Three-Period Model

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Introduction to the Model

To provide an economic intuition of the differential response of **cash holdings** to uncertainty and financial shocks, I present a properly augmented model in the spirit of Almeida, Campello, and Weisbach (2004).

It is a simple representation of a dynamic setting where a profit-maximizing firm have

- present and future investment opportunities
- current cash flow and external sources of finance might not be enough to fund all profitable projects

Main Features

- Model has three periods: 0, 1 and 2.
- There is one representative firm (or a continuum of it)
- Discount factor $\beta = 1$, but it can easily be relaxed
- In P0 firm can invest I_0 in a long-term project.
  - I_0 pays a deterministic return $g(I_0)$ in P2
- In P1 firm can invest I_1 in a short-term project.
  - I_1 pays a deterministic return $h(i_1)$ in P2
- Both $g(\cdot)$ and $h(\cdot)$ - display the following properties
  - $g'(\cdot)$ and $h'(\cdot)$ strictly positive
  - $g''(\cdot)$ and $h''(\cdot)$ strictly negative
  - $g'''(\cdot)$ and $h'''(\cdot)$ strictly positive

Period 0

- Firm enters the period with y_0 internal liquidity from past and current cash flows
- Firm chooses optimal level of investment I_0, cash holding C, and borrowing B_0
- Optimal choices are subject to nonnegative dividends constraint,

$$d_0 = y_0 + B_0 - I_0 - C \geq 0$$

and financial frictions since debt repayment in period 2 is

$$B_0(1 + r_0) \text{ where } r_0 = \alpha B_0$$

the economic intuition is that the larger the debt, the riskier the loan, the higher the rate.

Period 1

- Firm enters the period with $C + y_1$ internal liquidity where
  - C is optimal level of cash holding chosen in P0
  - $y_1 \sim F[\underline{y_1},\ \overline{y_1}] \geq 0$ is current cash flow
  - y_1 is unknown in P0 and drawn at the beginning of P1

- Firm chooses optimal schedules of both investment $I_1(c_1)$ and borrowing $B_1(c_1)$

- Optimal choices are subject to nonnegative dividends constraint,

$$d_1 = y_1 + B_1(c_1) - I_1(c_1) + C \geq 0$$

and financial frictions

$$B_1(1 + r_1) \quad \text{where} \quad r_1 = \alpha B_1$$

Period 2

- Firm receives deterministic returns $g(I_1)$ and $h(I_1(y_1))$
- Firm pays back $B_0(1 + r_0)$ and $B_1(y_1)(1 + r_1)$
- Dividends are defined as

$$d_2 = g(I_0) + h(I_1(y_1)) - B_0(1 + r_0) - B_1(y_1)(1 + r_1)$$

Firm's Problem

$$\max_{C, I_0, B_0, I_1(c_1), B_1(c_1)} d_0 + d_1 + d_2 \tag{1}$$

subject to

$$\begin{aligned}
d_0 &= y_0 + B_0 - I_0 - C \geq 0 \\
d_1 &= y_1 + B_1(y_1) - I_1(y_1) + C \geq 0 \\
d_2 &= g(I_0) + h(I_1(y_1)) - B_0(1 + r_0) - B_1(y_1)(1 + r_1) \qquad (2) \\
r_0 &= \alpha B_0 \\
r_1 &= \alpha B_1
\end{aligned}$$

Solution

I make the fair assumption that using only internal source of finance is not a profit maximizing solution. Mathematically,

$$g'(y_0) > 1 \quad \text{and} \quad h'(y_1) > 1$$

which means that the marginal return of investment is larger than the marginal cost of borrowing when debt is equal to zero.

This implies that $d_0 = d_1 = 0$, $B_0 > 0$, and $B_1 > 0$.

Thus, $I_0 = y_0 + B_0 - C$ and $I_1 = y_1 + B_1 + C$

Solution (cont.)

Problem can be rewritten as

$$\max_{B_0, B_1, C} \quad g(y_0 + B_0 - C) + \mathbb{E}\left[h(y_1 + B_1 + C)\right]$$
$$- B_0 - \alpha B_0^2 - B_1 - \alpha B_1^2 - y_0 - \mathbb{E}\, y_1$$

First Order Conditions imply

$$B_0 : \; g'(y_0 + B_0^* - C^*) = 1 + 2\alpha B_0^*$$

$$B_1 : \; \mathbb{E}\left[h'(y_1 + B_1^* + C^*)\right] = 1 + 2\alpha B_1^*$$

$$C : \; \mathbb{E}\left[h'(y_1 + B_1^* + C^*)\right] = g'(y_0 + B_0^* - C^*)$$

Solution (cont.)

FOC for C implies that

$$1 + 2\alpha B_0^* = 1 + 2\alpha B_1^* \quad \Rightarrow \quad B_0^* = B_1^*$$

since $Ey_1 = y_0$, this implies that

$$\mathbb{E}\left[h'(y_1 + B_1^*)\right] > g'(y_0 + B_0^*) \;\; \text{since} \;\; h''' > 0$$

which implies that in equilibrium $C^* > 0$.

Comparative Statics - Financial Shock

An unexpected financial shock implies that the cost of debt in period 0 increases of ε. First order conditions implies,

$$B_0 : \; g'(y_0 + B_0^{**} - C^{**}) = 1 + 2(\alpha + \varepsilon) B_0^{**}$$

$$B_1 : \; \mathbb{E}\left[h'(y_1 + B_1^{**} + C^{**}) \right] = 1 + 2\alpha B_1^{**}$$

$$C : \; \mathbb{E}\left[h'(y_1 + B_1^{**} + C^{**}) \right] = g'(y_0 + B_0^{**} - C^{**})$$

Comparative Statics - Financial Shock (cont.)

FOC for C implies that

$$1 + 2\alpha B_0^{**} + 2\varepsilon B_0^{**} = 1 + 2\alpha B_1^{**} \quad \Rightarrow \quad B_0^{**} < B_1^{**}$$

which implies that

$$\mathbb{E}\left[h'(y_1 + B_1^{**} + C^*)\right] < g'(y_0 + B_0^{**} - C^*)$$

which implies that in equilibrium $C^{**} < C^*$.

Comparative Statics - Uncertainty Shock

An uncertainty shock is defined as a mean preserving spread of the distribution of y_1. First order conditions implies,

$$B_0 : \; g'(y_0 + B_0^{***} - C^{***}) = 1 + 2\alpha B_0^{***}$$

$$B_1 : \; \mathbb{E}\left[h'(y_1 + B_1^{***} + C^{***})\right] = 1 + 2\alpha B_1^{***}$$

$$C : \; \mathbb{E}\left[h'(y_1 + B_1^{***} + C^{***})\right] = g'(y_0 + B_0^{***} - C^{***})$$

Comparative Statics - Uncertainty Shock (cont.)

FOC for C implies that

$$1 + 2\alpha B_0^{***} = 1 + 2\alpha B_1^{***} \quad \Rightarrow \quad B_0^{***} = B_1^{***}$$

since $Ey_1 = y_0$, this implies that

$$\mathbb{E}\left[h'(y_1 + B_1^{***} + C^*)\right] > g'(y_0 + B_0^{***} - C^*) \text{ since } \; h''' > 0$$

which implies that in equilibrium $C^{***} > C^*$.