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Source: *Journal of Money, Credit and Banking*, Vol. 28, No. 3, Part 1 (Aug., 1996), pp. 381-392
Published by: Ohio State University Press
Stable URL: https://www.jstor.org/stable/2077981
Accessed: 16-08-2018 15:09 UTC

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Disagreement as a Measure of Uncertainty

ECONOMIC LITERATURE ABOUNDS in models that predict that the current expectation of future inflation has an influence on current economic variables. The Fisher effect and the expectations augmented Phillips curve are the best-known examples. Since Fisher (1930) researchers have sought measures of the "representative" expectation of future inflation. A popular methodology is based on the rational expectations paradigm. A forecasting model of inflation is estimated over a historical period. The forecast the model provides for each date is identified with the rate of inflation expected by a representative agent as of that date.[1]

Recent research has focused on the uncertainty surrounding the expectation. Studies have examined the effect of such uncertainty on interest rates, the slope of the Phillips curve, the degree of indexation in contracts, etc. Estimates of uncertainty based on ARCH models are a natural extension of the rational expectations method. The estimated conditional variance at each date is assumed to reflect the uncertainty of a representative forecaster. Although devised to purge parameter estimates of heteroskedasticity effects, ARCH estimates are now used as measures of uncertainty.[2]

Survey forecasts of inflation provide an alternative method of measuring expectations. The mean of forecasts across forecasters is identified with expected inflation.

The author thanks the Federal Reserve Bank of Philadelphia for the use of the Livingston data. Thanks are also owed to Larry Kenny, Mark Rush, and especially David Denslow for helpful comments.

1. Although the method is often used to generate expected inflation, the most familiar application is by Barrow (1977) and Barro and Rush (1980) who generate an anticipated money growth series.

2. Although this mechanism was originally used in forecasting models of inflation (Engle 1982, 1983), it is now widely applied to financial markets. Bollerslev, Chu, and Kroner (1992) survey this extensive literature.

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Journal of Money, Credit, and Banking, Vol. 28, No. 3 (August 1996, Part 1)

More controversially, the variance across forecasters is used as a proxy for the uncertainty surrounding this forecast. There is no clear theoretical link between current disagreement and current uncertainty about the future, but a large number of papers have used this proxy.[3]

This paper examines the empirical validity of this assumed relationship for the Livingston data. Validity is assessed by precisely the criterion used in ARCH relationships: the conditional variance of forecast errors should be positively related to the disagreement among forecasters at the time of the forecast.

The results presented below indicate a significant and stable relationship between disagreement and uncertainty over the period 1946–94. Uncertainty, as measured by the conditional variance of inflation about an individual forecast, is approximately four times as large as disagreement, as measured by the variance of the individual forecasts about the mean. The large variation in disagreement over time corresponds to a comparable variation in uncertainty. Thus survey measures of disagreement provide a potentially useful basis for assessing the effects of uncertainty.

Section 1 discusses some issues of methodology. The data are summarized in section 2. Section 3 presents the empirical results. A concluding section follows.

1. UNCERTAINTY, ARCH, AND SURVEY MEASURES

A. ARCH Models and Uncertainty

Consider an individual, i, who provides a point forecast, p_{it}, of the inflation rate, π_t, during period t. The forecast leads to an error, $e_{it} = \pi_t - p_{it}$. For this individual, p_{it} could be labeled the expected inflation rate, and σ_{it}^2, the individual's perception of the variance of e_{it}, can be identified with inflation uncertainty. Researchers seeking to construct consensus measures of expected inflation and inflation uncertainty face two problems. First, p_{it} and σ_{it}^2 must be measured. Second, representative values p_t and σ_t^2, must be constructed from the individual values.

Engle (1982, 1983) solves both problems simultaneously. The procedure has been frequently repeated and generalized. A representative forecaster is presumed to be familiar with a model:

$$\pi_t = \boldsymbol{\alpha}\mathbf{x}_{t-1} + e_t; \qquad e_t \sim N(0, \sigma_t^2); \qquad \sigma_t^2 = \mathbf{a}\mathbf{z}_{t-1} \tag{1}$$

where $\boldsymbol{\alpha}$ and $\mathbf{a}$ are vectors of constants and $\mathbf{x}$ and $\mathbf{z}$ are vectors of predetermined variables known prior to period t ($\mathbf{z}$ includes lagged values of e^2). The econometrician estimates $\boldsymbol{\alpha}$ and $\mathbf{a}$ using the entire sample. The forecaster is presumed to know $\boldsymbol{\alpha}$ and $\mathbf{a}$. Thus, expected inflation and inflation uncertainty are given by

$$p_t = \hat{\boldsymbol{\alpha}}\mathbf{x}_{t-1} \text{ and } \sigma_t^2 = \hat{\mathbf{a}}\mathbf{z}_{t-1} . \tag{2}$$

3. These include Barnea, Dotan, and Lakonishok (1979), Bomberger and Frazer (1981), Hasbrouk (1984), Hendershott (1984), Holland (1984a, 1984b, 1986, 1993), Levi and Makin (1978, 1979), Makin (1982, 1983), Mullineaux (1978), Ratti (1985), and Wachtel (1977).

Despite the strong logic of the rational expectations concept, this approach has weaknesses. In particular, the resulting p_t and σ_t^2 are retrospective and model specific.

Survey results and common observation suggest that forecasts differ dramatically across forecasters at any given time. It is hard to account for this without assuming that forecasters use different models. Indeed, competing models fill the literature. Thus, uncertainty regarding the model which generates inflation may be a large part of inflation uncertainty. Conditional variances based one model are unlikely to capture this important phenomenon.

B. Survey Data and Uncertainty

The methodology described above contrasts with attempts to construct empirical counterparts for p_t and σ_t^2 from survey data. Survey data readily generate individual point forecasts of inflation p_{it}. Almost universally these are averaged across forecasters to construct a consensus forecast, p_t. This procedure is model free and involves no retrospective procedure.[4]

The NBER-ASA survey includes a forecast of the implicit GDP deflator and also asks forecasters about the distribution of possible outcomes. From these data individual measures of σ_{it}^2 can be generated with the aid of some assumptions. Both Lahiri, Tiegland, and Zaporowski (1988) and Zarnowitz and Lombros (1987), L-T-Z and Z-L hereafter, calculate values for σ_{it}^2. In both studies these are averaged over all forecasters for each period to arrive at a measure of σ_t^2.

It is unfortunate that such a measure is not available for the more widely used Livingston survey of CPI forecasts. The NBER-ASA data start more than twenty years after the Livingston data and the length of the forecast horizons depend (sometimes in an irregular way) on the quarter in which the forecast was made. Furthermore, the GDP deflator is of less general interest than the CPI.

Many studies have used the variance of point forecasts across forecasters, s_t^2, defined below, as a proxy for σ_t^2.

$$s_i^2 \equiv (1/n_t) \sum_i (p_{it} - p_t)^2 \tag{3}$$

where n_t is the number of respondents in period t. However, this measures disagreement, not uncertainty. Theoretical reasons for identifying one with the other are slim and the empirical validity of this identification has not been directly assessed. Only indirect evidence has been provided by comparisons of the s_t^2 from the Livingston survey with ARCH measures of σ_t^2 (Engle) or with contemporaneous survey measures of σ_t^2 (L-T-Z, Z-L).

This paper tests whether s_t^2 is a useful measure of the uncertainty of the Livingston point forecasts. The question asked is: "Are times at which the Livingston

4. However, the representative nature of the respondent is problematical, and can be viewed as a competing weakness. Anonymous forecasters may not have the same incentive to arrive at an optimal forecast as market participants.

forecasters disagree widely also times at which the errors generated from the Livingston forecasts seem to be drawn from a distribution with a larger variance?" This question can be addressed by examining e_{it} and s_t^2. These are readily calculated from individual forecasts and subsequent inflation. One can model the conditional variance of forecast errors without modeling the mechanism generating the forecasts.

C. Individual and Consensus Uncertainty

Before proceeding it is necessary to make a distinction between two concepts of uncertainty as derived from survey data. Uncertainty can be defined in relation to the mean forecast, p_t, and the subsequently observed mean error, e_t, or in relation to individual forecasts, p_{it}, and subsequently observed individual errors, e_{it}.

A measure of uncertainty of the first type answers the question, "What confidence would an observer have in the current mean forecast if he or she were familiar with the pattern of errors generated by such forecasts?". This is referred to as consensus uncertainty below and will be symbolized by θ^2. A measure of the second type answers the question, "What confidence would an observer have in a randomly selected individual forecast if he or she were familiar with the pattern of errors generated by such forecasts?" This is referred to as individual uncertainty below. It corresponds to the mean uncertainty of individual respondents in studies such as those by Z-L-T and Z-L and is identified with σ_t^2.

By construction the two measures are related. Using the following manipulation of the individual error:

$$e_{it} = \pi_t - p_t + p_t - p_{it} = e_t + p_t - p_{it} \, , \tag{4}$$

we calculate the average squared error of current forecasts:

$$\sum_i e_{it}^2/n_t = e_t^2 + 2\sum_i (p_t - p_{it})e_t/n_t + \sum_i (p_t - p_{it})^2/n_t = e_t^2 + s_t^2 \, . \tag{5}$$

Taking the conditional expectation of both sides,

$$\sigma_t^2 = \theta_t^2 + s_t^2 \, . \tag{6}$$

The uncertainty surrounding a (randomly drawn) individual forecast is greater than the uncertainty surrounding the mean. More concretely, an observer given a randomly drawn forecast, p_{it}, and knowing the current disagreement among point forecasts, s_t^2, would draw a confidence interval about the point forecast whose length increases with s_t^2 for a given θ_t^2. Thus, there is a positive effect of disagreement on individual uncertainty. Section 3 presents evidence of a positive relationship between consensus uncertainty, θ_t^2, and disagreement, s_t^2, which complements this effect.

2. DATA

Livingston respondents have provided forecasts published in June and December since 1946. We use forecasts of the level of the CPI for the next December and June, respectively. Using Carlson's (1977) assumption, that the most recent CPI levels available to the respondents were for April and October, responses are converted to expected annualized rates of inflation, p_{it}, for the overlapping eight-month periods April–December and October–June. The subsequently observed inflation, π_t, is the rate over the same period.

A total of 371 forecasters provided responses for 1946–94, generating 4,913 individual forecasts and 98 mean forecasts. For 1952–94, 348 forecasters generated 4,511 individual forecasts and 86 mean forecasts. Table 1 lists the data for the mean forecast, p_t, mean error, e_t, disagreement, s_t^2, and number of responses, n_t. As a measure of σ^2 in (6) the mean-squared error of individual forecasts is 15.4. The disagreement component (average of s^2) is 3.8 and the consensus component (average of e^2) is 11.6.

Two features of the data are revealed. First, sample averages are heavily influenced by large errors and disagreement in 1946–51 (MSE and its components are only 5.6, 3.8, and 1.8, respectively, for 1952–94). Below we see if relationships are unduly influenced by the first few observations. Second, consensus errors are much larger than disagreement, thus consensus uncertainty is the dominant component of individual uncertainty. If disagreement is to be good proxy for individual uncertainty, it must also track consensus uncertainty.

3. RESULTS

A. *Disagreement and Consensus Uncertainty*

The usefulness of disagreement as an indicator of consensus uncertainty is evaluated within the context of the specification

$$e_t \sim N(0, \theta_t^2) \qquad \theta_t^2 = a_0 + a_1 s_t^2 \,. \tag{7}$$

Table 2 presents maximum likelihood estimates of (7) (with various restrictions) for the full sample and the 1952–94 subsample.

Twice the difference in likelihoods between row 1 (homoskedastic case) and row 2 is 95.2. Under the null hypothesis that disagreement is unrelated to consensus uncertainty ($a_1 = 0$), this is distributed as a $\chi^2(1)$. The 1 percent significance value of 6.63 indicates a strong rejection of the null. The estimate of a_0 in row 2 is near zero suggesting that consensus uncertainty is *proportional* to disagreement. A comparison of rows 2 and 3 tests this conjecture. The restriction $a_0 = 0$ is not rejected ($\chi^2(1) = 1.4$ versus the 10 percent critical value 2.71).

TABLE 1

SUMMARY STATISTICS OF SURVEY DATA, 1946:1–1994:2

year	p	s^2	e	n	year	p	s^2	e	n
46	10.52	23.29	16.21	10		3.55	1.22	0.74	51
	−1.29	30.39	10.21	14	71	4.01	1.70	−0.32	45
47	−5.83	33.68	16.38	29		3.02	0.79	0.18	54
	2.51	56.47	4.81	33	72	3.59	0.99	0.08	50
48	−0.06	20.49	1.93	32		3.23	0.68	3.71	56
	−2.03	15.56	−1.41	35	73	3.99	1.70	5.09	53
49	−6.00	15.07	4.06	37		5.20	4.33	6.34	51
	−1.52	7.29	3.04	37	74	7.11	5.58	5.11	51
50	1.23	2.75	9.26	43		7.69	5.30	−0.15	54
	3.71	7.91	5.35	38	75	5.60	4.43	1.79	50
51	2.56	7.01	1.12	44		5.85	1.86	−0.80	51
	1.85	3.71	−0.09	50	76	5.30	1.65	0.17	46
52	0.32	3.44	1.36	46		5.00	0.82	2.42	49
	−0.14	5.26	0.54	55	77	5.92	1.82	−0.44	44
53	−0.99	1.77	2.57	42		5.98	1.48	2.91	55
	−1.28	4.03	0.89	54	78	6.39	2.41	2.64	47
54	−0.52	1.58	0.12	50		6.97	3.02	4.97	47
	0.16	1.18	−0.23	47	79	8.30	5.44	5.01	46
55	0.55	0.67	0.15	51		10.15	5.49	4.98	44
	0.75	1.07	0.97	55	80	10.38	2.75	−0.39	43
56	0.35	0.82	3.74	50		10.51	6.67	−0.05	54
	1.44	3.15	1.78	50	81	8.58	3.57	−0.19	52
57	1.06	1.24	1.81	55		6.95	4.79	−1.17	48
	0.06	2.61	3.15	60	82	5.32	3.73	−1.02	49
58	0.06	1.45	0.16	58		4.66	2.30	−2.60	52
	0.66	0.42	0.36	61	83	4.32	3.52	−0.25	58
59	0.58	1.12	1.31	60		5.05	1.30	−1.01	54
	0.99	0.57	0.23	56	84	5.21	1.59	−1.93	54
60	0.46	0.44	1.11	51		4.17	3.12	−0.82	56
	0.23	0.59	0.12	60	85	4.17	1.00	−0.73	53
61	1.00	0.38	−0.23	57		3.50	1.01	−2.39	54
	1.04	0.30	−0.13	62	86	3.18	1.60	−0.48	51
62	0.99	0.44	−0.16	59		3.26	0.97	1.13	60
	1.02	0.46	−0.13	61	87	4.05	1.48	−0.43	55
63	0.99	0.28	0.95	56		3.74	1.23	−0.11	60
	0.87	0.20	0.22	60	88	4.31	1.05	0.08	51
64	1.12	0.46	0.31	56		4.52	1.81	0.39	44
	1.26	0.38	0.97	59	89	4.86	1.94	−1.17	59
65	0.95	0.37	1.40	53		3.83	0.90	1.34	51
	1.54	0.49	1.86	64	90	3.89	0.66	1.86	62
66	1.80	0.94	1.13	48		3.90	1.45	−1.07	58
	2.04	1.57	−0.05	59	91	3.54	0.78	−0.52	64
67	2.12	0.66	1.71	52		3.07	0.71	0.00	41
	2.61	0.99	1.78	56	92	3.47	0.62	−0.87	42
68	3.06	1.11	1.68	54		3.10	0.53	−0.35	37
	2.72	1.07	3.05	57	93	3.26	0.50	−1.39	44
69	3.15	2.24	2.69	42		2.62	0.43	−0.24	51
	3.57	1.21	2.69	48	94	3.01	0.48	−0.66	54
70	3.54	2.26	1.52	48		3.32	0.78	−0.30	44

If aggregate uncertainty is proportional to disagreement, individual uncertainty is proportional as well. To demonstrate this, combine (6) with (7):

$$\sigma_t^2 = \theta_t^2 + s_t^2 = a_1 s_t^2 + s_t^2 = 3.65 s_t^2 \tag{8}$$

(based on the full sample estimate $a_1 = 2.65$). Thus individual uncertainty is approximately four times as great as disagreement using variances for each, or twice

TABLE 2

Disagreement as a Measure of Consensus Uncertainty

Estimates of $e_t \sim N(0, \theta_t^2)$ $\theta_t^2 = a_0 + a_1 s_t^2$

	a_0	a_1	log L
1946–94	11.57 (0.68)	0	−259.1
1946–94	−0.57 (0.30)	3.22 (0.49)	−211.5
1946–94	0	2.65 (0.30)	−212.2
1952–94	3.82 (0.43)	0	−179.7
1952–94	−0.19 (0.47)	2.52 (0.65)	−168.0
1952–94	0	2.31 (0.31)	−168.0

as great using standard deviations for each. This is consistent with the results of Z-L and L-T-Z who found that average individual uncertainty was larger than disagreement for forecasts of the GDP deflator. In particular, if one examines the results for two quarter forecasts of the GDP deflator which Z-L compare to the Livingston forecasts, their average value for σ_t^2 is between three and four times their average value for s_t^2. L-T-Z arrive at much higher values for s_t^2 from the same data, and therefore indicate a lower ratio of σ_t^2 to s_t^2.

A comparison with ARCH measures is also instructive. Engle points out that ARCH variances are larger than the Livingston disagreement measures for comparable periods despite the fact that the ARCH variances are based on a shorter, quarterly horizon (p. 297). This is not particularly meaningful since disagreement is, at best, a proxy for uncertainty, not a direct measure. If one accepts disagreement as a proxy for an underlying uncertainty four times as large, survey measures of uncertainty are larger than Engle's reported ARCH measures. This is consistent with the longer horizon and the fact that uncertainty with respect to model selection and parameters are not implicitly ruled out as in the ARCH procedure.

More importantly, the survey measure of uncertainty is more variable than the ARCH measures. Figure 1 displays graphs of the survey measure ($3.65s^2$) and the conditional variances reported by Engle (1983, pp. 298–300). Table 3 reports summary statistics of the two measures over the period for which the ARCH measure is available. The survey measure exhibits a higher ratio of its standard deviation to its mean, a lower minimum, and a higher maximum. This pattern is clearly exaggerated in the pre-1952 period. Therefore statistics are also reported for a sample that excludes those observations.[5] The greater variability of the survey measure is reduced for this period but is still evident. Thus, if one accepts the survey measures as valid, ARCH measures underestimate the variation in uncertainty over time.[6]

Returning to Table 2, there is no evidence that the very large levels of disagree-

5. The ARCH procedure is at a disadvantage during 1947–51. This is the beginning of the sample and the estimate conditional variances depend on arbitrary assumptions about initial values.

6. Disagreement can also be used to address another puzzle regarding survey forecasts, their deviation

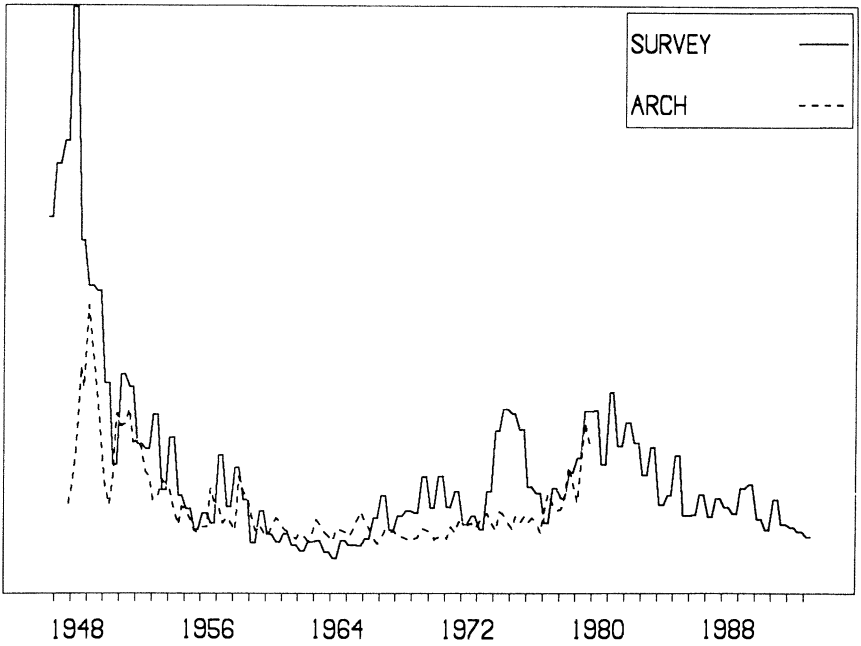

FIG. 1. Survey and ARCH Measures of Uncertainty

ment in the period before 1952 tend to distort the overall relationship. Assuming homoskedasticity, measured uncertainty falls by more than 60 percent if we exclude the pre-1952 period (comparing a_0 in row 1 with row 4). However, the estimated parameters of the disagreement-uncertainty relationship do not change significantly (comparing a_1 in rows 3 and 6). This is fortunate since 1946–51 seems to provide an interesting period for examining the effects of uncertainty.

C. *Disagreement and ARCH Models*

The relationship between disagreement and subsequent errors is suggestive, but the lack of a theoretical rationale suggests that care be taken to see if this relation-

from "rationality." If consensus errors are regressed on their lagged values, the constant term (bias) and lag coefficient (serial correlation) are large:

$$e_t = \underset{(.74)}{1.46} + \underset{(.06)}{.41}e_{t-1}; \qquad \sigma^2 = \underset{(.75)}{8.07}; \qquad \text{Log}\, L = -242.3.$$

This result is consistent with many previous estimates. However, when disagreement is used to correct for heteroskedasticity in the relationship, the bias and serial correlation are significantly reduced:

$$e_t = \underset{(.23)}{.69} + \underset{(.12)}{.27}e_{t-1}; \qquad \sigma_t^2 = \underset{(.44)}{2.14}s_t^2; \qquad \text{Log}\, L = -201.3.$$

TABLE 3

THE VARIABILITY OF SURVEY AND ARCH MEASURES OF UNCERTAINTY

	1947:4–1979:4				1952:1–1979:4			
	mean	stdv	min	max	mean	stdv	min	max
ARCH	6.2	7.7	1.6	54.1	4.1	3.1	1.6	19.0
Survey	14.0	29.6	0.7	206.7	6.3	5.4	0.7	20.4

ship is spurious. This section examines an alternative specification with ARCH effects. That is, the possibility that consensus errors are distributed with a variance that depends on lagged consensus errors is examined.

The motivation is twofold. First, ARCH may provide a useful model of the heteroskedasticity in survey errors. Hence, an ARCH measure could provide an additional conditioning variable for the variance of consensus errors. Second, the apparent role of disagreement may be due to its ability to mimic the underlying ARCH effects.

The results in Table 4 provide evidence on both possibilities. These conjectures can be evaluated within a general specification:

$$e_t \sim N(0, \theta_t^2);\ \theta_t^2 = a_0 + a_1 s_t^2 + a_2 s_{t+1}^2 + a_3 \theta_{t-1}^2 + a_4 e_{t-1}^2 \,. \tag{9}$$

If $a_2 = a_3 = a_4 = 0$, this is equivalent to (6). If $a_1 = a_2 = 0$, this is a GARCH relationship (Bollerslev 1986). Estimates of the GARCH are reported in row 1. A comparison with the results in the first row of Table 2 indicates that GARCH effects are significant (a statistic of 26.8 compared to 1 percent threshold for $\chi^2(2)$ of 9.21).

Row 2 reports estimates in which leading disagreement is added to contemporaneous disagreement as a conditioning variable for the variance of consensus errors. A comparison with row 2 of Table 2 indicates that leading disagreement is quite significant (the value for $\chi^2(1)$ is 19.6). Presumably, this reflects causality running from current errors to future disagreement.[7]

These two results suggest that disagreement may seem useful because of relationships that run from past consensus errors to current consensus errors (GARCH), and from past consensus errors to current disagreement. This conjecture can be assessed using the results reported in row 3 where current disagreement is added to GARCH. Compared to row 1, the addition of disagreement provides a significant improvement in the likelihood. As compared to the results in row 2 of Table 3, the addition of GARCH effects provides no explanatory power when disagreement is already included.

Rather than disagreement mimicking GARCH, the relationship seems to go the other way. Current disagreement provides useful information on consensus uncertainty, presumably because both quantities reflect some underlying, unobserved

7. Other estimates (not shown) indicate that lagged disagreement has no effect on the variance when current disagreement is included as a conditioning variable. This is not necessarily to be expected since we have no theory to suggest that only current disagreement is related to uncertainty.

TABLE 4

Disagreement versus ARCH as Measures of Consensus Uncertainty
Estimates of $e_t \sim N(0, \theta_t^2)$ $\quad \theta_t^2 = a_0 + a_1 s_t^2 + a_2 s_{t+1}^2 + a_3 e_{t-1}^2 + a_4 \theta_{t-1}^2$

a_0	a_1	a_2	a_3	a_4	Log L
.04	0	0	.39	.76	−243.4
(.68)			(.19)	(.08)	
−.21	−.68	2.94	0	0	−191.1
(.14)	(.37)	(.77)			
−.54	2.95	0	−.04	.10	−211.2
(.37)	(1.08)		(.15)	(.15)	

characteristic of the period in question. If disagreement were excluded, the process might be interpreted as GARCH due to serial correlation in disagreement.

D. Livingston Disagreement and Engle's Model

The evidence above indicates that disagreement is a useful measure of uncertainty. This result is not model specific, but it is specific to the forecast errors of this survey. To provide some evidence of its more general usefulness we examine the interaction of the Livingston disagreement measure and the model Engle (1983) used to generate ARCH measures of uncertainty with U.S. data.

Toward this end, estimates of Engle's model are reported in Table 5. Quarterly inflation, π_t, is a linear function of π_{t-1}, π_{t-2}, lagged import price inflation, i_{t-1}, lagged wage inflation, w_{t-1}, lagged M1 growth, m_{t-1}, and a trend. The error variance is a linear function of a weighted average of past squared errors, q_t^2.

$$v_t \sim N(0, \gamma_t^2); \qquad \gamma_t^2 = a_0 + a_1 q_t^2 \qquad q_t^2 = \sum_k^8 (9 - k) e_{t-k}^2 / 36. \tag{10}$$

Row 1 contains a reestimation of Engle's model applied to CPI data through 1994:4. The results are similar to Engle's (p. 291).[8]

Row 2 presents results from a specification that substitutes the Livingston disagreement measure, s_t^2, for the ARCH term, q_t^2.[9]

Estimates that allow terms in s_t^2 *and* q_t^2 are reported in row 3. s_t^2 accounts for heteroskedasticity better than past errors from the same model, q_t^2. Adding the Livingston measure to the ARCH model produces a significant improvement in the likelihood (rows 1 and 2). ARCH adds nothing once errors are conditioned on the Livingston measure (rows 2 and 3).[10]

8. Engle only reports parameter estimates for the GDP deflator version of the model. However, he states that the results were similar for the CPI and PPI versions.

9. Since s^2 is only available on a semiannual basis, it is inserted for both quarters included in the forecast horizon. The lag this introduces and the mismatch in forecast horizons puts the Livingston specification at a disadvantage in a quarterly model.

10. The hypothesis $a_2 = 0$ is rejected at a high level of confidence while $a_1 = 0$ is not rejected.

TABLE 5

ENGLE'S MODEL WITH ALTERNATIVE HETEROSKEDASTICITY MEASURES

$$\pi_t = \alpha_0 + \alpha_1\pi_{t-1} + \alpha_2\pi_{t-2} + \alpha_3 i_{t-1} + \alpha_4 w_{t-1} + \alpha_5 m_{t-1} + \alpha_6 t + u_t$$
$$u_t \sim N(0, \sigma_t^2) \qquad \sigma_t^2 = a_0 + a_1 q_t^2 + a_2 s_t^2$$

α_1	α_2	α_3	α_4	α_5	α_6	a_0	a_1	a_2	Log L
0.36	0.22	0.08	0.02	0.07	0.00	1.54	0.81	—	−429.7
(.10)	(.06)	(.02)	(.07)	(.05)	(.01)	(.70)	(.20)		
0.34	0.26	0.10	−0.01	0.08	0.00	1.28	—	2.14	−422.5
(.08)	(.07)	(.02)	(.07)	(.04)	(.01)	(.63)		(.51)	
0.35	0.28	0.09	−0.01	0.08	0.00	1.19	0.22	1.47	−421.7
(.09)	(.08)	(.02)	(.07)	(.04)	(.01)	(.62)	(.21)	(.67)	

4. CONCLUSION

Livingston forecaster disagreement is a potentially useful proxy for uncertainty. The variance of subsequent forecast errors is proportional to the cross-sectional variance of forecasts. The implied uncertainty is about four times as large as disagreement and is larger and more variable than ARCH measures over comparable periods. It is also consistently available from 1946 while ARCH is unreliable for the interesting immediate postwar period.

The relationship between disagreement and uncertainty is not due to an underlying ARCH mechanism. Indeed, disagreement tracks the uncertainty in forecasts better than ARCH and there is no evidence of remaining ARCH effects once disagreement is included. More surprisingly, disagreement dominates the ARCH mechanism within the context of the original quarterly ARCH model of U.S. inflation.

Absent from this study is an attempt to explain the source of the observed relationship. That requires a model of forecast formation explaining why forecasters disagree, and disagree more at some times than at others. I am aware of no model that has been convincingly applied to this endeavor.

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