The Determinants of Corporate Liquidity: Theory and Evidence

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Abstract

We model the firm's decision to invest in liquid assets when external financing is costly. The optimal amount of liquidity is determined by a tradeoff between the low return earned on liquid assets and the benefit of minimizing the need for costly external financing. The model predicts that the optimal investment in liquidity is increasing in the cost of external financing, the variance of future cash flows, and the return on future investment opportunities, while it is decreasing in the return differential between the firm's physical assets and liquid assets. Empirical tests on a large panel of U.S. industrial firms support the model's predictions.

I. Introduction

What is the value of liquidity? According to Brealey and Myers (1996), determining the value of liquidity is one of the 10 unsolved problems in finance. In practice, firms invest large sums of money in very liquid financial securities. For example, Business Week (1995) noted that Chrysler and Ford have \$7.6 billion and \$12.1 billion in cash and marketable securities, respectively. Kester (1986) reports that the average ratio of cash plus marketable securities to total assets is 8.6% in 1983 for a sample of 452 U.S. manufacturing firms in 27 different industries. Our own analysis shows that, in a sample of 915 industrial firms, the average ratio of cash plus marketable securities to total assets is 8.1% during the period from 1975 to 1994.

The purpose of this paper is to provide a theoretical and empirical investigation of the firm's decision to invest in liquid assets. A formal analysis requires careful consideration of both the costs and benefits of holding liquid assets. Investment in liquid assets (e.g., Treasury securities) is costly because the firm foregoes investment in less liquid but more productive assets, because the firm incurs transaction costs when buying and selling financial securities, and because they

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lead to higher taxation (relative to stockholders holding such securities directly).[1] In addition, a number of authors have noted that liquid assets may engender more severe agency problems than less liquid assets.[2]

Despite these costs, firms will generally maintain some cash and cash equivalents for business transactions needs. In addition, most textbooks argue that firms will maintain excess liquidity for "precautionary" and "speculative" motives. The precautionary motive argues that firms maintain excess liquidity to meet unexpected contingencies, while the speculative motive argues that firms maintain excess liquidity to take advantage of profitable future investment opportunities. However, in the absence of significant financial market frictions, neither motive is compelling since external funds for investment in production or to meet temporary operating cash flow shortfalls can always be obtained at a fair price. The implication is that the firm should optimally maintain zero *excess* liquidity.

However, the existence of capital market imperfections provides a rationale for significant and predictable amounts of excess liquid asset holdings by firms. In particular, if external financing is costly, then investment in liquid assets is an optimal response to having to seek costly external financing to fund future production needs.[3] The costs of external financing include the direct out-of-pocket expense to issue securities, the costs arising from potential agency conflicts, and the costs arising from adverse selection problems attributable to asymmetric information (see, e.g., Smith (1986)).[4] Thus, investment in excess liquidity can be viewed as an economically sensible way to reduce the firm's dependence on costly external financing. Of course, any such benefits must be balanced against the holding costs that liquid assets impose on the firm.

In our model, investment in liquidity is costly since liquid assets earn a low rate of return. However, given uncertain future internal funds and costly external financing, the firm may nevertheless decide to hold a positive amount of liquid assets.[5] Thus, there is a tradeoff between the holding cost of liquid assets (a low return) and the benefit of minimizing the need to seek costly external financing if internally generated funds are insufficient to finance future investment opportunities. We establish that the optimal amount of liquidity is increasing in the cost of external financing, the variance of future cash flows, and the profitability of future investment opportunities. Optimal liquidity is decreasing in the rate of return on current investment opportunities.

We test the implications of our analysis using a panel of 915 U.S. industrial firms during the period 1975 to 1994. We find that firms that face higher costs of external financing, have more volatile earnings, and that those having lower

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[1]See Miller (1986), Masulis and Trueman (1988), and Ang (1991) for discussions of the tax disadvantages of corporate investment in financial securities.

[2]See Huberman (1984), Ang (1991), and Myers and Rajan (1995).

[3]Firms may establish lines of credit with banks or commercial paper programs as an alternative to investing in liquid assets. However, lines of credit and commercial paper programs are costly. Lines of credit involve commitment fees and compensating balances, whereas the high fixed cost of establishing a commercial paper program excludes all but the largest firms from this source of financing.

[4]In addition, note that scale economies in floating securities may provide an incentive to hold excess liquid asset balances between security issuance dates and investment expenditure dates.

[5]The model ignores the pure transactions motive for holding cash, and so the optimal amount of liquid assets is zero in the absence of costly external financing.

returns on physical assets relative to those on financial securities tend to have significantly larger proportions of liquid assets to total assets. The empirical analysis also supports the model's prediction that firms build liquidity in anticipation of profitable future investment opportunities.

The remainder of the paper is organized as follows. Section II provides a brief review of related literature. Section III presents the model. Section IV solves the model and derives comparative static results. Section V develops and tests the model's empirical predictions. Section VI concludes.

II. Related Literature

The two most prominent points of view in the literature on the optimal amount of liquidity conclude that the firm should either hold large amounts of liquid assets or no liquid assets. Myers and Majluf (1984) argue that because of information asymmetry-induced financing constraints, firms should stock up on liquid assets to finance future investment opportunities with internal funds. Since there are no offsetting costs to liquid assets in their model, the optimal amount of liquidity is a corner solution. In contrast, Jensen (1986) argues that firms should be forced to pay out funds in excess of the amount necessary to finance all positive NPV investments to minimize the agency cost of free cash flow. In the absence of a benefit from liquid assets, Jensen's analysis implies that the firm would optimally carry no liquid assets. Our analysis includes both benefits and costs of holding liquid assets to develop predictions about the determinants of corporate liquidity.

Huberman (1984) also develops a model that can yield an interior optimal level of investment in liquid assets. He assumes that firms cannot use external financing to fund investment in production, requiring instead that such investment be financed with cash on hand. There are no costs, however, to issuing securities to finance investment in liquid assets. Not surprisingly, in this setting, he finds that firms invest in liquid assets to fund future investment opportunities. The critical difference between our model and Huberman's model is that we allow the firm to fund investment in production with external financing.

Similar to our model, Martin and Morgan (1988) examine optimal investment in liquid assets in a model where liquid assets earn a low return, but given uncertain future funding needs and costly external financing, may nevertheless be held. Adopting a cost minimization objective, they derive conditions under which corner solutions result, i.e., the firm either holds no liquid assets or holds more than enough liquid assets to cover any future funds shortfall.

III. The Model

Consider a three-date model with risk-neutral investors and a constant riskless rate of interest, ρ, each period. The firm chooses investment in production at time zero and time one. For an investment of I_t units at time t, $t \in \{0, 1\}$, the output at time $t + 1$ is

$$F(I_t) + \epsilon_{t+1}, \tag{1}$$

where $F(I)$ is twice continuously differentiable, strictly increasing, and strictly concave, with $F(0) = 0$, $F'(0) = \infty$, and $F'(z) \to 0$ as $z \to \infty$. In (1), the ϵ_{t+1} are independent and identically distributed output shocks with zero mean and support $[\underline{\epsilon}, \overline{\epsilon}]$.[6]

At time zero, the firm chooses investment in production, I_0, investment in liquid assets, L_0 (e.g., cash and marketable securities), and the dividend, D_0, to distribute to shareholders. As discussed above, investment in liquid assets is costly because of transaction costs, taxes, and agency costs. Denoting r_L as the rate of return on investment in liquid assets, we specify that $r_L < \rho$. This condition ensures that, in the absence of an offsetting benefit to liquidity, such investment has a negative net present value.

At time one, after observing the random output shock ϵ_1, the firm chooses investment in production, I_1, the amount of borrowing to finance the production plan, B_1, and the dividend level, D_1. We assume that the firm incurs a proportional cost $\xi \in (0, 1)]$ on borrowed funds. Thus, if the firm borrows B_1, the net amount of funds raised is $B_1(1 - \xi)$.[7]

We adopt a proportional financing cost structure for convenience. In reality, there may be significant economies of scale in direct financing costs—legal fees, accounting and printing costs, and underwriter fees—which suggest a fixed-cost component. In addition, one could argue that costs arising from information or incentive problems are likely to be convex instead of linear. Although including these features in a general financing cost function is certainly possible, it would not change the qualitative implications of our analysis.

A necessary condition for a nonzero investment in liquid assets at time zero is

$$F\left(I_0^*\right) + \underline{\epsilon}_1 \quad < \quad I_1^*, \tag{2}$$

where I_0^* and I_1^* are the optimal investment levels in production when external financing is costless, i.e., $\xi = 0$. (These optimal levels, I_0^* and I_1^*, will be defined later.) If this condition is not satisfied, then the firm will have sufficient cash on hand in period 1 to finance the first-best production plan even if it receives the worst possible output shock, as long as it invests I_0^* at time zero. Therefore, we assume (2) in order to focus on the more interesting case where the firm may need cash at time one.

Finally, at time two, conditional on the random output shock ϵ_2, the firm first pays off the bondholders and then issues a liquidating dividend D_2 (if any cash remains) to shareholders. Since the time two investment return is random, the firm may default on any borrowing undertaken at time one. Given limited liability of equityholders, bondholders will require an endogenously-determined default-risk premium π (expressed in return form) to compensate for the decreased return that they receive in bankruptcy.

[6]The stochastic production function in (1) has the unattractive property that uncertainty in the rate of return on investment is decreasing in the level of investment. However, the comparative static properties of the model are unaffected if we assume multiplicative uncertainty instead of additive uncertainty.

[7]Although our analysis assumes that external financing is debt, our results are unaffected if we allow for costly equity financing.

The firm's objective is to maximize the discounted expected value of the dividend stream to equityholders. This objective does not lead to a conflict of interest between bondholders and stockholders, because the required rate of return on debt reflects the possibility of default at time two and there is no private information. Thus, equity value maximization coincides with firm value maximization. The problem is solved using dynamic programming, by first solving the time two maximization problem, which is trivial and so is not shown. The time two solution is then used to solve the time one maximization problem, which is then folded back to time zero to determine optimal policies at that time.

The time one maximization problem is

$$\max_{I_1,B_1} \left\{D_1 + (1+\rho)^{-1}E_1[D_2]\right\}, \tag{3}$$

$$\text{subject to} \quad D_1 \geq 0, \quad I_1 \geq 0, \quad B_1 \geq 0, \tag{4}$$

$$\text{where} \quad D_1 = F(I_0) + \epsilon_1 + L_0(1+r_L) + B_1(1-\xi) - I_1, \tag{5}$$

$$\text{and} \quad E_1[D_2] = \int_{\underline{\epsilon}_2}^{\overline{\epsilon}_2} \max\left[F(I_1) + \epsilon_2 - (1+\rho+\pi)B_1, 0\right] g(\epsilon_2)d\epsilon_2. \tag{6}$$

In (3) and (6), $E_1[\cdot]$ denotes expectation conditional on information at time one, and $g(\cdot)$ is the probability density function of ϵ. The default risk premium on bonds, π, must be endogenously determined to ensure that the expected return on debt is equal to the risk-free rate of interest. Thus, the solution to (3) must also satisfy the condition that

$$B_1(1+\rho) = \int_{\underline{\epsilon}_2}^{\overline{\epsilon}_2} \min\left[F(I_1) + \epsilon_2, (1+\rho+\pi)B_1\right] g(\epsilon_2)d\epsilon_2. \tag{7}$$

Note that (6) and (7) can be combined to yield,

$$E_1[D_2] = F(I_1) - B_1(1+\rho). \tag{8}$$

Hence, the time one problem is to maximize (3) subject to (4), (5), and (8).

Given the solution to the time one problem, the time zero maximization problem is

$$\max_{I_0,L_0} \left\{D_0 + (1+\rho)^{-1}E_0[V_1]\right\}, \tag{9}$$

$$\text{subject to} \quad D_0 \geq 0, \quad I_0 \geq 0, \quad L_0 \geq 0, \tag{10}$$

$$\text{where} \quad D_0 = \overline{X}_0 - I_0 - L_0, \tag{11}$$

$$\text{and} \quad V_1 = \max_{I_1,B_1} \left\{D_1 + (1+\rho)^{-1}E_1[D_2]\right\}. \tag{12}$$

In (11), $\overline{X}_0$ is the firm's cash endowment coming into the period.

Note that the time one and time zero maximization problems do not allow the firm to issue liquid assets or invest in bonds, i.e., $L \geq 0$ and $B \geq 0$. Indeed, given that the expected return on bonds, ρ, exceeds the return on liquid asset, r_L, would not an unconstrained firm simply choose $B = L = -\infty$, arbitraging the spread between ρ and r_L? However, this would not be economically feasible. In our model, the firm's return from *investing* in bonds would be r_L and not ρ. The same transaction costs, taxes, and agency costs that produce a low return on liquid asset investments would also apply to corporate investment in bonds. By the same token, investors in our economy would require ρ and not r_L if the firm were to *issue* liquid assets (e.g., commercial paper). As a consequence, the non-negativity conditions on B and L are not simply imposed to rule out arbitrage opportunities.

IV. Analysis of the Model

A. Solution of the Time One Problem

Forming the Lagrangean,

$$Q(I_1, B_1, \lambda_1) \quad = \quad D_1 + (1+\rho)^{-1} E_1[D_2] + \lambda_1 D_1, \tag{13}$$

and substituting (5) and (8) into (13), we have the following Kuhn-Tucker first-order conditions,[8]

$$\frac{\partial Q}{\partial I_1} = \frac{F'(I_1)}{1+\rho} - (1+\lambda_1) \leq 0, \qquad I_1 \geq 0, \qquad I_1 \frac{\partial Q}{\partial I_1} = 0, \tag{14}$$

$$\frac{\partial Q}{\partial B_1} = -\xi + \lambda_1(1-\xi) \leq 0, \qquad B_1 \geq 0, \qquad B_1 \frac{\partial Q}{\partial B_1} = 0, \tag{15}$$

$$\frac{\partial Q}{\partial \lambda_1} = D_1 \geq 0, \qquad \lambda_1 \geq 0, \qquad \lambda_1 \frac{\partial Q}{\partial \lambda_1} = 0. \tag{16}$$

Since, by assumption, $F'(0) = \infty$, $I_1 > 0$, and by the complementary slackness condition, $I_1(\partial Q/\partial I_1) = 0$, the first-order condition for investment is satisfied at equality, $\partial Q/\partial I_1 = 0$. Also note that it is inefficient for the firm to finance a dividend with costly external financing, i.e., if $D_1 > 0$, then $B_1 = 0$.

There are three feasible combinations of dividend and financing decisions at time one, depending on the realization of the output shock ϵ_1. First, for a high enough shock, we have the case where $D_1 > 0$ and $B_1 = 0$. For this case, the firm has enough internal funds to make the first-best investment decision and money left over to pay a dividend. Using (14)–(16), we may determine that the first-best investment, I_1^*, satisfies:

$$F'(I_1^*) \quad = \quad (1+\rho). \tag{17}$$

[8]Since the objective function is concave, the Kuhn-Tucker conditions are both necessary and sufficient for a global optimum.

Second, if the shock is low enough, then we have the case where $D_1 = 0$ and $B_1 > 0$. From (14)–(16), we may deduce that investment is determined where

$$F'\left(\widehat{I}_1\right) = \frac{1+\rho}{1-\xi}. \tag{18}$$

This investment level, $\widehat{I}_1$, will be strictly less than I_1^* for $\xi > 0$. Thus, the firm invests less than it would if borrowing were costless.

Finally, we have the case where $D_1 = B_1 = 0$. For this case, the marginal internal rate of return on investment and, hence, the optimal investment level, falls within a range that is bounded by (17) and (18). From (14)–(16) we may determine that

$$1+\rho \leq F'\left(\widetilde{I}_1\right) \leq \frac{1+\rho}{1-\xi}, \tag{19}$$

or, in other words, $\widehat{I}_1 \leq \widetilde{I}_1 \leq I_1^*$. This is the intermediate case where the output shock is not low enough to justify borrowing, yet the firm is expected to underinvest since the cash on hand is insufficient to fund the first-best investment level, I_1^*.

Figure 1 displays the firm's investment, borrowing, and dividend decisions as a function of the output shock at time one. In the figure, $\widehat{\epsilon}_1$ is the output shock below which the firm borrows; and ϵ_1^* is the output shock above which the firm invests I_1^* and has enough internal funds left over to pay a dividend.[9] The upward sloping line in the figure shows the internal funds available for investment at time one, $X_1 = F(I_0) + \epsilon_1 + L_0(1+r_L)$, i.e., the cash flow from time zero investment in production, $F(I_0) + \epsilon_1$, and liquid assets, $L_0(1+r_L)$. The horizontal lines show the firm's choice of investment level. For $\epsilon_1 \in [\underline{\epsilon}_1, \widehat{\epsilon}_1)$, the firm borrows to invest at $\widehat{I}_1$. For $\epsilon_1 \in [\widehat{\epsilon}_1, \epsilon_1^*]$, the firm invests the cash on hand, X_1. Finally, for $\epsilon_1 \in (\epsilon_1^*, \overline{\epsilon}_1]$, the firm invests I_1^* and pays a dividend.

B. Solution of the Time Zero Problem

Forming the Lagrangean for the optimization problem in (9)–(12), we have the following Kuhn-Tucker conditions for the time zero optimization problem,[10]

$$\frac{\partial Q}{\partial I_0} = \frac{F'(I_0)}{1+\rho}Z - (1+\lambda_0) \leq 0, \qquad I_0 \geq 0, \qquad I_0\frac{\partial Q}{\partial I_0} = 0, \tag{20}$$

$$\frac{\partial Q}{\partial L_0} = \frac{1+r_L}{1+\rho}Z - (1+\lambda_0) \leq 0, \qquad L_0 \geq 0, \qquad L_0\frac{\partial Q}{\partial L_0} = 0, \tag{21}$$

$$\frac{\partial Q}{\partial \lambda_0} = D_0 \geq 0, \qquad \lambda_0 \geq 0, \qquad \lambda_0\frac{\partial Q}{\partial \lambda_0} = 0. \tag{22}$$

[9] Note in the figure that both $\hat{\epsilon}_1$ and ϵ_1^* are negative. In Appendix A, we prove this property of the solution.

[10] Appendix B provides the derivation of the first-order conditions for the time zero problem.

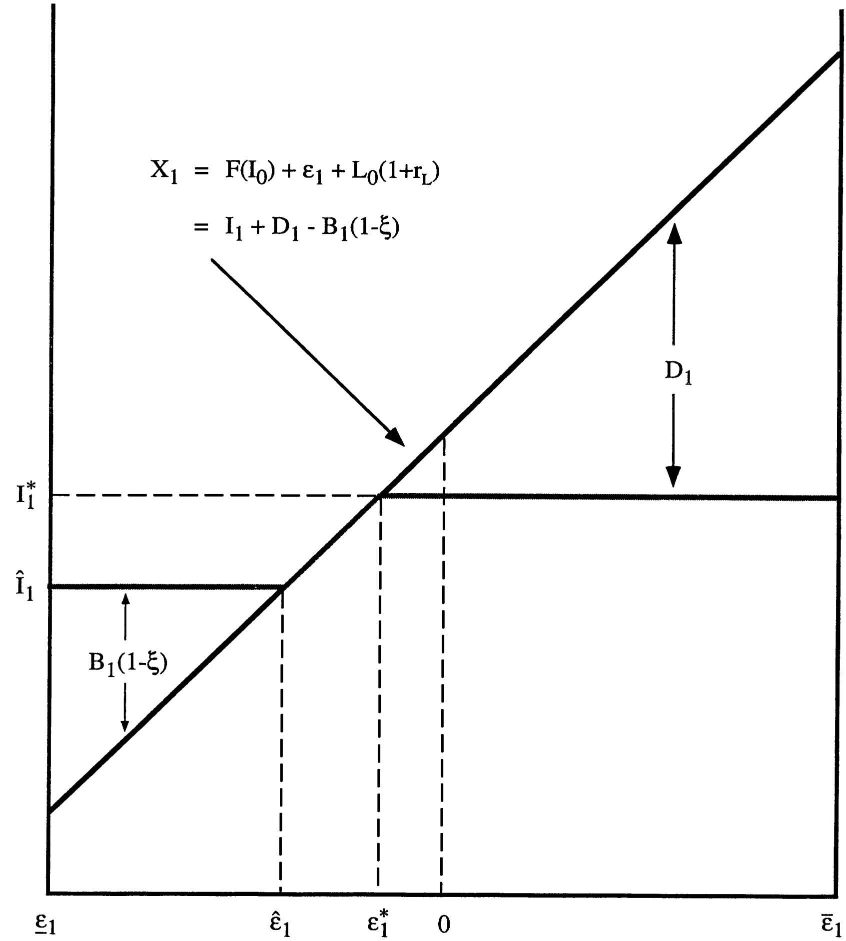

FIGURE 1

The Firm's Investment, Borrowing, and Dividend Decisions Conditional on the Realization of the Output Shock at Time One

When the time one shock is greater than ϵ_1^*, the firm has enough internal funds to invest at the first-best level, I_1^*, and to pay a dividend, D_1. If the shock is between $\hat{\epsilon}_1$ and ϵ_1^*, the firm invests the cash on hand, X_1. When the shock is less than $\hat{\epsilon}_1$, the firm borrows the amount $B_1(1-\xi)$ (net of borrowing costs, ξ), so as to invest the amount $\hat{I}_1$.

where λ_0 is the multiplier on D_0, and

$$(23) \quad Z \;=\; 1 + \int_{\underline{\epsilon}_1}^{\epsilon_1^*} \frac{F'(F(I_0) + \epsilon_1 + L_0(1+r_L) + B_1(1-\xi)) - (1+\rho)}{(1+\rho)} g(\epsilon_1) d\epsilon_1.$$

In (23), it is understood that $B_1 = 0$ for $\epsilon_1 \in [\hat{\epsilon}_1, \epsilon_1^*]$. Note that $Z > 1$, since the second term in (23) is positive for $\xi > 0$. The second term measures the present expected loss in value from underinvesting at time one.

First, consider the case where the firm's investment decisions are not constrained by available funds. If $\overline{X}_0$ is large enough so that $D_0 > 0$ for any (optimal) choice of I_0 and L_0, then by the complementary slackness condition in (22), $\lambda_0=0$. Further, since $F'(0) = \infty$, $I_0 > 0$, which, by the complementary slackness condition in (20), requires that the first-order condition for investment in production is satisfied at equality.

For this case, there are two subcases: $L_0 = 0$ and $L_0 > 0$. If $L_0 = 0$, then, from (20) and (21), we have that investment in production must satisfy

$$(24) \qquad 1 + r_L \leq F'(I_0) = \frac{1+\rho}{Z}.$$

Since $Z > 1$ for $\xi > 0$, the time zero investment level that satisfies (24) exceeds the first-best investment level, I_0^*, where $F'(I_0^*) = 1 + \rho$. Thus, with no constraint on available funds at time zero and costly external financing at time one, the firm overinvests in production at time zero relative to I_0^*.

Is there an upper bound, $\overline{I}_0$, on the amount of investment? To answer this question, consider the subcase where $L_0 > 0$. Again, from (20) and (21), we find that

$$(25) \qquad F'\left(\overline{I}_0\right) = 1 + r_L.$$

Thus, the upper bound on time zero investment, $\overline{I}_0$, is determined by the rate of return on liquid assets.

In the case where the firm has a shortage of funds at time zero, we have $D_0=0$ and $\lambda_0 \geq 0$. As before, there are two subcases. If $L_0=0$, then it is straightforward to show that the firm invests more or less than I_0^* depending on the amount of available funds, $\overline{X}_0$. Alternatively, if $L_0 > 0$, then it is also straightforward to show that investment in production is again determined according to (25). For this subcase, the firm first invests available funds in production until condition (25) is satisfied, and then invests what remains, $\overline{X}_0 - \overline{I}_0$, in liquid assets.

There are, of course, other cases that can also be examined. In particular, although the model is setup so that the firm does not have access to external financing at time zero, it is relatively straightforward to include this in the model. For the subsequent comparative static analysis, we consider the case where the firm has no shortage of funds at time zero and, therefore, is not constrained over its choice of time zero investment levels. This is only for convenience. The qualitative implications of the analysis are the same if instead we analyze the case where there is a shortage of funds at time zero and the firm has the option to seek costly external financing.

C. Comparative Static Properties of the Model

We first examine the effect of time one financing costs (ξ) on time zero investment in production and liquid assets. For $L_0=0$, using the first-order condition for investment in (20) and the definition of Z in (23), we find that

$$(26)\quad \frac{dI_0}{d\xi} = \frac{-(1+\rho)F'(I_0)\int_{\underline{\epsilon}_1}^{\hat{\epsilon}_1} g(\epsilon_1)d\epsilon_1}{(1-\xi)^2\left[F''(I_0)Z(1+\rho)+\{F'(I_0)\}^2\int_{\hat{\epsilon}_1}^{\epsilon_1^*}F''(F(I_0)+\epsilon_1)g(\epsilon_1)d\epsilon_1\right]},$$

which is positive because $F' > 0$, $F'' < 0$, and $Z > 1$. Similarly, for $L_0 > 0$, using the first-order condition for liquidity in (21) and the definition of Z in (23), we can determine that

$$(27)\quad \frac{dL_0}{d\xi} = \frac{-(1+\rho)\int_{\underline{\epsilon}_1}^{\hat{\epsilon}_1} g(\epsilon_1)d\epsilon_1}{(1-\xi)^2\left[(1+r_L)\int_{\hat{\epsilon}_1}^{\epsilon_1^*}F''\left(F\left(\bar{I}_0\right)+\epsilon_1+L_0(1+r_L)\right)g(\epsilon_1)d\epsilon_1\right]},$$

which also is positive. Thus, optimal L_0 and I_0 are increasing in time one financing costs. Note, however, that (26) is defined only for $L_0 = 0$, since when $L_0 > 0$, optimal time zero investment is fixed at $\bar{I}_0$, and is, therefore, independent of ξ. By the same token, (27) is only defined for $I_0 = \bar{I}_0$.

The last observation implies that there must be a critical level of financing costs, call it $\underline{\xi}$, above which the firm invests in liquid assets. To see this, note that L_0 is positive only when $I_0 = \bar{I}_0$. Thus, since $\bar{I}_0 > I_0^*$ only when $\xi > 0$, and since $dI_0/d\xi > 0$ for $I_0^* < I_0 < \bar{I}_0$, it follows that $\underline{\xi} > 0$. Since ξ is bounded at 1, an important question is whether, and under what conditions, $\underline{\xi}$ is less than one. Without appealing to numerical computations, we can only say that $\underline{\xi} < 1$ critically depends on the degree to which liquid assets earn a low rate of return. In particular, letting $\Delta = \rho - r_L$, it can be shown that $d\underline{\xi}/d\Delta > 0$. In other words, as the cost of investing in liquid assets increases, a larger cost of external financing is required before the firm is compelled to invest in liquidity.

Another property of the model can be gleaned from an examination of the numerator in (27). As seen there, the derivative of L_0 with respect to ξ will vanish when $\hat{\epsilon}_1 = \underline{\epsilon}_1$. Since $\hat{\epsilon}_1$ decreases as L_0 increases, this implies that the firm will eventually stop investing in liquid assets when ξ reaches some critical upper bound, $\bar{\xi}$. Specifically, since there is a positive probability that the firm will have to seek costly external financing at time one when $\hat{\epsilon}_1 > \underline{\epsilon}_1$, and since the firm can drive $\hat{\epsilon}_1$ down until it eventually equals $\underline{\epsilon}_1$ by investing in liquid assets at time zero, there must exist a pair, $\bar{\xi}$ and $\bar{L}_0$, at which the firm stops investing in liquid assets.

Figure 2 illustrates these properties by displaying optimal time zero investment in production and liquid assets as a function of the time one cost of external financing. The upper line represents time zero investment in production, and the lower line represents time zero investment in liquid assets. Proceeding from left to right, note that when $\xi = 0$, the firm invests I_0^* and does not invest in liquidity.

As ξ increases, investment in production increases, reaching a maximum of $\overline{I}_0$ when the cost of external financing equals $\underline{\xi}$. For values of ξ above $\underline{\xi}$, the firm invests in liquid assets, and stops at $\overline{L}_0$ when the cost of external financing equals $\overline{\xi}$.

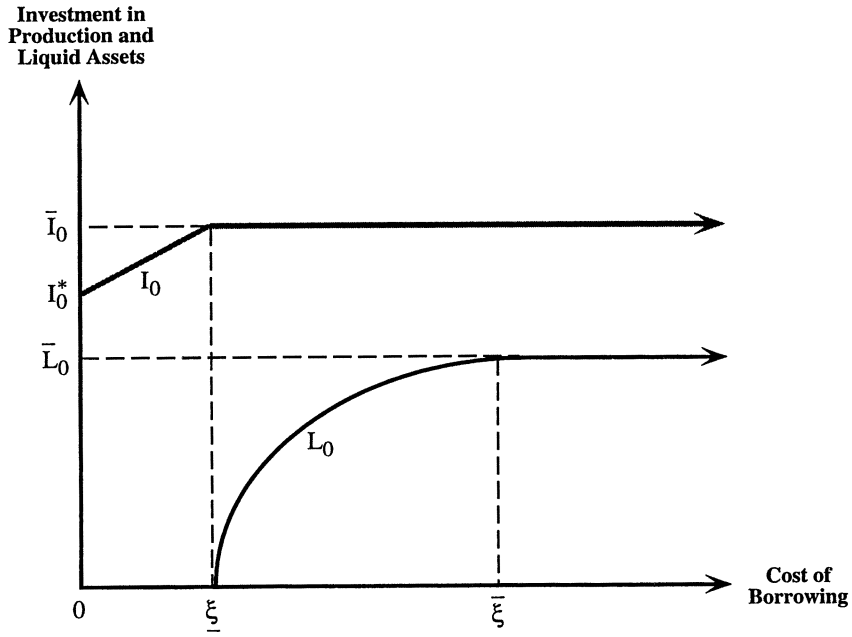

FIGURE 2

Time Zero Investment in Production and Liquid Assets as a Function of the Time One Cost of Borrowing

When the time one borrowing cost (ξ) is zero, the firm invests at the first-best level, I_0^*, and does not invest in liquid assets. As ξ increases, investment in production increases, reaching a maximum of $\overline{I}_0$ when the borrowing cost is $\underline{\xi}$. For values of ξ larger than $\underline{\xi}$, investment in liquid assets quickly accelerates, reaching a maximum of $\overline{L}_0$ when the cost of borrowing equals $\overline{\xi}$.

We now present the remaining comparative static results for liquid assets. Consider the effect of r_L on optimal L_0. From the first-order condition for L_0 in (21), we have that

$$\frac{dL_0}{d(1+r_L)} = \frac{-Z^2}{(1+r_L)\int_{\widehat{\epsilon}_1}^{\epsilon_1^*} F''\left(F\left(\overline{I}_0\right)+\epsilon_1+L_0(1+r_L)\right) g(\epsilon_1)d\epsilon_1} - \frac{L_0}{1+r_L)}. \tag{28}$$

Equation (28) is difficult to sign analytically, because the first term is positive and the second term is negative. If L_0 is small, an increase in r_L will encourage the firm, at the margin, to increase L_0. Alternatively, if L_0 is large, an increase in

r_L may allow the firm to decrease optimal L_0 without impairing liquidity at time one. However, extensive numerical simulations (not reported) using a variety of specifications for the production function, F, and the distribution for the output shock, ϵ_1, show that the relation between L_0 and r_L is always positive.

The effect of a change in either current (time zero) or future (time one) investment opportunities on optimal investment in liquid assets can be examined by rewriting the production function as $F(I) = \alpha\widehat{F}(I)$, where $\alpha > 0$ is a shift parameter. Note that as α increases, the expected return on investment increases.

First, consider the effect of a shift in future investment opportunities on time zero investment in liquidity. Substituting $F(I_1) = \alpha_1\widehat{F}(I_1)$ into equation (23), and using the first-order condition for L_0 in (21), we find that

$$\text{(29)} \qquad \frac{dL_0}{d\alpha_1} = \frac{-\int_{\hat{\epsilon}_1}^{\epsilon_1^*} \widehat{F}'(F(I_0)+\epsilon_1+L_0(1+r_L))g(\epsilon_1)d\epsilon_1}{(1+r_L)\int_{\hat{\epsilon}_1}^{\epsilon_1^*} \alpha_1\widehat{F}''(F(I_0)+\epsilon_1+L_0(1+r_L))g(\epsilon_1)d\epsilon_1},$$

which is positive. Note that as α_1 increases, the expected time one investment level will increase, since the return on investment is directly proportional to α_1. Thus, given costly external financing, the firm will increase investment in liquid assets as α_1 increases to offset the higher likelihood of having insufficient internal funds to finance the increased expected time one investment level.

Next, consider the effect of a shift in time zero investment opportunities on optimal investment in liquidity. Substituting $F(I_0) = \alpha_0\widehat{F}(I_0)$ into equation (23), and using the first-order condition for L_0 in (21), we find that

$$\text{(30)} \qquad \frac{dL_0}{d\alpha_0} = \frac{-\widehat{F}(I_0)}{1+r_L},$$

which is negative. Intuitively, the higher the return on investment in production relative to that on investment in liquid assets, the smaller the investment in liquidity, all else being the same.

Finally, we examine the effect of an increase in the variance of ϵ_1 on optimal time zero investment in production and liquid assets. In Appendix C, we show that both I_0 and L_0 are increasing functions of $\sigma^2(\epsilon_1)$. The greater the uncertainty about the need for costly external financing at time one, the more the firm will invest in production and liquid assets at time zero.

V. Empirical Investigation

A. Testable Implications

The analysis has several empirically testable implications. First, in a cross-section of firms, we would expect a positive relation between the cost of external financing and investment in liquid assets. We use two proxies for the cost of external financing. Recent research shows that small firms are more likely to face borrowing constraints than large firms (see, e.g., Gertler and Hubbard (1988), Whited (1992), and Fazzari and Petersen (1993)). In addition, Barclay and Smith (1996) argue that the cost of external financing is smaller for larger firms because of scale economies resulting from a substantial fixed cost component of security issuance costs. Thus, we would expect a negative relation between firm size and investment in liquid assets.

The literature on asymmetric information argues that firms with more severe information asymmetry between insiders and outside investors face higher costs of external financing. In particular, Myers and Majluf (1984) argue that firms whose values are largely determined by growth opportunities face more severe asymmetric information-induced financing constraints. Growth opportunities are proxied by the ratio of the market value of the firm's assets to the book value of its assets, and as such, the theory would predict that liquid asset holdings are positively related to the market-to-book ratio.[11]

In addition, Myers (1977) argues that risky debt financing may engender suboptimal investment incentives when a firm's investment opportunity set includes growth options. Managers acting on behalf of equityholders may fail to exercise profitable investment options because debt captures a portion of equityholders' return in the form of a reduction in the probability of default. The firm can reduce the risk of financial distress and thereby mitigate the incentive to underinvest in growth options by maintaining excess liquidity.[12] This view also predicts a positive relation between corporate liquidity and the market-to-book ratio.

The second empirical prediction is that the greater the firm's cash flow variability, the larger the investment in liquid assets. We investigate this prediction by computing the variability of both operating cash flow and free cash flow (operating cash flow minus capital expenditures). Note that if capital expenditures are highly variable and are primarily financed with internally generated funds, then the variability of free cash flow may provide a better proxy for liquidity risk. We subtract nonoperating income from both cash flow measures to extract the endogenous effect of liquid asset holdings on cash flow variability.

Third, the analysis shows that investment in liquid assets is negatively related to the current rate of return on investment in production and positively related to the return on liquid assets. We use the difference between the return on the firm's current assets and the return on short-term Treasury bills to measure the relative attractiveness of investment in production vs. investment in liquid assets. We predict that investment in liquid assets will be negatively related to this return spread measure. Similar to the construction of the cash flow variability measures, we subtract nonoperating income when computing the rate of return on assets to extract the endogenous effect of current liquid assets on firm returns.

[11]Smith and Watts (1992) and Stohs and Mauer (1996) use the market value of the firm's assets to the book value of its assets as a proxy for growth options in the firm's investment opportunity set. They reason that since the book value of assets does not reflect intangible assets such as growth options, the more growth options in the firm's investment opportunity set, the higher will be the firm's market value in relation to its book value.

[12]Similarly, Shleifer and Vishny (1992) argue that financial distress will be more costly for firms with a large proportion of assets that are intangible or linked to the value of the firm as a going concern, i.e., firms with high market-to-book ratios. Their analysis predicts that these types of firms will maintain a larger proportion of liquid assets to minimize the cost of financial distress.

Fourth, the analysis predicts that the more promising are future investment opportunities, the larger the current holdings of liquid assets. Thus, we would expect a positive relation between investment in liquid assets and forecasts of future economic conditions. Following Choe, Masulis, and Nanda (1993), we use the logarithmic growth rate in the index of leading economic indicators as a measure of the forecast of future economic conditions.

Our theory and corresponding empirical predictions are intended to capture factors that drive the demand for *excess* corporate liquidity. To control for the level of liquid asset holdings justified by normal transaction needs, we include a number of additional control variables in our empirical tests.

We include the firm's cash cycle and the variability of the cash cycle. The cash cycle is measured as the sum of average inventory age and receivables collection period minus the average payment period for accounts payable. Corporate liquidity is affected by the cash cycle because it measures the average amount of time that cash is tied up in operations. Thus, a firm with a long cash cycle is expected to have lower levels of cash and marketable securities, all else being equal. Similarly, we expect a positive relation between corporate liquidity and the variability of the cash cycle; firms with more variable cash cycles must maintain larger balances of cash and marketable securities to hedge uncertain transactional demand for liquidity.

The firm's debt ratio is expected to be negatively related to liquid assets. There are at least two plausible reasons. Baskin (1987) argues that as the firm's debt ratio increases, the cost of funds used to invest in liquidity increases thereby reducing funded liquidity. Additionally, John (1993) argues that firms with access to debt markets—as proxied by the debt ratio—can use borrowing as a substitute for maintaining a stock of liquid assets.[13]

Operating cash flow and especially free cash flow provide a ready source of liquidity to meet operating expenditures and maturing liabilities. Accordingly, firms with high cash flows can afford to keep lower levels of cash and marketable securities. We expect a negative relation between cash flow measures and liquid asset holdings.

Firms with a greater likelihood of financial distress are expected to have lower levels of liquidity. Following MacKie-Mason (1990), we measure the probability of financial distress by the *inverse* of Altman's (1968), (1993) ZSCORE.[14] We expect a negative relation between the inverse of a firm's ZSCORE and its level of liquidity.

Finally, we control for industry classification to capture differences in liquidity levels across industries. Damodaran (1997) shows that there is substantial variation in cash and marketable securities as a proportion of assets across industry groupings. He argues that industry differences in liquidity are a natural reflection of the transaction demands for cash and cash equivalents in different lines of business.

[13]Note that firms with access to long-term debt markets are also more likely to have commercial paper programs that provide short-term financing for current liquidity needs.

[14]Recall that the higher is Altman's ZSCORE, the lower is the probability of financial distress. We, therefore, use the inverse of the ZSCORE as a proxy for the probability of financial distress.

B. Data and Descriptive Statistics

The sample consists of panel data on 915 industrial firms during the 20-year period from 1975 to 1994. To be included in the sample, each firm must have at least 10 years of data on the Compustat industrial annual file for each one of the variables used in our empirical analysis. We do not require complete data for all 20 years because such a requirement may introduce a survivorship bias. However, as discussed below, there are very few firms in the sample that do not have at least 15 years of data for each variable.

For each firm in the sample and for each year during the sample period, we measure liquidity (LIQRAT) as the ratio of cash plus marketable securities to the book value of total assets.[15] We relate LIQRAT to the following proxies for the hypothesized determinants of liquidity.

1. Cost of External Financing

Firm Size. Firm size (SIZE) is measured by the natural logarithm of the market value of the firm's assets in constant 1987 dollars, where the market value of assets is estimated as the book value of assets plus the difference between the market and book values of equity. The producer price index serves as the deflator. We expect a negative relation between LIQRAT and SIZE.[16]

Growth Opportunities. The ratio of our estimate of the market value of the firm's assets to the book value of its assets (MV/BV). We expect a positive relation between LIQRAT and MV/BV.[17]

2. Cash Flow Uncertainty

We use two variables to measure cash flow uncertainty. The first is the variability of operating cash flow (VARCF), measured as the standard deviation of the first difference in earnings before interest, depreciation, and taxes minus nonoperating income, scaled by the average book value of assets. The second is the variability of free cash flow (VARFCF), measured as the standard deviation of the first difference in earnings before interest, depreciation, and taxes minus nonoperating income and capital expenditures, scaled by the book value of assets. Both VARCF and VARFCF are expected to be positively related to LIQRAT.

3. Current and Future Investment Opportunities

Return Spread. The attractiveness of investment in physical assets vs. liquid assets is measured by the difference between the return on the firm's assets and the return on Treasury bills (RSPREAD). The return on assets is measured by the

[15]The predictions from the theoretical model are for the level of liquid assets and not for the proportion of liquid assets. However, the model implicitly assumes that all firms have the same scale of operations. To test the model's predictions empirically, we, therefore, normalize liquid assets by total assets to account for differences in liquid asset holdings that are driven purely by differences in the scale of operations across firms.

[16]The results reported below are the same if instead we measure size by the natural logarithm of firm sales.

[17]The regression results reported below are similar if instead we use the ratios of advertising expenditures to sales and research and development expenditures to sales as proxies for growth options.

ratio of earnings before interest, depreciation, and taxes minus nonoperating income to the book value of assets. Treasury bill rates are from Ibbotson Associates (1995). We expect a negative relation between LIQRAT and RSPREAD.

Forecast of Future Economic Conditions. Future economic conditions are proxied by the logarithmic growth rate of the index of leading economic indicators (GLEI). The index of leading economic indicators is from the Survey of Current Business. The model predicts a positive relation between LIQRAT and GLEI.[18]

4. Control Variables (Proxies for the Transactions Demand for Liquidity)

Average Cash Cycle. The average cash conversion cycle (CASHCC) is measured by the sum of average inventory age and receivables collection period minus the average payment period for accounts payable. We expect a negative relation between LIQRAT and CASHCC.

Cash Cycle Variability. The variability of the cash cycle (VARCC) is measured by the standard deviation of CASHCC. We expect a positive relation between LIQRAT and VARCC.

Debt Ratio. The debt ratio (DEBTRAT) is measured by the ratio of total debt (long-term debt plus debt in current liabilities) to the book value of assets.[19] We expect a negative relation between LIQRAT and DEBTRAT.

Cash Flow. We use two cash flow measures: the ratio of earnings before interest, depreciation, and taxes to sales (CF), and the ratio of earnings before interest, depreciation, and taxes minus capital expenditures to sales (FCF). Both CF and FCF are expected to be negatively related to LIQRAT.[20]

Bankruptcy Predictor. We measure the probability of financial distress by the inverse of Altman's (1968) ZSCORE,

$$\begin{aligned}\text{ZSCORE} \;=\; & (3.3)\frac{\text{EBIT}}{\text{total assets}} + (1.0)\frac{\text{sales}}{\text{total assets}} + (1.4)\frac{\text{retained earnings}}{\text{total assets}} \\ & + (0.6)\frac{\text{market value equity}}{\text{book value total debt}},\end{aligned}$$

where EBIT is earnings before interest and taxes. Altman's (1968) ZSCORE includes a measure of liquidity. We exclude that term because the analysis is intended to explain the determinants of liquidity. We expect a negative relation between LIQRAT and 1/ZSCORE.[21]

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[18]We use the logarithmic growth rate of industrial production (GIP) as an alternative proxy for future economic conditions. The results using GIP are virtually identical to those using GLEI and, therefore, are not reported.

[19]Although we choose to use a broad measure of debt, the results reported below are insensitive to how debt is measured.

[20]The results reported below are insensitive to whether the cash flow measures are scaled by sales or assets.

[21]Note that the coefficient estimates of Altman's ZSCORE are potentially dated because the original model was estimated using data from the 1940s through the 1960s. However, the results for 1/ZSCORE reported below are similar if we use the updated coefficient estimates of ZSCORE reported in Begley, Ming, and Watts (1996).

Industry Classification. Sample firms are grouped into industry categories using three-digit SIC codes. We use industry dummy variables to control for industry-specific determinants of liquidity.[22]

Table 1 reports descriptive statistics for LIQRAT, SIZE, MV/BV, VARCF, VARFCF, RSPREAD, CASHCC, VARCC, DEBTRAT, CF, FCF, and 1/ZSCORE for the pooled time-series cross-sectional data. Note that the statistics for SIZE are in millions of constant 1987 dollars. We use the logged value of this variable in the regressions. There are a possible 18,300 (915 firms $\times$ 20 years) firm-year observations for each variable. However, because of missing observations, all time-series cross-sectional variables have less than 18,300 firm-year observations. The smallest number of observations is for 1/ZSCORE, with 14,402 firm-year observations, or an average of 15.74 observations per firm.

TABLE 1

Descriptive Statistics of Variables Used in the Regression Analysis

Variable	Mean	Standard Deviation	First Quartile	Median	Third Quartile	Sample Size
LIQRAT	0.081	0.092	0.017	0.047	0.115	15240
SIZE ($ M)	2062.871	7291.887	78.390	311.279	1410.707	17626
MV/BV	1.389	1.503	0.961	1.141	1.493	17069
VARCF	0.051	0.045	0.026	0.041	0.064	17029
VARFCF	0.063	0.049	0.033	0.051	0.078	16995
RSPREAD	0.089	0.078	0.051	0.087	0.128	16731
CASHCC	67.262	67.479	31.206	61.639	99.008	17356
VARCC	28.749	57.307	10.560	16.875	27.294	17356
DEBTRAT	0.518	0.164	0.406	0.537	0.632	17024
CF	0.137	0.425	0.076	0.124	0.193	17348
FCF	0.046	0.498	0.022	0.059	0.102	16888
1/*Z*SCORE	0.421	0.738	0.230	0.334	0.510	14402

The variables are defined as follows: LIQRAT is the ratio of cash plus marketable securities to total assets; SIZE is the market value of the firm's assets in constant 1987 dollars using the producer price index deflator, where the market value of assets is estimated as the book value of assets plus the difference between the market and book values of equity; MV/BV is the ratio of the estimate of the market value of the firm's assets to the book value of its assets; VARCF is the ratio of the standard deviation of the first difference in earnings before interest, depreciation, and taxes minus nonoperating income to the average of total assets over the sample period; VARFCF is the ratio of the standard deviation of the first difference in earnings before interest, depreciation, and taxes minus nonoperating income and capital expenditures to the average of total assets over the sample period; RSPREAD is the difference between the return on assets and the return on short-term Treasury bills, where the return on assets is the ratio of earnings before interest, depreciation, and taxes minus nonoperating income to total assets; CASHCC is the sum of average inventory age and receivables collection period minus the average payment period for accounts payable; VARCC is the standard deviation of CASHCC; DEBTRAT is the ratio of total debt (long-term debt plus debt in current liabilities) to total assets; CF is the ratio of earnings before interest, depreciation, and taxes to sales; FCF is the ratio of earnings before interest, depreciation, and taxes minus capital expenditures to sales; and 1/*Z*SCORE is the inverse of Altman's *Z*SCORE. The descriptive statistics are for the pooled time-series cross-sectional data.

[22] Alternatively, we industry-adjust firm-year liquidity ratios by subtracting the industry median liquidity ratio. The results for this alternative measure of liquidity are similar to those using industry dummies and so are not reported.

The mean and median values for LIQRAT are 8% and 5%, respectively. Thus, for the typical sample firm, liquid assets are a non-trivial component of total assets. However, there is considerable variability of liquid asset holdings in the sample. In particular, LIQRAT ranges from a minimum of zero to a maximum of 40% of total assets.

Table 2 reports Pearson and Spearman-rank correlation coefficients for the pooled time-series cross-sectional data. All of the correlation coefficients between LIQRAT and the explanatory variables have the predicted signs and are statistically significant.

C. Determinants of Liquidity

Table 3 reports cross-sectional and pooled time-series cross-sectional regressions of LIQRAT on the various explanatory variables.[23] The first column of the table lists the independent variables, and the second column displays the predicted signs for the coefficient estimates. The t-statistics of the coefficient estimates in the cross-sectional regressions are computed using White's (1980) heteroskedasticity-consistent standard errors, and the t-statistics of the coefficient estimates in the pooled regressions are computed using robust standard errors with Newey and West (1987) heteroskedasticity and serial correlation corrections. All t-statistics are reported in parentheses below the parameter estimates. Models (4) and (6) include a set of industry dummy variables. For these models, the table reports an F-statistic for a test of the null hypothesis that all industry dummy variable coefficients are equal to zero.

The regressions provide reasonably strong support for the model's prediction of a positive relation between the cost of external financing and investment in liquidity. The coefficient estimates on the market-to-book ratio (MV/BV) are positive and significant in all of the regressions. In addition, the coefficient estimates on firm size (SIZE) are always negative, but are never significant. To gauge the economic significance of the influence of the market-to-book ratio on liquidity, consider the coefficient estimate on MV/BV in model (4). All else being equal, a one standard deviation increase in MV/BV increases LIQRAT by 14.8%.

Consistent with the model's cash flow uncertainty prediction, the coefficient estimates on VARCF and VARFCF are positive. However, only the coefficient estimates in the pooled regressions of model (3) using VARCF and model (5) using VARFCF are statistically significant. Observe that when industry dummy variables are included in the pooled regressions (models (4) and (6)), the coefficient estimates on VARCF and VARFCF are substantially reduced and are no longer significant. Thus, industry classification subsumes much of firm level cash flow uncertainty.

The coefficient estimates on the difference between the return on assets and the return on Treasury bills (RSPREAD) are significantly negative in all of the regressions. For example, the pooled regression's coefficient estimate of model (4)

[23]The cross-sectional regressions use the time-series averages for each firm's variables. Note that GLEI is not included in the cross-sectional regressions since it does not vary across firms. We also estimate fixed effects regressions (not reported), which are pooled regressions with firm-specific intercepts. The coefficient estimates of the fixed effects regressions are qualitatively similar to those reported Table 3.

TABLE 2

Correlation Matrix (Lower Triangle: Pearson / Upper Triangle: Spearman-Rank)

	1	2	3	4	5	6	7	8	9	10	11	12	13
1. LIQRAT		−0.137	0.220	0.239	0.169	−0.054	0.054	0.068	0.121	−0.425	−0.095	0.125	−0.471
		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
2. SIZE	−0.150		0.243	−0.483	−0.534	−0.032	−0.004	−0.253	−0.172	0.214	0.388	0.189	0.124
	(0.000)		(0.000)	(0.000)	(0.000)	(0.000)	(0.573)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
3. MV/BV	0.248	0.060		−0.023	−0.058	−0.202	0.057	−0.058	−0.011	−0.189	0.281	0.321	−0.530
	(0.000)	(0.000)		(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.116)	(0.000)	(0.000)	(0.000)	(0.000)
4. VARCF	0.244	−0.388	0.410		0.807	−0.129	0.003	0.307	0.398	−0.212	−0.326	−0.175	−0.192
	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	(0.665)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
5. VARFCF	0.215	−0.423	0.374	0.867		−0.110	0.005	0.118	0.287	−0.145	−0.222	−0.234	−0.074
	(0.000)	(0.000)	(0.000)	(0.000)		(0.000)	(0.457)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
6. RSPREAD	−0.098	0.009	−0.093	−0.177	−0.158		−0.111	−0.042	−0.212	−0.113	0.255	0.342	−0.152
	(0.000)	(0.211)	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
7. GLEI	0.047	−0.004	0.005	−0.000	0.002	−0.115		−0.008	0.002	−0.033	0.016	0.044	−0.016
	(0.000)	(0.591)	(0.474)	(0.986)	(0.831)	(0.000)		(0.299)	(0.784)	(0.000)	(0.026)	(0.000)	(0.041)
8. CASHCC	−0.017	−0.175	−0.039	0.105	−0.000	−0.070	−0.003		0.348	−0.259	−0.208	0.078	−0.119
	(0.026)	(0.000)	(0.000)	(0.000)	(0.999)	(0.000)	(0.676)		(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
9. VARCC	0.088	−0.102	0.051	0.137	0.172	−0.142	0.006	0.010		−0.035	−0.026	−0.013	0.127
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.441)	(0.182)		(0.000)	(0.000)	(0.073)	(0.000)
10. DEBTRAT	−0.432	0.246	−0.102	−0.154	−0.111	−0.103	−0.036	−0.176	−0.011		−0.054	−0.196	0.679
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.138)		(0.000)	(0.000)	(0.000)
11. CF	−0.103	0.121	0.017	−0.055	−0.042	0.125	−0.013	−0.116	−0.074	0.025		0.606	0.113
	(0.000)	(0.000)	(0.023)	(0.000)	(0.000)	(0.000)	(0.064)	(0.000)	(0.000)	(0.001)		(0.000)	(0.000)
12. FCF	−0.051	0.066	0.032	−0.039	−0.080	0.143	−0.010	−0.002	−0.128	−0.002	0.906		−0.236
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.175)	(0.745)	(0.000)	(0.788)	(0.000)		(0.000)
13. 1/*Z*SCORE	−0.134	0.036	−0.081	−0.046	0.019	−0.102	−0.008	−0.013	0.053	0.199	0.034	0.060	
	(0.000)	(0.000)	(0.000)	(0.000)	(0.016)	(0.000)	(0.332)	(0.096)	(0.000)	(0.000)	(0.000)	(0.000)	

The variables are defined as follows: LIQRAT is the ratio of cash plus marketable securities to total assets; SIZE is the natural logarithm of the market value of the firm's assets; MV/BV is the ratio of the market value of the firm's assets to the book value of its assets; VARCF is the ratio of the standard deviation of the first difference in earnings before interest, depreciation, and taxes minus nonoperating income to the average of total assets over the sample period; VARFCF is the ratio of the standard deviation of the first difference in earnings before interest, depreciation, and taxes minus nonoperating income and capital expenditures to the average of total assets over the sample period; RSPREAD is the difference between the return on assets and the return on short-term Treasury bills, where the return on assets is the ratio of earnings before interest, depreciation, and taxes minus nonoperating income to total assets; GLEI is the logarithmic growth rate in the annual index of leading economic indicators; CASHCC is the sum of average inventory age and receivables collection period minus the average payment period for accounts payable; VARCC is the standard deviation of CASHCC; DEBTRAT is the ratio of total debt to total assets; CF is the ratio of earnings before interest, depreciation and taxes to sales; FCF is the ratio of earnings before interest, depreciation, and taxes minus capital expenditures to sales; and 1/*Z*SCORE is the inverse of Altman's *Z*-score. Numbers in parentheses are *p*-values.

TABLE 3

Regressions of Liquidity Ratio on Explanatory Variables for 915 Firms during 1975–1994

Independent Variable	Predicted Sign	Cross-Sectional Regressions (1)	Cross-Sectional Regressions (2)	Pooled Times-Series Cross-Sectional Regressions (3)	(4)	(5)	(6)
Intercept		0.257 (14.83)***	0.252 (13.91)***	0.208 (23.88)***	0.234 (14.83)***	0.207 (22.57)***	0.236 (15.48)***
SIZE	−	−0.001 (−0.80)	−0.001 (−0.66)	−0.001 (−0.87)	−0.001 (−0.79)	−0.001 (−1.10)	−0.001 (−1.05)
MV/BV	+	0.013 (8.19)***	0.013 (8.41)***	0.009 (5.09)***	0.008 (6.05)***	0.009 (5.86)***	0.008 (6.76)***
VARCF	+	0.030 (0.47)		0.154 (2.87)***	0.046 (0.74)		
VARFCF	+		0.038 (0.63)			0.113 (2.49)**	0.019 (0.36)
RSPREAD	−	−0.337 (−6.60)***	−0.342 (−6.74)***	−0.124 (−8.35)***	−0.117 (−8.37)***	−0.128 (−8.65)***	−0.120 (−8.57)***
GLEI	+			0.078 (4.20)***	0.084 (4.63)***	0.078 (4.15)***	0.084 (4.63)***
CASHCC	−	−0.0002 (−3.39)***	−0.0001 (−2.56)***	−0.0001 (−5.63)***	−0.0002 (−7.12)***	−0.0001 (−4.62)***	−0.0002 (−6.60)***
VARCC	+	0.00004 (0.74)	0.00002 (0.40)	0.00006 (2.04)**	−0.00003 (−0.89)	0.00005 (1.61)	−0.00005 (−1.15)
DEBTRAT	−	−0.264 (−13.60)***	−0.261 (−13.43)***	−0.236 (−25.04)***	−0.216 (−22.45)***	−0.235 (−24.95)***	−0.214 (−22.11)***
CF	−	−0.019 (−22.61)***		−0.014 (−10.57)***	−0.013 (−10.19)***		
FCF	−		−0.015 (−10.60)***			−0.006 (−1.14)	−0.006 (−1.25)
1/*Z*SCORE	−	−0.024 (−3.12)***	−0.026 (−2.95)***	−0.005 (−2.80)***	−0.002 (−1.47)	−0.005 (−2.69)***	−0.002 (−1.16)
Industry *F*					24.07***		24.94***
Adjusted R^2		0.58	0.57	0.29	0.35	0.28	0.35
Regression *F*		139.25***	134.76***	575.05***	114.42***	550.29***	111.40***
No. of Obs.		915	913	13954	13954	13864	13864

The dependent variable LIQRAT is the ratio of cash plus marketable securities to total assets. The explanatory variables are defined as follows: SIZE is the natural logarithm of the market value of the firm's assets; MV/BV is the ratio of the market value of the firm's assets to the book value of its assets; VARCF is the ratio of the standard deviation of the first difference in earnings before interest, depreciation, and taxes minus nonoperating income to the average of total assets over the sample period; VARFCF is the ratio of the standard deviation of the first difference in earnings before interest, depreciation, and taxes minus nonoperating income and capital expenditures to the average of total assets over the sample period; RSPREAD is the difference between the return on assets and the return on short-term Treasury bills, where the return on assets is the ratio of earnings before interest, depreciation, and taxes minus nonoperating income to total assets; GLEI is the logarithmic growth rate in the annual index of leading economic indicators; CASHCC is the sum of average inventory age and receivables collection period minus the average payment period for accounts payable; VARCC is the standard deviation of CASHCC; DEBTRAT is the ratio of total debt to total assets; CF is the ratio of earnings before interest, depreciation, and taxes to sales; FCF is the ratio of earnings before interest, depreciation, and taxes minus capital expenditures to sales; and 1/*Z*SCORE is the inverse of Altman's *Z*-score. *t*-statistics (in parentheses) are calculated using White's (1980) heteroskedasticity-consistent standard errors (1) and (2) and Newey and West (1987) heteroskedasticity and serial correlation corrections (3)–(6). The industry *F*-statistic tests the null hypothesis that the industry dummy variable coefficients are equal to zero. Asterisks indicate significance at the 1% (***), 5% (**) levels in a two-tail test.

indicates that a one standard deviation increase in RSPREAD decreases LIQRAT by 11.1%. The negative relation between liquidity and return spread is consistent with the prediction that the higher the return on physical assets relative to that on liquid assets, the smaller the investment in liquid assets.

Finally, there is a significant positive relation between liquidity and the growth rate in the index of leading economic indicators (GLEI) in all of the regressions. Focusing again on the pooled regression of model (4), a one standard deviation increase in GLEI increases LIQRAT by 6.2%. The positive relation between LIQRAT and GLEI supports the model's prediction that firms build liquidity in anticipation of favorable economic conditions to minimize the risk of not having enough internal funds to finance profitable investment opportunities.

The signs of the coefficient estimates on the control variables are generally in line with predictions. In particular, the regressions provide strong support for a negative relation between liquidity and the cash conversion cycle (CASHCC), debt ratio (DEBTRAT), cash flow measures (CF and FCF), and probability of financial distress (1/ZSCORE). Furthermore, note that industry classification provides significant additional explanatory power; tests of the null hypothesis that industry dummy variable coefficients are equal to zero are easily rejected.

VI. Conclusions

In this paper, we develop a model of optimal corporate investment in liquid assets based on a cost-benefit tradeoff between the holding cost of liquid assets (a low return) and the benefit of minimizing the need to fund profitable future investment opportunities with costly external financing. The model predicts that the optimal investment in liquidity is increasing in the cost of external financing, the variance of future cash flows, and the return on future investment opportunities, while it is decreasing in the return differential between physical assets and liquid assets. We test these predictions using a large panel of U.S. industrial firms over the period from 1975 to 1994.

We find that firms with larger market-to-book ratios have significantly larger positions in liquid assets. In addition, firm size tends to be negatively related to liquidity. These results support the model's prediction of a positive relation between liquidity and the cost of external financing to the extent that the market-to-book ratio and firm size are reasonable proxies for the cost of external financing. We also find that firms with more volatile earnings and lower returns on physical assets relative to those on liquid assets tend to have significantly larger positions in liquid assets. Finally, we find a positive and significant relation between liquidity and measures of future economic conditions, which supports the model's prediction that firms build liquidity in anticipation of promising future investment opportunities.

Appendix A: Proof that $\widehat{\epsilon}_1$ and ϵ_1^* are Negative

To prove that $\epsilon_1^* = I_1^* - F(I_0) - L_0(1 + r_L) < 0$, we only need to show that $I_1^* - F(I_0) < 0$, since $L_0 \geq 0$. Analysis of the time zero problem will establish that $I_0 > I_1^*$ for $\xi > 0$. We can use that result and the mean-value theorem to prove that $\epsilon_1^* < 0$. The mean-value theorem establishes that if the function f is continuous on the closed interval $[a, b]$ and has a derivative on the open interval (a, b), then there exists at least one point c in the open interval (a, b) such that $f(b) = f(a) + f'(c)(b-a)$. Since $F(\cdot)$ satisfies the necessary continuity properties, we can apply the theorem for $a = 0$, $b = I_0$, and $c = I_1^*$, to determine that

$$\begin{aligned} I_1^* - F(I_0) &= I_1^* - \{F(0) + F'(I_1^*)(I_0 - 0)\} \\ &= I_1^* - F'(I_1^*) I_0 \\ &= I_1^* - (1+\rho) I_0 \\ &< 0, \end{aligned}$$

where the second equality follows from the property of the production function that $F(0) = 0$, the third equality follows from the condition that $F'(I_1^*) = (1+\rho)$, and the inequality follows from the result that $I_0 > I_1^*$ and the assumption that $\rho \geq 0$. Proof that $\widehat{\epsilon}_1 < 0$ is immediate since $\widehat{\epsilon}_1 < \epsilon_1^*$.

Appendix B: Derivation of the First-Order Conditions for the Time Zero Problem

From (9)–(12) and the solution to the time one problem, we may form the Lagrangean,

$$\begin{aligned} \text{(B-1)}\quad Q(I_0, L_0, \lambda_0) &= \left[\overline{X}_0 - I_0 - L_0\right] \\ &+ \frac{1}{(1+\rho)} \int_{\epsilon_1^*}^{\overline{\epsilon}_1} \{F(I_0) + \epsilon_1 + L_0(1+r_L) - I_1^*\} g(\epsilon_1) d\epsilon_1 \\ &+ \frac{1}{(1+\rho)^2} \int_{\underline{\epsilon}_1}^{\hat{\epsilon}_1} \int_{\epsilon_2^b}^{\overline{\epsilon}_2} \left\{F\left(\widehat{I}_1\right) + \epsilon_2 - (1+\rho+\pi) B_1\right\} g(\epsilon_2) g(\epsilon_1) d\epsilon_2 d\epsilon_1 \\ &+ \frac{1}{(1+\rho)^2} \int_{\hat{\epsilon}_1}^{\epsilon_1^*} \int_{\underline{\epsilon}_2}^{\overline{\epsilon}_2} \{F(F(I_0) + \epsilon_1 + L_0(1+r_L)) + \epsilon_2\} g(\epsilon_2) g(\epsilon_1) d\epsilon_2 d\epsilon_1 \\ &+ \frac{1}{(1+\rho)^2} \int_{\epsilon_1^*}^{\overline{\epsilon}_1} \int_{\underline{\epsilon}_2}^{\overline{\epsilon}_2} \{F(I_1^*) + \epsilon_2\} g(\epsilon_2) g(\epsilon_1) d\epsilon_2 d\epsilon_1 + \lambda_0 \left[\overline{X}_0 - I_0 - L_0\right]. \end{aligned}$$

Using the results that

$$\text{(B-2)}\quad \int_{\epsilon_2^b}^{\overline{\epsilon}_2} \left\{F\left(\widehat{I}_1\right) + \epsilon_2 - (1+\rho+\pi) B_1\right\} g(\epsilon_2) d\epsilon_2 = F\left(\widehat{I}_1\right) - B_1(1+\rho),$$

$$\text{(B-3)}\quad \text{and} \quad B_1 = \frac{\widehat{I}_1 - F(I_0) - \epsilon_1 - L_0(1+r_L)}{1-\xi},$$

and noting that $E[\epsilon_2] = 0$, we may rewrite (B-1) as

$$\text{(B-4)}\quad Q(I_0, L_0, \lambda_0) = [\overline{X}_0 - I_0 - L_0] + \frac{1}{(1+\rho)} \int_{\epsilon_1^*}^{\bar{\epsilon}_1} \{F(I_0) + \epsilon_1 + L_0(1+r_L) - I_1^*\}\, g(\epsilon_1) d\epsilon_1$$

$$+ \frac{1}{(1+\rho)^2} \int_{\epsilon_1^*}^{\bar{\epsilon}_1} F(I_1^*)\, g(\epsilon_1) d\epsilon_1$$

$$+ \frac{1}{(1+\rho)^2} \int_{\hat{\epsilon}_1}^{\epsilon_1^*} F(F(I_0) + \epsilon_1 + L_0(1+r_L)) g(\epsilon_1) d\epsilon_1$$

$$+ \frac{1}{(1+\rho)^2} \int_{\underline{\epsilon}_1}^{\hat{\epsilon}_1} F\left(\widehat{I}_1\right) g(\epsilon_1) d\epsilon_1$$

$$- \frac{1}{(1+\rho)} \int_{\underline{\epsilon}_1}^{\hat{\epsilon}_1} \frac{\left[\widehat{I}_1 - F(I_0) - \epsilon_1 - L_0(1+r_L)\right]}{1-\xi} g(\epsilon_1) d\epsilon_1$$

$$+ \lambda_0 \left[\overline{X}_0 - I_0 - L_0\right],$$

where $\hat{\epsilon}_1 \equiv \widehat{I}_1 - F(I_0) - L_0(1+r_L)$ and $\epsilon_1^* \equiv I_1^* - F(I_0) - L_0(1+r_L)$.

Using the restated Lagrangean in (B-4) and Leibnitz's rule, the first-order conditions for I_0 and L_0, respectively, are

$$\text{(B-5)}\quad \frac{\partial Q}{\partial I_0} = \frac{F'(I_0)}{1+\rho} Z - (1+\lambda_0) \leq 0,$$

$$\text{(B-6)}\quad \text{and} \quad \frac{\partial Q}{\partial L_0} = \frac{1+r_L}{1+\rho} Z - (1+\lambda_0) \leq 0,$$

$$\text{(B-7)}\quad \text{where} \quad Z = 1 + \int_{\hat{\epsilon}_1}^{\epsilon_1^*} \frac{F'(F(I_0) + \epsilon_1 + L_0(1+r_L)) - (1+\rho)}{(1+\rho)} g(\epsilon_1) d\epsilon_1$$

$$+ \frac{\xi}{1-\xi} \int_{\underline{\epsilon}_1}^{\hat{\epsilon}_1} g(\epsilon_1) d\epsilon_1.$$

Noting that $F'(\widehat{I}_1) = (1+\rho)/(1-\xi)$ and $\widehat{I}_1 = F(I_0) + \epsilon_1 + L_0(1+r_L) + B_1(1-\xi)$, we can rewrite the third term on the right-hand side of (B-7) as

$$\text{(B-8)}\quad \frac{\xi}{1-\xi} \int_{\underline{\epsilon}_1}^{\hat{\epsilon}_1} g(\epsilon_1) d\epsilon_1 = \int_{\underline{\epsilon}_1}^{\hat{\epsilon}_1} \frac{F'\left(\widehat{I}_1\right) - (1+\rho)}{(1+\rho)} g(\epsilon_1) d\epsilon_1$$

$$= \int_{\underline{\epsilon}_1}^{\hat{\epsilon}_1} \frac{F'(F(I_0) + \epsilon_1 + L_0(1+r_L) + B_1(1-\xi)) - (1+\rho)}{(1+\rho)} g(\epsilon_1) d\epsilon_1.$$

Combining (B-8) with the second term on the right-hand side of (B-7) allows us to write Z as given in equation (23) in the text. The complete set of Kuhn-Tucker conditions for the time zero optimization problem are given in equations (20)–(22) in the text.

Appendix C: Derivation of the Effect of an Increase in the Variance of ϵ_1 on L_0 and I_0

Let the time one production shock be specified as $\epsilon_1 = \sigma u_1$, where u_1 is distributed such that $E[u_1] = 0$ and $\mathrm{var}(u_1) = 1$. Performing a change of variables, we may rewrite Z in equation (B-7) as

$$\text{(C-1)} \qquad Z = 1 + \int_{\hat{\epsilon}_1/\sigma}^{\epsilon_1^*/\sigma} \frac{F'(F(I_0) + \sigma u_1 + L_0(1+r_L)) - (1+\rho)}{(1+\rho)} \widehat{g}(u_1) du_1 + \frac{\xi}{1-\xi} \int_{\underline{\epsilon}_1/\sigma}^{\hat{\epsilon}_1/\sigma} \widehat{g}(u_1) du_1,$$

where $\widehat{g}(u_1)$ is the p.d.f. of u_1. For $L_0 > 0$, we can, using the first-order condition for liquid assets in (21) and Z in (C-1), determine that

$$\text{(C-2)} \qquad \frac{dL_0}{d\sigma} = \frac{-\int_{\hat{\epsilon}_1/\sigma}^{\epsilon_1^*/\sigma} F''\left(F\left(\bar{I}_0\right) + \sigma u_1 + L_0(1+r_L)\right)(u_1)\widehat{g}(u_1)du_1}{(1+r_L)\int_{\hat{\epsilon}_1/\sigma}^{\epsilon_1^*/\sigma} F''\left(F\left(\bar{I}_0\right) + \sigma u_1 + L_0(1+r_L)\right)\widehat{g}(u_1)du_1},$$

which is positive, since both the numerator and denominator are negative. The numerator is negative since $\epsilon_1 < 0$ for $\epsilon_1 \in (\widehat{\epsilon}_1, \epsilon_1^*)$—and, therefore, $u_1 < 0$ for $u_1 \in (\widehat{\epsilon}_1/\sigma, \epsilon_1^*/\sigma)$—as proved in Appendix A. Similarly, for $L_0 = 0$, using the first-order condition for investment in production in (20) and Z in (C-1), we find that

$$\text{(C-3)} \qquad \frac{dI_0}{d\sigma} = \frac{-\int_{\hat{\epsilon}_1/\sigma}^{\epsilon_1^*/\sigma} F''\left(F\left(\bar{I}_0\right) + \sigma u_1 + L_0(1+r_L)\right)(u_1)\widehat{g}(u_1)du_1}{F''(I_0)Z^2 + F'(I_0)\int_{\hat{\epsilon}_1/\sigma}^{\epsilon_1^*/\sigma} F''\left(F\left(\bar{I}_0\right) + \sigma u_1 + L_0(1+r_L)\right)\widehat{g}(u_1)du_1},$$

which also is positive.

References

Altman, E. I. "Financial Ratios, Discriminant Analysis, and the Prediction of Corporate Bankruptcy." *Journal of Finance,* 23 (1968), 589–609.

_________________. *Corporate Financial Distress.* New York, NY: John Wiley & Sons (1993).

Ang, J. "The Corporate-Slack Controversy." *Advances in Working Capital Management,* 2 (1991), 3–14.

Barclay, M. J., and C. W. Smith, Jr. "On Financial Architecture: Leverage, Maturity, and Priority." *Journal of Applied Corporate Finance,* 8 (Winter 1996), 4–17.

Baskin, J. "Corporate Liquidity in Games of Monopoly Power." *Review of Economics and Statistics,* 64 (1987), 312–319.

Begley J.; J. Ming; and S. Watts. "Bankruptcy Classification Errors in the 1980s: An Empirical Analysis of Altman's and Ohlson's Models." *Review of Accounting Studies,* 1 (1996), 267–284.

Brealey A., and S. Myers. *Principles of Corporate Finance,* Fifth Edition. New York, NY: McGraw-Hill Book Co. (1996).

Business Week. "A Bittersweet Year for Corporate America." (March 27, 1995), 90–95.

Choe, H.; R. W. Masulis; and V. Nanda. "Common Stock Offerings across the Business Cycle." *Journal of Empirical Finance,* 1 (June 1993), 3–31.

Damodaran, A. *Corporate Finance: Theory and Practice.* New York, NY: John Wiley & Sons (1997).

Fazzari, S. M., and B. C. Petersen. "Working Capital and Fixed Investment: New Evidence on Financing Constraints." *Rand Journal of Economics,* 23 (Autumn 1993), 328–342.

Gertler, M., and R. G. Hubbard. "Financial Factors in Business Fluctuations." *In Financial Market Volatility—Causes, Consequences, and Policy Responses.* Kansas City, MO: Federal Reserve Bank of Kansas City (1988).

Huberman, G. "External Financing and Liquidity." *Journal of Finance,* 39 (July 1984), 895–908.

Ibbotson Associates. *Stocks, Bonds, Bills, and Inflation 1995 Yearbook.* Chicago, IL: Ibbotson Associates, Inc. (1995).

Jensen, M. C. "Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers." *American Economic Review,* 76 (May 1986), 323–329.

John, T. A. "Accounting Measures of Corporate Liquidity, Leverage, and Costs of Financial Distress." *Financial Management* (Autumn 1993), 91–100.

Kester, C. W. "Capital and Ownership Structure: A Comparison of United States and Japanese Manufacturing Corporations." *Financial Management,* 15 (Spring 1986), 5–16.

MacKie-Mason, J. K. "Do Taxes Affect Corporate Financing Decisions?" *Journal of Finance,* 45 (Dec. 1990), 1471–1493.

Martin, J. D., and G. E. Morgan. "Financial Planning where the Firm's Demand for Funds is Nonstationary and Stochastic." *Management Science,* 34 (Sept. 1988), 1054–1066.

Masulis, R. W., and B. Trueman. "Corporate Investment and Dividend Decisions under Differential Personal Taxation." *Journal of Financial and Quantitative Analysis,* 23 (Dec. 1988), 369–385.

Miller, M. H. "Behavioral Rationality in Finance: The Case of Dividends." *Journal of Business,* 59 (Oct. 1986), S451–S468.

Myers, S. C. "Determinants of Corporate Borrowing." *Journal of Financial Economics,* 5 (Nov. 1977), 147–175.

Myers, S. C., and N. S. Majluf. "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have." *Journal of Financial Economics,* 13 (June 1984), 187–221.

Myers, S. C., and R. G. Rajan. "The Paradox of Liquidity." Working Paper, Univ. of Chicago (April 1995).

Newey, W. K., and K. D. West. "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Matrix." *Econometrica,* 55 (1987), 703–708.

Shleifer, A., and R. Vishny. "Liquidation Values and Debt Capacity: A Market Equilibrium Approach." *Journal of Finance,* 47 (Sept. 1992), 1343–1366.

Smith, C. W. "Investment Banking and the Capital Acquisition Process." *Journal of Financial Economics,* 15 (Jan./Feb. 1986), 3–29.

Smith, C. W., and R. L. Watts. "The Investment Opportunity Set and Corporate Financing, Dividend, and Compensation Policies." *Journal of Financial Economics,* 32 (Dec. 1992), 263–292.

Stohs, M. H., and D. C. Mauer. "The Determinants of Corporate Debt Maturity Structure." *Journal of Business,* 69 (July 1996), 279–312.

White, H. "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity." *Econometrica,* 48 (May 1980), 817–838.

Whited, T. M. "Debt, Liquidity Constraints, and Corporate Investment: Evidence from Panel Data." *Journal of Finance,* 47 (Sept. 1992), 1425–1460.