row and first column of  $\mathbf{S} = \frac{1}{n-1} \mathbf{Y}'\mathbf{Y}$

*The element on the first row and first column is found by multiplying  $(y_{11} \ y_{21} \ \dots \ y_{n1})$  by its transpose.*

*This is the same as  $y_{11}^2 + y_{21}^2 + \dots + y_{n1}^2$ . Putting everything together it is  $\frac{1}{n-1} \sum_{i=1}^n y_{i1}^2$ . If the data have mean zero, then this is the sample variance of the first variable.*