HDDA Tutorial: MatrixBasics : Solutions

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Tutorial 7

Vectors

Consider the vectors

$$\mathbf{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

Work out the following (without using R). All multiplication is matrix multiplication.

1. $\mathbf{a} + \mathbf{b}$
2. $\mathbf{a}'\mathbf{a}$
3. $\mathbf{a}'\mathbf{b}$
4. $\mathbf{a}'\mathbf{c}$
5. $\mathbf{a}\mathbf{b}'$

Solutions provided in section using R

Matrices

Consider the matrices

Work out the following (without using R). All multiplication is matrix multiplication.

$$\mathbf{X} = \begin{pmatrix} 1 & 2 \\ 1 & 4 \\ 0 & -1 \end{pmatrix} \quad \mathbf{Y} = \begin{pmatrix} 2 & -1 \\ 3 & 0 \\ 3 & -1 \end{pmatrix}$$

1. $\mathbf{X} + \mathbf{Y}$
2. $\mathbf{X}\mathbf{Y}$
3. $\mathbf{X}'\mathbf{Y}$

Solutions provided in section using R

Vectors and Matrices in R

Repeat all above questions using R. Useful functions are `c` for setting a vector, `matrix` for setting a matrix and `t` for the transpose. Also note that `*` does NOT do matrix multiplication. Instead use `%*%`.

```
#Vectors

a<-c(2,4)
b<-c(1,0)
c<-c(-2,1)

a+b
```

```
## [1] 3 4
```

```r
t(a)%*%a
```

```
##      [,1]
## [1,]   20
```

```r
t(a)%*%b
```

```
##      [,1]
## [1,]    2
```

```r
t(a)%*%c
```

```
##      [,1]
## [1,]    0
```

```r
a%*%t(b)
```

```
##      [,1] [,2]
## [1,]    2    0
## [2,]    4    0
```

```r
#Matrices

#Make sure you set byrow=T
X<-matrix(c(1,2,
            1,4,
            0,-1),3,2,byrow = T)
X
```

```
##      [,1] [,2]
## [1,]    1    2
## [2,]    1    4
## [3,]    0   -1
```

```r
Y<-matrix(c(2,-1,
            3,0,
            3,-1),3,2,byrow = T)
Y
```

```
##      [,1] [,2]
## [1,]    2   -1
## [2,]    3    0
## [3,]    3   -1
```

```r
X+Y
```

```
##      [,1] [,2]
## [1,]    3    1
## [2,]    4    4
## [3,]    3   -2
```

```r
# X%*%Y This is non-conformable
t(X)%*%Y
```

```
##      [,1] [,2]
## [1,]    5   -1
## [2,]   13   -1
```

Data matrix

Consider the data matrix $\mathbf{Y}$

$$\mathbf{Y} = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{21} & y_{22} & \cdots & y_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{np} \end{pmatrix}$$

where y_{ij} is the value of variable j for observation i

1. How many rows are there in $\mathbf{Y}$?

There are n rows, i.e. each observation is a row

2. How many columns are there in $\mathbf{Y}$?

There are p columns, i.e. each variable is a column

3. What are the dimensions of $\mathbf{Y}$?

The matrix is an $n \times p$ matrix

4. Find an expression for the first row and first column of $\mathbf{S} = \frac{1}{n-1}\mathbf{Y}'\mathbf{Y}$

The element on the first row and first column is found by multiplying $\begin{pmatrix} y_{11} & y_{21} & \ldots & y_{n1} \end{pmatrix}$ by its transpose. This is the same as $y_{11}^2 + y_{21}^2 + \ldots + y_{n1}^2$. Putting everything together it is $\frac{1}{n-1}\sum_{i=1}^{n} y_{i1}^2$. If the data have mean zero, then this is the sample variance of the first variable.