Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity

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We study the cyclical implications of credit market imperfections in a quantitative dynamic, stochastic general equilibrium model wherein firms face persistent shocks to aggregate and individual productivity. In our model economy, optimal capital reallocation is distorted by two frictions: collateralized borrowing and partial investment irreversibility. We find that a negative shock to borrowing conditions can, on its own, generate a large and persistent recession through disruptions to the distribution of capital. This recession, and the subsequent recovery, is distinguished both quantitatively and qualitatively from that driven by an exogenous shock to total factor productivity.

I. Introduction

Can a large shock to an economy's financial sector produce a large and lasting recession? Over the past few years, events in the real and financial sectors of the United States and other large, developed economies have been difficult to disentangle. In this paper, we develop a quantitative, dynamic stochastic general equilibrium (DSGE) model to explore how

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real and financial shocks affect the size and frequency of aggregate fluctuations. Firms in our model experience persistent shocks to both aggregate and individual productivity, while credit market frictions interact with real frictions to yield persistent disruptions to the efficient allocation of capital, and thus persistent reductions in aggregate total factor productivity (TFP). Calibrating the model to aggregate and microeconomic data, we use it as a laboratory in which to examine the question raised above.

Capital reallocation is impeded by two frictions in our model, one financial and one real. First, collateralized borrowing constraints limit investment loans. Second, partial irreversibilities in investment lead firms to follow (S, s) rules with respect to their capital. The model gives rise to a rich distribution of firms over idiosyncratic productivity, capital, and financial assets. Within this distribution, a subset of firms have investment curtailed by their current ability to borrow, while a second subset have sufficient resources as to have permanently outgrown the implications of collateral constraints. Most firms fall into a third class, one in which borrowing constraints do not currently bind, but the prospect that they may bind in the future affects current decisions.

We are to our knowledge the first to explore the endogenous TFP channel in a quantitative DSGE setting in which real frictions slow the reallocation of capital across firms and in which reallocation is essential in determining aggregate TFP. We show that a shock to the availability of credit can, on its own, generate a large and protracted recession because it induces changes in the distribution of firms. These changes bring about a large, but gradual, deterioration in aggregate productivity by disrupting the allocation of capital further from that implied by firm productivities.

Our model is consistent with evidence from the Flow of Funds that, in the aggregate, nonfinancial firms can finance investment expenditures from cash flows (Chari 2012). Furthermore, our average cash holdings closely match the high corporate cash ratios in Compustat just prior to the 2007 US recession (Bates, Kahle, and Stulz 2009). Nonetheless, because individual firms have differing credit needs and access, the model predicts a sharp reduction in real economic activity when hit by a credit shock sufficient to reproduce observed recent declines in lending.

The extent of capital reallocation in ordinary times influences how much a shock to the availability of credit disrupts real economic activity. While partial capital irreversibility complicates our analysis, it is essential in allowing the model to reproduce important moments from the distribution of microeconomic investment rates and, thus, for measuring the stochastic process governing firm-level productivity shocks. The model's ability to capture the effective importance of financial frictions is further demonstrated by its success in reproducing the average cross-sectional size-leverage correlation and standard deviation of cash ratios in micro-level financial data.

When we consider a temporary shock affecting individual firms' access to credit, our model predicts aggregate changes resembling those from the 2007 US recession in several respects. It delivers a gradual deterioration of GDP, a delayed decline in consumption, and an unusually steep fall in investment. It also captures the magnitudes of decline in US GDP and investment, alongside reductions in total lending consistent with several measures from the data. Further, the resulting shifts in the distribution of production across firms drive endogenous changes in aggregate TFP explaining more than half of the observed decline. Finally, employment declines among small firms exceed those among large firms, consistent with census evidence we present. By contrast, an exogenous shock to TFP generates declines in GDP, employment, investment, and lending far weaker than those in the 2007 recession, and it delivers an equal incidence of employment decline among small versus large firms.

We argue that our analytical framework captures real and financial frictions that are vital in explaining actual microeconomic reallocation and may thus shape macroeconomic outcomes. Our findings suggest that changes in firms' access to credit are important in understanding the recent US recession. However, we would not claim that our model explains the 2007 recession. We explore one channel, increased capital misallocation, but abstract from others, such as changes in households' access to credit and labor market imperfections, that also seem to have been important. Thus, the simple credit crisis exercise considered here cannot, in itself, explain the US recovery episode beginning in the second half of 2009. Most notably, while its GDP recovery is gradual, our model cannot simultaneously account for the growth in TFP alongside much weaker growth in employment and investment.

The numerical method used to solve our model may be of independent interest. We first identify a set of firms with sufficient assets so that collateralized borrowing limits can never again affect their choice of capital. Using nonlinear methods to approximate their value functions, we characterize the behavior of these firms, including both their investment in physical capital and their debt or savings. We use the resulting value function as the starting point to solve the decision rules of other firms in the economy. Since our aggregate state involves a distribution of firms over productivity, capital, and financial assets, we also must extend existing aggregate state space approximation methods to compute equilibria in the presence of both real and financial shocks.

The remainder of the paper is organized as follows. Section II summarizes the literature most related to our work. Section III presents the model economy, and Section IV provides analysis useful in developing a numerical algorithm capable of its solution. In Section V, we describe our calibration to moments drawn from postwar US aggregate and firm-level data and discuss the solution method. Section VI presents results, and Section VII presents concluding remarks.

II. Related Literature

There is a large related literature exploring how financial frictions influence the aggregate response to nonfinancial shocks. Leading this literature, Kiyotaki and Moore (1997) develop a model of credit cycles and show that collateral constraints can have a large role in amplifying and propagating shocks to the value of collateral.[1] Our own work follows in the spirit of Kiyotaki and Moore in that the financial frictions we explore are collateralized borrowing constraints. However, our collateral constraints are anchored on firms' existing, rather than future, capital stocks. This gives rise to firm-level dynamics reminiscent of those in models with constrained optimal dynamic contracts.[2]

While we assume collateral constraints, there are well-known alternative approaches. Cooley, Marimon, and Quadrini (2004) study constrained-optimal dynamic contracts under limited enforceability. Elsewhere, a large literature examines agency costs as the source of financial frictions (see Bernanke and Gertler 1989; Carlstrom and Fuerst 1997; Bernanke, Gertler, and Gilchrist 1999). These papers do not consider financial shocks as such, and they abstract from potentially important heterogeneity under which the allocation of capital, and thus credit, becomes relevant.

Over the past few years, several studies have begun exploring how financial shocks affect aggregate fluctuations. Our emphasis on firm borrowing subject to collateral constraints is complementary to results by Boz and Mendoza (2012), who study borrowing by a representative household in a small open economy in which land serves as collateral. Examining the effect of financial innovation with uncertain persistence, and assuming Bayesian learning, they explain large increases in household debt and land prices given optimistic priors.

[1] Kocherlakota (2000) and Cordoba and Ripoll (2004) argue that these effects are quantitatively minor in calibrated versions of the model. Kiyotaki (1998) extends Kiyotaki and Moore (1997) to accommodate log utility, differing productivities between productive and unproductive agents, and the switching of types.

[2] For example, our young firms grow as their ability to borrow rises, and mean growth rates fall with age and size. Albuquerque and Hopenhayn (2004) derive these regularities in a model with limited enforceability, while Clementi and Hopenhayn (2006) derive them under private information.

Our focus on the aggregate implications of financial shocks is also shared by the work of Jermann and Quadrini (2012). They develop a representative firm model in which investment is financed using both debt and equity, and costs of adjusting dividends prevent the avoidance of financial frictions. These frictions stem from limited enforceability of debt contracts, which gives rise to endogenous borrowing limits. If the firm chooses to default, the lender can recover only a fraction of its net worth, and shocks to the fraction the lender can confiscate alter the severity of borrowing limits. Measuring these credit shocks, Jermann and Quadrini find that they have been an important source of business cycles.[3]

In contrast to Jermann and Quadrini's model, the financial frictions in our setting do not significantly dampen aggregate responses to nonfinancial shocks. This may be useful in light of findings by Reinhart and Rogoff (2009) and Bianchi and Mendoza (2012) that large financial shocks are rare in postwar US history. We also capture important aspects of microeconomic investment behavior by including both heterogeneous firms and partial capital irreversibility.[4]

Firms in our model accumulate capital and may borrow or save. They have a natural maturing phase and tend to eventually overcome the effect of collateral constraints on investment. Thus, the incidence of a credit shock differs, and we can explore the extent to which small firms with greater reliance on investment loans are disproportionately affected. Moreover, following such shocks, shifts in the distribution of capital drive endogenous movements in aggregate TFP through misallocation. By accommodating differences in financial capital, our model matches both aggregate debt and the cash held by nonfinancial firms in the data. Thus, we can consider how much firms' savings mitigate the effect of a fall in lending. However, because we do not have a working capital constraint, borrowing limits do not directly affect our firms' employment decisions. This is the main channel through which financial shocks have real effects in Jermann and Quadrini (2012).

Our paper contributes to a body of work exploring the effects of financial frictions in models with heterogeneous firms. Arellano, Bai, and Kehoe (2012) examine the role of uncertainty shocks in a model with noncontingent debt and equilibrium default. Gomes and Schmid (2010) develop a model with endogenous default, where firms vary with respect

[3] Jermann and Quadrini (2009) adapt this model to address the variability of real and financial variables in the past 25 years.

[4] See Veracierto (2002) for a DSGE analysis of how these frictions affect aggregate responses to productivity shocks. Caggese (2007) considers irreversible capital and collateral constraints; our study is distinguished from his by general equilibrium analysis, partial reversibility in investment, and frictionless within-period borrowing.

to their leverage, and study the implication for credit spreads. Gilchrist, Sim, and Zakrajšek (2011) study credit spreads under uncertainty shocks in a model with default.[5]

Buera and Moll (2013) share our focus on collateral constraints in an economy with production heterogeneity. They assume entrepreneurs operating constant returns to scale production technologies subject to independent and identically distributed productivity shocks. When investment loans are subject to collateral constraints requiring that debt not exceed a proportion of future capital, and future productivity is known before capital is allocated, Buera and Moll establish that shocks to collateral constraints are isomorphic to shocks to aggregate TFP.

In our model, credit shocks lead to gradual reductions in aggregate TFP that are qualitatively different from persistent shocks to its exogenous component. We assume firms operating decreasing returns to scale production technologies subject to persistent idiosyncratic shocks not observed in advance, and we assume that existing, not future, capital serves as collateral. These assumptions together imply that firms eventually overcome the effect of collateral constraints and adopt efficient levels of capital. By introducing entry and exit, and assuming that entrants have less capital than the typical firm, we ensure that the aggregate effects of collateral constraints persist over time. Credit shocks in our model drive nonmonotone responses in aggregate TFP because the misallocation of resources grows as a growing fraction of firms find it increasingly difficult to finance investment. Furthermore, our approach allows us to use micro data to measure the idiosyncratic productivity process using several moments from the distribution of firm-level investment. Because the extent of the misallocation of capital hinges on the distribution of productivity across firms, this is a potentially important prerequisite for quantitatively assessing the aggregate importance of credit shocks.

III. Model

In our model economy, firms face both partial capital fixity and collateral constraints, which together compound the effects of persistent differences in their total factor productivities to yield substantial heterogeneity in production. We begin our description of the economy with an initial look at the optimization problem facing each firm and then follow with a discussion of households and equilibrium. Next, in Section IV, we use a simple implication of equilibrium alongside some immediate observations about firms' optimal allocation of profits across dividends and retained earnings to characterize the capital adjustment decisions of

[5] Credit spreads are also emphasized by Gertler and Kiyotaki (2010) in a model in which such spreads are driven by agency problems arising with financial intermediaries.

our firms. This analysis allows us to derive a convenient, computationally tractable algorithm to solve for equilibrium allocations in our model, despite its three-dimensional heterogeneity in production.

A. *Production, Credit, and Capital Adjustment*

We assume a large number of firms, each producing a homogeneous output using predetermined capital stock k, and labor n, via an increasing and concave production function, $y = z\varepsilon F(k, n)$. The variable z represents exogenous stochastic TFP common across firms, while ε is a firm-specific counterpart. We assume that ε is a Markov chain; $\varepsilon \in \mathbf{E} \equiv \{\varepsilon_1, \ldots, \varepsilon_{N_\varepsilon}\}$, where $\Pr(\varepsilon' = \varepsilon_j | \varepsilon = \varepsilon_i) = \pi_{ij} \geq 0$ and $\sum_{j=1}^{N_\varepsilon} \pi_{ij} = 1$ for each $i = 1, \ldots, N_\varepsilon$. Similarly, $z \in \{z_1, \ldots, z_{N_z}\}$, where $\Pr(z' = z_g | z = z_f) = \pi^z_{fg} \geq 0$ and $\sum_{g=1}^{N_z} \pi^z_{fg} = 1$ for each $f = 1, \ldots, N_z$.

Because our interest is in understanding how financial constraints shape the investment decisions made by firms in our economy, we must prevent firms from accumulating sufficient resources that none will ever again experience a binding borrowing limit. To ensure that this does not occur, we impose exit and entry in the model. In particular, we assume that each firm faces a fixed probability, $\pi_d \in (0, 1)$, that it will be forced to exit the economy following production in any given period. Within a period, prior to investment, firms learn whether they will survive to produce in the next period. Exiting firms are replaced by an equal number of new firms whose initial state will be described below.

At the beginning of each period, a firm is defined by its predetermined stock of capital, $k \in \mathbf{K} \subset \mathbf{R}_+$, by the level of one-period debt it incurred in the previous period, $b \in \mathbf{B} \subset \mathbf{R}$, and by its current idiosyncratic productivity level, $\varepsilon \in \mathbf{E}$. Immediately thereafter, the firm learns whether it will survive to produce in the next period.[6] Given this individual state and the current aggregate state, the firm takes a series of actions to maximize the expected discounted value of dividends returned to its shareholders, the households in our economy. First, it chooses its current level of employment, undertakes production, and pays its wage bill. Thereafter, it repays its existing debt and, conditional on survival, it chooses its investment, i, current dividends, and the level of debt with which it will enter into the next period, b'. For each unit of debt it incurs for the next period, a firm receives q units of output that it can use toward paying current dividends or investing in its future capital. The relative price q^{-1}

[6] For computational tractability, we make the exit shock orthogonal so that all firms borrow at a common interest rate. Our timing is to ensure that there is no equilibrium default. Because the only firms borrowing are those that will produce next period and the debt they take on is limited by a collateral constraint, firms always repay their debt in our quantitative exercises.

reflecting the interest rate at which firms can borrow and lend is a function of the economy's aggregate state, as is the wage rate ω paid to workers. For expositional convenience, we suppress the arguments of these equilibrium price functions until we have described the model further.

Our model economy is distinguished from most in allowing an interaction between real and financial frictions. Two external forces together determine what fraction of its capital stock a firm can borrow against: the degree of specificity in capital and the enforceability of financial arrangements. Firms face collateralized borrowing constraints of the form $b' \leq \zeta\theta_k k$ in each period, where $\theta_k \in [0, 1]$ is a parameter reflecting the fraction of a firm's capital stock that survives when it is uninstalled, and $\zeta \in \{\zeta_1, \ldots, \zeta_{N_\zeta}\}$ is the fraction of that collateral firms can borrow against. The credit variable, ζ, may be interpreted as reflecting the efficiency of the economy's financial sector. We assume that it is common across firms and follows a Markov chain, with $\Pr(\zeta' = \zeta_k | \zeta = \zeta_h) = \pi^{\zeta}_{hk}$ and $\sum_{k=1}^{N_\zeta} \pi^{\zeta}_{hk} = 1$ for $h = 1, \ldots, N_\zeta$. For convenience below, we summarize the economy's exogenous aggregate state by $s = (z, \zeta)$ and its transition probabilities by $\pi^s_{lm} = \Pr(s' = (z, \zeta)_m | s = (z, \zeta)_l)$, where each π^s_{lm} is derived from the transition probabilities π^{ζ}_{hk} and π^z_{fg}, and we denote the real and financial shocks in realized state s_l as z_l and ζ_l, for each $l = 1, \ldots, N_s$ (where $N_s = N_z N_\zeta$).

If a firm undertakes any nonnegative level of investment, $i \geq 0$, its capital stock at the start of the next period is determined by a familiar accumulation equation, $k' = (1 - \delta)k + i$, where $\delta \in (0, 1)$ is the rate of capital depreciation, and primes indicate one-period-ahead values. Because there is some degree of specificity in capital, the same equation does not apply when the firm undertakes negative investment. In this case, the effective relative price of investment is θ_k rather than 1, so the accumulation equation is $\theta_k k' = \theta_k(1 - \delta)k + i$ when $i < 0$.

As noted in Section I, we include partial irreversibility so our model can be calibrated to reproduce important moments from the distribution of establishment investment rates. We adopt this real friction over others because it allows us to match the frequency of large (spike) investments alongside the weakly positive autocorrelation of investment rates observed in microeconomic data. Firm-level quadratic adjustment costs can achieve the latter only at the expense of the former, since they imply an increasing marginal cost of adjustment. Randomly drawn nonconvex costs of capital adjustment are associated with investment spikes but do not overturn negative serial correlation in investment rates; the implied increasing returns adjustment technology makes a firm with positive investment in one period unlikely to invest in the next.

We show in Section IV that partial irreversibility (a linear cost) naturally yields two-sided (S, s) investment decision rules; firms have nonzero investment only when their capital falls outside an (S, s) inactivity band.[7] A firm in our model finding itself with an unacceptably high capital stock (given its current productivity) will reduce its stock only to the upper bound of its inactivity range. Similarly, a firm with too little capital invests only to the lower bound of its inactivity range to reduce the linear penalty it will incur if it later chooses to shed capital. Thus, partial irreversibility can deliver persistence in firms' investment rates by encouraging repeated small investments at the edges of inactivity bands.

As mentioned above, a firm's capital adjustment may also be influenced by its ability to borrow. This is in turn affected by the capital (collateral) it currently holds. Further, the firm's current investment decision may influence the level of debt it carries into the next period. These observations imply that we must monitor the distinguishing features of firms along three dimensions: their capital, k, their debt, b, and their idiosyncratic productivity, ε. Thus, in contrast to models with loan market frictions, but without capital adjustment frictions, a firm's cash on hand is an insufficient description of its state; capital and debt are distinct state variables.

We summarize the distribution of firms over (k, b, ε) using the probability measure μ defined on the Borel algebra, $\mathcal{S}$, generated by the open subsets of the product space, $\mathbf{S} = \mathbf{K} \times \mathbf{B} \times \mathbf{E}$. The aggregate state of the economy is (s, μ), and the distribution of firms evolves over time according to a mapping, Γ, from the current aggregate state; $\mu' = \Gamma(s, \mu)$. The evolution of the firm distribution is determined in part by the actions of continuing firms and in part by entry and exit. As mentioned above, a fraction π_d of firms exit the economy after production in each period. These firms are replaced by the same number of new firms. Each new firm has zero debt and productivity $\varepsilon_0 \in \mathbf{E}$ drawn from the ergodic distribution implied by $\{\pi_{ij}\}$, and each enters with an initial capital stock $k_0 \in \mathbf{K}$.

We now turn to the problem solved by each firm in our economy. Let $v_0(k, b, \varepsilon_i; s_l, \mu)$ represent the expected discounted value of a firm that enters the period with (k, b) and firm-specific productivity ε_i, when the aggregate state of the economy is (s_l, μ), just before it learns whether it will survive into the next period. We state the firm's dynamic optimization problem using a functional equation defined by (1)–(4) below:

$$v_0(k, b, \varepsilon_i; s_l, \mu) = \pi_d \max_n [z_l \varepsilon_i F(k, n) - \omega(s_l, \mu) n + \theta_k (1 - \delta) k - b] + (1 - \pi_d) v(k, b, \varepsilon_i; s_l, \mu). \tag{1}$$

[7] The problem of costly investment reversibility is solved by Abel and Eberly (1996). Dixit and Pindyck (1994) develop an analysis emphasizing the option value of waiting to invest.

After the start of the period, the firm knows which part of (1) will prevail. If it is not continuing beyond the period, the firm simply chooses labor to maximize its current dividend payment to shareholders. Because it will carry no capital or debt into the future, an exiting firm's dividends are its output, less wage payments and debt repayment, together with the capital it can successfully uninstall at the end of the period. The problem conditional on continuation is more involved because a continuing firm must choose its current labor and dividends alongside its future capital and debt. For expositional convenience, given the partial irreversibility in investment, we begin to describe this problem by defining the firm's value as the result of a binary choice between upward versus downward capital adjustment in (2), then proceed to identify the value associated with each option in (3) and (4):

$$v(k, b, \varepsilon_i; s_l, \mu) = \max\{v^u(k, b, \varepsilon_i; s_l, \mu), v^d(k, b, \varepsilon_i; s_l, \mu)\}. \tag{2}$$

Assume that $d_m(s_l, \mu)$ is the discount factor applied by firms to their next-period expected value if the exogenous aggregate state at that time is s_m and the current aggregate state is (s_l, μ). Taking as given the evolution of ε and s according to the transition probabilities specified above, and given the evolution of the distribution, $\mu' = \Gamma(s, \mu)$, the firm solves the following two optimization problems to determine its values conditional on (weakly) positive and negative capital adjustment. In each case, the firm selects its current employment and production, alongside the debt and capital with which it will enter into next period and current dividends, D, to maximize its expected discounted dividends. As above, dividends are determined by the firm's budget constraint as the residual of current production and borrowing after its wage bill and debt repayment have been covered, net of investment expenditures.

Conditional on an upward capital adjustment, the firm solves the following problem constrained by (i) the fact that investment must be nonnegative, (ii) a borrowing limit determined by its collateral, and (iii)–(iv) the requirements that dividends be nonnegative and satisfy the firm's budget constraint:

$$\begin{aligned} &v^u(k, b, \varepsilon_i; s_l, \mu) \\ &\quad = \max_{n,k',b',D} \left[D + \sum_{m=1}^{N_s} \pi^s_{lm} d_m(s_l, \mu) \sum_{j=1}^{N_\varepsilon} \pi_{ij} v_0(k', b', \varepsilon_j; s_m, \mu') \right] \end{aligned} \tag{3}$$

subject to

$$\begin{aligned} &k' \geq (1-\delta)k,\ b' \leq \zeta_l \theta_k k, \\ &0 \leq D \leq z_l \varepsilon_i F(k,n) - \omega(s_l,\mu)n \\ &\quad + q(s_l,\mu)b' - b - [k' - (1-\delta)k], \\ &\text{and } \mu' = \Gamma(s_i,\mu). \end{aligned}$$

The downward adjustment problem differs from that above only in that investment must be nonpositive, and thus, its relative price is θ_k:

$$\begin{aligned} &v^d(k,b,\varepsilon_i;s_l,\mu) \\ &\quad = \max_{n,k',b',D}\left[D + \sum_{m=1}^{N_s}\pi^s_{lm} d_m(s_l,\mu)\sum_{j=1}^{N_\varepsilon}\pi_{ij} v_0(k',b',\varepsilon_j;s_m,\mu')\right] \end{aligned} \tag{4}$$

subject to

$$\begin{aligned} &k' \leq (1-\delta)k,\ b' \leq \zeta_l \theta_k k, \\ &0 \leq D \leq z_l \varepsilon_i F(k,n) - \omega(s_l,\mu)n + q(s_l,\mu)b' \\ &\quad - b - \theta_k[k' - (1-\delta)k], \\ &\text{and } \mu' = \Gamma(s_i,\mu). \end{aligned}$$

Notice that there is no friction associated with the firm's employment choice since the firm pays its current wage bill after production takes place, and its capital choice for next period also has no implications for current production. Thus, irrespective of their current debt or continuation into the next period, all firms sharing in common the same (k,ε) combination select employment $N(k,\varepsilon;s,\mu)$ and production $y(k,\varepsilon;s,\mu)$.[8] By contrast, the intertemporal decisions of continuing firms depend on the full firm-level state, given the presence of both borrowing limits and irreversibilities. Let $K(k,b,\varepsilon;s,\mu)$ and $B(k,b,\varepsilon;s,\mu)$ represent the choices of next-period capital and debt, respectively, made by firms of type (k,b,ε). We characterize these decision rules below in Section IV.

B. Households

The economy is populated by a unit measure of identical households. Household wealth is held as one-period shares in firms, which we iden-

[8] Here forward, except where necessary for clarity, we suppress the indices for the current exogenous aggregate state (l) and firm productivity (i).

tify using the measure λ, and in one-period noncontingent bonds, ϕ.[9] Given the (dividend-inclusive) values of their current shares, $\rho_0(k, b, \varepsilon; s, \mu)$, the bond price $q(s, \mu)^{-1}$, and the real wage $\omega(s, \mu)$, households determine their current consumption, c, hours worked, n^h, new bond holdings ϕ', and the numbers of new shares, $\lambda'(k', b', \varepsilon')$, to purchase at ex-dividend prices $\rho_1(k', b', \varepsilon'; s, \mu)$.[10] The lifetime expected utility maximization problem of the representative household is listed below:

$$V^h(\lambda, \phi; s_l, \mu) = \max_{c, n^h, \phi', \lambda'} \left[u(c, 1 - n^h) + \beta \sum_{m=1}^{N_s} \pi^s_{lm} V^h(\lambda', \phi'; s_m, \mu') \right] \quad (5)$$

subject to

$$c + q(s_i, \mu)\phi' + \int_{\mathbf{S}} \rho_1(k', b', \varepsilon'; s_l, \mu)\lambda'(d[k' \times b' \times \varepsilon'])$$
$$\leq \omega(s_l, \mu)n^h + \phi + \int_{\mathbf{S}} \rho_0(k, b, \varepsilon; s_l, \mu)\lambda(d[k \times b \times \varepsilon])$$
$$\text{and } \mu' = \Gamma(s_i, \mu).$$

Let $C^h(\lambda, \phi; s, \mu)$ describe the household decision rule for current consumption, and let $N^h(\lambda, \phi; s, \mu)$ be the rule determining the allocation of current available time to working. Let $\Phi^h(\lambda, \phi; s, \mu)$ describe the household decision rule for bonds, and let $\Lambda^h(k', b', \varepsilon', \lambda, \phi; s, \mu)$ be the quantity of shares purchased in firms that will begin the next period with k' units of capital, b' units of debt, and idiosyncratic productivity ε'.

C. *Recursive Equilibrium*

A *recursive competitive equilibrium* is a set of functions,

$$(\omega, q, (d_m)_{m=1}^{N_s}, \rho_0, \rho_1, v_0, N, K, B, D, V^h, C^h, N^h, \Phi^h, \Lambda^h),$$

that solve firm and household problems and clear the markets for assets, labor, and output, as described by the following conditions.

[9] Households also have access to a complete set of state-contingent claims. As there is no household heterogeneity, these assets are in zero net supply in equilibrium; thus, for simplicity's sake, we do not explicitly model them here.

[10] In equilibrium, $\rho_1(k', b', \varepsilon'; s, \mu)$ is the expected discounted value of owning a share in type (k', b', ε') firms at the start of the next period. Although ε' is drawn by individual firms at the start of the next period, the household can choose its ownership of type (k', b', ε') firms in the current period since it knows the ε transition probabilities and the law of large numbers applies.

i. v_0 solves (1)–(4), N is the associated policy function for exiting firms, and (N, K, B, D) are the associated policy functions for continuing firms;

ii. V^h solves (5), and (C^h, N^h, Λ^h) are the associated policy functions for households;

iii. $\Lambda^h(k', b', \varepsilon_j, \mu, \phi; s, \mu) = \mu'(k', b', \varepsilon_j; s, \mu)$ for each $(k', b', \varepsilon_j) \in \mathbf{S}$;

iv. $N^h(\mu, \phi; s, \mu) = \int_{\mathbf{S}} [N(k, \varepsilon; s, \mu)]\mu(d[k \times b \times \varepsilon])$;

v.

$$\begin{aligned} C^h(\mu, \phi; s, \mu) = \int_{\mathbf{S}} \{ & z\varepsilon F(k, N(\varepsilon, k; s, \mu)) \\ & - (1 - \pi_d)\mathcal{J}(K(k, b, \varepsilon; s, \mu) - (1 - \delta)k) \\ & \times [K(k, b, \varepsilon; s, \mu) - (1 - \delta)k] \\ & + \pi_d[\theta_k(1 - \delta)k - k_0]\}\mu(d[k \times b \times \varepsilon]), \end{aligned}$$

where $\mathcal{J}(x) = 1$ if $x \geq 0$ and θ_k otherwise;

vi.

$$\begin{aligned} \mu'(A, \varepsilon_j) = (1 - \pi_d) & \int_{\{(k,b,\varepsilon_i)|(K(k,b,\varepsilon_i;s,\mu),B(k,b,\varepsilon_i;s,\mu))\in A\}} \pi_{ij}\mu(d[k \times b \times \varepsilon_i]) \\ & + \pi_d \chi(k_0) H(\varepsilon_j), \end{aligned}$$

for all $(A, \varepsilon_j) \in \mathcal{S}$, defines Γ, where $\chi(k_0) = \{1 \text{ if } (k_0, 0) \in A; 0 \text{ otherwise}\}$.

The bond market–clearing condition,

$$\Phi^h(\mu, \phi; s, \mu) = \int_{\mathbf{S}} [B(k, b, \varepsilon; s, \mu)]\mu(d[k \times b \times \varepsilon]),$$

is satisfied by Walras's law.

IV. Analysis

To solve for recursive competitive equilibrium, we begin by devising a reformulation of firms' problems that subsumes the efficiency conditions arising from the household problem. Let C and N describe the market-clearing values of household consumption and hours worked satisfying conditions iv and v above, and denote next period's equilibrium consumption and hours worked when $s' = s_m$ as C'_m and N'_m, respectively. It is straightforward to show that market clearing requires that (*a*) the real wage equal the household marginal rate of substitution between leisure and consumption, $\omega(s, \mu) = D_2U(C, 1 - N)/D_1U(C, 1 - N)$, (*b*) the bond price, q^{-1}, equal the expected gross real interest rate,

$$q(s,\mu) = \beta \sum_{m=1}^{N_s} \pi^s_{lm} D_1 U(C'_m, 1 - N'_m)/D_1 U(C, 1 - N),$$

and (c) firms' state-contingent discount factors agree with the household marginal rate of substitution between consumption across states,

$$d_m(s,\mu) = \beta D_1 U(C'_m, 1 - N'_m)/D_1 U(C, 1 - N).$$

We compute equilibrium by solving the firm problem with these implications of household utility maximization imposed, effectively subsuming households' decisions into the problems faced by firms.

Without loss of generality, we assign $p(s,\mu)$ as an output price at which firms value current dividends and payments and correspondingly assume that firms discount their future values by the household subjective discount factor. Given this alternative means of expressing firms' discounting, the following three conditions ensure that all markets clear in our economy:

$$p(s,\mu) = D_1 U(C, 1 - N), \tag{6}$$

$$\omega(s,\mu) = D_2 U(C, 1 - N)/p(s,\mu), \tag{7}$$

$$q(s,\mu) = \beta \sum_{m=1}^{N_s} \pi^s_{lm} p(s_m, \mu')/p(s,\mu). \tag{8}$$

Our reformulation of (1)–(4) below yields an equivalent description of the firm-level problem in which each firm's value is measured in units of marginal utility rather than output, with no change in the resulting decision rules. Here, we exploit the fact that the choice of n is independent of the k' and b' choices and use the indicator function $\mathcal{J}(x) = \{1 \text{ if } x \geq 0; \theta_k \text{ if } x < 0\}$ to distinguish the relative price of nonnegative versus negative investment:

$$\begin{aligned} V_0(k, b, \varepsilon_i; s_l, \mu) = \pi_d \max_n p(s_l,\mu)[z_l \varepsilon_i F(k,n) - \omega(s_l,\mu) n \\ + \theta_k (1-\delta) k - b] + (1 - \pi_d) V(k, b, \varepsilon_i; s_l, \mu), \end{aligned} \tag{9}$$

where

$$\begin{aligned} &V(k, b, \varepsilon_i; s_l, \mu) \\ &\quad = \max_{n,k',b',D} \left[p(s_l,\mu) D + \beta \sum_{m=1}^{N_s} \sum_{j=1}^{N_\varepsilon} \pi^s_{lm} \pi_{ij} V_0(k', b', \varepsilon_j; s_m, \mu') \right] \end{aligned} \tag{10}$$

subject to

$$0 \leq D \leq z_l \varepsilon_i F(k, n) - \omega(s_l, \mu) n + q(s_l, \mu) b' \\ - b - \mathcal{J}(k' - (1-\delta)k)[k' - (1-\delta)k] \tag{11}$$

and

$$b' \leq \zeta_l \theta_k k. \tag{12}$$

The problem listed in equations (9)–(12) forms the basis for solving equilibrium allocations in our economy, as long as the prices p, ω, and q taken as given by our firms satisfy the restrictions in (6)–(8) above.[11] From here, we begin to characterize the decision rules arising from this problem. A firm of type (k, b, ε) chooses its labor $n = N(k, \varepsilon; s, \mu)$ to solve $z\varepsilon D_2 F(k, n) = \omega$, thereby determining its current production, $y(k, \varepsilon) = z\varepsilon F(k, N(k, \varepsilon; s, \mu))$, and its earnings net of labor costs and debt, $\pi(k, b, \varepsilon)$:

$$\pi(k, b, \varepsilon, s, \mu) \equiv z\varepsilon F(k, N(k, \varepsilon; s, \mu)) \\ - \omega(s, \mu) N(k, \varepsilon; s, \mu) - b. \tag{13}$$

The more challenging objects to determine are D, k', and b' among continuing firms. To disentangle these, we use a simple observation about the implications of borrowing constraints for the valuation of retained earnings versus dividends and sort firms into two categories.

If a firm places nonzero probability weight on encountering a state in which its collateral constraint will bind, we identify it as *constrained*; otherwise, it is *unconstrained*. To be clear, a constrained firm need not face a binding collateral constraint in the current period; our definition includes firms perceiving risk of a binding constraint in the future. The shadow value of retained earnings for a constrained firm includes the discounted value of at least one strictly positive multiplier on a future borrowing constraint, so it exceeds the valuation of current dividends, p.[12] Thus, any such firm sets $D = 0$, and its choice of k' directly implies a level of debt for next period through the binding budget constraint in (11). We will return to the resulting univariate problem of a constrained firm below. However, it is useful to first characterize the decisions of unconstrained firms, whose capital choices are unaffected by borrowing limits.

[11] Here, and in many instances below, we suppress the s, μ arguments of price functions, decision rules, and firm-level state vectors to reduce notation.

[12] This is easily proved using a sequence approach with explicit multipliers on each constraint; see Caggese (2007).

A. Decisions among Unconstrained Firms

An unconstrained firm has accumulated sufficient capital or financial wealth to ensure that collateral constraints will never again affect its investment activities. For any such firm, the multipliers on all future borrowing constraints are zero. Thus it is indifferent between financial savings and dividends; its marginal value of retained earnings equals the household valuation, p.

Let W_0 represent the beginning-of-period expected value of an unconstrained firm and W be the firm's value if it will continue beyond the current period. These functions are analogous to those defined for any firm in (1) and (2), with W^u and W^d denoting the continuing firm's value conditional on upward and downward capital adjustment, respectively:

$$\begin{aligned} W_0(k, b, \varepsilon_i; s_l, \mu) = \pi_d p[\pi(k, b, \varepsilon_i) + \theta_k(1-\delta)k] \\ + (1-\pi_d)W(k, b, \varepsilon_i; s_l, \mu), \end{aligned} \tag{14}$$

$$W(k, b, \varepsilon_i; s_l, \mu) = \max\{W^u(k, b, \varepsilon_i; s_l, \mu), W^d(k, b, \varepsilon_i; s_l, \mu)\}. \tag{15}$$

The crucial identifying features of an unconstrained firm are that (i) its capital choices here forward are independent of its financial position; thus (ii) it is indifferent about b' as (iii) it has the same marginal valuation of savings as a household. Any such firm's b affects its value only through current earnings defined in (13). As these are valued by p, we can express the value of a type (k, b, ε) continuing unconstrained firm as $W(k, 0, \varepsilon) - pb$, and we can write the beginning-of-period expected value as $W_0(k, 0, \varepsilon) - pb$. Given these observations, we have

$$\begin{aligned} W^u(k, b, \varepsilon_i; s_l, \mu) = p\pi(k, b, \varepsilon_i) + p(1-\delta)k \\ + \max_{k' \geq (1-\delta)k}\left[-pk' + \beta\sum_{m=1}^{N_s}\sum_{j=1}^{N_\varepsilon}\pi^s_{lm}\pi_{ij}W_0(k', 0, \varepsilon_j; s_m, \mu')\right], \end{aligned} \tag{16}$$

$$\begin{aligned} W^d(k, b, \varepsilon_i; s_l, \mu) = p\pi(k, b, \varepsilon_i) + p\theta_k(1-\delta)k \\ + \max_{k' \leq (1-\delta)k}\left[-p\theta_k k' + \beta\sum_{m=1}^{N_s}\sum_{j=1}^{N_\varepsilon}\pi^s_{lm}\pi_{ij}W_0(k', 0, \varepsilon_j; s_m, \mu')\right], \end{aligned} \tag{17}$$

where (13) defines $\pi(k, b, \varepsilon)$, and $\mu' = \Gamma(s, \mu)$.

We characterize the capital decision rule for an unconstrained firm by reference to two target capital stocks that would solve the problems in (16) and (17), respectively, if the sign restrictions on investment were removed. Specifically, define the upward target, k^*_u, as the capital a firm would choose given a unit relative price of investment, and define the downward target, k_d^*, as the capital a firm would choose given a relative price at θ_k:

$$k_u^*(\varepsilon_i) = \arg\max_{k'} \left[-pk' + \beta \sum_{m=1}^{N_s} \sum_{j=1}^{N_\varepsilon} \pi^s_{lm} \pi_{ij} w_0(k', \varepsilon_j; s_m, \Gamma(s_l, \mu)) \right], \tag{18}$$

$$k_d^*(\varepsilon_i) = \arg\max_{k'} \left[-p\theta_k k' + \beta \sum_{m=1}^{N_s} \sum_{j=1}^{N_\varepsilon} \pi^s_{lm} \pi_{ij} w_0(k', \varepsilon_j; s_m, \Gamma(s_l, \mu)) \right]. \tag{19}$$

Appendix A establishes that the firm adopts a capital decision rule of the following (S, s) form:[13]

$$K^w(k, \varepsilon; s, \mu) = \begin{cases} k_u^*(\varepsilon; s, \mu) & \text{if } k < \dfrac{k_u^*(\varepsilon; s, \mu)}{1-\delta} \\ (1-\delta)k & \text{if } k \in \left[\dfrac{k_u^*(\varepsilon; s, \mu)}{1-\delta}, \dfrac{k_d^*(\varepsilon; s, \mu)}{1-\delta} \right] \\ k_d^*(\varepsilon; s, \mu) & \text{if } k > \dfrac{k_d^*(\varepsilon; s, \mu)}{1-\delta}. \end{cases} \tag{20}$$

By definition, unconstrained firms are indifferent between financial savings and dividends. To guarantee that they remain so, we assign each such firm a savings rule that implies zero probability of a binding borrowing constraint in every possible future date and state. More specifically, we assign a *minimum savings policy* exactly ensuring that, under all possible paths of (ε, s), the firm will have sufficient wealth to implement its optimal investment plan without borrowing more than is permitted by (12). Provided that an unconstrained firm maintains a level of debt not exceeding the threshold defined by the minimum savings policy, it will remain indifferent to its financial position. Thus, by construction, the savings rule we impose is an optimal policy.

We derive the minimum savings policy, $B^w(k, \varepsilon; s, \mu)$, recursively as the solution to (21)–(22). Define $\tilde{B}(K^w(k, \varepsilon; s, \mu), \varepsilon_j; s_m, \Gamma(s, \mu))$ as the largest debt level at which a firm entering next period with capital $K^w(k, \varepsilon; s, \mu)$ can be unconstrained, given realized exogenous state (ε_j, s_m). Taking the minimum such $\tilde{B}(\cdot)$ over all possible (ε_j, s_m), we identify the largest debt with which a firm can exit this period and be sure to remain unconstrained next period, $B^w(k, \varepsilon; s, \mu)$:[14]

[13] We use w superscripts on unconstrained decision rules to link them to the value function W.

[14] The assumption that ε, z, and ζ are Markov chains permits our proceeding as we do. A solution to (21)–(22) requires that all shocks have finite support.

$$B^w(k,\varepsilon;s,\mu) = \min_{\{\varepsilon_j|\pi_{ij}>0 \text{ and } s_m|\pi^s_{lm}>0\}} \tilde{B}(K^w(k,\varepsilon),\varepsilon_j;s_m,\Gamma(s,\mu)), \quad (21)$$

$$\begin{aligned}\tilde{B}(k,\varepsilon;s,\mu) = {} & z\varepsilon F(k,N(k,\varepsilon)) - \omega N(k,\varepsilon) \\ & + q\min\{B^w(k,\varepsilon;s,\mu),\zeta\theta_k k\} \\ & - \mathcal{J}(K^w(k,\varepsilon)-(1-\delta)k)[K^w(k,\varepsilon)-(1-\delta)k].\end{aligned} \quad (22)$$

Equation (22) defines the beginning-of-period maximum debt level under which a firm can adopt the unconstrained capital rule and debt not exceeding that identified by the minimum savings policy without paying negative dividends and, hence, satisfy the definition of an unconstrained firm. Notice that $\tilde{B}$ is increasing in the firm's current earnings since these may be used to cover outstanding debt. The minimum operator imposes the borrowing constraint; if the firm does not have sufficient collateral to borrow to B^w, it can still be unconstrained if it has sufficient savings to finance its investment. Finally, given the decision rule for capital and the minimum savings policy, we retrieve unconstrained firms' dividend payments as

$$\begin{aligned}D^w(k,b,\varepsilon,s,\mu) = {} & \pi(k,b,\varepsilon,s,\mu) - \mathcal{J}(K^w(k,\varepsilon)-(1-\delta)k) \\ & \times [K^w(k,\varepsilon)-(1-\delta)k] \\ & + q\min\{B^w(k,\varepsilon,s,\mu),\zeta\theta_k k\}.\end{aligned} \quad (23)$$

B. *Decisions among Constrained Firms*

We next consider the decisions of a continuing firm of type (k,b,ε) that has been constrained until now. We begin by evaluating whether the firm has crossed the relevant wealth threshold to become unconstrained. If the firm can adopt the capital rule $K^w(k,\varepsilon)$ in (20) and a level of debt not exceeding that identified by $B^w(k,\varepsilon)$ in (21), while maintaining nonnegative dividends, then it achieves W in (15), and it becomes effectively indistinguishable from other unconstrained firms entering the period with (k,ε):

$$V(k,b,\varepsilon;s,\mu) = \begin{cases} W(k,b,\varepsilon;s,\mu) & \text{iff } D^w(k,b,\varepsilon;s,\mu)\geq 0 \\ V^c(k,b,\varepsilon;s,\mu) & \text{otherwise.}\end{cases} \quad (24)$$

Any firm that can permanently adopt the unconstrained firm decision rules will do so since $V\leq W$. However, this is impossible when the inequality in the top line of (24) is not satisfied. In that case, the firm is constrained and achieves value $V^c(k,b,\varepsilon;s,\mu)$ determined below.

We approach a constrained firm's problem as follows. First, given its (k, ε), we isolate a cutoff debt level under which nonnegative investment is possible without violating the borrowing constraint in (12). The maximum b at which the firm can afford $k' = (1-\delta)k$ while avoiding negative dividends is easily identified from (11) as

$$b = q\zeta\theta_k k + z\varepsilon F(k, N(k,\varepsilon)) - \omega N(k,\varepsilon).$$

If the firm's debt exceeds this cutoff, it must undertake downward capital adjustment, and its value is given by $V^d(k, b, \varepsilon; s, \mu)$ in (27) below; otherwise, it solves the full problem starting in equation (25). In either case, $\mu' = \Gamma(s, \mu)$ is taken as given, and V_0 is defined by (9):

$$V^c(k, b, \varepsilon; s, \mu) = \max\{V^u(k, b, \varepsilon; s, \mu), V^d(k, b, \varepsilon; s, \mu)\}, \tag{25}$$

$$\begin{aligned} V^u(k, b, \varepsilon; s, \mu) &= \max_{k' \in \Omega^u(k,b,\varepsilon)} \beta \sum_{m=1}^{N_s} \sum_{j=1}^{N_\varepsilon} \pi^s_{lm} \pi_{ij} V_0(k', b'_u(k'), \varepsilon_j; s_m, \mu') \\ \text{subject to } b'_u(k') &= \frac{1}{q}\{-\pi(k, b, \varepsilon) + [k' - (1-\delta)k]\}, \end{aligned} \tag{26}$$

$$\begin{aligned} V^d(k, b, \varepsilon;\ s, \mu) &= \max_{k' \in \Omega^d(k,b,\varepsilon)} \beta \sum_{m=1}^{N_s} \sum_{j=1}^{N_\varepsilon} \pi^s_{lm} \pi_{ij} V_0(k', b'_d(k'), \varepsilon_j; s_m, \mu') \\ \text{subject to } b'_d(k') &= \frac{1}{q}\{-\pi(k, b, \varepsilon) + \theta_k[k' - (1-\delta)k]\}. \end{aligned} \tag{27}$$

The constraint sets for the upward and downward adjustment problem are, respectively,

$$\begin{aligned} \Omega^u(k, b, \varepsilon) &= [(1-\delta)k, \bar{k}_u(k, b, \varepsilon)], \\ \Omega^d(k, b, \varepsilon) &= [0, \max\{0, \min\{(1-\delta)k, \bar{k}_d(k, b, \varepsilon)\}], \end{aligned}$$

where $\bar{k}_u$ and $\bar{k}_d$ represent the maximum affordable capital stocks under each option:

$$\begin{aligned} \bar{k}_u(k, b, \varepsilon) &\equiv (1-\delta)k + [q\zeta\theta_k k + \pi(k, b, \varepsilon)], \\ \bar{k}_d(k, b, \varepsilon) &\equiv (1-\delta)k + \frac{1}{\theta_k}[q\zeta\theta_k k + \pi(k, b, \varepsilon)]. \end{aligned}$$

Let the capital stocks solving the conditional adjustment problems in (26) and (27) be denoted by $\hat{k}_u(k, b, \varepsilon)$ and $\hat{k}_d(k, b, \varepsilon)$. The constrained firm sets $D = 0$ and implements the following decision rules for capital and debt:

$$K^c(k, b, \varepsilon; s, \mu) = \begin{cases} \hat{k}_u(k, b, \varepsilon) & \text{if } V^c(k, b, \varepsilon; s, \mu) \\ & \quad = V^u(k, b, \varepsilon; s, \mu) \\ \hat{k}_d(k, b, \varepsilon) & \text{if } V^c(k, b, \varepsilon; s, \mu) \\ & \quad = V^d(k, b, \varepsilon; s, \mu), \end{cases} \tag{28}$$

$$\begin{aligned} B^c(k, b, \varepsilon; s, \mu) = \frac{1}{q}\{&\mathcal{J}(K^c(k, b, \varepsilon; s, \mu) - (1-\delta)k)[K^c(k, b, \varepsilon; s, \mu) \\ &- (1-\delta)k] - \pi(k, b, \varepsilon; s, \mu)\}. \end{aligned} \tag{29}$$

V. Calibration and Solution

The data on establishment-level investment dynamics are reported annually. As the mechanics of the reallocation of capital across firms are at the core of our model, we reproduce salient empirical regularities from these data. Accordingly, we set the length of a period to 1 year. We assume that the representative household's period utility is the result of indivisible labor (Rogerson 1988): $u(c, L) = \log c + \varphi L$. The firm-level production function is Cobb-Douglas: $z\varepsilon F(k, n) = z\varepsilon k^\alpha n$. The initial capital stock of each entering firm is a fixed χ fraction of the typical stock held across all firms in the long run of our full economy; that is, $k_0 = \chi \int k\tilde{\mu}(d[k \times b \times \varepsilon])$, where $\tilde{\mu}$ represents the steady-state distribution therein.

A. *Aggregate Data*

We determine the values of β, ν, δ, α, and φ using moments from the aggregate data as follows. First, we set the household discount factor, β, to imply an average real interest rate of 4 percent, consistent with recent findings by Gomme, Ravikumar, and Rupert (2011). Next, the production parameter, ν, is set to yield an average labor share of income at 0.60 (Cooley and Prescott 1995). The depreciation rate, δ, is taken to imply an average investment-to-capital ratio of roughly 0.069, which corresponds to the average value for the private capital stock between 1954 and 2002 in the US Fixed Asset Tables, controlling for growth. Given this value, we determine capital's share, α, so that our model matches the average private capital-to-output ratio over the same period, at 2.3, and we set the parameter governing the preference for leisure, φ, to imply that an average of one-third of available time is spent in market work.

Exact aggregation obtains in the version of our model without real or financial frictions; in particular, it has an aggregate production function. We use this reference model to estimate an exogenous stochastic process for aggregate productivity. We begin by assuming a continuous shock following a mean zero AR(1) process in logs: $\log z' = \rho_z \log z + \eta_z'$ with $\eta_z' \sim N(0, \sigma_{\eta_z}^2)$. Next, we estimate the values of ρ_z and σ_{η_z} from Solow residuals measured using data on real US GDP and private capital, together with the total employment hours series constructed by Cociuba, Prescott, and Ueberfeldt (2012) from Current Population Survey household data, over the years 1959–2012, and we discretize the resulting productivity process using a grid with three shock realizations ($N_z = 3$) to obtain (z_f) and (π_{fg}^z).

We will see below that there are no perceptible changes in the endogenous component of aggregate TFP in our full model when aggregate fluctuations are driven by exogenous shocks to TFP. Such changes arise only with fluctuations in the credit variable, ζ. We specify ζ as a two-state Markov chain with realizations ζ_o and ζ_l and transition matrix

$$\Pi^\zeta = \begin{bmatrix} p_o & 1 - p_o \\ 1 - p_l & p_l \end{bmatrix}.$$

The realization ζ_o corresponds to ordinary borrowing conditions, while ζ_l is a low realization corresponding to crisis conditions. In the transition matrix, p_o is the probability of continuing in ordinary borrowing conditions, $\Pr\{\zeta' = \zeta_o | \zeta = \zeta_o\}$, while $1 - p_l$ is the probability of escape from crisis conditions, $\Pr\{\zeta' = \zeta_o | \zeta = \zeta_l\}$.

We calibrate our model to reproduce the average aggregate indebtedness of firms in the US economy. We assume $\zeta = \zeta_o$ in calibrating our steady state, and we choose ζ_o to imply a steady-state debt-to-assets ratio at 0.372, matching the average for nonfarm nonfinancial businesses over 1954–2006 in the Flow of Funds. Next, we choose ζ_l so that, when subjected to a credit shock alone, our model delivers a 26 percent drop in debt. This choice is guided by evidence on the reductions in syndicated lending and commercial and industrial loans over the 2007 US recession, which we discuss below in Section VI.

We select the parameters of the Π^ζ matrix using evidence on banking crises from Reinhart and Rogoff (2009). Their definition of a banking crisis includes episodes in which bank runs lead to the closure or public takeover of financial institutions as well as those without bank runs in which the closure, merging, takeover, or government bailout of one important financial institution is followed by similar outcomes for others. They document 13 crises in the United States since 1800 and the share of years spent in crises at 13 percent, which together imply an average crisis duration of 2.09 years. Given our use of postwar targets to calibrate the remaining parameters of our model, the more appropriate statistics for our purposes are those from the period 1945–2008, wherein the United States has had two banking crises (the 1989 savings and loan crisis and the 2007 subprime lending crisis).[15] Unfortunately, it is not possible to determine the average length of a US crisis from this sample period without knowing the ending date of the most recent crisis. Given this difficulty, alongside Reinhart and Rogoff's argument that the incidence and number of crises are similar across the extensive set of countries they consider, we focus instead on their data for advanced economies. The average number of banking crises across advanced economies over 1945–2008 was 1.4, while the share of years spent in crisis was 7 percent. Combining these observations, we set $p_o = 0.9765$ and $1 - p_l = 0.3125$ so that the average duration of a credit crisis in our model is 3.2 years, and the economy spends 7 percent of time in the crisis state.

B. *Firm-Level Data*

We set the exit rate, π_d, so that 10 percent of firms enter and exit the economy each year. Next, we set the fraction of the steady-state aggregate capital stock held by each entering firm, χ, at 0.10 so that, in an average date, each entering firm begins with an initial capital that is one-tenth the size of the aggregate stock. If we had assumed constant returns to scale in production, this would imply an employment size of entering firms averaging one-tenth the size of the typical firm in our economy, matching the Davis and Haltiwanger (1992) data.[16]

The costly reversibility of investment and the dispersion of firm-level TFP are calibrated to reproduce microeconomic evidence on establishment-level investment dynamics. We begin by assuming that firm-specific productivity follows an AR(1) lognormal process, $\log \varepsilon' = \rho_\varepsilon \log \varepsilon + \eta'$, with $\eta' \sim N(0, \sigma^2_{\eta_\varepsilon})$. Next we choose θ_k, ρ_ε, and $\sigma_{\eta_\varepsilon}$ jointly so that our steady state reproduces three aspects of establishment-level investment data documented by Cooper and Haltiwanger (2006) based on a 17-year sample drawn from the Longitudinal Research Database. These targets are (i) the average standard deviation of investment rates (i/k), 0.337; (ii) the average serial correlation of investment rates, 0.058; and (iii) the frequency of lumpy investments, 0.186, which is the fraction of establishment-year observations with a positive investment spike $(i/k > 0.20)$. While not a target in the calibration, we also closely match the average mean investment rate from the Cooper and Haltiwanger study, 0.122; our counterpart is 0.11. In implementation, for each postulated ρ_ε and $\sigma_{\eta_\varepsilon}$, we dis-

[15] These observations are consistent with findings by Bianchi and Mendoza (2012); they document a frequency of financial crisis at 3 percent, consistent with three financial crises in the United States over the past 100 years. Mendoza (2010) estimates a crisis frequency of 3.6 percent across emerging economies since 1980.

[16] Our model has returns to scale at 0.87, so the relative employment size of a new firm is 21 percent. To match Davis and Haltiwanger's (1992) relative size of an entrant, we would have to assume that an entrant has 3.3 percent of the capital of the typical firm. In that case, firms would take longer to mature, amplifying the effects of financial frictions and financial shocks.

cretize firms' lognormal productivity process using seven values ($N_\varepsilon = 7$) to obtain $\{\varepsilon_i\}_{i=1}^{N_\varepsilon}$ and $(\pi_{ij})_{i,j=1}^{N_\varepsilon}$.

While our model has life cycle aspects affecting firms' investments, the Cooper and Haltiwanger (2006) data set includes only large manufacturing establishments that remain in operation throughout their sample period. Thus, for this part of our calibration, we must generate a comparable model data set. We do so by simulating a large sample of unconstrained firms over 30 years, retaining only those firms that survive throughout, and discarding the investment rates from early years. This restricts attention to firms whose investment decisions are unaffected by their borrowing limits and eliminates additional life cycle aspects arising from irreversibility.

The idiosyncratic shock process we calibrate has a persistence of 0.659 and a standard deviation of innovations of 0.118.[17] As firms in the model sample are unaffected by borrowing constraints, their investments would respond immediately to changes in their TFPs in the absence of costs of uninstalling capital. This implies a negative autocorrelation in investment rates since capital is determined by lagged investment. The costly reversibility of capital is then essential in reproducing the investment moments reported above, and we set $\theta_k = 0.954$ to reproduce the serial correlation of investment rates in the data.[18]

At 0.046, our calibrated irreversibility might appear inconsequential. However, in its absence, firm-level capital reallocation changes dramatically. If we maintained our idiosyncratic shock process while setting $\theta_k = 1$, the mean and standard deviation of firm investment rates would rise by roughly 150 percent (reaching 0.29 and 0.83, respectively), while the serial correlation would fall from the observed 0.058 to -0.15, and the frequency of investment spikes would nearly double (0.31). Alternatively, if we reset $\sigma_{\eta_\varepsilon}$ to maintain the observed standard deviation of i/k, the resulting $\sigma_{\eta_\varepsilon}$ would be 0.483 times the calibrated value, the frequency of investment spikes would still be overstated (0.32), and serial correlation would again be negative (-0.16).

[17] Moll (2012) and Midrigan and Xu (forthcoming) stress that the persistence of idiosyncratic shocks is important for the implications of borrowing constraints. This may be more so in settings in which productivities are known in advance, borrowing limits depend on future capital, and firms face no other frictions hindering reallocation. Since our model generates a natural firm life cycle and differences in size distinct from productivity differences, we need not rely as heavily on ρ_ε to hit our micro-investment targets as would otherwise be the case.

[18] At roughly 5 percent, our irreversibility is far lower than the 40+ percent friction Ramey and Shapiro (2001) infer from aerospace plants' capital resale activities, low relative to the roughly 34 percent estimated by Bloom (2009), and high relative to the 2.5 percent estimated by Cooper and Haltiwanger (2006). Any such parameter inference is sensitive to the model through which it is obtained; more important than the raw number is its consequence in observable firm-level outcomes. Our firms' investment decisions are very responsive to the real friction, so only a modest friction is needed in our calibration.

Ideally, a heterogeneous firm setting devised to explore the implications of credit shocks should be consistent with not only microeconomic evidence on real reallocation but also micro-level evidence on firms' financing decisions. Because it implies firm life cycle dynamics, one natural target for our model is the relation between firm size and leverage. In Appendix C, we examine a sample of nonfinancial firms from Compustat very similar to that in Bates et al. (2009), and we find that the average cross-sectional correlation of size (book assets) with leverage (the ratio of total debt to book assets) over 1954–2011 is 0.022. This finding of a modest positive relation is consistent with empirical evidence presented by Fama and French (2002) and Rajan and Zingales (1995); examining regressions of leverage on size, Fama and French identify coefficients ranging from 0.02 to 0.04, while Rajan and Zingales report a 0.03 coefficient. A second target for our model is the cross-sectional variation in firms' cash-to-asset ratios; the average standard deviation in cash ratios from Compustat over 1954–2011 is 0.161.

As in the Compustat data, we identify size in our model as the total value of a firm's capital and financial savings ($|b < 0|$) less its debt ($b > 0$), leverage as the ratio of a firm's debt to its size, and the cash ratio as the ratio of financial savings to size. If we draw a large random sample of firms from the stationary distribution of the model we have described to this point, we obtain a correlation between size and leverage at roughly -0.2. Because we have no theory of firm ownership, all leverage in our model is associated with investment loans. By contrast, the data reflect borrowing not only for investment activities but also for other activities such as restructuring and mergers and acquisitions; Ivashina and Scharfstein (2010) report that two-thirds of syndicated loans in 2007 were associated with these other activities. Our model is not designed to explain such activities; however, with a simple modification, it can be made to accommodate their implications.

Alongside the firms described above, we achieve consistency with the Compustat observations by introducing an additional group of firms that do not face collateral constraints. We will refer to these as *no-constraint firms*. Whereas unconstrained firms are firms that have accumulated sufficient wealth over time to outgrow financial frictions, our no-constraint firms are unaffected by borrowing constraints throughout their entire lifetimes. Each period, they optimally adopt the capital policies of unconstrained firms, setting $k' = K^w(k, \varepsilon; s, \mu)$. However, because they are impervious to financial frictions, they need not adopt the minimum savings policy in (21)–(22).

For simplicity, we assign our no-constraint firms a debt policy implying that their leverage and capital are linearly related; $b' = \alpha_e(k')^2$. Next, we assume that, of the new firms entering the economy each period, fraction ω_e are firms of this type. We choose α_e and ω_e so that a large random sample drawn from our stationary distribution reproduces the Compustat size-leverage correlation and standard deviation of cash ratios.

Table 1 lists the parameter set obtained from our calibration. Given ordinary credit conditions, note that these parameters imply a moderate degree of financial frictions, with firms able to take on debt up to 138 percent of the value of their tangible assets. Also note that firm-level shocks are more volatile and less persistent than aggregate shocks.

Our calibrated model gives rise to a stationary distribution of firms over (k, b, ε) wherein roughly 9 percent of firms are unconstrained and 62 percent are constrained (using the definition from Sec. IV), whereas the fraction of firms facing a currently binding borrowing limit is 17 percent. While we have not targeted information on aggregate cash holdings in the calibration, the aggregate cash-to-asset ratio in our model's steady state (0.12) is close to the Compustat ratio from 2006 (0.102). Bates et al. (2009) have stressed that cash ratios rose substantially between 1980 and the late 2000s. Indeed, relative to the postwar average (0.082), they were unusually high as the United States entered the recent financial crisis. When we examine the effects of a credit shock below, the firm distribution in the first date will be the steady-state distribution. Thus, for comparability with the US experience, it is important that our steady state is consistent with the high cash ratio preceding the 2007 recession.

C. *Numerical Overview*

The numerical algorithm we use to solve our model builds on that described in Khan and Thomas (2003, 2008) using the analysis from Section IV. However, the discrete choices and three-dimensional hetero-

TABLE 1
Parameter Values

				A.				
β	ν	δ	α	φ	χ	θ_k	ω_e	α_e
.960	.600	.065	.270	2.150	.100	.954	.291	.225
				B.				
π_d	ρ_ε	$\sigma_{\eta_\varepsilon}$	ρ_z	σ_{η_z}	ζ_o	ζ_l	p_o	p_l
.100	.659	.118	.909	.014	1.380	.500	.977	.688

Note.—In panel A, β is discount factor, ν is labor share, δ is depreciation rate, φ is leisure preference parameter, χ is capital per new firm relative to aggregate, θ_k is degree of reversibility, ω_e is fraction of firms without collateral constraints, and α_e is ratio of leverage to capital among them. In panel B, π_d is exit rate, ρ_ε and $\sigma_{\eta_\varepsilon}$ are persistence and innovation volatility of firm productivity shock, ρ_z and σ_{η_z} are persistence and innovation volatility of aggregate productivity shock, ζ_o is ordinary (and steady-state) credit value, ζ_l is low value corresponding to a crisis, p_o is probability of continuing ordinary credit conditions, and p_l is probability of crisis next period conditional on current crisis.

geneity arising here from the presence of investment irreversibility and collateralized borrowing, alongside the firm-level productivity shocks, necessitate a nonlinear solution method that is more involved than that used in these papers.

Our model includes a distribution of firms over capital, debt, and idiosyncratic productivity, (k, b, ε). We compute equilibrium by solving the problems of firms in a setting in which prices are consistent with market clearing. Because the distribution in the model's aggregate state is a high-dimensional object, we approximate it with the first moment of the distribution of capital and two dummy variables reflecting credit conditions in recent dates, applying the algorithm of Krusell and Smith (1998).[19] Specifically, we assume that agents perceive $(s, m, \vartheta_1, \vartheta_2)$ as the economy's aggregate state instead of (s, μ). The variable m represents the unconditional mean of the distribution of capital across firms, while ϑ_1 and ϑ_2 are lagged crisis dummies. The dummy ϑ_1 takes the value one if the economy was in a credit crisis in the previous period (if $\zeta_{t-1} = \zeta_l$) and zero otherwise. Similarly, $\vartheta_2 = 1$ in the event of a credit crisis two periods in the past (if $\zeta_{t-2} = \zeta_l$).

The solution method is iterative. In each iteration, there are three steps. First, firms' value functions are solved in an inner loop using existing forecasting rules for m' and p. We interpolate these functions using a set of knots in individual and aggregate states. We do not restrict current or future firm choices to these points; we use multivariate piecewise polynomial cubic splines to interpolate the value function off knot points. Second, an outer loop solves equilibrium quantities and relative prices over a 10,000-period simulation, date by date, using the forecasting rule for m' and firms' value functions solved in the inner loop. Each period of the simulation determines the equilibrium p using the actual distribution of firms in place at the start of the period. The third step updates forecasting rules using the simulation. Appendix B describes our numerical method in greater detail, presents forecasting rules, and discusses accuracy.

VI. Results

A. *Steady State*

Figure 1 overviews the stationary distribution of firms in our model, displaying the distribution of firms over capital and debt-to-capital levels at the median productivity. Figure 1 effectively contains three distributions.

[19] We solve the model using MPI with 128 computational cores in a Beowulf cluster. Parallel methods are required, despite our use of the Krusell-Smith algorithm, because the dependency of constrained firms' decisions on their productivity, debt, and capital implies a computationally intensive numerical algorithm.

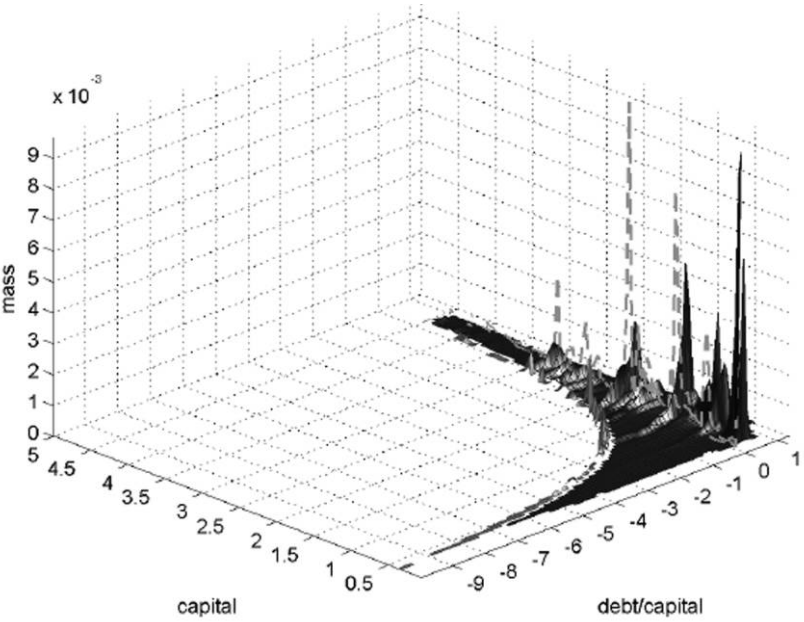

FIG. 1.—Steady-state distribution: median productivity. Capital increases right to left. Debt/capital ratio increases front to back; negative values reflect financial savings.

The first, in the foreground, has a curved shape reflecting an inverse relation between firms' capital stocks and their financial savings. This corresponds to older, wealthier firms that are unconstrained and following the minimum savings policy from Section IV.[20] Such firms have higher capital or higher savings relative to constrained firms, which are the bulk of firms distributed near the back of the figure. Finally, the distribution depicted with dashed lines near the back corresponds to no-constraint firms that never face borrowing limits; these firms select the same capitals as unconstrained firms and, by assumption, they adopt debt/capital levels in proportion to their capital stocks.

While not shown here, the stationary distributions over $(k, b/k)$ at other productivity levels are similar to figure 1. The 10 percent of firms entering the economy each period are dispersed over ε according to the ergodic distribution of productivities. These firms enter with zero debt and low initial capital (0.15); see the spike near the left edge of figure 1. After its first date in production, a new firm starts taking on debt to raise its capital. In the absence of collateral constraints, it would immediately take on a large, temporary debt to jump to the stock selected by uncon-

[20] Unconstrained firms that have had a run of low productivities leading to low current capitals have amassed sufficient financial savings through retained previous earnings to satisfy (21)–(22).

strained firms with the same current productivity. Here, however, firms with little collateral have limited ability to borrow, so their capital accumulation is more gradual. Young firms slowly move into higher ranges of k and b/k as they age, steadily raising their capital while maintaining a roughly constant borrowing rate typically below the maximum permitted. Those surviving long enough eventually adopt the unconstrained capital choices consistent with their current ε while beginning to reduce debt. Those surviving longer will, at some point, attain capital and savings sufficient to shield all future investments from the consequences of borrowing limits; at that point, they join the distribution of unconstrained firms in the foreground.

The life cycle aspects of our model may be seen from figure 2, which displays the average capital and debt choices for a cohort of (initially) 50,000 firms as they age. Note that the typical firm raises capital and debt over its first five periods of life. Thereafter, starting in period 7, it begins to reduce its debt and finances the remaining rise in its capital stock out of earnings. By period 16, the typical firm has become a net saver and thereafter joins the distribution of permanently unconstrained firms.[21]

To be clear, figure 2 does not imply that the model predicts a negative relation between size and leverage. Recall that this figure is not drawn from a balanced panel of firms; given the constant exit rate, there are fewer firms in the right half of the figure than there are on the left, where leverage is roughly constant. In fact, when we draw a large random sample of firms from our stationary distribution, the sample correlation between size and leverage is 0.022. This is by design in that we have calibrated the number of no-constraint firms and their linear leverage rule to match the empirical size-leverage relation and cross-sectional volatility of cash ratios.

While relatively simple in its microeconomic elements, our existing model is consistent with various aspects of firm-level behavior observed in the data. For example, our unconditional stationary firm size distribution is right-skewed, firm employment growth is negatively correlated with age (Dunne, Roberts, and Samuelson 1989), and larger and older firms pay more dividends (Fama and French 2001). Moreover, recall that the model is calibrated to reproduce aspects of firm behavior that are crucial in affecting the core misallocation mechanism therein; specifically, it reproduces the empirical variability and autocorrelation of microeconomic investment rates, alongside the frequency of spikes, and it closely matches the mean investment rate.

For a large fraction of our economy's firms, financial considerations interfere with the optimal investment responses to information about the future marginal product of capital conveyed by current productivity. Even

[21] The cohort's average net debt in fig. 2 does not become negative until period 18 because of no-constraint firms, the group of firms without borrowing limits defined above.

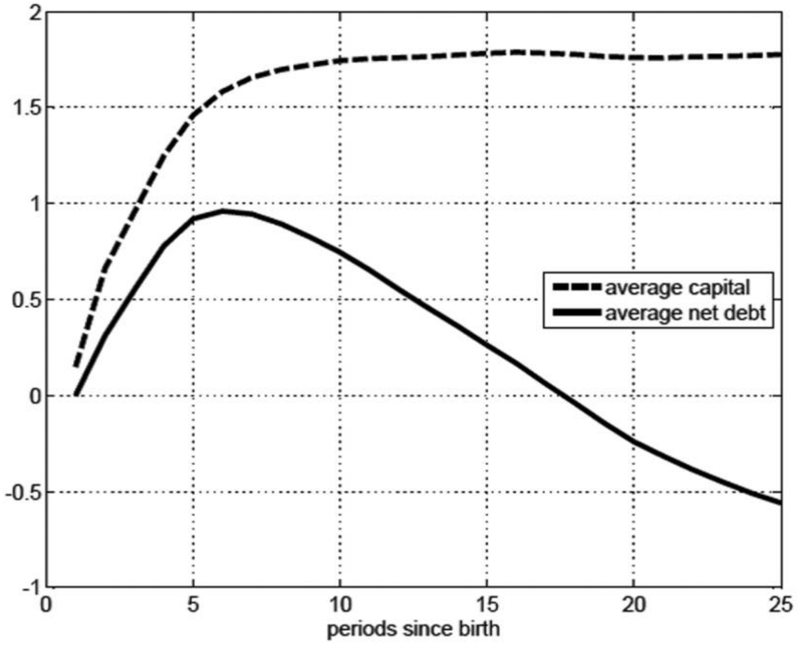

FIG. 2.—Cohort in steady state. Cohort average capital and net debt are taken from an unbalanced panel of firms born at date 1. Net debt is defined as debt less financial savings.

in ordinary times, this generates misallocation. One indication of this is the fact that the average capital stock among unconstrained firms in our model's stationary distribution is 1.98, while the average stock of constrained firms is 1.17. Despite the presence of 0.291 no-constraint firms that accumulate capital rapidly in youth, the typical old firm in our economy has much more capital than the typical young firm. This reflects capital misallocation since each age group draws from the same productivity distribution. In the absence of financial frictions, steady-state output would be 2.3 percent higher, and measured TFP would rise by 0.6 percent.[22]

Old firms in our economy do not carry excess capital; the inefficiency lies in the fact that young, small firms carry too little. This is clarified by figure 3, which again examines a cohort of (initially) 50,000 firms, this time focusing on the expected discounted return to investment for the cohort as it ages over time. In the absence of real and financial frictions, firms would always select investment to equate this return to the unit purchase price of investment goods, so the mean investment return across firms in the top panel of our figure would be constant at 1, and the co-

[22] If we omitted no-constraint firms from our model, collateral constraints would have larger steady-state effects, reducing output by 4 percent and TFP by 1 percent. We discuss steady-state capital choices in online App. D.

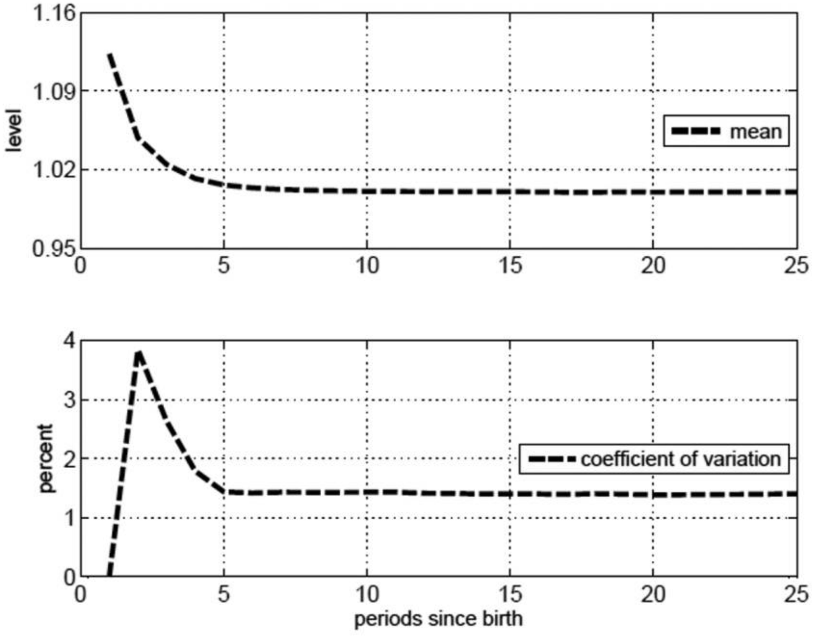

Fig. 3.—Cohort expected discounted return to capital in steady state. The top panel is period-by-period mean expected discounted marginal value of capital for an unbalanced panel of firms born in period 1. The bottom panel is the coefficient of variation in expected return from the same sample.

efficient of variation in this return in the bottom panel would always be 0. By contrast, in our model, the mean expected discounted return to investment is 1.12 for a cohort as it ends its first year of production, and the coefficient of variation in this return is 3.85. Thereafter, over each subsequent year of life, we see ever less dispersion in the return to investment across surviving members of the cohort and the mean expected return falling toward 1.

After a certain age, surviving members of the cohort have sufficient assets that financial frictions no longer affect their investment decisions. Thus, we see the mean expected investment return for the cohort ultimately reach 1. Even then, there remains some variation in the expected return, given the implications of irreversibility.

B. *Business Cycles*

We begin to examine business cycle results by first considering the implications of credit shocks in our economy's typical business cycle. Table 2 presents moments derived from a Hodrick-Prescott (HP) filtered 10,000-period simulation of our full model driven by aggregate produc-

TABLE 2
Business Cycles in the Full Economy

	$x =$					
	Y	C	I	N	K	r
Mean(x)	.578	.485	.094	.333	1.323	.042
σ_x/σ_Y	(2.046)	.514	4.106	.599	.517	.467
Corr(x, Y)	1.000	.880	.945	.914	.094	.657

Note.—Row 1 lists means of GDP, consumption, investment, hours worked, capital, and the real interest rate from a 10,000-period simulation of our full model with both z and ζ shocks. Rows 2 and 3 are relative standard deviations and contemporaneous correlations with GDP for HP-filtered simulation data (smoothing parameter 100); HP-filtered GDP volatility is listed in parentheses.

TABLE 3
Business Cycles without Credit Shocks

	$x =$					
	Y	C	I	N	K	r
Mean(x)	.583	.488	.096	.334	1.354	.042
σ_x/σ_Y	(1.997)	.503	3.860	.562	.485	.453
Corr(x, Y)	1.000	.931	.967	.945	.073	.671

Note.—Results from the model without ζ shocks. Row 1 lists simulation means; rows 2 and 3 report relative standard deviations and contemporaneous correlations with GDP for HP-filtered simulation data with smoothing parameter 100; HP-filtered GDP volatility is listed in parentheses.

tivity shocks and credit shocks. Table 3 presents the same moments when aggregate productivity shocks are the only source of aggregate fluctuations.

Comparing the two tables, we see that credit shocks reduce the average levels of output, capital, and consumption somewhat. In terms of second moments, these shocks raise overall output volatility, as well as the relative volatilities of consumption, investment, and hours worked, while they weaken contemporaneous correlations with output.

We emphasize that the differences across tables 2 and 3 would be more pronounced if credit shocks happened more often.[23] Recall that our full model economy is calibrated to reproduce the observation that credit crises occur in only 7 percent of years among advanced economies. The fact that differences are even discernible in these moments tables indicates that, when they occur, credit shocks have large effects on the real economy. This will be confirmed when we consider the responses following a credit shock below in subsection C. There, we will see both a

[23] The similarities here are reminiscent of results of Mendoza (2010), which finds that business cycle moments are largely unaffected by a collateral constraint in a small open economy.

large recession (hence the greater volatility in table 2) and a gradual unraveling of real economic activity led by sharp declines in investment and employment.

Table 4 presents the business cycle moments for a 10,000-period simulation of our model driven only by credit shocks. These foreshadow some important differences in our economy's response to a credit shock versus a TFP shock. Because credit shocks happen rarely, HP-filtered output volatility is very low over this simulation. Nonetheless, comparing table 4 with tables 2 and 3, note the greater relative volatility in investment and hours worked, these series' reduced contemporaneous correlations with GDP, and the essentially acyclical consumption series. These differences will be explained below when we compare impulse responses following a negative TFP shock (fig. 4) with those following a credit shock (figs. 6 and 9).

Having briefly considered how the business cycle moments change when one or the other shock is eliminated from our economy, we emphasize two further points. First, the second moments from our full model economy in table 2 are broadly similar to those from a typical real business cycle model. Output volatility is roughly 2 percent, and consumption is about half as volatile as output, while investment is roughly four times as volatile. We also see the customary strong contemporaneous correlations with output in consumption, investment, and hours worked. While the usual difficulties of excessive investment volatility and weak hours volatility are a bit more pronounced relative to some real business cycle models, these distinctions come from our differing returns to scale in production rather than either friction we mean to study; the same features are present when we strip both frictions away (see table E3 in online App. E).

Second, despite the differences noted above, the second moments across tables 2 and 3 are similar on the whole. As long as the credit shocks in our model are calibrated to reproduce the frequency and length of banking crises observed in advanced countries, real shocks are, on aver-

TABLE 4
Business Cycles without Real Shocks

	$x =$					
	Y	C	I	N	K	r
Mean(x)	.577	.484	.093	.333	1.319	.042
σ_x/σ_Y	(.434)	.625	7.000	1.145	.907	.644
Corr(x, Y)	1.000	.056	.880	.843	.274	.501

Note.—Results from the model with only ζ shocks (no z variation). Row 1 lists simulation means; rows 2 and 3 report relative standard deviations and contemporaneous correlations with GDP for HP-filtered simulation data with smoothing parameter 100; HP-filtered GDP volatility is listed in parentheses.

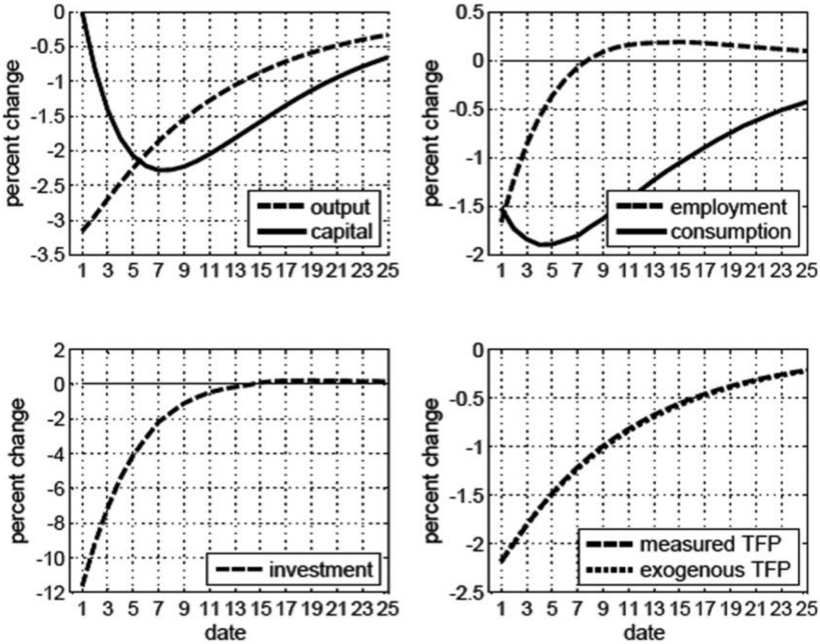

FIG. 4.—Negative technology shock. Response to 2.18 percent productivity shock with persistence ρ_z. The y-axes measure percentage deviations from simulation means.

age, dominant in driving aggregate fluctuations. Thus, our model naturally delivers plausible business cycles because its responses to real shocks are very similar to those in a frictionless business cycle model. This observation is reinforced by figure 4, which presents our full model economy's impulse responses following a persistent negative shock to the exogenous component of TFP.

The close match between the exogenous and measured TFP series in the bottom-right panel reveals that a persistent technology shock has no perceptible implications for the endogenous component of aggregate productivity. As a result, the responses in aggregate output, consumption, employment, and investment closely resemble those from an economy without real or financial frictions. Just as in a frictionless business cycle model, there are immediate declines in all four series. Further, aside from the customary U-shaped consumption response, we see the largest declines at the impact of the shock, with each series thereafter reverting toward its long-run level. Note also that the largest declines in GDP and investment are roughly 3.2 and 11.8 percent, respectively, while the fall in hours worked is half that in GDP.

The 2007Q4–2009Q2 US recession exhibits several notable differences relative to the recession following a technology shock in our model (fig. 4) or the canonical equilibrium business cycle model. Figure 5 reports the movements in detrended GDP, consumption, investment, employment hours, and measured TFP, plotting each series' deviations relative to their 2007Q4 levels. We defer discussion of the data beyond 2009Q2 for now and focus here on the economic downturn.

Starting at 2007Q4, the initial decline in GDP was small relative to its ultimate drop. Consumption actually rose by 0.6 percent and stayed above its initial (detrended) level until 2008Q4. Moreover, the early declines in investment were modest relative to what came later. While total private investment fell immediately, this was initially driven by housing. Nonresidential investment did not begin to fall until 2008Q3, at which point it began to drop off sharply relative to the more gradual declines in GDP and consumption. Measured TFP fell until the first quarter of 2009, though its greatest declines happened starting in 2008Q3. The greatest decline in hours worked came even later. As of 2009Q2, GDP was roughly 5.6 percent below its initial level, total hours had fallen by 6 percent, and in-

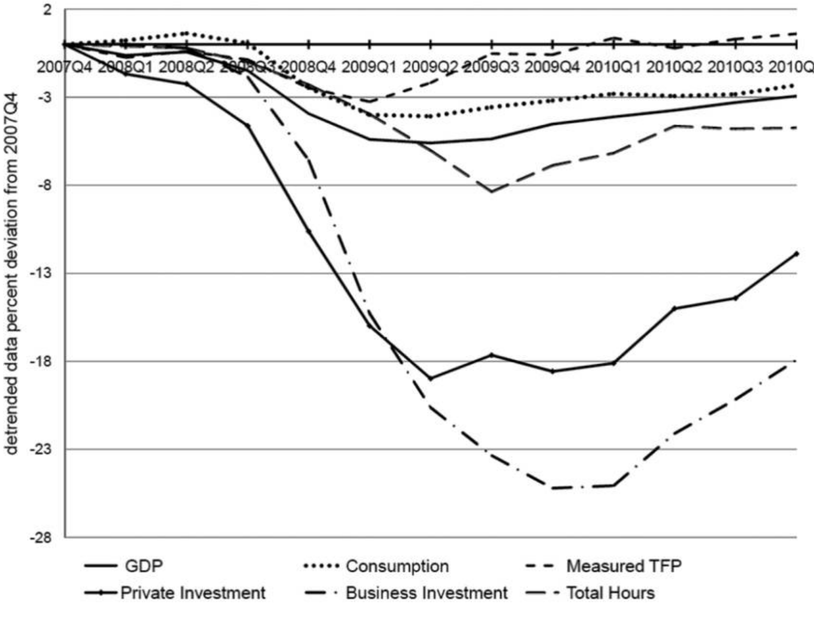

FIG. 5.—The recent recession. Real GDP, consumption, private and business fixed investment series from Bureau of Economic Analysis (BEA) GDP tables. Consumption is nondurable goods and services; private investment is business fixed investment, residential investment, and consumer durables. Total hours is civilian and military hours worked by noninstitutional population aged 16–64 from Cociuba et al. (2012). Measured TFP is a direct Solow residual calculation using the capital and labor shares to which our model is calibrated, with private capital taken from BEA Fixed Assets Tables. All series are in logs, detrended using the HP filter with weight 1,600, and plotted as percentage point deviations from 2007Q4 values; the filter is constructed using data from 1954Q1–2012Q4.

vestment had fallen by 19–20.6 percent.[24] Meanwhile, the drop in measured TFP was only 2.18 percent, precisely the size of the shock depicted in figure 4.

The recent US economic downturn presents several challenges for any equilibrium business cycle model driven by aggregate productivity shocks alone. First, it shows an initial rise in consumption where the model would predict a clear decline. Next, the ultimate losses in GDP, employment, and investment by the end of the recession are greater than a productivity shock would imply, despite the same drop in measured TFP. Third, the trough dates differ across series, whereas a technology shock–driven model would always predict the greatest fall in GDP, hours worked, investment, and TFP at a common date. Fourth, while not shown in figure 5, there was a very sharp reduction in debt over this period that would be impossible to reproduce with any plausible-sized technology shock. We discuss this below in subsection C.

C. Credit Crisis

Clearly, the challenges regarding the latest US recession apply not only to a standard business cycle model but also to ours, as long as it is driven by a shock to exogenous productivity. Here we consider what happens when financial frictions become unusually severe. Whereas a standard business cycle model is unaffected by such events, this is not the case in our setting where firms have different access to, and need for, credit.

We begin by discussing the evidence for an exogenous shock to the availability of credit that, in our model, corresponds to a drop in ζ. As is now well understood, it is hard to conclusively establish that the United States experienced an exogenous reduction in lending to businesses in the late 2000s. In a very early exploration of the matter, Chari, Christiano, and Kehoe (2008) argued that there was little evidence that the financial crisis had affected lending to nonfinancial firms. Examining the Flow of Funds, they found that the stock of commercial and industrial loans across regulated banks had actually risen as of the third quarter of 2008. They also argued that, in the aggregate, business fixed investment is less than firms' revenues net of labor costs; that is, the mean firm can self-finance investment. It is worth noting that our calibrated model is consistent with this observation in the aggregate data. Nonetheless, borrowing limits bind for some smaller firms, and this leads to insufficient capital in those firms, reducing both aggregate TFP and GDP.

Ivashina and Scharfstein (2010) reexamine the issue of lending over the financial crisis. They study Reuters DealScan data on syndicated loans,

[24] Ohanian (2010) presents evidence that the magnitudes of these declines in investment and hours are dramatic in comparison with those seen in previous postwar US recessions, as well as other Group of 7 countries' 2007–9 recessions.

which capture new lending to large corporations. While these loans originate with banks, the pool of lenders, which includes nonbank financial institutions, is larger. Moreover, the data on syndicated lending cover new loans, as opposed to the stock of outstanding debt that is reported in the Flow of Funds. Ivashina and Scharfstein find strong evidence of a reduction in lending around the start of the 2007 recession. Between 2007 and 2008, they report that total syndicated lending fell 54 percent, while loans used to fund investment in equipment and structures fell 48 percent.

Koepke and Thomson (2011) examine loans from Federal Deposit Insurance Corporation insured commercial banks and savings institutions. They point out that the stock of these loans, an important source of borrowing for small and medium-sized businesses, fell 18.7 percent between 2008Q4 and 2009Q4 and fell yet further through 2010Q2, when they were 19.1 percent below their 2007Q4 level. Reexamining commercial and industrial loans at commercial banks deflated by the GDP deflator series, we find that real lending continued falling through the fourth quarter of 2011, when it was 26 percent below its level from the end of 2008. Since then, the series has gradually risen, but it was still more than 11 percent below the 2008Q4 level as of the first quarter of 2013.

While these investigations provide evidence of a reduction in lending to nonfinancial firms, they do not establish whether this represents an exogenous reduction in credit or instead an equilibrium response to reductions in business fixed investment. Almeida et al. (2009) and Duchin, Ozbas, and Sensoy (2010) provide support for a credit shock interpretation. Controlling for firm characteristics using a matching estimator, Almeida et al. study the investment behavior of firms that, given their existing loan maturity structures, needed to refinance a substantial fraction of their long-term debt over the year following August 2007. They find that investment spending among such firms fell by one-third. By contrast, other firms with similar characteristics, but without a large refinancing in the period following the start of the financial crisis, showed no investment reduction. Since the fraction of long-term debt maturing after August 2007 was likely exogenous to the financial crisis, this suggests an exogenous reduction in the supply of credit. Similarly, Duchin et al. compare the behavior of firms that were carrying more cash prior to the onset of the crisis with that of firms carrying less cash. Using a difference-in-difference approach, they find that firms with less liquid assets before the financial crisis exhibited a larger reduction in investment.

Figure 6 depicts our model economy's response to a credit crisis, without any exogenous TFP shock. It is the response to an 88 percentage point drop in the value of firms' collateral from ζ_o to ζ_l, which we will see below implies an eventual 26 percent reduction in debt. This reduction is consistent with the actual declines in various series reflecting lending dis-

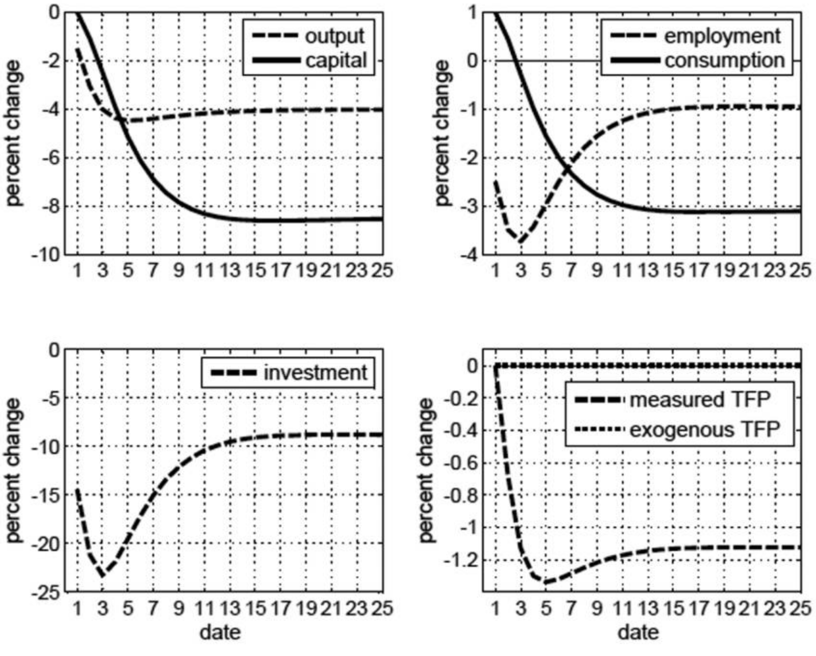

FIG. 6.—Persistent financial crisis. Response to a drop in the credit variable from ζ_o to ζ_l. Expectations are consistent with the calibrated shock process; however, the credit variable is maintained at its crisis level through period 25. The y-axes measure percentage deviations from simulation means.

cussed above; it matches our measure of the ultimate fall in commercial and industrial loans, and it is substantially smaller than the fall in syndicated investment loans reported by Ivashina and Scharfstein (2010). From the first date of the credit shock onward, households and firms expect a return to normal financial conditions with probability $1 - p_l$ consistent with our calibrated Π^ζ matrix. Thus, when the shock occurs in period 1, they expect that it will persist for 3.2 years. For now, we focus exclusively on the downturn following the credit shock, thus omitting recovery from this figure. We show in online Appendix F that there is little interaction if the credit shock we examine here is compounded by a negative shock to the exogenous component of TFP.

Although the distribution of capital is predetermined when the financial shock hits in year 1, the top-left panel of figure 6 reveals that aggregate production immediately falls by about 1.5 percent relative to its simulated mean in normal financial times. This is a direct consequence of the 2.5 percent fall in the labor input (top-right panel), which is, in turn, a reaction to the reduced expected return to investment. With the sudden reduction in credit, there is a drop in the fraction of firms that are financially unconstrained and a sharp rise in the fraction of firms facing currently binding borrowing limits. Underlying these changes, young firms are now more hindered in their investment activities relative to the preshock economy and thus will take longer to outgrow financial frictions and begin producing at a scale consistent with their productivities. If these financial conditions persisted permanently, the resulting stationary distribution would have 43 percent of firms constrained in their current capital adjustments, whereas this fraction is 17 percent in ordinary financial times.

In contrast to the response following a negative productivity shock, consumption does not initially fall when the credit shock hits our economy. Anticipating a more distorted distribution of production over coming years, and thus unusually low TFP (in the lower-right panel), the representative household in our economy expects a lowered return to saving.[25] This leads to an almost 1 percent rise in consumption at the impact of the shock and a fall in hours worked. The fall in investment (at the lower left) does not support consumption for long, though; consumption falls to its preshock level by date 3, then steadily declines over 8 more years before it levels off. Elsewhere, labor falls at the impact of the shock as described above. Thereafter, given increased misallocation of capital at the start of date 2, alongside reductions in total capital, the marginal product of labor drops, yielding further large reductions in hours worked. By year 3, the series is 3.7 percent below its preshock level, and it does not rise to the level consistent with the new financial setting until around period 15. This long adjustment period is a reflection of the time it takes for the capital distribution to settle, as may be inferred from the measured TFP response in the lower-right panel. The entire reduction in measured TFP is an endogenous response to a growing misallocation of productive resources.

Figure 7 shows how the credit shock distorts the allocation of production in our economy. The top panel plots the distribution of firms over capital and productivity in place in date 1 when the shock occurs. The lower panel is the same distribution at the start of date 2.[26] Comparing the two panels, we see increased dispersion within just one period. The mass of firms with capital stocks between 1.5 and 3 in the top panel is reduced, with much of that mass pushed into lower regions of capital by

[25] Fernald and Matoba (2009) argue that utilization-adjusted TFP rose after the start of the 2007 US recession. However, the series they construct falls sharply after 2009; we thank an anonymous referee for pointing this out. The counterpart to their series in our model, the exogenous component of TFP, is held constant over our credit shock exercise.

[26] Because there are some firms transiting to any given current productivity from each of seven different last-period productivity levels, it is difficult to distinguish the three values of capital selected by unconstrained firms drawing a common ε last period in this figure. It is harder still to line up current productivities with the capitals selected by constrained firms last period, since those decisions were affected by the full (k, b, ε) state.

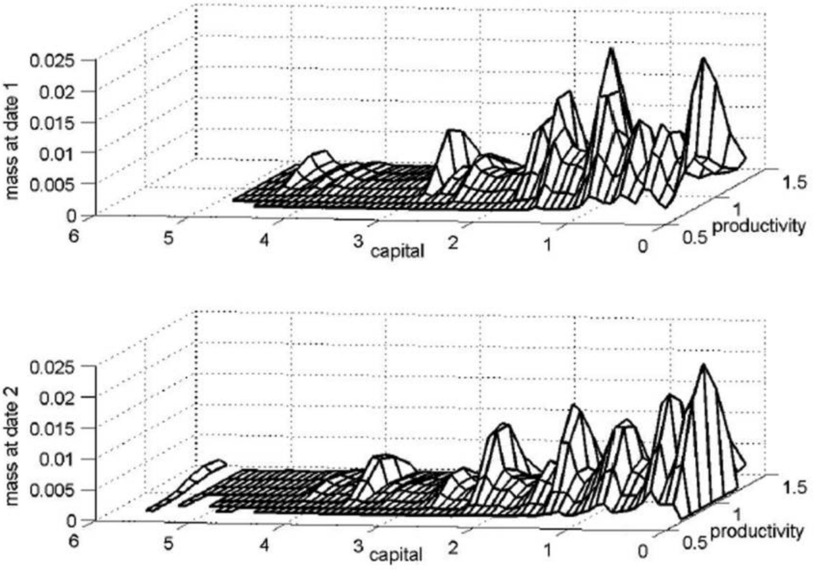

Fig. 7.—Persistent financial crisis: capital-productivity distribution. Response to a drop in the credit variable from ζ_o to ζ_l. Expectations are consistent with the calibrated shock process. The top panel is the predetermined distribution of firms over capital and productivity at the date of the shock; the bottom panel is the same distribution at the start of the next period. The y-axes measure the mass at firms at each (k, ε) combination.

date 2. In other words, the shock creates fewer medium-sized firms and more small firms. At the same time, we see a few firms, the very largest, growing larger.[27]

Given a heightened misallocation of production coming in date 2, the largest firms, otherwise unaffected by credit concerns, respond to the reduced real interest rate by expanding in size. Such firms adopt the efficient capital levels dictated by their (S, s) investment policies, given the productivities and real interest rate they face. The increased inefficiency that reduces TFP arises because small firms, now facing unusually severe collateral requirements, see the gap between their expected discounted return to capital and the real rate widen. These problems grow in subsequent dates, and the cross-sectional mean and coefficient of variation of the ex post marginal product of capital continue rising (figure available on request).

Critically, the increase in the mean marginal product of capital coincides with a fall in the ex post real interest rate. The marginal product of capital across large, unconstrained firms falls, with the contrasting rise

[27] Our increased dispersion in production is consistent with evidence from Bloom, Floetotto, and Jaimovich (2009) that various measures of firm-level dispersion rise during recessions.

in the economywide average driven entirely by tighter borrowing limits for other firms. Capital falls in these firms over time, so their marginal product of capital rises. With the increased dispersion in the returns to capital, the coefficient of variation is seen to rise. These results in our model following a credit shock are reminiscent of empirical results of Eisfeldt and Rampini (2006), who show that the benefits of capital reallocation rise in recessions while the level of reallocation falls. In our model, both forces operate.

Given the discussion above, a credit crisis in our model clearly has a disproportionately negative impact on smaller, younger firms. Note that this happens only following a drop in ζ; firms are evenly affected following a shock to TFP. This same distinctive feature of a credit shock appears in the 2007 US recession data, as may be seen from figure 8.

To consider more specifically how small firms are affected by a credit shock, we require a definition. We use Business Employment Dynamics (BED) data from the Bureau of Labor Statistics to compare small versus large firm employment changes in the United States and there identify small firms as those with fewer than 100 employees. The employment share across such firms was roughly 38 percent at the start of 2007 (and, on average, over 1993–2006). Thus, we identify small firms in our model as firms with (k, ε) pairs that, in an average date, constitute the bottom 38 percent of employment. Defined as such, one period after the impact of our credit shock, the number of small firms rises by 1.3 percentage points, and their average employment falls by 6.7 percent, whereas economywide employment falls by 3.5 percent.

Figure 8 displays net employment changes in the BED from quarter to quarter between 1992Q3 and 2011Q3. The solid series is the change in total employment, the dotted series is the change among firms with fewer than 100 employees, and the dashed series is the change among firms with over 1,000 employees. During the 2001 recession, small firms and large firms initially contracted roughly equally, as would be consistent with the implications of a productivity shock in our model. By contrast, note that small firms reduced their employment roughly twice as much as large firms did over the 2007 recession, as consistent with the implications of a credit shock in our model. The relative employment decline among small firms over 2007Q4–2009Q2, the decline among small firms relative to the total decline, was roughly 47 percent. This is almost 10 percent more than the fall consistent with their initial employment share. Examining a peak-to-trough comparison in our model, we find a relative employment decline of roughly 61 percent across small firms.

On balance, we take the following observations from figures 6–8. A tightening of collateral constraints alone, a purely financial shock, drives large and persistent real effects in our model economy. It does so because it moves the distribution of firm-level capital further away from the

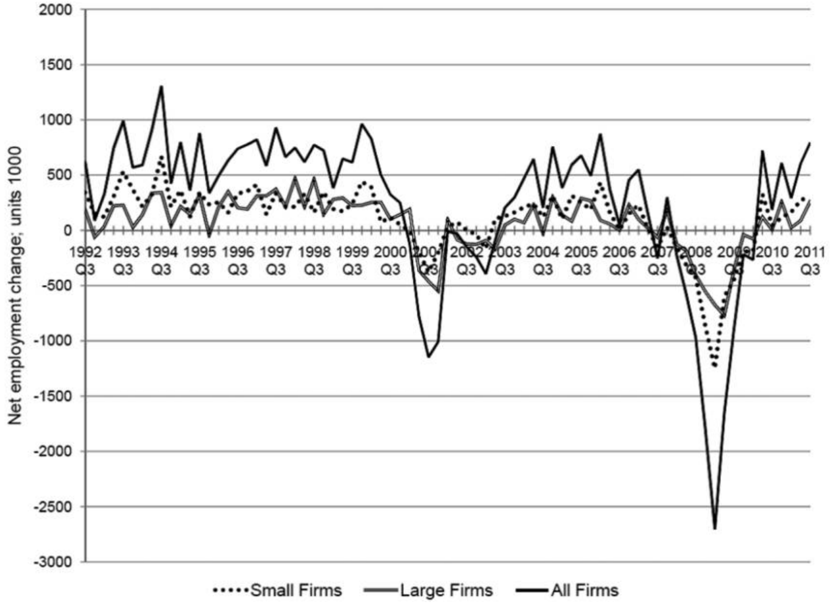

FIG. 8.—Net employment changes by size. Net employment changes (gross job growth less gross losses), measured in thousands. Small firms are defined as firms with employment under 100 and large firms as firms with employment over 1,000. Data source is BED, constructed from the quarterly *Census of Employment and Wages*. Data include all firms covered by state unemployment insurance programs, which is roughly 98 percent of nonfarm payrolls.

efficient one consistent with the firm-level productivity distribution (and capital specificity), allocating insufficient capital to more small firms and putting downward pressure on interest rates. This implies a disproportionate decline in production among small firms relative to large firms consistent with that observed over the 2007 US recession. In the example we have shown here, the misallocation of production arising from tight financial conditions is compounded by the reductions in aggregate capital and labor that it causes. There are protracted adjustments in aggregate quantities lasting a decade or more, and GDP is ultimately reduced by 4 percent, while aggregate consumption is reduced by 1 percent.

Until now, we have considered the implications of a protracted financial crisis, in that credit conditions do not improve. We next consider the recovery. We assume that the same financial shock we studied in figure 6 lasts for four periods; thereafter, beginning in date 5, we allow a gradual recovery of credit conditions, returning the value of collateral to ζ_o at rate $1 - p_l$.

In table 5, we compare the peak-to-trough results of our model with the 2007 recession. Focus first on row 2, the four-period credit crisis de-

TABLE 5
PEAK-TO-TROUGH DECLINES: US 2007 RECESSION AND MODEL

	Trough	GDP	I	N	C	TFP	Debt
Data	2009Q2	5.59	18.98	6.03	4.08	2.18	25.94
Four-period crisis	4	4.38	21.88	3.42	.99	1.30	25.96
z-shock (ρ_z)	1	3.16	11.63	1.66	1.52	2.18	.24
ζ-shock (ρ_z)	4	3.80	18.40	2.85	.97	1.09	22.14

NOTE.—Debt data entry is the fall in real stock of commercial and industrial loans at commercial banks between 2008Q4 and 2011Q4; all other series are measured at the GDP trough date listed in col. 2. Excluding debt, data in row 1 are described in fig. 5. Row 2 is the four-period credit crisis described in the text; results here are the same whether the financial recovery starting in date 5 is immediate or gradual. Row 3 is a negative productivity shock with persistence ρ_z and date 1 value chosen to match the empirical drop in measured TFP. Row 4 is a credit shock with persistence ρ_z and date 1 value chosen to match the empirical drop in debt.

scribed above wherein ζ falls to ζ_l in date 1 and stays there until date 5. Notice that, driven only by this simple financial shock, our model generates 78 percent of the empirical drop in GDP, 57 percent of the observed fall in hours worked, and 60 percent of the fall in measured TFP. As noted above, the empirical drop in debt is reproduced by construction. However, because our households are very responsive to anticipated changes in the returns to saving, it produces an investment response slightly larger than in the data and about 25 percent of the observed consumption decline. Our consumption series ultimately falls by 2.5 percent; however, that happens two periods after the GDP trough in date 4.

The third row of table 5 presents the model's peak-to-trough results in response to a persistent technology shock. This supports our earlier discussion of the challenge the 2007 US recession presents for models driven exclusively by aggregate productivity shocks. On its own, a TFP shock selected to match the empirical drop in measured TFP fails across the board in explaining the magnitude of this recession. The predicted GDP drop is 56.5 percent of that in the data, while the model explains only 61.3 and 27.5 percent of the declines in US private investment and total employment hours, respectively. The response in consumption is comparable to that following a credit shock. Because the reduction in debt under a TFP shock is driven purely by changes in firms' investment demand, the model fails badly there, generating less than 1 percent of the observed decline.

The final row of table 5 further illustrates the distinct nature of our model's response to financial versus real shocks. There, we subject the economy to the same date 1 credit shock as in row 2. However, for comparability with the TFP shock in row 3, we assume that ζ immediately thereafter begins moving back toward its ordinary value at rate $1 - \rho_z$. Two findings from this exercise are interesting. First, although ζ has the same reversion rate as z in row 3, the GDP trough occurs in date 4

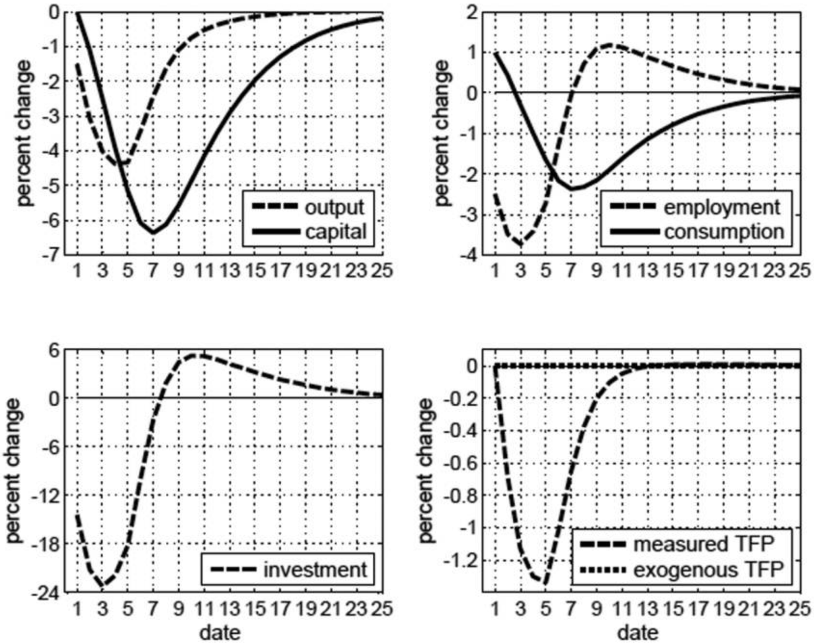

FIG. 9.—Financial crisis and recovery. Response to a drop in the credit variable from ζ_o to ζ_l. Expectations are consistent with calibrated shock processes. The credit variable is at crisis level through date 4, then recovers at rate $1 - p_l$ (0.3125) starting in date 5. The y-axes measure percentage deviations from simulation means.

rather than in date 1. Second, the ultimate fall in each series is far closer to the four-period credit crisis than the TFP shock exercise.

Finally, our complete credit-driven recession is shown in figure 9. In dates 1–4, $\zeta = \zeta_l$; thereafter, ζ reverts toward ζ_o at the rate $1 - p_l$. Three aspects of this figure are worthy of note. First, as long as GDP or consumption is adopted as our measure, the effects of a large credit shock are not rapidly reversed. While loan markets start improving in year 5, GDP remains 4.3 percent below average in that date. Measuring from date 4, the shock's half-life is just under 2 years, while GDP's is just over 3.[28] Consumption takes longer to return to normal, with a half-life around five periods. The slow recovery of output and consumption arises in part from the fact that the distribution of capital is slow to settle back

[28] The gradual recovery in our model is even more striking when we fully restore financial conditions in date 5. In that case, GDP recovers only 16 percent in date 5, and its half-life is 1.5 years. By contrast, if we instead delivered the economy a one standard deviation TFP shock for four periods and then fully eliminated it in date 5, GDP would complete 94 percent of its recovery instantaneously, while recoveries in employment and investment would be even faster. These figures are available on request.

to its preshock state. As a result, aggregate productivity remains below average for several periods, as seen in the third panel of the figure. Beyond this, it takes many periods to rebuild the aggregate capital stock; capital is 5 percent below normal as of date 5, and it falls another 1.5 percent over the following two dates while investment remains very low.

Second, once the credit recovery begins, aggregate productivity starts improving. However, consumption does not begin to recover in date 5. Given raised demand for investment goods and output's failure to rebound rapidly, households allow their consumption to fall for two more periods and thereafter raise it only very slowly.

Third, it is the labor input that drives the recovery. Anticipating rises in endogenous productivity, and thus improved returns to saving, households raise their hours worked by about 26 percent between years 4 and 5. In the next date, the allocation of capital across firms begins moving back toward normal, and the resulting improvement in productivity directly encourages a further large rise in the labor input to feed investment. By year 7, hours worked is back to normal. Thereafter, it overshoots its average level by just over 1 percent and remains high for many periods while the capital stock is being rebuilt.

VII. Concluding Remarks

We have developed a dynamic stochastic general equilibrium model with persistent, firm-level shocks to total factor productivity, costly investment reversibility, and collateralized borrowing constraints. We have calibrated the model to microeconomic investment and financial data, as well as overall measures of borrowing by nonfinancial firms. Our resulting economy is characterized by a nontrivial distribution of firms over productivity, debt, and capital that shapes aggregate output and TFP.

Firms respond endogenously to the frictions they face and, over time, build sufficient precautionary savings so as to ensure that borrowing limits will not affect their investment. Only a small subset of the firms in our economy have investment activities curtailed by their current ability to borrow. Nonetheless, without any real shock to the economy, we find that a credit crisis can generate a recession that is not only large but persistent. Because tight borrowing conditions deliver a long-lived disruption to the distribution of capital, and thus to endogenous aggregate productivity, their aftermath is a gradual recovery in output.

The recession generated in our model by a credit shock is qualitatively different from that following a negative shock to aggregate productivity, and it more closely resembles the 2007 US recession in several respects. The decline in GDP is gradual. Consumption initially rises. The responses in investment, employment, and GDP are unusually severe relative to the fall in TFP. The decline in measured TFP that accompanies these movements is similar to that in the data. Moreover, employment declines among small firms are disproportionately large.

While capturing several aspects of the recent US recession, the credit shock we have considered here does not deliver the unusually slow recovery in investment and employment over the 18 months of data since the trough of the recession in 2009Q2. It is possible that tight credit has affected not only business fixed investment but also firms' ability to finance working capital used to pay wages, and we know that lending conditions did not fully recover by the end of 2010. However, no such explanation is likely to reconcile the observed changes in investment and employment with the growth in TFP. A more likely suggestion proposed by Ohanian (2010) is that time-varying distortions in the labor market have been important in shaping employment over this recession. Given the complexity of our current model, introducing additional frictions to drive such distortions is beyond the scope of this paper; we leave this to future research.

Appendix A

Capital Decision Rules

The capital rules for constrained firms are described in Section IV.B. As stated in Section IV.A, the decision rule for an unconstrained firm is characterized by reference to two target capitals, an upward target, k_u^*, and a downward target, k_d^*, solving

$$k_u^*(\varepsilon; s, \mu) = \arg\max_{k'} \left[-p(s,\mu)k' + \beta \sum_{m=1}^{N_s} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^s \pi_{ij} W_0(k', 0, \varepsilon_j; s_m, \Gamma(s_l, \mu)) \right],$$

$$k_d^*(\varepsilon; s, \mu) = \arg\max_{k'} \left[-p(s,\mu)\theta_k k' + \beta \sum_{m=1}^{N_s} \sum_{j=1}^{N_\varepsilon} \pi_{lm}^s \pi_{ij} W_0(k', 0, \varepsilon_j; s_m, \Gamma(s_l, \mu)) \right].$$

Each target depends only on the aggregate state and the firm's ε. Further, the downward target exceeds the upward target: $k_d^* > k_u^*$, since $\theta_k < 1$ and W_0 is strictly increasing in k. The latter is confirmed by inspection of (14)–(17), given the strictly increasing net earnings function, (13).

Given a constant price associated with raising its capital stock, and because W_0 is increasing in k, the firm solves the upward adjustment problem in (16) by setting its future capital as close to its upward target as nonnegative investment permits:

$k_u(\varepsilon) = \max\{(1-\delta)k,\ k_u^*(\varepsilon)\}$. Similarly, it solves the downward adjustment problem in (17) by selecting a capital as near the downward target as nonpositive investment permits: $k_d(\varepsilon) = \min\{(1-\delta)k, k_d^*(\varepsilon)\}$.

From the conditional adjustment rules above, it is clear that an unconstrained firm of type (k, b, ε) will select one of three capital levels, $k' \in \{k_u^*(\varepsilon), k_d^*(\varepsilon), (1-\delta)k\}$. Which one it selects depends only on where its current capital lies in relation to the targets. Recall that $k_u^*(\varepsilon) < k_d^*(\varepsilon)$. If $(1-\delta)k < k_u^*(\varepsilon)$, then $k_u(\varepsilon) = k_u^*(\varepsilon)$, $k_d(\varepsilon) = (1-\delta)k$, and the firm adopts $k' = k_u(\varepsilon)$ (since $(1-\delta)k$ lies in the $k_u(\varepsilon)$ choice set). If $(1-\delta)k > k_d^*(\varepsilon)$, then $k_u(\varepsilon) = (1-\delta)k$, $k_d(\varepsilon) = k_d^*(\varepsilon)$, and the firm sets $k' = k_d(\varepsilon)$. Finally, if $k_u^*(\varepsilon) < (1-\delta)k < k_d^*(\varepsilon)$, then $k' = k_u(\varepsilon) = k_d(\varepsilon) = (1-\delta)k$. Collecting these observations, we obtain the (S, s) decision rule for capital listed in equation (20).

Appendix B

Numerical Method

The solution algorithm replaces the aggregate law of motion Γ with forecasting rules conditional on the current exogenous state, $\Gamma_m^n(s_l, m, \vartheta_1, \vartheta_2)$, $l = 1, \ldots, N_s$, using the approximate endogenous aggregate state $(m, \vartheta_1, \vartheta_2)$ instead of μ. As described in the text, m is the unconditional mean of the distribution of capital and the variable $\vartheta_j = 1$ iff there was a credit shock $j = 1$ or 2 periods ago. We iterate over forecasting rules, $n = 1, 2, \ldots,$ until they converge.

In each iteration, we solve for firm value functions in the inner loop.[29] First, the unconstrained firm value function W is determined using (14)–(17).[30] This yields decision rules for K^w, B^w, and D^w from (20)–(23), which are used to solve V, the constrained firm value function, using (9) and (24)–(27), with W_0 serving as the initial guess for V_0. In each case, we solve firm values at a set of points from the firm-level state vector; then we use nonlinear multivariate piecewise polynomial spline interpolation to approximate the value function consistent with these data. When solving the constrained firm problem, we find it useful to solve for a value function $\hat{V}(k, b/k, \varepsilon; \cdot)$ rather than the primitive $V(k, b, \varepsilon; \cdot)$; this lets us restrict the knot points to a feasible set of (k, b, ε).

In the outer loop, we simulate the model for $T = 10,000$ periods and assume an initial distribution of firms consistent with the steady state of the model. The distribution over (k, b, ε) is stored using a large three-dimensional grid. Firms' decisions are interpolated onto this grid using weights so as to preserve their average values.

At each date of the simulation, the distribution μ_t implied by the previous date's equilibrium decisions is used to compute m_t, while the s_t realization es-

[29] The inclusion of indicator variables in forecasting rules implies that they are part of the aggregate exogenous state vector. Consistent with this, in our model solution, firms' value functions include ϑ_1 and ϑ_2, alongside z and ζ, as arguments. That implies that the size of the exogenous aggregate state perceived by firms is actually $N_z \times N_\zeta \times 2 \times 2$. Further, $\vartheta_2' = 1$ if $\vartheta_1 = 1$ and $\vartheta_1' = 1$ if current ζ is at its low value. Since ϑ_1 and ϑ_2 evolve over time as deterministic functions of ζ and ζ_{-1}, it is unnecessary to describe them as exogenous state variables in our exposition.

[30] We do not need a separate forecasting rule for w; given the form of our utility function, w is determined by p.

tablishes z_t and ζ_t. On the basis of this information, alongside ϑ_{1t} and ϑ_{2t}, firms use W, $\hat{V}$, and Γ_m^n to forecast their future values associated with any feasible choice of k' and b'. This lets them select their optimal production, debt, and investment in response to any given set of prices (p, q, w). We solve for the equilibrium (p_t, q_t, w_t) so that the market-clearing conditions iv–vi in Section III.C are satisfied and asset markets clear. This give us μ_{t+1}, the distribution of firms at the start of the next period. After we have completed the simulation, we use the stored data $\{s_t, m_t, \vartheta_{1,t}, \vartheta_{2,t}, m_{t+1}, p_t\}_{t=1}^{T}$ to estimate new forecasting rules with which to begin the $n+1$ iteration of the algorithm.

Accuracy.—Despite the rich distribution of firms in our economy, agents predict prices and the future proxy aggregate state well with no more information on the current distribution than the mean capital stock and two indicators of recent credit conditions. Tables B1 and B2 present the forecasting rules for the future mean capital stock, m', and the marginal valuation of output, p.

As credit shocks are part of the exogenous stochastic process of the model and take on one of two values, we have achieved accurate forecasting rules by introducing two indicator variables, ϑ_1 and ϑ_2. The resulting regressions are more accurate than alternatives that add additional endogenous state variables, for example, the debt held by ordinary firms facing collateral constraints or the

TABLE B1
Conditional Forecasting Rules: Aggregate Capital

Observations	β_0	β_1	β_2	β_3	Standard Error	Adjusted R^2	Maximum Error
	A. Conditional on $s_1 = (z_1, \zeta_o)$						
2,287	.03742	.79925	−.00566	−.00338	.00056	.99966	.00432
	(.00007)	(.00031)	(.00008)	(.00007)			
	B. Conditional on $s_2 = (z_1, \zeta_l)$						
185	.02926	.79324	−.00231	−.00363	.00094	.99931	.00353
	(.00039)	(.00170)	(.00020)	(.00019)			
	C. Conditional on $s_3 = (z_2, \zeta_o)$						
4,724	.05716	.79827	−.00609	−.00369	.00055	.99964	.00388
	(.00006)	(.00023)	(.00006)	(.00005)			
	D. Conditional on $s_4 = (z_2, \zeta_l)$						
324	.04846	.79386	−.00262	−.00346	.00100	.99913	.00338
	(.00043)	(.00145)	(.00015)	(.00014)			
	E. Conditional on $s_5 = (z_3, \zeta_o)$						
2,316	.07793	.79627	−.00601	−.00407	.00051	.99970	.00327
	(.00010)	(.00029)	(.00008)	(.00007)			
	F. Conditional on $s_6 = (z_3, \zeta_l)$						
164	.07130	.78481	−.00295	−.00386	.00106	.99876	.00343
	(.00088)	(.00248)	(.00022)	(.00021)			

Note.—Regressions: $\ln(m') = \beta_0 + \beta_1 \ln(m) + \beta_2 \vartheta_1 + \beta_3 \vartheta$, where m is aggregate capital and ϑ_1 and ϑ_2 are crisis dummies taking on value 1 if $\zeta_{t-1} = \zeta_l$ and if $\zeta_{t-2} = \zeta_l$, respectively. Standard errors are reported in parentheses.

TABLE B2
Conditional Forecasting Rules: Output Valuation

Observations	β_0	β_1	β_2	β_3	Standard Error	Adjusted R^2	Maximum Error
	A. Conditional on $s_1 = (z_1, \zeta_o)$						
2,287	.87276	−.40899	.00360	.00116	.00008	.99997	.00061
	(.00001)	(.00005)	(.00001)	(.00001)			
	B. Conditional on $s_2 = (z_1, \zeta_l)$						
185	.86616	−.41771	.00361	.00275	.00027	.99981	.00075
	(.00011)	(.00048)	(.00006)	(.00005)			
	C. Conditional on $s_3 = (z_2, \zeta_o)$						
4,724	.83748	−.40234	.00349	.00115	.00008	.99997	.00075
	(.00001)	(.00003)	(.00001)	(.00001)			
	D. Conditional on $s_4 = (z_2, \zeta_l)$						
324	.83128	−.41220	.00372	.00254	.00028	.99976	.00076
	(.00012)	(.00041)	(.00004)	(.00004)			
	E. Conditional on $s_5 = (z_3, \zeta_o)$						
2,316	.80195	−.39617	.00355	.00126	.00007	.99998	.00061
	(.00001)	(.00004)	(.00001)	(.00001)			
	F. Conditional on $s_6 = (z_3, \zeta_l)$						
164	.79657	−.40865	.00392	.00264	.00026	.99974	.00074
	(.00022)	(.00061)	(.00005)	(.00005)			

Note.—Regressions: $\ln(p) = \beta_0 + \beta_1 \ln(m) + \beta_2\vartheta_1 + \beta_3\vartheta$, where m is aggregate capital and ϑ_1 and ϑ_2 are crisis dummies taking on value 1 if $\zeta_{t-1} = \zeta_l$ and if $\zeta_{t-2} = \zeta_l$, respectively. Standard errors are reported in parentheses.

aggregate debt-to-assets ratio. If we omit ϑ_j, then such additional regressors improve the accuracy of our forecasting rules. They do not improve forecasting accuracy once the indicator variables are introduced. Further, the inclusion of additional endogenous aggregate variables in forecasting rules requires their introduction as additional arguments of firms' value functions. This is numerically cumbersome given the dimensionality of the nonlinear problem. Specifically, we use tensor-product piecewise polynomial cubic splines to approximate firms' expected future value functions, and these are already functions of six variables, three of which are continuous.

Table B2 shows the forecasting rule for p. As explained above, this is used only in an inner loop that solves for firms' value functions using forecasting rules for both m' and p; the p forecasts are not used in the outer loop. Table B2 shows that all R^2's are high (above .99974) and standard errors are small (below 0.028 percent).[31] In the context of accuracy of forecasting rules for heterogeneous agent models, Den Haan (2010) notes that R^2's are averages and are scaled by the variance of the dependent variable. We provide a robust statistic by reporting the maximum forecast error for each regression. The maximum percentage error

[31] As dependent variables are in logs, we interpret 100 times the reported value as percentage errors.

for p is 0.076 percent. The forecasting rules for p used to solve firms' value functions are extremely precise.

We assess the accuracy of the forecasting rule for m' in table B1. Again, the R^2's are high and the percent standard errors are small. The maximum error, $\max_t |\log \hat{m}_t - \log m_t|$, is 0.432 percent. For an additional assessment of the accuracy of our forecast rule, we calculate Den Haan's (2010) maximum forecast error for a multistep forecast. In the model solution algorithm described above, the forecasting rule used to derive $\log \hat{m}'$ is

$$\log \hat{m}_{t+1} = \beta_0^i + \beta_1^i \log m_t + \beta_2^i \vartheta_{1,t} + \beta_3^i \vartheta_{2,t}, \quad i = 1, \ldots, 6.$$

Summarize this function as $\log \hat{m}_{t+1} = B(i, \log m_t, \vartheta_{1,t}, \vartheta_{2,t})$, and notice that m_t represents the actual mean of the distribution of capital while $\hat{m}_{t+1}$ is a one-period forecast. We emphasize that only one-step-ahead forecasts are ever used to solve the model. Den Haan argues that a proper accuracy test should generate $\{\tilde{m}_t\}_{t=1}^{T}$ using $\log \tilde{m}_{t+1} = B(i, \log \tilde{m}_t, \vartheta_{1,t}, \vartheta_{2,t})$, for $t = 1, \ldots, T$, with $\tilde{m}_1 = m_1$. This involves forecasting an entire history for capital without ever updating the forecast. That is, the mean of the actual distribution of capital is never used to update a forecasted value; rather, the forecasted value for period t is used to forecast a value for period $t + 1$. Den Haan suggests evaluating accuracy using the maximum error $e_{\max} = \max_t |\log \tilde{m}_t - \log m_t|$; we also report the average error

$$e_{\text{mean}} = \frac{1}{T-1} \sum_{t=2}^{T} |\log \tilde{m}_t - \log m|.$$

Our simulation length is $T = 10,000$ and, for the forecasting rule in table B1, $e_{\max} = 0.008149$ and $e_{\text{mean}} = 0.001003$. Toward establishing the accuracy of our forecast, we compare these values with those for two models in Krusell and Smith (1998). The first is their baseline model, solved using a 2,500-period simulation. It has a maximum error of 0.003 and a mean error of 0.0008. The second is their stochastic beta model; it has corresponding error values of 0.0202 and 0.0027. Both our maximum and mean errors are in the middle of the range defined by these two models. Since these two models have been found to be solved extremely accurately, we conclude that our forecast rule for the approximate aggregate state, and thus our model solution, is highly accurate.

Appendix C

Financial Data

We report three moments in the text associated with corporate sector financing behavior: the time-averaged cross-sectional correlation of size (book assets) with leverage (the ratio of debt to book assets) over 1954–2011 (0.022), the average standard deviation in firms' cash-to-asset ratios over 1954–2011 (0.161), and the aggregate cash-to-asset ratio in 2006 (0.102). These moments are drawn from an unbalanced sample of nonfinancial firms similar to that examined by Bates et al. (2009). In each year, our sample includes all Compustat firms incorporated in the United States that have positive book assets (data item 6) and positive sales

TABLE C1
SIZE, LEVERAGE, AND CASH OVER 1954–73

Year	Observations	Aggregate Cash Ratio	Standard Deviation (Cash Ratio)	Correlation (Size, Leverage)
1954	538	.162	.104	.052
1955	549	.174	.105	.023
1956	571	.127	.097	.016
1957	591	.121	.093	.008
1958	608	.122	.094	.024
1959	627	.124	.099	.031
1960	1,124	.117	.107	.020
1961	1,246	.113	.107	.014
1962	1,446	.113	.101	.009
1963	1,630	.113	.103	−.002
1964	1,776	.100	.105	−.005
1965	1,933	.093	.104	−.008
1966	2,095	.077	.098	−.002
1967	2,286	.072	.098	.009
1968	2,877	.074	.109	.017
1969	3,075	.062	.106	.022
1970	3,246	.055	.094	.023
1971	3,352	.064	.097	.023
1972	3,433	.066	.098	.025
1973	3,729	.068	.095	.009

TABLE C2
SIZE, LEVERAGE, AND CASH OVER 1974–93

Year	Observations	Aggregate Cash Ratio	Standard Deviation (Cash Ratio)	Correlation (Size, Leverage)
1974	4,953	.061	.103	−.004
1975	4,958	.067	.105	.000
1976	4,946	.076	.110	−.005
1977	4,920	.068	.109	−.010
1978	4,778	.066	.116	−.015
1979	4,671	.059	.119	−.021
1980	4,748	.056	.142	−.016
1981	4,771	.052	.156	−.001
1982	4,967	.054	.153	−.006
1983	5,174	.067	.187	−.005
1984	5,146	.061	.174	−.006
1985	5,331	.062	.175	−.016
1986	5,458	.069	.189	−.005
1987	5,441	.070	.189	−.002
1988	5,259	.058	.175	.029
1989	5,139	.051	.176	.031
1990	5,130	.049	.178	.034
1991	5,234	.053	.195	.041
1992	5,545	.054	.198	.045
1993	5,828	.055	.203	.048

TABLE C3
Size, Leverage, and Cash over 1994–2011

Year	Observations	Aggregate Cash Ratio	Standard Deviation (Cash Ratio)	Correlation (Size, Leverage)
1994	6,107	.053	.191	.044
1995	6,755	.055	.215	.043
1996	6,900	.060	.229	.046
1997	6,734	.062	.228	.049
1998	6,734	.062	.234	.043
1999	6,637	.070	.250	.041
2000	6,263	.067	.246	.039
2001	5,703	.073	.238	.041
2002	5,327	.084	.233	.044
2003	5,076	.098	.240	.044
2004	4,893	.106	.245	.044
2005	4,727	.103	.243	.046
2006	4,503	.102	.246	.046
2007	4,275	.096	.243	.050
2008	4,051	.097	.222	.053
2009	3,976	.116	.226	.057
2010	3,851	.121	.228	.059
2011	3,568	.118	.227	.055

(item 12), excluding financial firms (Standard Industrial Classification [SIC] 6000–6999) and utilities (SIC 4900–4999). Cash is defined as cash and marketable securities (data item 1), and debt is long-term debt (item 9) plus current liabilities (item 34). Tables C1–C3 report moments from each year in our sample.

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