A Non-Linear Observer Implementation for Magnetic Needle

Vineet Pandey, Ezra Olagoke

***Abstract*—This paper presents an overview of development in the field of Observers and Estimators for Linear and Non-linear systems. Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) are found to be suitable for the magnetic needle [18].Both the filters are implemented and their performances are compared. It is found that UKF outperforms EKF. Detailed implementation and performance have been described in the implementation section.**

***Index Terms*—Steerable Needle, Magnetic manipulation,Filters, Unscented Kalman Filter, Kalman Filter, Extended Kalman Filter**

I. INTRODUCTION

It is important to monitor all the states of a system during operation in order to predict its possible characteristics in future. However, it is difficult to measure all the states using sensors due to the following reasons -difficulty in placing the sensors inside closed systems, cost of the sensor, unavailability of the sensor to work at specific scales and the possibility of damage to the sensor while the system is in operation. Hence, it is critical to bring forth a solution that addresses this problem. State observers and estimators are an efficient answer to the problem. These are essentially a mathematical model estimating the internal state of a system from noisy input measurements, outputs, as well as an indefinite knowledge of the initial conditions of the state. This method of determining the states has been extensively used in various industries [28][6][16]. State observation and estimation became a popular field of research with the advent of faster computers. Extensive studies have been carried out on linear systems which have proven to be extremely useful in observer-based controllers [9]. Development of Kalman filter and Luenberger Observer in the early 1960s and 1970 respectively [12] [14] introduced robust algorithms to estimate states in higher order linear systems. Observers for Non-linear systems became the center of attraction for research after Extended Kalman Filtering was introduced in [3]. This fostered a mathematically strong algorithm to linearize a non-linear system to determine its states. Using EKF, the system is linearized about the mean of the state estimated after propagating it through the nonlinear function. Hence, Kalman Filter became the center of research and many observers were developed around the idea of recursive Kalman Filtering [22][15][3][4][11][21].Concurrently, other observers like High gain observers were developed [13]. These proved to be extremely useful in systems which have a slow response time. Nevertheless, they came with an innate property to amplify the noise in the state estimation model and hence failed on multiple proving grounds [27]. Unbiased Finite Impulse Response filter is another estimator which works on an iterative algorithm like Kalman filter but has been largely been used for linear systems [24] and mostly modeled using batches of state estimates. Hence, they posed a major drawback in terms of Mean Square Error (MSE) when compared with Kalman Filter Variants and presented a difficulty in selection of the optimal length of a batch [22]. Receding Horizon Filters [7] were designed to address this limitation. However, these are computationally expensive and difficult to implement [23]. Information filters are another set of filters which are similar to KFs but modeled in a different space. Hence, alternating between spaces to extract the value in the required form proved to be computationally expensive[1]. Hence, Kalman Filter and its variants proved to be better estimation models for dynamic systems. Although Extended Kalman Filter proved better for nonlinear systems on several occasions [29][21][8], its estimations were largely deviated from the real values when it was used for second or higher order non-linear systems[20]. Ensemble Kalman Filter is a variant of Kalman filter that was introduced to resolve two major problems with Extended Kalman Filter-(a) Use of an approximate closure scheme i.e, the first order Taylor series approximation and (b) storage and forward integration of the error covariance matrix [4].In this algorithm, an ensemble of several possible state vectors is generated using Monte Carlo approach, which represent the current estimated state vector. Another variant of Kalman Filter called Unscented Kalman Filter was developed which worked on a recursive algorithm like Kalman but modified the filtering by the application of unscented transformation to normally distributed points (known as Sigma points) propagated through the nonlinear function [11] [26] whose error convergence is analyzed and proven in [5]. UKF and Ensemble Kalman Filter are compared and contrasted in [10], and it was found that although Ensemble Kalman filter performed better than UKF in presence of larger ensembles, it took longer than UKF to converge to zero error . The following sections describe the basic working and algorithm of linear Kalman Filter (Section-II), Extended Kalman Filter(Section-III) and Unscented Kalman Filter(Section-IV), followed by sections describing the implementation of EKF and UKF for the needle observer in [18].In the end, a comparative study on the performance of UKF and EKF justifies the selection of the suitable observer for the magnetic needle.

II. LINEAR KALMAN FILTER

Kalman Filter is a bayesian Infinite Impulse response filter for linear systems. It starts at time zero and hence requires initial value of state $\hat{X}_0$ and intial estimation error P_0. It requires the process noise and and measurement noise covariances Q1 and R1 respectively, to predict the next state $\hat{X}_1$ and error covariance P_1.Hence, initial value estimation plays an important role in this filter as they affect the estimates over long transients due to infinite impulse response. Consider the following set of linear system:

$$X_k = A_k X_{k-1} + B_k u_k + E_k w_k \tag{1}$$

$$Y_k = C_k X_k + V_k \tag{2}$$

A is a state transition matrix of size $S \times S$ and is applied to the previous state X_{k-1} to determine the current state estimate X_k, where S is the number of state. u_k is the input vector to the system and is of size L. B is the input control matrix of size $S \times L$. E_k is the noise matrix of size $S \times M$. Y_k is the current measurement vector and C_k is the observation matrix of size $P \times S$, which maps X_k to Y_k. Process noise w_k is assumed to be zero mean Gaussian, $w_k \sim N(0, Q_k)$, with covariance Q_k. Observation noise v_k is also assumed to be zero mean Gaussian , $v_k \sim N(0, R_k)$, with covariance R_k. In recursive Kalman filter algorithm, there two phases :

1) Prediction phase
2) Update Phase

In the prediction phase, the current state estimate is determined using the state at the previous time step and current input. This predicted state is called a priori state estimate as it does not use an observation of current time step . Following estimates are produced in this step:

- a priori estimate:

$$\hat{X}^- = A_k \hat{X}_{k-1} + B_k u_k \tag{3}$$

- priori error covariance matrix:

$$P_k^- = A_k P_{k-1} A_k^T + E_k Q_k E_k^T \tag{4}$$

In the update phase , the priori predictions are combined with the state observations to refine state estimate and the error covariance matrix. The estimate that uses current observation is the posteriori state estimate. Following updates are produced [24]in this phase:

- the Innovation residual :

$$Z_k = y_k - C_k \hat{X}_k^- \tag{5}$$

- Innovation Covariance Matrix:

$$S_k = C_k P_k^- C_k^T + R_k \tag{6}$$

- The Kalman Gain :

$$K_k = P_k^- C_k^T S^- 1_k \tag{7}$$

- The posteriori state estimate:

$$\hat{X}_k = \hat{X}_k^- + K_k Z_k \tag{8}$$

- posteriori error covariance matrix:

$$P_k = (I - K_k C_k) P_k^- \tag{9}$$

Larger the error between the estimate and the real system states, higher is the penalty incurred on the state estimate function. Hence, Kalman gain K_k controls the penalty on the estimated value in every iteration to give better estimates in the next iteration.

III. Extended Kalman Filter

The Kalman Filter gives a good approach to provide an estimate for linearly varying systems. It is the ideal estimator for the cases of Gaussian distributions of linear models. However, for our particular project our system will vary nonlinearly; hence a more advanced filtration is required. One solution lies with the enhancement of the Kalman Filter: The Extended Kalman Filter. Much of this resembles the basic Kalman Filter with a similar structure of states and covariances. The main models that alter are the state matrix "A" and the input measurement function as they contain nonlinear functions. Gains and covariances are computed in the same way. Consider a known process nonlinear vectoral function "f" used in computation with the state vector as shown below [25].

$$x_k = f_{k-1} + w_{k-1} \tag{10}$$

For a nonlinear system, the vectoral function can be expanded in a Taylor Series as shown below.

$$f(x_{k-1}) \equiv f(x_{k-1}^a) + J_f(x_{k-1}^a)(x_{k-1} - x_{k-1}^a) \tag{11}$$

$$J_f = \begin{matrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_n} \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_n} \end{matrix} \tag{12}$$

Equation 13 then becomes the following:

$$f(x_{k-1}) \approx f(x_{k-1}^a) + J_f(x_{k-1}^a) e_{k-1} \tag{13}$$

where

$$e_{k-1} \equiv x_{k-1} - x_{k-1}^a \tag{14}$$

The following equation then becomes the forecasted value for the state vector

$$x_k^f \approx f(x_{k-1}^a) \tag{15}$$

The following equation describes the forecasted error:

$$e_k^f \equiv x_k - x_k^f \approx J_f(x_{k-1}^a) e_{k-1} + w_{k-1} \tag{16}$$

The forecasted covariance is given by:

$$P_k^f = J_f(x_{k-1}^a) P_{k-1} J_f^T(x_{k-1}^a) + Q_{k-1} \tag{17}$$

In climax, the Kalman Gain is given by Equation 18 below:

$$K_k = P_k^f J_h^T(x_k^f)(J_h(x_k^f) P_k^f J_h^T(x_k^f) + R_k)^{-1} \tag{18}$$

$$J_h = \begin{matrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \frac{\partial h_1}{\partial x_n} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \frac{\partial h_2}{\partial x_n} \\ \frac{\partial h_n}{\partial x_1} & \frac{\partial h_n}{\partial x_2} & \frac{\partial h_n}{\partial x_n} \end{matrix} \tag{19}$$

$$P_k = (I - K_k J_h(x_k^f))P_k^f \tag{20}$$

Equation 23 is the Jacobian for the observed nonlinear vector function. Equation 20 shows the updated covariance matrix. As is observed, the Extended Kalman Filter is the Kalman Filter simply protracted with additional operations to particularly linearize the nonlinear functions embedded through the filter. By the same token, once the nonlinear model characterizing the measurements has been linearized about each state estimate, the linear Kalman Filter is then implemented to produce the next predicted state estimate [2].

On behalf of our project, considering an implementation of the Extended Kalman Filter, nine states will be evaluated in the ”A” matrix including needle position, velocity, and magnetic field. Insertion velocity is a control input. The needle position will be evaluated using a three dimensional Cartesian coordinate model [19]. Head position and velocity are evaluated by a function that of which will undergo a Jacobian transformation for linearization purposes. Permeating circumferential noise effects, the filter will then give a precise estimate of the nine states specified previously. Our model will exhibit the following system of governing equations:

$$\dot{P} = h * v_{el} \tag{21}$$

$$\dot{H} = -H_x^2 B * v_{el} e + H(1 - H^T H) \tag{22}$$

$$\dot{B} = -B_x \overrightarrow{w} + B(1 - B^T B) \tag{23}$$

$$H^2 = (HH^T - (H^T H)I) \tag{24}$$

Equations 21 and 22 represent the head dynamics of the needle; Equation 23 represents the magnetic field surrounding the needle.

IV. Unscented Kalman Filter

Unscented Kalman Filter (UKF) was introduced by Julier and Uhlmann in [11]. It is based on the idea that approximation of Gaussian distribution is easier than approximating a non-linear function (like in EKF).It addresses the major drawbacks of Extended Kalman Filter by utilizing sigma points from the distribution of the state vector at the beginning of every time step and then propagating them through the non-linear function. Number of sigma points equals 2N+1, where N is the dimension of the state variable x of the state vector X. If state variable x is scalar then the number of sigma points equal to 5. All the sigma points selected are weighted points and always add up to 1, i.e, if χ^i is the vector of sigma points and w^i are the weights corresponding to each sigma point selected, then,

- $\Sigma w^i = 1$
- mean, $\mu = \Sigma w^i \chi^i$
- covariance vector, $\Sigma = \Sigma w^i(\chi^i - \mu)(\chi^i - \mu)$

These sigma points are chosen such that the first element of χ is μ and the rest of the points are determined around μ. Hence, they are evaluated as:
$\chi^0 = \mu$
$\chi^i = \mu \pm (\sqrt{(n+\lambda)\sigma})$, where i=1...2n , λ is the scaling factor that decides the spread of sigma points around the mean μ and n is the dimension of the variable in state matrix vector.
Similarly, weights of the sigma points are determined as follows:
$w_m^0 = \lambda/n + \lambda$
$w_c^0 = w_m^0 + (1 - \alpha^2 + \beta)$
$w_n^i = w_c^i = 1/2(n+\lambda)$
, where w_m^0 and w_c^0 are the weights of mean point and the error covariance, & $\alpha \in (0, 1]$, $\lambda = \alpha^2(n+k) - n$, where $k \geq 0$, and β in general equals to 2, are selection parameters. Unlike in linear systems which have A, B & C matrices to represent the system, non-linear systems have a function G(X,u) which represents the state estimation equation and H(X,u) which represents function similar to observation matrix. Error covariance matrices are modelled similar to linear systems. UKF, like KF, has two phases during estimation - Prediction and Gain& Update. Following is the mathematical model and algorithm of UKF:

- $\chi_{t-1} = (\mu_{t-1}, \mu_{t-1} \pm \gamma\sqrt{\Sigma_{t-1}})$
- $\overline{\chi}^* = G(u_t, \chi_{t-1})$
- $\overline{\mu}_t = \sum_{i=0}^{2n} w_n^i \overline{\chi}_t^i$
- $\overline{\Sigma}_t = \sum_{i=0}^{2n} w_c^i(\overline{\chi}_t^* - \overline{\mu}_t)(\overline{\chi}_t^* - \overline{\mu}_t)^T + R_t$
- $\overline{\chi}_t = (\overline{\mu}_t, \overline{\mu}_t \pm \gamma\sqrt{\overline{\Sigma}_t})$
- $\overline{Z}_t = H(\overline{\chi}_t)$
- $\hat{Z}_t = \sum_{i=0}^{2n} w_m^i Z_t^i$
- $S_t = \sum_{i=0}^{2n} w_c^i(\overline{Z}_t^i - \hat{Z}_t)(\overline{Z}_t^i - \hat{Z}_t)^T + Q_t$
- $\overline{\Sigma}_t^{x,z} = \sum_{i=0}^{2n} w_c^i(\overline{\chi}_t^i - \overline{\mu}_t)(\overline{Z}_t^i - \hat{Z}_t)^T$
- $K_t = \overline{\Sigma}_t^{x,z} S_t^{-1}$
- $\mu_t = \overline{\mu}_t = K_t(Z_t - \overline{Z}_t)$
- $\Sigma_t = \overline{\Sigma}_t - K_t S_t K_t^T$
- return μ_t, Σ_t

V. Implementation

Both, Extended Kalman Filter and Unscented Kalman Filter were implemented for the needle observer system that is governed by the equations below. $G(X, u)$:

$$\dot{P} = H * V \tag{25}$$

$$\dot{H} = -[H]_X^2 B * V * e + H(1 - H^T H) \tag{26}$$

$$\dot{B} = -[B]_X \vec{\omega} + B(1 - B^T B) \tag{27}$$

$$Z(X, u) = C * \begin{pmatrix} P \\ B \end{pmatrix} \tag{28}$$

Where G(X,u) is the state estimation equation in continuous time, and Z(X,u) is the measurement equation based on the predicted state. This system has 9 states i.e, three components of Position P, Heading H, and Magnetic field B, each. Position and Magnetic field are measured using the respective sensors and compared against the estimated value to determine the Kalman gain K_t.Scaling factor, e is assumed to be equal to $1/3$. Following subsections describe the implementation of Extended Kalman Filter and Unscented Kalman Filter.

A. Implementation of Extended Kalman Filter

The concept of the Extended Kalman Filter is utilized in our state estimation for the trajectory of magnetic needle; MATLAB being the software tool. The entire code is classified into five separate functional scripts; all however are within the same folder. Only one specified script is required to be executed for the entire code to dispatch. As stated previously, the nine states evaluated in the **A** matrix of the emblematic Extended Kalman Filter include the respective needle position, velocity, and magnetic field. Insertion velocity is a control input. The needle position is evaluated using a three dimensional Cartesian coordinate model. Head position and velocity are evaluated by a function that of which undergoes a Jacobian transformation for linearization purposes. Permeating circumferential noise effects, the filter then gives a precise estimate of the nine states.

Our observed measurements, state covariance, Kalman Gain, output covariance, and output state are given by the following respectively.

$$Y_e = HX_f \tag{29}$$

$$P_x = J_x P_1 J_x^T + Q \tag{30}$$

$$K = P_x H^T/(HP_x H^T + R) \tag{31}$$

$$P_o = (I - KH)P_x \tag{32}$$

$$X_o = X_f + K(Y_m - Y_e) \tag{33}$$

Here, "H" is a 9x9 matrix that filters out the needle position and magnetic field. Equation 9 denotes the final state added with the product of the Kalman Gain and the difference between the actual measurement and the observed measurement respectively. In our model, the Jacobian state is a 9x9 matrix; each individual element is a differentiation of one state with respect to another. Ultimately, all nine states are partially differentiated. This script, denoted Kalman, incorporates much of the user interface with the trajectory setting, initial conditions, as well as the state control. Plenty of executions were run with this code. Setting a trajectory length of two hundred as well as a chiefly grounded initial condition, the following XY position, the top graph, and filtered trajectory error plots, the bottom graph were acquired in Figure 1.

Most of the initial conditions were set at zero, hence, for the XY Trajectory graph, the Extended Kalman Filter and Measurement outputs were identical.

Figure 2 shows a plot with altered initial conditions, insertion velocity, and magnetic field. Here, it can be seen how the

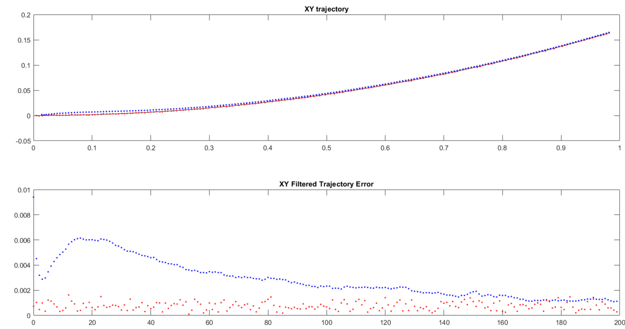

Fig. 1. Trajectory and error convergence

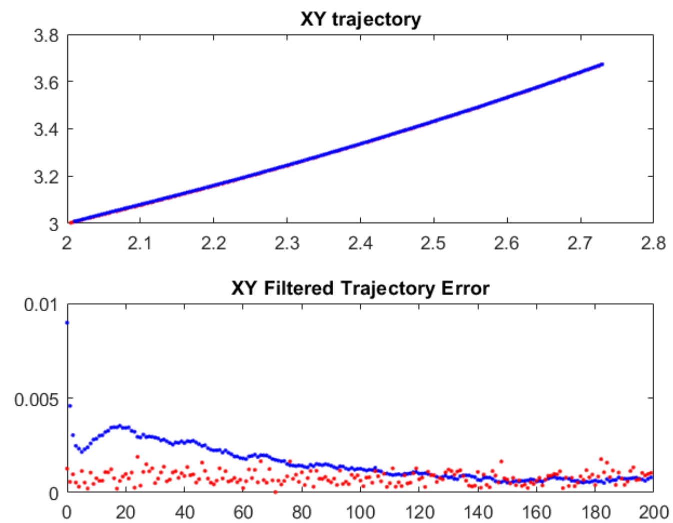

Fig. 2. Trajectory and error convergence at distorted initial condition

measurement output and Extended Kalman Filter on the top graph vary initially due to the change in the initial state guess. The Extended Kalman Filter is able not only to account for noise surroundings but also account for deviations between the initial state estimate and the actual initial state [17], [**hamzah**]. Moreover, its error rapidly converges to zero as can be seen on the bottom graphs of all three preceding figures.

B. Implementation of Unscented Kalman Filter

The principle and algorithm of Unscented Kalman Filtering explained in section IV were used to implement an observer on the system described above, in MATLAB.A process covariance of R and a measurement covariance of Q are modelled in the observer.A needle velocity of $V = 3ms^{-1}$ and an angular velocity of magnetic field $\omega = [1, 2, 3]'\ rads^{-1}$ are assumed. The Actual system is discretized to a small period of 0.025 seconds and ODE45 MATLAB solver is used to integrate the states over time to predict the consecutive states and measurement output.Fig 3 shows the filter implementation schematic.

A trajectory generator function was developed to generate the desired trajectory in the absence of actual system. Various

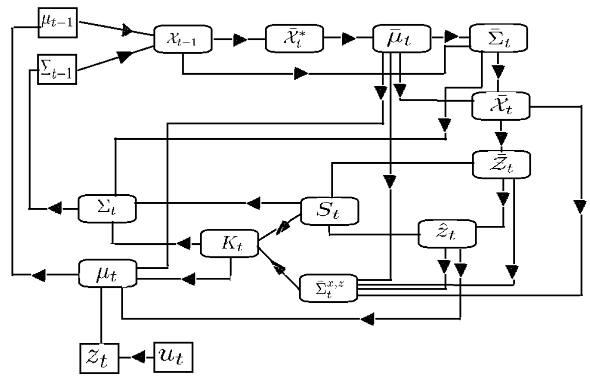

Fig. 3. Unscented Kalman Filter implementation schematic

estimated trajectories were plotted in MATLAB for different values of actual & estimated values and covariances. One of the trajectories is as shown in fig 4 and fig 5.

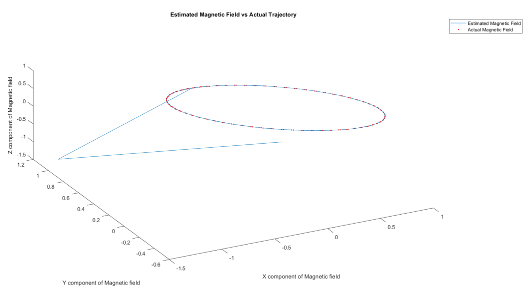

Fig. 4. Trajectory of estimated Magnetic field with time

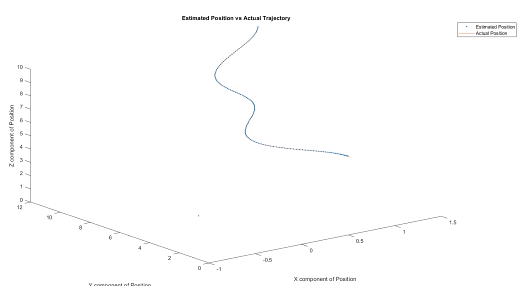

Fig. 5. Trajectory of estimated Magnetic field with time

Kalman gains K_t, Mean state estimate μ_t and Measurements $Z(X, u)$ are tracked over time to analyze the performance of the filter. Fig 6 and Fig 7 show the error convergence of the system when UKF is implemented.

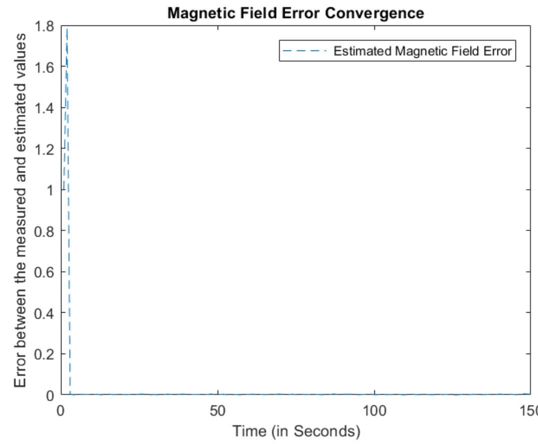

Fig. 6. Error convergence of estimated Magnetic field with time

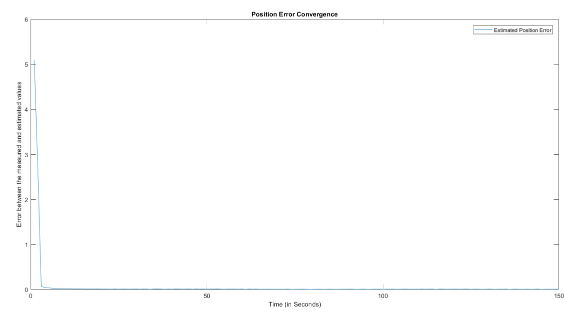

Fig. 7. Error convergence of estimated Position with time

VI. Discussion & Conclusion

From the observations from graphs plotted for Unscented Kalman Filter and Extended Kalman Filter, it can be inferred that UKF outperforms EKF in many aspects. Although its easier to implement EKF, better error convergence and estimates are obtained from UKF as it renders the non-linearity of the dynamic system to the estimated sigma points which in turn help in better estimates, unlike in EKF, where the estimates are determined about the mean of the curve.Hence, UKF is the better observer to predict the states of the magnetic needle. Implementation codes for EKF & UKF have been placed in the folder along with the report.

References

[1] Nicholas J. Belkin and W. Bruce Crof. "Information filtering and information retrieval: Two sides of the same coin?" In: *Association for Computing Machinery* (1992).

[2] G. A. Einicke and L. B. White. "Robust extended kalman filtering". In: *IEEE Transactions On Signal Processing VOL 47, NO. 9* (1999).

[3] Geir Evensen. "Sequential Data Assimilation with a non Linear quasi geostropic model using Monte Carlo Methods to forecast error statistics". In: *Journal of geophysical Research, Vol.99* (1994).

[4] Geir Evensen. "The Ensemble Kalman Filter: theoretical formulation and practical implementation". In: *Ocean Dynamics , Volume 53* (2003).

[5] Jiuchao Feng, Hongjuan Fan, and Chi K. Tse. "Convergence Analysis of the Unscented Kalman Filter for Filtering Noisy Chaotic Signals". In: *2007 IEEE International Symposium on Circuits and Systems* (2007).

[6] Nikolay Gnezdov, Aleksey Kolganov, and Sergey Lebedev. "Application of Astatic State Observers in Electromechanotronic Modules". In: *Scientific Journal of Riga Technical University* (2010).

[7] Soo Hee Han, Wook Hyun Kwon, and Pyung Soo Kim. "Receding-Horizon Unbiased FIR Filters for Continuous- Time State-Space Models Without a priori Initial State Information". In: *IEEE Transactions On Automatic Control, Vol. 46* (2001).

[8] C. Navarro Hernandez, S.P. Banks, and M. Aldeen. "Observer Design for Nonlinear Systems using Linear Approximations". In: *IMA Journal of Mathematical Control and Information, Volume 20* (2003).

[9] Z. Hidayat et al. "Observers for Linear Distributed-Parameter Systems: A Survey". In: *IEEE International Symposium on Robotic and Sensors Environments* (2011).

[10] Anneke Hommels, Akira Murakami, and Shin-Ichi Nishimura. "Comparison of the Ensemble Kalman filter with the Unscented Kalman filter: application to the construction of a road". In: *19th European Young Geotechnical Engineers Conference* (2008).

[11] Simon J. Julier and Jeffrey K. Uhlmann. "A New Extension of the Kalman Filter to Nonlinear Systems". In: *Proceedings Volume 3068, Signal Processing, Sensor Fusion, and Target Recognition VI* (1997).

[12] R. E. kalman and R. S. Bucy. "New Results in Linear Filtering and Prediction Theory". In: *Journal of Basic Engineering, ASME* (1961).

[13] Hassan K. Khalil. "High gain Observers in Feedback Control". In: *IEEE Control Systems Magazine* (2017).

[14] David G. Luenberger. "An Introduction to Observers". In: *IEEE Transactions On Automatic Controls* (1971).

[15] J. M. Maestre, P. Giselsson, and A. Rantzer. "Distributed Receding Horizon Kalman Filter". In: *49th IEEE Conference on Decision and Control* (2010).

[16] H. J. Marquez and M. Riaz. "Robust State Observer Design with Application to an Industrial Boiler System". PhD thesis. University of Alberta, 2004.

[17] P. Poignet P. Mignon and J. Troccaz. "Beveled-tip needle-steering using 3d ultrasound, mechanical-based kalman filter and curvilinear roi prediction". In: *14th International Conference on Control, Automation, Robotics and Vision (ICARCV)* (2016).

[18] Andrew J. Petruska et al. "Magnetic Needle Guidance for Neurosurgery: Initial Design and Proof of Concept". In: *International Conference on Robotics and Automation* (2016).

[19] J. Memisevic R. J. Webster and A. M. Okamura. "Design considerations for robotic needle steering". In: *Proceedings of the 2005 IEEE International Conference on Robotics and Automation* (2005).

[20] Manojkumar Rampelli and Debashisha Jena. "Advantage of Unscented Kalman Filter Over Extended Kalman Filter in Dynamic State Estimation of power Sytem Networks". In: *Michael Faraday IET International Summit* (2015).

[21] S. Koteswara Rao. "Modified gain extended Kalman filter with application to bearings-only passive manoeuvring target tracking". In: *IEE Proceedings - Radar, Sonar and Navigation* (2005).

[22] Yuriy S. Shmaliy. "An Iterative Kalman-Like Algorithm Ignoring Noise and Initial Conditions". In: *IEEE Transactions On Signal Processing* (2011).

[23] Yuriy S. Shmaliy and Daniel J. Simon. "Iterative Unbiased FIR State Estimation: A Review of Algorithms". In: *EURASIP Journal on Advances in Signal Processing* (2013).

[24] Yuriy S. Shmaliy, Shunyi Zhao, and Choon Ki Ahn. "Unbiased Finite Impulse Response Filtering". In: *IEEE Control Systems Magazine* (2017).

[25] G. A. Terejanu. "Extended kalman filter tutorial". In: (2017).

[26] Eric A. Wan and Rudolph van der Menve. "The Unscented Kalman Filter for Nonlinear Estimation". In: *Proceedings of the IEEE 2000 Adaptive Systems for Signal Processing, Communications, and Control Symposium* (2000).

[27] Weiwen Wang and Zhiqiang Gao. "A Comparison Study of Advanced State Observer Design Techniques". In: *American Control Conference* (2003).

[28] Ming Xin and S.N. Balakrishnan. "A New State Observer and Flight Control of Highly Maneuverable Aircraft". In: *American Control Conference* (2009).

[29] Yongchang Zhang et al. "A Comparative Study of Luenberger Observer, Sliding Mode Observer and Extended Kalman Filter for Sensorless Vector Control of Induction Motor Drives". In: *IEEE Energy Conversion Congress and Exposition* (2009).