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BY FORCE OF HABIT: A CONSUMPTION-BASED EXPLANATION OF AGGREGATE STOCK MARKET BEHAVIOR

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ABSTRACT

We present a consumption-based model that explains the procyclical variation of stock prices, the long-horizon predictability of excess stock returns, and the countercyclical variation of stock market volatility. Our model has an i.i.d. consumption growth driving process, and adds a slow-moving external habit to the standard power utility function. The latter feature produces cyclical variation in risk aversion, and hence in the prices of risky assets. Our model also predicts many of the difficulties that beset the standard power utility model, including Euler equation rejections, no correlation between mean consumption growth and interest rates, very high estimates of risk aversion, and pricing errors that are larger than those of the static CAPM. Our model captures much of the history of stock prices, given only consumption data. Since our model captures the equity premium, it implies that fluctuations have important welfare costs. Unlike many habit-persistence models, our model does not necessarily produce cyclical variation in the risk free interest rate, nor does it produce an extremely skewed distribution or negative realizations of the marginal rate of substitution.

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1. Introduction

A number of empirical phenomena suggest tantalizing links between asset markets and macroeconomics. Most importantly, equity risk premia seem to be higher at business cycle troughs than they are at peaks. The evidence comes from several sources. Regressions show that excess returns on common stocks are forecastable, and many of the variables that predict high excess stock returns move countercyclically (Ferson and Merrick 1987, Fama and French 1989). The volatility test literature mirrors this conclusion: price-dividend ratios move procyclically, but this movement is hard to explain by variation in expected dividends or real interest rates, indicating large countercyclical variation in expected excess returns (Campbell and Shiller 1988a,b, Shiller 1989, Campbell 1991, Cochrane 1991a,b). Estimates of conditional variances of returns also change through time (see Bollerslev, Chou, and Kroner 1992 for a survey), but they do not move one-for-one with estimates of conditional mean returns. Hence the slope of the conditional mean-variance frontier, a measure of the price of risk, changes through time with a business cycle pattern (Harvey 1989, 1991, Chou, Engle, and Kane 1992).

As yet, we have no accepted economic explanation for these observations. In the language of finance, we lack a successful theory and measurement procedure for the fundamental sources of real, aggregate macroeconomic risks that drive expected returns. In the language of macroeconomics, standard business cycle models utterly fail to reproduce the level, variation, and cyclical co-movement of equity premia. These phenomena are among the most dramatically counterfactual predictions of current business cycle models.

We argue that many of the puzzles in this area can be understood using a simple modification of the standard consumption-based asset pricing model. Our model reconciles the level, volatility, and cyclical variation in asset returns with aggregate consumption via a representative agent utility function. The central ingredient is a slow-moving habit, or time-varying subsistence level, added to the basic power utility function. As consumption declines toward the habit in a business cycle trough, risk aversion rises, so expected returns on risky assets rise and risky asset prices fall. There need not be any strong effect of habit on the riskless interest rate.

We model consumption growth as an i.i.d. lognormal process, with the same mean and standard deviation as postwar consumption growth. Our model can easily accommodate more complex consumption processes, including processes with predictability, conditional heteroskedasticity, and non-normality. But these features are not salient characteristics of consumption data. More importantly, we want to emphasize that the model generates interesting asset price behavior internally, without building it from exogenous variation in the consumption growth distribution. In this, our approach is the opposite of that of Kandel and Stambaugh (1990, 1991), who use fairly standard preferences, but derive a more restricted range of phenomena by movement over time in the conditional moments of consumption growth.

Initially we choose our model's parameters so that the risk free rate is constant. We do this for several reasons. First, estimated risk free rates do not vary much in the data, so this is not a bad approximation. Second, we want to show how our model can deliver the observed variation in returns and expected returns entirely by variation in risk premia without any movement in the risk free rate. Third, many habit persistence models with exogenous consumption give rise to wild variation in risk free rates. When production is added, consumers smooth away the consumption fluctuations. A constant risk free rate is consistent with a linear production technology. In fact, we could present the constant risk free rate version of the model as a linear production economy in which consumers choose i.i.d. consumption growth. This fact gives us hope that the addition of a production sector and endogenous consumption choice will not change the qualitative results.

Following the tradition in the equity premium literature, we model stocks as a claim to the consumption stream. We choose a discount rate of 0.97 and a utility curvature parameter of 2.37 so that the model produces a 1% interest rate and so that it matches the ratio of mean to standard deviation of excess stock returns found in postwar U.S. data. Our model accounts for the equity premium and the level of the risk free interest rate.

To study the dynamic implications of our model, we generate artificial data and calculate a battery of popular statistics. Our model replicates the level of mean excess returns and their standard deviation, and the level and volatility of price-dividend ratios. It replicates the long-horizon forecastability of stock returns from price-dividend ratios, despite a constant risk free rate; it produces persistent variation in return volatility; and it replicates the negative results of volatility tests. It produces large coefficients in regressions of stock price changes on dividend changes. When we allow small variation in interest rates, the model displays a time-varying term structure, and yield spreads forecast long-horizon stock and bond returns.

Our model also displays much of the behavior that bedevils consumption-based models with power utility. In our model, interest rate variation has nothing to do with variation in expected consumption growth, power utility risk-aversion estimates are high, and power utility Euler equations are violated. A conditional CAPM with extra factors formed from information variables outperforms the static CAPM, and both do much better than the standard power utility consumption-based model.

We feed our model actual consumption data, and we find that the price-consumption ratios and returns predicted by our model provide a surprisingly good account of fluctuations in stock prices and returns over the last century.

Finally, we examine the welfare costs of fluctuations. Lucas (1987) made a celebrated calculation that the welfare consequences of fluctuations are orders of magnitude lower than those of long-term growth. However, Lucas based his calculation on a power utility function that dramatically fails to explain asset market data, and the state-prices implicit in asset price data measure much greater welfare costs of fluctuations. Our model, which does explain the equity premium, implies that fluctuations are as important as growth. The exact tradeoff depends on the current state of the economy. At the steady state, consumers would roughly trade all growth for the elimination of fluctuations. In the depths of recessions they would accept negative growth to eliminate fluctuations.

Habit formation

Habit formation has a long history in the study of consumption. Deaton and Muellbauer (1980) survey early work in the area, while Deaton (1992) gives a more recent overview. Constantinides (1990), Ryder and Heal (1973), and Sundaresan (1989) are major theoretical papers on the subject. Recently the idea of habit formation has been applied to broader issues. Carroll, Overland, and Weil (1994) and Chatterjee (1994), for example, argue that habit formation may have important effects in deterministic growth models. In a sense, habit persistence is a fundamental feature of psychology: repetition of a stimulus diminishes the response, and even the perception of the stimulus.

Our empirical specification has three distinctive features. First, we specify that habit formation is *external,* as in Abel's (1990) "keeping up with the Joneses" formulation or Duesenberry's (1949) "relative income" model (see also Gali 1994). An individual's habit level depends on the history of aggregate consumption rather than the individual's own past consumption. This specification simplifies our analysis and helps us to reconcile a constant riskless interest rate with a random walk consumption process. With an internal habit and a constant interest rate, a consumer who receives a sudden windfall will increase consumption slowly and predictably, so that consumption today does not blunt future enjoyment by forming habits too quickly (Constantinides 1990).

Second, we specify that habit moves slowly in response to consumption, unlike empirical specifications in which today's habit is proportional to yesterday's consumption (Ferson and Constantinides 1990). This feature produces slow mean reversion in the price-dividend ratio, long-horizon return forecastability and persistent movements in volatility.

Third, we specify that habit adapts nonlinearly to the history of consumption, in a way that keeps habit always below consumption and keeps marginal utility always finite and positive. Previous authors including Constantinides (1990), Sundaresan (1989), Ferson and Constantinides (1990) and Heaton (1993) have struggled with this issue but have not found an entirely satisfactory solution. Abel (1990) changes utility from $u(C - X)$ to $u(C/X)$ (a member of a general class of models considered by DeTemple and Zapatero 1991), but this specification eliminates the changing risk aversion that is central to our account of asset price behavior. Chapman (1994) argues that the problem is simply intractable. We hope that this paper provides a counterexample.

2. Modeling habit formation and asset prices

2.1. Consumption and habit

We model consumption growth as an i.i.d. lognormal process. We use lower-case letters to denote the logs of corresponding upper-case letters, so that for example C_t is consumption and c_t is log consumption. With this notation, our consumption process is

$$\Delta c_{t+1} = g + v_{t+1}; \quad v_{t+1} \sim i.i.d.\, \mathcal{N}(0, \sigma^2). \tag{2.1}$$

We specify a representative agent utility function with a habit,

$$E \sum_{t=0}^{\infty} \delta^t \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}.$$

Here X_t is the level of habit, δ is the subjective discount factor, and γ is the utility curvature parameter.

It is convenient to capture the relation between consumption and habit by the *surplus consumption ratio*

$$S_t \equiv \frac{C_t - X_t}{C_t}.$$

The surplus consumption ratio increases with consumption. $S = 0$ corresponds to extreme bad times in which consumption is equal to the habit; as consumption rises relative to habit, S approaches 1. If habit X is held fixed as consumption C varies, the local coefficient of relative risk aversion is

$$\frac{-C u_{CC}}{u_C} = \frac{\gamma}{S}.$$

Thus, risk aversion rises as consumption declines toward habit, or as the surplus consumption ratio declines.

To complete the description of preferences, we specify how the habit X responds to consumption. We specify that the log surplus consumption ratio s_t evolves as

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)(c_{t+1} - c_t - g) \tag{2.2}$$

The function $\lambda(s)$ controls the sensitivity of s_{t+1} and thus habit x_{t+1} to contemporaneous consumption c_{t+1}. We call $\lambda(s)$ the *sensitivity function,* and we specify it further below.

Equation (2.2) specifies that today's habit is a complex nonlinear function of current and past consumption. A linear approximation may help to understand it. Near the steady state, the specification (2.2) is approximately a standard linear specification in which habit responds slowly to consumption,[1]

$$x_{t+1} \approx [(1-\phi)\,h + g] + \phi x_t + (1-\phi)\,c_t \tag{2.3}$$

or

$$x_{t+1} \approx \left[h + \frac{g}{1-\phi}\right] + (1-\phi)\sum_{j=0}^{\infty}\phi^j c_{t-j}.$$

where $h = \ln(1-\bar{S})$ is the steady state value of $x-c$. One can make a similar approximation in which the level (rather than the log) of habit is a slowly decaying moving average of the level of past consumption.

Why use the nonlinear process in equation (2.2), rather than the more traditional specification of equation (2.3)? With the specification (2.3), consumption can fall below habit, resulting in infinite or negative marginal utility. A process for $s = \ln(S)$ defined over the real line implies that consumption can never fall below habit. Furthermore, a linear specification like (2.3) typically implies interest rates that vary a great deal over time. Allowing a non-constant sensitivity function $\lambda(s)$ allows us to control interest rate variation, and (as we discuss below) is essential in order to generate time-varying risk premia.[2]

[1] We take a loglinear approximation around the steady state $s = \bar{s}$ and $c_{t+1} - c_t = g$. The first-order approximations are deraived as follows. The definition of s_t can be written

$$s_t = \ln\left(\frac{e^{c_t} - e^{x_t}}{e^{c_t}}\right).$$

Taking a linear approximation,

$$s_t - \bar{s} \approx \left(1 - \frac{1}{\bar{S}}\right)(x_t - c_t - h)\ .$$

Similarly,

$$\lambda(s_t)\ (c_{t+1} - c_t - g) \approx (\lambda(\bar{s}) + \lambda'(\bar{s})(s_t - \bar{s}))(c_{t+1} - c_t - g) \approx \lambda(\bar{s})(c_{t+1} - c_t - g).$$

Then, equation (2.2) becomes

$$(1 - 1/\bar{S}\,)(x_{t+1} - c_{t+1} - h) \approx \phi\,(1 - 1/\bar{S}\,)(x_t - c_t - h) + \lambda(\bar{s})(c_{t+1} - c_t - g)\,.$$

We impose the condition $\lambda(\bar{s}) = 1/\bar{S} - 1$ below, which ensures that x_{t+1} does not respond to c_{t+1} at the steady state $s = \bar{s}$. This substitution gives equation (2.3).

[2] Of course, one could specify a traditional process like (2.3) and a constant interest rate, and allow the consumer to *choose* the consumption process. He would never choose consumption below habit. But he would also choose a consumption process that is not a random walk. One might be able to construct models

2.2. Calculating asset prices

Marginal utility and marginal rate of substitution

Following Abel (1990), we specify that the habit X_t is external; it depends on aggregate (or "the Joneses") consumption rather than on an individual's own consumption. Thus the marginal utility of consumption today does not reflect the expected effect of consumption today on future habits.

The marginal utility of consumption is

$$u'(C_t) = (C_t - X_t)^{-\gamma} = S_t^{-\gamma} C_t^{-\gamma}.$$

The marginal rate of substitution is then

$$M_{t+1} \equiv \delta \frac{u'(C_{t+1})}{u'(C_t)} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}.$$

In terms of the state variable s_t, the marginal rate of substitution simplifies to

$$M_{t+1} = \delta G^{-\gamma} e^{-\gamma(s_{t+1} - s_t + v_{t+1})}. \tag{2.4}$$

Equations (2.1), (2.2), and (2.4) allow us to calculate moments of the marginal rate of substitution and find asset prices.

Consumption claim

We model stocks as a claim to the consumption stream. This is the simplest approach. It is traditional in the equity premium literature, so it highlights the fact that our results do not come from extra modeling assumptions. Finally, it poses the strongest challenge: if you can get reasonable results from a claim to the consumption stream, adding leverage, a dividend process, and so forth can only help.

From the basic pricing relation and the definition of returns,

$$1 = E_t [M_{t+1} R_{t+1}]; \quad R_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t},$$

along this line, and hope that the required temporal dependence in consumption is not counterfactually large, but we have followed the opposite modeling strategy.

price-dividend ratios satisfy

$$\frac{P_t}{D_t} = E_t\left[M_{t+1}\left(1+\frac{P_{t+1}}{D_{t+1}}\right)\frac{D_{t+1}}{D_t}\right].$$

The state variable for our economy is s_t, so, using equation (2.4) to substitute for M_{t+1}, the price-consumption ratio P/C of a claim to the consumption stream from $t+1$ onward satisfies the functional equation

$$\frac{P}{C}(s_t) = \delta G^{1-\gamma} E\left[e^{-\gamma(s_{t+1}-s_t)} e^{(1-\gamma)v_{t+1}}\left(1+\frac{P}{C}(s_{t+1})\right)\Big| s_t\right] \tag{2.5}$$

We solve this functional equation numerically on a grid for the state variable s_t, starting at the nonstochastic solution,[3] and using numerical integration to evaluate the conditional expectation. Given the price-consumption ratio as a function of state, expected returns, the conditional standard deviation of returns, and other interesting quantities follow naturally.

[3]The discounted present value is

$$\frac{P_t}{C_t} = E_t\sum_{j=1}^{\infty}\delta^j\frac{u'(C_{t+j})}{u'(C_t)}\frac{C_{t+j}}{C_t} = E_t\sum_{j=1}^{\infty}\delta^j\left(\frac{S_{t+j}}{S_t}\right)^{-\gamma}\left(\frac{C_{t+j}}{C_t}\right)^{1-\gamma}$$

For the nonstochastic case, we use $\sigma^2 = 0$, so $c_{t+1} = g + c_t$, and

$$\frac{P_t}{C_t} = \sum_{j=1}^{\infty}\left(\delta G^{1-\gamma}\right)^j e^{-\gamma(s_{t+j}-s_t)} = \sum_{j=1}^{\infty}\left(\delta G^{1-\gamma}\right)^j e^{\gamma\left(1-\phi^j\right)(s_t-\bar{s})}.$$

Slope of mean-standard deviation frontier

The slope of the conditional mean-standard deviation frontier can be found from the conditional moments of the marginal rate of substitution,[4]

$$Slope = \max_{\{\text{all assets}\}} \frac{E_t(R^e)}{\sigma_t(R^e)} = \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})} = \left(e^{\gamma^2\sigma^2[1+\lambda(s_t)]^2} - 1\right)^{\frac{1}{2}}. \tag{2.6}$$

The first equality defines the slope of the frontier, where R^e denotes the difference between two returns, called an excess return.

This formula helps us to specify the model. Since we observe a time-varying slope of the mean-variance frontier, $\lambda(s)$ must vary with s. To get risk prices that are higher in bad times, when s is low, $\lambda(s)$ must decline with s.

Risk free rate

The real risk free rate is the inverse of the price of a sure unit of consumption,

$$R^f = 1/E_t\left(M_{t+1}\right).$$

[4]A simple argument: Start with the pricing equation for an excess return,

$$0 = E_t\left(M_{t+1}R^e_{t+1}\right) = E_t(M_{t+1})E_t(R^e_{t+1}) + \rho_t\sigma_t(M_{t+1})\sigma_t(R^e_{t+1}).$$

Rearranging,

$$\frac{E_t(R^e_{t+1})}{\sigma_t(R^e_{t+1})} = -\frac{\rho_t\sigma_t(M_{t+1})}{E_t(M_{t+1})}$$

Since $-1 \leq \rho_t \leq 1$,

$$\frac{E_t(R^e_{t+1})}{\sigma_t(R^e_{t+1})} \leq \frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})},$$

so the slope of the mean-standard deviation frontier is the above with equality. See Hansen and Jagannathan (1992) for a more general discussion.

To find the ratio $\sigma_t\left(M_{t+1}\right)/E_t\left(M_{t+1}\right)$, we use the identity that for normal x,

$$\sigma\left(e^x\right) = \left[Ee^{2x} - \left(Ee^x\right)^2\right]^{\frac{1}{2}} = \left[e^{2Ex+2\sigma_x^2} - e^{2Ex+\sigma_x^2}\right]^{\frac{1}{2}} =$$

$$= \left[e^{2Ex}\left(e^{2\sigma_x^2} - e^{\sigma_x^2}\right)\right]^{\frac{1}{2}} = \left[e^{2Ex+\sigma_x^2}\left(e^{\sigma_x^2} - 1\right)\right]^{\frac{1}{2}} = E(e^x)\left(e^{\sigma_x^2} - 1\right)^{\frac{1}{2}}.$$

Hence,

$$\sigma_t(M_{t+1}) = E_t(M_{t+1})\left(e^{\gamma^2\sigma^2(1+\lambda(s_t))^2} - 1\right)^{\frac{1}{2}}.$$

Using (2.2), (2.4), and the consumption growth process, the log risk free rate is

$$r_t^f = -\ln(\delta) + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}\left[\lambda(s_t)+1\right]^2 \tag{2.7}$$

The $(s_t - \bar{s})$ term reflects intertemporal substitution, or mean-reversion in marginal utility. If the surplus consumption ratio is low, the marginal utility of consumption is high. However, the surplus consumption ratio is expected to revert to its mean, so marginal utility is expected to fall in the future. Therefore, the consumer would like to borrow and this drives up the equilibrium risk free interest rate. The last term in equation (2.7) is a precautionary savings term. As uncertainty increases, consumers become more willing to save and this drives down the equilibrium risk free interest rate.

In the data, we notice relatively little variation in risk free rates. This means that the serial correlation parameter ϕ must be near one, and/or $\lambda(s)$ must decline with s so that uncertainty is high when s is low and the precautionary saving term offsets the intertemporal substitution term. Note that $\lambda(s)$ declining with s is the same condition we need to get countercyclical variation in the price of risk.

2.3. Choosing the sensitivity function $\lambda(s)$

We choose the sensitivity function $\lambda(s)$ to satisfy three conditions: 1) The real risk free rate is constant. 2) Habit is predetermined at the steady state $s = \bar{s}$. 3) Habit is predetermined near the steady state, or, equivalently, habit moves non-negatively with consumption everywhere.

We motivated the constant risk free rate above. Habit must not stay completely fixed, or a sufficiently low realization of consumption growth would leave consumption below habit. On the other hand, habit must not move one-for-one with consumption everywhere (a constant $\lambda(s)$), or we would not see time-varying risk premia. Hence, we require that habit is predetermined, but only at and near the steady state. Finally, the intuitive stories behind habit persistence would be strained if we allowed habit to *decline* as consumption rises.

These three considerations lead us to a restriction that must hold between the steady state surplus consumption ratio $\bar{S} = e^{\bar{s}}$ and the other parameters of the model,

$$\bar{S} = \sigma\sqrt{\frac{\gamma}{1-\phi}}, \tag{2.8}$$

and they lead us to a specification of the sensitivity function,

$$\lambda(s) = \begin{cases} \frac{1}{\bar{S}}\sqrt{1-2(s-\bar{s})}-1, & s \leq s_{\mathbf{max}} \\ 0 & s \geq s_{\mathbf{max}} \end{cases} \tag{2.9}$$

where $s_{\mathbf{max}}$ is simply defined as the value of s at which the square root specification runs into zero,

$$s_{\mathbf{max}} \equiv \bar{s} + \frac{1}{2}\left(1-\bar{S}^2\right). \tag{2.10}$$

For s above $s_{\mathbf{max}}$ we take $\lambda(s) = 0$, and abandon the constant risk free rate. In the continuous time limit, the s process never attains this region. In discrete time, s can occasionally get into the $\lambda = 0$ region via an unusually high realization of consumption, and then s moves back deterministically.

It's fairly easy to verify that this specification achieves the three objectives set out above. First, simply plugging (2.8) and (2.9) into the formula for the risk free rate, (2.7), we see that the risk free rate is a constant. Second, differentiating the transition equation (2.2), we obtain

$$\frac{dx_{t+1}}{dc_{t+1}} = 1 - \frac{\lambda(s_t)}{e^{-s_{t+1}}-1} \approx 1 - \frac{\lambda(s_t)}{e^{-s_t}-1}. \tag{2.11}$$

The latter approximation holds since $\phi \approx 1$. (We use $\phi = 0.97$ below.) To obtain $dx/dc = 0$ at $s = \bar{s}$, we require

$$\lambda(\bar{s}) = \frac{1}{\bar{S}} - 1. \tag{2.12}$$

Evaluating at $\bar{s}$, equation (2.9) satisfies this condition. Third, to ensure that habit is predetermined in a neighborhood of the steady state, we add the requirement

$$\frac{d}{ds}\left(\frac{dx}{dc}\right)\bigg|_{s=\bar{s}} = 0.$$

This condition also implies that habit moves *non-negatively* with consumption everywhere, since dx/dc is a U-shaped function of s. Taking the derivative $d/ds\,(dx/dc)$ of the expression in (2.11) and setting it to zero at $s = \bar{s}$, we obtain[5]

$$\lambda'(\bar{s}) = -1/\bar{S}$$

Equation (2.9) satisfies this condition. Since the function $\lambda(s)$ was already determined by the first two conditions, this condition determines the constraint (2.8) on the parameters of the model.

Figure 2.1 plots sensitivity λ as a function of the surplus consumption ratio, given the parameter values described below. λ is a shifted square root function of $-s$, so λ increases to infinity as s declines to minus infinity. The surplus consumption ratio is $S = e^s$, so λ increases to infinity as S declines to zero in the figure. As we discussed above, a declining $\lambda(s)$ is needed to produce a constant risk free interest rate and a countercyclical price of risk. Where $\lambda(s)$ hits zero, we see the upper bound of the surplus consumption ratio, $S_{\max}$.

The right hand panel of Figure 2.1 plots the derivative of log habit with respect to log consumption implied by our specification of $\lambda(s)$, as given by equation (2.11). The figure verifies that dx/dc is zero at and near the steady state (vertical line), and that $dx/dc \geq 0$ everywhere. As the surplus consumption ratio declines to zero, or increases to its upper bound (the vertical dashed line), dx/dc rises to $+1$. Log habit starts to move one-for-one with log consumption, in order to keep habit below consumption and the surplus consumption ratio below its upper bound.

2.4. Choosing parameters

We take the mean and standard deviation of log consumption growth, g and σ, directly from consumption data. We determine the subjective discount rate δ given all other parameters to deliver a real risk free rate of 1% per year. We choose the serial correlation parameter ϕ to match the serial correlation of log price-dividend ratios. This leaves only the curvature

[5] Differentiating,

$$\frac{d}{ds}\left(\frac{dx}{dc}\right) = \frac{d}{ds}\left[1 - \frac{\lambda(s)}{e^{-s}-1}\right] = -\frac{\lambda(s)e^{-s}}{(e^{-s}-1)^2} - \frac{\lambda'(s)}{e^{-s}-1}.$$

Setting this derivative to zero at $s = \bar{s}$, we obtain

$$\frac{\lambda'(\bar{s})}{\lambda(\bar{s})} = -\frac{1/\bar{S}}{1/\bar{S}-1}$$

We already have $\lambda(\bar{s}) = 1/\bar{S} - 1$, so our extra condition is $\lambda'(\bar{s}) = -1/\bar{S}$.

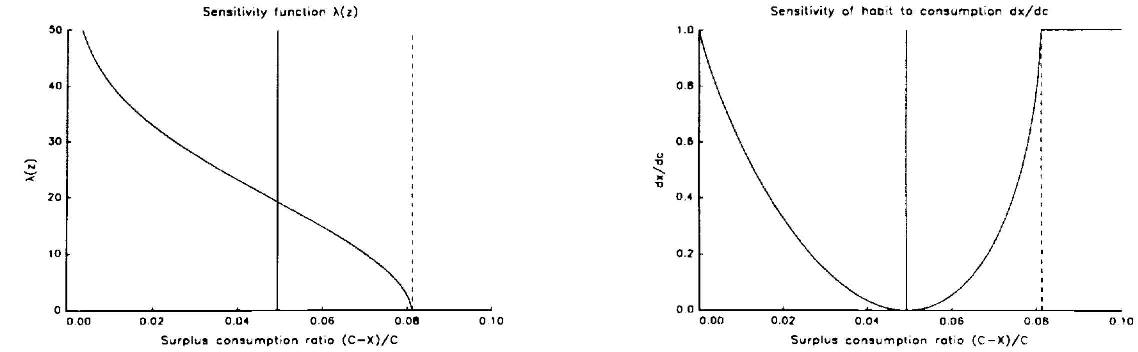

Figure 2.1: Left panel: Sensitivity λ as a function of the surplus consumption ratio. Right panel: derivative of log habit with respect to log consumption, The solid vertical line indicates the steady state surplus consumption ratio. The dashed vertical line indicates the maximum surplus consumption ratio $S_{\mathbf{max}}$.

parameter γ to be determined. Since the ratio of unconditional mean to unconditional standard deviation of excess returns is the heart of the equity premium puzzle (as distilled by Hansen and Jagannathan 1992 from Mehra and Prescott 1985), we search for a value of γ that matches this ratio in the data. Other parameters follow from these choices. In particular, the steady state surplus consumption ratio follows using equation (2.8).

Table 2.1 presents summary statistics for postwar quarterly nondurable and services per capita consumption and value-weighted NYSE stock data, and Table 2.2 presents statistics for long-term annual S&P500 and consumption data. In postwar data, the mean excess return is 1.6% per quarter or 7.4% per year. Divided by the also substantial standard deviation of stock returns, the unconditional price of risk is 0.21 per quarter or 0.42 on an annual basis. However, the table sounds a warning to equity premium calibrators (including ourselves): the mean excess return is imprecisely measured. A 95% confidence interval extends from about 0.5% to 2.7% per quarter. The annual data give an easier target: the mean excess return and price of risk are about half their values in postwar data (and also imprecisely measured). Furthermore, the standard deviation of consumption growth in the long annual data set is also about double its value in quarterly postwar data.

Since the postwar quarterly data give a harder target, we use them for our calibration,

Table 2.1: Postwar quarterly statistics.

	Mean	S.e.	Std. dev.	Annualized
Ln VW - ln TB return (%)	1.61	0.56	7.74	6.45
Sharpe ratio = mean/s.d.	0.21			0.42
Ln consumption growth (%)	0.44	0.04	0.56	1.78
VW P/D	24.9	0.44	6.05	
AR(1) from annual p-d	0.97	0.02		0.88

Stock return = quarterly value weighted NYSE (CRSP). T-bill rate from quarterly SBBI (CRSP). Consumption = real nondurable+services/capita (CITIBASE series GCNQ+GCSQ/GPOP). AR(1) is the quarterly statistic implied by a regression using annual data. Sample 1947:1-1993:4.

Table 2.2: Long term annual statistics.

	Mean	S.e.	Std. dev.
Ln S&P return - ln cp (%)	3.69	1.59	17.39
Sharpe ratio = mean/s.d.	0.21		
Ln consumption growth (%)	1.72	0.33	3.32
S&P P/D	22.3	0.56	6.06
AR(1) in p-d	0.72	0.06	

Campbell-Shiller (1988) data, updated. Stocks = S&P 500,1872-1990. Consumption = nondurables 1889-1992.

picking $g = 0.44\%$ and $\sigma = 0.56\%$ to match the moments of postwar quarterly consumption growth. Since there is a strong seasonal in quarterly price-dividend ratios, and hence deseasonalized data are artificially smooth, we match the quarterly serial correlation coefficient implied by *annual* postwar data, $0.88^{1/4} = 0.97$.

Table 2.3 presents the effects of varying the curvature parameter γ on our model's predictions for the ratio of mean to standard deviation of excess returns. A value $\gamma = 2.372$ delivers the ratio 0.21 found in postwar data. Given this value of γ, a discount factor $\delta = 0.973$ produces the desired 1% risk free rate.

Table 2.3 also shows that the price of risk is not very sensitive to the value of γ in our model. In a power utility model, risk premia move in proportion with risk aversion γ. In our model, risk aversion at the steady state is given by $\gamma/\bar{S}$. The relation between $\bar{S}$ and the other parameters of the model, equation (2.8), implies that

$$\frac{\gamma}{\bar{S}} = \frac{\sqrt{\gamma(1-\phi)}}{\sigma}. \tag{2.13}$$

Table 2.3: Effect of curvature parameter γ on model predictions for the ratio of mean to standard deviation of excess stock returns.

	$E(r-r^f)/\sigma(r-r^f)$
Postwar data	0.21
$\gamma = 1$	0.14
2	0.19
2.372	0.21
3	0.25
4	0.30
5	0.34

Thus local risk aversion at the steady-state rises only with the square root of γ. We see in Table 2.3 that the equity premium roughly doubles when γ increases from 1 to 4, consistent with this analysis. Our other results are similarly insensitive to the choice of γ. This insensitivity is comforting. It means that the structure of the model rather than specific parameter choices drive our qualitative results. Also, it means that we obtain quite similar results with $\gamma = 1$, which is the only value consistent with balanced growth and a utility function separable between consumption and leisure.

Table 2.4 summarizes the values we assume for the parameters of our model, and gives the implied values of the subjective discount factor and steady state surplus consumption ratio.

Table 2.4: Assumptions and derived parameters.

Assumptions	
Constant interest rate r^f (%)	0.250
Mean ln consumption growth rate g (%)	0.444
Std. dev. ln consumption growth σ (%)	0.555
Curvature γ	2.372
AR(1) coefficient ϕ	0.970
Derived parameters	
Discount rate δ	0.973
Steady state surplus consumption ratio $\bar{S}$	0.049
Maximum surplus consumption ratio $\bar{S}_{\mathbf{max}}$	0.081

2.5. The equity premium puzzle: a comment

Once we have picked parameters $\gamma = 2.372$ and $\delta = 0.973$ to match the slope of the unconditional mean-standard deviation frontier with a constant 1% risk free rate, our model fits the unconditional equity premium. Since the equity premium puzzle has attracted so much attention, it is worth reflecting on what our model does and does not say about it, even though the focus of this paper is time-variation in the equity premium.

In a sense, we account for the high unconditional equity premium by high risk aversion. Recall from (2.8) that our model imposes a restriction on the steady state surplus consumption ratio given other parameters. With our parameter values $\gamma = 2.372$, $\phi = 0.97$, and $\sigma = 0.0056$, the implied steady state surplus consumption ratio is $\bar{S} = 0.049$. The local risk aversion coefficient at the steady state is $\gamma/\bar{S} = 48.4$. (The mean surplus consumption ratio is considerably above the steady state, and habit is not fixed when consumption changes away from the steady state. Both effects lower mean risk aversion but all measures of average risk aversion in our model are well above 10, the highest value considered plausible by Mehra and Prescott 1985.)

There is an important difference between our model and a power utility model with a high risk aversion coefficient γ, however. In the power utility model with lognormal consumption growth, the log interest rate is

$$r_t^f = -\ln(\delta) + \gamma g - \gamma^2 \frac{\sigma^2}{2}.$$

A high value of γ makes the term γg very large. To get a realistically low average risk free interest rate, one often must make δ implausibly large. With $g = 0.44\%, \sigma = 0.56\%$, and $\gamma = 48.4$, for example, one needs $\delta = 1.19$ to get a 1% risk free rate; imposing $\delta = 1$, one predicts a risk free rate of 19% per quarter. Weil (1989) emphasizes this "risk free rate puzzle".[6] Perhaps more serious than the intuitive implausibility of $\delta = 1.19$, high γ implies that the risk free interest rate should be quite sensitive to the mean consumption growth

[6]Kocherlakota (1990) argues that $\delta > 1$ is possible in growth economies. Of course, even higher values of γ or high values of σ^2 can result in low risk free rates through the $-\gamma^2\sigma^2/2$ term. Extremely risk-averse agents have an important precautionary savings motive when they are faced with even minor uncertainty in consumption growth, and this helps to reduce the equilibrium risk free interest rate as in Kandel and Stambaugh (1991). But with high risk aversion and power utility, the predicted level of the risk free rate is always a delicate balancing act; it depends sensitively on the parameters g and σ^2. See Cochrane and Hansen (1992) for an extended discussion of predictions of high-risk-aversion models for the level and variability of risk free interest rates.

rate, which does not seem to be the case in either time-series or cross-country data.

In our model, by contrast, we can evaluate equation (2.7) at the steady state surplus consumption ratio and use the restriction (2.12) to write the log risk free rate as

$$r_t^f = -\ln(\delta) + \gamma g - \left(\frac{\gamma}{\bar{S}}\right)^2 \frac{\sigma^2}{2}.$$

The curvature parameter γ, much lower than the risk aversion coefficient at the steady state $\gamma/\bar{S}$, controls the effect of average consumption growth on the risk free rate. Thus we avoid the risk free rate puzzle: In our model a much lower value of the discount factor δ is consistent with the average level of the risk free interest rate, and we get a much less sensitive relationship between mean consumption growth and interest rates.[7]

The level of risk aversion may still seem intuitively troubling, although the major piece of evidence against it–sensitivity of interest rates to mean consumption growth–is no longer a problem. It may seem that we have traded an implausibly high γ for an implausibly small $\bar{S}$. But we have chosen the sample period and variable definitions that give the smoothest consumption and highest equity premium, we have ignored sampling variation in mean returns, and we have not included any of the usual devices to boost the equity premium, such as modeling stocks as a levered claim to an unusually volatile part of the capital stock (industrial capital, not housing or human capital), recognizing that dividends are more variable than consumption, introducing occasional extremely bad states in the consumption distribution, to say nothing of transactions costs, heterogeneous consumers, monetary frictions, and so forth. A more complex model that incorporates some of these features is likely to allow substantially lower risk aversion, and hence a lower power parameter γ and higher values of the surplus consumption ratio.

Furthermore, high risk aversion may not be an implausible description of representative agent preferences in the first place. Kandel and Stambaugh (1991) subject some common thought experiments to a careful sensitivity analysis, showing that high risk aversion is not as implausible as one might have believed. And the literature is full of examples in which representative agent preferences are different from individual preferences.

[7]The preferences suggested by Epstein and Zin (1989) and Weil (1989), and studied by Kandel and Stambaugh (1991) and Campbell (1993), distinguish the coefficient of relative risk aversion and the elasticity of intertemporal substitution. These preferences, like ours, allow average consumption growth to have a much smaller effect than consumption volatility on the risk free interest rate. However Epstein-Zin-Weil preferences do not display the time-varying risk aversion that is the central feature of our model.

2.6. Stationary distribution

Figure 2.2 presents the stationary distribution of the surplus consumption ratio, calculated from a simulation. The appendix calculates the stationary distribution of the continuous time version of our process, and this distribution and the corresponding probability density are also included in Figure 2.2. The continuous time distribution is visually indistinguishable from the simulated distribution, suggesting that the continuous time density is a good approximation to the density of the discrete-time process. The continuous time process does not go beyond the upper limit. There is a small leakage where the discrete time distribution can go beyond the limit in response to unusually large consumption innovations. The distribution is skewed, and the surplus consumption ratio spends much of its time near the upper boundary. The mean is somewhat greater than the steady state. Thus, the high expected returns and low prices seen in deep recessions are unusual events.

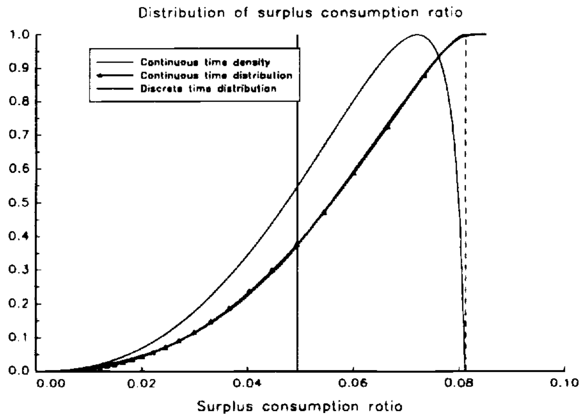

Figure 2.2: Distribution of surplus consumption ratio in 40,000 quarter simulation, and in the continuous time limit. The solid vertical line indicates the steady state surplus consumption ratio $\bar{S}$ and the dashed vertical line indicates the upper bound of the surplus consumption ratio $S_{\max}$.

2.7. Conditional mean and variance

Figure 2.3 presents the expected stock return and risk free rate as a function of the surplus consumption ratio. As consumption declines towards the habit, consumers become very risk averse, so the expected return rises dramatically over the constant risk free rate.

The risk free rate ceases to be constant for values of the surplus consumption ratio above the upper limit $S_{\max}$. This is not a concern for our model, since the process (almost) never gets into this region. But it does imply a cautionary tale. Intertemporal substitution effects—the $s - \bar{s}$ term in equation (2.7)—vary a great deal with the state variable. They are offset by equally large and variable precautionary savings effects—the $\lambda(s)$ term—resulting in a constant risk free rate. Absent the precautionary savings effects, interest rates would vary with the state variable s as they do in the region $s > s_{\max}$. This variation is greater than the variation of expected returns with the state variable in our stochastic model. Therefore, a researcher who analyzed data from our economy with a nonstochastic model would be puzzled that large interest rate variation was absent.

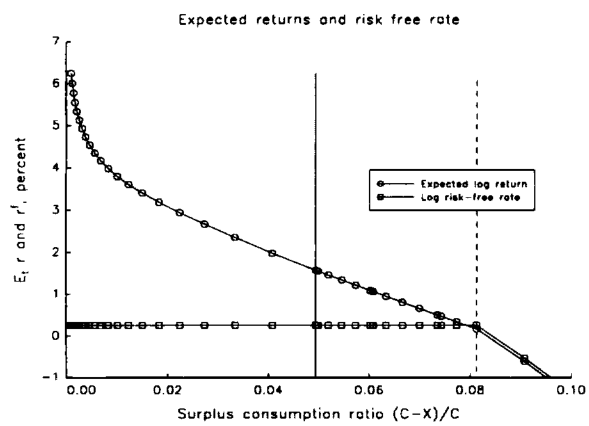

Figure 2.3: Expected log return and log risk free rate as functions of surplus consumption ratio.

Figure 2.4 presents the conditional standard deviation of the log stock return as a function of the surplus consumption ratio. As consumption declines towards habit, the conditional variance of returns increases as well as the conditional mean. Thus, our model will produce highly autocorrelated conditional variance in stock returns, as found by the ARCH literature.

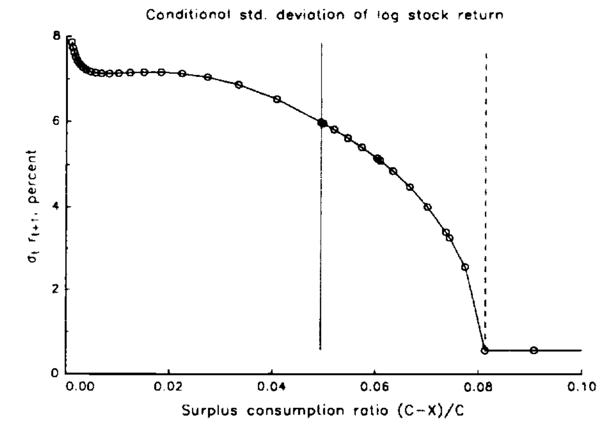

Figure 2.4: Conditional standard deviation of log returns as function of surplus consumption ratio.

Comparing figure 2.3 and figure 2.4, we see that means and standard deviations do not move in the same way, so the slope of the mean-standard deviation frontier or the price of risk increases as the surplus consumption ratio declines.

3. Statistics from simulated data.

We simulate 40,000 artificial data points to calculate predicted values for popular statistics.

Means and standard deviations

Table 3.1 presents means and standard deviations in our simulated data.

Table 3.1: Means and standard deviations in simulated data.

	Mean	S.d.
$r - r^f$	1.15	5.55
P/C	27.0	4.86
$E_t r - r^f$	1.15	0.84

Returns are percent per quarter.

We picked γ to match the ratio of mean to standard deviation for excess stock returns, but this choice does not say anything about the level of mean and standard deviation. The mean and standard deviation of excess returns are in fact close to, but still a little below, the corresponding values in the postwar data shown in Table 2.1. For several reasons the discrepancy does not worry us. First, we close half the gap when we add a small amount of interest rate variation to the model below. Second, the model's mean excess return is less than one standard error below the mean in the data. Third, the value weighted stock return is a leveraged claim, since most firms are also financed by debt. Leverage increases both the mean and the standard deviation of returns but has little effect on their ratio. The consumption claim in our model corresponds to a broader concept of wealth, including stocks, bonds, human capital etc., that is likely to have a smaller mean and standard deviation than the NYSE portfolio. Fourth, a small increase in consumption growth volatility or a decrease in measured expected returns, as found in our long horizon data set, can eliminate or reverse this gap.

The mean and standard deviation of the price-consumption ratio are also quite similar to what we found for price-dividend ratios in the data. Matching the mean basically reflects our choice of risk free rate, the similarity of our excess return to that in the data, and the fact that our assumed mean consumption growth is similar to mean dividend growth in the data. Matching the standard deviation means that our model explains the volatility of prices. We take up this issue in detail below, by running a volatility test.

The standard deviation of the conditional expected return is much smaller than the standard deviation of returns, 0.84% rather than 5.55% per quarter. However, 0.84% per quarter or 2.5% per year is a large variation in expected returns, about the magnitude indicated by regression evidence in the data.

Autocorrelations and cross-correlations

Table 3.2 presents autocorrelations from our simulated data. We picked ϕ to generate the 0.97 first order autocorrelation of the price-consumption ratio seen in the table. Higher autocorrelations decay slowly, as in the data. Returns display a series of small negative autocorrelations that generate univariate mean reversion (see Fama and French 1988a and Poterba and Summers 1988 for univariate mean reversion in stocks, Cochrane 1988 for the relation between autocorrelation and mean-reversion). The negative autocorrelation of returns also generates observations that price changes tend to be reversed. *Expected* returns are highly positively autocorrelated. They are a function of the state variable, and so inherit its autocorrelation. The autocorrelations of squared returns and absolute returns reveal long-horizon conditional heteroskedasticity. The ARCH literature finds higher values for these autocorrelations in very high frequency data, but values similar to these at the quarterly frequency we study.

Table 3.2: Autocorrelations and cross-correlations in simulated data.

	Lag (quarters)							
Variable	1	2	3	4	8	12	16	20
$p-c$	0.97	0.94	0.91	0.88	0.78	0.69	0.61	0.54
r	-0.01	-0.03	-0.02	-0.02	-0.01	-0.01	-0.00	-0.00
$E_t r$	0.97	0.93	0.90	0.88	0.77	0.68	0.60	0.53
r^2	0.06	0.05	0.04	0.04	0.04	0.03	0.03	0.03
$\|r\|$	0.08	0.07	0.06	0.07	0.06	0.04	0.04	0.04
$p_t - c_t,\ r_{t+j}$	-0.15	-0.15	-0.14	-0.13	-0.12	-0.10	-0.09	-0.08
$p_t - c_t,\ r^2_{t+j}$	-0.28	-0.27	-0.26	-0.25	-0.22	-0.19	-0.17	-0.15
$r_t,\ r^2_{t+j}$	-0.07	-0.06	-0.05	-0.01	-0.05	-0.06	-0.03	-0.02

The cross-correlation between the price-consumption ratio and returns verifies that the price-consumption ratio forecasts long-horizon returns, with the right sign: high prices forecast low returns. The cross-correlations between the price-consumption ratio or returns and future squared returns shows that a low price-consumption ratio or a big price decline signal high volatility for many quarters ahead, the "leverage effect" that Black (1976), Schwert (1989), Nelson (1991) and many others have found in the data.

Vector autoregressions

Table 3.3 presents a first order VAR of the log price-consumption ratio and log returns from our simulated data. The pattern is very similar to what one sees in data (Campbell 1991, Hodrick 1992). We see again the strong autocorrelation of the price-consumption ratio, and very little feedback from returns to the price-consumption ratio. Returns have very little forecast power to forecast returns. The R^2 for the price-consumption ratio is quite high, as in the data. The R^2 for one quarter returns is 0.15, roughly what one finds in the data. The serial correlation of the right hand variable — price-consumption ratio — means that the coefficients and R^2 statistics rise with horizon, as we see next.

Table 3.3: Regression coefficients and R^2 in a VAR from simulated data.

	$p - c_{t-1}$	r_{t-1}	R^2
$p - c_t$	0.97	0.00	0.97
r_t	-0.17	0.00	0.15

Long-horizon regressions

Table 3.4 presents long-horizon regressions of log excess stock returns on the log price-consumption ratio in our simulated data. We use excess returns to emphasize that risk premia vary over time. We see the classic pattern documented by Campbell and Shiller (1988b) and Fama and French (1988b). The coefficients are negative: high prices imply low expected returns. They increase linearly at first, and then less quickly; the R^2 start low, but then rise to impressive values.

Table 3.4: Long horizon return regressions in simulated data.

Horizon	10 x coef.	R^2
1 Qtr.	-0.42	0.15
2	-0.83	0.21
1 Year	-1.60	0.30
2	-2.98	0.41
3	-4.18	0.48
5	-6.21	0.58
10	-9.61	0.72

Regression equation: $r^e_{t \to t+k} = a + b(p_t - c_t) + \epsilon_{t+k}$.

Volatility tests

Our model features constant interest rates and completely unpredictable dividend (consumption) growth. Yet it produces about the same variance of the price-dividend ratio as seen in the data. Hence, our model produces the violation of variance bounds tests documented by Shiller (1981), LeRoy and Porter (1981), West (1988), Campbell and Shiller (1988a, 1988b) and many others. Variation in prices cannot be explained by varying expectations of cash flows or interest rates.

To show the point quantitatively, we replicate one of the most recent versions of such tests. Campbell (1991) and Cochrane (1991a) calculate decompositions of the variance of price-dividend ratios and returns. These decompositions try to answer the questions, "How much of the variance of prices or *ex-post* returns is due to news about future cash flows, and how much is due to news of future expected returns?" They find that almost all of the variation in prices and returns has to be accounted for by news of future expected excess returns, and hence unmodeled variation in risk premia.

We use a variance decomposition that is based on the approximate identity[8]

$$var\left(p_t - d_t\right) \approx \sum_{j=1}^{\infty} \rho^j \left[cov\left(p_t - d_t, \Delta d_{t+j}\right) - cov\left(p_t - d_t, r_{t+j}\right)\right]$$

This identity says that prices can only vary insofar as they forecast subsequent dividend growth or returns. Using a similar identity, Cochrane (1991b) finds that essentially all the variance of price-dividend ratios is due to changing expectations about future excess returns. Price-dividend ratios have little forecasting power for subsequent dividend growth or interest rates. The sum of the dividend and return terms is nearly equal to 100%, suggesting that price volatility is not due to bubbles (violations of the transversality condition used to derive the present value model). However, expected return forecasts are so slowly mean-reverting that 15 annual covariances are needed before the decomposition adds up to 100% of the price-dividend ratio variance. Campbell (1991) and Campbell and Ammer (1993) report similar results.

Table 3.5 presents the results of the price-consumption ratio variance decomposition in our simulated data. As in the actual data, it takes about 15 years of forecasts to account for the variance of the price-consumption ratio. Return forecasts account for all the price-consumption ratio variation. Since consumption growth is completely unpredictable in our model, consumption growth accounts for exactly 0% of the price-consumption ratio variance. In a sample, of course, one will find some contribution due to sampling variation. Since the real interest rate is constant in our model, the forecasts of returns are forecasts of excess returns and not forecasts of changing risk free interest rates.

---

[8]Start with the identities

$$1 = E_t\left(R_{t+1}^{-1} R_{t+1}\right); \;\; R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}.$$

Iterating, and imposing a no-bubbles or transversality condition, we obtain the identity

$$\frac{P_t}{D_t} = E_t \sum_{j=1}^{\infty} \prod_{k=1}^{j} R_{t+k}^{-1} \Delta D_{t+k}.$$

Linearizing about a steady state P/D, and with $\rho = \frac{P/D}{1+P/D}$, we obtain an expression relating price today to expected future returns and dividend growth,

$$p_t - d_t \approx const. + \sum_{j=0}^{\infty} \rho^j \left(\Delta d_{t+j} - r_{t+j}\right)$$

Multiplying by $p_t - d_t - E\left(p_t - d_t\right)$ and taking expectations, we obtain the decomposition.

Table 3.5: Price/consumption ratio variance decomposition.

No. of cov's (quarters)	Percent of var(P/C) due to Returns	Dividends
1	4	0
4	16	0
8	29	0
20	57	0
40	82	0
60	94	0
80	100	0

Table entries are the percentage of price/consumption ratio variance explained by excess return forecasts and consumption growth forecasts,
$100 \times \sum_{j=1}^{k} \rho^j cov(p_t - d_t, r_{t+j})/var(p_t - d_t)$ and
$100 \times \sum_{j=1}^{k} \rho^j cov(p_t - d_t, \Delta d_{t+j})/var(p_t - d_t)$.

Thus, our model replicates volatility test rejections. But the variation in prices is, by construction, entirely rational and driven by economic fundamentals. Of course, as emphasized by Campbell and Shiller (1988b), Shiller (1989), and Cochrane (1991a), volatility test rejections convey the same information as long-horizon return regressions, so the result is not surprising.

Regressions of long-horizon price growth on long-horizon dividend growth

Barsky and DeLong (1993) note that regressions of multi-year log stock price changes on multi-year log dividend changes yield coefficients greater than 1; their largest coefficient is 1.61 at a 20-year horizon. They attribute this phenomenon to "extrapolative forecasting"—investors who see several years of high dividend growth raise their forecasts of trend growth, yielding a large change in stock price. For example, they claim that "to account for the actual correlations of multi year price and dividend changes, a little less than half of any shift in average dividend growth rates over a twenty-year period must be expected to persist indefinitely." (p.297.)

Our model displays the same behavior, but as a result of countercyclical risk aversion rather than extrapolative forecasting. If consumption increases while habit adjusts slowly, the surplus consumption ratio will increase, expected returns will fall, and the price-consumption ratio will rise. Hence, prices will rise by more than the change in consumption.

In the very long run, of course, habits adjust and prices and consumption are cointegrated, so the coefficient tends to 1.0 for very long-run changes or for a regression in levels.

Table 3.6 presents regressions of multi-year log price changes on multi-year log consumption changes in our simulated data. We obtain coefficients as large as 9.65, with a 20-year coefficient still as high as 4.31. Our coefficients are higher than Barsky and DeLong's, since we do not model the difference between consumption and dividends. In our model, a one-quarter change in dividends is equivalent to a change in consumption; it changes risk aversion and hence the price-dividend ratio right away. In the data, several years of dividend growth may be required before consumption changes.

Table 3.6: Regression of long-horizon log price change on long-horizon log consumption growth in simulated data.

	Horizon in years						
	1/4	1	2	5	10	20	40
Coefficient	9.65	9.27	8.77	7.51	6.03	4.31	2.77

Regression equation: $p_t - p_{t-k} = \alpha + \beta\left(c_t - c_{t-k}\right) + \epsilon_t$

Regressions of stock returns on yield spreads and the level of the interest rate.

Yield spreads and the level of the short rate forecast stock and bond returns and macroeconomic variables. These facts constitute a suggestive and important link between macroeconomics and finance. To address this phenomenon, we modify our model to include a small amount of interest rate variation.

We now choose $\lambda(s)$ so that the interest rate is a linear function of the state s,[9]

$$r^f = r_0^f - B\left(s - \bar{s}\right). \tag{3.1}$$

The only difference this makes to our previous analysis is that we generalize equation (2.8)

[9]Since the risk-free rate is a linear function of the state variable s, and since the volatility of the state variable s is very close to a square root function of s (see equation 2.2), our model is very close to square root models of the term structure such as the model of Cox, Ingersoll and Ross (1985). Square root models (and the slightly more general ***affine yield models*** discussed by Duffie and Kan 1993) are particularly tractable and have dominated recent research on the term structure of interest rates. In fact, at our parameter values, a plot of the conditional variance of the interest rate against the level of the interest rate is visually indistinguishable from a straight line, as is a plot of bond yields against s or the interest rate.

to

$$\bar{S} = \sigma\sqrt{\frac{\gamma}{1-\phi-B/\gamma}}. \tag{3.2}$$

The definition of $\lambda(s)$ given in (2.9) now produces a risk free rate of the linear form (3.1), and satisfies the conditions $\lambda(\bar{s}) = 1/\bar{S} - 1$ and $\lambda'(\bar{s}) = -1/\bar{S}$.

We pick the intercept r_0^f to correspond to a 1% per year real rate. We pick B so that the lower bound of the real risk free rate is zero. Endowment economies can have negative net risk free rates, but storage makes this unlikely in a production economy. Hence, we determine B from the condition[10]

$$r^f(s_{\max}) = r_0^f - B\left[s_{\max}(B) - \bar{s}\right] = 0.$$

We calculate a term structure recursively, from

$$P^{(1)}(s_t) = E\left[M_{t+1}\middle| s_t\right]$$

$$P^{(j)}(s_t) = E\left[M_{t+1}P^{(j-1)}(s_{t+1})\middle| s_t\right], \quad j = 2, 3, 4, \ldots$$

We could not find analytic solutions past $j = 2$, so we calculate these prices on a grid for the state variable s_t as we did for the consumption claim. Bond yields, returns, and expected returns follow from prices.

Table 3.7 presents means and standard deviations in our simulated data. The mean excess stock return is now 1.40% per quarter which closes about half the gap between the previous 1.15% and the 1.61% in the postwar quarterly data. Other predictions for stock prices and returns are almost completely unaffected by the addition of interest rate variation, so we do not repeat them. The interest rate now has a standard deviation of 0.21% per quarter or

[10] Alas, $s_{\max}$ depends on B, since $\bar{S}$ depends on B. We find B as follows. Substituting for $s_{\max}$ and $\bar{S}$ from equations (2.10) and (3.2), we obtain

$$r_0^f + \frac{B}{2}\left[\frac{\sigma^2\gamma}{1-\phi-B/\gamma} - 1\right] = 0$$

Solving for B, we obtain the inelegant but still operational expression

$$B = \frac{1}{2}\left\{\gamma(1-\phi) - \gamma^2\sigma^2 + 2r_0^f - \left[\left(\gamma(1-\phi) - \gamma^2\sigma^2 + 2r_0^f\right)^2 - 8\gamma(1-\phi)r_0^f\right]^{\frac{1}{2}}\right\}.$$

0.42% per year. This is much smaller than the variation in expected stock returns, which is why the predictions of our model for stock market behavior are not substantially affected.

Table 3.7 shows that the means and standard deviations of bond returns and yields rise with maturity in our model. The increasing standard deviation of bond returns is consistent with the data, but measured yields on long bonds vary less than yields on short bonds. Our model also exaggerates the upward slope of the average yield curve. These features reflect a well-known tension between time-series and cross-sectional predictions of single-factor term structure models (Campbell, Lo, and MacKinlay 1994). For example, we could reduce the volatility of long-term bond yields by lowering the persistence parameter ϕ, but then we would not fit the time-series behavior of interest rates or stock prices. In the end, one needs more than a single-factor square-root model of real bonds to match both time-series and cross-sectional aspects of nominal bond price data. However, the yield spread varies with the state variable s, which is enough for our present purposes.

Table 3.7: Means and standard deviations in simulated data, model with interest rate variation.

	Mean	S.d.
Excess return $r - r^f$	1.40	6.52
Risk free rate (1Q)	0.25	0.21
1 Year bond return	0.29	0.29
3 "	0.41	0.73
5 "	0.56	1.32
10 "	1.02	3.32
1 Year bond yield	0.27	0.22
3 "	0.33	0.24
5 "	0.39	0.27
10 "	0.59	0.34

Table 3.8 presents long-horizon regressions of stock and bond excess returns on the yield spread in our simulated data. For stock returns we see almost exactly the same results as in the regressions on the price-consumption ratio: The coefficients and R^2 statistics rise slowly with the horizon. Since expected one-year bond returns vary less than stock returns, the bond return forecasting coefficients are much lower. But interestingly, the R^2 for bond returns is even higher than that for stock returns. The coefficients and R^2 again rise with maturity. Longer maturity (5 year) bonds have more predictable variation in returns than short maturity bonds. In summary, high yield spreads forecast high excess stock and bond

returns as documented by Shiller, Campbell, and Schoenholtz (1983), Fama and Bliss (1986), Keim and Stambaugh (1986), Campbell (1987), Fama and French (1989), and Campbell and Shiller (1991).

Table 3.8: Regressions of long horizon excess returns on yield spread in simulated data.

	Stock return		1 year bond		5 year bond	
Horizon	coef.	R^2	coef.	R^2	coef.	R^2
1 Qtr	6.66	0.13	0.28	0.22	2.12	0.21
2	13.2	0.18	0.55	0.30	4.18	0.30
1 Year	25.3	0.25	1.07	0.42	8.05	0.41
2	47.0	0.35	2.05	0.57	15.1	0.56
3	71.3	0.42	2.90	0.66	22.0	0.65
5	98.8	0.51	4.23	0.78	31.9	0.77
10	154	0.63	6.55	0.88	49.5	0.88

Regression equation: $r^e_{t\to t+k} = a + b(y_t^{10year} - y_t^1) + \epsilon_{t+k}$.

The level of interest rates is also a popular forecasting variable for stock and bond returns. Since the interest rate is a linear function of the surplus consumption ratio, the interest rate reveals the surplus consumption ratio just like the dividend-price ratio or yield spread. Hence the level of interest rates produces similar results to those in Table 3.8. We spare the reader an extra table of results.

Consumption and interest rates

Our model also generates many of the puzzles that researchers face in evaluating the power utility consumption-based model. Of course, our model generates an equity premium puzzle and a risk free rate puzzle. More generally, our model generates violations of moment condition tests as in Hansen and Singleton (1982, 1983).

The relation between consumption growth and interest rates is one of the most interesting moments. The standard model predicts that expected consumption growth should vary with risk-free interest rates. For example, with lognormal i.i.d. consumption growth, the consumer's first order condition specializes to a regression equation relating consumption growth to interest rates:

$$\Delta c_{t+1} = \left(\frac{\ln\delta}{\gamma} + \frac{\gamma\sigma^2_{\Delta c}}{2}\right) + \frac{1}{\gamma} r_t^f + \epsilon_{t+1}; \quad E_t\epsilon_{t+1} = 0.$$

Hansen and Singleton (1983) and many subsequent authors find that this equation does not work well in the data. In fact, the lack of a consistent relation between consumption growth rates and interest rates, over time or across countries, is often held up as evidence of the hopelessness of consumption-based models of asset returns.

In our model with interest rate variation, the consumption growth rate is i.i.d., even though interest rates vary through time. Thus, the model predicts *no* relation between consumption growth and beginning of period interest rates. (Consumption growth does affect subsequent interest rates, through its effect on the surplus consumption ratio.) To dramatize this point, Figure 3.1 plots consumption growth against the riskless real interest rate in 5000 of our simulated data points. This might represent a typical plot of consumption growth vs. interest rates across time or countries. The lack of correlation is evident.

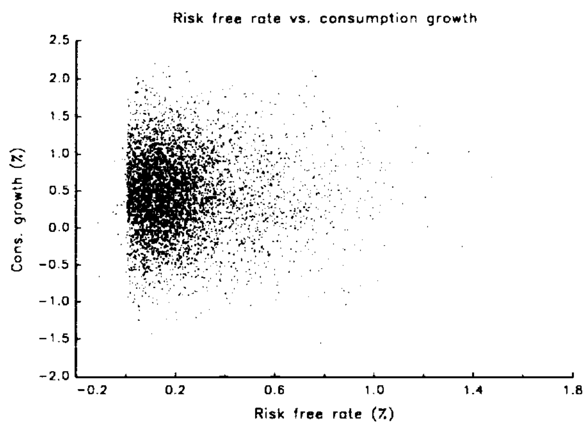

Figure 3.1: Risk free rate vs. consumption growth in 5000 simulated data points from the model with interest rate variation.

Correlation of consumption growth with stock returns

Equilibrium consumption-based models typically imply that consumption growth and stock returns are almost if not perfectly correlated. For example, with log utility, the return on the consumption claim should equal consumption growth, data point for data point, an implication that is often overlooked! As Cochrane and Hansen (1992) emphasize, the low correlation between actual stock returns and consumption growth lies at the heart of many empirical failures of the consumption-based model.

In our model, consumption growth and returns are conditionally perfectly correlated, since consumption growth is the only source of uncertainty. But the relation between consumption growth and returns varies over time with the surplus consumption ratio. Hence the unconditional correlation between consumption growth and returns is not perfect, as in the data.

To illustrate this point, Figure 3.2 plots consumption growth against ex post stock returns in 5000 simulated data points. In the standard power utility model, the points would all lie on a straight line. One can see the data fill a region bounded by two straight lines, each of which corresponds to one limit of the surplus consumption ratio.

The unconditional correlation between stock returns and consumption growth is still 0.92 in our model. This value is less than the 1.0 of the standard model, but much more than the 0.3 or less found in the data. Thus, our model helps but does not fully resolve Cochrane and Hansen's (1992) "correlation puzzle." Part of the reason, of course, is that the "dividend" in our model is consumption itself. A model that has more shocks and hence less than perfect conditional correlation between consumption growth and stock returns should do even better.

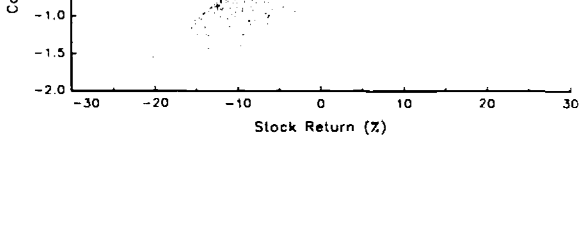

Figure 3.2: Stock return vs. consumption growth in 5000 simulated data points.

Performance of standard asset pricing models.

The CAPM is a standard benchmark asset pricing model, that performs reasonably well for many assets. Recently, multi-factor extensions of the CAPM have been found to price assets better than the static CAPM, especially when factors and asset portfolios that include information variables are considered (Chen, Roll, and Ross 1986, Cochrane 1992, Fama and French 1993, Jagannathan and Wang 1994). On the other hand, even the static CAPM seems to outperform the power-utility consumption-based model (Mankiw and Shapiro 1986). This observation has been interpreted as evidence against consumption-based models in general rather than one particular utility function. We find the same ranking of these models when applied to artificial data from our model.

We consider the following cross-section of assets: the excess stock return, the one and five year excess bond returns and the excess return of a one period consumption claim. In addition, we look at two managed portfolios, $R^e_{t+1} \times (p_t - d_t)$, where R^e_{t+1}denotes an excess return, and $R^e_{t+1} \times \{1 \text{ if } (p_t - d_t) < \text{mean } (p-d), \, 0 \text{ else}\}$. The first transformation captures a portfolio that varies linearly with the price-dividend ratio, while the second is the return to an investor who jumps in and out of the market depending on the price-dividend ratio.

We express each asset pricing model as a model for the stochastic discount factor M. The static CAPM is

$$M_{t+1} = 1 + bR^{e,stock}_{t+1}.$$

Since we only examine excess returns, the constant is irrelevant; 1 is as good as any other value. We pick b, and the free parameters in all subsequent models, to minimize the sum of squared pricing errors $\sum_i E\left(MR^{e,i}\right)^2$. This can be thought of as a first-stage GMM parameter estimate.

A conditional CAPM—the statement that the market return is conditionally mean-variance efficient—does not imply an unconditional CAPM.[11] Cochrane (1992) suggests that one can partially overcome this problem by adding scaled factors, representing the conditional CAPM by the discount factor model

$$M_{t+1} = 1 + b_1 R^{e,stock}_{t+1} + b_2 \left[R^{e,stock}_{t+1} \times (p_t - d_t)\right].$$

[11] Hansen and Richard (1987). In finance language, the essence of the point is that betas and factor risk premia may vary over time, and $E_t\left(R_{t+1}\right) = R^f_t + \beta_t \lambda_t$ does not imply $E\left(R_{t+1}\right) = R^f + E(\beta_t)E(\lambda_t)$ with $E(\beta_t)$ the unconditional regression coefficient.

He finds that this scaled CAPM produces smaller pricing errors than the static CAPM and produces no small firm residual. Jagannathan and Wang (1994) argue that this kind of conditional CAPM captures the effects of additional factors such as those advocated by Fama and French (1993), which are portfolios of stocks sorted on the basis of market value and book/market ratios.

The power utility consumption-based model is, of course,

$$M_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

The discount factor δ is unidentified since we only examine excess returns.

To evaluate the performance of each model, we calculate the quantity

$$\frac{E\left(MR^{e,i}\right)}{E(M)E\left(R^{e,i}\right)} \times 100.$$

$E\left(MR^{e,i}\right)$ is the pricing error for the excess return on asset i, which should be zero. Dividing by $E(M)$ transforms the pricing error into expected return units, i.e. actual less predicted expected excess return or "alpha," since

$$\frac{E\left(MR^{e,i}\right)}{E(M)} = E(R^{e,i}) - \frac{Cov(M, R^{e,i})}{E(M)}.$$

Dividing by $E\left(R^{e,i}\right)$ and multiplying by one hundred, we measure the percentage of each expected excess return that is not explained by each asset pricing model. We also report the root mean square pricing error–the root mean squared value of $E(MR^{e,i})/E\left(M\right)$ across assets i.

Table 3.9 presents these pricing error measures in our artificial data. The first column gives mean excess returns of these portfolios, to see what there is to be explained. Examining the columns of the table, we see that the CAPM is not a bad approximation. It prices the market quite well, but this is almost by construction. Other pricing errors are about 5% of mean excess returns. The conditional CAPM still prices the market almost by construction. More importantly, the other pricing errors are reduced from about 5% to about 2.5%, and the root mean square pricing error is more than halved. Hence, the time-varying risk aversion in our model implies that multi-factor extensions of the CAPM designed to incorporate conditioning information outperform the static CAPM. The power utility consumption-based model produces a high estimate of the risk aversion parameter, as many researchers find in actual data. [12] Most importantly, the power utility model has pricing errors that are two to three times those of the static CAPM. As Mankiw and Shapiro (1986) found in real data, the static CAPM works much better than the power utility consumption-based model.

Table 3.9: Pricing errors of approximate models.

Portfolio	Mean return	Static CAPM	Scaled CAPM	Consumption $\gamma = 46$
Stock	1.65	0.4	-0.12	1.3
Scaled stock	7.07	-0.4	-0.08	-0.75
Stock x (pd < mean)	1.11	7.3	-1.1	23
1 year bond	0.04	6.0	2.5	9.2
5 year bond	0.32	6.3	2.7	9.8
Scaled 1 year bond	0.17	5.0	2.4	7.0
Scaled 5 year bond	1.34	5.3	2.6	7.7
1 q cons. claim	0.13	1.3	3.3	-8.0
Scaled 1 q cons. claim	0.57	0.4	3.3	-11
RMS pricing error (%)		0.11	0.04	0.29

"Mean return" reports the mean simple (not log) excess return. Table entries are expected return pricing error (or "alpha") divided by mean excess return, in percent, $100 \times E(MR^e)/[E(R^e) \times E(M)]$, from simulated data. The bottom row gives the percent root mean square pricing error $100 \times E(MR^e)/E(M)$ across assets. Model free parameters are picked to minimize the sum of squared pricing errors, i.e. the first-stage GMM estimate.

Our model has only one shock, so the discount factor proxies are almost perfectly conditionally correlated, and thus all the models conditionallyprice assets perfectly. However, variation in the hidden state variable s means that unconditional first and second moments used in unconditional tests are not related in the same way as the conditional first and second moments.

[12] This is the result in the data when relatively few assets are included, so GMM spends a lot of effort in trying to match unconditional equity premia. The slight predictability of consumption growth about matches the slight predictability of some bond returns, leading to lower estimates of γ in Hansen and Singleton (1983). The erratum to Hansen and Singleton (1982) shows both sets of estimates.

4. A view of historical consumption and price data

Instead of simulating artificial consumption data, we now feed our model actual postwar data on nondurables and services consumption per capita, and study the model's predictions for stock prices and returns.

Figure 4.1 plots the consumption series and the implied level of habit, starting the simulation at the steady state surplus consumption ratio. Habit is much smoother than consumption. One can see consumption drop near habit in the troughs of the last four recessions. These are periods in which our model predicts high risk aversion, low stock prices, and high expected stock returns. Finally, the recent recession saw an unusually large decline in consumption; our model therefore predicts a large increase in risk aversion.

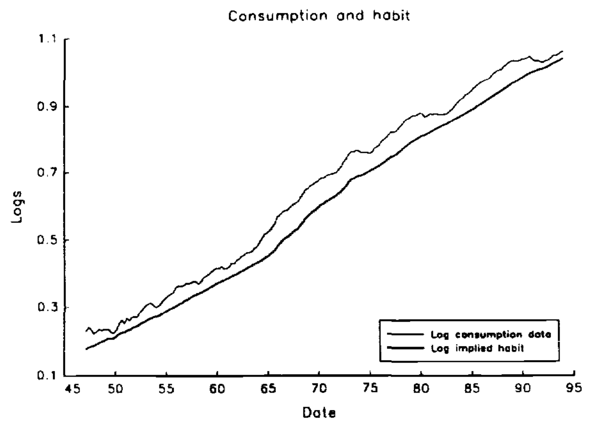

Figure 4.1: Nondurable + services consumption per capita and implied habit.

The left panel of Figure 4.2 presents our model's prediction for the price-consumption ratio, together with the value-weighted NYSE price-dividend ratio. To our eyes, the model provides a tantalizingly good account of the cyclical and longer-term fluctuations in stock prices over the postwar period, using only the history of consumption.[13]

[13] It is a little embarassing that the worst performance occurs in the last few years. Consumption growth has been slow in the early 1990's, so our model predicts a fall in price-dividend ratios, rather than the increase we see in the data. Part of the story may be repurchases: Many companies are undergoing a fundamental restructuring, repurchasing shares rather than paying dividends. This skews measured price-dividend ratios upward, but it does not distort the one-year returns shown in the right panel of the figure.

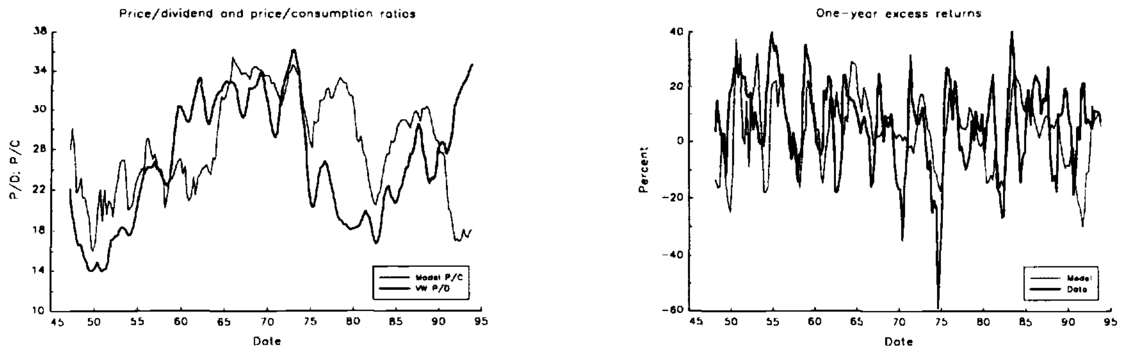

Figure 4.2: Left panel: Price consumption ratio predicted by the model based on nondurable and services per capita consumption data, and value-weighted NYSE price- dividend ratio. Right panel: One-year excess returns in the model and data.

To emphasize the business cycle frequency at which the model's price-consumption variation and the NYSE price-dividend variation seem to line up best, the right hand panel of Figure 4.2 presents the model's predictions for one-year excess returns together with the NYSE excess returns over the Treasury bill rate. Again, we see a tantalizing account of major swings in stock returns. The correlation between predicted and actual excess returns is 0.32. This is comparable to the correlation between stock returns and returns on capital inferred from investment growth in Cochrane (1991c).

We also feed the long annual consumption series through the model. Since our model is quarterly, we split each year's consumption growth evenly over four quarters. This consumption series is much more volatile, which together with a lower price of risk might suggest a recalibration of the model with lower power γ and higher surplus consumption ratios. However, preferences should be stable, and our model is fairly insensitive to parameters in any case. Hence, we retain the parameters used for the quarterly postwar data.

Figure 4.3 presents our implied price-consumption ratio and the S&P500 price-dividend ratio. Again, the model seems to account for many cyclical swings, as well as long term movements such as the high prices of 1890-1907, the 1920's and the 1960's, based entirely on the history of consumption.

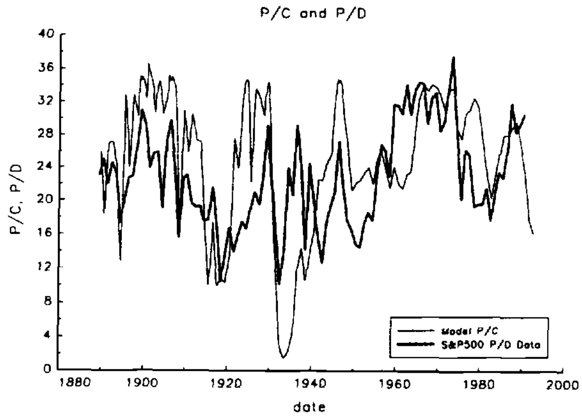

Figure 4.3: Price consumption ratio predicted by the model based on annual nondurable consumption data, and S&P500 price-dividend ratio.

5. The welfare costs of fluctuations

Lucas (1987) made a provocative calculation that the welfare costs of fluctuations are trivial compared to the welfare implications of growth. Here is a simplified version of his calculation: If consumers have power utility $u(C) = C^{1-\gamma}$ and consumption growth is i.i.d. lognormal, then the reduction in the log mean growth rate of consumption ($\alpha \equiv \ln E\left(C_{t+1}/C_t\right)$) that consumers would accept along with the elimination of all fluctuations is[14]

$$\Delta\alpha = \frac{\gamma\sigma^2}{2}.$$

With σ and g on the order of one percent annually and γ near 2, consumers would only trade

[14]Up to a constant, expected utility is

$$\frac{1}{1-\gamma}E\sum_j \delta^j C_{t+j}^{1-\gamma} = \frac{C_t^{1-\gamma}}{1-\gamma}E\sum_j \delta^j \left(\frac{C_{t+j}}{C_t}\right)^{1-\gamma} = \frac{C_t^{1-\gamma}}{1-\gamma}\sum_j \delta^j e^{(1-\gamma)j\left[g+(1-\gamma)\frac{\sigma^2}{2}\right]} =$$

$$= \frac{C_t^{1-\gamma}}{(1-\gamma)\left\{1-\delta e^{(1-\gamma)\left[g+(1-\gamma)\frac{\sigma^2}{2}\right]}\right\}}.$$

The log mean consumption growth rate is

$$\alpha = \ln E\left(\frac{C_{t+1}}{C_t}\right) = g + \frac{\sigma^2}{2}.$$

Following Lucas, we compare economies by their log mean consumption growth rather than mean log consumption growth. Increasing the variance of log consumption growth also increases mean consumption growth, and consumers like this increase. If one holds the mean log consumption growth constant, then log utility consumers are indifferent to fluctuations! In terms of log mean gross consumption growth α, then, utility is

$$\frac{C_t^{1-\gamma}}{(1-\gamma)\left\{1-\delta e^{(1-\gamma)\left(\alpha-\frac{\sigma^2}{2}\right)+(1-\gamma)^2\frac{\sigma^2}{2}}\right\}} = \frac{C_t^{1-\gamma}}{(1-\gamma)\left\{1-\delta e^{(1-\gamma)\alpha-(1-\gamma)\gamma\frac{\sigma^2}{2}}\right\}}$$

Define α^* as the growth rate in a nonstochastic economy that gives the same level of utility. Then

$$\frac{C_t^{1-\gamma}}{(1-\gamma)\left\{1-\delta e^{(1-\gamma)\alpha^*}\right\}} = \frac{C_t^{1-\gamma}}{(1-\gamma)\left\{1-\delta e^{(1-\gamma)\alpha-(1-\gamma)\gamma\frac{\sigma^2}{2}}\right\}}$$

which implies

$$\alpha^* = \alpha - \frac{\gamma\sigma^2}{2}.$$

one hundredth of a percentage point of growth for the complete elimination of fluctuations.

Lucas' calculation appears to have been accepted by many eminent economists. Krugman (1990), for example, writes that "Productivity isn't everything, but in the long run it is almost everything" (p.9), while Barro (1994) claims that "Economic growth is the part of macroeconomics that really matters" (p.7). Lucas' calculation is a powerful challenge that research such as this, with one foot in the grave of macroeconomic fluctuations and the other in the tomb of aggregate asset pricing, is a waste of time and paper.

But Lucas' calculation ignores observable market prices that measure the costs of fluctuations. The equity premium measures the marginal rate of substitution or transformation between growth and volatility. Each of us can achieve a tradeoff orders of magnitude greater than Lucas' calculation suggests by shouldering the volatility of the stock market. For example, by borrowing and investing in the stock market, a consumer with nonstochastic consumption growth can get a 0.4 percentage point increase in mean annual consumption growth for every 1 percent standard deviation of consumption growth that he accepts—far above the 0.01 percentage point increase in mean consumption growth that Lucas' calculation suggests would leave him equally well-off.[15] This unexploited profit opportunity is just the equity premium puzzle: Lucas' calculation uses a utility function that is at a loss to explain asset market data.

Our model implies much larger welfare costs of fluctuations. Figure 5.1 presents a calculation. For each value of the surplus consumption ratio, we calculate the level of expected utility numerically. Then we search for the log mean growth rate in a nonstochastic economy that gives the same level of expected utility. The graph presents the log mean growth rate in the utility-equivalent nonstochastic economy. Like Lucas' calculation, this is not a calculation of welfare costs, but a comparison of two economies that give the same level of welfare. For reference, the log mean consumption growth rate of the stochastic economy,

[15]To be specific, consider a trust-fund consumer with wealth W_t invested at a constant risk free rate R. The consumer consumes a constant fraction α of wealth each period. Hence, wealth and consumption grow at the constant rate $R-\alpha$. If the consumer now borrows a fraction β of his wealth and invests it in stocks yielding a random excess return R^e_{t+1}, his consumption (proportional to wealth) evolves as

$$\frac{C_{t+1}}{C_t} = R - \alpha + \beta R^e_{t+1}$$

Thus, the tradeoff of mean for standard deviation in the stock market—$E(R^e)/\sigma(R^e)$—is also the tradeoff of mean for standard deviation of consumption growth faced by this consumer.

equal to the postwar US consumption growth rate, is shown in the figure as a horizontal dashed line. Finally, Lucas' calculation—the nonstochastic growth rate that leaves Lucas' consumers equally well off—is shown as the horizontal solid line with boxes. Since it is only a hundredth of a percentage point below the actual consumption growth rate, the latter two lines are hard to distinguish visually.

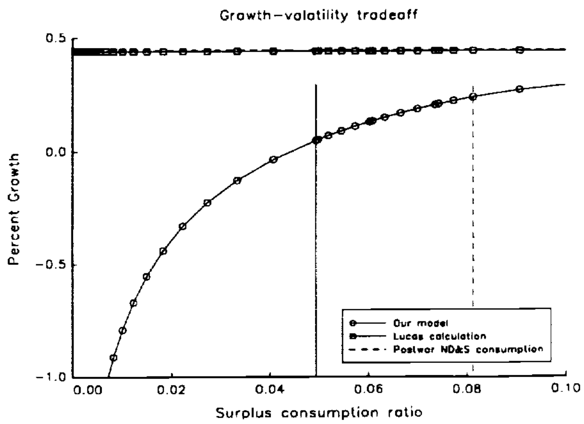

Figure 5.1: Growth-volatility tradeoff. The lines plot the log growth rate of nonstochastic economy that gives agents the same utility level as in the stochastic economy.

At the steady state surplus consumption ratio, consumers in our model would be willing to trade all growth for the elimination of fluctuations. When the surplus consumption ratio is low, risk aversion is very high, so consumers would accept large negative consumption growth rates if they could get rid of fluctuations.

Our result does not come by disparaging the benefits of growth. Habits do not catch up with the consumer, rendering consumption growth useless. Utility $(C-X)^{1-\gamma}$ depends on the difference between consumption and habit, yet the habit/consumption ratio is stationary. Thus, if one doubles consumption and habit, one multiplies utility by $2^{1-\gamma}$, exactly as if utility was of the power form $C^{1-\gamma}$. Similarly, on a nonstochastic growth path, marginal utility is $u'(C_t) = \bar{S}C_t^{-\gamma}$. Our model just multiplies marginal utility by a constant, which has no effects on the marginal tradeoff between consumption and growth. As a third way to see the point, the present value of the consumption stream in a nonstochastic growth path is[16]

$$PV_0 \equiv \sum_{t=0}^{\infty} \delta^t \frac{u'(C_t)}{u'(C_0)} C_t = \frac{C_0}{1 - \delta G^{1-\gamma}}.$$

This marginal tradeoff between consumption and growth is governed by the power parameter γ, not by the risk aversion coefficient $\gamma/\bar{S}$. It is the same whether or not habit is in the model.

Instead, our model increases the welfare costs of fluctuations. Our model changes non-marginal trade-offs between current consumption and future growth. As current consumption is reduced to pay for future growth, the marginal utility of current consumption rises much more rapidly in our model than it would do in a power utility model with the same curvature parameter. This feature is what makes fluctuations so much more important in our model. As we noted in discussing the determination of the risk free real interest rate, marginal analysis around a nonstochastic steady state is a very bad approximation to the world of our model.

Our result is also not an artifact of a particular utility function and habit formation process. As Atkeson and Phelan (1994) emphasize, our conclusion must result from any utility function that explains the level and time-variation of the equity premium. The only way to ignore the calculation is to argue that, because of transactions costs or other frictions, stock markets are de-linked from the real economy. But if this is so, then what is the invisible mechanism that equates marginal rates of substitution to marginal rates of transformation? If one believes in such large frictions, one cannot go on to study frictionless intertemporal models for quantities.

[16]The present value is

$$PV = \sum_{t=0}^{\infty} \delta^t \left(\frac{C_t - X_t}{C_0 - X_0} \right)^{-\gamma} C_t = \sum_{t=0}^{\infty} \delta^t \frac{C_t^{1-\gamma}}{C_0^{-\gamma}} \left(\frac{S_t}{S_0} \right)^{-\gamma}.$$

In a steady state, the surplus consumption ratio S_t is constant, and consumption grows at the rate G, so we obtain

$$PV = C_0 \sum_{t=0}^{\infty} \delta^t G^{j(1-\gamma)} = \frac{C_0}{1 - \delta G^{1-\gamma}}.$$

6. Concluding remarks

Summary and conclusions

We have documented a broad variety of empirical successes for our consumption-based model with external habit formation. After calibrating our model to fit the unconditional equity premium and risk free interest rate, we capture many dynamic features of stock market behavior. The model generates long-horizon predictability of excess stock and bond returns from the dividend-price ratio, yield spread, and short-term interest rate; it generates a high average level of stock return volatility; and it generates persistent movements in return volatility. All of these phenomena are linked to economic fluctuations: Expected returns, return volatility, and the price of risk are high when consumption falls for a few quarters, and price-dividend ratios are low. The model predicts many puzzles that face the standard power utility consumption-based model, and in particular that the CAPM and its multifactor extensions outperform the power utility consumption-based model. Fed actual consumption data, the model captures the main swings in stock prices in both quarterly postwar and long-run annual data. All this, despite constant conditional moments for consumption growth and a constant or mildly countercyclical real interest rate.

Our model gives some hope that finance can productively search for fundamental risk factors that explain at least the time-series behavior of stock returns, rather than just relate some asset returns to other asset returns.

Our model also has some lessons for macroeconomics. It captures welfare costs of fluctuations on the order of the benefits of growth, as measured by stock prices. Perhaps more importantly, we were forced to construct a model in which important intertemporal substitution effects are largely offset by precautionary savings effects, in order to capture stock price behavior with relatively constant interest rates. Our world would be very poorly understood by a nonstochastic model or near-steady-state approximations. Approaching data from our world with this conventional analysis would leave one puzzled why large interest rate variation is absent.

Directions for research

A skeptic might ask, why does the model not fit the data even better? The correlation between the model's predicted annual stock returns and actual stock returns is 0.32 in postwar data: why is it not 1? Where is the likelihood ratio test of the model's predictions for the joint time-series behavior of consumption, and stock returns? Why do we not report an asset pricing test using consumption data and an interesting cross-section of stock and bond returns?

Our answer is that the model is simplified, in order to clearly view basic forces at work. It is not embellished with details that, though necessary for it to be a complete description of reality, would cloud a view of the basic mechanisms. Even admitting the representative consumer abstraction, the model uses a simple AR(1) process to infer habit from consumption, it omits leverage, it makes no distinction between dividends and consumption, and it omits market frictions of all kinds. The model has only one shock, so it produces stochastic singularities—the dividend-price ratio, yield spread, interest rate level, and habit consumption ratio are all deterministically related. All quantities are perfectly correlated, conditioned on any of these state variables. A likelihood ratio test would seize on these facts to reject the model with a zero p-value, ignoring the quality of the model's match to the interesting time-series features of the data.

Furthermore, cross-sectional asset pricing tests check whether mean returns are correctly related to the covariance of returns with a measured marginal rate of substitution. Covariances are sensitive to time-aggregation, the precise timing of variables, measurement problems (see Wilcox 1992 for a sobering account), and the precise extraction of each period's habit level. The comparison between the model's time-series behavior and that of the data which we have studied in this paper is much less sensitive to these issues.

But with the basic mechanisms now clear, we hope that extensions of our model can incorporate other features of the data and provide a useful framework for more formal tests. For example, multi-factor models currently dominate cross-sectional research (Chen, Roll, and Ross 1986 and Fama and French 1993 are notable examples), but the factors in these models are specified in an ad hoc manner and their risk prices are treated as free parameters to be estimated. Once our model is generalized so that the stochastic singularity is broken, dividend-price ratios, yield spreads, the level of interest rates, and the history of past consumption will all show up in asset pricing tests as indicators of one underlying risk factor, rather than as separate ad-hoc risk factors. Campbell (1993) argues that a complete cross-sectional model should derive the factors and their risk prices from some underlying structure; it should be possible to do this in our framework. Also, a cross-sectional implementation of our model will allow us to study a broader variety of assets than those considered here.

Another direction for future research will be to include production in the model. With production, we can address more tantalizing evidence for relations between stock returns and macroeconomics; among others, Fama's (1990) work relating stock returns to leads and lags of output growth and Cochrane's (1991c) findings that investment growth is highly correlated with stock returns and that expected investment growth matches expected stock returns. We can check that the preferences we have used to explain asset market data do not lead to grossly counterfactual quantity dynamics in an equilibrium business cycle framework. The fact that our model is consistent with a risk free rate and hence linear technology gives us hope in this dimension.

Aggregation is another important question that we do not address. Our goal in this paper is to reconcile aggregate consumption data with stock returns via a representative consumer utility function. This is a legitimate procedure; it can be defended as a model of the social welfare function of an economy of agents with unknown preferences, as the utility function of a marginal investor who consumes aggregate consumption and holds the market portfolio of assets, or by the assumption of identical agents. But it is important to study aggregation, using results like those of Rubinstein (1976), if one wishes to draw lessons from asset markets for the preferences that one uses in micro data.

However, the literature is full of "mongrel aggregation" results in which individual preferences are quite different from representative consumer preferences. Our "habit" might represent the aggregated effects of idiosyncratic income variation in a model such as Constantinides and Duffie's (1992); or it might capture the effects of debt, built up in booms, in incomplete markets with frictions. Marcus (1989) and Grossman and Zhou (1994) show how risk aversion rises when stocks fall as a result of portfolio insurance or stop-loss rules that they suspect come from the principal-agent relationship in delegated financial management. Or perhaps the representative agent results from the interactions of heterogeneous agents, some with time-varying risk aversion and others with constant risk aversion. Campbell and Kyle (1993), for example, estimate a "noise trader" model of aggregate stock prices and find that noise-trader demand for stock increases when aggregate dividends increase. They suggest that the noise traders may be investors with wealth-varying risk-aversion, of the type modeled formally here.

These considerations suggest that drawing direct implications of our results for micro data may be difficult. On the other hand, readers who find our external habit specification to be a counterintuitive description of individual preferences may be able to accept it as a description of representative agent preferences.

7. References

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8. Appendix: Density of s in continuous time.

The continuous time version of the model is

$$ds_t = (1-\phi)(\bar{s} - s_t)\, dt + \lambda(s_t)\sigma dB_t$$

$$dc_t = g dt + \sigma dB_t$$

and

$$u' = (SC)^{-\gamma} = e^{-\gamma(s+c)}. \tag{8.1}$$

Let V denote cumulated value, so the basic first order condition in continuous time is

$$\rho dt = E\left(\frac{du'V}{u'V}\right) = E\left(\frac{du'}{u'}\right) + E\left(\frac{dV}{V}\right) + E\left(\frac{du'dV}{u'V}\right).$$

For the instantaneous risk free rate, the return is certain, $dV/V = r^f dt$,so

$$r^f = \rho - \frac{1}{dt}E\left(\frac{du'}{u'}\right).$$

Using the utility function for our model equation (8.1) and Ito's formula,

$$r^f = \rho + \gamma g - \gamma(1-\phi)(s_t - \bar{s}) - \frac{\gamma^2\sigma^2}{2}(\lambda(s)+1)^2.$$

This is the same formula as in discrete time, so our choices of $\lambda(s)$ give a constant instantaneous risk free rate and a risk free rate linear in s for the continuous time version of the model.

The forward equation implies that the stationary density $q(s)$ of a diffusion $ds = \mu(s)dt + \sigma(s)dB$, if it exists, satisfies

$$\mu(s)q(s) = \frac{1}{2}\frac{d}{ds}\left(\sigma^2(s)q(s)\right)$$

The solution of this differential equation can be expressed as

$$q(s) = \frac{z(s)}{\int z(s)ds}$$

where

$$z(s) = \frac{e^{2\int^{s} dv \frac{\mu(v)}{\sigma^2(v)}}}{\sigma^2(s)}.$$

Plugging in the drift and diffusion coefficients of our process, and performing a messy integral, we obtain

$$\ln z(s) = \frac{\gamma^4\sigma^2(\phi-1)}{(\gamma(1-\phi)-B)^2}\left[\frac{\lambda(s)^2}{2} + 3\lambda(s) + \frac{N^2-1}{\lambda(s)}\right] + \left[\frac{(3-N^2)\gamma^4\sigma^2(\phi-1)}{(\gamma(1-\phi)-B)^2} - 2\right]\ln\lambda(s)$$

where $N = 1/S$.

Figure 2.2 plots this distribution for our parameter values. Perhaps the most important thing we learn is that the process does not pile up at the boundary.