Noise Shocks and Heterogeneous Agents

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Research Question (Preliminary)

The key empirical question is how **information dispersion** across agents affect economic fluctuations.

Two possible analysis

1. Identifying an **information-dispersion shock**
2. Analyzing the effect of fundamental shocks conditioning on different level of information dispersion

Background (I)

A standard **noise shock** is defined as a noisy public signal regarding aggregate future fundamentals.

In this case agents **coordinate** their choices with respect to an aggregate biased signal.

Underlying assumption is that the **quality** of the signal is low.

Background (II)

An **information-dispersion shock** is defined as a signal which spread out expectations regarding aggregate future fundamentals.

In this case agents **fail to coordinate** their choices with respect to expected future fundamentals.

Underlying assumption is that the **quantity** of the signal is low.

Formalization

Consider a simple economy populated by I agents where each agent i attempt to forecast fundamental variable x_{t+1} given the information set at time t.

Define the mean squared forecast error across agents as follows,

$$\phi_t = I^{-1} \sum_{i=1}^{I} \left\{ E_t^i[x_{t+1}] - x_{t+1} \right\}^2 \tag{1}$$

where

- $E_t^i[x_{t+1}]$ is the expectation of agent i on x_{t+1}
- $E_t^i[x_{t+1}] - x_{t+1}$ is the forecast error of agent i on x_{t+1}

Intuition. ϕ_t represents the precision of agents' expectations in the whole economy.

Decomposition

Now, the interesting part of Equation 1 is that can be decomposed as follows,

$$
\begin{aligned}
\phi_t &= I^{-1} \sum_{i=1}^{I} \left\{ E_t^i(x_{t+1}) - x_{t+1} \right\}^2 \\
&= I^{-1} \sum_{i=1}^{I} \left\{ E_t^i(x_{t+1})^2 - 2E_t^i(x_{t+1})x_{t+1} + x_{t+1}^2 \right\} \\
&= I^{-1} \sum_{i=1}^{I} E_t^i(x_{t+1})^2 - 2\bar{x}_{t,t+1}^i x_{t+1} + x_{t+1}^2 \\
&= I^{-1} \sum_{i=1}^{I} E_t^i(x_{t+1})^2 - (\bar{x}_{t,t+1}^i)^2 + (\bar{x}_{t,t+1}^i)^2 - 2\bar{x}_{t,t+1}^i x_{t+1} + x_{t+1}^2 \\
&= Var_t^i(x_{t+1}) + (\bar{x}_{t,t+1}^i - x_{t+1})^2,
\end{aligned}
\tag{2}
$$

where $\bar{x}_{t,t+1}^i = I^{-1} \sum_{i=1}^{I} E_t^i(x_{t+1})$ is the average expectation across agents of x_{t+1} given the information set at time t.

Intuition

From previous slide,

$$\phi_t = Var_t^i(x_{t+1}) + (\bar{x}_{t,t+1}^i - x_{t+1})^2 \tag{3}$$

Equation 3 is divided into two parts:

1. $Var_t^i(x_{t+1})$, which is the variance across agents of expectations of x_{t+1} given information set at time t,
2. $(\bar{x}_{t,t+1}^i - x_{t+1})^2$, which is the square of difference between the average expectation across agents of x_{t+1} at time t and its actual realization.

An **information-dispersion shock** can be identified as

$$\iota_t = Var_t^i(x_{t+1}) \tag{4}$$

and a standard **noise shock** as

$$\eta_t = \bar{x}_{t,t+1}^i - x_{t+1} \tag{5}$$

Thus, ϕ_t can be represented as the sum of two shocks

$$\phi_t = \iota_t + \eta_t^2 \tag{6}$$

which is simply the Variance-Bias decomposition.

Intuition. Information-dispersion shocks can be interpreted as situations where aggregate information is weak across agents while **noise shocks** as situation where aggregate information is biased.