Three-Period Model

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Dissertation Project

Introduction to the Model

To provide an economic intuition of the differential response of **cash holdings** to uncertainty and financial shocks, I present a properly augmented model in the spirit of

- Almeida, Campello, and Weisbach (2004)
- Han and Qiu (2007)

It is a simple representation of a dynamic setting where a profit-maximizing firm have

- present and future investment opportunities
- current cash flow and external sources of finance might not be enough to fund all profitable projects

Main Features

- Model has three periods: 0, 1 and 2.
- There is one representative firm (or a continuum of it)
- Discount factor $\beta = 1$, but it can easily be relaxed
- In P0 firm can invest I_0 in a long-term project.
  - I_0 pays a deterministic return $g(I_0) = G(I_0) + qI_0$ in P2
- In P1 firm can invest I_1 in a short-term project.
  - I_1 pays a deterministic return $h(i_0) = H(i_0) + qI_0$ in P2
- Both $G(\cdot)$ and $H(\cdot)$ - and thus $g(\cdot)$ and $h(\cdot)$ - display the following properties
  - $G'(\cdot)$ and $H'(\cdot)$ strictly positive
  - $G''(\cdot)$ and $H''(\cdot)$ strictly negative
  - $G'''(\cdot)$ and $H'''(\cdot)$ strictly positive

Period 0

- Firm enters the period with c_0 internal liquidity from past and current cash flows
- Firm chooses optimal level of investment I_0, cash holding C, and borrowing B_0
- Optimal choices are subject to nonnegative dividends constraint,

$$d_0 = c_0 + B_0 - I_0 - C \geq 0$$

and borrowing constraint

$$0 \leq B_0 \leq (1 - \tau) q I_0$$

where

  - $q \in (0, 1]$ is the part of I_0 that can be liquidated after usage
  - $1 - \tau$ is the part of the liquidation value of I_0 that can be captured by creditors

Period 1

- Firm enters the period with $C + c_1$ internal liquidity where
  - C is optimal level of cash holding chosen in P0
  - $c_1 \sim F[\underline{c_1},\ \overline{c_1}] \geq 0$ is current cash flow
  - c_1 is unknown in P0 and drawn at the beginning of P1

- Firm chooses optimal schedules of both investment $I_1(c_1)$ and borrowing $B_1(c_1)$

- Optimal choices are subject to nonnegative dividends constraint,

$$d_1 = c_1 + B_1(c_1) - I_1(c_1) + C \geq 0$$

and borrowing constraint

$$0 \leq B_1(c_1) \leq (1 - \tau_1) q I_1(c_1)$$

Period 2

- Firm receives deterministic returns $g(I_0)$ and $h(I_1(c_1))$
- Firm pays back loans B_0 and $B_1(c_1)$
- Dividends are defined as

$$d_2 = g(I_0) + h(I_1(c_1)) - B_0 - B_1(c_1)$$

Firm's Problem

$$\max_{C, I_0, B_0, I_1(c_1), B_1(c_1)} d_0 + d_1 + d_2 \tag{1}$$

subject to

$$\begin{aligned}
d_0 &= c_0 + B_0 - I_0 - C \geq 0 \\
d_1 &= c_1 + B_1(c_1) - I_1(c_1) + C \geq 0 \\
d_2 &= g(I_0) + h(I_1(c_1)) - B_0 - B_1(c_1) \\
0 &\leq B_0 \leq (1-\tau) q I_0 \\
0 &\leq B_1(c_1) \leq (1-\tau) q I_1(c_1)
\end{aligned} \tag{2}$$

Solution - Unconstrained Firms

A firm is financially unconstrained if it has enough financial resources such that

$$g'(I_0^*) = 1$$

and

$$h'(I_1^*(c_1)) = h'(I_1^*) = 1 \quad \forall\ c_1 \in [\underline{c_1},\ \overline{c_1}]$$

- Both I_0^* and I_1^* are independent of c_1
- Firm is indifferent on optimal C^*, B_0^*, and B_1^*

Solution - Constrained Firms (I)

A firm is financially constrained if its investment levels are always lower than the first-best levels I_0^* and I_1^*.

Constraints are always expected to bind since it is not profitable to

- paying out dividends in the first two periods
- borrowing less than the maximum amount

Solution - Constrained Firms (II)

Since constraints bind,

$$I_0 = \frac{c_0 - C}{1 - q + q\tau} \quad \text{and} \quad I_1(c_1) = \frac{c_1 + C}{1 - q + q\tau}$$

Firm faces now a trade of on choosing optimal cash holding C^*

- the cost of saving an additional unit of cash holding C is forgoing a unit of current investment projects
- the benefit of saving an additional unit of cash holding C is the higher ability to fund future investment projects

Solution - Constrained Firms (III)

Optimal $C^*(0, F)$ should be chosen to maximize

$$\begin{aligned}
\text{Objective} &= g(I_0) - I_0 + \mathbb{E}\left[h(I_1) - I_1 | F\right] \\
&= g\left(\frac{c_0 - C}{1 - q + q\tau}\right) - \frac{c_0 - C}{1 - q + q\tau} \\
&+ \mathbb{E}\left[h\left(\frac{c_1 + C}{1 - q + q\tau}\right) - \frac{c_1 + C}{1 - q + q\tau}\middle| F\right]
\end{aligned} \tag{3}$$

Solution for $C^*(0, F)$ solves

$$g'\left(\frac{c_0 - C^*(0, F)}{1 - q + q\tau}\right) = \mathbb{E}\left[h'\left(\frac{c_1 + C^*(0, F)}{1 - q + q\tau}\right)\middle| F\right] \tag{4}$$

where SOC is negative by assumption on $G(\cdot)$ and $H(\cdot)$.

Analysis

I analyze the effect of an uncertainty and credit supply shocks in period 0 in order to see the differential response of cash holding C^*

Uncertainty Shock. Mean-preserving spread of perceived distribution F of c_1.

Credit Supply Shock. Decrease by ε of the part of the liquidation value of I_0 that can be captured by creditors,

$$B_0 \leq (1 - (\tau + \varepsilon))qI_0$$

Uncertainty Shock (I)

Assume that $C^*(0, F)$ is the optimal cash holding when distribution of c_1 is F and no credit supply shocks are in place.

Then the following two relations hold,

$$g'\left(\frac{c_0 - C^*(0,F)}{1-q+q\tau}\right) = \mathbb{E}\left[h'\left(\frac{c_1 + C^*(0,F)}{1-q+q\tau}\right)\middle| F\right] \tag{5}$$

and

$$\mathbb{E}\left[h'\left(\frac{c_1 + C^*(0,F)}{1-q+q\tau}\right)\middle| Q\right] > \mathbb{E}\left[h'\left(\frac{c_1 + C^*(0,F)}{1-q+q\tau}\right)\middle| F\right] \tag{6}$$

where

- Equation 5 holds because $C^*(0, F)$ is optimal cash flow with F
- Equation 6 holds because of Jensen's inequality.

Combining 5 and 6 yields

$$g'\left(\frac{c_0 - C^*(0,F)}{1-q+q(\tau+\varepsilon)}\right) > \mathbb{E}\left[h'\left(\frac{c_1 + C^*(0,F)}{1-q+q\tau}\right)\middle| F\right] \tag{7}$$

which implies that $C^*(0,Q) > C^*(0,F)$.

Result. In response to an uncertainty shock firm tends to accumulate additional cash holding from the motive of precautionary savings.

Credit Supply Shock (I)

Given $C^*(0, F)$, then the following two relations hold,

$$g'\left(\frac{c_0 - C^*(0,F)}{1 - q + q\tau}\right) = \mathbb{E}\left[h'\left(\frac{c_1 + C^*(0,F)}{1 - q + q\tau}\right)\middle| F\right] \tag{8}$$

and

$$g'\left(\frac{c_0 - C^*(0,F)}{1 - q + q(\tau + \varepsilon)}\right) > g'\left(\frac{c_0 - C^*(0,F)}{1 - q + q\tau}\right) \tag{9}$$

where

- Equation 8 holds because $C^*(0, F)$ is optimal cash flow with F
- Equation 9 holds because of Jensen's inequality.

Credit Supply Shock (II)

Combining 8 and 9 yields

$$g'\left(\frac{c_0 - C^*(0,F)}{1-q+q\tau}\right) < \mathbb{E}\left[h'\left(\frac{c_1 + C^*(0,F)}{1-q+q\tau}\right)\middle| Q\right] \tag{10}$$

which implies that $C^*(\varepsilon, F) < C^*(0, F)$.

Result. In response to a negative credit supply shock firm tends to accumulate less cash to finance current projects which are more financially constrained.

Lesson learned

As a precautionary motive, firms prefer to increase cash holdings if they expect future outcomes to be more volatile.

After a financial shock, firms substitute more cash today with less cash tomorrow because they expect to me more financially constrained today than tomorrow.