Advanced Asset Pricing

Lecture 7

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Overview

- Motivation and basic idea
- Model
- Numerical solution
- Simulated data
- Historical data
- Extensions and additional evidence
- Main literature

Intro

- Key mechanism of the ▸ Campbell and Cochrane (1999) habit model is business cycle related time varying risk aversion.
  - In good times, when consumption is well above the habit level, relative risk aversion is low.
  - In bad times, when consumption gets close to the habit level, relative risk aversion is high.
- Investors fear stocks in recessions when risk aversion is high.

Intro

The habit model matches:

- The level of the equity premium and its volatility.
- The level of the risk-free rate.
- Countercyclical variation of stock market volatility.
- Time-series predictability of returns from dividend-price ratios.

Model

- The utility function of the representative investor is:

$$E_t \sum_{j=0}^{\infty} \delta^j \frac{\left(C_{t+j} - X_{t+j}\right)^{1-\gamma} - 1}{1-\gamma} \tag{1}$$

C_t is real consumption, X_t is the external habit level, δ is the impatience parameter, and γ is the utility curvature parameter.

- External habit: A person's habit level depends on the past history of aggregate consumption and not the person's own past consumption.
- Campbell and Cochrane capture the relation between consumption and habit through the surplus consumption ratio:

$$S_t \equiv \frac{C_t - X_t}{C_t} \tag{2}$$

Model

- The logarithm of the surplus consumption ratio $s_t = \ln(S_t)$ is specified as a stationary first-order autoregressive process:

$$s_{t+1} = (1-\phi)\,\bar{s} + \phi s_t + \lambda(s_t)\, v_{t+1} \tag{3}$$

  where $0 < \phi < 1$ is the habit persistence parameter, $\bar{s}$ is the steady state level of s_t, and $\lambda(s_t)$ is the sensitivity function that determines how innovations in consumption growth v_{t+1} influence s_{t+1}.

- The consumption growth process is given by:

$$\triangle c_{t+1} = g + v_{t+1}, \qquad v_{t+1} \sim niid\left(0, \sigma_c^2\right) \tag{4}$$

  where $c_t = \ln(C_t)$.

Model

- Habit is a non-linear function of current and past consumption.
- We can substitute $s_t \equiv \ln\left(1 + \exp\left(x_t - c_t\right)\right)$ into (3) and then linearize around the steady state (see page 6 in ▸ Campbell and Cochrane (1995, NBER)):

$$x_{t+1} \approx \alpha + (1 - \phi) \sum_{j=0}^{\infty} \phi^j c_{t-j} \tag{5}$$

- From the linear approximation, we see that habit adjusts slowly to consumption.
- Note that if we use a linear model for habit as in (5), then habit can go below consumption. CC (1999) avoids this problem by using (3).

Model

Time-varying relative risk aversion

- In the habit model relative risk-aversion is no longer measured by γ but as $\frac{\gamma}{S_t}$:

$$\eta_t \equiv -\frac{C_t u_{cc}(C_t, X_t)}{u_c(C_t, X_t)} = \frac{\gamma C_t (C_t - X_t)^{-\gamma-1}}{(C_t - X_t)^{-\gamma}} = \frac{\gamma C_t}{(C_t - X_t)} = \frac{\gamma}{S_t}$$

- This shows that relative risk-aversion is time-varying and counter-cyclical: when consumption is high relative to habit, relative risk-aversion is low.
- By contrast, when consumption is low and close to habit, relative risk-aversion is high.

Model

Stochastic discount factor

- The stochastic discount factor:

$$M_{t+1} \equiv \delta \frac{u_c\left(C_{t+1}, X_{t+1}\right)}{u_c\left(C_t, X_t\right)} = \delta \left(\frac{C_{t+1}}{C_t} \frac{S_{t+1}}{S_t} \right)^{-\gamma}$$

- Note that:

$$\begin{aligned} M_{t+1} &= \delta G^{-\gamma} \exp\left(-\gamma\left(s_{t+1} - s_t + v_{t+1}\right)\right) \\ s_{t+1} - s_t &= (1-\phi)\left(s_t - \bar{s}\right) + \lambda\left(s_t\right) v_{t+1} \\ M_{t+1} &= \delta G^{-\gamma} \exp\left(-\gamma\left[(\phi - 1)\left(s_t - \bar{s}\right) + \left(1 + \lambda\left(s_t\right)\right) v_{t+1}\right]\right) \end{aligned}$$

Model

Expected excess return

- If excess returns on the stock market and consumption growth are jointly conditionally lognormally distributed, the habit model implies:

$$E_t\left(r_{t+1}^e\right)+\frac{1}{2}\sigma_t^2=\gamma\left[1+\lambda\left(s_t\right)\right] cov_t \tag{6}$$

  where $E_t\left(r_{t+1}^e\right)$ is the expected log excess stock return and $\frac{1}{2}\sigma_t^2$ is a Jensen's inequality term.
- Eq. (6) shows that the expected excess stock return is given by the state-dependent price of risk, $\gamma\left[1+\lambda\left(s_t\right)\right]$, times the amount of risk, cov_t (the conditional covariance between returns and consumption growth).
- Since $\lambda\left(s_t\right)$ is decreasing in s_t, it follows that expected excess stock returns vary counter-cyclically with s_t. Thus, investors require a higher expected excess stock return in recession times when consumption is close to habit.

Model

The risk-free rate

- The log real risk-free rate is:

$$\begin{aligned} r_{f,t} &= \ln\left[\frac{1}{E_t\left[M_{t+1}\right]}\right] \\ &= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[1+\lambda(s_t)\right]^2 \end{aligned} \quad (7)$$

- First two terms are similar to power utility model, although γ is not equal to the risk aversion coefficient as in the power utility model.
- $\gamma(1-\phi)(s_t - \bar{s})$: For low values of s_t, there is an incentive to borrow and the risk-free rate increases (intertemporal substitution).
- $\frac{\gamma^2\sigma^2}{2}\left[1+\lambda(s_t)\right]^2$: Higher uncertainty increases the willingness to save and the risk-free rate decreases (precautionary savings).
- Campbell and Cochrane choose $\lambda(s_t)$ such that the two effects offset and the risk-free rate is constant.

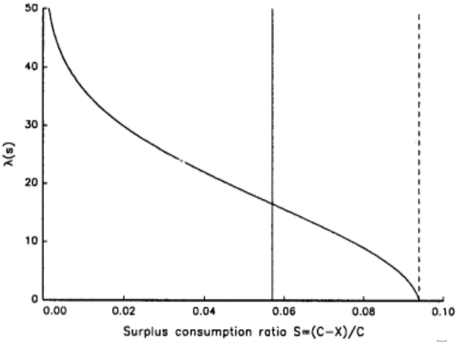
Model

Sensitivity function

The sensitivity function $\lambda(s_t)$ is specified as follows:

$$\lambda\left(s_t\right)=\left\{\begin{array}{ll}\frac{1}{\bar{S}}\sqrt{1-2\left(s_t-\bar{s}\right)}-1, & s_t\leq s_{\max} \\ 0 & s_t\geq s_{\max}\end{array}\right\}, \tag{8}$$

where $\overline{S}=\sigma_c\sqrt{\frac{\gamma}{1-\phi}}$, $s_{\max}=\overline{s}+\frac{1}{2}(1-\overline{S}^2)$, and $\overline{s}=\ln(\overline{S})$.



Model

Constant risk-free rate

$$
\begin{aligned}
r_{f,t} &= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[1 + \lambda(s_t)\right]^2 \\
&= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[\frac{1}{\bar{S}}\sqrt{1 - 2(s_t - \bar{s})}\right]^2 \\
&= -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[\frac{1-\phi}{\sigma^2\gamma}(1 - 2(s_t - \bar{s}))\right] \\
&= -\ln(\delta) + \gamma g - \frac{\gamma}{2}(1-\phi)
\end{aligned}
$$

Model

Time-variation in the risk-free rate

- Extra parameter B:

$$\overline{S} = \sigma_c \sqrt{\frac{\gamma}{1 - \phi - B/\gamma}}$$

$$r_{f,t} = -\ln(\delta) + \gamma g - \frac{\gamma(1-\phi) - B}{2} - B(s_t - \overline{s})$$

- Positive values of B imply a counter-cyclical real risk-free rate and an upward-sloping yield curve (more on this later).
- $B = 0$ corresponds to the baseline version of the Campbell-Cochrane model with a constant real risk-free rate and a flat term structure.

Model

Consumption claim

- The aggregate market is represented as a claim to the future consumption stream. The price-consumption ratio for a consumption claim satisfies:

$$\frac{P_t}{C_t}(s_t) = E_t\left[M_{t+1}\frac{C_{t+1}}{C_t}\left[\frac{P_{t+1}}{C_{t+1}}(s_{t+1}) + 1\right]\right] \tag{9}$$

- The surplus consumption ratio is the only state variable in the model. Thus, the price-consumption ratio can therefore be written as a function of s_t only.
- Note that since consumption is the dividend paid by the market, the price-consumption ratio is analogous to the price-dividend ratio. Thus, the return on the aggregate market is given as:

$$R_{t+1} = \frac{(P_{t+1}/C_{t+1}) + 1}{P_t/C_t}\frac{C_{t+1}}{C_t} \tag{10}$$

Model

Dividend claim

- Alternatively, the model can be solved using the dividend claim:

$$\frac{P_t}{D_t}(s_t) = E_t\left[M_{t+1}\frac{D_{t+1}}{D_t}\left[\frac{P_{t+1}}{D_{t+1}}(s_{t+1}) + 1\right]\right] \tag{11}$$

- Dividend process:

$$\triangle d_{t+1} = g + w_{t+1}, \qquad w_{t+1} \sim niid\left(0, \sigma_w^2\right)$$

where $corr\left(v_{t+1}, w_{t+1}\right) = \rho$.

Model

Parameters

TABLE 1

PARAMETER CHOICES

Parameter	Variable	Value
Assumed:		
Mean consumption growth (%)*	g	1.89
Standard deviation of consumption growth (%)*	σ	1.50
Log risk-free rate (%)*	r^f	.94
Persistence coefficient*	ϕ	.87
Utility curvature	γ	2.00
Standard deviation of dividend growth (%)*	σ_w	11.2
Correlation between Δd and Δc	ρ	.2
Implied:		
Subjective discount factor*	δ	.89
Steady-state surplus consumption ratio	$\bar{S}$	.057
Maximum surplus consumption ratio	$S_{\max}$	.094

* Annualized values, e.g., $12g$, $\sqrt{12}\sigma$, $12r^f$, ϕ^{12}, and δ^{12}, since the model is simulated at a monthly frequency.

Numerical solution

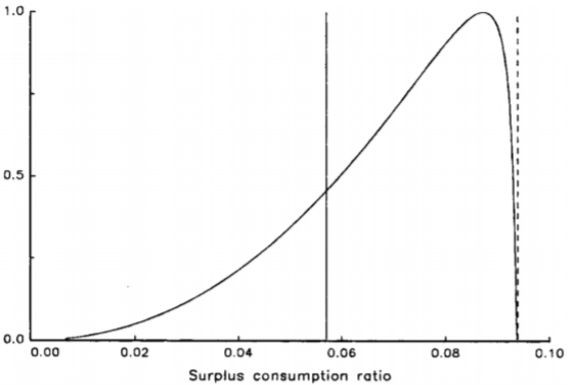


FIG. 2.—Unconditional distribution of the surplus consumption ratio. The solid vertical line indicates the steady-state surplus consumption ratio $\bar{S}$, and the dashed vertical line indicates the upper bound of the surplus consumption ratio $S_{\max}$.

Numerical solution

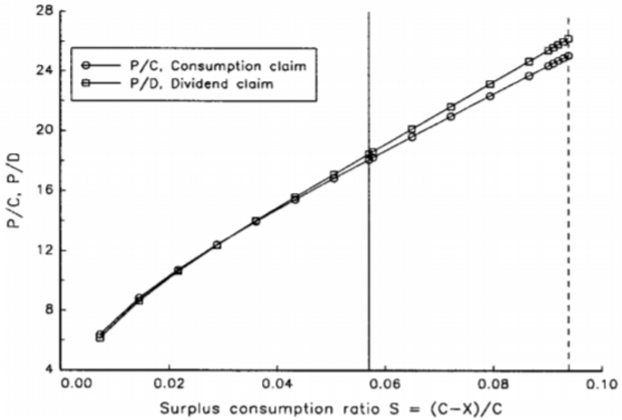


FIG. 3.—Price/dividend ratios as functions of the surplus consumption ratio

Numerical solution

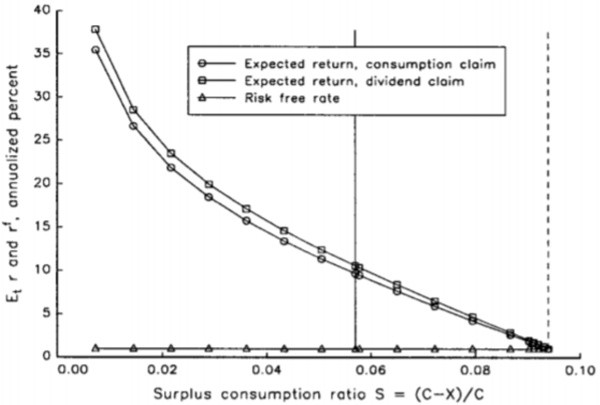


FIG. 4.—Expected returns and risk-free rate as functions of the surplus consumption ratio.

Numerical solution

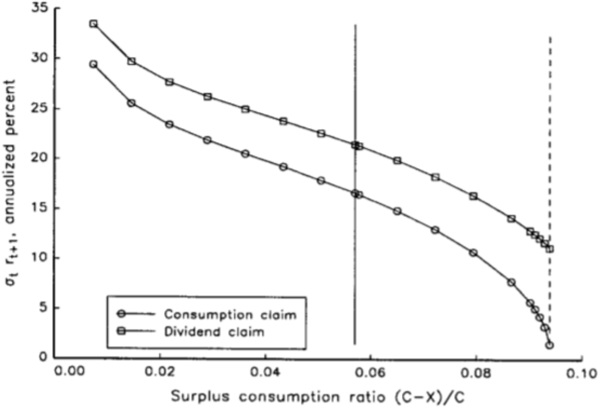


FIG. 5.—Conditional standard deviations of returns as functions of the surplus consumption ratio.

Simulated data

TABLE 2

MEANS AND STANDARD DEVIATIONS OF SIMULATED AND HISTORICAL DATA

Statistic	Consumption Claim	Dividend Claim	Postwar Sample	Long Sample
$E(\Delta c)$	1.89*		1.89	1.72
$\sigma(\Delta c)$	1.22*		1.22	3.32
$E(r^f)$	.094*		.094	2.92
$E(r - r^f)/\sigma(r - r^f)$	.43*	.33	.43	.22
$E(R - R^f)/\sigma(R - R^f)$	.50		.50	
$E(r - r^f)$	6.64	6.52	6.69	3.90
$\sigma(r - r^f)$	15.2	20.0	15.7	18.0
$\exp[E(p - d)]$	18.3	18.7	24.7	21.1
$\sigma(p - d)$	.27	.29	.26	.27

NOTE.—The model is simulated at a monthly frequency; statistics are calculated from artificial time-averaged data at an annual frequency. All returns are annual percentages.

* Statistics that model parameters were chosen to replicate.

Simulated data

TABLE 5

LONG-HORIZON RETURN REGRESSIONS

	CONSUMPTION CLAIM		DIVIDEND CLAIM		POSTWAR SAMPLE		LONG SAMPLE	
HORIZON (Years)	10 × Coefficient	R^2	10 × Coefficient	R^2	10 × Coefficient	R^2	10 × Coefficient	R^2
1	−2.0	.13	−1.9	.08	−2.6	.18	−1.3	.04
2	−3.7	.23	−3.6	.14	−4.3	.27	−2.8	.08
3	−5.1	.32	−5.0	.19	−5.4	.37	−3.5	.09
5	−7.5	.46	−7.3	.26	−9.0	.55	−6.0	.18
7	−9.4	.55	−9.2	.30	−12.1	.65	−7.5	.23

Historical data

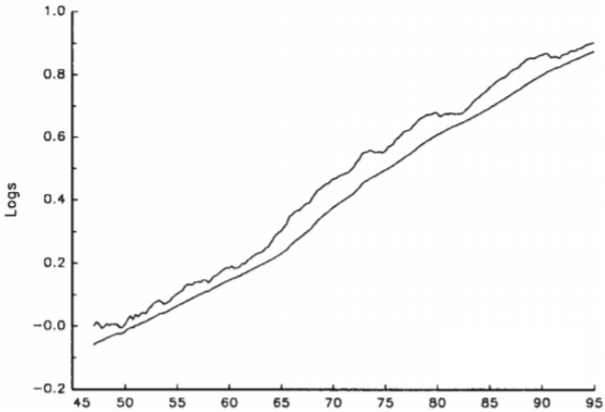


FIG. 8.—Nondurable and services consumption per capita and habit level implied by the model, under the assumption that the surplus consumption ratio starts at the steady state.

Historical data

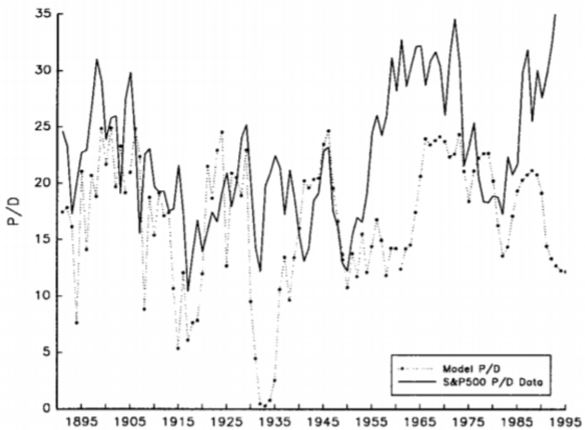


FIG. 9.—Historical price/dividend ratio and model predictions based on the history of consumption.

Extensions and additional evidence

Bond market

- Campbell and Cochrane (1999) have their main focus on explaining stylized facts on the equity market. They set $B = 0$, which implies a constant real risk-free rate and a flat yield curve.
- Despite high relative risk aversion, the model implies a plausible low value for the real risk-free rate. Thus, the model explains the equity premium puzzle without facing a risk-free rate puzzle.
- Campbell and Cochrane (1995, NBER) consider an extension of the model where the risk-free rate is time-varying, which leads to countercyclical variation in bond risk premia.

Extensions and additional evidence

Bond market

- CC (1995, NBER) specify the real risk-free rate to be a linear function of s_t:

$$r_{f,t} = -\ln(\delta) + \gamma g - \frac{\gamma(1-\phi) - B}{2} - B(s_t - \bar{s}) \qquad (12)$$

- B controls the precautionary savings and intertemporal smoothing effects of surplus consumption on the real risk-free rate.
- To get an upward sloping yield curve, we need a positive value of B.
- $B > 0$ implies that the intertemporal substitution effect is larger than the precautionary savings effect.

Extensions and additional evidence

Bond market

- CC (1995) show that $B > 0$ generates yield spreads that forecast bond returns.
- The relation between yield spreads and future bond returns is positive as we see in the data.
- Allowing for a nonzero value of B does not impact the model predictions for stock prices and returns.

Extensions and additional evidence

Bond market

The Euler equation for bonds is:

$$P_{n,t} = E_t \left[P_{n-1,t+1} M_{t+1} \right]$$

where $P_{n,t}$ is the time t real price of a bond with maturity n, and the log real yield is given by $y_{n,t} = -\frac{1}{n} \log P_{n,t}$. It is possible to find closed form solutions for $n = 1$ and $n = 2$, but not for higher values of n. For $n = 1$ the real price is:

$$\begin{aligned} P_{1,t} &= E_t \left[P_{0,t+1} M_{t+1} \right] \\ &= E_t \left[M_{t+1} \right] \\ &= e^{-r_{f,t+1}} \end{aligned}$$

where $r_{f,t+1}$ is known at time t.

Extensions and additional evidence

Bond market

For $n = 2$ the real price is:

$$
\begin{aligned}
P_{2,t} &= E_t\left[P_{1,t+1} M_{t+1}\right] \\
&= E_t\left[e^{-\bar{r}+B(s_{t+1}-\bar{s})}\delta e^{-\gamma\{g+(\phi-1)(s_t-\bar{s})+[1+\lambda(s_t)]v_{t+1}\}}\right] \\
&= \delta e^{-\bar{r}} e^{-\gamma g} e^{(\gamma(1-\phi)+B\phi)(s_t-\bar{s})} e^{0.5\sigma_c^2[B\lambda(s_t)-\gamma-\gamma\lambda(s_t)]^2}
\end{aligned}
$$

where $\bar{r} = -\ln(\delta) + \gamma g - \frac{\gamma(1-\phi)-B}{2}$.

Extensions and additional evidence

Bond market

- The implied two-period real yield spread is:

$$
\begin{aligned}
y_{2,t} - y_{1,t} &= y_{2,t} - r_{f,t+1} \\
&= -\frac{1}{2}\left[\begin{array}{c} -0.5(\gamma\left(1-\phi\right) - B) \\ +(\gamma\left(1-\phi\right) + B(\phi - 2))(s_t - \bar{s}) \\ +0.5\sigma_c^2\left[B\lambda\left(s_t\right) - \gamma - \gamma\lambda\left(s_t\right)\right]^2 \end{array}\right].
\end{aligned}
$$

- Here we see that:
  - $B = 0$ gives a yield spread of zero.
  - $B > 0$ gives a positive yield spread.
- There is no closed form expression for yields with more than two periods to maturity, and we can instead use numerical methods in order to calculate the bond prices.

Extensions and additional evidence

Bond market

- CC (1995) only analyze the real term structure.
- Wachter (2006) analyze both the nominal and real term structure.
- Wachter considers an extension of the habit model where nominal bonds depend on past consumption growth through habit and on expected inflation.
- Wachter shows that the model has the ability to produce a reasonable fit of the means and volatilities of nominal bond yields.
- In addition, the model has the ability to explain the failure of the expectations hypothesis.

Extensions and additional evidence

FX market

- ▸ Verdelhan (2010) considers another extension of the model to study the forward premium puzzle.
- In contrast to Wachter (2006), Verdelhan sets up a model with pro-cyclical interest rates.
- Note that depending on the value of the B, the CC model implies constant ($B = 0$), pro-cyclical ($B < 0$) or counter-cyclical interest rates ($B > 0$).
- Verdelhan (2010) shows that the modified CC model explains the forward premium puzzle.

Extensions and additional evidence

Predictability of stock returns from past consumption

- A key implication of the CC model is predictability of stock returns from past consumption.
- In good times, when consumption is high relative to the past history of consumption (habit), and hence the marginal utility of present consumption is low, investors are less risk averse and willing to give up current consumption and invest. This in turn forces stock prices to increase and future expected returns to decrease.
- Conversely, in bad times, when consumption gets close to habit (which is determined by the past history of consumption), investors are risk averse and require high expected stock returns to invest.
- Consistent with this theoretical prediction, Atanasov et al. (2019) suggest a new consumption-based predictive variable that has strong predictive power for future stock returns.

Extensions and additional evidence

Cross section

- The standard consumption-based model with power utility model has been shown to perform poorly in explaining the cross-sectional variation in returns. The empirical evidence has shown that multifactor versions of the CAPM outperform the standard CCAPM.
- Campbell and Cochrane (2000) show that this is to be expected if the true model is the external habit-based model of CC (1999).
- CC (2000) simulate data from the CC (1999) model and then consider a number of false models:
  - the CAPM
  - a scaled version of the CAPM
  - the canonical power utility consumption-based CAPM
  - a linear version of the standard CCAPM
- Measure pricing error for each model based on the Hansen-Jagannathan distance.

Extensions and additional evidence

Cross section

- Two interesting findings:
  - the CAPM delivers lower pricing errors than the canonical consumption-CAPM.
  - a scaled version of the CAPM improves on the static CAPM.
- Implications:
  - a good fit of multifactor extensions of the CAPM based on real data compared to the power utility model cannot be used as evidence against the consumption-based framework in general.
  - asset pricing models that take account of time-varying conditioning information are likely to perform better than models that do not do so.

Extensions and additional evidence

Cross section

- Does habit formation models explain the actual cross section of expected returns?
- This question has been analyzed in a number of papers. For example, Chen and Ludvigson (2009), who treat the functional of habit as unknown.
- They provide empirical evidence that a SDF based on habit formation does well in explaining a cross-section of size and book-market sorted equity returns.

Main literature

- Campbell, J., Cochrane, J. (1999). By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy* 107, 205-251.
- Campbell, J., Cochrane, J. (2000). Explaining the Poor Performance of Consumption-Based Asset Pricing Models. *Journal of Finance* 55, 2863-2878.