FUNDAÇÃO GETÚLIO VARGAS
ESCOLA DE PÓS-GRADUAÇÃO EM ECONOMIA
PROGRAMA DE MESTRADO ACADÊMICO EM ECONOMIA

Jivago B. Ximenes de Vasconcelos

CAN A HABIT FORMATION MODEL REALLY EXPLAIN THE FORWARD PREMIUM ANOMALY?

Rio de Janeiro-RJ
2009

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Dissertação apresentada à Escola de Pós-graduação em Economia da Fundação Getúlio Vargas como requisito parcial para obtenção de título de Mestre em Economia.

Orientador: Prof. Carlos Eugênio da Costa

Rio de Janeiro-RJ
2009

*Para meu amor, minha esposa Isabel,
que tudo me deu sem eu nada pedir.*

*A meus pais, Antemar e Annete, pelo
apoio incondicional, paciência e
carinho.*

*A Enrico, Mônica e Gabriel, pelo
incentivo, zelo e descontração nos
momentos mais difíceis.*

*A meus sogros, Odenir e Luzia, pela
ajuda e palavras de perseverança.*

AGRADECIMENTOS

Este trabalho é fruto de uma trajetória de esforços ao longo dos últimos 12 meses do programa de mestrado em economia *strictu sensu* realizado na Fundação Getúlio Vargas. Nesta trajetória, professores, colegas, familiares e amigos participaram direta, ou indiretamente, com sugestões e críticas que ajudaram a conformar este projeto. Portanto, este trabalho também pertence a todos aqueles que influenciaram minha vida neste último ano. Nas poucas palavras que disponho aqui, será uma difícil tarefa elencar todos a quem tenho de agradecer. Só me resta prometer que tentarei fazê-la da maneira mais atenciosa.

Dentre os professores, merecem especial destaque: meu orientador, Prof. Carlos Eugênio da Costa, pela irrestrita atenção no desenvolver deste trabalho e zelo pelo seu formato e conteúdo; ao Professor Caio Almeida, pelo entusiasmo e conselhos fundamentais; ao Prof. Fernando Holanda Barbosa, pelas recomendações de leitura no ínterim deste projeto e conversas descontraídas sobre a "real economia"; e a José Valetim, do Banco Central do Brasil, pelo interesse na leitura desta dissertação e comentários.

Aos meus colegas, agradeço a Jaime Filho, pela ajuda na aquisição de dados; Leonardo Silva, Marcus Studart, Marinho Bertanha, Rafael Mourão e Vitor Farinha, pela atenção e profícuos comentários. Em especial, a Jefferson Bertolai e João Ayres que demonstraram imensa paciência assistindo, por mais de uma vez, minhas apresentações parciais, "estressando" meu domínio sobre os fundamentos econômicos inerentes a este trabalho.

À minha família, não há como resumir toda sua importância nesta folha. Sem o apoio financeiro e estímulo dados pelos meus pais e sogros, ao longo de todo o programa de mestrado, não estaria escrevendo estas palavras. Às tias, Auristela e Zezé, pelo terno acolhimento e pelas "consultas econômicas" que me faziam pensar, ser eu, um experiente economista. A meu irmão, e colega, Enrico, agradeço o incentivo e ajuda em todo este percurso, principalmente durante o difícil e árido "primeiro ano da Fundação", mas acima de tudo, agradeço pelo exemplo. À minha esposa Isabel, agradeço pelo seu companheirismo e compreensão, pelos muitos dias e noites de minha ausência, pelas risadas das más notícias e pelas lágrimas das boas, e, principalmente por sempre acreditar em mim, mesmo em meus momentos de total incredulidade.

Aos amigos do Vieira, meus agradecimentos pela eterna amizade e descontração nos momentos de descanso. Aos amigos da Engenharia, agradeço o apoio e incentivo, apesar do estranhamento que a abstração econômica traz a um engenheiro. Aos novos amigos cariocas, obrigado pelo acolhimento e por terem me proporcionado a sensação de estar em casa.

Institucionalmente, agradeço à Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) pelo auxílio financeiro recebido e ao programa de Mestrado Acadêmico em Economia da Escola de Pós-Graduação em Economia (EPGE) pela confiança em mim depositada.

Finalmente, agradeço a Deus, pela força concedida sempre e, principalmente, nos momentos mais difíceis, nos quais nunca me faltou.

LISTA DE FIGURAS

LISTA DE TABELAS

LISTA DE ABREVIATURAS

CAPES – Coordenação de Aperfeiçoamento de Pessoal de Nível Superior

EPGE – Escola de Pós-Graduação em Economia

FPP – Forward Premium Puzzle

EPP – Equity Premium Puzzle

UIP – Uncovered Interest Parity

C&C – Campbell and Cochrane (1999) paper

CCAPM – Consumption Capital Asset Pricing Model

JIT – Jensen Inequality Term

PPP – Purchase Power Parity

CRRA – Constant Relative Risk Aversion

US – United States of America

90-day T-Bill – US Treasury bill with a maturity of three months

CPI – Consumer Price Index

CRSP – Center Research of Stock and Prices

SUMÁRIO

Modelos com formação de hábitos podem realmente explicar o *Forward Premium Anomaly*? [*]

Jivago Vasconcelos[†]

Fundação Getulio Vargas

23 de Maio de 2009

Resumo

Verdelhan (2009) mostra que desejando-se explicar o comportamento do prêmio de risco nos mercados de títulos estrangeiros usando-se o modelo de formação externa de hábitos proposto por Campbell e Cochrane (1999) será necessário especificar o retorno livre de risco de equilíbrio de maneira pró-cíclica. Mostramos que esta especificação só é possível sobre parâmetros de calibração implausíveis. Ainda no processo de calibração, para a maioria dos parâmetros razoáveis, a razão preço-consumo diverge. Entretanto, adotando a sugestão proposta por Verdelhan (2009) - de fixar a função sensibilidade $\lambda(s_t)$ no seu valor de steady-state durante a calibração e liberá-la apenas durante a simulação dos dados para se garantir taxas livre de risco pró-cíclicas - conseguimos encontrar um valor finito e bem comportado para a razão preço-consumo de equilíbrio e replicar o *foward premium anomaly*. Desconsiderando possíveis inconsistências deste procedimento, sobre retornos livres de risco pró-cíclicos, conforme sugerido por Wachter (2006), o modelo utilizado gera curvas de *yields* reais decrescentes na maturidade, independentemente do estado da economia - resultado que se opõe à literatura subjacente e aos dados reais sobre *yields*.

Palavras-Chaves: Forward Premium Puzzle, Equity Premium Puzzle, Formação de Hábito, Asset Pricing

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1

Can a Habit Formation Model really explain the Forward Premium Anomaly? *

Jivago Vasconcelos[†]

Fundação Getulio Vargas

23 de Maio de 2009

Abstract

Verdelhan (2009) shows that if one is to explain the foreign exchange forward premium behavior using Campbell and Cochrane (1999)'s habit formation model one must specify it in such a way to generate pro-cyclical short term risk free rates. At the calibration procedure, we show that this is only possible in Campbell and Cochrane's framework under implausible parameters specifications given that the price-consumption ratio diverges in almost all parameters sets. We, then, adopt Verdelhan's shortcut of fixing the sensitivity function $\lambda(s_t)$ at its steady state level to attain a finite value for the price-consumption ratio and release it in the simulation stage to ensure pro-cyclical risk free rates. Beyond the potential inconsistencies that such procedure may generate, as suggested by Wachter (2006), with pro-cyclical risk free rates the model generates a downward sloped real yield curve, which is at odds with the data.

Keywords: Forward Premium Puzzle, Equity Premium Puzzle, Habit formation, Asset Pricing

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1   Introduction

The forward premium anomaly or Forward Premium Puzzle – FPP, henceforth - is closely related to the failure of Uncovered Interest Parity – UIP henceforth – relation, i.e. if covered interest parity holds then the forward discount and the interest differential should be unbiased predictors of the ex-post change in the spot rate, assuming rational expectations. The UIP arises from a simple arbitrage argument in a risk neutral world. Higher interest rates imply currency devaluation so that the expected returns on domestic and foreign short-term bonds are equalized. Once we depart from risk neutrality, violations of UIP need not be puzzling, it is always possible to define a risk premium that accommodates the behavior of excess returns on foreign markets.

The relevant questions are whether this risk premium is derived from a sensible model and whether it is compatible with the behavior of other asset prices. The main difficulty one faces in trying to accommodate foreign market facts with a risk model is the poor performance of most asset pricing models, as evidenced by the Equity Premium Puzzle—EPP.

It is in this context that the results in Verdelhan (2009) seem very heartening. There it is shown that the external habit model of Campbell and Cochrane (1999) may, at the same time, account for most stylized facts in both domestic and foreign markets.

Campbell and Cochrane (1999) external habit formation model delivers time-varying countercyclical risk premia. In bad times investor's risk aversion is higher, so he demands a larger risk premium. Hence, when consumption falls, expected returns, return volatility and the price of risk rise. Economic fluctuations generate in the model important aspects of asset prices behavior: (a) long-horizon predictability of excess returns from consumption-price ratio; (b) mean reversion in returns; (c) high stock price and return volatility despite smooth consumption growth. All these characteristics make the model a suitable candidate for the exercise conducted by Verdelhan (2009).

In this paper, we follow Verdelhan's lead and calibrate Campbell and Cochrane (1999) model for US post-war data and created endowment processes for fictitious countries based on US data. The process of each fictitious country is generated arbitrarily aiming to reach a target correlation with US consumption data. Thus, we can use the same model to generate the stochastic discount factor for each fictitious country, which allow us to evaluate the real exchange rate between US and its counterparts. So, we are able

2

to evaluate an appropriate forward premia in currency markets which fits domestic markets asset behavior.

Interest rates generated by the model must be pro-cyclical to reproduce the forward discount anomaly, as shown by Verdelhan (2009). The problem with this imposing a parametrization that generates pro-cyclical risk-free rates is that the price-consumption ratio delivered by the model diverges for almost all calibration parameters sets. Moreover, not only a very narrow range of model parameters results in price-consumption ratio convergence, but also all parameters in that range are not capable of generating a reasonable behavior of other variables. How, one might ask, is this compatible with Verdelhan's claim of successful results? When we adopt a stricter assumption on the time series behavior of the surplus function (which relates consumption and habit in a tractable manner) at model calibration stage, we are able to match a finite price-consumption ratio, now with reasonable parameters. In such a way, we can calibrate the model under pro-cyclical interest rates and match the forward discount anomaly simultaneously.

As we will show, this assumption has a significant impact on real yields. In contrast with empirical data, which usually exhibits an upward sloping real yield curve, we show that when risk free rates are pro-cyclical the habit formation model delivers a downward sloping real yield curve. This behavior is at odds with evidence in almost all data sets.

In a broad sense, although our results seem to indicate that the original Campbell and Cochrane consumption-based model is not able to produce the behavior of exchange markets we cannot state it for sure, given the inconsistency in the procedure. Indeed, this another drawback of this application of the model: the calibration shortcut used to reach a finite price-consumption ratio and all its potential inconsistencies. These potential inconsistencies should result given that stock market returns depend on price-consumption ratio.

The remainder of the paper is organized as follows. Section 2 explains the FPP puzzle and presents UIP null hypothesis in nominal and real terms. Section 3 describes the model and how the exchange rates are attained. Moreover, is derived a sufficient and necessary condition to reproduce the FPP evidence in simulated data. Section 4 exhibits the methodology and simulation results after having imposed a constant sensibility function, besides a critical analysis about that assumption. Section 5 concludes.

3

2 Literary Review: a glance at the FPP

2.1 The null hypothesis in nominal terms

Following the seminal work of Hodrick (1987), a large volume of research aiming at evaluating the efficiency of forward markets for foreign exchange has been produced. The focus of such agenda has been to explain the most puzzling aspect of this market's behavior: an apparent large conditional bias in the use of forward rates to forecast the future spot exchange rates.

If a market is said to be efficient, in equilibrium, all agents have access to all relevant information in the market and possibilities of excesses of returns by arbitrage are impossible.[1]

In an efficient market with rational expectations and that risk neutrality by the agents, the expected earnings in keeping a foreign currency more valued than the domestic will be compensated by the opportunity cost of maintaining foreign assets instead domestic assets.

The following expression,

$$1 + i_t = (1 + i_t^*)\frac{E_t \mathcal{E}_{t+1}}{\mathcal{E}_t}, \tag{1}$$

for UIP holds, where i_t is the domestic nominal interest rate; i_t^* is the equivalent foreign nominal interest rate, $\mathcal{E}_t$ and $E_t \mathcal{E}_{t+1}$ are, respectively, the spot exchange rate at time t and the expected future spot exchange rate evaluated in units of domestic currency. In all that follows, E_t represents the mathematical expectation conditional on information available to the market at time t.

Thus, in logs, and if expectations are rational, $\mathsf{e}_{t+1} - \mathsf{e}_t = E_t(\mathsf{e}_{t+1} - \mathsf{e}_t) + \epsilon_{t+1}$ with ϵ_{t+1} uncorrelated with time t variables representing the error on future spot exchange rate expectations. Using log-approximation $\log(1+x) \sim x$, the expression (1) becomes

$$E_t(\mathsf{e}_{t+1} - \mathsf{e}_t) = E_t \Delta \mathsf{e}_{t+1} = i_t - i_t^* \tag{2}$$

with $\mathsf{e}_t = \log(\mathcal{E}_t)$. Using the covered interest parity relation in logs $f_t = i_t - i_t^* + \mathsf{e}_t$, where f_t represents the log of exchange rate one period forward, the expression (2) can be rewritten as the forward discount version of UIP

$$\Delta \mathsf{e}_{t+1} = \alpha + \beta(f_t - \mathsf{e}_t) + \varepsilon_{t+1} \tag{3}$$

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[1] See Taylor (1995), for a very enlightening survey.

4

This regression appears frequently in the literature. Generally, the null hypothesis tested is $\alpha = 0$ and $\beta = 1$ assuming that $E_t[\varepsilon_{t+1} \mid \Omega_t] = 0$ and $E_t[\varepsilon_{t+1}(f_t - \mathsf{e}_t) \mid \Omega_t] = 0$ where Ω_t is the available information set in period t. Under this null hypothesis, f_t is an unbiasedness predictor of spot exchange rate one period forward.

However, empirical studies like Hansen and Hodrick (1980) and many others,[2] have produced strong evidence against the unbiasedness hypothesis showing an apparent large conditional bias in models which use forward rates to forecast the future spot exchange rates. A large volume of studies have found that currency devaluation are negatively correlated with the cross-country interest differentials, i.e. currencies with higher interest rates tend to appreciate. This outcome is often referred to as the forward discount anomaly. Indeed, according to Froot (1990), the mean estimate of β over 75 empirical works is -0.88, challenging on empirical grounds the unbiasedness hypothesis.

Because this widespread evidence poses a serious challenge to our understanding of international financial markets, the nature of this anomaly has been the focus of an enormous literature concerning the efficiency of these markets. We can mention three main lines of work aiming at explaining the FPP. First, the existence of a distortion on the component of rational expectations carried by exchange market participants turns the forward discount anomaly a rejection of the rational expectation hypothesis. Second, market imperfections as source of asymmetries in foreign exchange markets, which are supported by evidence of conditional-mean nonlinearities in risk premia. Third, the presence of a risk premium not appraised in the UIP expression.

Concerning the first research line, one can mention as example Rogoff (1979) who suggested that, in a context where agents attribute a low probability for rare nature states (for instance, great changes in economy foundations) an asymmetry will appear in a prior event distribution, the well-known *Peso Problem*. Froot and Frankel (1989) attributed the anomaly to expectation errors. In a recent work, Bacchetta and Van Wincoop (2006) developed a model of rational inattention that produces sticky prices and it is consistent with the FPP. The role of market imperfections in FPP explanation, as the second research line, could be seen, for instance, in Coakley and Fuertes (2001) where authors considered an extension of the Dornbusch exchange rate

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[2]Hodrick (1987) has surveyed a large number of works on efficiency tests of foreign exchange forward markets.

5

overshooting theory as a special case of asymmetric or nonlinear behavior in foreign exchange markets that fits the FPP.

The third line of research, perhaps the most natural explanation for why the forward premium predicts the wrong direction of exchange rate movements, focuses on a wedge between expected changes and actual changes driven by a risk premium. How to model the risk premium is the challenge of this literature and the purpose of our work. Hodrick (1987) concluded: "We do not yet have a model of expected returns that fits the data" (p.157). Engel (1996) provides a survey.

Many researchers tried to reproduce models of asset pricing that taking into account the forward premium correctly in the cross-country exchange market. Those models reproduced a negative slope coefficient when the theoretical spot rate changes are regressed on the theoretical lagged forward premium. See e.g. Backus et al. (1995), Bekaert (1996), Bekaert et al. (1997) and Macklem (1991). Unfortunately, many of these models cannot explain most of already mentioned foreign exchange market stylized facts.

A specific subgroup of rational models that departs from risk neutrality is that formed by consumption-based models. Unfortunately most of these models cannot reproduce many stylized facts from domestic asset markets, the most recognized of these failures being the Equity Premium Puzzle – EPP. In few words, it is based on the observation that in order to reconcile the much higher return on equity stock compared to US government bonds,[3] individuals must have implausible high risk aversion according to classical models. In fact, the consumption-based asset pricing model (CCAPM) with power utility fails to explain important facts about stock returns, including the high equity premium, the high volatility of returns and the countercyclical variation in the equity premium.[4]

However, consumption-based models have been resurrected for forward risk premia evaluations by appealing to more sophisticated preferences. Moore and Roche (2007) relies upon external habit preferences imbedding them into a monetary model. More recently, Verdelhan (2009) has also forwarded a

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[3]This feature prevails in many other industrialized countries, as pointed out by Kocherlakota (1996).

[4]See Grossman and Shiller (1981), Shiller (1981) and Kandel and Stambaugh (1990). For a deeper discussion about EPP, see Kocherlakota (1996) and Mehra and Prescott (2003). Both reviews present a detailed analysis of these explanations in financial markets and conclude that the puzzle is real and remains unexplained. Subsequent reviews of the literature have similarly found no agreed upon resolution.

model using external habit preferences, which led to quantitatively large risk premia and matched the variance of real exchange rates. The purpose of our work is very similar to the forthcoming work by Verdelhan. Contrary to him, our findings are not favorable to the model.

2.2 The null hypothesis in real terms

The condition for the absence of profit opportunities by a risk neutral domestic investor endowed with rational expectations from forward market speculation is

$$E_t \left(\frac{F_t - \mathcal{E}_{t+1}}{P_{t+1}} \right) = 0,$$

where P_{t+1} is the domestic price of goods and F_t is the foward rate level from exchange market.

For simplicity, assume that all variables above are conditionally lognormally distributed.[5] We may, in this case, rewrite the expression above as

$$E_t(\mathsf{e}_{t+1}) = f_t - 0.5 Var_t(\mathsf{e}_{t+1}) + Cov_t(\mathsf{e}_{t+1}, p_{t+1}).$$

Note that the risk premium $rp_t = f_t - E_t(\mathsf{e}_{t+1})$ for the risk neutral investor is not zero because the term $0.5 Var_t(\mathsf{e}_{t+1}) - Cov_t(\mathsf{e}_{t+1}, p_{t+1})$ is not zero. This term is the Jensen inequality term—JIT.[6] Thus, f_t is not necessarily a conditional predictor of e_{t+1} despite the very small absolute value of JIT. According to Engel (1996), most empirical works do not consider the JIT due to its very small size.

The true risk premium can, nonetheless, be calculated as follows

$$trp_t \equiv f_t - E_t(\mathsf{e}_{t+1}) - 0.5 Var_t(\mathsf{e}_{t+1}) + Cov_t(\mathsf{e}_{t+1}, p_{t+1}) = rp_t - JIT.$$

By the same token, a foreign risk neutral investor would require a risk premium given by

$$trp_t = f_t - E_t(\mathsf{e}_{t+1}) - 0.5 Var_t(\mathsf{e}_{t+1}) + Cov_t(\mathsf{e}_{t+1}, p_{t+1}^* + \mathsf{e}_{t+1}),$$

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[5] An analytically convenient assumption, very common in finance literature, though not very successful on empirical grounds.

[6] This subsection is based on a survey by Engel (1996), where the reader can find a deeper discussion about the Jensen inequality term.

where p_{t+1}^* is the log of the foreign price in foreign currency. The two expressions above will be the same when the purchasing power parity—PPP—condition holds, i.e.,

$$p_{t+1} = p_{t+1}^* + \mathsf{e}_{t+1}.$$

If PPP does not hold, then domestic and foreign investors evaluate real returns differently, so that there would be no equilibrium with risk neutral investors in each country.

The null hypothesis described in the latter subsection is equivalent to

$$rp_t = f_t - E_t(\mathsf{e}_{t+1}) = 0.$$

Considering the covered interest parity again and risk neutrality of domestic investor we can rewrite the null hypothesis as

$$i_t - i_t^* = E_t(\mathsf{e}_{t+1} - \mathsf{e}_t) + 0.5 Var_t(\mathsf{e}_{t+1}) - Cov_t(\mathsf{e}_{t+1}, p_{t+1}). \qquad (4)$$

When a foreign investor is assumed to be risk neutral, the null is

$$i_t - i_t^* = E_t(\mathsf{e}_{t+1} - \mathsf{e}_t) + 0.5 Var_t(\mathsf{e}_{t+1}) - Cov_t(\mathsf{e}_{t+1}, p_{t+1}^* + \mathsf{e}_{t+1})) \qquad (5)$$

So, we can express each the nulls (4), (5) as

$$i_t - E_t(p_{t+1} - p_t) = \left[i_t^* - E_t(p_{t+1}^* - p_t^*) \right] + E_t \left[(p_{t+1}^* + \mathsf{e}_{t+1} - p_t^* - \mathsf{e}_t) - (p_{t+1} - p_t) \right] + JIT \qquad (6)$$

where the second term in right side of (6) could be referred as PPP deviation term. Note that the JIT terms will be different between foreign and domestic investors, unless PPP holds.

Using the Fisher relation between nominal and real interest rates, then

$$r_t - r_t^* = E_t \left[(p_{t+1}^* + \mathsf{e}_{t+1} - p_t^* - \mathsf{e}_t) - (p_{t+1} - p_t) \right] + JIT.$$

Assuming there are no arbitrage opportunities in forward exchange market, the law of one price holds, then $p_t = p_t^*$, $\forall t$. Besides, following a large number of papers in this literature, we will assume that $JIT \approx 0$. Thus,

$$E_t \Delta \mathsf{e}_{t+1} = r_t - r_t^*.$$

Further in section 3, under complete markets assumption we will be able to rewrite the UIP expression in real terms

$$E_t \Delta q_{t+1} = r_t - r_t^* \qquad (7)$$

where q_{t+1} denotes the log of real exchange rate express in domestic goods.

3 The model

To depart from risk neutrality, one would like to have a model for the risk premium. We have chosen to use Campbell and Cochrane (1999) external habit model. The model is, in some sense, a reverse engineering procedure aimed at reproducing many of the stylized facts in domestic markets. Our idea is to calibrate the model as in Campbell and Cochrane (1999) to handle the behavior of US domestic asset markets, then, to assume complete foreign markets in order to make predictions about foreign exchange.

We start by presenting Campbell and Cochrane (1999)'s model.

3.1 An external habits pricing model

Campbell and Cochrane (1999) is a variant of presented by Abel (1990) external habit model. Identical investors have preferences over consumption that depend on a reference point X_t, the habit. Preferences are represented by a CRRA utility function,

$$U(C_t, X_t) = E\left[\sum_{t=0}^{\infty} \delta^t . \frac{(C_t - X_t)^{1-\gamma} - 1}{1 - \gamma}\right],$$

where $\delta > 0$ is the subject discount parameter of time preferences and $\gamma > 0$ is the utility curvature parameter.

It is convenient to model the behavior of habits indirectly through another variable called *consumption surplus ratio*, S_t, defined as

$$S_t \equiv \frac{C_t^a - X_t}{C_t^a},$$

where C_t^a is the aggregate consumption.

In equilibrium, given identical agents, $C_t^a = C_t$. So, the agent's coefficient of relative risk aversion rate is simply

$$-\frac{C_t u_{cc}}{u_c} = \frac{\gamma}{S_t}.$$

The expression above shows that S_t works as a proxy variable for economic recession periods, in which the risk aversion raises because S_t becomes low and vice-versa. This feature provides a time-variant risk aversion for investors which, as we are going to see soon, results in countercyclical risk premia for

9

asset returns. So, the agent's main concern is the decline of consumption relative to the external habits. This latter feature characterizes this model as a *Catching Up with the Joneses* model, to use the terminology of Abel (1990). This model differs, however, from Abel's model in two aspects. First, the agent's risk aversion varies with the level of consumption relative to habit, whereas risk aversion is constant in Abel's model. And, second, consumption must always be above habit for utility to be well defined, whereas this is not required in Abel's model.

To ensure that consumption is always above habit, the model specifies a non-linear process by which habit adjusts to consumption, remaining below consumption level at all times. We specify this process next.

Consumption growth is an i.i.d. log-normal process $\Delta c_{t+1} = g + v_{t+1}$, where $\Delta c_{t+1} = \log(C_{t+1}) - \log(C_t)$, g is the mean of the process and $v_{t+1} \sim i.i.d.\mathcal{N}(0, \sigma_v^2)$. The process for the log surplus consumption ratio, $s_t = \log(S_t)$, is assumed to be a heteroskedastic AR(1) model,

$$s_{t+1} = (1 - \phi)\overline{s} + \phi s_t + \lambda(s_t)v_{t+1}, \tag{8}$$

where $\overline{s}$ represents steady-state log consumption surplus, ϕ the habits persistence parameter and $\lambda(s_t)$ a sensitivity function to innovations in consumption growth v_{t+1}.

Notice that the process of s_t is heteroskedastic and perfectly conditionally correlated with innovations in consumption process. The sensitivity function is originally built to completely offset the intertemporal substitution and precautionary savings effects in Campbell and Cochrane (1999), thus making the real risk-free rate constant. However, we will specify $\lambda(s_t)$ so that the real risk-free rate is linear in s_t and $x_{t+1} \equiv \log X_{t+1}$ is a deterministic function of past consumption on a neighborhood of $\overline{s}$, $s_t \approx \overline{s}$.[7]

Taking a linear approximation around the steady state in (8) the model can be shown to be a traditional habit formation model in which log habit responds slowly to log consumption

$$x_{t+1} \approx \left[(1 - \phi)\ln(1 - \overline{S}) + g\right] + \phi x_t + (1 - \phi)c_t$$

$$= \left[\ln(1 - \overline{S}) + \frac{g}{1 - \phi}\right] + (1 - \phi)\sum_{i=0}^{\infty} \phi^i c_{t-i},$$

where $\ln(1 - \overline{S})$ is the steady-state value of $(x_t - c_t)$.

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[7]This specification is originally due to Campbell and Cochrane (1995).

These features imply that

$$\lambda(s_t) = \begin{cases} \frac{1}{\overline{S}}\sqrt{1 - 2(s_t - \overline{s})} - 1 & ; \quad s_t \leq s_{\max} \\ 0 & ; \quad s_t > s_{\max} \end{cases}, \tag{9}$$

$$\overline{S} = \sigma_v \sqrt{\frac{\gamma}{1 - \phi - \frac{b}{\gamma}}}, \tag{10}$$

and

$$s_{\max} = \overline{s} + \frac{1}{2}(1 - \overline{S}^2), \tag{11}$$

where b is a preference parameter that determines the behavior of interest rates and has an important economic interpretation that we shall explained soon.

Given that the s_{t+1} process is driven by (8), using (9), and knowing that habits are external, the marginal utility of consumption is given by $u'(c_t) = (C_t - X_t)^{-\gamma} = (S_t C_t)^{-\gamma}$. We, thus, obtain the pricing kernel

$$M_{t+1} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} = \delta e^{-\gamma[g - (1-\phi)(s_t - \overline{s}) + (1 + \lambda(s_t))v_{t+1}]}. \tag{12}$$

The reverse engineering nature of the sensitivity function is easily understood by taking into account that it was built to satisfy four properties: i) the domain of the pricing kernel is $\mathbb{R}_+$; ii) the natural logarithm of the risk-free rate, r_t^f, is linear in s_t; iii) the derivative of x_t with respect to c_t is zero at $\overline{s}$, and; iv) the second derivative of x_t with respect to c_t is zero at $\overline{s}$. Note that the last two properties implies that habits are predetermined at the steady-state and near it, so it moves non-negatively with consumption everywhere.

Using the fundamental pricing equation for any asset returns R_{t+1}

$$E_t\left[M_{t+1}R_{t+1}\right] = 1$$

and denoting $r_{t+1}^f \equiv \ln(R_{t+1}^f)$ as the one-period real risk-free rate, taking logs and using (12) and (8) the model delivers

$$r_t^f = \ln(1/E_t\left[M_{t+1}\right]) \tag{13}$$

$$= -\ln\delta + \gamma g - \gamma(1 - \phi)(s_t - \overline{s}) - \frac{(\gamma\sigma_v)^2}{2}(1 + \lambda(s_t))^2$$

11

The first two terms are familiar from the power utility. The third term reflects intertemporal substitution, or mean-reversion in marginal utility. If the surplus consumption ratio is low, the marginal utility of consumption is high. However, the surplus consumption ratio is expected to revert to its mean, so marginal utility is expected to fall in the future. Therefore, the consumer would like to borrow and this drives up the equilibrium risk-free rate. The fourth term reflects precautionary savings. As uncertainty increases, consumers become more willing to save and this drives down the equilibrium risk-free rate.

Although (13) is, in fact, an approximation, for it is derived under the assumption that there is zero probability that s_t exceeds $s_{\max}$, the approximation is highly accurate, as discussed by Wachter (2005).

Substituting $\lambda(s_t)$ from (9) in (13) we obtain

$$r_t^f = -\ln\delta + \gamma g - \frac{\gamma(1-\phi) - b}{2} - b(s_t - \overline{s}). \tag{14}$$

As mentioned before, b ascribes important economic interpretations to the model. If $b > 0$, the intertemporal smoothing effect dominates the precautionary savings effect and an increase in the surplus consumption ratio, s_t, drives down the interest rate, so that interest rates are anti-cyclical. If $b < 0$, the precautionary savings effect dominates and an increase in the surplus consumption ratio, s_t, reduces the sensitivity function and drives up the interest rate so that interest rates are pro-cyclical. Setting $b = 0$ results in constant real interest rates because the two effects cancel each other and drives the results presented in Campbell and Cochrane (1999).

While the functional form of $\lambda(s_t)$ is chosen to match the behavior of the risk-free rate, it has important implications for returns on risky assets too. It follows from Euler equation that the Sharpe ratio of any asset return must obey

$$\frac{E_t(R_{t+1}^e)}{\sigma_t(R_{t+1}^e)} = -\rho_t(M_{t+1}, R_{t+1}^e)\frac{\sigma_t(M_{t+1})}{E_t(M_{t+1})}$$

where $R_{t+1}^e = R_{t+1} - R_{t+1}^f$ and ρ_t denotes conditional correlation.

As M_{t+1} is lognormally distributed, we have the following approximation

using (12),[8]

$$\frac{E_t(R^e_{t+1})}{\sigma_t(R^e_{t+1})} \approx -\gamma \rho_t(M_{t+1}, R^e_{t+1})\sigma_v \left(1 + \lambda(s_t)\right). \qquad (15)$$

Because $\lambda(s_t)$ is decreasing in s_t, the ratio of the volatility of the stochastic discount factor with respect to its mean varies countercyclically. This provides a mechanism by which Sharpe ratios and risk premia vary countercyclically over time.

Campbell and Cochrane models stocks as a claim to the consumption stream, taking stocks to represent the wealth portfolio.[9] Using the Euler equation, one can verify that the price-consumption ratio for a consumption claim satisfies

$$\frac{P_t}{C_t}(s_t) = E_t \left[M_{t+1} \frac{C_{t+1}}{C_t} \left[1 + \frac{P_{t+1}}{C_{t+1}}(s_{t+1}) \right] \right], \qquad (16)$$

where P_t denotes the ex-dividend price of this claim.

Note that s_t is the only state variable in expression above. The model is solved by substituting the stochastic discount factor expression (12) and endowment process $\exp(\Delta c_{t+1}) = \exp(g + v_{t+1})$ into (16) and solving it by numerical integration for a grid of s_t over the normally distributed shock v_{t+1}. This allows one to evaluate conditional expectations and to determine the price-consumption ratio fixed-point. Then, we must calculate expected and conditional standard deviations of returns to match calibration parameters over the real data.

Evaluation of simulated stochastic discount factors and, consequently, risk-free rates, are easier. We simulate the model by drawing consumption shocks v_{t+1} and feeding (8) with these draws. In this fashion, we obtain draws for s_t and use them to attain the simulated risk-free rate and stochastic discount factor.

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[8] Assume $\log M \sim iid(\mu, \sigma^2)$. Then,

$$\frac{\sigma(M)}{E(M)} = \sqrt{\frac{E(M^2) - E(M)^2}{E(M)^2}} = \sqrt{\frac{e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2}}{e^{2\mu+2\sigma^2}}} = \sqrt{e^{\sigma^2} - 1} \approx \sigma$$

[9] This strong assumption represents an extra challenge to the model since, in their callibration exercise, stock prices are generated by the model.

3.2 Exchange rates

Some extra notation is needed to introduce foreign markets. We following the presentation of Lustig and Verdelhan (2006) and Lustig and Verdelhan (2007). Let $\mathcal{E}_t^i$ be the nominal exchange rate defined at amount of domestic currency that one must pay for a unit of country i currency as described in Section 2, and $R_{t,t+1}^{f,i}$, the one-period return of the risk-free rate in country i currency units. We shall associate this latter with the nominal interest rate that drives the returns of a one-year discount bond. Let finally $R_{t,t+1}^{i,\$}$ denotes the domestic investor exchange risk return from buying a foreign one-year discount bond in country i, selling the payoff - one unit of foreign currency - after one year and converting the proceeds back into domestic currency. Using this notation the following expression obtains

$$R_{t,t+1}^{i,\$} = R_{t,t+1}^{f,i} \left(\frac{\mathcal{E}_{t+1}^i}{\mathcal{E}_t^i} \right).$$

Assuming no-arbitrage in bond markets, absence of frictions (e.g. *bid-ask spreads* and *short-sale constraints*), and market completeness, we have:

1. Domestic investors who acquire a foreign one-year discount bond in country i have the following Euler equation:

$$E_t \left[M_{t+1} R_{t,t+1}^{i,\$} \left(\frac{P_t}{P_{t+1}} \right) \right] = E_t \left[M_{t+1} R_{t,t+1}^{f,i} \left(\frac{\mathcal{E}_{t+1}^i}{\mathcal{E}_t^i} \right) \left(\frac{P_t}{P_{t+1}} \right) \right] = 1 \tag{17}$$

where

$$R_{t,t+1}^{i,\$} \left(\frac{P_t}{P_{t+1}} \right)$$

denotes the return in domestic consumption units from foregoing investment and P_t the price of a domestic consumption unit in period t. By the same token, the Euler equation for a foreign investor's who made the same investment in country i bonds is

$$E_t \left[M_{t+1}^i R_{t,t+1}^{f,i} \left(\frac{P_t^i}{P_{t+1}^i} \right) \right] = 1, \tag{18}$$

where P_t^i denotes the price of a country's i consumption unit in period t.

14

2. If no-arbitrage assumption holds it implies the law of One Price. Consequently,

$$\frac{P_t}{P_{t+1}} = \frac{P_t^i}{P_{t+1}^i}$$

and we obtain the equivalence between nominal and real exchange rates, i.e.:

$$\frac{\mathcal{E}_{t+1}^i}{\mathcal{E}_t^i} = \frac{Q_{t+1}^i}{Q_t^i},$$

where Q_t^i denotes the real exchange rate express in domestic goods.

3. Because markets are complete, there is only one stochastic discount factor in each country,[10] which allows us to match (17) and (18). Hence,

$$\frac{Q_{t+1}^i}{Q_t^i} = \frac{M_{t+1}^i}{M_{t+1}}.$$

Taking logs in expression above we obtain

$$\Delta q_{t+1}^i = m_{t+1}^i - m_{t+1}. \tag{19}$$

Interpreting $log(1 + R_{t,t+1}^{i,\$}) = r_t^i$ - the country's i one-year discount bond log-return - as the abroad interest rate, substituting (7) above and considering that $r_t^i = -E_t m_{t+1}^i$, $\forall i$, we attain

$$E_t \Delta q_{t+1}^i = r_t - r_t^i \tag{20}$$

the expression that represents UIP for real terms.

3.3 A necessary and sufficient condition to reproduce the FPP

Here, we will show conditions under which the Campbell and Cochrane external habits model is able to reproduce a negative slope coefficient in regression

$$\Delta q_{t+1}^i = (m_{t+1}^i - m_{t+1}) = \alpha + \beta^{UIP}(r_t - r_t^i) + \epsilon_{t+1} \tag{21}$$

where ϵ_{t+1} is the regression error.

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[10]See Cochrane (2001), chapter 4.

Finding $\beta^{UIP} < 0$ in simulated data would be an evidence that the FPP is a phenomenon, at least in part, explained by a model that fits coherently the risk premia. In our environment it is necessary to force interest rates to be pro-cyclical. That is we set the preference parameter that determines the interest rates behavior, b, to be negative. To show this dependence, we begin explaining how the model generates exchange risk premia.

The exchange risk premium is the excess returns obtained by an investor that makes the following operation: i) borrows resources in domestic currency to purchase foreign bonds; ii) converts these resources into foreign currency; iii) buy bonds and receives the yields at foreign risk-free rates; iv) resells the bond after a reference period, converting it again into domestic currency.

Excess returns can be described by three components: the real exchange rate de/appreciation during the income period; the yield rate from foreign country; and the opportunity cost of not having acquired a domestic bond. In logs we obtain:

$$E_t r^e_{t+1} = E_t \Delta q^i_{t+1} + r^i_t - r_t. \tag{22}$$

Note that if there is no risk premium, the expression (22) is given by the UIP real version (20). So, given the log-normality of stochastic discount factor (12) we obtain

$$\log R^{f,i}_{t,t+1} = r^i_t = -\log E_t M^i_{t+1} = -E_t m^i_{t+1} - \frac{1}{2} Var_t(m^i_{t+1}).$$

Using the previous expression to evaluate domestic interest rate r_t, we can express $r_t - r^i_t$ as

$$r_t - r^i_t = E_t m^i_{t+1} - E_t m_{t+1} + \frac{1}{2} \left[Var_t(m^i_{t+1}) - Var_t(m_{t+1}) \right].$$

In section 3.2, we have shown that $\Delta q^i_{t+1} = m^i_{t+1} - m_{t+1}$. Using this expression, we obtain the expectations form of (22),

$$E_t \Delta q^i_{t+1} = (r_t - r^i_t) - \frac{1}{2} \left[Var_t(m^i_{t+1}) - Var_t(m_{t+1}) \right],$$

or

$$E_t r^e_{t+1} = \frac{1}{2} \left[Var_t(m_{t+1}) - Var_t(m^i_{t+1}) \right]. \tag{23}$$

In Campbell and Cochrane's model,

$$Var_t(m_{t+1}) = (\gamma \sigma_v)^2 \left[1 + \lambda(s_t) \right]^2 = \frac{(\gamma \sigma_v)^2}{\bar{S}^2} \left[1 - 2(s_t - \bar{s}) \right]. \tag{24}$$

Henceforth, we will adopt a simplifying assumption by imposing the same model parameters $(g, \sigma_v, \gamma, \phi, \overline{S})$ for home and foreign countries. Given that s_t comes from stochastic process, it allows for each country to have its own s_t. Substituting (22), (24) in (23), the exchange risk premia may be found as

$$E_t \Delta q_{t+1}^i = \left(r_t - r_t^i \right) + \frac{(\gamma \sigma_v)^2}{\overline{S}^2}(s_t^i - s_t) \tag{25}$$

As we have seen in (14), the model delivers linear interest rates and is straightforward to see that $r_t - r_t^i = b(s_t^i - s_t)$. So, the expression above could be rewritten as

$$E_t \Delta q_{t+1}^i = \left[1 + \frac{(\gamma \sigma_v)^2}{b \overline{S}^2} \right] (r_t - r_t^i).$$

Finally, replacing $\overline{S}$ with (10), we obtain

$$E_t \Delta q_{t+1}^i = \frac{\gamma (1 - \phi)}{b}(r_t - r_t^i).$$

This calculation is found in Verdelhan (2009).

Note that the coefficient

$$\beta^{UIP} = \frac{\gamma (1 - \phi)}{b}$$

must be negative to reproduce empirical findings in UIP estimation. This can be accomplished setting $b < 0$. Verdelhan gives a nice intuition to why pro-cyclical interest rates causes $\beta^{UIP} < 0$ in Campbell and Cochrane model.

In periods in which economy does poorly, domestic investors are more risk averse than foreign investor, so $s_t < s_t^i$. This delivers a higher conditional variance of stochastic discount factor at home than abroad as it can be seen in (24), due to $\lambda'(s_t) < 0$. So, the domestic consumption shocks effects on the real exchange rate dominate their foreign counterparts shocks. This relation can be seen by merging (12) and (19),

$$\Delta q_{t+1}^i = \gamma(1 - \phi)(s_t^i - s_t) + \gamma[1 + \lambda(s_t)](\Delta c_{t+1} - g) - \gamma[1 + \lambda(s_t^i)](\Delta c_{t+1}^i - g).$$

The exchange rate decreases in response to a domestic negative consumption growth shock and *vice-versa*. Foreign currency is riskier the more risk averse the domestic investor relative to his foreign counterpart. Then, a

17

greater risk premium is required by domestic investors for them to hold foreign bonds. As a result, the domestic investor gets a positive excess return if he is more risk averse than his foreign counterpart. Further, times of high risk aversion correspond to low interest rates. In other words, when s_t is low enough and is imposed pro-cyclical interest rates ($b < 0$) the equation (14) provides low interest rates.

Summarizing, domestic investors expects positive foreign bonds excess returns when two situations happen at the same time: they are more risk averse than their foreign counterparts and domestic interest rates are low and foreign ones are high. This results in $\beta^{UIP} < 0$. As we have seen, the Campbell and Cochrane model guarantees that these two situations always happened simultaneously when interest rates runs pro-cyclically.

3.4 Business cycle behavior of real interest rates

Interest rates primarily depend on policy and expectations, thus the relationships with the business cycle depend on explicit decisions and subjective judgements of key players (Central Banks, Holders, etc...). At this moment, it is important to verify stylized facts about real interest rates to establish if the main hypothesis in last subsection is reasonable, i.e., if real interest rates keep up with economic cycles.

Ang and Bekaert (2002) summarized a set of stylized facts about nominal and real interest rates and inflation that are seen commonly in empirical papers. We quote them:

> Interest rates are often associated with the business cycle. According to the conventional wisdom, interest rates are pro-cyclical and spreads counter-cyclical (see, for example, Fama (1990)). In fact, interest rates are overall larger during recessions. However, when we focus on real rates, the conventional story is right.

The same conclusion is found in Veronesi and Yared (1999).[11,12]

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[11] In the work quoted above, Fama (1990), shows that the one-year U.S. interest rate is lower at the business trough than at the preceding or following peak in every business cycle of the 1952-1988 period.

[12] Other policy rules would imply different behavior. For instance, if the target is mainly inflation, during a stagflation period (a depressed GDP with high inflation) the interest rates may be particularly high. Thus, a counter-cyclical pattern would emerge.

4 Methodology and Results

In what follows, we present our results and our interpretation of their meaning for the model's capacity of accommodating the forward premium puzzle.

4.1 Creating fictitious countries

As in subsection 3.3, we assume that each pair, US/fictitious country i, can be characterized by the same set of parameters $(g, \sigma_v, \gamma, \phi, b)$. We draw values for v_{t+1} to form the US endowment shock process Δc_{t+1} and use it to generate the other country's consumption processes by setting their correlation ρ^i in the following manner:

$$\Delta c_{t+1}^i = \rho^i \Delta c_{t+1} + \sqrt{1 - (\rho^i)^2} \Delta c_{t+1} \tag{26}$$

where Δc_{t+1}^i is the consumption process for fictitious foreign country "i".

There is a subtle difference in these two simulated variables Δc_{t+1} presented in (26). Both represents the same US consumption process but are generated by different *seeds* in their Normal random process.[13] This assures that the two process are instantaneously different from one another, which allows us to create Δc_{t+1}^i with a controlled correlation value ρ^i. We input 21 different correlation values equally spread between -1 and 1. So, we have 21 different countries to evaluate exchange rates. Note that (26) did not attribute correlation values exactly as those we have input but they become very alike as the number of v_{t+1} draws grows.

4.2 Calibration

The task here is to calibrate the external-habit asset pricing model to US post-war data.[14] Following Campbell and Cochrane (1999), Wachter (2005) and Verdelhan (2009), we fix the intertemporal elasticity, γ, to 2. For other model parameters we adjust them to fit the mean and volatility of US consumption process, 90-day T-Bill mean rate, as the US risk-free asset, and

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[13]**Seed:** an integer used to set the starting point for generating a series of random numbers. The seed sets the generator to a random starting point. A unique seed returns a unique random number sequence.

[14]We are avoiding the well acknowledged structural break between pre and post-war data.

average equity excess returns. The parameter ϕ is the first-order autocorrelation of price-consumption ratio verified in the data. It determines the speed of mean reversion exhibited by the s_t stochastic process described in (8).

The preference's free parameter b is quite difficult to determine and has been shown to be very controversial. Its absolute value does not make much difference in equity returns, as Wachter (2006) has pointed out, but its sign determines the slope of the term structure of interest rates generated by the model. In order to generate an upward-sloping yield curve and anti-cyclical interest rates, it is necessary that $b > 0$, i.e., the risk-free rate loads negatively on b and, consequently, negatively correlated with surplus consumption ratio. Wachter (2006) estimate $b = 0.011$ for US data in the 1952-2004 quarterly sample.[15]

However, in contrast to what is claimed by Wachter, there is also large empirical evidence in favor of pro-cyclical interest rates, as we have already discussed. Indeed, Ma (2006) uses pro-cyclical interest rates to investigate the macroeconomic conditions that accompany high interest rates and currency appreciation by observing the cyclical behavior of cross-country interest differentials and exchange rate movements.

4.3   Data sources

Annual US population and consumption expenditures data are from Robert Shiller's database between 1947 and 2004.[16] We use the return of 90-day Treasury bill deflated by the CPI index as our measure of US real interest rate. Stock market excess returns, Treasury bill rates and CPI are from CRSP.

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[15]Wachter (2006) conducted a brief investigation of real risk-free rate time-series implications. She ran the following regression

$$r_{t+1}^f - \Delta\pi_{t+1} = a_0 + a_1 \sum_{k=1}^{40} \phi^k \Delta c_{t-k} + \varepsilon_{t+1},$$

where the regressed variable is the ex-post real interest rate and the regressor is a consumption surplus ratio proxy for quarterly simulation, which is approximately equal to s_t. Data is quarterly, beginning in the second quarter of 1952 and ending in the second quarter of 2004.

[16]http://www.econ.yale.edu/~shiller/data/chapt26.xls

4.4 Simulation

We simulate 100,000 quarters of data. To check the calibration of US data, the price-consumption ratio and, consequently, stock market returns and real yields we use a numerical algorithm based on a fixed-point evaluation of Euler equation (16), as in Campbell and Cochrane (1999).[17] For numerical procedures, we have used a grid of state variable S_t with 24 grid points, where 15 of them are equally distributed between $(0, S_{max}]$, 4 additional points were added at intervals of 0.01 slightly bellow S_{max} and 5 other discretional points closer to zero.[18]

4.5 Is it possible to impose $b < 0$?

We have imposed $b < 0$ in the calibration, which we have already shown, in section 3, to be necessary and sufficient to reproduce a negative value for β^{UIP}. However, we were not capable of finding a finite value for the price-consumption ratio. The price-consumption ratio diverges for any reasonable set of calibration parameters when b is negative. Thus, we did not get to match other real data such as mean and volatility of equity returns, risk-free rate, etc..

To show the robustness of our calculations and rule out the possibility of computational errors we ran our algorithm using the calibration parameters of two well-known works: Campbell and Cochrane (1999) and Wachter (2005). Given that we are capable to find the fixed point of (16) only when $b \geq 0$, these papers are useful benchmarks for a robustness test, because b is non-negative in both works. The annualized parameters for each paper are summarized in table bellow.

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[17]To check the robustness of our results, we have also computed the equilibrium price-consumption ratio through the series algorithm proposed by Wachter (2005). All results are similar to the results found by the method of fixed point used in this paper.

[18]By discretional points we mean [.0005 .0015 .0025 .0035 .0045], as in Wachter (2005). We added additional density on grid ends following Campbell and Cochrane's advice. They argue that this procedure will improve numerical fixed-point calculation in artificial data.

Parameter	Symbol	Cochrane	Wachter
Used:			
Consumption growth mean	g	1.89	2.20
Consumption growth volatility	σ_v	1.5	0.86
Risk-free rate	r^f	0.94	–
Habits persistence	ϕ	0.87	0.89
Utility curvature	γ	2	2
Interest behavior coeff.	b	0	0.011
Derived parameters:			
Intertemporal discount factor	δ	0.89	0.93
Steady consumption surplus	$\overline{S}$	0.057	0.04
Max. consumption surplus	$S_{\max}$	0.094	0.07

Tab. 1 - Calibration parameters from Campbell and Cochrane (1999) and Wachter (2005)

Feeding our program with these parameters we reach the following results.[19]

Campbell and Cochrane (1999) - C&C

Statistic	C&C	Simulated
$E(\Delta c_{t+1})$ (%)	1.89	1.90
$\sigma(\Delta c_{t+1})$ (%)	1.22	1.20
$E(r^f)$ (%)	0.94	0.94
$\sigma(r^f)$ (%)	0.00	0.00
$E(r^m)$ (%)	6.64	6.68
$\sigma(r^m)$ (%)	15.20	15.08
Sharpe ratio	0.44	0.44
$E(p-d)$	2.91	2.90
$\sigma(p-d)$	0.27	0.27
$Corr(p-d)$	0.87	0.87

Tab. 2 - Comparison between simulation results from Campbell and Cochrane (1999)
and our computational calculations at same calibration parameters set.

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[19]In Campbell and Cochrane (1999), the authors have simulated 500,000 months of artificial data. We adopted the same procedure. Later, 400,000 quarters of artificial data were generated. Wachter (2005) has also used many different grid sets. Here, we used as benchmark the second grid set suggested in her work.

Wachter (2005)

Statistic	Wachter	Simulated
$E(\Delta c_{t+1})$ (%)	2.20	2.20
$\sigma(\Delta c_{t+1})$ (%)	~ 0.86	0.71
$E(r^f)$ (%)	1.47	1.46
$\sigma(r^f)$ (%)	–	1.91
$E(r^m - r^f)$ (%)	5.43	5.25
$\sigma(r^m - r^f)$ (%)	16.07	15.53
Sharpe ratio	0.34	0.34
$E(p-d)$	3.10	3.15
$\sigma(p-d)$	0.31	0.32
$Corr(p-d)$	0.89	0.89

Tab. 3 - Comparison between simulation results from Wachter (2005).

Taking into account the fact that in a computer simulation there are many variables that we cannot match exactly, for instance: virtual random machine used in normal v_{t+1} draws, decimal number approximation rules and so many others issues that make two computer simulations different from one another, we do not believe that program errors underlie the divergence that we found with $b < 0$. Our program seems appropriate as we can see for its ability to reproduce finely the results in Campbell and Cochrane (1999) and Wachter (2005).

Unfortunately, we do not have a reasonable explanation as of why we cannot reach finite values for the price-consumption ratio when interest rates are increasing in s_t. Technically, by focusing on (24) one can see that the absolute value of $\lambda(s_t)$ drives positively the pricing kernel volatility. We can see in expression (10) that a negative b decreases the steady-state consumption surplus ratio $\overline{S}$ and, therefore, reduces $S_{\max}$. Thus, the state-variable grid is filled with lower values. Given that λ is decreasing in s_t, the sensitivity function will be higher than when a positive b is set on consumption surplus grid points raising the stochastic discount factor volatility. How this increased volatility produces a divergence of our calibration procedure is of as this moment still not understood.

In another effort to understand the computational problems with $b < 0$, we have used an approach which is similar to the one proposed by Campbell and Cochrane (1995), footnote 10. When we impose $b < 0$, the risk free rate is pro-cyclical and a linear increasing function of consumption surplus. In

this case, if we fix an upper bound for the risk-free rate,[20] we were able to find a closed expression for b in terms of γ, ϕ, σ_v, r_0^f and the upper bound using (14) as follows:

$$r^f(s_{\max}) = r_0^f - b\left[s_{\max} - \overline{s}\right] \leq r_{upper\ bound}.$$

We then created three grids for (γ, ϕ, σ_v) [21] and used all possible combinations of these parameters to find which ones have implied negative values for b. Our outcomes point to the fact that there is a very narrow range of parameters $(0,98\%)$ that delivers $b < 0$. Moreover, all parameters combinations which yielded $b < 0$ displayed $\phi = 1$, implying that the consumption surplus is a random-walk. This feature creates many difficulties to accommodate asset pricing phenomena other than the one the model is aiming at matching.

These results reinforce our perception that Campbell and Cochrane's consumption-based model does not deal with pro-cyclical interest rates, at least in a well-behaved way, i.e. with $\phi < 1$. To overcome this difficulty, Verdelhan (2009) proposed an *ad hoc* approach to the original external habits model, which consists in making the sensitivity function, λ, described in (9), constant, at least during the price-consumption ratio computation from (16). The approach and its consequences will be the issue of our next subsection.

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[20] *Here, we chose high values for upper bounds in way to guarantee that the model's curve of the risk free return has positive values in all grid points. For instance, we have used never reached values 5%, 10%, and 15% by the US real interest rate in our sample data.*

[21] We established grids for (γ, ϕ, σ_v) using 50 points equally-spaced, respecting each one of the following intervals $\sigma_v(\%) \in [0, 10]$; $\gamma \in (0, 20]$; $\phi \in [0, 1]$ and set r_0^f at 0,9%.

4.6 Verdelhan's assumption

How is it possible to reconcile our negative results with Verdelhan's well succeeded use of Campbell and Cochrane's approach? That is, how did Verdelhan succeed in finding a finite value for the consumption-price ratio while at same time imposing $b < 0$? The answer lies in the sensitivity function, $\lambda(s_t)$. To find a closed form expression for the risk-free volatility, Verdelhan (2009) assumes that $\lambda(s_t) \approx \lambda(\bar{s})$, i.e., $\lambda(s_t)$ is a constant. With a constant λ no divergence problem arises and we can calibrate the model.[22]

After finding an appropriate price-consumption ratio, we release the sensitivity function λ in the simulations to vary as the original model does. In this manner, we are ensuring that risk free rate is pro-cyclical and, consequently, the UIP slope coefficient in simulated data is negative. In contrast, with a fixed λ, the risk fee rate is driven by $r_t^f = -\gamma(1-\phi)(s_t-\bar{s})+\kappa$, where $\kappa \equiv -\ln\delta + \gamma g - .5(\gamma\sigma_v)^2 (1 + \lambda(\bar{s}))^2$. Because $\gamma(1-\phi) > 0$, r_t^f is still decreasing in s_t, the model would not generate a negative β^{UIP}. Also, fixing λ in simulation stage implies that risk premium no longer varies counter-cyclically. This can be verified by examination of the Sharpe ratio expression for risky asset returns, (15).

To summarize, the procedure is potentially problematic because we are computing the price-consumption ratio under a stochastic discount factor that leads to anti-cyclical risk free rates although we intend to set interest rates pro-cyclically. We guess that this shortcut may generate inconsistencies for the model, mainly in realized equity returns that are driven by price-consumption's time series.[23]

Instead of focusing on this potential source of inconsistencies we shall focus on other aspects of the model's output. So, we pursue the strategy of fixing λ in the calibration part of the exercise, and letting it vary in the simulations.

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[22]Note,that, if one does not impose constancy of λ, $\lambda(.)$ exhibits significant variation in s_t. For instance, in the parameter set used in Campbell and Cochrane's calibration parameters, between the first and last s_t grid terms, we have a difference roughly of 38 in λ values.

[23]Realized equity returns are computed as follows

$$R_t = \frac{C_{t+1}}{C_t} \left(\frac{\frac{P_{t+1}}{C_{T+1}}(s_{t+1})}{\frac{P_t}{C_t}(s_t)} \right),$$

so, any price-consumption inconsistency will affect realized returns.

By running the calibration parameters from table II of his working paper, our program has been successful in reproducing his simulation results. Verdelhan draws 10,000 endowment shocks to create artificial quarterly data. Unfortunately, we do not know which state variable grid format was assumed in his working paper, thus little differences between his results and ours could happen at his parameters set.[24]

Verdelhan (2009)'s [25]

Calibration parameters	Verdelhan		Statistic	Verdelhan	Simulated
g	2.12		$E(\Delta c_{t+1})$ (%)	2.13	2.13
σ_v	1.02		$\sigma(\Delta c_{t+1})$ (%)	1.04	0.84
r^f	1.36		$E(r^f)$ (%)	1.65	1.40
ϕ	0.96		$\sigma(r^f)$ (%)	2.54	2.35
γ	2		$E(r^m - r^f)$ (%)	3.98	3.93
b	-0.01		$\sigma(r^m)$ (%)	8.72	13.62
Implied parameters			$Sharpe\ ratio$	0.46	0.29
δ	1.00		$E(p-d)$	3.44	3.60
$\overline{S}$	0.07		$\sigma(p-d)$	0.49	0.49
$S_{\max}$	0.12		$Corr(p-d)$	0.97	0.97

Tab. 4 - Comparison between simulation results from Verdelhan (2009) working paper and our computational calculations at same annualized calibration parameters set.

---

[24]How much the use of different grids could change simulation results in this framework may be seen in Wachter (2005). The author uses 3 different grid types and summarizes her simulation results in tables 2 and 3 where great differences can be seen.

[25]Notice that the difference in simulated consumption growth deviations displayed here and in Wachter (2005)'s comparison exercise is due the Campbell and Cochrane's annualizing procedure. Put differently, if we would evaluate the deviation from quarterly artificial data and later multiply it by $\sqrt{4}$, we could match Wachter and Verdelhan's value for annualized $\sigma(\Delta c_{t+1})$. As evidence, in table 5 we performed this same comparison exercise but using the latter annualizing procedure.
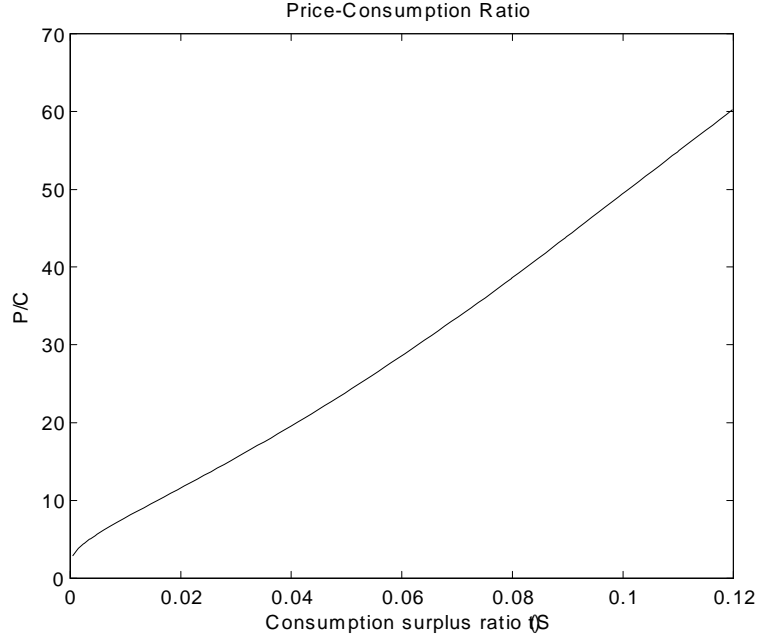
Fig. 1 - Price-consumption ratio simulated under Verdelhan (2009)'s calibration parameters and a fixed sensivity function. Although fixing the sensivity function in calibration stage seems to be a misleading procedure, it is display a well-behaved price consumption ratio after all.

Our numerical exercise seems to match all results from Verdelhan (2009), but equity returns volatility, and, consequently, the Sharpe ratio. However, we ought to highlight the low numbers of draws used by Verdelhan (10,000), which can make such results less reliable. To see this, we ran the model with the same calibration parameters used in latter comparison exercise but different seeds (See footnote 13). There are significant differences in such results as one can see in table 5. Whether one is matching or not the results becomes a more fuzzy matter.

Statistics	Verdelhan	seed1	seed2	seed3	seed4	seed5
$E(\Delta c_{t+1})$ (%)	2.13	2.10	2.06	2.10	2.12	2.15
$\sigma(\Delta c_{t+1})$ (%)	1.04	1.04	1.02	1.03	1.02	1.02
$E(r^f)$ (%)	1.65	1.13	0.72	0.99	1.24	1.93
$\sigma(r^f)$ (%)	2.54	2.07	2.51	2.51	2.32	1.47
$E(r^m - r^f)$ (%)	3.98	4.32	5.10	4.66	4.17	2.82
$\sigma(r^m)$ (%)	8.72	14.08	14.49	13.78	13.48	12.02
$Sharpe\ ratio$	0.46	0.31	0.34	0.34	0.31	0.25
$E(p - d)$	3.44	3.53	3.44	3.51	3.57	3.72
$\sigma(p - d)$	0.49	0.49	0.54	0.55	0.52	0.47

Tab. 5 - Comparison between simulation results from Verdelhan (2009) working paper
using 10,000 draws for consumption artificial data with different random seeds.

Nevertheless, the equity returns volatility found in simulations are consistently higher than his. Apparently, this is the only difference between the results in exercises using our algorithm and those elsewhere, given that we have succeeded in replicating Campbell and Cochrane (1999) and Wachter (2005) results and almost all statistics from Verdelhan's paper.

Now, assuming that λ is constant and equal to its steady-state value, $\lambda(\bar{s})$, allows us to calibrate the model and match the US real data.[26] The parameter values and simulation results are described in tables 6, below.

Calibration parameters		Statistic	Real data	Simulated
g	2.19	$E(\Delta c_{t+1})$ (%)*	2.21	2.21
σ_v	2.02	$\sigma(\Delta c_{t+1})$ (%)*	1.73	1.73
r^f	0.98	$E(r^f)$ (%)*	1.02	1.02
ϕ	0.931	$\sigma(r^f)$ (%)	2.96	2.16
γ	2	$E(r^m - r^f)$ (%)*	6.27	6.27
b	−0.01	$\sigma(r^m - r^f)$ (%)	15.15	17.32
Implied parameters		$Sharpe\ ratio$	0.41	0.36
δ	0.95	$E(p - d)$	3.33	3.11
$\overline{S}$	0.095	$\sigma(p - d)$	0.44	0.45
$S_{\max}$	0.156	$Corr(p - d)$	0.915	0.93

[26]Setting $\lambda(s_t) = \lambda(\bar{s})$ is just an intuitive choice. Any other constant value, $\bar{\lambda}$, results in model convergence. However, the choice of $\bar{\lambda}$ affects dramatically the equity returns volatility. Indeed, for $\lambda(\bar{s})$ values above 30 the equilibrium price-consumption ratio seems to blow up despite still converging.

Tab. 6 - Annualized calibration parameters set and simulation results chosen to match 1947-2004 US consumption, price and equity real data. Statistics that calibration parameters were chosen to replicate.

The UIP coefficients we found confirms that the model delivers a consistent negative bias, whatever correlation between consumption processes of different nations is used. Therefore, the *ad hoc* Campbell and Cochrane model version proposed by Verdelhan is able to reproduce the FPP. Its values are summarized below.

UIP Coefficients				
ρ^i	β^{UIP}		ρ^i	β^{UIP}
-1.0	-3.2444		0.9	-4.4325
-0.9	-3.3499		0.8	-4.8277
-0.8	-3.4146		0.7	-5.0623
-0.7	-3.4911		0.6	-5.0707
-0.6	-3.5485		0.5	-4.9533
-0.5	-3.6016		0.4	-4.8047
-0.4	-3.6726		0.3	-4.6212
-0.3	-3.7593		0.2	-4.4344
-0.2	-3.8693		0.1	-4.2607
-0.1	-3.9948		0.0	-4.0883

Tab.7 - Respective UIP coefficients in relation to ρ^i. All values are significant under an 95% confidence interval. The level coefficient α was statistically null in every regression.

What consequences this assumption about the sensitivity function will have for other asset prices? Is it a reasonable shortcut?

In their original paper, Campbell and Cochrane (1999) built the sensitivity function λ to reach 3 main goals: *i)* to make risk-free rate linear in the state variable s_t, *ii)* to guarantee that habits are always lower than consumption, and; *iii)* to impose a non-negative co-movement between habits and consumption. The first objective is attained and can be seen in (14). For our calibration parameters, the latter two objectives are intact as can be seen in figure below. So, up to this moment, imposing a constant sensitivity function does not hurt the stated goals of Campbell and Cochrane (1999).
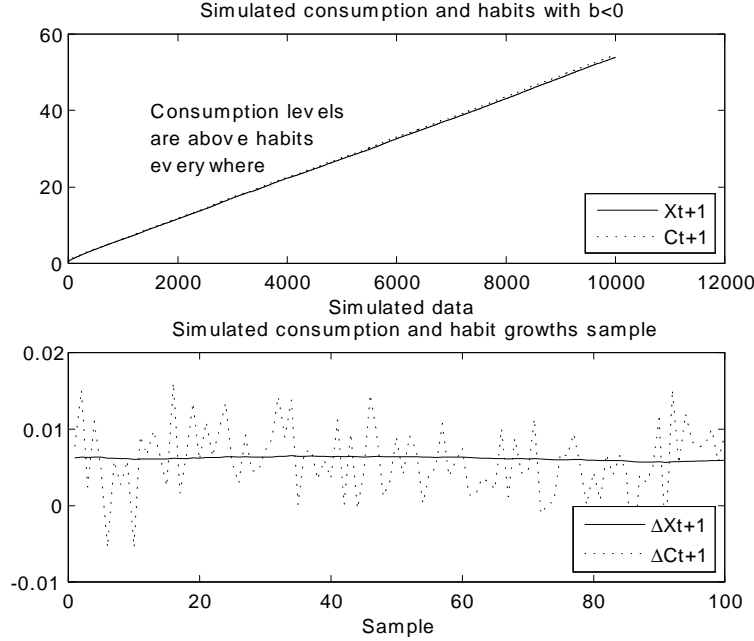
Fig. 2 - Assuming Verdelhan's shortcut in model evaluation does not seem to compromise original sensivity function's objectives. Habits remain below consumption levels everywhere and habits comoves slowly and nonnegative with consumption.

4.7 Real bond yields

Nevertheless, impose pro-cyclical interest rates to match the forward discount anomaly in habits formation framework results in a detachedness between the average slope of the real yield curve delivered by model's simulation and that observed in data. Wachter (2006), like us, used Campbell and Cochrane external habits model to fit main features of the US nominal and real term structure of interest rates. She pointed out that under a positive b, i.e. when intertemporal substitution effects dominates precautionary savings effects in investor preferences, this model generates a positive real bond premia that increases with maturity. Hence, an upward-sloped yield curve, a feature which Boudoukh et al. (1999) have found support in real data, is generated by the model when $b > 0$.

As mentioned in Andersen and Lund (1996), inspection of historical U.S. interest rates reveals that the real yield curve tends to be upward-sloping

and quite steep at the short end (0 to 5 years), while it is relatively flat for maturities in excess of 5 years. To check this observation in more recent data, figure 3 displays the real yields at 2-yr and 20-yr maturities for the period that runs from Apr/99 to Jan/04.[27]
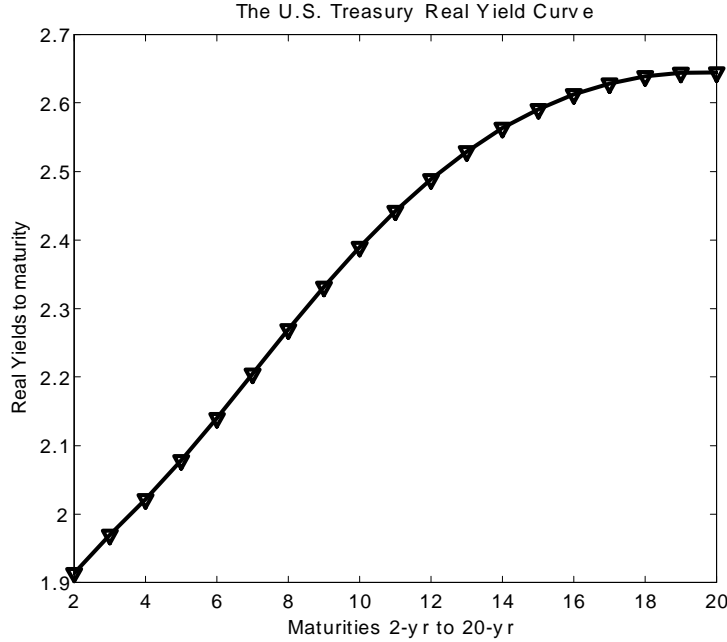
Fig. 3 - Real yields from nonofficial Federal Reserve statistical release presented in http://www.federalreserve.gov/econresdata/researchdata/feds200805.xls. We have used inflation compensated zero-coupon yield series within 2-years and 20-years maturities.

To understand the link between the sign of b and the slope of the real yield curve we follow Wachter (2006) and write the standing representative investor's Euler equation in the covariance form,

$$E(R_{n,t} - R_{1,t}) = -Cov(R_{n,t} - R_{1,t}, M_t)\frac{\sigma(M_t)}{E(M_t)}. \qquad (27)$$

[27]You may find this US real yield data in FED's webpage at http://www.federalreserve.gov/econresdata/researchdata/feds200805.xls

31

Return at t on a bond maturing n periods in the future is $R_{n,t}$, the real short-term interest rate is the one-period maturity return $R_{1,t}$ and M_t is the pricing kernel.

The reason why this model generates a positively sloped yield curve is the combination of two facts. First, the fact that bonds returns move in the opposite direction from short term returns. Second, the fact that, with $b > 0$, short term returns is negatively related to s_t, as one can see in (14).

Adjusting a positive b and calibrating the model to match real data we were able to reproduce an upward-sloping real yield curves for 1-year and 5-year bonds coinciding with Wachter (2006) and Andersen and Lund (1996) findings. These curves are presented below.
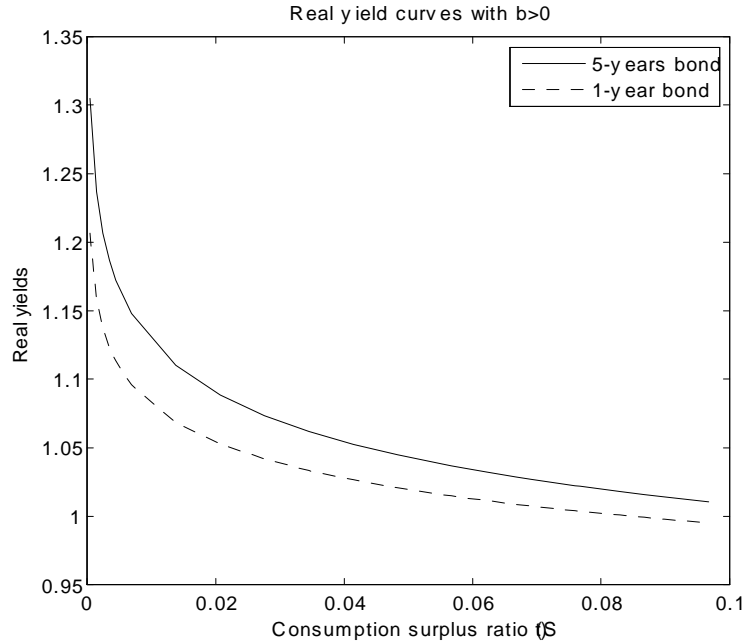
Fig. 4 - Simulated real yields when $b > 0$ is imposed from 1-year and 5-year long-term bonds. This curves are closely related to figure 4 from Wachter (2006) which shows that under anti-cyclical risk free rates the real yield curve is upward-sloped.

Turning back to our calibration parameters, i.e with $b < 0$, the model attain the following yield curves for same long-term bonds
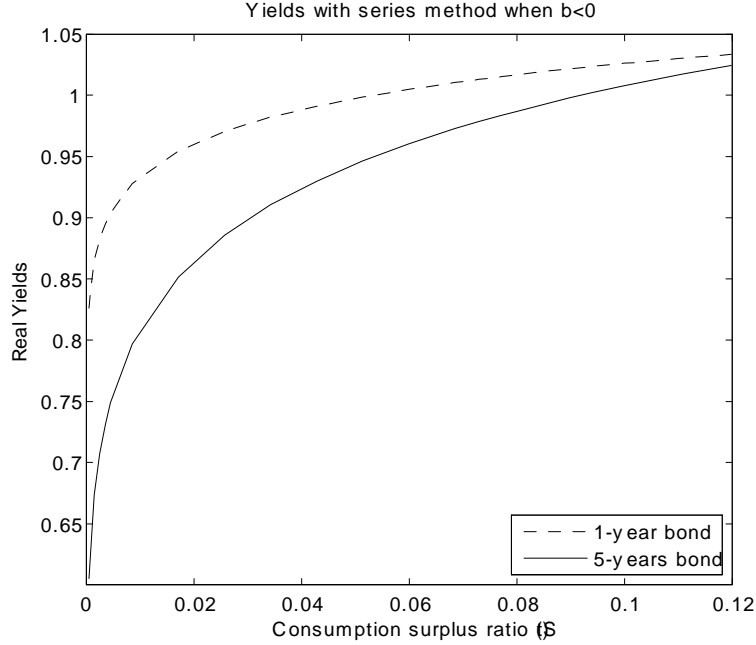
Fig. 5 - The same simulated long-term bond real yields from figure 4 but under $b < 0$. In opposite to stylized facts from the structure term literature, real yields became downward-sloped at any state when pro-cyclical risk free rates were imposed.

In figure 5 we have a downward-sloping real yield curve with negative values. Using the symmetrical argument discussed above in case $b > 0$, the pro-cyclical behavior of interest rates results in a positive covariance between economic cycles and short term interest rate. Since bond returns move in opposite direction from the short term interest rate, when economy goes badly bond returns are higher. This behavior brings real bonds closer to insurance assets. Then, investors demands smaller risk premia or even negative to hold them. Because long-term bonds have smaller expected returns than if there were no risk premia, they must have smaller yields.

This real yield curve behavior contradicts the well documented empirical statements discussed before. The reader can find out more about this in Mishkin (1990) and Piazzesi and Schneider (2006).

5　Conclusion

The presence of a forward premium anomaly in foreign exchange currency markets is tightly associated with the pro-cyclical behavior of equilibrium interest rates in Campbell and Cochrane (1999)'s habit formation model framework. The model seemed incapable to reproduce the US equity and risk-free rates in post-war data when pro-cyclical interest rates were imposed in our calibration process. This outcome is strengthened by the lack of other works in literature that have used such model with pro-cyclical risk-free rates.

In a recent work, Verdelhan has had success using Campbell and Cochrane's model to reproduce FPP findings in exchange markets calibrating it for industrialized economies. In the same way, our paper could match real data for US economy and reply the stylized fact of a negative UIP slope coefficient. However, this triumph is attained at some costs.

First, the shortcut proposed by Verdelhan to deal with the difficulties in the calibration stage may lead to inconsistencies on the model's time series, given that the price-consumption ratio is computed under an anti-cyclical environment. We did not explore this path so we do not provide any evidence of inconsistencies.

Another notable cost, not necessarily due Verdelhan's shortcut, is the real yield downward-sloped shape delivered by the model when pro-cyclical risk free rates are imposed. It is well-known in literature that real yields almost always display an upward-sloping curve or a humped curve, but only seldom a downward-sloped, whilst here this shape shows always under pro-cyclical interest rate assumption.

Campbell and Cochrane's consumption-based model seems not to be able to reproduce simultaneously stylized facts in bond markets and in exchange markets because pro-cyclical interest rates must be settled to match the forward discount anomaly. Our results are not very conclusive, however, since we used Verdelhan's shortcut. Unfortunately, as of this moment we do not have a satisfactory explanation as of why the price-consumption ratio lack of convergence during original model's solution when $b < 0$ is imposed. We know that there are definitely parameter values that can lead to non-convergence.

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APPENDIX A - Algorithm's Processing Flow
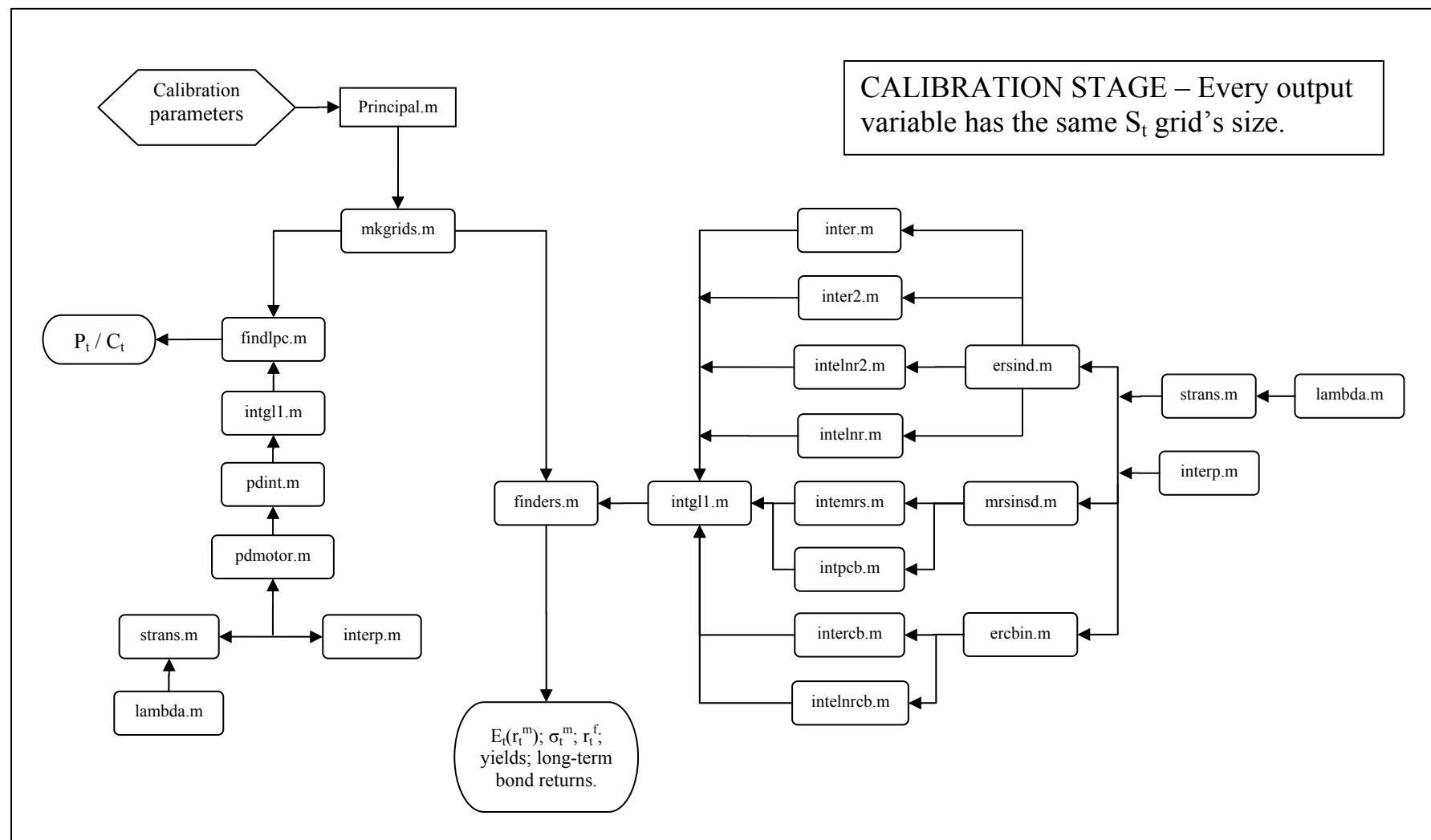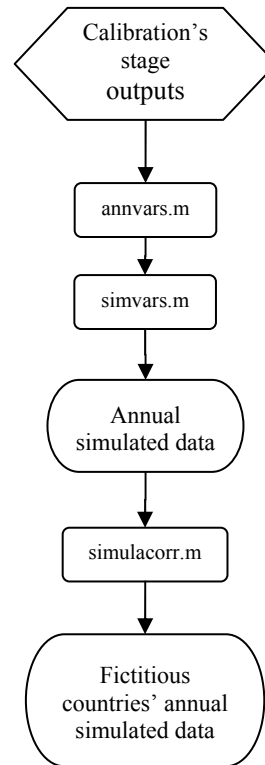
SIMULATION STAGE – Every output variable is annualized and its size depends on *ncalc*'s value.

Calibration's stage outputs

↓

annvars.m

↓

simvars.m

↓

Annual simulated data

↓

simulacorr.m

↓

Fictitious countries' annual simulated data

APPENDIX B – Routines

Every routine presented here was written in MatLab language.

Principal.m

```
% A Model with external habits formation with time varying risk free rate
% By Jivago Ximenes

%% ECHO beginig
clear all
format long
tic

%% Escolha do método de solução de P/C (Ponto Fixo ou Séries)
method = 1;      % Escolha:  0 método por ponto fixo;
                 %           1 método por séries de Wachter (2005)
                 %           2 comparar os dois métodos
%% Escolha a calibração
calib=1;         % 0 - Minha calibração para b>0
                 % 1 - Minha calibração para b<0
                 % 2 - Paper Campbell & Cochrane (1999)
                 % 3 - Paper Wachter (2005)
                 % 4 - Working Paper Verdelhan (2008)


%% Início do programa - parâmetros de calibração
global g sig delta phi gamma S_bar s_bar S_max s_max tsc sg B maxcb ncalc ...
    bondsel rho seedval verd debug ann lnpca con

% resumo=zeros(13,100);
% for loop=1:100

% Parâmetros para b>0
if calib == 0;
tsc = 4;                   % Intervalo de tempo para simulação (no caso,
mensal)
g=0.0228/tsc;              % Média da evolução do consumo
sig=0.009/sqrt(tsc);       % Desvio padrão da evolução do consumo
rf0=0.0127/tsc;            % Log da taxa livre de risco
B=0.009;                   % Coeficiente angular de rf
gamma=2.5;                 % Curvatura da função utilidade
phi=0.902^(1/tsc);         % Taxa de persistência
verd=0;                    % Hipótese de Verdelhan
ann=0;                     % Anualização segundo Verdelhan
end

% Parâmetros para b<0
if calib == 1
tsc = 4;
g=0.0219/tsc;
sig=0.0202/sqrt(tsc);
rf0=0.0098/tsc;
```

```matlab
phi=0.931^(1/tsc);
gamma=2;
B=-0.01;
verd=1;
ann=0;
end


% Parâmetros Campbell & Cochrane (1999)
if calib == 2
tsc = 12;
g=0.0189/tsc;
sig=0.015/sqrt(tsc);
rf0=0.0094/tsc;
phi=0.87^(1/tsc);
gamma=2;
B=0;
verd=0;
ann=0;
end


% Parâmetros Wachter (2005)
if calib == 3
tsc = 4;
g=0.022/tsc;
sig=0.0086/sqrt(tsc);
rf0=0.0147/tsc;
phi=(0.89)^(1/tsc);
gamma=2;
B=0.011;
verd=0;
ann=0;
end


% Parâmetros Verdelhan (WP)
if calib == 4
tsc = 4;
g=0.02115/tsc;
sig=0.0102/sqrt(tsc);
rf0=0.0136/tsc;
phi=0.97^(1/tsc);
gamma=2.4;
B=-0.01;
verd=1;
ann=1;
end


%% Parâmetros derivados do modelo
rho = (-1:.1:1);
S_bar=sig*sqrt(gamma/(1-phi-B/gamma));
s_bar = log(S_bar);
s_max = s_bar + (1-S_bar^2)/2;
S_max = exp(s_max);
delta=exp(gamma*g-.5*((1-phi)*gamma-B)-rf0); % Equação (12) do paper C&C-
1999.

szgrid=15;                        % Tamanho do grid
```

```
ncalc = 100000;                    % Número de simulações
bondsel = [1 2 3 4 5 7 10 20];     % Maturidades dos bonds a serem calculados
maxcb = max(bondsel);
seedval = 123;                     % Número do gerador randômico
chk = 1;                           % Se 1 irá simular dados pós-guerra.
                                   % Se 0, irá simular dados a partir de 1926.1
flag1 = 0;                         % Simulará dados fictícios.
flag2 = 1;                         % 1 simulará dados anuais. 0 seria trimestral.


%ann = 0;                          % 1 anualiza dados de consumo e risk-free
                                   % da maneira do Verdelhan.
con=0;                             % Usa na simulação dos países o rf vindo de
simulação.
                                   % Se con = 0 usa rf interpolado na curva
                                   % calibrada.
%% Criando o grid de log(S)
sg = mkgrids(szgrid,0);
S=exp(sg);


%% Encontrando o valor de P/C e das perpetuidades pelo método de ponto fixo
if method == 0 || method == 2
    [lnpca ctrindx]=findlpc(sig,g,s_bar);
    PC_ratio=exp(lnpca);
    plot(S,PC_ratio/tsc,'k');     % plotando o P/C ratio anualizado (dividido
por tsc)
end
lnpca_pf=lnpca;
%% Encontrando o valor de P/C e das perpetuidades pelo método de séries
if method == 1 || method == 2
    [W_PC_ratio]=WfindFn(sig,sg);
    plot(S,W_PC_ratio/tsc,'k');
    lnpca_s=log(W_PC_ratio);
    lnpca=lnpca_s;
end
% Comparando gráficos dos dois métodos
if method == 2
    plot(S,W_PC_ratio/tsc,'r',S,PC_ratio/tsc,'g');
    legend('Series method','Fixed-point method',2);
    xlabel('Consumption surplus ratio (S{t})');
    ylabel('Price-consumption ratio (P{t}/C{t})');
    comp=max(abs((W_PC_ratio-PC_ratio)./PC_ratio));
end


%% Encontrando os retornos esperados e desvios condicionais do consumption
claim
verd=0;
% Pelo método de ponto fixo
if method == 0 || method == 2
[er_pf elnr_pf sdr_pf sdlnr_pf lnrf_pf lnrf1_pf lny_pf elnrcb_pf sdlnrcb_pf
slpmv_pf] = finders(sg);
end
% Pelo método de Séries
if method == 1 || method == 2
[er_s elnr_s sdr_s sdlnr_s lnrf_s lnrf1_s lny_s elnrcb_s sdlnrcb_s slpmv_s] =
finders(sg);
end
```

```
%% Ajustando o input para a simulação

if flag1 == 0 && flag2 == 0
%     [qlnc,qlndc,qvwret,qvwretx,qlnrf]=loadqdata;
    dc = 0;

elseif flag1 == 1 && flag2 == 0
%     [qlnc,qlndc,qvwret,qvwretx,qlnrf]=loadqdata;
    dc = exp(qlndc);

elseif flag1 == 0 && flag2 == 1
%     [alnc,alndc,avwret,avwretx,alnrf]=loadadata;
    dc = 0;

elseif flag1 == 1 && flag2 == 1
%     [alnc,alndc,avwret,avwretx,alnrf]=loadadata;
    dc = exp(alndc);
end


%% Simulando séries temporais e anualizando-as
% (Caso não se simule através de dados reais, isto é, flag1 == 0)
% verd=0;
% Pelo método de ponto fixo
if method == 0 || method == 2
randn('seed',seedval);        % Ajustando o seed randômico
[alndctsim_pf astsim_pf alnpctsim_pf alnrtsim_pf alnrfsim_pf asdlnrtsim_pf
...
    alnchpsim_pf alnysim_pf aelnrcbsim_pf asdlnrcbsim_pf
atesterfsim_pf]=annvars(dc,...

lnpca_pf,er_pf,elnr_pf,sdr_pf,sdlnr_pf,elnrcb_pf,sdlnrcb_pf,lny_pf,lnrf1_pf);
end
% Pelo método de Séries
if method == 1 || method == 2
randn('seed',seedval);        % Ajustando o seed randômico
[alndctsim_s astsim_s alnpctsim_s alnrtsim_s alnrfsim_s sdlnrtsim_s
alnchpsim_s ...
    alnysim_s aelnrcbsim_s asdlnrcbsim_s
atesterfsim_s]=annvars(dc,lnpca_s,er_s,elnr_s,sdr_s,...
    sdlnr_s,elnrcb_s,sdlnrcb_s,lny_s,lnrf1_s);
end


%% Criando as estatísticas de interesse

if method == 0 || method == 2
    if ann == 1
        Edc_pf = tsc*mean(alndctsim_pf);
        Stdc_pf = sqrt(tsc)*std(alndctsim_pf);
    else
        Edc_pf = mean(alndctsim_pf);    % Esperança da evolução do log do
consumo
        Stdc_pf = std(alndctsim_pf);    % Desvio da evolução do log do consumo
    end
```

43

```matlab
Erf_pf = mean(alnrfsim_pf);        % Esperança do log da taxa livre de
risco
Stdrf_pf = std(alnrfsim_pf);       % Desvio do log da taxa livre de risco
Erfinterp_pf = mean(atesterfsim_pf);
Stdrfinterp_pf = std(atesterfsim_pf);

exrett_pf = alnrtsim_pf - alnrfsim_pf; % Excesso de retonos
exrettinterp_pf = alnrtsim_pf - atesterfsim_pf;

Shpr_pf = mean(exrett_pf)/std(exrett_pf); % Razão de Sharpe para log dos
retornos
ShpR_pf = mean(exp(alnrtsim_pf)-exp(alnrfsim_pf))/std(exp(alnrtsim_pf)-
exp(alnrfsim_pf));
Shprinterp_pf = mean(exrettinterp_pf)/std(exrettinterp_pf);
ShpRinterp_pf = mean(exp(alnrtsim_pf)-
exp(atesterfsim_pf))/std(exp(alnrtsim_pf)-exp(atesterfsim_pf));

Eexrett_pf = mean(exrett_pf);   % Média do excesso de retornos (em log)
Stdexrett_pf = std(exrett_pf);  % Desvio do excesso de retornos (em log)
Eexrettinterp_pf = mean(exrettinterp_pf);
Stdexrettinterp_pf = std(exrettinterp_pf);

Ep_d_pf = mean(alnpctsim_pf);   % Média do log da razão preço-consumo simulada
Stdp_d_pf = std(alnpctsim_pf); % Desvio do log da razão preço-consumo
simulada

table = zeros(13,1);

table(1,1) = Edc_pf;
table(2,1) = Stdc_pf;
table(3,1) = Erf_pf;
table(4,1) = Stdrf_pf;
table(5,1) = Shpr_pf;
table(6,1) = ShpR_pf;
table(7,1) = Eexrett_pf;
table(8,1) = Stdexrett_pf;
table(9,1) = Ep_d_pf;
table(10,1) = Stdp_d_pf;
table(11,1)= S_max;
table(12,1)= S_bar;
table(13,1)= delta^tsc;
end

if method == 1 || method == 2
    if ann == 1
        Edc_s = tsc*mean(alndctsim_s);
        Stdc_s = sqrt(tsc)*std(alndctsim_s);
    else
        Edc_s = mean(alndctsim_s);   % Esperança da evolução do log do
consumo
        Stdc_s = std(alndctsim_s);   % Desvio da evolução do log do consumo
    end

Erf_s = mean(alnrfsim_s);          % Esperança do log da taxa livre de risco
Stdrf_s = std(alnrfsim_s);         % Desvio do log da taxa livre de risco
```

```
Erfinterp_s = mean(atesterfsim_s);
Stdrfinterp_s = std(atesterfsim_s);


exrett_s = alnrtsim_s - alnrfsim_s; % Excesso de retonos
exrettinterp_s = alnrtsim_s - atesterfsim_s;


Shpr_s = mean(exrett_s)/std(exrett_s); % Razão de Sharpe para log dos
retornos
ShpR_s = mean(exp(alnrtsim_s)-exp(alnrfsim_s))/std(exp(alnrtsim_s)-
exp(alnrfsim_s));
Shprinterp_s = mean(exrettinterp_s)/std(exrettinterp_s);
ShpRinterp_s = mean(exp(alnrtsim_s)-exp(atesterfsim_s))/std(exp(alnrtsim_s)-
exp(atesterfsim_s));


Eexrett_s = mean(exrett_s);  % Média do excesso de retornos (em log)
Stdexrett_s = std(exrett_s); % Desvio do excesso de retornos (em log)
Eexrettinterp_s = mean(exrettinterp_s);
Stdexrettinterp_s = std(exrettinterp_s);


Ep_d_s = mean(alnpctsim_s);  % Média do log da razão preço-consumo simulada
Stdp_d_s = std(alnpctsim_s); % Desvio do log da razão preço-consumo simulada


table = zeros(13,1);


table(1,1) = Edc_s;
table(2,1) = Stdc_s;
table(3,1) = Erf_s;
table(4,1) = Stdrf_s;
table(5,1) = Shpr_s;
table(6,1) = ShpR_s;
table(7,1) = Eexrett_s;
table(8,1) = Stdexrett_s;
table(9,1) = Ep_d_s;
table(10,1) = Stdp_d_s;
table(11,1)= S_max;
table(12,1)= S_bar;
table(13,1)= delta^tsc;
end


if method == 2
table(1,1) = Edc_pf; table(1,2)=Edc_s;
table(2,1) = Stdc_pf; table(2,2) = Stdc_s;
table(3,1) = Erf_pf; table(3,2) = Erf_s;
table(4,1) = Stdrf_pf; table(4,2) = Stdrf_s;
table(5,1) = Shpr_pf; table(5,2) = Shpr_s;
table(6,1) = ShpR_pf; table(6,2) = ShpR_s;
table(7,1) = Eexrett_pf; table(7,2) = Eexrett_s;
table(8,1) = Stdexrett_pf; table(8,2) = Stdexrett_s;
table(9,1) = Ep_d_pf; table(9,2) = Ep_d_s;
table(10,1) = Stdp_d_pf; table(10,2) = Stdp_d_s;
table(11,1)= S_max; table(11,2)= S_max;
table(12,1)= S_bar; table(12,2)= S_bar;
table(13,1)= delta^tsc; table(13,2)= delta^tsc;
end
```

```matlab
% resumo(:,loop)=table;
% end
%% Tentativa de Simulação dos SDF's

% Simulando os dados novamente sob a mesma semente aleatória
if method == 0 || method == 2
randn('seed',seedval);        % Ajustando o seed randômico
[stsim vtsim lndctsim lnpctsim lnrtsim lnrfsim ertsim elnrtsim sdrtsim...
    sdlnrtsim elnrcbsim sdlnrcbsim lnysim lnrcbsim testerfsim]=...

simvars(dc,lnpca_pf,er_pf,elnr_pf,sdr_pf,sdlnr_pf,elnrcb_pf,sdlnrcb_pf,lny_pf
,lnrf1_pf);

% Fator estocástico de desconto dos USA
SDFus = delta*exp(-g*gamma)*exp(-gamma*vtsim).*exp(-
gamma*(stsim(2:length(stsim))...
     -stsim(1:length(stsim)-1)));

% Vamos gerar um SDF que tenha os choques vtsim correlacionados com o vtsim
% utilizado para construir SDFus. Cada país será identificado pelas
% correlações dispostas no vetor rho acima descrito.

for k=1:length(rho)
    [stx vtx lndctx lnrfx]=simulacorr(rho(k),lnrf1_pf);

    SDFx(:,k)= delta*exp(-g*gamma)*exp(-gamma*vtx).*exp(-
gamma*(stx(2:length(stx))...
     -stx(1:length(stx)-1)));

    stsimx(:,k)=stx;
    vtsimx(:,k)=vtx;
    lndctsimx(:,k)=lndctx;
    lnrfsimx(:,k)=lnrfx;
end

%% Encontrando as taxas de câmbio real

deltaq = log(SDFx) - kron(ones(1,size(SDFx,2)),log(SDFus));

%% Regressões da UIP
if con == 1
for k=1:length(rho)
    diff(:,k)=lnrfsim-lnrfsimx(:,k);
    regressor=cat(2,ones(length(diff(:,k)),1),diff(:,k));
    [betas interv]=regress(deltaq(:,k),regressor(1:length(regressor)-1,:));
    Betas_pf(k,1)=rho(k);
    Betas_pf(k,2)=betas(1,1);
    Betas_pf(k,3)=betas(2,1);
    if k==1
        Interv_pf=interv;
    else
        Interv_pf=cat(1,Interv_pf,interv);
    end
end
end
```

```
elseif con == 0

 for k=1:length(rho)
    diff(:,k)=testerfsim-lnrfsimx(:,k);
    regressor=cat(2,ones(length(diff(:,k)),1),diff(:,k));
    [betas interv]=regress(deltaq(:,k),regressor(1:length(regressor)-1,:));
    Betas_pf(k,1)=rho(k);
    Betas_pf(k,2)=betas(1,1);
    Betas_pf(k,3)=betas(2,1);
    if k==1
        Interv_pf=interv;
    else
        Interv_pf=cat(1,Interv_pf,interv);
    end
end
end
end


if method == 1 || method == 2
randn('seed',seedval);        % Ajustando o seed randômico
[stsim vtsim lndctsim lnpctsim lnrtsim lnrfsim ertsim elnrtsim sdrtsim...
    sdlnrtsim elnrcbsim sdlnrcbsim lnysim lnrcbsim testerfsim]=...

simvars(dc,lnpca_s,er_s,elnr_s,sdr_s,sdlnr_s,elnrcb_s,sdlnrcb_s,lny_s,lnrf1_s
);

% Fator estocástico de desconto dos USA
 SDFus = delta*exp(-g*gamma)*exp(-gamma*vtsim).*exp(-
gamma*(stsim(2:length(stsim))...
      -stsim(1:length(stsim)-1)));

% Vamos gerar um SDF que tenha os choques vtsim correlacionados com o vtsim
% utilizado para construir SDFus. Cada país será identificado pelas
% correlações dispostas no vetor rho acima descrito.

for k=1:length(rho)
    [stx vtx lndctx lnrfx]=simulacorr(rho(k),lnrf1_s);

    SDFx(:,k)= delta*exp(-g*gamma)*exp(-gamma*vtx).*exp(-
gamma*(stx(2:length(stx))...
      -stx(1:length(stx)-1)));

    stsimx(:,k)=stx;
    vtsimx(:,k)=vtx;
    lndctsimx(:,k)=lndctx;
    lnrfsimx(:,k)=lnrfx;
end

%% Encontrando as taxas de câmbio real

deltaq = log(SDFx) - kron(ones(1,size(SDFx,2)),log(SDFus));

%% Regressões da UIP
if con == 1
for k=1:length(rho)
```

```
        diff(:,k)=lnrfsim-lnrfsimx(:,k);
        regressor=cat(2,ones(length(diff(:,k)),1),diff(:,k));
        [betas interv]=regress(deltaq(:,k),regressor(1:length(regressor)-1,:));
        Betas_s(k,1)=rho(k);
        Betas_s(k,2)=betas(1,1);
        Betas_s(k,3)=betas(2,1);
        if k==1
            Interv_s=interv;
        else
            Interv_s=cat(1,Interv_s,interv);
        end
    end

elseif con == 0
    for k=1:length(rho)
    diff(:,k)=testerfsim-lnrfsimx(:,k);
    regressor=cat(2,ones(length(diff(:,k)),1),diff(:,k));
    [betas interv]=regress(deltaq(:,k),regressor(1:length(regressor)-1,:));
    Betas_s(k,1)=rho(k);
    Betas_s(k,2)=betas(1,1);
    Betas_s(k,3)=betas(2,1);
    if k==1
        Interv_s=interv;
    else
        Interv_s=cat(1,Interv_s,interv);
    end
end
end
end

if method == 2
Betas = cat(2,Betas_pf,NaN*ones(size(Betas_pf,1),1));
Betas = cat(2,Betas,Betas_s);

Intervalo = cat(2,Interv_pf,NaN*ones(size(Interv_pf,1),1));
Intervalo = cat(2,Intervalo,Interv_s);
end

%% Teste da hipótese das expectativas
bn=zeros(length(bondsel)-1,2);
bnconf=zeros(length(bondsel)-1,4);
for i=2:length(bondsel)
chgyield=lnysim(2:size(lnysim,1),i-1)-lnysim(1:size(lnysim,1)-1,i);
spreadyield=(lnysim(1:size(lnysim,1)-1,i)-lnysim(1:size(lnysim,1)-
1,1))./(bondsel(i)-1);

[aux1 aux2]=regress(chgyield,[ones(length(lnysim)-1,1) spreadyield]);
bn(i-1,:)=aux1';
bnconf(i-1,:)=reshape(aux2,1,4);
end

% Fim do programa
load gong
wavplay(y,Fs)
toc
```

mkgrids.m

```
function [sg] = mkgrids(szgrid,flag)

% Construirá o grid de s de uma maneira eficiente.                         %
% ------------------------------------------------------------------------- %
global s_max S_max s_bar

if flag == 0

sg = linspace(0,S_max,szgrid);
aux = [(sg(szgrid)-.01) (sg(szgrid)-.02) (sg(szgrid)-.03) (sg(szgrid)-.04)];
sg = cat(2,sg,aux);
sg = sort(sg);
sg = log(sg(2:size(sg,2)))'; % Já entendeu o por quê do szgrid+1 acima, não??
                             % Foi para eliminar log(0) no grid de log(S). %

if max(sg == s_bar) == 0     % Certificando-se que s_bar vai estar no grid.
    sg = cat(1,sg,s_bar);
    sg = sort(sg);
end
if max(sg == s_max) == 0
    sg = cat(1,sg,s_max);
    sg = sort(sg);
end

% Colocar mais densidade no início do grid para melhorar a                 %
% iteração durante o procedimento de ponto fixo.                           %
% ------------------------------------------------------------------------- %
idens=log([.0005 0.0015 .0025 .0035 .0045])';
sg=cat(1,sg,idens);
sg = sort(sg);        % Ordenando os valores presentes no grid

end
% Grid 3 da Wachter (2005)
if flag == 1
sg = linspace(0,S_max,szgrid);
sg = log(sg(2:size(sg,2)))'; % Já entendeu o por quê do szgrid+1 acima, não??
                             % Foi para eliminar log(0) no grid de log(S). %

if max(sg == s_bar) == 0     % Certificando-se que s_bar vai estar no grid.
    sg = cat(1,sg,s_bar);
    sg = sort(sg);
end
if max(sg == s_max) == 0
    sg = cat(1,sg,s_max);
    sg = sort(sg);
end
u=min(sg);
aux = linspace(-300,u,200)';
sg = cat(1,sg,aux);
sg = sort(sg);
end
end
```

findlpc.m

```matlab
function [lnpca ctrindx]=findlpc(sig,g,s_bar)

% Este é o procedimento que irá calcular o ponto fixo P/C.               %
% --------------------------------------------------------------------- %
global s sg lnpc delta gamma phi B debug

%% S_bar
% Agora precisamos achar o index do valor de s_bar para ser utilizado em
% gráficos e outras estatísticas
if max(sg == s_bar) == 1
    ctrindx = find(sg == s_bar);
else
    disp('ERRO: O valor estacionário de log(S) não está no grid');
end

%% Vetores da função valor P/C
lnpca = zeros(size(sg,1),1);        % Estamos começando com P/C = 1
lnpc = lnpca;
newlnpc = lnpc;

%% Loop: achar ln(P/C) a partir do grid de s
iter = 1;
erro = 1;

while iter < 10000 && erro > 1e-6

    for i=1:size(sg,1)
        s = sg(i);
     if exp(-log(delta)+gamma*g-(gamma*(1-phi)-B)/2-B*(s-s_bar)) < g
        disp('\t ATENÇÃO: Rf < g \n');
        fprintf('valor de st: %g',s);
     end
% Gerar o Log da taxa de juros variável no tempo

        newlnpc(i) = log(intgl1(@pdint,abs(sig)*(-8),abs(sig)*8));
        debug;
    end
    tv = max(abs((exp(newlnpc)-exp(lnpc))./exp(newlnpc)));
    lnpc = newlnpc;
    erro = max(tv);
    iter = iter + 1;
end
    lnpca = lnpc;
end
```

intgl1.m

```
function [fx] = intgl1(f,a,b)

% Rotina que desenvolve o método de integração de Gauss-Legendre.
%
% Nesta rotina, o vetor 'e' compõe-se de valores entre 0 e 1 (e ~ (0,1))
% e o vetor 'w' das raízes do polinômio de Legendre cujos coeficientes vem
% de 'e'. Estes valores estão contidos no arquivo ewquad.txt e foram
% processados pelo programa 'gausslege.m' que não faz parte deste trabalho
% por fugir muito do seu escopo.

    [e w] = textread('ewquad.txt','%f %f');

    e = sort(cat(1,e,-e));
    w = cat(1,sort(w),w);

    diff = b - a;
    xc = 0.5*( (b+a)+(diff * e'));
    fx = zeros(size(xc)) + f(xc);
    fx = ((diff/2)* (fx*w));
end
```

pdint.m

```
function [inside] = pdint(v)

% Irá criar uma nova densidade normal de acordo com as inovações de v{t+1}
% para integrar o funcional de P/C.
% ----------------------------------------------------------------------- %
global sig debug

inside = (1/(sig*(2*pi)^(.5)))*exp(-.5*(v/sig).^2).*pdmotor(v);

debug(:,3)=inside';
end
```

pdmotor.m

```
function [inside] = pdmotor(v)

% Procedimento para ser usado na integração numérica quando for se calcular
% o ponto fixo. Tem como argumento apenas v ~ N(0,sig). Retorna o VALOR de
% P/C para cada iteração sobre o valor corrente de s{t} em cada iteração.
% ----------------------------------------------------------------------- %
```

```
global delta g gamma s sg lnpc debug
s1=strans(s,v);
inside = delta*exp(g*(1-gamma))*exp(-gamma*(s1-s)).*...
    (1+exp(interp(s1,sg,lnpc)))'.*exp((1-gamma)*v);

debug(:,2)=inside';
end
```

strans.m

```
function [news]=strans(s,v)

% Procedimento "strans"                                          %
%                                                                %
% Esta função retornará o valor de s{t+1} = log(S{t+1})          %
%                                                                %
% s{t+1} = (1-phi)*s_bar + phi*s{t} + lambda(s{t})*v{t+1};        %
% -------------------------------------------------------------- %

global s_bar phi debug

news = (1-phi)*s_bar + phi*s + lambda(s)*v;

debug(:,1)=news';
end
```

lambda.m

```
function [y]=lambda(s)
% Função Lambda
%
% Retorna o valor da função sensibilidade

global S_bar s_bar s_max verd

if verd == 0
if (s <= s_max)
            y = (1 / S_bar)*sqrt(max(0 , 1-2*(s-s_bar)))-1;
        else
            y = 0;
end;
elseif verd == 1
    y=(1-S_bar)/S_bar;
end
end
```

interp.m

```matlab
function [fofs indx] = interp(sv,x,fx)

% Procedimento de interpolação da distribuição de s                        %
%                                                                          %
% sv -> vetor de valores quaisquer onde se quer gerar f(s).                %
% x -> vetor de pontos do grid de s. Deve ser monotônico.                  %
% fx -> vetor de valores atuais de f(x) no grid.                           %
%                                                                          %
% Acha a inclinação e o intercepto locais para ser usado sobre o grid de   %
% log(S), de tal forma que retorne f(x) = a + b*x                          %
% ------------------------------------------------------------------------ %
if isempty(min(find(fx == 0))) == 0
    fofs = 0*sv';
else
    T= size(x,1);
% Algumas garantias sobre os inputs
    if x(2) < x(1)
     disp('O grid deve ser monótono e crescente');
    end

    if size(sv,2) > 1 && size(sv,1) == 1
        sv=sv';
        chk = 1;
    elseif size(sv,2) > 1 && size(sv,1) > 1
        disp('ERRO: Vetor sv não pode ser uma matriz');
    elseif size(sv,2) == 1 && size(sv,1) > 1
        chk = 0;
    elseif size(sv,2) == 1 && size(sv,1) == 1
        chk = 2;          % Para o caso de sv = s %
    end
gradf = (fx(2:T)-fx(1:T-1))./(x(2:T)-x(1:T-1));
const = cat(1,(fx(1:T-1)-gradf.*x(1:T-1)),(fx(T)-gradf(T-1)*x(T)));
const = cat(1,fx(1)-gradf(1)*x(1),const);
slope = cat(1,gradf(1),gradf);
slope = cat(1,slope,gradf(T-1));
% Estamos interessados em achar o menor índice do vetor (sv(i)-x) de forma
% a obter um vetor indx que contenha estes índices e possamos organizar
% monotonicamente os vetores de slope e intercepto.
indx = zeros(size(sv,1),1);
for i = 1 : size(sv,1)
   if sv(i)> x(1)
        indx(i)= max(find((sv(i)-x) > 0))+1;
   else
        indx(i)=1;
   end
end
fofs = const(indx)+ slope(indx).*sv;
if chk == 0
    fofs = fofs';
end
end
end
```

53

finders.m

```matlab
function [er elnr sdr sdlnr lnrf lnrf1 lny elnrcb sdlnrcb slpmv] =
finders(sg)

% Procedimento que calcula os retornos esperados dos ativos de consumo. ele
% nos fornecerá E(R), SD(R), lnrf, a curvatura da fronteira média variância
% dada pela variável (slpmv) e integrará a razão de Sharpe do vetor de   %
% ativos de consumo.                                                     %
% -------------------------------------------------------------------- %

global g gamma sig phi s maxcb s_bar delta tsc lnpcb matur

% Inclinação da Fronteira de Média-Variância                           %
%                                                                      %
% slpmv = (exp((gamma*sig)^2.*(1+lambda(sg)).^2)-1).^.5               %
% -------------------------------------------------------------------- %

slpmv = (exp((gamma*sig)^2*(1+lambda(sg)).^2)-1).^(0.5);

%% Estrutura termo da taxa de juros dada por:                          %
%                                                                      %
% lnrf = -ln(delta) + gamma*g -                                        %
% gamma*(1-phi)*(s{t}-s_bar)-.5((gamma*sig)^2)*(1+lambda(s{t}))^2
% - B*(sg - s_bar)             --> Variável no estado
% -------------------------------------------------------------------- %

lnrf = -log(delta) + gamma*g - gamma*(1-phi)*(sg-s_bar)...
    - 0.5*(gamma*sig*(1+lambda(sg))).^2;

%% Bonds
% Matriz de todos os preços dos títulos. Sua dimensão será
% N(sg) x (maxcb*tsc)
lnpcb = [];
lnpcb(:,1) = -lnrf;
lnp = zeros(size(sg,1),1);

    for j = 2:(maxcb*tsc)
        for i = 1:length(sg)
            s = sg(i);
            lnp(i) = log(intgl1(@intpcb,abs(sig)*(-8),abs(sig)*8));
        end
        lnpcb = cat(2,lnpcb,lnp);
    end

% Yields
lny = -
lnpcb./kron(ones(size(sg,1),1),linspace(1/tsc,(maxcb*tsc)/tsc,(maxcb*tsc)));

%% Retornos e Desvios Padrão Esperados                                 %
% -------------------------------------------------------------------- %

lnrf1 = zeros(size(sg,1),1);
```

```
er = zeros(size(sg,1),1);
elnr = zeros(size(sg,1),1);
sdr = zeros(size(sg,1),1);
sdlnr = zeros(size(sg,1),1);
elnrcb = zeros(size(sg,1),maxcb*tsc);        % zero-coupon bonds %
sdlnrcb = zeros(size(sg,1),maxcb*tsc);

for i=1:size(sg,1)
    s = sg(i);

    lnrf1(i)= - log(intgl1(@intemrs,abs(sig)*(-8),abs(sig)*8));
    er(i)= intgl1(@inter,abs(sig)*(-8),abs(sig)*8);
    elnr(i)= intgl1(@intelnr,abs(sig)*(-8),abs(sig)*8);
    sdr(i) = intgl1(@inter2,abs(sig)*(-8),abs(sig)*8);
    sdr(i) = (sdr(i) - er(i).^2).^(.5);
    sdlnr(i) = intgl1(@intelnr2,abs(sig)*(-8),abs(sig)*8);

    % Bonds
    matur = maxcb*tsc;
    elnrcb(i,1) = lnrf(i);
    for k = 2:matur
        elnrcb(i,k) = intgl1(@intelnrcb,abs(sig)*(-8),abs(sig)*8);
        sdlnrcb(i,k) = intgl1(@intelnrt2,abs(sig)*(-8),abs(sig)*8);
        sdlnrcb(i,k) = (sdlnrcb(i,k) - elnrcb(i,k).^2).^(.5);
    end
end
end
```

inter.m

```
function [out] = inter(v)

% Fornece o valor do retorno esperado de acordo com uma distribuição normal
% ----------------------------------------------------------------------- %
global sig
out = (1/(sig*(2*pi)^(.5)))*exp(-.5*(v/sig).^2).*erinsd(v);
end
```

inter2.m

```
function [out] = inter2(v)

% Fornece o valor do retorno esperado de acordo com uma distribuição normal
% ----------------------------------------------------------------------- %
global sig
out = (1/(sig*(2*pi)^(.5)))*exp(-.5*(v/sig).^2).*erinsd(v).^2;
end
```

intelnr.m

```
function [out] = intelnr(v)

% Fornece o valor do retorno esperado de acordo com uma distribuição normal
% ------------------------------------------------------------------------ %
global sig
out = (1/(sig*(2*pi)^(.5)))*exp(-.5*(v/sig).^2).*log(erinsd(v));
end
```

intelnr2.m

```
function [out] = intelnr2(v)

% Fornece o valor do retorno esperado de acordo com uma distribuição normal
% ------------------------------------------------------------------------ %
global sig
out = (1/(sig*(2*pi)^(.5)))*exp(-.5*(v/sig).^2).*log(erinsd(v)).^2;
end
```

intemrs.m

```
function [out]=intemrs(v)

% Retornará a taxa marginal de substituição de tal forma que possa ser     %
% utilizada na integração do ponto fixo.                                   %
% ------------------------------------------------------------------------ %
global sig
out = (1/(sig*(2*pi)^(.5)))*exp(-.5*(v/sig).^2).*mrsinsd(v);
end
```

intpcb.m

```
function out = intpcb(v)

% Função que fornece o preço dos bonds para cada maturidade %

global s sg lnpcb sig
out = pdf('norm',v,0,sig).*mrsinsd(v).*...
    exp(interp(strans(s,v),sg,lnpcb(:,size(lnpcb,2)))))';
end
```

intercb.m

```
function out = intercb(v)

% Integrando dos retornos esperados dos títulos públicos %
% ou, da estrutura a termo.                              %
global sig
out = pdf('norm',v,0,sig).*ercbin(v);
end
```

intelnrcb.m

```
function out = intercb(v)

% Integrando dos retornos esperados dos títulos públicos %
% ou, da estrutura a termo.                              %
global sig
out = pdf('norm',v,0,sig).*log(ercbin(v));
end
```

ersind.m

```
function [er]=erinsd(v)

% Procedimento para calcular os retornos do consumption claim.          %
% --------------------------------------------------------------------- %
global sg s lnpca g
er=((1+exp(interp(strans(s,v),sg,lnpca)))'./(ones(size(v))...
    *exp(interp(s,sg,lnpca)))).*exp(g+v);
end
```

ercbin.m

```
function out = ercbin(v)

% Retornos esperados da estrutura a termo %
global s sg lnpcb matur
out = exp(interp(strans(s,v),sg,lnpcb(:,matur-1)))./...
    exp(interp(s,sg,lnpcb(:,matur)));
end
```

mrsinsd.m

```
function [out]=mrsinsd(v)

% Retorna a taxa marginal de substituição intertemporal no modelo.      %
% --------------------------------------------------------------------- %
global delta g gamma s
out = delta*exp(-gamma*g)*exp(-gamma*v).*exp(-gamma*(strans(s,v)-s));
end
```

annvars.m

```
function [alndctsim astsim alnpctsim alnrtsim alnrfsim asdlnrtsim alnchpsim
...
    alnysim aelnrcbsim asdlnrcbsim
atesterf]=annvars(dc,lnpc,er,elnr,sdr,sdlnr,elnrcb,sdlnrcb,lny,lnrf1)

% Este programa trabalha sobre os dados vindos de simvars quando estes são
% simulados fora dos dados reais, isto é, ncalc dados. Serão anualizados
% os retornos esperados. Falta anualizar os retornos dos títulos (bonds)
% simulados.

global tsc bondsel ann
%% Rodando as séries temporais artificiais
[stsim vtsim lndctsim lnpctsim lnrtsim lnrfsim ertsim elnrtsim sdrtsim...
    sdlnrtsim elnrcbsim sdlnrcbsim lnysim lnrcbsim
testerf]=simvars(dc,lnpc,er,elnr,sdr,sdlnr,elnrcb,sdlnrcb,lny,lnrf1);

T = size(stsim,1);

%% Consumo
if ann == 1
    alndctsim=lndctsim;
else
    alnctsim = cumsum(lndctsim);      % Torna mensal a evolução do log do
consumo
    alnctsim = log(chgfreq(exp(alnctsim),tsc,tsc,0));
    alndctsim = alnctsim(2:size(alnctsim,1))-alnctsim(1:(size(alnctsim,1)-
1));
end
%% Variável de estado do modelo
if T > 1
    astsim = chgfreq(stsim(2:T),1,tsc,0);
    astsim = astsim(2:size(astsim,1));
end

%% Razão P/C
if size(lnpctsim,1) > 1
    alnpctsim = chgfreq(lnpctsim(2:T),1,tsc,0)-log(tsc);
    alnpctsim = alnpctsim(2:size(alnpctsim,1));
```

```
    end

%% Vamos multiplicar os retornos para anualizá-los
if size(lnrtsim,1) > 1
    alnrtsim = chgfreq(lnrtsim,tsc,tsc,0);
    alnrtsim = alnrtsim(2:size(alnrtsim,1));
end

% Retorno livre de risco
% Simulado
if size(lnrfsim,1) > 1
    alnrfsim = chgfreq(lnrfsim(1:T-1),tsc,tsc,0);
    alnrfsim = alnrfsim(2:size(alnrfsim,1));
end
% Interpolado
if size(testerf,1) > 1
    atesterf = chgfreq(testerf(1:T-1),tsc,tsc,0);
    atesterf = atesterf(2:size(atesterf,1));
end
%% Desvios condicionais dos retornos
if size(sdlnrtsim,1) > 1
    asdlnrtsim = chgfreq(sdlnrtsim,tsc,tsc,0);
    asdlnrtsim = asdlnrtsim(2:size(asdlnrtsim,1));
end

%% Evoluçao dos preços
if size(lnpctsim,1) > 1
    lnchpsim = lnpctsim(2:T)-lnpctsim(1:T-1)+lndctsim;
    alnchpsim = chgfreq(lnchpsim,tsc,tsc,0);
    alnchpsim = alnchpsim(2:size(alnchpsim,1));
end

%% Yields
if size(lnysim,1) > 1
    for i=1:length(bondsel)+1
        alnysim(:,i) = chgfreq(lnysim(1:T-1,i),tsc,tsc,0);
    end
    alnysim = alnysim(2:size(alnysim,1),:);
end

%% Bonds
% Retorno médio
if size(elnrcbsim,1) > 1
    for i=1:length(bondsel)+1
        aelnrcbsim(:,i) = chgfreq(elnrcbsim(1:T-1,i),tsc,tsc,0);
    end
    aelnrcbsim = aelnrcbsim(2:size(aelnrcbsim,1),:);
end
% Desvio
if size(sdlnrcbsim,1) > 1
    for i=1:length(bondsel)+1
        asdlnrcbsim(:,i) = chgfreq(sdlnrcbsim(1:T-1,i),tsc,tsc,0);
    end
    asdlnrcbsim = asdlnrcbsim(2:size(asdlnrcbsim,1),:);
end
end
```

simvars.m

```
function [stsim vtsim lndctsim lnpctsim lnrtsim lnrfsim ertsim elnrtsim
sdrtsim...
    sdlnrtsim elnrcbsim sdlnrcbsim lnysim lnrcbsim
testerf]=simvars(dc,lnpca,er,elnr,sdr,sdlnr,elnrcb,sdlnrcb,lny,lnrf1)
%
% Esta rotina simulará séries temporais das principais grandezas deste    %
% modelo e terá a opção de utilizar dados reais ou fictícios.             %
%                                                                          %
% Simulará:                                                                %
% - processo de s=log(S);                                                  %
% - P/C;                                                                    %
% - R{t+1};                                                                 %
% - E{t}[R{t+1}];                                                           %
% - SD{t}[R{t+1}];                                                          %
% - USA Rf{t+1} calibrado;                                                  %
% - Rf{t+1} com consumo correlacionado com os USA                          %
% - Bonds                                                                   %
%                                                                          %
% Todas as séries simuladas serão sufixadas por 'tsim' e terão a mesma    %
% nomeação herdada dos programas anteriores (findlpc e finders).           %
%                                                                          %
% Se o input de simvars é nulo, que dizer que este programa irá simular o %
% processo de consumo do modelo. Caso o usuário queira inserir os dados    %
% reais da economia americana, as outras variáveis serão simuladas através%
% destes dados de consumo.                                                 %
% ----------------------------------------------------------------------- %
global ncalc gamma sig g phi delta s_max s_bar sg maxcb tsc bondsel
%% Ajustando o input do programa de simulação dos dados Americanos
if dc == 0                         % Simvars irá simular os dados de consumo.
    T=ncalc;
    vtsim = sig*randn(T,1);
    lndctsim = g + vtsim;
else                               % Simvars irá utilizar dados reais de
consumo.
if min(dc) <= 0
        disp('simvars: Você inseriu o log do crescimento do consumo.');
        disp('Preciso que você insira os dados de crescimento do consumo ');
        disp('bruto, isto é, nem log nem crescimento líquido.');
end
T = length(dc);
lndctsim = log(dc);
vtsim = lndctsim - g;
end


%% Simular variável de estado log(S)


stsim = zeros(T+1,1);
stsim(1) = s_bar;            % A economia começa no seu estado estacionário.


for i=2:T+1
    if strans(stsim(i-1),vtsim(i-1)) <= s_max
        stsim(i) = strans(stsim(i-1),vtsim(i-1));
```

```
    else
        stsim(i)=(1-phi)*s_bar+phi*stsim(i-1);
    end
end

%% Simulando a razão P/C %

lnpctsim = interp(stsim,sg,lnpca)';

%% Retornos ex-post
%                                          %
% R = (C'/C){(1+(P/C)')/(P/C)}       %
% ----------------------------- %

lnrtsim = log(1+exp(lnpctsim(2:T+1))) - lnpctsim(1:T) + lndctsim;

%% Log de Rf variante no tempo

lnrfsim = -log(delta) + gamma*g - gamma*(1-phi)*(stsim-s_bar)...
    - 0.5*(gamma*sig*(1+lambda(stsim))).^2;

%% Retornos esperados e desvio condicional
testerf = interp(stsim,sg,lnrf1)';
ertsim = interp(stsim,sg,er)';
elnrtsim = interp(stsim,sg,elnr)';
sdrtsim = interp(stsim,sg,sdr)';
sdlnrtsim = interp(stsim,sg,sdlnr)';

%% Retornos esperados das T-Bill 90

elnrcbsim = interp(stsim,sg,elnrcb(:,1))';   % Retornos esperados do bonds
sdlnrcbsim = interp(stsim,sg,sdlnrcb(:,1))';
lnysim = interp(stsim,sg,lny(:,1))';          % Bond yields
lny2sim = zeros(size(lnysim,1),1);

for i = 2:(maxcb*tsc)
        if find(i == bondsel*tsc)
         lnysim = cat(2,lnysim,interp(stsim,sg,lny(:,i))');
         lny2sim = cat(2,lny2sim,interp(stsim,sg,lny(:,i-1))');
         elnrcbsim = cat(2,elnrcbsim,interp(stsim,sg,elnrcb(:,i))');
         sdlnrcbsim = cat(2,sdlnrcbsim,interp(stsim,sg,sdlnrcb(:,i))');
        end
end

% Retornos das maturidades ajustadas em bondsel = [1 2 4 8 12 16 20]
lnrcbsim = cat(1,0,lnysim(1:T-1,1));

for i = 2:length(bondsel)
    lnrcbsim = cat(2,lnrcbsim,cat(1,0,(-lny2sim(2:T,i)*((bondsel(i)-
1/tsc))+...
        lnysim(1:T-1,i)*bondsel(i))/tsc));
end
end
```

simulacorr.m

```matlab
function [stsim vtsim lndctsim lnrfsim]=simulacorr(rho)

global ncalc gamma sigma g phi beta B s_bar seedval

%% Smulando choques do processo de consumo com correlação controlada rho
T=ncalc;
randn('seed',seedval);
x = sigma*randn(T,1);
y = sigma*randn(T,1);

vtsim = rho*x + sqrt(1-rho^2)*y;    % Controlando a correlação dos processos
de consumo.
lndctsim = g + vtsim;

%% Simular variável de estado log(S)

stsim = zeros(T+1,1);
stsim(1) = s_bar;            % A economia começa no seu estado estacionário.

for i=2:T+1
    stsim(i) = strans(stsim(i-1),vtsim(i-1));
end

%% Log de Rf variante no tempo

lnrfsim = -log(beta)+gamma*g-(gamma*(1-phi)-B)/2-B*(stsim-s_bar);

end
```