

Structural Form

Assume an economy with two variables: X_1 and X_2 ; and two structural shocks: s_1 and s_2 . Law of motions of X_1 and X_2 has the following process:

$$X_t = B(L)X_{t-1} + As_t$$

which is

$$\begin{cases} X_{1,t} = B_{1,1}(L)X_{1,t-1} + B_{1,2}(L)X_{2,t-1} + A_{1,1}s_{1,t} + A_{1,2}s_{2,t} \\ X_{2,t} = B_{2,1}(L)X_{1,t-1} + B_{2,2}(L)X_{2,t-1} + A_{2,1}s_{1,t} - A_{2,2}s_{2,t} \end{cases}$$

I assume to know exactly the values of matrix $B(L)$ and to only know signs of each element of matrix A . Without loss of generality I will assume that each element of A is strictly positive. Objective is to recover from the reduced-form VAR,

$$\begin{cases} X_{1,t} = B_{1,1}(L)X_{1,t-1} + B_{1,2}(L)X_{2,t-1} + i_{1,t} \\ X_{2,t} = B_{2,1}(L)X_{1,t-1} + B_{2,2}(L)X_{2,t-1} + i_{2,t} \end{cases}$$

structural matrix A . In particular, I want to be able to solve the system

$$As_t = i_t$$

which is

$$\begin{cases} A_{1,1}s_{1,t} + A_{1,2}s_{2,t} = i_{1,t} \\ A_{2,1}s_{1,t} - A_{2,2}s_{2,t} = i_{2,t} \end{cases}$$

Since by assumption, $s_t's_t = I_2$, we have that $A'A = i_t'i_t$ which is

$$\begin{pmatrix} A_{1,1} & A_{2,1} \\ A_{1,2} & A_{2,2} \end{pmatrix} \begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} = \begin{pmatrix} \sigma_{1,1}^2 & \sigma_{1,2}^2 \\ \sigma_{1,2}^2 & \sigma_{2,2}^2 \end{pmatrix}$$

which is

$$\begin{pmatrix} A_{1,1}^2 + A_{2,1}^2 & A_{1,1}A_{1,2} + A_{2,1}A_{2,2} \\ A_{1,1}A_{1,2} + A_{2,1}A_{2,2} & A_{1,2}^2 + A_{2,2}^2 \end{pmatrix} = \begin{pmatrix} \sigma_{1,1}^2 & \sigma_{1,2}^2 \\ \sigma_{2,1}^2 & \sigma_{2,2}^2 \end{pmatrix}$$

which boils down to the following system,

$$\begin{cases} A_{1,1}^2 + A_{2,1}^2 = \sigma_{1,1}^2 \\ A_{1,1}A_{1,2} + A_{2,1}A_{2,2} = \sigma_{1,2}^2 \\ A_{1,2}^2 + A_{2,2}^2 = \sigma_{2,2}^2 \end{cases}$$