The Value of Financial Flexibility

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ABSTRACT

We develop a model that endogenizes dynamic financing, investment, and cash retention/payout policies in order to analyze the effect of financial flexibility on firm value. We show that the value of financing flexibility depends on the costs of external financing, the level of corporate and personal tax rates that determine the effective cost of holding cash, the firm's growth potential and maturity, and the reversibility of capital. Through simulations, we demonstrate that firms facing financing frictions should simultaneously borrow and lend, and we examine the nature of dynamic debt and liquidity policies and the value associated with corporate liquidity.

RECENT SURVEYS OF AMERICAN AND EUROPEAN CFOs suggest that the most important driver of firms' capital structure decisions is the desire to attain and preserve financial flexibility.[1] Financial flexibility represents the ability of a firm to access and restructure its financing at a low cost. Financially flexible firms are able to avoid financial distress in the face of negative shocks, and to readily fund investment when profitable opportunities arise. While a firm's financial flexibility depends on external financing costs that may reflect firm characteristics such as size, it is also a result of strategic decisions made by the firm related to capital structure, liquidity, and investment. In this paper, we explore how firms should optimally manage their financial flexibility in the face of various transaction costs and taxes, and in turn examine the value of financial flexibility under different conditions.

We particularly focus on the strategic management of corporate liquidity and its relationship with the firm's financing and investment policies. A pervasive, and perhaps puzzling, aspect of corporate financial policy is that most firms that employ debt financing simultaneously hold cash balances. While equivalent borrowing and lending positions offset each other from a tax perspective, there may be other reasons why different combinations of debt and cash positions that lead to the same net debt value are not necessarily neutral permutations. We

[1] See Graham and Harvey (2001), Brounen, de Jong, and Koedijk (2004), and Bancel and Mittoo (2004).

show that transaction costs such as debt issuance costs can explain this finding, and we systematically analyze optimal liquidity policies and their resulting effects on firm value.

In order to properly capture the management of financial flexibility, we construct a dynamic structural model of the firm. Dynamic models have two important features that result in more realistic characterizations of firm decision making than do static models. First, they recognize that a firm's investment and financing decisions are marginal decisions that depend on the firm's current state. This state reflects not only the current levels of uncertain variables such as profitability, but also the firm's current financial structure and capital in place, which are a result of past decisions taken along a particular path of uncertainty resolution. Second, this intertemporal link between decisions also means that financial and investment decisions should be forward-looking in nature. In other words, the impact of current decisions on the firm's future states and corresponding state-dependent decisions are considered when making decisions today. These two features of dynamic models capture the complex link that exists between investment and financing decisions over time, one that becomes particularly interesting in the presence of transaction costs such as security issuance costs, taxes, and distress costs.

We build on the model of Hennessy and Whited (2005), which has a rich set of features including endogenous investment, financing *and* payout decisions, graduated corporate taxes, investor taxes on interest and equity distributions, equity issuance costs, and financial distress costs (a fire-sale discount on capital).[2] However, we relax three key assumptions, which generates our distinct results. First, we separately control for the borrowing and lending decisions of the firm rather than tracking only the net debt balance of the firm. Second, we introduce an issuance cost for debt. Third, capital is sold at a discount to its depreciated value. The first two features allow us to address the simultaneous existence of debt and cash balances in firms, while the third feature allows us to explore the interactions between financial and investment flexibility under the more realistic assumption of partial reversibility.

We show that the presence of debt issuance costs leads firms to retain cash even while having debt outstanding. In times of low profitability, when the firm wishes to decrease its net debt position to avoid triggering financial distress costs, the firm should increase its cash balance rather than paying down debt. Since the firm may later wish to restore its net debt to a higher level to take

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[2] Cooley and Quadrini (2001) have a similar model structure, though they exclude taxes and impose a rate of return shortfall on corporate cash that induces firms to pay out dividends rather than save cash, which creates an upper bound on cash and equity. They do, however, incorporate risky debt, as do the more recent works of Hennessy and Whited (2007), Obreja (2006), and Moyen (2007). Other papers that endogenize both dynamic financing and investment policies include Brennan and Schwartz (1984), Mauer and Triantis (1994), Gomes (2001), Titman and Tsyplakov (2007), and Tserlukevich (2005). However, none of the models in these papers attempt to endogenize the firm's liquidity policy (or equivalently its payout policy), even treating liquidity as negative debt as do Hennessy and Whited (2005) and Cooley and Quadrini (2001).

advantage of interest tax shields, it will be better off paying out cash to shareholders at that time rather than issuing new debt and incurring issuance costs. The implication of this insight is that different combinations of cash and debt that produce the same net debt level may lead to significantly different firm values, which we illustrate through simulations.

We also examine the marginal benefit of cash, which reflects the relative benefit of avoiding issuance and financial distress costs versus the tax disadvantage of cash being held by the firm rather than by investors who are subject to lower tax rates. We illustrate how this tradeoff results in an interior solution for the optimal liquidity of the firm.

We quantify the value of financial flexibility by comparing firm values with and without security issuance costs, and show how the value of financial flexibility depends on taxes, growth opportunities, profitability, and reversibility of capital. Costly external financing has a relatively small negative impact on the value of a mature firm that continues to contract and expand its capacity in response to productivity shocks, but can usually finance its investment internally. Allowing the firm to manage its cash balance can significantly alleviate the impact of external financing costs, though this depends critically on the size of the tax disadvantage associated with cash holdings.

The effect of financial flexibility on firm value can be quite large, however, when there is significant opportunity for growth on the upside, or when the firm is performing poorly on the downside. High volatility in the firm's profitability thus magnifies the value of financial flexibility. We also find that firms with more flexible capital can partially compensate for costly external financing, indicating that investment and financial flexibility are substitutes to some extent.

Finally, we simulate a large cross-section of firms based on optimal investment, financing, and payout policies in order to examine the evolution of firm dynamics, and highlight several differences between young and mature firms in terms of their financing, liquidity, and payout policies, as well as firm characteristics such as leverage and cash to value ratios. We also provide a measure of financial slack and illustrate how firms with higher risk manage their financing and liquidity decisions in order to preserve more slack.

Two recent papers on corporate liquidity examine issues that are closely related to those in our paper. Acharya, Almeida, and Campello (2007) also examine why cash is not the same as negative debt. Their model emphasizes that cash is retained when investment opportunities are likely to occur in low cash flow states and the firm has external financial constraints, whereas if investment opportunities occur in high cash flow states, cash flow is directed toward paying down debt. In our setting, which incorporates additional features such as flexible investment, taxes, distress costs, and equity issuance costs, we find that cash flow is frequently used to increase a firm's liquidity even though investment opportunities are perfectly correlated with cash flow.[3]

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[3] Kim, Mauer, and Sherman (1998) also examine the interplay between financing and liquidity. Their three-period model imposes a rate of return shortfall on lending relative to borrowing in

Faulkender and Wang (2006) empirically examine the marginal value of liquidity for constrained firms. Their findings are consistent with our results: The marginal value of liquidity is higher for firms with lower liquidity, greater investment opportunities, and higher external financing constraints. However, they do not explicitly examine the impact of taxes and distress costs, which we find to have a significant effect on firms' liquidity decisions.[4]

Finally, we should note that agency issues are absent from our model. The level of liquidity and the net benefit of financial flexibility that result from our model are likely to be overstated if managers are tempted to opportunistically exploit this flexibility for their own private benefit. Several empirical papers, including Dittmar, Mahrt-Smith, and Servaes (2003), Harford (1999), Kalcheva and Lins (2007), Pinkowitz, Stulz, and Williamson (2006), and Mikkelson and Partch (2003), find that excess cash can lead to value decreasing decisions, and that the market value of cash reserves is lower when firms are poorly governed and there is weak shareholder protection.[5] Debt agency problems could also affect the firm's financial policy.[6] In our model, managers maximize shareholder value and debt is riskless, and thus no agency problems arise.

Section I presents a simple example to illustrate the intuition behind our key results. Section II develops our full model. Section III describes the numerical implementation of the model. Section IV provides results related to the value and management of financial flexibility. Section V summarizes our key findings.

I. A Simple Example

To illustrate the essence of our results, we construct a simple three-period (3-year) model with some of the key features found in our general model. The firm begins with one unit of capital ($K = 1$), and maintains this level throughout the 3-year period (with no depreciation and no liquidation value), though it has an option to expand its capacity at the beginning of the third year, as will be

order to derive an internal solution for liquidity, whereas we attain an internal solution based on the tax structure in our model. Our model also includes various other features, particularly flexible investment, and we analyze the management and value of financial flexibility in greater depth.

[4] Billett and Garfinkel (2004) examine the interaction between liquidity and the cost of external access to capital for U.S. bank holding companies. They find that banks with lower access costs have greater value and hold fewer liquid assets. Sapriza and Zhang (2004) address the impact of financial flexibility on firm value by estimating the difference in value between a constrained and an unconstrained firm in a simulated model. However, they do not allow the firm to manage its financial flexibility through an internal cash balance, and they do not explore how investment flexibility interacts with financial flexibility.

[5] In contrast, Opler et al. (1999) find little evidence that excess cash leads to managerial agency problems. Rather, they find support for a more traditional static tradeoff model of cash holdings related to factors such as growth opportunities, risk, and access to external financing, which we capture in our model.

[6] Debt agency problems have been examined in a dynamic setting by Mello and Parsons (1992), Childs, Mauer, and Ott (2005), Titman and Tsyplakov (2007), and Moyen (2007). None of these papers, however, explicitly models the firm's liquidity policy.
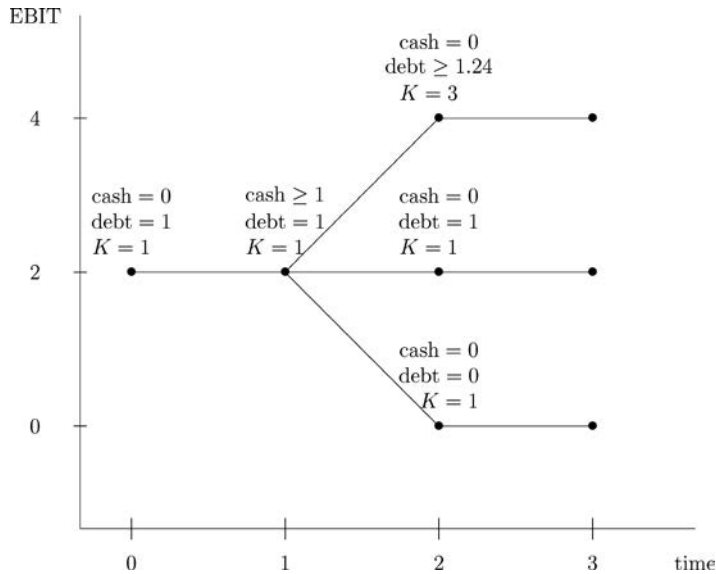
Figure 1. Three-period example. This figure shows the evolution of EBIT over the three periods in our simple example. There is uncertainty only during the second period. The firm begins with capital (K) equal to one unit, and with no debt or cash. At $t = 1$, the firm leaves its debt level unchanged, and decides to save either all of its after-tax cash flow of $1.56, or only $1 (see text for more details). At the beginning of the last period ($t = 2$), the firm makes the following decisions: If $EBIT = 4$, the firm adds two more units of capacity and increases its debt financing to either $1.24 or $1.80 (depending on if it retained $1.56 or $1 at $t = 1$, respectively); if $EBIT = 2$, the firm leaves its debt level unchanged and pays out all its cash to shareholders; if $EBIT = 0$, the firm uses its cash to pay off its debt.

described later. This unit of capital produces a known earnings before interest and taxes (EBIT) equal to $2 at the end of the first year, and an uncertain EBIT in the second and third years of $0, $2, or $4 (see Figure 1). Furthermore, we assume that the EBIT in the third year is identical to the EBIT in the second year, that is, there is uncertainty resolution only during the second year as to whether profitability in the last 2 years will be low, medium, or high.

The firm has one dollar of debt at $t = 0$. We assume that the firm must always be able to make its principal and coupon payments, and thus the debt is risk-free. The debt matures at the end of the 3-year horizon, but may be paid back (at par value) at any time. The risk-free rate (and thus the firm's cost of debt) is 5%. The firm begins with a zero cash balance at $t = 0$, but it can choose to retain cash at a later date.

Borrowing is motivated by an interest tax shield. We assume a corporate tax rate of 20% on earnings net of interest, and no personal taxes (one can alternatively view the 20% as the relative tax differential between corporate and personal taxes). Out of the $EBIT = 2$ at $t = 1$, the firm will pay $0.05 of interest (generating a tax shield of $0.20 \times 0.05 = 0.01$) and $0.39 (= 0.20 \times (2 - 0.05))$ of tax, leaving a net income (free cash flow) of $1.56. The firm has three

choices for how to deploy this cash: (1) pay it out to shareholders; (2) pay down the debt; or (3) retain as cash inside the firm.

In making this cash deployment decision at $t = 1$, the firm needs to be forward-looking. If there is a negative shock to profitability during the second year, the firm will have zero EBIT for the next 2 years. Given that there is debt outstanding, the firm cannot simply pay out the first-year earnings to its shareholders. Rather, it needs to take one of two precautionary measures: Either pay down its debt, or retain one unit of cash, investing it in a risk-free security that can be used to pay off the debt at the end of the second year if the firm ends up in the zero EBIT state. So far, it would seem that the firm would be indifferent between these two choices, since they both result in a zero net debt balance and thus avoid default in the lowest profitability state.

However, consider what happens when debt is costly to issue. In either the medium or high EBIT states in the last 2 years, the firm could support positive net debt and would gain from having a debt tax shield. If the firm has chosen at $t = 1$ to keep its debt and cash levels both at one, then it can simply pay out its cash balance at $t = 2$ when EBIT is positive, thus costlessly restoring its net debt level back to one. If the firm instead has paid off its debt at $t = 1$, it would need to issue new debt at a cost. Thus, the firm is clearly better off by keeping a cash balance at $t = 1$ rather than paying off debt and then reissuing it later.

Now recall that the firm generated $1.56 of cash at the end of the first period. While we have provided a rationale for saving $1 of this cash from a precautionary standpoint, what about the remaining $0.56? It may be useful to save this additional cash in case the firm has a profitable investment opportunity that can be financed internally rather than facing issuance costs associated with external financing. But, there is also a tax disincentive to holding cash, since (in the present example) interest is taxed at the corporate level, but not at the personal level.

To illustrate, assume that the firm is able to add two more units of capital at $t = 2$ at a cost of $2.5 per unit. When $EBIT = 4$, the net income at $t = 3$ of each additional unit (not including any additional interest tax shield) is $3.20 ($= 4 \times (1 - 0.2)$), yielding a discounted value at $t = 2$ of $3.05 (=$3.20/1.05$). Thus, adding capacity is a positive NPV opportunity. Since the firm earns a net income of $3.20 at $t = 2$, it can finance the first unit of additional capacity from its cash flow. A second unit of new capacity could be financed using the remaining $0.70 of cash flow at $t = 2$, and $1.80 from a combination of the retained cash from $t = 1$ and new debt financing. If the firm retains $1 of cash at $t = 1$, it would need to borrow $0.80 (bringing the total debt at $t = 2$ to $1.80); if it retains the full $1.56 at $t = 1$, it would only need $0.24 of new debt (total debt at $t = 2$ would then be $1.24).

Was it worthwhile to have the extra $0.56 of cash saved to provide internal financing? It depends. Having this extra cash in the high EBIT state avoids the costs of issuing an extra $0.56 of debt (net of the extra interest tax shield). However, there is an effective tax penalty of 1% (20% tax on 5% of interest) on the additional cash retained during the second year. If the probability of the

high EBIT state times the net cost on the $0.56 of new debt is greater than the effective tax penalty, for example, if there is a 60% probability of high EBIT and a 2% net cost of debt (3% proportional debt flotation cost minus a 1% interest tax shield benefit), then it is indeed worthwhile to save the extra $0.56 of cash at $t = 1$.

Our example illustrates that saving cash rather than paying it out to shareholders can increase firm value by (1) decreasing net debt to prevent default in low profitability states, in a way that allows for a costless increase in net debt when profitability recovers, and (2) potentially avoiding external financing costs when investment occurs in high profitability states. While this simple example demonstrates the key insights of our paper, it also shows that the firm's liquidity strategy depends on a large number of factors, including issuance costs associated with external financing, relative taxation at the corporate and personal levels, the resolution of uncertainty over time, and the nature of the firm's investment opportunities. We now present a more general dynamic model that includes several additional features in order to better understand the drivers of liquidity policy, and more generally to prescribe how firms should jointly determine their financing, liquidity, and investment policies in order to optimally manage financial flexibility and maximize firm value.

II. The Model

The model uses discrete time and has infinite horizon. The source of uncertainty driving the dynamic policies in our model is the productivity of the firm, denoted θ. We assume that θ, under the risk-neutral probability measure, follows the process[7]

$$\log \theta_{t+1} = \eta + \rho \log \theta_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \quad \text{i.i.d.,} \tag{1}$$

where ε has compact support and $|\rho| < 1$.

The operating cash flow of the firm, $\pi(k, \theta)$, depends on the book value of assets in place, $k > 0$, and the productivity parameter, θ. It can take on either sign, implicitly reflecting the presence of fixed operating costs. We assume that π is an increasing function of k and θ and is a concave function of k satisfying the usual conditions

$$\lim_{k \to 0^+} \pi_1(k, \theta) = \infty \qquad \text{and} \qquad \lim_{k \to \infty} \pi_1(k, \theta) = 0.$$

The level of capital, k, can vary over time as a consequence of investment and disinvestment decisions. As will be seen shortly, the latter is done either on a voluntary basis, because the current return on invested capital is too low, or as a result of financial distress. Capital is homogeneous across date of purchase, and depreciates both economically and for accounting purposes at a constant rate $\delta > 0$.

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[7] This choice is rather popular in the literature; see, for example, Hennessy and Whited (2005), Moyen (2004), and Sapriza and Zhang (2004).

The firm can issue perpetual debt (consol bonds) with face value $p \geq 0$. The lender imposes a collateral constraint ensuring that the firm can always meet its repayment obligations, and thus the debt pays a coupon rate equal to the risk-free rate r. The firm may simultaneously decide to lend at the risk-free rate r by accumulating a cash balance, $b \geq 0$, which can be augmented over time by retaining cash, or drawn down as needed (as detailed below).

Corporate taxes are a convex function g of taxable earnings. The convexity of g approximates a limited loss offset provision, that is, there is a tax credit associated with negative earnings, but at a lower rate than for positive earnings.

The Earnings Before Taxes (EBT) is equal to the firm's EBITDA, $\pi(k, \theta) + rb$, minus depreciation and interest:

$$y(k, p, b, \theta) = \pi(k, \theta) + rb - \delta k - rp.$$

Subtracting taxes and adding back depreciation gives the after corporate tax cash flow to equity holders:

$$\pi(k, \theta) + rb - rp - g(y(k, p, b, \theta)). \tag{2}$$

The dynamics of the firm can be generally described as follows. As θ evolves over time, investment, financing, and retention decisions are made in order to optimize shareholder value. While the evolution of the firm is thus highly path dependent, the cash flow as well as the policy decisions will depend on the state of the firm, given by the levels of productivity, θ, capital, k, debt, p, and cash balance, b.

At a given date, after observing θ, the firm chooses a new level of book value of capital, k', debt, p', and cash balance, b', for the next period. Focusing first on investment, if $k' = k$, investment is set equal to depreciation, δk, in order to maintain the book value of capital. In general, if there is positive investment, the cost is $k' - k(1 - \delta) > 0$. The cash to finance investment may come from current cash flow, from liquidating some of the firm's cash balance, or from issuing debt and/or equity. If the firm instead decides to sell off some of its capital, we assume that the asset is sold at a liquidation price $\ell \leq 1$, so that the cash inflow from divestment is $\ell(k(1 - \delta) - k')$. For notational convenience, for a general ξ, we define the function $\chi(\xi, \ell)$ as

$$\chi(\xi, \ell) = \begin{cases} \xi & \text{if } \xi \geq 0 \\ \xi\ell & \text{if } \xi < 0. \end{cases}$$

The firm may choose to decrease its debt level by paying down debt using current cash flow, drawing down its cash balance, or issuing equity. There is no direct cost associated with paying down debt. There is, however, a proportional cost on new debt issued,

$$q(p, p') = \begin{cases} q(p' - p) & \text{if } p' > p \\ 0 & \text{if } p' \leq p \end{cases} \tag{3}$$

for given parameter $q \geq 0$.[8] Note that since debt is risk-free in our model, there is no agency problem associated with the firm increasing its leverage to expropriate wealth from existing creditors.

To implement the collateral restriction imposed by the lender, we use the fact that the state variable θ is bounded in a compact set so that, at all dates, $\theta \in [\theta_d, \theta_u]$. Furthermore, we can confine our analysis to $k \in [0, k_u]$, where k_u is the maximum amount of production capacity such that the operating cash flow exceeds depreciation and the opportunity cost of capital under the best-case scenario θ_u, that is, $\pi(k_u, \theta_u) - (\delta + r)k_u = 0$.[9]

The investment and financing policy (k', p', b') chosen by the firm must satisfy the collateral constraint[10]

$$p'(1+r) \leq b'(1+r) + sk'(1-\delta) + \pi(k', \theta_d) - g(y(k', p', b', \theta_d)), \qquad (4)$$

where $0 < s \leq \ell$ is a discount when capital must be sold to cover debt obligations, and θ_d is the worst-case productivity scenario. Thus, the end-of-period cash balance plus the fire-sale value of the depreciated asset plus the after corporate tax operating cash flow must always be greater than the end-of-period debt value (the face value of the debt plus the coupon payment).[11]

The firm is considered to be in financial distress in a period where its operating cash flow, together with the available cash balance, are insufficient to cover the coupon payment. In this case, we assume that the firm must sell a fraction of its assets at a discount s in order to pay the coupon.[12]

Given the above, the residual cash flow to equityholders at a given state (k, p, b, θ), assuming a particular set of investment, financing, and retention

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[8] Our model is flexible enough to accommodate other cost structures, such as including a fixed cost of debt issuance. Since reducing debt does not require calling back all debt at par value and then reissuing new debt, as in Fischer, Heinkel, and Zechner (1989) and Titman and Tsyplakov (2007), we do not need to impose a cost to decrease debt.

[9] Since π is increasing with respect to θ, and increasing and concave with respect to k, for any other scenario $\theta < \theta_u$ and any k, $\pi(k, \theta) < \pi(k, \theta_u)$ and hence k_u is the maximum possible level of production capacity.

[10] See Hennessy and Whited (2005). In our case, the presence of a cash balance may allow the firm to take on additional debt. We later examine the debt level net of the cash balance.

[11] By defining C as the feasible set of triples (k', p', b') that satisfy (4), we can assume that the choice (k', p', b') and the current state of the firm (k, p, b) are always in C with no loss of generality. We can thus limit our numerical computations to the set C instead of exploring at any date the hyper-rectangle $[0, k_u] \times [0, \overline{p}] \times [0, \overline{b}]$.

[12] Strebulaev (2007) and Titman and Tsyplakov (2005) use a similar definition of distress, requiring that an operating cash flow to coupon coverage ratio be met. Our approach differs somewhat in that our firm has a cash balance that it can also use to cover the coupon. In Titman and Tsyplakov (2007), distress leads to a loss in operating cash flow, while in our model assets must be sold at a discount. This is similar to Strebulaev (2007), though asset sales are based on a discount to market value in his model, as opposed to a discount to book value in our model.

decisions (k', p', b'), is

$$
\begin{aligned}
cf&(k, p, b, k', p', b', \theta) \\
&= \max\{(\pi(k, \theta) - g(y(k, p, b, \theta)) + rb - rp) + b, 0\} - b' - p + p' - q(p, p') \\
&\quad - \chi\left(k' - k(1 - \delta) + \max\left\{\frac{(rp + g(y(k, p, b, \theta)) - \pi(k, \theta) - rb) - b}{s}, 0\right\}, \ell\right).
\end{aligned}
\tag{5}
$$

Interpreting this equation, if there is no financial distress, then the residual cash flow to equityholders is the after tax flow from operations plus the cash flows from changes in the debt, the cash balance, and the book value of assets (if there is a reduction in the asset value, this occurs at the liquidation discount ℓ). If financial distress occurs, the Net Operating Profit After Tax (NOPAT = EBITDA – Taxes – Depreciation) plus the current cash balance is negative and the firm is forced to sell part of its existing assets at the fire-sale discount s to pay the coupon. These two separate cases can be seen more clearly by expressing (5) as follows:

1. In the case in which there is no current financial distress (no fire sales), that is, $rp < \pi(k, \theta) - g(y(k, p, b, \theta)) + (1 + r)b$,

$$
\begin{aligned}
cf(k, p, b, k', p', b', \theta) &= (\pi(k, \theta) - g(y(k, p, b, \theta)) + rb - rp) \\
&\quad + b - b' - p + p' - q(p, p') - \chi(k' - k(1 - \delta), \ell).
\end{aligned}
\tag{6}
$$

2. In the case in which the firm is in financial distress, that is, $rp \geq \pi(k, \theta) - g(y(k, p, b, \theta)) + (1 + r)b$, and there is a fire sale of $(rp - \pi(k, \theta) + g(y(k, p, b, \theta)) - (1 + r)b)/s$ units of capital,

$$
\begin{aligned}
cf&(k, p, b, k', p', b', \theta) \\
&= p' - p - q(p, p') - b' \\
&\quad - \chi\left(k' - k(1 - \delta) + \frac{(rp + g(y(k, p, b, \theta)) - \pi(k, \theta) - rb) - b}{s}, \ell\right).
\end{aligned}
\tag{7}
$$

The cash distributed to equityholders is subject to personal taxes levied at a constant rate τ_e.[13] If the residual cash flow is instead negative, funds are raised by issuing new equity. In this case, the cash flow received from equityholders is reduced by a given proportion $\lambda > 0$ representing issuance transaction costs.[14] Hence, the payout function for equity holders is denoted Γ, and is defined, for a generic pre-personal tax flow ξ, as

$$
\Gamma(\xi, \tau_e, \lambda) = \begin{cases} \xi(1 - \tau_e) & \text{if } \xi \geq 0 \\ \xi(1 + \lambda) & \text{if } \xi < 0 \end{cases}.
$$

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[13] τ_e can be considered a blend of the tax rate of dividends and tax rate of capital gains. See Graham (2003). As in Hennessy and Whited (2005), we do not distinguish between alternative forms of cash distributions. Lewellen and Lewellen (2005) examine the effect of differential taxes on dividends and capital gains.

[14] While costs associated with debt and equity issuance may be in part attributable to asymmetric information, we do not explicitly model asymmetric information in our paper.

Given this definition, the actual cash flow to equity holders is

$$e(k, p, b, k', p', b', \theta) = \Gamma(cf(k, p, b, k', p', b', \theta), \tau_e, \lambda).$$

Let $E(k, p, b, \theta)$ denote the value of the equity of the firm at state (k, p, b, θ). We solve for E by the method of successive approximations as described in Section III. At every date, the value of the equity is the after personal tax net optimal cash flows to shareholders for the current period plus the optimal continuation value (i.e., the discounted present value, at the after personal tax risk-free rate, of the expected optimal future cash flows) stemming from the current decision (k', p', b', θ'), assuming that it satisfies the collateral constraint

$$E(k, p, b, \theta) = \max_{(k', p', b') \in C} \{e(k, p, b, k', p', b', \theta) + \beta \mathbb{E}_{k,p,b,\theta}[E(k', p', b', \theta')]\}, \quad (8)$$

where $\beta = (1 + r^z(1 - \tau_e))^{-1}$, with r^z denoting the certainty equivalent rate of return on equity flows,[15] and the expectation is computed under the risk-neutral probability measure, conditional on the current state of the firm. The value of debt at any state (k, p, b, θ) is given simply by the face value, p, and the value of the firm is thus $V = p + E$.

In addition to measuring value effects, we are interested in examining the underlying investment, financing, and retention policies and their interactions. Given the state (k, p, b, θ), from the Bellman condition in (8) we derive the optimal policy $\varphi(k, p, b, \theta) \in C$:

$$\varphi(k, p, b, \theta) = (k^*, p^*, b^*) = \arg\max_{(k', p', b') \in C} \{e(k, p, b, k', p', b', \theta) \\ + \beta \mathbb{E}_{k,p,b,\theta}[E(k', p', b', \theta')]\}. \quad (9)$$

Given $\varphi(k, p, b, \theta) = (k^*, p^*, b^*)$, we denote the optimal investment policy as $K(k, p, b, \theta)$

$$= k^* - \left(k(1 - \delta) - \max \left\{ \frac{(rp + g(y(k, p, b, \theta)) - \pi(k, \theta) - rb) - b}{s}, 0 \right\} \right), \quad (10)$$

the debt policy as $P(k, p, b, \theta) = p^* - p$, and the cash retention policy as $B(k, p, b, \theta) = b^* - b$.

In a subsequent section we will solve the optimization problem (8) for a set of parameters, and we will analyze the interplay between the state of the firm, the value of the firm, and the investment, financing, and retention policies.

As a benchmark case, we will also study a simplified version of our model where we assume that a positive cash balance cannot be held when debt is positive, and cash is used to immediately reduce the debt. Hence, as in Hennessy and Whited (2005) and Cooley and Quadrini (2001), when debt is positive there is no cash balance, and similarly, a negative value of debt implies that the firm

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[15] In a generalized Miller equilibrium economy, the certainty equivalent rate of return on equity flows, r^z, is determined as $r^z = r(1 - \tau_d)/(1 - \tau_e)$, where τ_d is the personal tax on debt income, and τ_e is the personal tax on equity income assuming an accrual-based capital gains tax. See Sick (1990) for details.

has a positive cash balance. This reduced dimensionality model is formally presented in the Appendix.

III. Numerical Implementation of the Model

A. Quadrature

The solution method is based on a numerical approximation of the infinite-horizon dynamic programming problem in (8) by a discrete state space and successive approximation method.[16]

First, the quadrature method of Tauchen (1986) is used to approximate the dynamics of the logarithmic AR(1) in (1), where we assume $\varepsilon \sim \mathcal{N}(0, \sigma^2)$, with a finite-state Markov chain.[17] According to this method, the discrete abscissae of the Markov chain and the transition probabilities are found by a Gauss–Hermite quadrature rule.[18]

Specifically, by defining $y = \log(\theta)$, we take S discrete abscissae in an interval of semiwidth $I_p = 3\sigma/\sqrt{1 - \rho^2}$, and centered on the long-term mean of process, $\eta/(1 - \rho)$. The set of the discretized state variable is $\widetilde{Y} = \{\widetilde{y}(s) \mid s = 1, \ldots, S\}$, where

$$\widetilde{y}(s) = \eta/(1 - \rho) - \max\left\{\left(\frac{S-1}{2} + 1\right) - s, 0\right\} u + \max\left\{s - \left(\frac{S-1}{2} + 1\right), 0\right\} u,$$

with $u = 2I_p/S$.

Next, we define the cells for the state variable as $c(j) = [Y(j), Y(j+1)]$, for $j = 1, \ldots, S$, where

$$
\begin{aligned}
Y(1) &= -\infty, \\
Y(j) &= \frac{\widetilde{y}(j) + \widetilde{y}(j-1)}{2}, \quad j = 2, \ldots, S, \\
Y(S+1) &= +\infty.
\end{aligned}
$$

To obtain the transition probability matrix under the risk-neutral probability, we have to determine the probability, conditional of the current state y, that the future state is y'. Given the above approximation, this is equivalent to the probability $\Pi(i,j)$ that y' falls into cell $c(j)$, given the current state $y = \widetilde{y}(i)$, for

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[16] See, for example, Burnside (1999).

[17] The logarithmic AR(1) process in equation (1), with $\varepsilon \sim \mathcal{N}(0, \sigma^2)$ i.i.d. and for $\rho > 0$, can be thought of as the discrete-time version of the continuous-time process $d\theta_t = \kappa \left(\log \theta_L - \log \theta_t\right) \theta_t dt + \overline{\sigma} \theta_t dZ_t$ under the equivalent martingale measure, where κ is the speed of mean reversion, θ_L is the long-term mean, $\overline{\sigma}$ is the instantaneous volatility, and Z is a Brownian motion. With this notation, assuming $\Delta t = 1$, we have $\rho = e^{-\kappa}$, $\eta = (1 - \rho)(\log \theta_L - \overline{\sigma}^2/(2\kappa))$, and $\sigma = \overline{\sigma}\sqrt{(1 - e^{-2\kappa})/(2\kappa)}$.

[18] For the problem at hand, a Markov chain approach provides some benefits over a lattice approach, because it permits us to keep the set of discrete states constant through the whole time span. Moreover, it is less computationally demanding than a Monte Carlo simulation approach as long as the exogenous state variable is one-dimensional. Lastly, discrete Markov chains are more flexible to implement a dynamic programming problem with multiple (controlled) state variables such as that presented in Section II.

all $j = 1, \ldots, S$ and all $i = 1, \ldots, S$:

$$
\begin{aligned}
\Pi(i, j) &= \Pr\{y' \in c(j) \mid y = \widetilde{y}(i)\} \\
&= \Pr\{Y(j) \le y' < Y(j+1) \mid y = \widetilde{y}(i)\} \\
&= \mathcal{N}\left(\frac{Y(j+1) - \eta - \rho\,\widetilde{y}(i)}{\sigma}\right) - \mathcal{N}\left(\frac{Y(j) - \eta - \rho\,\widetilde{y}(i)}{\sigma}\right).
\end{aligned}
$$

The transition probability matrix is $\Pi = (\Pi(i, j), i, j = 1, \ldots, S)$. In our computation we will use the values $\widetilde{\theta} = \exp(\widetilde{y})$, collected in the set $\widetilde{X}$, with the transition probability matrix Π. The proposed method converges as $S \to \infty$, as shown by Tauchen (1990).

B. Solving for Firm Value and Optimal Policies

We implement a dynamic programming approach by first discretizing our state space (k, p, b, θ). The set $[\theta_d, \theta_u]$ is discretized into S values, as described in the previous section. The book value of assets k is bounded in the interval $[0, k_u]$, as shown earlier, and we can determine numerically the values p_u and b_u that are never binding for the optimal choices of p and b determined by the simulation procedure described in Section III.C below. We discretize each of these three sets into N_k, N_p, and N_b values, respectively. We denote $\widetilde{K}$ to be the discretized set for capital stock, defined as

$$
\widetilde{K} = \{\widetilde{k}_j = k_u(1 - \delta)^j \mid j = 1, \ldots, N_k\};
$$

$\widetilde{P}$ is the set of discrete values for debt and $\widetilde{B}$ is the discretized set for the cash balance. The sets $\widetilde{P}$ and $\widetilde{B}$ are obtained by taking equally spaced values of the debt and cash balance, respectively, in the relevant set. In the same manner, we will denote $(\widetilde{k}, \widetilde{p}, \widetilde{b})$ as the discretized state variable. Hence, the controlled state space has size $N_k \cdot N_p \cdot N_b$.

We assume that the operating cash flow rate of the firm is $\pi(k, \theta) = \theta k^\alpha - F$, where $\alpha < 1$ models decreasing returns to scale and F is a fixed cost, to capture operating leverage. As in Hennessy and Whited (2005), we specify the corporate tax function g as

$$
g(y) = \begin{cases} \displaystyle\int_0^y \tau_c(\zeta)\,d\zeta & \text{if } y \ge 0 \\ \displaystyle -\int_y^0 \tau_c(\zeta)\,d\zeta & \text{if } y < 0, \end{cases}
$$

where $\tau_c(y) = 0.35 \cdot \phi(y, \mu_\tau, \sigma_\tau)$ is the marginal tax rate function, ϕ is the Normal cumulative probability distribution, $\mu_\tau = -12.267$, and $\sigma_\tau = 9.246$. To speed up numerical computations, we approximate g with a piecewise linear function that has a negligible impact on the accuracy of values or policies.

Since the collateral constraint in (4) is independent of the state (k, p, b, θ), we can accelerate the computational analysis by focusing only on the subset

of state values for the discretized capital, debt, and cash balance such that $(\widetilde{k}, \widetilde{p}, \widetilde{b}) \in C$.

Given the setup described above, the approximated value function E and the related optimal policy function φ are computed using a successive approximation approach by means of a *policy iteration method*,[19] as described in chapter 12 of Judd (1998).

By denoting the state $x = (k, p, b, \theta)$, and because the set of states is finite and is the same at every step of the procedure, the problem at hand can be written as

$$E(x) = \max_{\varphi \in C} \left\{ e(\varphi, x) + \beta \sum_{\theta'} \Pi(\theta, \theta') E(\varphi(x), \theta') \right\} \text{ for all } x. \qquad (11)$$

We can think of E as a vector that has as many components as the number of states, x. Accordingly, we define also the transition probability from x to x' when the feasible policy φ is applied:

$$Q^{\varphi}(x, x') = \begin{cases} \Pi(\theta, \theta') & \text{if } (k', p', b') = \varphi(x) \\ 0 & \text{otherwise.} \end{cases}$$

For a policy $\varphi \in C$, we denote the corresponding cash flow to equity holders $e^{\varphi} = e(\varphi, x)$. The value of this policy, denoted E^{φ}, is the solution of the system of linear equations[20]

$$E^{\varphi} = e^{\varphi} + \beta Q^{\varphi} E^{\varphi}. \qquad (12)$$

Hence, the solution method based on policy function iteration proceeds as follows. The value of equity is initialized. At the nth step of the procedure, given a value E_n, the related *greedy* policy, denoted φ_n, is found:

$$\varphi_n(x) = \arg\max_{\varphi \in C} \left\{ e(\varphi, x) + \beta \sum_{\theta'} \Pi(\theta, \theta') E_n(\varphi(x), \theta') \right\} \text{ for all } x.$$

By solving equation (12), the new value $E_{n+1} = E^{\varphi_n}$ is determined and we are ready for the subsequent step. The procedure is repeated until convergence of the value function (and hence of the policy function).[21]

This method converges because it is based on Bellman equation (11). Moreover, this method converges faster because it uses a given policy for an infinite

---

[19] We observe that a policy function iteration method based on the Euler equation of the dynamic programming problem, as proposed by Coleman (1990), or an approach based on a discrete state space Euler equation approach, as proposed by Baxter, Crucini, and Rouwenhorst (1990), are not feasible in this case, because the payoff function e is nonsmooth with respect to the control variables.

[20] This is the critical step of the procedure. When the state space is large, as it is in our case, a standard solution technique either based on matrix inversion or iterative methods like Gauss–Jacobi or Gauss–Seidel may be impractical. Under these circumstances, a more efficient approach is based on a modified value iteration using equation (12) recursively.

[21] In our computations we repeat the procedure until $\max\|E_{n+1} - E_n\| < 10^{-5}$. We also use a much more stringent tolerance level (10^{-14}) for some cases, and find equivalent results.

Table I

Base-case Parameter Values for the Model

ρ	AR(1) persistence	0.62
θ_0	Initial value of state variable	1
η	Parameter of state variable	0
σ	Annual volatility of state variable	0.15
r	Annual risk-free borrowing rate	5%
τ_e	Personal tax rate on equity cash flows	12%
τ_b	Personal tax rate on bond coupons	25%
μ_T	Parameter for corporate tax function	-12.267
σ_T	Parameter for corporate tax function	9.246
α	Production return-to-scale parameter	0.45
δ	Annual depreciation rate	0.1
F	Fixed cost of production	1.3
s	Fire-sale discount for asset sales	0.5
ℓ	Liquidation value for voluntary asset sales	0.75
λ	Variable flotation cost for equity	6%
q	Variable flotation cost for debt	2%

number of steps, as opposed to only one step as in the value function iteration method.

We solve the model using $S = 9$ points for $\widetilde{X}$, $N_k = 25$ points for $\widetilde{K}$, $N_p = 13$ points for $\widetilde{P}$, and $N_b = 13$ points for $\widetilde{B}$. To smooth the results, multidimensional linear interpolation is used extensively both for the value function and for the optimal policy function.

C. Simulating Values and Policies

We simulate 10,000 firms (paths) for $T = 60$ years for the base case and for each of the alternative cases described in Section III.D. Each path for the state variable θ is obtained by iterating equation (1) using Monte Carlo simulation, for T time steps. The simulated paths for θ are restricted to a set of discrete values $\widetilde{X}$.

For each step along each simulated path, the optimal policy φ in equation (9) is applied for the current state of the firm (k, p, b, θ, t), and the state is updated accordingly. The initial state of the firm $(k_0, p_0, b_0, \theta_0)$ for all paths is assumed to be the intermediate point of $\widetilde{K} \times \widetilde{P} \times \widetilde{B} \times \widetilde{X}$.

When analyzing the results produced by the simulation, we focus primarily on two dates, $t = 1$ and 50, in order to understand the differing characteristics and decisions of a start-up versus a mature firm.

D. Parameter Values for Analysis

The base-case parameters for our analysis are shown in Table I. These parameter values are largely based on values used in related papers, specifically, Hennessy and Whited (2005), Titman and Tsyplakov (2007), and Moyen (2004).

The base case represents a firm that faces reasonable financial constraints: a proportional debt issuance cost (q) of 2%; an equity issuance cost (λ) of 6%; and a 50% fire-sale discount (s) on forced asset sales triggered by financial distress. To gauge the effects of financial flexibility, we adjust the level of the fire-sale discount to $s = 1$, and drop the equity and debt issuance costs to zero one at a time to focus on their relative impact, as well as setting both equal to zero, implying no direct costs associated with external financing. While several papers focus on fully constrained firms to contrast the extremes of access to financing (e.g., Almeida, Campello, and Weisbach (2004) and Moyen (2004)), we examine the effects of financing frictions in a range that appears to be more characteristic of publicly traded firms that constitute the standard sample for most empirical tests.

The base case also captures a firm whose investment opportunities are somewhat irreversible ("inflexible" capital), in that 25% of the value of capital is lost upon liquidation. The alternative we consider in our analysis below is a firm whose capital is fully reversible or "flexible" ($\ell = 1$, that is, its assets can be sold at book value). In addition to combining high and low levels of investment flexibility together with the different levels of financial flexibility, we also examine in some parts of our analysis the case in which there are no taxes, in order to better isolate how taxes drive our results. We also conduct other robustness analyses, such as changing the coefficient of mean reversion, volatility, and fixed costs, but report only a subset of these in the paper given that the rest generate qualitatively similar results.

IV. Results

A. *Value of Financial Flexibility*

We begin by measuring the effect of financial flexibility on firm value under different scenarios; we then focus specifically on the value of liquidity. Figure 2 shows the firm value under costly financing as a percentage of the value of an otherwise identical firm that has access to costless financing ($\lambda = q = 0$ and $s = \ell$).[22] This allows us to measure the value of financial flexibility by observing the percentage value loss due to the presence of issuance and distress costs, as well as financing constraints. The value comparisons are shown for different levels of capital (k), assuming zero debt, zero cash balance, and the mean value of the state variable ($\theta = 1$). Three cases are examined: inflexible capital, flexible capital, and inflexible capital assuming no personal or corporate taxes, which allows us to better isolate the effect of taxes on financial flexibility.

Figure 2 shows that the value loss due to costly financing is quite substantial for low values of k. When capacity is low, the growth opportunity to add capacity is large, but the investment in new capacity must be financed externally given that there is not sufficient cash being generated from the firm's current

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[22] The condition that $s = \ell$ for the costless financing benchmark case is imposed to ensure that there are no distress costs associated with debt financing in the form of a fire-sale discount on capital that is greater than the regular discount on selling capital.
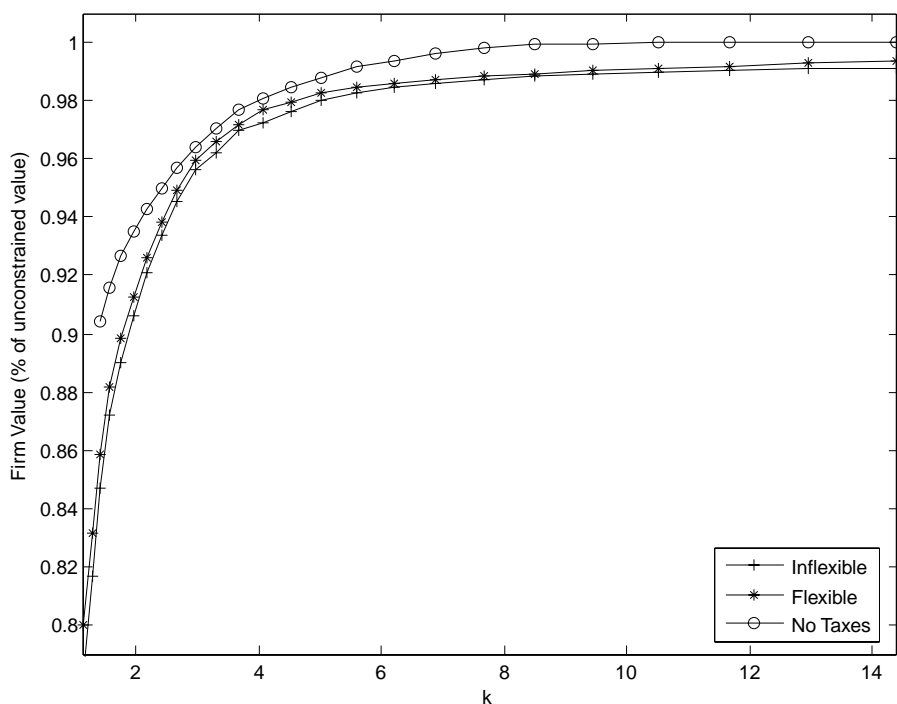
Figure 2. Value of financial flexibility vs. capital. This figure shows the value of a firm under costly financing as a percentage of the value of an otherwise equivalent firm with costless access to financing, for different capital levels. The three cases shown are: *Inflexible*, which is the base case of inflexible capital ($\ell = 0.75$) and costly financing ($q = 0.02$ and $\lambda = 0.06$); *Flexible*, flexible capital ($\ell = 1.0$) with $q = 0.02$ and $\lambda = 0.06$; and *No Taxes*, where capital is inflexible ($\ell = 0.75$) but there are no personal or corporate taxes ($\tau_e = \tau_b = 0$, $g \equiv 0$). Current values for debt and cash balance are $p = 0$ and $b = 0$, respectively. The productivity state variable is at the long-term mean value, $\theta = 1$. All the other parameter values are shown in Table I.

operations to finance the investment internally. Thus, the firm may be forced to pay the costs of external financing in order to profit from higher production. The firm could instead choose to delay investment until it is able to build up enough of a cash balance to finance the capacity addition internally, but this delay would result in forgone profits, and thus lost value. Since a low level of capacity is more likely when the firm is young, the results support the reasonable conclusion that even if financing costs are similar across all companies, they will have a much larger impact on firm value for early-stage companies than for more mature companies that have reached their steady-state production levels. We explore the effects of a firm's maturity in more detail later.

From Figure 2, one can also observe that the presence of taxes can have a significant effect on the value of financial flexibility. As will become more clear shortly when we analyze our other results, a firm can compensate for high costs of external financing by building up its cash stock in order to internally finance investment in new capacity. However, there is an implicit tax cost associated
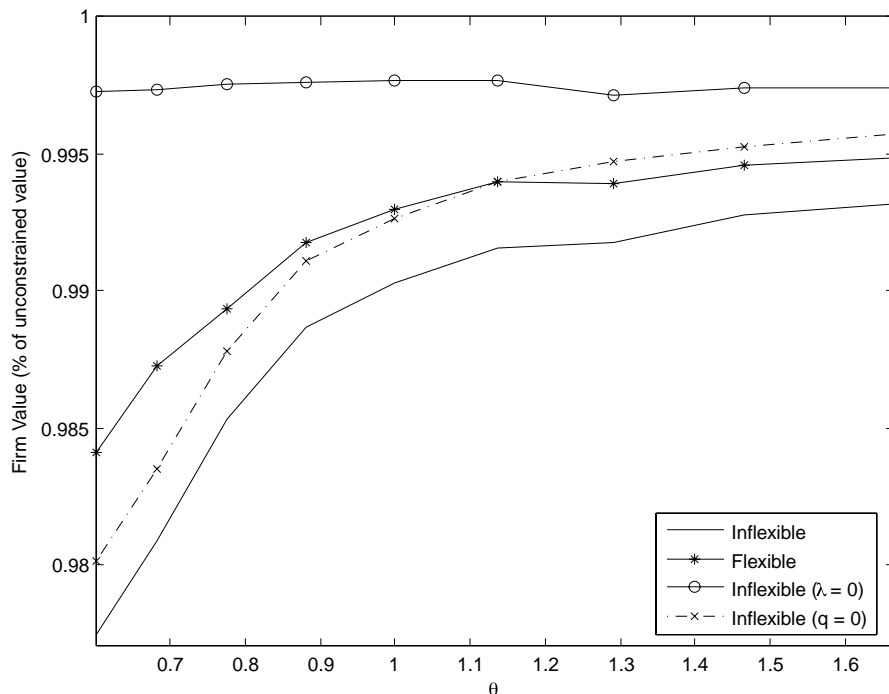
Figure 3. Value of financial flexibility vs. productivity. This figure shows the value of a firm under costly financing as a percentage of the value of an otherwise equivalent firm with costless access to financing, for different profitability levels (θ). The four cases shown are: *Inflexible*, which is the base case of inflexible capital ($\ell = 0.75$) and costly financing ($q = 0.02$ and $\lambda = 0.06$); *Inflexible* ($\lambda = 0$), inflexible capital with $q = 0.02$ but $\lambda = 0$; *Inflexible* ($q = 0$), inflexible capital with $q = 0$ and $\lambda = 0.06$; and *Flexible*, flexible capital ($\ell = 1.0$) with $q = 0.02$ and $\lambda = 0.06$. Current values for debt and cash balance are $p = 3$ and $b = 0$, respectively. Capital is at the intermediate level of $k = 6.9$. All the other parameter values are shown in Table I.

with this cash balance since the effective tax rate on interest income is higher when the cash is kept in the firm versus directly in the hands of investors. In the case in which there are no personal and corporate taxes, and thus there is no tax disadvantage to retaining cash, the negative impact of external financing costs can be significantly mitigated by creating internal financing flexibility through managing the cash balance.

Figure 3 also measures the effect of financial flexibility on firm value, this time plotted against the productivity variable θ, and focusing on a firm that already has a significant amount of capital in place ($k = 6.9$). Three of the cases represent firms with inflexible capital and with different levels of issuance costs: *Inflexible* is our base case ($q = 0.02$ and $\lambda = 0.06$); *Inflexible* ($\lambda = 0$) eliminates the equity issuance cost, but retains $q = 0.02$, which could represent the case of a closely held firm that may be able to raise equity financing at low cost, but still incurs significant costs in issuing new public or private debt; and *Inflexible* ($q = 0$) eliminates the debt issuance cost, but retains $\lambda = 0.06$, representing the case of a firm that may have relatively easy access to debt

financing (e.g., if it draws down a line of credit (see Sufi (2007)) rather than issue new public debt), but would still incur large costs when issuing equity. The fourth case we examine (*Flexible*) pertains to a firm with flexible capital, but with costly external financing ($q = 0.02$ and $\lambda = 0.06$).

Several observations can be drawn from Figure 3. First, consistent with what we saw in Figure 2, the value loss due to costly financing is relatively small for an intermediate capital level, since the firm is operating more or less in a steady state, often just replacing depreciated capital by reinvesting some of its current operating profits. However, the impact is somewhat larger for lower θ values. Since the firm's cash flow becomes smaller as θ decreases, the firm is less able to support its debt, implying that it must recapitalize in order to continue to satisfy the collateral constraint and to avoid financial distress. Observe that when there are no equity issuance costs, there is no drop in value for lower θ values since the firm can raise equity to recapitalize without incurring any transaction costs.

Second, note that while eliminating debt issuance costs increases the value of the firm, there is a much larger value gain from eliminating equity issuance costs. In either of these cases there is a source of external financing available without issuance costs. While at first blush the cost of the other source of financing would seem irrelevant, this is not the case. In the case in which there is no issuance cost of debt, there still is an indirect cost of additional debt in the form of an increasing marginal expected cost of financial distress, and thus after a particular debt level the firm will prefer to access equity financing.

Third, observe that the value loss from the lack of perfect financial flexibility is somewhat mitigated when capital is more flexible ($\ell = 1.0$). The reversibility of investment in capital compensates to some degree for the loss in financial flexibility, primarily because the firm can more readily reduce its capital when productivity is low, providing an alternative to holding cash as a buffer to deal with the firm's debt obligation. This partial substitutability between investment and financial flexibility also suggests that one of the benefits of flexible capital may not be captured in traditional real options models that implicitly assume costless financing.

B. Value of Liquidity

Since financial flexibility thus depends not only on external financing costs, but also on the firm's liquidity policy, it is important to explicitly examine the relationship between firm value and liquidity. Figure 4 shows the value of the firm net of its cash balance, typically referred to as a firm's "Enterprise Value," for different levels of cash balance (assuming an intermediate level of capital, zero debt, and $\theta = 1$). It therefore addresses the question of whether an equity holder would benefit by infusing an additional dollar into the company to be held in its cash balance, for various levels of current liquidity. The value of the firm would certainly increase, but the equity holder would be short one dollar, so the net effect may be positive or negative, which is precisely what we measure.

In Panel A of Figure 4, we present several cases in which there are no taxes, and thus there is no tax penalty associated with keeping a cash balance. As expected, we find that there is no benefit to having a cash balance when the firm does not face issuance costs, regardless of whether or not there is a fire-sale discount associated with the collateral constraint and distress. The firm will simply raise capital externally at no cost whenever the need arises.

When the firm faces external financing costs, however, enterprise value increases with the cash balance, though at a decreasing marginal rate, since the value of an additional dollar of cash balance will be close to zero when the firm already has significant liquidity. In fact, as the cash balance gets very high, the enterprise value of the firm subject to costly external financing asymptotes to the value assuming costless financing. Thus, the presence of high internal liquidity provides the same financial flexibility benefits as does the absence of external issuance costs.

Consider the firm that faces issuance costs but no fire-sale discount. When it has a zero cash balance, the NPV of a dollar of cash should be between 0.02 (debt issuance cost) and 0.06 (equity issuance cost), which is precisely what we find. Given that the firm has a low capital level, it will seek financing to increase its capital. A dollar of cash balance can save the firm from paying the debt issuance cost of 0.02. However, since the company is limited in its ability to finance with debt due to the collateral constraint, it may at some point issue equity, and thus the dollar in cash balance today is effectively saving the firm from the issuance cost on some implicit combination of debt and equity financing for the firm. Note that for the case of $s = 1$ (no fire-sale discount), the incremental value of a dollar of cash balance is somewhat lower than when $s = 0.5$. There are two reasons for this. First, the distress cost, and thus the all-in transaction cost of debt, is lower when $s = 1$. Second, the collateral constraint is looser, thereby helping the firm to avoid equity flotation costs.[23]

Panel B of Figure 4 shows the effect of a cash balance when there are corporate and personal taxes (our base-case assumption). In general, there is an interior optimum, where a positive cash balance level maximizes enterprise value. Internal liquidity is a source of financial flexibility for two reasons: On the upside, the firm can access financing in order to invest without having to pay external issuance costs; on the downside, the firm can avoid "distress costs" associated with a fire-sale discount of its capital. For low cash balance values, the marginal benefit of an increase in cash balance exceeds the tax disadvantage associated with the additional retained cash. However, as the marginal benefit of cash declines with higher cash balance levels, the tax penalty of retained cash overtakes the financial flexibility benefit of cash, and enterprise value gradually declines.

The only case in Panel B of Figure 4 that does not lead to an internal cash flow optimum is that in which $s = \ell = 1$. Since capital is perfectly reversible in

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[23] A cash balance of at least one is required when $s = 0.5$ in order to satisfy equation (4) given the low level of capital.
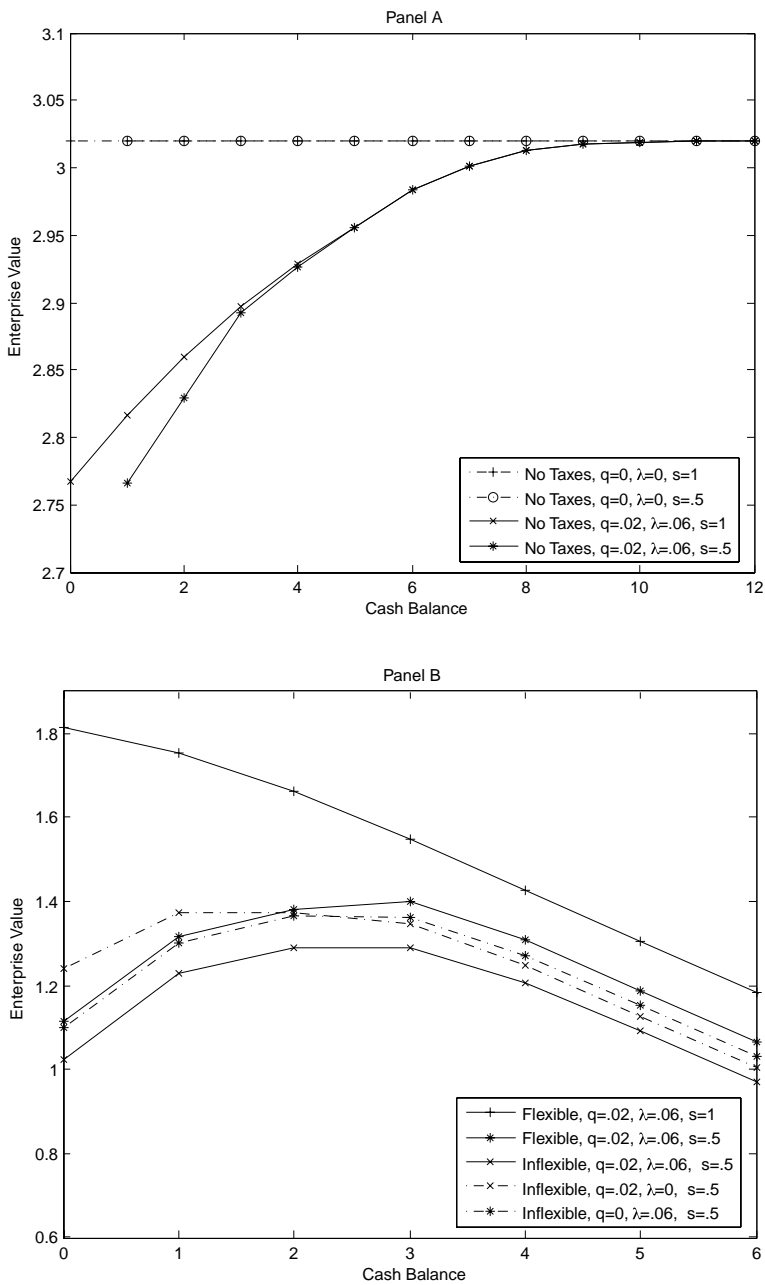
Figure 4. Value of liquidity. This figure shows the relationship between enterprise value (firm value net of the cash balance) and the level of cash balance. In Panel A, there are neither personal taxes ($\tau_e = \tau_b = 0$), nor corporate taxes ($g \equiv 0$), and the firm has flexible capital ($\ell = 1.0$). In Panel B, the base-case tax assumptions are used, and various cases are illustrated, as shown in the legend. In all cases, current debt is $p = 0$, the capital level is $k = 1.2$, and the profitability state variable is at the long-term value, $\theta = 1$. All the other parameter values are shown in Table I.
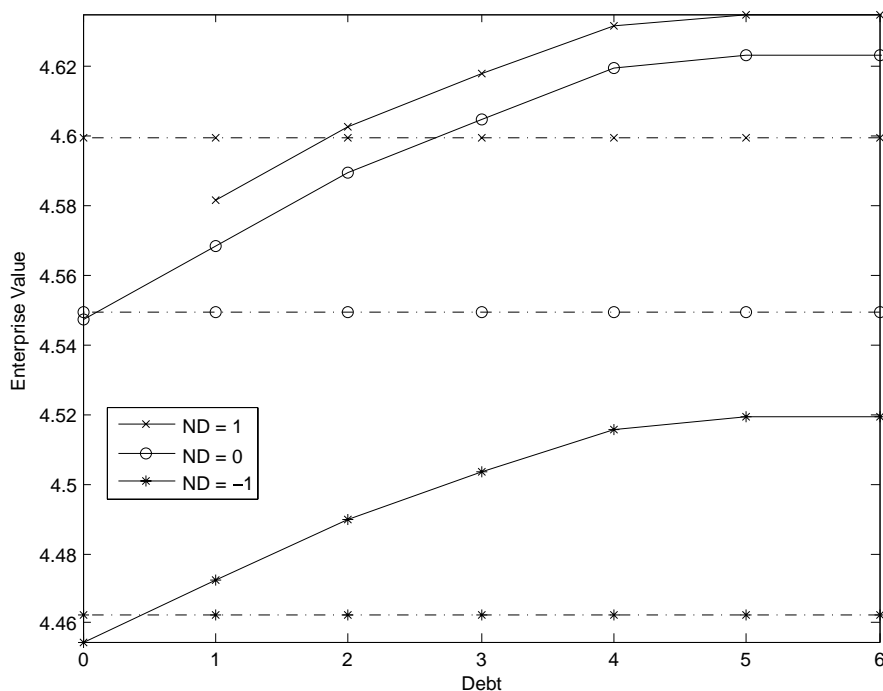
Figure 5. Incremental value from separate control of liquidity. This figure shows the joint impact of cash balance and debt on enterprise value (firm value net of the cash balance). Enterprise value is shown for three levels of net debt (ND), and is plotted against different debt, and thus cash balance, levels. The solid lines represent values from the more general model with both cash balance and debt level controls. For example, the solid line with circles shows the enterprise value of the firm for different combinations of debt and cash balance levels such that the difference between debt and cash balance is equal to zero. The dotted lines are horizontal benchmark values from the restricted model (in the Appendix) that allows for only a single "net debt" variable. All graphs are for the base case (inflexible capital and costly financing). The capital level is $k = 4.1$, and the profitability state variable is $\theta = 1$. All other input parameter values are shown in Table I.

this case, it acts as a close substitute to cash when the firm is in a distressed condition. Thus, the benefit of retaining more cash does not outweigh the significant tax penalty of cash even at low cash balance levels.[24]

Given that our model allows the firm to separately control its cash retention and debt policies, as opposed to having a single variable that allows either borrowing *or* lending but not both (as in Hennessy and Whited (2005) and Cooley and Quadrini (2001)), it is useful to examine whether firm value can be enhanced with the extra lever of control. In other words, while many alternative combinations of cash and debt levels lead to the same "net debt" amount (i.e.,

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[24] The present value of avoiding a fire-sale of capital is significantly smaller when there are no taxes, as shown in Panel A, Figure 4. Since the absence of taxes makes the firm much more profitable, it is much less likely that distress is ever triggered.

debt minus cash balance), are some combinations superior? Figure 5 addresses this issue by comparing enterprise values from the model with both cash and debt controls to enterprise values obtained from an otherwise equivalent model that has only a single net-debt control variable, as detailed in the Appendix. We focus on the base case of inflexible capital and issuance costs for both debt and equity.

Examine, for instance, the case in Figure 5 where the net debt value (ND) is equal to zero. When the debt and cash levels are both zero, the net firm value is approximately 4.55, which is equal to the value obtained from the model with only a single net debt control variable (shown for illustration purposes as a horizontal "base line"). As the debt level increases along the x-axis, and the cash balance increases in step with debt, since the net debt level remains at zero for the $ND = 0$ "iso net debt" curve, the net firm value increases to a level of approximately 4.62. Thus, by allowing the firm to take on positive levels of both cash and debt, the firm value increases by approximately 2% relative to if it can only lend or borrow, but not both. Even though the cash balance and debt amounts are identical in this case, and hence there is no net debt tax shield, the firm is better off having a positive cash balance to draw down on in order to finance additional investment in capacity or to later gain from interest tax shields from positive net debt, rather than having to subsequently issue debt and incur an issuance cost. Put somewhat differently, the firm would not want to pay down debt with available cash since it is costly to replace debt later when financing is needed. Thus, in the presence of debt flotation costs, for example, underwriting fees, it can be value enhancing to retain cash even if the firm simultaneously has debt outstanding.

Note that after a certain level of cash balance has been attained (in the example shown in Figure 5, once the debt and cash levels exceed five), there is no additional benefit to further increasing the level of cash and debt. At the same time, however, there is no value loss from doing so either, and thus there will be an indeterminate number of debt and cash pairings that yield the same net firm value. Equivalent upward or downward changes in cash and debt levels will not affect shareholder value. This type of neutral mutation makes it difficult to conduct empirical tests on either debt or cash balances separately. Of course, for lower cash balance levels, our results show that using the net debt level may also not completely capture the effect of financing policy on firm value. Thus, debt and cash balance should both be simultaneously considered, rather than netted out, when conducting careful empirical tests of financing policy.

C. Cash Generation and Deployment Strategies

In order to gain more insight into the effect of financial flexibility on the dynamic financing, investment, and retention policies of firms, we use the simulation procedure described in Section II to measure various characteristics of firms following simulated paths of the state variable over time. In each case, the firms make optimal forward-looking decisions over time, as specified in the
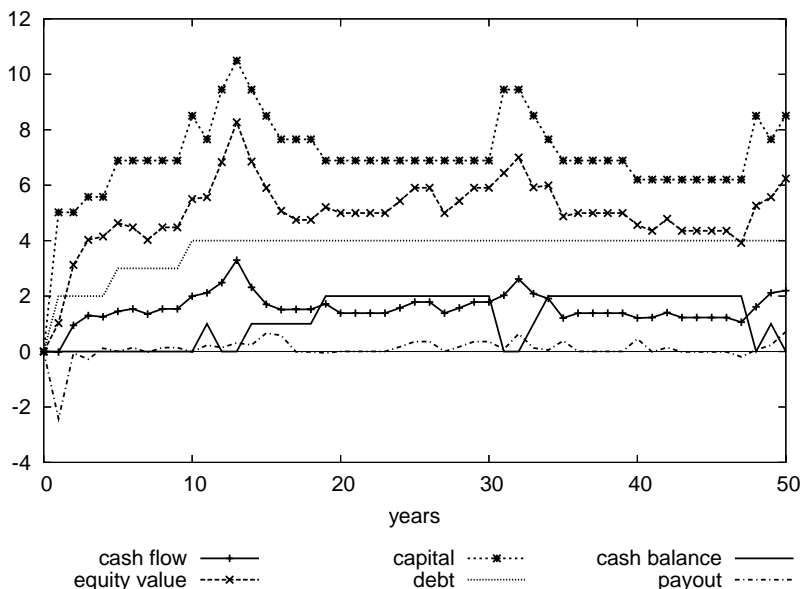
Figure 6. Example of dynamic evolution of a firm. This figure shows the dynamic evolution of a base case firm (inflexible capital and costly financing) over a 50-year period, starting from initial conditions of zero debt, zero cash balance, zero capital, and $\theta = 1$. All the other parameter values are shown in Table I.

optimization in equation (8). We examine various flow and stock characteristics based on 10,000 simulated paths (i.e., 10,000 different firms tracked over time).

Figure 6 illustrates the evolution of a firm, beginning as a start-up with no debt, no cash, and no capital, and emerging over time into a mature firm, responding in an optimal fashion to productivity shocks as they occur. While the dynamics shown in Figure 6 are simply meant to illustrate the evolution of a firm in our model under a particular scenario, and thus do not permit us to make generalizations about the nature of the policies and the resulting characteristics of firms, it is worth making some quick observations.

First, note that the firm immediately adds capacity, since our model does not include convex adjustment or financing costs that would spread out investment more over time. The firm finances investment initially using both debt and equity (negative payout), in approximately equal proportion. It subsequently borrows some more as it invests in additional capacity (given the positive productivity shocks), which in turn can support a higher debt level. Eventually, the firm ceases to borrow more, and maintains a constant debt level. We will explore this characteristic of the debt policy in greater detail later in this section.

The cash balance in the first 10 years is zero, but then starts to build up to a higher level, co-existing with debt in an optimal fashion as described earlier. The cash balance does drop to zero, however, during time periods where the
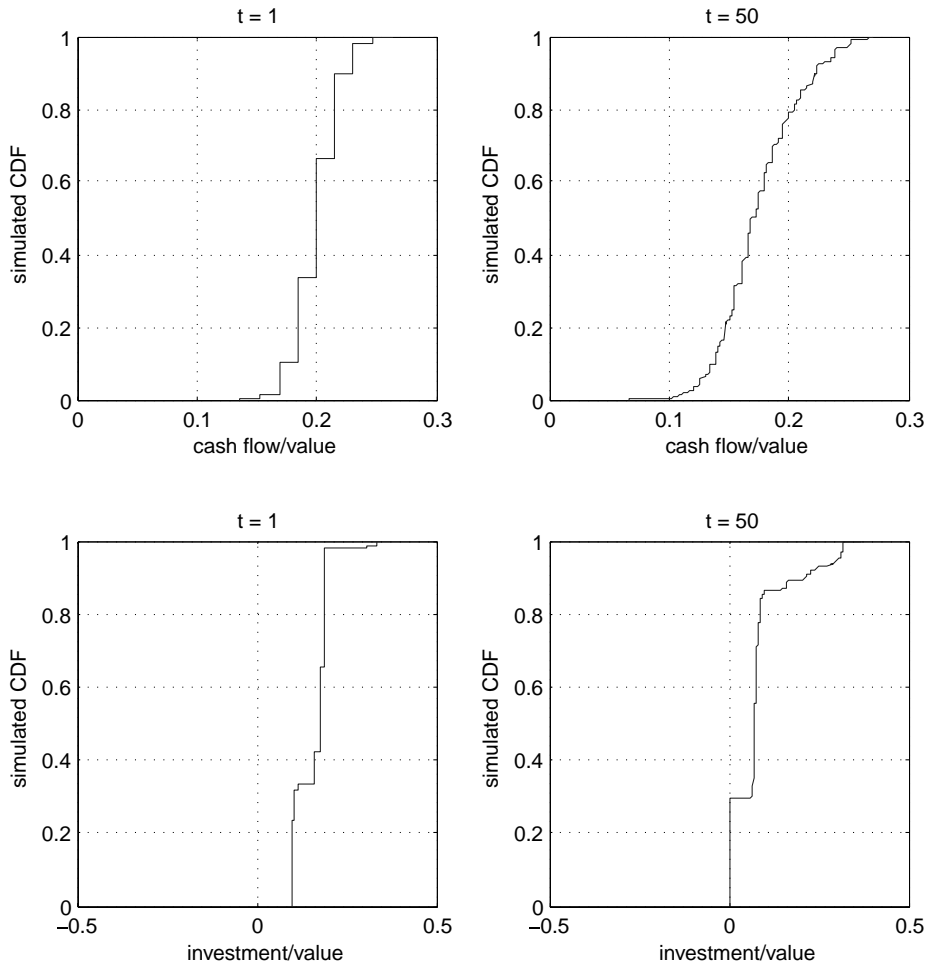
Figure 7. Distributions of cash flow and investment. This figure illustrates the cumulative distributions of cash flow and investment (each normalized by firm value), based on 10,000 simulation runs. The left-hand column shows cross-sectional distributions for 1-year-old ($t = 1$) firms, while the right-hand column is for mature firms ($t = 50$). All base-case assumptions hold, as shown in Table I.

productivity shock is significantly positive. In these periods, the firm uses its liquidity to finance profitable investment internally, while also benefiting from increased tax shields on a higher net debt level, which is backed by increased capital as collateral.

We turn now to look more systematically at the generation and deployment of cash by firms. Figures 7 and 8 illustrate the distributions of five flow variables across the 10,000 scenarios (firms), each normalized by firm value: cash flow, investment, change in debt ("Debt Policy"), change in cash

balance ("Cash Policy"), and payout.[25] The first variable captures the cash generation of the firm, while the other four variables describe the firm's cash deployment strategy: The firm's cash flow can be used to pay back debt, to pay out dividends or repurchase equity (we don't distinguish between these two forms of payouts), to invest in new capacity, or to increase the firm's cash balance.

We examine the distribution of each flow variable across two cross-sections of firms: firms that have been active for only 1 year (i.e., $t = 1$, shown in the left-hand column panels), and firms that have been around for 50 years (i.e., $t = 50$, in the right-hand column panels). The latter group of firms have settled well into a steady-state situation, affected by recent productivity shocks but no longer by the idiosyncrasies of the start-up phase. All distributions are for the base case of inflexible capital and costly financing.

Figure 7 shows that the after-tax cash flow (before any capital expenditure) is distributed smoothly between 10% and 25% of firm value when the firm is in a mature stage. The start-up firm has a similar, but tighter, distribution given that there has only been one period's worth of variance. Investment ranges between 0% to +30% of firm value for the mature firm, but roughly half the time the investment is very close to the mean of 7.8% of firm value.[26] In separate calculations, we find that as a percentage of the book value of capital, this is close to 11.1%. This spike in the cumulative distribution is due to the fact that approximately half the time the firm is simply replacing its capacity lost due to depreciation (depreciation is 10%, and thus the replacement investment needs to be 11.1% of the depreciated capital). The firm also responds, however, to the stochastic changes in productivity by increasing capital, or by letting its capacity gradually depreciate over time.[27] The standard deviation of investment/value from our simulation is 0.081, which is reasonably close to the standard deviation of 0.077 for nonfinancial COMPUSTAT firms during 1985 to 2005. In contrast, the standard deviation for investment/value found in our simulations using fully reversible capital (0.126) is much higher than the data seem to suggest.

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[25] Though normalization by the book value of capital is common in the literature for certain variables, particularly investment, since the cash balance of the firm is a productive asset of sorts in that it can enable production to occur at optimal capacity levels, it seems reasonable to focus on a value measure that incorporates the value of cash. The value of the firm appears most appropriate for this purpose.

[26] Note that the start-up invests more (though much investment has already taken place at $t = 0$), but the probability of very high investment levels is lower given that the productivity parameter is less likely to hit high levels after just 1 year.

[27] When capital is flexible, we find that the firm occasionally sells off part of its capital in response to negative productivity shocks. In other unreported results, we examine the frequently studied relationship between investment and cash flow. Using a sorting procedure similar to that in Moyen (2004), we find, as she does, that there is a nonmonotonic relationship between investment and cash flow, regardless of whether the firm faces costly or costless financing. We extend these results by looking at the relationship between investment and the sum of current cash flow plus cash balance, as well as between investment and financial slack (to be defined below). We find these relationships to be nonmonotonic as well. These results are available from the authors.
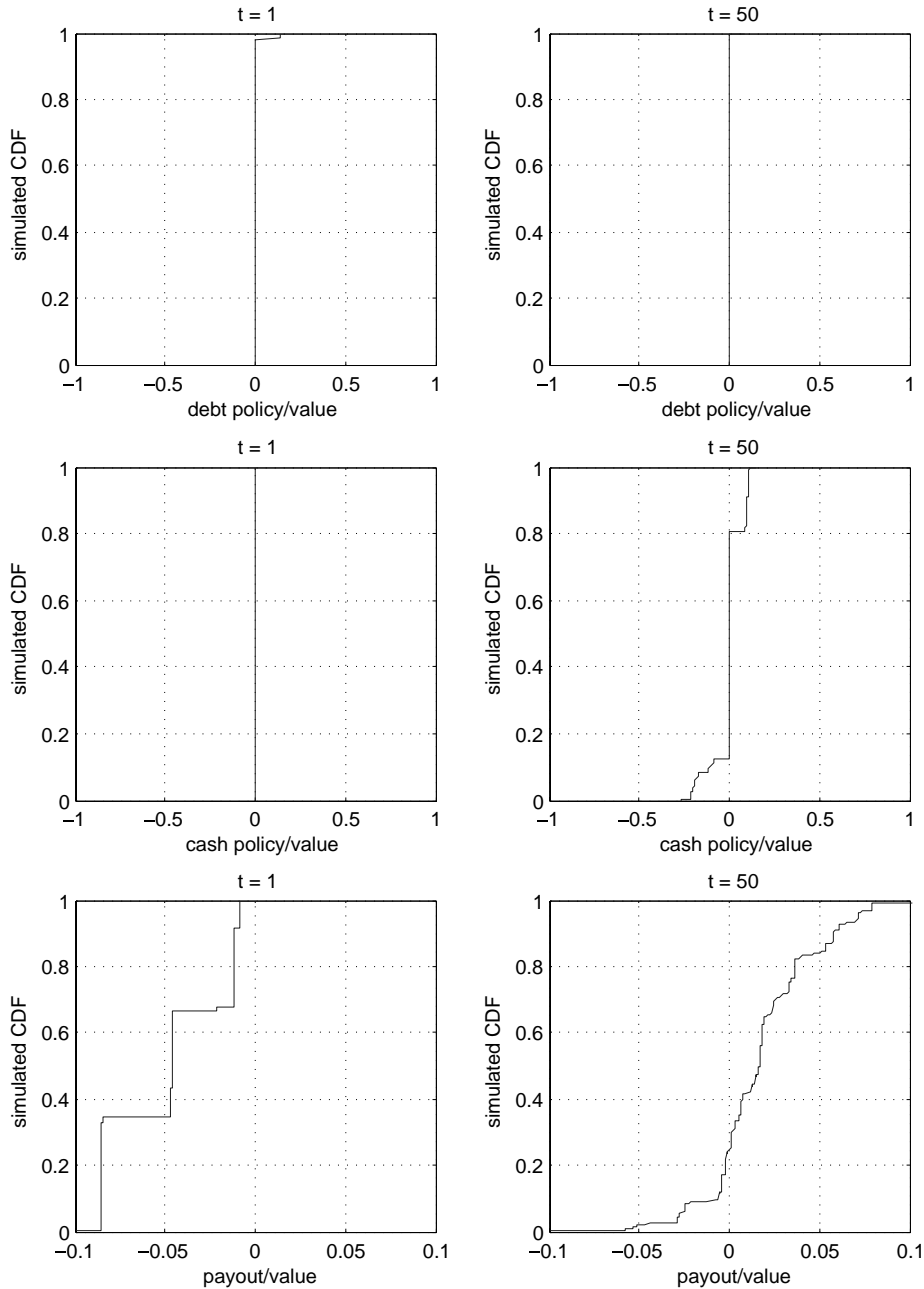
Figure 8. Distributions of financing flow variables. This figure illustrates the cumulative distributions of change in debt ("debt policy"), change in cash balance ("cash policy"), and payout (each normalized by firm value), based on 10,000 simulation runs. The left-hand column shows cross-sectional distributions for 1-year-old ($t = 1$) firms, while the right-hand column is for mature firms ($t = 50$). All base-case assumptions hold, as shown in Table I.

Figure 8 shows that, under the assumptions of the base model, once the constrained firm reaches its steady state, it neither increases nor decreases its level of debt. The firm picks a debt level that it can support even if the firm's profitability drops substantially. Since increasing debt financing is costly, the firm not only avoids increases in debt, but it eschews decreases as well since the debt level would subsequently need to be increased again once profitability returns to a higher level.[28]

Instead, the firm adjusts its net debt by altering its cash balance level, since this incurs no adjustment costs. Observing the firm's liquidity policy in Figure 8, we see that the firm actively changes its cash balance level about one-third of the time, with approximately equal incidence of increases and decreases in liquidity. Decreases in cash balance occur if the firm draws on internal liquidity to finance investment, and if the firm pays shareholders excess cash that is not needed to help support its outstanding debt and future investment. Increases in cash balance enable the firm to decrease its net debt position in response to negative productivity shocks, and to provide additional liquidity for future investment, particularly if the cash balance has been drawn down. Note that when the firm is still starting up (left-hand column), it issues debt about a third of the time in order to help build up the firm's capital. In contrast, the firm is not yet actively managing its liquidity. It uses its cash flow during the first period to help support its investment, rather than to build up a cash balance.

The final component of the cash deployment strategy shown in Figure 8 is the payout policy. The start-up firm does not pay out any cash to its shareholders. Rather, the firm at this stage is always raising equity (which appears as a negative payout in the figure) in order to finance investment. Given that the start-up firm issues debt only some of the time at this early stage of its life, this may appear to be a violation of a pecking order based on the direct issuance costs of debt and equity. However, as mentioned earlier, one must also consider the indirect costs to debt due to the fire-sale discount.[29] At the optimal level of debt that is chosen, equity financing has a lower all-in transaction cost than debt financing, and thus is chosen ahead of debt. Thus, the impact of financial distress is reflected in the dynamic pecking order choice at the margin.

The bottom right-hand graph in Figure 8 shows that once the firm matures, it has a positive net payout to shareholders approximately 70% of the time. The payout yield is typically in the 0% to 5% range, with a mean of 1.8%. These payout yields seem to be roughly in line with payout policies observed in practice. We find (in unreported results) that the distribution of payouts is relatively insensitive to whether the firm has costly or costless external financing, supporting the view of Kaplan and Zingales (1997) and others who argue

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[28] While the constant debt level result may seem extreme, Lemmon, Roberts, and Zender (2006) find that debt is relatively stable over time. This should be even more so for mature firms operating in a steady-state environment.

[29] In addition, the marginal tax rate is low for a start-up firm due to its lower profits, thus leading to a smaller debt tax shield benefit.

that the payout policy of a firm may not be a very good proxy for the presence of financial constraints. Finally, note that the mature firm also has occasional, but rather small, issuances of equity. If we were to include a fixed cost of issuance in our model, the frequency of issuance would likely be even lower, though the amounts would be larger, as observed in practice.[30]

D. Debt, Liquidity, and Financial Flexibility

In addition to studying optimal *incremental* investment, financing, liquidity, and payout decisions made over time, as we have done in the last section, it is also informative to analyze resulting firm characteristics pertinent to financial flexibility, such as the firm's leverage ratio, its cash to value ratio, and its financial slack to value ratio. Figure 9 presents cross-sectional distributions of these ratios based on two sets of optimizations and simulation runs (10,000 runs each). The distributions in the left-hand column capture firms at $t = 50$ under our base-case assumptions. The right-hand column distributions are for firms at $t = 50$ that are subject to higher persistence for the productivity variable ($\rho = 0.85$), which in turn creates higher effective volatility for these firms.

The leverage ratio in our base case ranges from 30% to 60%, with a mean of 45% and a standard deviation of 6%. The variability in leverage ratio is driven here by the variation in firm value. With greater productivity persistence, the average leverage is almost identical (44%), but the standard deviation is much higher (15%), again driven primarily by the variance of firm value. While it may seem surprising that the firm does not have lower leverage when exposed to more underlying uncertainty, the financial conservatism of the high risk firm is reflected primarily in the level of liquidity chosen by the firm (mean of 27% versus 17% for the base case). Taken together, the net debt of the firm is quite sensitive to volatility: The average net debt level is 17% when persistence in productivity is high versus 28% under the base-case persistence assumption.[31]

It is also useful to examine the notion of "financial slack," which is often discussed in the context of financial flexibility. To provide a precise definition

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[30] Kurshev and Strebulaev (2006) also show that fixed debt issuance costs in a dynamic capital structure model induce size effects on leverage.

[31] While these comparisons are illustrative of the general effects of volatility, both the debt and cash balance levels produced by our simulations are higher than those found in practice. For instance, the 25, 50, and 75 percentile points for Debt/Firm Value (using long-term plus short-term debt in the numerator, and debt plus market value of equity in the denominator) are 0.02, 0.15 and 0.38 for COMPUSTAT firms (excluding financial firms and utilities) during 1985 to 2005 (and the mean leverage ratio is 0.23), as compared to 25, 50, and 75 percentile points of 0.40, 0.44, and 0.48 (and a mean of 0.45) for our simulated data. Similarly, the 25, 50, and 75 percentile points for Cash Balance/Firm Value from COMPUSTAT are 0.02, 0.07, 0.16, with a mean of 0.15, as compared to the 25, 50, and 75 percentile points of the simulated distribution of 0.01, 0.20, and 0.23, with a mean of 0.17. Our simulated distributions are based on a steady-state environment using a single set of parameters, and thus don't capture the extent of firm heterogeneity present in reality. Furthermore, there are some important features missing from our model, most notably agency problems as discussed earlier, which could well lead to the lower debt and cash values observed in practice.
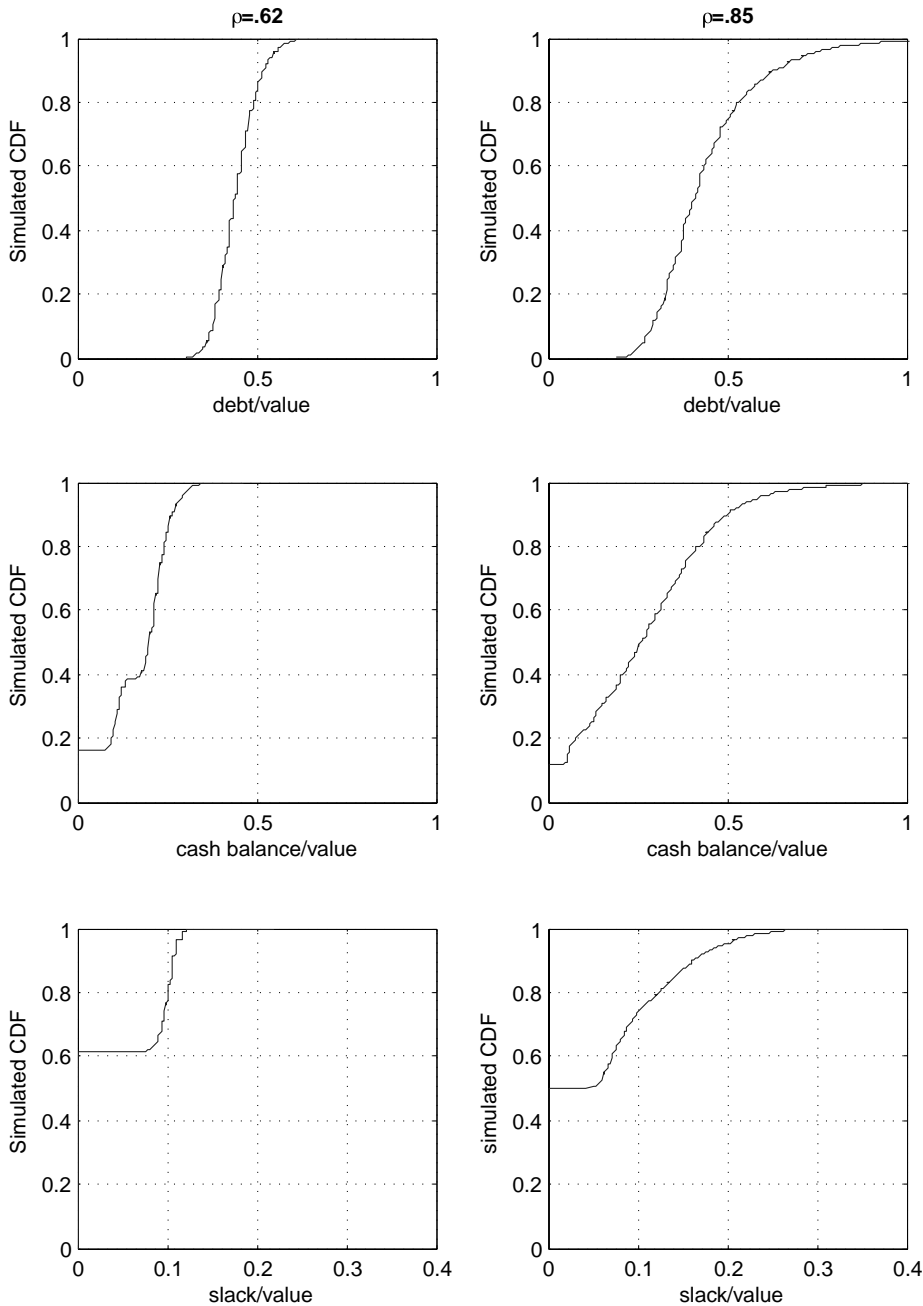
Figure 9. Distributions of leverage, liquidity, and slack ratios. This figure illustrates the cumulative distributions at $t = 50$ of leverage, cash balance, and financial slack ratios based on 10,000 simulation runs. The left-hand column assumes base-case parameters shown in Table I, including a persistence factor of $\rho = 0.62$. The right-hand column is based on a higher persistence factor of $\rho = 0.85$.

of financial slack, we first need to define the maximum debt capacity of the firm. Given

$$C(k, b) = \{(k', p', b') \in C \mid k' = k, b' = b\},$$

the (k, b)-section of the feasible set, the maximum debt capacity at *(k,b)* is defined as

$$DC(k, b) = \max\{p' \mid p' \in C(k, b)\}.$$

Debt capacity *DC(k)* can be interpreted as the maximum debt that can be issued by the firm without violating the collateral constraint. Given the state *(k, p, b)*,

$$DC(k, b) - p \tag{13}$$

defines the financial slackness of the firm.

The bottom graphs in Figure 9 show the effect of the productivity persistence factor on the distribution of financial slack. Given the conservative debt capacity we calculate based on the collateral constraint, it is perhaps not surprising that the firm uses its debt capacity fully approximately 60% of the time under the base-case assumptions. However, it retains a reasonable amount of financial slack the other 40% of the time. The firm leaves itself significantly more slack when the productivity level is more persistent, with net leverage hitting the maximum level only 50% of the time, while keeping up to a 30% slack to value ratio under other scenarios.

V. Conclusions

In our study, we examine the impact of financial flexibility on firm value and on dynamic investment, financing and cash retention policies. Financial flexibility depends not only on direct costs of external financing, but also on corporate and personal tax rates and the liquidation value of capital. We show how a firm can compensate for low exogenous financial flexibility (i.e., high transaction costs) by optimally managing its liquidity policy. If there are no taxes, or more practically, if a tax structure does not create a disincentive to holding cash within the firm, then the firm's liquidity policy can significantly mitigate the effects of external financing costs. We find that simultaneous borrowing and lending by the firm can be optimal in the presence of financing frictions, and that controlling leverage and liquidity policies separately can increase firm value relative to controlling only the net debt level.

Our model indicates that firms with high levels of financial flexibility should be valued at a premium relative to less flexible firms. For mature firms with low growth opportunities, the premium is very small, whereas it can be much larger for firms with either low capital (e.g., younger firms) or low current profitability. The premium is also more substantial if the production technology is relatively inflexible, suggesting that financial and investment flexibility are substitutes to some degree.

While our model provides a rich dynamic framework that yields normative guidelines for managing financial flexibility, and offers an explanation for the observed simultaneous borrowing and lending by firms, it is limited in its ability to accurately match empirical findings related to corporate financial policy. For instance, the liquidity levels observed in practice are lower than those we find in our model. This is perhaps not surprising given the typical concerns expressed by shareholders regarding the potential misallocation of free cash by management. Natural extensions of our model would involve relaxing the restriction that debt is riskless and examining both managerial and debt-related agency problems, as well as allowing for a richer set of investment opportunities and underlying stochastic variables.

Appendix: Model with Single "Net Debt" Variable

To better compare some of our results to those from existing models that assume that the firm either only borrows or only lends (or, equivalently, that it simultaneously borrows and lends, but only the difference between the two positions is relevant), we examine a simplified model where there is a single variable for "net debt," rather than two separate controls for debt and liquidity decisions as we have in our main model. We introduce the state variable m, with unrestricted sign: The negative part of m is the cash balance, $b = \max\{-m, 0\} = m^-$; the positive part is the debt, $p = \max\{m, 0\} = m^+$, with $m = m^+ - m^- = p - b$. Hence, the EBT is $y(k, m, \theta) = \pi(k, \theta) - \delta k - mr$, and the after tax cash flow from operations is $\pi(k, \theta) - mr - g(y(k, m, \theta))$. The state of the firm is described by the triple (k, m, θ). The feasible set with respect to the collateral constraint is now the set of (k', m') such that

$$m'(1 + r) \le sk'(1 - \delta) + \pi(k', \theta_d) - g(y(k', m', \theta_d)).$$

Since now we use the same variable to describe both the debt and the cash balance, the debt issuance cost function in (3) must be modified as follows:

$$q(m, m') = \begin{cases} q_0 + q_1 \max\{(m')^+ - m^+, 0\} & \text{if } m^+ \ne (m')^+ \\ 0 & \text{if } m^+ = (m')^+. \end{cases}$$

Given the above specifications, the cash flow to equity holders becomes

$$\begin{aligned} cf&(k, m, k', m', \theta) \\ &= \max\{(\pi(k, \theta) - g(y(k, m, \theta)) - mr) + m^-, 0\} + m' - m^+ - q(m, m') \\ &\quad - \chi\left(k' - k(1 - \delta) + \max\left\{\frac{(mr + g(y(k, m, \theta)) - \pi(k, \theta)) - m^-}{s}, 0\right\}, \ell\right). \end{aligned}$$

From this point on, the model is the same as our main model described in the text with the control (k', m') in place of (k', p', b').

The optimal policy is defined as follows. Given the state (k, m, θ), the optimal policy is $\varphi(k, m, \theta) \in C$:

$$\varphi(k, m, \theta) = (k^*, m^*) = \arg\max_{(k', m') \in C} \{e(k, m, k', m', \theta) + \beta \mathbb{E}_{k,m,\theta}[E(k', m', \theta')]\}.$$

In particular, given $\varphi(k, m, \theta) = (k^*, m^*)$, we denote the optimal investment policy as

$$K(k, m, \theta) = k^* - \left(k(1 - \delta) - \max \left\{ \frac{(mr + g(y(k, m, \theta)) - \pi(k, \theta)) - m^-}{s}, 0 \right\} \right),$$

and the debt/cash policy as $M(k, m, \theta) = m^* - m$. Notice that $M > 0$ represents either a debt increment (if $m > 0$) or a cash reduction (if $m < 0$).

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