How do investors perceive the risks from macroeconomic and financial uncertainty? Evidence from 19 option markets*

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Abstract

This paper studies the pricing of shocks to implied and realized volatility using options in 19 different markets, covering financials, metals, energies, and agricultural products. The markets are directly related to the state of the macroeconomy and financial markets, and investors can use the options to separately hedge shocks to real uncertainty and to the realization of volatility.

Historically, realized volatility has earned a robustly negative risk premium, indicating that high macroeconomic volatility is associated with high marginal utility. However, models are driven by forward-looking conditional variances, which can be proxied by implied volatility. Over the same period, the cost of hedging shocks to implied volatility in commodity markets has been negative: portfolios with returns that are positively correlated with shocks to implied volatility have earned positive average returns. That result is inconsistent with the view that periods of high uncertainty, as measured by forward-looking implied volatility, are "bad" states of the world with high marginal utility. The result is, however, potentially consistent with models in which uncertainty is high in periods of high innovation.

1 Introduction

The goal of this paper is to understand how investors perceive the risks associated with shocks to realized and implied volatility. If high uncertainty leads to declines in economic activity (e.g. Bloom (2009)), we would expect assets that hedge uncertainty shocks (i.e. shocks to implied volatility) to earn negative average returns. On the other hand, uncertainty might be high in periods of high innovation and growth (Pastor and Veronesi (2009)), which could cause uncertainty shocks to earn positive premia.

While there is a large literature that estimates the risk premium for S&P 500 uncertainty, recent evidence suggests there are multiple types of uncertainty shocks that can hit the economy (Ludvigson, Ma, and Ng (2015)). S&P 500 uncertainty is related to conditions in the financial sector, but it is possible that the driving force in the economy is actually uncertainty about the

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macroeconomy. This paper contributes to the literature by estimating risk premia associated with uncertainty in 19 different markets – including markets for many real assets like oil and gold – which allows us to quantify the premia associated with not just financial uncertainty, but also uncertainty about the macroeconomy and goods prices.

Our empirical analysis yields three key findings. First, for markets associated with macroeconomic uncertainty – futures with nonfinancial underlyings – shocks to uncertainty carry a positive risk premium. That fact implies that marginal utility covaries negatively with uncertainty in those markets: uncertainty is high in good times. Second, for financial underlyings, uncertainty has a risk premium that is not significantly different from zero, indicating that it has no average correlation with marginal utility. Third, for both financial and nonfinancial underlyings, realized volatility – a measure of the magnitude of realized movements in underlying prices – carries a negative premium. So while forward-looking uncertainty carries a positive or zero premium, surprise jumps in prices robustly earn negative premia.

The latter result is commonly referred to as the variance risk premium, and has been widely studied in the past literature. The observed negative variance risk premium in the data means that investors are willing to pay to hedge realized volatility shocks, and is often interpreted to imply that uncertainty has negative effects on the economy. Berger, Dew-Becker, and Giglio (2017) argue, though, that the negative variance risk premium actually just shows that investors are willing to pay to hedge extreme realizations of shocks, since that is what realized volatility measures. Uncertainty, though, is fundamentally forward-looking: it can be thought of as a conditional variance, where as realized volatility is a (backward-looking) sample variance.

The correct way to study how uncertainty shocks are perceived by investors is therefore to examine portfolios that hedge *implied volatility* shocks, since implied volatility reflects the uncertainty investors face about future shocks. We use the term structure of option market returns to construct portfolios that give pure exposures to either realized or implied volatility and then examine their relative returns. A similar decomposition has been explored in the existing literature using S&P 500 options. A number of papers find evidence that while realized volatility in the S&P 500 carries a large negative premium, shocks to implied volatility do not, suggesting that uncertainty about the future might not be an important driver of marginal utility.¹

The contribution of this paper is to extend our knowledge of the pricing of implied and realized volatility to a broader array of underlying variables beyond the S&P 500. Rather than focusing just on a single market, we draw from as many markets as possible. In our most general results, we have information on volatility premia in 19 different markets spanning the S&P 500, Treasury bonds, currencies, metals, energies, and agricultural products. We also have data on option prices at multiple maturities for each market, ranging from one to at least five months. Having multiple maturities allows us to distinguish the premia associated with the realization of volatility from those associated with shocks to uncertainty (implied volatility).

¹See Egloff, Leippold, and Wu (2010), Dew-Becker et al. (2017), Van Binsbergen and Koijen (2017), Andries et al. (2015), and Ait-Sahalia, Karaman, and Mancini (2015).

The first step in our analysis is documenting a strong relationship between the implied volatility in the 19 options markets and measures of aggregate uncertainty. In particular, we examine correlations between the implied volatilities for each market and the different measures of uncertainty developed by Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (LMN; 2017). Perhaps unsurprisingly, implied volatility for our financial underlyings, the S&P 500 and Treasury bonds in particular, is primarily associated with the LMN financial uncertainty index. At the same time, implied volatility for our nonfinancial underlyings is much more strongly associated with uncertainty about the real economy and goods prices. This confirms that hedging shocks to implied volatility in these markets represents a good way to hedge aggregate uncertainty shocks. So an important contribution of our work is to study implied volatilities that reflect uncertainty about the real economy rather than financial markets, thus helping to bridge a gap between theories of macroeconomic uncertainty and the real-world data.

We next examine how shocks to realized and implied volatility are priced across the various futures markets using two methods. First, drawing on Cremers, Halling, and Weinbaum (2015), a simple expansion of option returns in the Black–Scholes (1973) model yields a pair of portfolios that, in theory, yield pure exposure to realized and implied volatility. The portfolio that gives exposure to realized volatility is long short-term straddles and short an equal amount of long-term straddles. The short position hedges the exposure of the straddles to implied volatility shocks, leading to pure, and symmetrical, exposure to price changes. Under the Black–Scholes model, the simplest way to obtain pure exposure to implied volatility is to buy a straddle with the longest possible maturity. In our case, that is a five month straddle. Residual exposure to price movements is then hedged by shorting a small amount of one-month straddles.

We show that the Black–Scholes predictions for the loadings of the straddle returns on realized and implied volatility shocks are very good approximations in the data. Nevertheless, to avoid any reliance on the predictions of the Black-Scholes model, we also estimate linear factor models in which realized and implied volatility are priced factors. The results are consistent across the two methods.

For the S&P 500, we replicate the result from past work that shocks to realized volatility earn large negative risk premia (the negative variance risk premium result) while shocks to implied volatility earn premia near zero. An alternative way of stating the result is to say that long-term straddles for the S&P 500 earn average returns close to zero, while short-term straddles earn highly negative average returns. Similar results are obtained for the other financial underlyings.

For nonfinancial underlyings – those whose implied volatilities are more strongly associated with the real economy, such as energies, metals, and agricultural products – realized volatility continues to earn a negative premium on average, but implied volatility actually earns a *positive* premium. In other words, over our data sample, an investor who bought portfolios of straddles that yielded exposure to implied volatility – i.e. insurance against increases in uncertainty – earned significantly positive average returns. As before, another way to describe the result is to say that while short-term straddles for nonfinancials earn negative average returns, long-term straddles actually tend

to earn *positive* average returns.

That result is obviously difficult to reconcile with the view that innovations in economic uncertainty are contractionary. Long-term straddles, and our implied volatility portfolios, provide insurance against periods of high uncertainty. If increases in uncertainty were viewed as "bad" in the sense of raising marginal utility, then we would find a negative premium on implied volatility (investors are willing to accept negative average returns on assets that are hedges). Instead, our results imply that investors have historically viewed periods of high uncertainty and implied volatility as being good, in the sense that they are associated with low marginal utility.

Our work is related to three main strands of literature. The first studies the relationship between uncertainty and the macroeconomy. There are numerous channels that have been proposed through which uncertainty about various aspects of the aggregate economy may have real effects. There is a related literature that tries to measure whether uncertainty does in fact have contractionary effects.² This paper builds on that work by providing measures of risk premia that help indicate how investors perceive the effects of aggregate uncertainty shocks. The next section discusses the theoretical background of our analysis in much greater detail.

The second literature that we build on estimates the pricing of volatility risk in financial markets. As discussed above, that literature primarily studies S&P 500. There are various papers that have studied specific markets, such as individual equities (e.g. Bakshi, Kapadia, and Madan (2003)) or Treasury bonds (Mueller, Vedolin, and Yen (2017)). Perhaps the closest paper to our work in this area is Prokopczuk and Simen (2014), which examines the variance risk premium across many of the same markets that we study (see also Trolle and Schwartz (2010), who examine the variance risk premium for energy futures). Our additional contribution beyond that work is to examine multiple maturities in each market, which is crucial in order to isolate the premium on implied volatility as opposed to just the (realized) variance risk premium. We also provide evidence on the relationship between implied volatility and the macroeconomy.

Finally, our work is related to past research into factor models and risk premia in option markets. Most importantly, Jones (2006) and Constantinides, Jackwerth, and Savov (2013) estimate factor models highly similar to the specifications that we examine here.

The remainder of the paper is organized as follows. Section 1.1 provides a review of the theoretical background. Section 2 describes the data and its basic characteristics. Sections 3, 4, and 5 study the pricing of realized and implied volatility using straddle returns and report our main results on risk premia. Finally, section 6 concludes.

1.1 Theoretical background

Theoretical research on the effects and pricing of volatility, both realized and implied, can be divided into two classes: work in which consumption, and hence marginal utility, is taken as endogenous

²Recent examples include Berger, Dew-Becker, and Giglio (2017), Jurado, Ludvigson, and Ng (2015), Ludvigson, Ma, and Ng (2015), Baker, Bloom, and Davis (2015), and Alexopoulos and Cohen (2009), among many others.

and potentially driven by volatility, and models of endowment economies with exogenously specified consumption and volatility.

The first class of papers studies the effects of uncertainty shocks on the macroeconomy, where macroeconomic variables respond to uncertainty shocks. Those studies typically posit some exogenous uncertainty process, which is represented by variation in the conditional variance of fundamental shocks over time. The literature studies various mechanisms through which variation in uncertainty could have real effects on the economy, such as a Keynesian demand channel (Basu and Bundick (2017), real options effects on investment (Bloom (2009), Bloom et al. (2017)), effects on labor search (Leduc and Liu (2015)), or through financial frictions and credit spreads (Gourio (2013)). Importantly, these models do not generate a uniform prediction that uncertainty shocks are necessarily contractionary. While there are contractionary forces, such as wait-and see effects and Keynesian demand channels, there are also forces through which uncertainty can be expansionary, including precautionary saving and the Oi–Hartmann–Abel effect that is extensively discussed by Bloom et al. (2017) (see also Gilchrist and Williams (2005))

In addition to the fact that models typically find ambiguous effects of volatility on real activity, there is also a theoretical literature that looks at the reverse channel – how changes in the economy can lead to changes in volatility. Pastor and Veronesi (2009) study how technological revolutions – associated with high growth – can generate high asset price volatility (see also Garleanu, Panageas, and Yu (2012) and Kung and Schmid (2015)). Jones et al. (2005) argue for similar effects in a model with endogenous growth.

Overall, then, theory is ambiguous on the correlation of innovations in uncertainty with real activity. An empirical estimate of whether uncertainty shocks – which we will measure by changes in option implied volatility – are associated with contractions (or with high marginal utility states) can help distinguish among different theories. Models in which increases in conditional variances are contractionary predict that implied volatility should carry a negative premium, while models in which technological progress endogenously causes volatility imply a positive premium.

Since the key driver of the models discussed so far is variation in conditional variances, the related empirical work tests whether increases in uncertainty, often measured by S&P 500 option-implied volatility (the VIX), cause declines in real activity (e.g. Bloom (2009) and Basu and Bundick (2017)). An important concern with such work, though, is that the S&P 500 volatility only measures one type of uncertainty, which may not be the relevant type that drives the real economy. Ludvigson, Ma, and Ng (2015), building on Jurado, Ludvigson, and Ng (2015) treat macro and financial uncertainty as two separate concepts, and show, using a VAR analysis, that they have different relationships with subsequent economic activity. It is an important question, then, how different measures of uncertainty are priced in option markets, instead of just S&P 500 uncertainty.

The second class of theoretical studies focuses on endowment economies in which consumption is taken as exogenous. This literature asks what combination of preferences and endowment dynamics can rationalize observed asset prices. These models have been augmented with jumps, e.g. by Naik and Lee (1990), Drechsler and Yaron (2011), Wachter (2013) and Schreindorfer (2016), to help match the behavior of asset prices. Those models have different predictions for the relative pricing of implied volatility versus jumps or realized volatility depending on preferences. Dew-Becker et al. (2017) show that in standard models with Epstein–Zin (1991) preferences, the pricing of realized and implied volatility tends to be linked, in the sense that they either both receive large or small negative premia. That link can be broken when agents are given some sort of explicit aversion to jumps or crashes, e.g. Bates (2008), Liu, Pan, and Wang (2004), and Schreindorfer (2016). Cremers, Halling and Weinbaum (2015) test these models by examining the pricing of realized and implied volatility for the S&P 500. There is no work, though, that examines the relative pricing of realized and implied volatility in contexts outside the S&P 500.

2 Basic characteristics of option and futures data

This section describes our data sources, examines the basic characteristics of the option returns, their liquidity, and their link to macroeconomic uncertainty.

2.1 Data sources

We obtain data on prices of options and their underlying futures from the end-of-day database from the CME group. The data includes daily closing settlement prices of forwards and options across all tradable maturities and strikes. It also reports daily volume and open interest, which we use to filter the observations (see appendix A.3). The CME data includes information from a number of different individual exchanges, including the Chicago Mercantile Exchange, the Kansas City and Chicago Boards of Trade, and the New York Mercantile and Commodity Exchanges.

The data has been collected in ways that has clearly changed over time, meaning that it requires a large amount of cleaning by hand. For example, the prices of the underlying futures are quoted in a variety of ways – pennies, eighths, sixteenths, etc. – and there are frequent decimal shifts. As a threshold set of filters, we study only observations of option closing prices from days on which volume and open interest are both positive, maturity is greater than nine days, and prices are at least five ticks above zero. There are also numerous extreme outliers in the data. When options are very far from maturity, for example, we frequently observe implausible implied volatilities. We implement a range of systematic filters to eliminate questionable data. The filters are described in the appendix and code is available on request.

Table 1 lists the set of underlyings that we study. We limit our attention to those contracts for which we have at least 15 years of data and maturities for options extending to at least six months. This leaves us with 14 commodity and 5 financial underlyings.

2.2 Behavior of realized and implied volatility

The major distinction that we draw in this paper is between realized and implied volatility (RV and IV, respectively). We begin by analyzing how they relate to each other and to measures of volatility in the macroeconomy.

By realized volatility we mean the actual squared changes in a time series over some period. For example, daily realized volatility for the S&P 500 for a particular month is the sum of daily squared returns on the index in that month. We calculate daily realized volatility on all our other underlyings in the same way, using the squared futures returns. Realized volatility is a sample variance. To an investor, it is a backward-looking measure: it describes how extreme returns have been in the recent past. While past volatility can obviously be informative about volatility in the future, realized volatility itself does not literally measure forward-looking risk or uncertainty.

For all our underlyings we calculate implied volatility using the Black (1976) model (which corresponds to a special case of the Black–Scholes (1973) model in which the dividend is equal to the interest rate). We construct at-the-money Black IVs using the observed option prices nearest to the current underlying futures price.³ Implied volatility is fundamentally forward-looking. It is a measure, based on prices, of a conditional variance – specifically, the conditional variance of futures prices at maturity under the pricing measure (or marginal utility-adjusted probability measure).

Tables A.1 and A.2 report pairwise correlations of realized and implied volatility across the 19 underlyings. The various markets are sorted into related categories, with the result that the largest correlations are generally along the main diagonal. Shading denotes the degree of correlation, with darker cells representing greater correlation. The largest correlations in implied volatility are among similar underlyings – crude and heating oil, the agricultural products, gold and silver, and the British Pound and Swiss Franc. Correlations outside those groups are notably smaller.

The correlations are also generally smaller for realized than implied volatility. That result is not surprising when realized volatility is interpreted as a sample statistic that is, in some sense, subject to measurement variation. That is, given a conditional variance at the beginning of a particular month, there will generally be a divergence between conditional and the realized sample variance due simply to random variation. So even if all markets shared the same conditional variance process (i.e. they shared the same implied volatility), we would not necessarily expect that they would have the same realized variances.

While there is substantial commonality across the markets, tables A.1 and A.2 show that there is also substantial independent variation – and hence independent information – in them. For IV, five principal components are needed to explain 75 percent of the variation across the underlyings and nine are needed to explain 90 percent of the variation. For realized volatility, there is less commonality: 7 and 12 principal components are now needed to explain 75 and 90 percent of

³The majority of the options that we study have American exercise, while the Black model technically refers to European options. We examine IVs calculated assuming both exercise styles (we calculate American IVs using a binomial tree) and obtain nearly identical results. Since there are no dividends on futures contracts, early exercise is only rarely optimal for the options we study.

the variation, respectively. This finding is typical, as realized volatility is driven by the largest movements in returns, which appear to be more idiosyncratic. To give further context, note that these principal components decompositions imply far less common variation than is observed in the term structure of bond returns, where three principal components explain over 99 percent of the total variation (see for example Joslin et al. (2014)).

2.3 Relationship of volatility with macroeconomic uncertainty

More important than the commonality of RV and IV across the underlyings is how they relate to the aggregate economy. The most direct channel through which volatility risk would be priced is if it is correlated with consumption uncertainty. We study the relationship of our RV and IV measures with the measure of aggregate uncertainty developed in Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (2017). Those two papers measure uncertainty in two steps. On a data set of 280 individual macro and financial time series, they first construct forecast errors from an AR(4) model augmented with principal components of the time series. We treat the squared forecast errors from this step as representing realized volatility for the macroeconomy. Next, they estimate for each individual data series a time series of uncertainty using a two-sided filter applied to the squared forecast errors. Finally, the aggregate uncertainty series are constructed as the first principal component of the cross-section of the uncertainty series.

LMN divide the 280 indicators into two sets, corresponding to macroeconomic and financial time series. We further subdivide the macro set into indicators measuring prices and indicators measuring real quantities. As in LMN, we construct macro, financial, real, and price uncertainty as the first principal component of the uncertainty series in each category.

To explore the relationship between the option-implied volatilities and the LMN measures, table 2 reports results from two regressions,

$$\frac{IV_{i,t}}{SD(IV_{i,t})} = a_{1,i} + b_{1,i}U_t^{\text{Financial}} + b_{2,i}U_t^{\text{Macro}} + \varepsilon_{1,i,t}$$
(1)

$$\frac{IV_{i,t}}{SD(IV_{i,t})} = a_{1,i} + b_{1,i}U_t^{\text{Financial}} + b_{2,i}U_t^{\text{Macro}} + \varepsilon_{1,i,t}$$

$$\frac{IV_{i,t}}{SD(IV_{i,t})} = a_{2,i} + b_{3,i}U_t^{\text{Financial}} + b_{4,i}U_t^{\text{Real}} + b_{5,i}U_t^{\text{Price}} + \varepsilon_{2,i,t}$$
(2)

where $IV_{i,t}$ denotes at-the-money implied volatility for underlying i averaged over month t, $SD(IV_{i,t})$ is the sample standard deviation of $IV_{i,t}$, and the various U_t series are the LMN uncertainty series. The uncertainty series all have unit standard deviations by construction, and we normalize the implied volatilities here to also have unit standard deviations for the regressions (1–2). We average the implied volatilities within each month because the uncertainty series are constructed from a mix of stock and flow variables. Standard errors are computed using the Newey-West (1987) method with a lag window of 6 months, and indicate in the table coefficients that are significant at the five-percent level by shading them in blue. Since all the variables in the regressions have unit standard deviations, the b coefficients can be interpreted as partial standard deviations – $b_{1,i}$ is the number of standard deviations that $IV_{i,t}$ rises for a unit standard deviation increase in $U_t^{\text{Financial}}$,

etc.

For the S&P 500 and US Treasury bonds, implied volatilities are strongly related to financial uncertainty. That result is not surprising since measures of aggregate stock pries and interest rates (though not implied volatilities themselves) are included in LMN's set of financial indicators. The results for the currencies are mixed: the British Pound is more correlated with macro uncertainty, while the Yen is more correlated with financial uncertainty, but in both cases the R²s are relatively small.

Among the nonfinancial underlyings, the loadings almost entirely favor macro uncertainty — in 12 of 14 cases, the coefficient on macro uncertainty is larger than that on financial uncertainty (including for gold and silver which could plausibly be thought of as closer to financial assets). Moreover, the coefficients are generally economically large. The average coefficient on macro uncertainty among the nonfinancial underlyings is 0.32. The coefficients are relatively larger for industrial products like energies and metals — all above 0.4 except natural gas; they are somewhat smaller for the agricultural products, averaging 0.23. More concretely, a one-standard-deviation increase in macro uncertainty is associated with a 0.6 standard-deviation increase in copper implied volatility and a 0.44 standard-deviation increase in crude oil volatility, implying that options on those two underlyings can provide substantial protection against increases in macro uncertainty.

The average R^2 across the first set of regressions is 0.26, indicating that the LMN series account for a substantial fraction of the variation in the implied volatilities. The last row of the table reports results from a regression of the average of all 19 implied volatilities on the LMN series. In that case, the coefficients on the two LMN series are similar, both close to 0.4, and the R^2 rises to 0.54. The common component of the implied volatilities is thus strongly related to the LMN series, while the implied volatilities in the individual markets contain notable idiosyncratic variation.

To further decompose those results, the right-hand side of table 2 reports results from the regression (2) that replaces the macro uncertainty time series with its real and price components. The nonfinancial underlyings are nearly evenly split, with six having larger loadings on the price component and eight having larger loadings on the real component. The energies, perhaps naturally, are more associated with price uncertainty, with coefficients near 0.5. Metals and agricultural products, on the other hand, are more associated with macro uncertainty, with coefficients near 0.4.

Table 3 reports results analogous to those in table 2, but replacing implied volatility on the left-hand side with normalized realized volatility in each month. On the right hand side, we replace the uncertainty indexes, which are principal components of the smoothed squared forecast errors of the aggregate series, with realized volatility indexes calculated as principal components of the raw squared forecast errors. So the aggregate realized volatility series essentially measure the average magnitude of squared forecast errors of macro and financial time series in each month.

The results in table 3 are qualitatively similar to those in table 2. We again find that the S&P 500 and the Treasuries are primarily related to financial indicators, while the nonfinancial underlyings are more related to macro volatility. The main difference in this case is that the relationship with

the macroeconomy now seems to come primarily through price volatility, as opposed to volatility in the real variables. The R^2 s for the individual underlyings are smaller on average by half, at 0.15, but the R^2 for the average of realized volatility across the markets remains high, at 0.42.

Overall, tables 2 and 3 show that there is a strong relationship between implied and realized volatility measured in futures markets and what is constructed from aggregate time series, both in terms of statistical and economic significance. Past work has focused on S&P 500 implied and realized volatility, which we find (consistent with arguments in LMN) primarily measures financial uncertainty.

The tables show that the wide range of underlyings that we study is valuable for two key reasons. First, they provide exposure to more than just financial uncertainty – they allow investors to obtain exposure to macro uncertainty, both in its real and price components. Second, by averaging across many underlyings, idiosyncratic variation in implied volatility (e.g. due to weather shocks) is diversified away, leaving relatively pure exposures to aggregate uncertainty series with high R²s. We conclude from tables 2 and 3 that returns on the options we study can effectively reveal how marginal utility covaries with aggregate uncertainty.

2.4 Calculation of straddle returns

We study two-week returns on straddles with maturities between one and six months. Past work on option returns and volatility risk premia has examined returns at frequencies of a day (e.g. Andries et al. (2017)), a week (Coval and Shumway (2001), who use both weekly and daily returns), a month (Dew-Becker et al. (2017); Constantinides, Jackwerth, and Savov (2013)), and holding the options to maturity (Bakshi and Kapadia (2003)). The precision of our estimates of the riskiness of the straddles (both variances and factor loadings) is, all else equal, expected to be higher when we use shorter windows. And the accuracy of the expansion for the risk of the portfolios in (3) is only locally valid, meaning that we should examine the shortest holding period possible. On the other hand, shorter windows cause any measurement error in option prices to have larger effects. We choose two-week windows because they are short enough to allow us to still calculate returns on relatively short-maturity options.⁴ The appendix reports results using other time windows and shows that they are similar.⁵

To account for the fact that the returns are overlapping (since we have daily data), we use Hansen–Hodrick (1980) standard errors with a lag window of 14 days in all time-series calculations reported below, unless otherwise specified.

Table 1 reports basic descriptive statistics for the straddle returns in the 19 markets. The length

⁴Some of the existing literature, beginning with Bakshi and Kapadia (2003), examines delta-hedged returns. In our main specifications we use unhedged returns, but we have replicated the results using delta-hedging as well, and report the main results in the appendix.

⁵Examining returns to maturity in our setting can be particularly dangerous because we examine relatively long maturities. Bakshi and Kapadia (2003), study returns to maturity, but only on options with maturities shorter than 60 days. Even with delta hedging, the higher-order risk exposures (gamma, vega, and others) of the straddles change substantially as the spot changes. Higher-frequency returns avoid that problem.

of the time series for each contract covered by the data ranges between 18 and 31 years, with most of the contracts having at least 25 years of data available. The standard deviations and skewness are surprisingly consistent across the various markets. Standard deviations tend to cluster around 140 percent per year for the one-month maturity, and fall to 50 percent per year at six months. Skewness is, similarly, consistent across most markets, ranging typically between 2 and 3 across all maturities. Positive skewness is to be expected since straddle returns have no upper bound, while one can lose no more than 100 percent. Straddles on crude oil and S&P 500 futures have skewness that are much larger than for other underlyings, indicating much larger jumps in the underlyings.

2.5 Measures of liquidity

We measure liquidity across maturities using two methods. First, we estimate the standard Roll (1984) effective spread. The Roll model assumes that there is an unobservable midquote that follows a random walk in logs. Observed prices are then assumed to have equal probability of being from a buy or sell order, with buys and sells occurring at prices equal distances above and below the midquote. Bid-ask bounce then induces negative autocorrelation in returns. Under the Roll model, the half-spread, in percentage points, can be estimated as the square root of minus the first autocovariance of returns. When the first sample autocovariance is zero, we simply report a half-spread of zero.

As discussed below, our empirical analysis focuses on returns of straddle portfolios (long a put and a call). We calculate the autocovariance of returns on all the straddles in our data, and then average it within monthly maturity bins. For this exercise we limit ourselves to contracts whose strikes are within 0.5 standard deviations of the spot measured by the at-the-money implied volatility (i.e. where $|\log{(K/S)}|/(\sigma/\sqrt{T}) < 0.5$, where K is the strike, S the spot, T the time to maturity, and σ the at-the-money implied volatility).

The top panel of figure 1 plots the effective half-spreads at maturities of 1, 3, and 5 months for the 19 contracts that we study. At the one-month maturity, the half-spreads are all around 3 percent. Interestingly, the effective spreads actually decline at longer maturities, indicating that there is less observed negative autocovariance in returns for options at those maturities. For the three- and five-month options, the spreads are smaller by about half, averaging 1 to 1.5 percent.

As a second measure of liquidity, we obtained posted bid/ask spreads for the options closest to the month on Friday, 8/4/2017 for our 19 contracts at maturities of 1, 4, and 7 months. We report the results in the bottom panel. The bid/ask spreads have two major drawbacks: we have them only on a single day, and they do not measure the behavior of prices that investors actually pay. It is well known that the prices at which investors trade often lie in a narrower range than the bid/ask spread (Petersen and Fialkowski (1994)). Nevertheless, the observed bid/ask spreads are still useful as a check on our results for effective spreads. For the majority of the options, the spreads are less than 3 percent. More importantly, though, we see that across nearly all the contracts, the spreads actually decline with maturity. That said, for some of the contracts, there were no available bids

or asks at the 4- and 7-month maturities. Those contracts also had zero volume, and thus would be dropped from our analysis.

To give further context to the magnitude of the posted bid/ask spreads, we note that for 10 of the 19 contracts, the one-month spreads are nearly indistinguishable from that for the S&P 500, which is typically viewed as a highly liquid market.⁶ For crude oil, which we study in detail below, the spreads at all three maturities are essentially identical to those for the S&P 500.

Figure 1 gives us two important results. First, it shows that the liquidity of the straddles is reasonably high, in the sense that effective and posted spreads are both relatively narrow in absolute terms for most of the contracts that we study. Second, liquidity does not appear to deteriorate as the maturity of the options grows, and in fact in many cases we see improvements with increasing maturities.

3 Mean returns on straddles

Having established that realized and implied volatility in futures markets is related to macroeconomic volatility and uncertainty, we now proceed to construct portfolios of straddles that yield exposure to realized and implied volatility in order to measure the associated risk premia.

3.1 Risk exposures

Since the final payoff of a straddle depends on the magnitude of returns of the underlying at maturity, its price at any time depends on the dispersion in the conditional distribution of the futures price at maturity. That is, the price of a straddle depends on uncertainty. If a straddle is sold before maturity, then, the return partly depends on how that uncertainty changes during the holding period. For example, one might buy a six-month straddle and hold it for a single day before selling. In that case, the primary determinant of the change in the price of the straddle will be the change in other investors' uncertainty. When uncertainty rises, the price of the straddle will also rise, giving the portfolio a positive return. Straddles are therefore interesting for our purposes because they give exposure both to realized and implied volatility.

There are various ways to formalize that intuition; the simplest method, used in Coval and Shumway (2001), Bakshi and Kapadia (2003), and Cremers, Halling, and Weinbaum (2015), is to examine the exposures based on the derivative of the price of a straddle in the Black–Scholes model. Specifically, the appendix shows that the return can be approximated using a local expansion as

$$r_{n,t} \approx 0 \times f_t + \frac{1}{2} \frac{1}{n} \left(\frac{f_t}{\sigma_{t-1}} \right)^2 + \frac{\Delta \sigma_t}{\sigma_{t-1}}$$
 (3)

where $r_{n,t}$ is the return on date t of a straddle with maturity n, f_t is the return on the underlying future, σ_t is the volatility of the underlying, and Δ is the first-difference operator.⁷

⁶See Bondarenko (2014) for a discussion of the liquidity of S&P 500 options at the CME and CBOE.

⁷We ignore here the fact that options at different maturities have different underlying futures contracts. If that

According to the first term, straddles all have zero local exposure to the spot return, since their payoff is a symmetrical function of the underlying return. The second term shows that straddles are exposed to higher-order components of returns. The exposure to squared returns on the underlying (scaled by volatility) is inversely proportional to time to maturity. That decline comes partly from the fact that the price of a straddle increases with time to maturity and partly from the fact that a given movement in the underlying is less beneficial when the time to maturity is long. Intuitively, for a one-day option, if the spot price rises, there is little chance it will revert prior to maturity. With a very long maturity, though, a price movement of a given magnitude is much more likely to revert. So the straddles most highly exposed to realized volatility are those with the shortest maturities.

Throughout the paper, we interpret exposures to squared returns as representing exposure to realized volatility, since realized volatility is calculated based on squared returns over some period. Here we see that what a straddle specifically gives an investor exposure to is squared returns normalized by implied volatility. That is, straddles pay essentially for returns that turn out to be surprisingly large compared to initial implied volatility.

The third term shows that straddles are also exposed to changes in expected future volatility, through $\frac{\Delta \sigma_t}{\sigma_{t-1}}$, and that exposure is in fact constant across maturities. Intuitively, the raw sensitivity of the price of a straddle to a change in volatility is higher at longer maturities, but the price itself is also higher, so in percentage terms the return sensitivity is constant.

As discussed above, implied volatility is closely related to uncertainty: it measures how dispersed the distribution of final outcomes for the futures price is. Whereas realized volatility measures the magnitude of the shocks that actually occur – since it is a function of the realized return on the underlying future – implied volatility measures the magnitude of the shocks that are expected to occur in the future: all straddles provide insurance against shocks to uncertainty.

Another way to summarize the results in (3) in standard option terminology is that we interpret gamma as exposure to realized volatility and vega as exposure to uncertainty or implied volatility. Relative to the price of a straddle, gamma shrinks with maturity while vega remains constant.

3.2 RV and IV portfolios

The approximation in (3) gives a method for constructing portfolios that the Black-Scholes model says should give exposures directly to realized volatility – squared returns, measured by $(f_{n,t}/\sigma_{t-1})^2$ – and implied volatility or uncertainty, measured by $\Delta \sigma_t/\sigma_{t-1}$. Since the Black-Scholes model is not a perfect description of the world this is certainly not a perfect description of returns, as we show below, but we will see that it is a good approximation. Cremers, Halling, and Weinbaum (CHW; 2015) use exactly the same idea to construct risk factors for S&P 500 realized and implied volatility (though they call the term related to gamma jump risk, rather than realized volatility;

elision is important, it can be expected to appear as a deviation of the estimated factor loadings from the predictions of the approximation (3). Note also that the units of n and σ must match. So if σ is expressed annual units, then a one-month straddle has n = 1/12 and would have a loading of 6 on $(f_{n,t}/\sigma_{t-1})^2$.

we are comfortable with either descriptor).

As in CHW, we examine two particular combinations of short-and long-term straddles. To construct a portfolio that has positive exposure to realized volatility but zero exposure to implied volatility, one clearly needs to be long short-term straddles. The exposure to implied volatility is then eliminated by shorting a longer-term straddle. Specifically, we define the return on the RV portfolio in market i on date t to be

$$RV_{i,t} = \frac{5}{24} \left(r_{i,1,t} - r_{i,5,t} \right) \tag{4}$$

$$\approx \left(\frac{f_{n,t}}{\sigma_{t-1}}\right)^2 \tag{5}$$

Since there are equal amounts of money invested in the two straddles, the exposure to implied volatility is zero, and since the portfolio is long the shorter-maturity straddle, the exposure to the squared return is positive. We use the one- and five-month straddles as those are the shortest and longest maturities that we consistently observe in the data (CHW use just one- and two-month straddles, giving them a smaller maturity gap).

To construct a portfolio that is theoretically exposed only to implied volatility, we go long $\frac{5}{4}$ units of the five-month straddle and short $\frac{1}{4}$ of one unit of the one-month straddle. This portfolio then has zero exposure to realized volatility and the same exposure to implied volatility that the underlying straddles themselves do. Intuitively, at long maturities, returns are dominated by the exposure to implied volatility; the exposure to realized volatility eventually declines to zero. If we had an infinite-maturity straddle, it would yield positive exposure to implied volatility with no exposure to realized volatility. In practice we have to short a small amount of the short-term straddle to completely eliminate the realized volatility exposure. The IV portfolio is defined as

$$IV_{i,t} = \frac{5}{4}r_{i,5,t} - \frac{1}{4}r_{i,1,t} \tag{6}$$

$$\approx \frac{\Delta \sigma_t}{\sigma_{t-1}}$$
 (7)

So the RV portfolio is long and short equal amounts of the one- and five-month straddles, while the IV portfolio is primarily long the five-month straddle, with a small short position in the one-month straddle. We can thus think of the IV portfolio as being a claim on the long end of the maturity curve, while the RV portfolio is a bet on the slope. Whereas CHW used similarly constructed portfolios as pricing factors for individual equity returns, our analysis will focus on the average returns of the portfolios themselves, which directly reveal their risk premia.

3.3 Mean returns and Sharpe ratios

Figure 2 plots average annualized returns on straddles at maturities from one to six months for the 19 underlyings. Across the various contracts, the straddles generally earn negative returns at short maturities; even in the cases where the sample means are positive, the confidence intervals (constructed from a block bootstrap with 50-day blocks and 5,000 bootstrap samples) at least contain zero, meaning that we can never reject the hypothesis that average returns are negative at short maturities.

As the maturity increases, the average returns on the straddles also increase. For six of the underlyings, all nonfinancials, the returns on straddles at the 5 or 6-month maturity are actually statistically significantly positive. So while we observe a negative variance risk premium for both financial and non-financial underlyings – consistent with the well-known negative variance risk premium of the S&P 500 – the sign of the return on long-term straddles differs. The bottom-right panel of figure 2 summarizes that result by plotting the average of the average returns across the financial and non-financial underlyings. At all maturities, the average returns on the financial underlyings are less than those on the nonfinancials. Moreover, at all maturities above just two months, the average return for the straddles with non-financial underlyings are actually positive. So a person can buy a straddle and get positive exposure to volatility – returns that are high when volatility rises – and also positive average returns. That behavior is notably different from what is observed for financial underlyings, where all maturities earn negative average returns.

While the means vary across maturities, the volatility of the returns also tends to vary, declining with maturity (see table 1). To measure risk premia more formally, figure 3 plots sample Sharpe ratios on the various straddles. The variation in Sharpe ratios across maturities is qualitatively similar to the variation in mean returns: the curves generally slope upward and they turn significantly positive at long maturities for the nonfinancial underlyings. The shape of the curve for the S&P 500 straddles is similar to what is reported by Andries et al. (2015). Compared to their results, we find less negative returns on straddles at all maturities, likely because our sample runs further back, including the 1987 crash, which gave straddles very high returns.

In terms of magnitudes, the Sharpe ratios appear generally reasonable. The largest Sharpe ratio for short-term straddles is for the S&P 500 at -0.9. Some of the nonfinancials have Sharpe ratios for six-month straddles as large as 0.8, but the confidence bands are wide, with uncertainty of ± 0.5 for most contracts.

The mean returns and Sharpe ratios in figures 2 and 3 contain the main information that will allow us to document premia on realized and implied volatility. The key result is that there are economically large and statistically significant (as we will document more formally below) differences in returns between short- and long-maturity straddles. The appendix shows that nearly identical results are obtained when we examine raw straddle returns with delta hedging, or using a 7-day instead of 14-day holding period return.

3.4 Returns on RV and IV portfolios

We now examine the returns on the RV and IV portfolios, which theoretically give pure exposure to realized and implied volatility. The top two panels of figure 4 plot sample Sharpe ratios and

confidence bands for the RV and IV portfolios across the underlyings. Across the various markets, the RV portfolio consistently earns negative average returns while the IV portfolio earns positive returns. Furthermore, the returns on the IV portfolios appear to be higher on average for the nonfinancial than the financial underlyings.

For the RV portfolio, the most negative Sharpe ratio is for the S&P 500. The other Sharpe ratios are generally smaller, and none individually are economically implausible. The Sharpe ratios on the IV portfolios are generally positive and smaller than 1. Moreover, the confidence bands are all rather wide – between 0.5 and 1 – and almost all contain Sharpe ratios of 0.5 or less, again implying that the magnitudes of the risk premia are not implausible on their own.

The bottom panel of figure 4 plots the correlation of the RV and IV portfolio returns within each market. It is well known in many markets that unexpectedly high realized volatility is often followed by continued high volatility in the future (e.g. Engle (1982) and Bollerslev (1986)). That fact implies that shocks to realized volatility should be positively correlated with shocks to implied volatility, and that is precisely what we observe empirically, though not particularly strongly. The correlations are mostly positive, averaging 18 percent, and are no higher than 65 percent.

The fact that the RV and IV portfolios are positively correlated with each other makes it even more surprising that their Sharpe ratios generally have opposing signs. A mean-variance optimal portfolio will generally then be long one of the two and short the other. Moreover, since the returns are imperfectly correlated across markets, an optimal portfolio would hold straddles in many markets. We study these issues in detail in section 4.3.

The RV and IV portfolios are constructed to give pure exposures to specific risks, but one might worry that they do not obtain the desired exposures in practice. Appendix table A.3 reports results of regressions for each underlying of the returns on the two portfolios on the underlying futures return, the squared futures return, and the change in implied volatility. The table shows that while the Black–Scholes predictions do not hold perfectly, it is true that the RV portfolio is much more strongly exposed to realized than implied volatility, and the opposite holds for the IV portfolio. The coefficients on $(f_t/\sigma_{t-1})^2$ average 0.76 for the RV portfolio and 0.10 for the IV portfolio (though that average masks some variation across markets). Conversely, the coefficients on $\Delta \sigma_t/\sigma_{t-1}$ average 0.03 for the RV portfolio and 0.79 for the IV portfolio Furthermore, the R^2 s are large, averaging 72 percent across the various portfolios, implying that their returns are well described by the approximation (3).

So far, then, we see that the RV portfolios tend to earn negative returns, while the IV portfolios tend to earn positive returns, and the RV and IV portfolios do in fact approximately yield the desired exposures. We now proceed to document the average behavior of the returns across markets more formally.

4 Combining information across markets

The previous section establishes the basic facts about the means and Sharpe ratios of the returns on the RV and IV portfolios in each market. Our overall goal, though, is to make general statements about how those risks are priced on average across different markets. Making such statements requires combining information across the various markets, and doing so in a way that takes into account the facts that the means and Sharpe ratios are estimated with error, the variances of the errors differ across markets, and that the errors are correlated across markets. We do so using a standard random effects (or hierarchical) model. The model is useful more generally in settings where researchers have data from many markets, especially where they face an unbalanced panel.

4.1 Random effects model

Denote the vector of true average returns for the straddles in market i as \bar{r}_i . Our goal is to estimate the distribution of \bar{r}_i across the various underlyings and then to ask whether the \bar{r}_i are significantly different for financial and nonfinancial underlyings. A natural benchmark distribution for the means is the normal distribution. \bar{r}_i is therefore modeled as

$$\bar{r}_i \sim N\left(\mu_{\bar{r}}, \Sigma_{\bar{r}}\right)$$
 (8)

Our goal in this section is to estimate $\mu_{\bar{r}}$ and $\Sigma_{\bar{r}}$. The parameter $\mu_{\bar{r}}$ is an estimate of the high-level mean of returns across all the markets; it describes how means behave across maturities for the average market, while \bar{r}_i is the mean for a specific market. Our goal is to estimate the mean of the market-specific means.

 $\Sigma_{\bar{r}}$ describes how the market-specific means vary. Our estimates in the previous section differ noticeably across markets, but much of that is variation is likely driven by sampling error. $\Sigma_{\bar{r}}$ is an estimate of how much the *true* mean returns differ across markets, as opposed to the sample means. We assume that the \bar{r}_i are drawn from the distribution (8) independently across markets. Intuitively, that assumption is simply a default – it is no different from assuming that residuals in a regression are uncorrelated. Here, we are essentially regressing mean returns on a constant and treating the residuals as independent (it is straightforward to extend the analysis here to allow for explanatory variables that would help predict the \bar{r}_i). The difference between the random effects model and a regression is that the random effects model uses information about the estimation of the sample means to improve efficiency, similar to generalized least squares.

Furthermore, note that we can also transform the vector $\mu_{\bar{r}}$ to obtain estimates of the distribution of the returns on the RV and IV portfolios. The high-level mean return on the RV portfolio is $(5/24)[1,0,0,0,-1]\mu_{\bar{r}}$ and the mean for the IV portfolio is $[-1/4,0,0,0,5/4]\mu_{\bar{r}}$. It is straightforward to allow for a difference in the means of those portfolios across financials and nonfinancials and also to estimate the variance of the means across underlyings by transforming $\Sigma_{\bar{r}}$.

Denote the sample mean of the vector of returns in each market as \hat{r}_i , and the stacked vector

of sample means as $\hat{\mathbf{r}} \equiv [\hat{r}'_1, \hat{r}'_2, ...]'$. Similarly, denote the vector of true means as $\bar{\mathbf{r}} \equiv [\bar{r}'_1, \bar{r}'_2, ...]'$. Under the central limit theorem,

$$\hat{\mathbf{r}} \Rightarrow N\left(\bar{\mathbf{r}}, \Sigma_{\hat{r}}\right)$$
 (9)

where \Rightarrow denotes convergence in distribution and the covariance matrix $\Sigma_{\hat{r}}$ depends on the covariance between all the returns, across both maturities and underlyings. The appendix describes how we construct $\Sigma_{\hat{r}}$ (the method involves principal components analysis to control the number of parameters that must be estimated).

Using the asymptotic distribution as an approximation to the small-sample distribution, we can eliminate the unobservable parameter $\bar{\mathbf{r}}$ and write the distribution of $\hat{\mathbf{r}}$ as a function of the hyperparameters $\mu_{\bar{r}}$ and $\Sigma_{\bar{r}}$,

$$\hat{\mathbf{r}} \sim N\left(\left[\mu_{\bar{r}}', \mu_{\bar{r}}', \ldots\right]', \Sigma_{\hat{r}} + \Sigma_{\bar{r}}\right) \tag{10}$$

(10) represents a fully specified distribution for the data as a function of $\mu_{\bar{r}}$ and $\Sigma_{\bar{r}}$. It is then straightforward to calculate the likelihood of the data as a function of those parameters and obtain point estimates and confidence intervals.

To allow for the possibility that average returns differ between the financial and nonfinancial underlyings, the mean in the likelihood can be replaced by $\mu_{\bar{r}} + \mu_D I_F$, where μ_D is the difference in means and I_F is a 0/1 indicator for whether the associated underlying is financial

There are two ways to approximate the sampling distribution of the estimated parameters ($\mu_{\bar{r}}$, μ_D , and $\Sigma_{\bar{r}}$). The standard method when using maximum likelihood estimation is to calculate the curvature of the likelihood function at the optimum. That is an asymptotic approximation. An alternative small-sample method is to note that the likelihood is proportional to the Bayesian posterior density when the priors on the parameters are flat. Under that interpretation, a sampling distribution for the parameters can be calculated using standard Monte Carlo methods. We use the latter method here because it allows us to avoid calculating extremely large matrices of second derivatives (which can be numerically unstable) and because it is valid and more likely to be accurate in small samples.

A final point is how we parameterize $\Sigma_{\bar{r}}$. We find that when $\Sigma_{\bar{r}}$ is left unconstrained, the optimization drives it to degeneracy because the distribution of mean returns, \bar{r}_i , does not appear to be of full rank. One way to see that is to examine the eigenvalues of the correlation matrix of the market-specific estimates of \bar{r}_i . Those eigenvalues are 1.089, 0.040, 0.005, 0.002, and 0.0006. Even the third eigenvalue is two orders of magnitude smaller than the first. We thus model $\Sigma_{\bar{r}}$ as

$$\Sigma_{\bar{r}} = VDV' \tag{11}$$

where V is a 5 × 2 matrix of the eigenvectors associated with the two largest eigenvalues of the average covariance matrix of the market-specific estimates of \bar{r}_i . We then parameterize D as a diagonal matrix. Empirically, perhaps unsurprisingly, we find that the two columns of V appear to capture the level and slope of the elements of \bar{r}_i , as is often found in term structures. This

specification yields a simple two-parameter family for $\Sigma_{\bar{r}}$.

The estimation in this section is performed using the Bayesian computation engine Stan, which provides functions that both maximize the likelihood and rapidly sample from the posterior distribution. Sample Stan code for implementing the models estimated here is included in the appendix (the code for the full analysis is available on request).

Below we also report estimates of a random-effects model for the Sharpe ratios of the various portfolios. That model is estimated using the same specification as we use for the means.⁸

4.2 Estimates

Tables 4 and 5 report results from the random-effects estimation for the mean returns and Sharpe ratios across maturities and for the RV and IV portfolios.

Consistent with the results in the previous sections, the estimated mean term structure is substantially upward sloping for both financials and nonfinancials. In both cases is it significantly negative at the one-month maturity, showing that there is a variance risk premium on average across the various markets. For financials, mean returns are statistically significantly negative at all maturities, while for nonfinancials they turn positive at months 4 and 5. The difference in the means, in the third row of the table, is marginally significant at the 10-percent level for maturities between three and five months.

For both financials and nonfinancials, we observe a statistically significant negative risk premium for the RV portfolio. The IV portfolio, on the other hand, has an average return close to zero for financials and 18 percent for nonfinancials. That difference is itself statistically significant at the 10-percent level.

In terms of signs and significance, the results for the Sharpe ratios are generally similar. For financials, the average Sharpe ratio at the one-month maturity is -0.19, similar to -0.20 for nonfinancials, while at long horizons financials have a Sharpe ratio near zero and it is strongly positive for nonfinancials. Again, the differences between the two sets of markets is significant at the 10-percent level for maturities between 3 and 5 months.

For the RV portfolio the Sharpe ratios are again similar between the two sets of markets at -0.23 and -0.36. The magnitude of this premium is economically reasonable – it is approximately the same magnitude as the historical premium on the aggregate equity market.

Where we again see a statistically and economically significant difference in risk premia is in the Sharpe ratio on the IV portfolio, which is 0.48 for the nonfinancials but only 0.13 for the financials. So we find that one may earn a large positive premium when buying protection against fluctuations in implied volatility, but that effect is much stronger for the nonfinancial underlyings.

The bottom two lines of tables 4 and 5 report the estimated standard deviation of the (true) mean returns and Sharpe ratios across the markets. In table 5, the estimated standard deviation

 $^{^{8}}$ In that case, though, the RV and IV portfolios cannot be obtained as linear combinations of the mean Sharpe ratios for the monthly straddles. We therefore estimate a separate random effects model for the RV and IV Sharpe ratios.

of the mean return on the IV portfolio for nonfinancials is less than half that of the mean of the means. In other words, then, the data imply that only in very few markets (1.51 percent at the point estimates) would the IV portfolio have a true Sharpe ratio that is negative. For the RV portfolio there is relatively more dispersion: the estimates imply that 22 percent of underlyings (both financial and nonfinancial) will have Sharpe ratios that are positive. That result is consistent with the fact that 5 of our 19 point estimates are actually positive (though none statistically significantly).

Tables 4 and 5 formally confirm the basic intuition that is obtained from visual inspection of figure 4: exposure to realized volatility on average earns a negative risk premium in options markets, while exposure to implied volatility carries a statistically and economically significant positive premium among nonfinancial underlyings.

The findings in tables 4 and 5 are our central results on risk premia. They show that investors have historically demanded a premium for exposure to shocks to realized volatility, but assets that are exposed to implied volatility, thus providing insurance against high uncertainty, have in generally earned zero or even positive returns. Section 5 below provides an alternative method of confirming the same finding that does not rely on the approximation (3). We first, though, provide some more detail on the mean returns.

4.3 Portfolios combined across markets and risk factors

Since, as we saw above, realized and implied volatility are imperfectly correlated across markets, even larger returns can be earned by holding portfolios that diversify across the various underlyings. Table 6 reports results of various implementations of such a strategy. The first row reports results for portfolios that put equal weight on every available straddle in each period, the second row uses only nonfinancial underlyings, and the third row only financial underlyings. The columns report Sharpe ratios for various combinations of the RV and IV portfolios. The first two columns report Sharpe ratios for strategies that hold only the RV or IV portfolio, the third column uses a strategy that is short RV and long IV in equal weights, while the final column is short RV and long IV, but with weights inversely proportional to their variances (i.e. a simple risk parity strategy).

The Sharpe ratios reported in table 6 are generally substantially larger than what we obtain in figure 3 or in the averages from the random effects model reports in table 5. The portfolios that are short RV and long IV are able to attain Sharpe ratios well above 1. The largest Sharpe ratios come in the portfolios that combine RV and IV, which follows from the fact that they are positively correlated, so going short RV and long IV leads to internal hedging. All of that said, these Sharpe ratios remain generally plausible. Values near 1 are observed in other contexts (e.g. Broadie, Chernov, and Johannes (2009) for put option returns, Asness and Moskowitz (2013) for global value and momentum strategies, and Dew-Becker et al. (2017) for variance swaps).

The portfolios that take advantage of all underlyings simultaneously seem to perform best, presumably because they are the most diversified. While holding exposure to implied volatility

among the financials earns a relatively small premium, it is still generally worthwhile to include financials for the sake of hedging (at least for the RV part of the strategy).

Finally, it is important to note that the combined portfolios have returns that are much less skewed than those on the market-specific RV and IV portfolios. The bottom panel of table 6 reports the skewness of the various strategies from above, and, for the portfolios that include both RV and IV, they range between -0.77 and 3.27. That degree of skewness is substantially less than what we observe for the individual straddle returns in table 1. So while there may be some skewness, it is not particularly large in either direction – negative with equal weighting of the RV and IV portfolios, positive for the variance weighting. That suggests that the premia from these factors can be earned without necessarily holding a portfolio that is substantially negatively skewed (as with writing puts or straddles). In fact, the risk-parity strategy that holds both financials and nonfinancials has earned a historical Sharpe ratio of 1.26 with positive skewness of 0.94.

4.4 Case study: crude oil

It is worthwhile to briefly delve more deeply into one market to build confidence in the robustness of our basic results. We choose the crude oil market for this exercise because it has one of the longest time series available with the most maturities of any of the markets that we study, it is highly liquid (e.g. Gibson and Schwartz (1990) and Trolle and Schwartz (2010)), and it has a strong link to the macroeconomy.

Figures 7 and 8 contain several plots that help illustrate the historical behavior of the crude oil market.

Panel A of figure 7 plots the history of total volume for one- and five-month options (specifically, average daily volume of all contracts with maturities between 15 and 45 or 135 and 165 days to maturity, respectively). The volume of contracts in both maturity bins has risen over time, peaking in 2008, with a subsequent decline. On average, there is about 15 times more volume in the one-than the five-month option, though the volume in the five-month option has been trending upward. Average daily volume in recent years for one-month options is around 20,000 contracts, while for five-month options it is closer to 2,000 contracts. With prices averaging approximately \$75 per barrel over the last decade and since each contract represents 1,000 barrels, this corresponds to options with exposure to \$1.5 billion worth of oil for the one-month maturity and \$150 million for the five-month maturity.

Panel B of figure 7 plots 5-year rolling sample Sharpe ratios for the IV and RV portfolios. The left-hand section plots results for crude oil, while, for reference, the right-hand panel plots results for the S&P 500. For crude oil, we see that the RV portfolio had negative average returns in almost all five-year periods in our data, while the IV portfolio had positive returns in almost all five-year periods. The RV returns trend down over time, implying that the variance risk premium may have been growing. The IV returns are somewhat more consistent, though the returns were close to zero or even negative for short periods at the beginning and end of the sample.

The right-hand side of panel B gives further context to those results by plotting the RV and IV returns for the S&P 500 options. For the S&P we see that the RV portfolio has relatively more negative returns than for crude, while the IV portfolio has average returns that are generally centered on zero, rather than staying consistently positive as we observe for crude oil.

Panel C of figure 7 is similar to panel B, except instead of plotting returns on the RV and IV portfolios, it plots their constituents, the returns on the one- and five-month straddles. For crude oil, the five-month straddle has consistently *positive* returns, unlike the S&P 500, for which the five-month straddle tends to have negative returns. In both cases we see negative returns for the one-month straddle, though that effect is stronger for the S&P 500.

Overall, panels B and C have two uses. First, they show that the returns that we observe on the IV and RV portfolios are not driven by a small number of outliers; rather, they are fairly consistent over time. Second, they provide further detail on the divergences between the behavior of straddle returns for the S&P 500 compared to crude oil.

Next, to help understand how crude oil volatility relates to macroeconomic uncertainty, the top panel of figure 8 plots one-month at-the-money implied volatility for crude oil along with the LMN financial and price uncertainty series. The correlation of oil price uncertainty with the two series is immediately apparent. The various spikes upward in crude oil volatility are all traceable to spikes in either price or financial uncertainty. This figure thus underscores the utility to an investor of buying five-month crude oil straddles: they provide good protection against increases in the LMN uncertainty indexes and at the same time earn positive average returns.

Because the crude oil market is so large, it has relatively more traded maturities than the other underlyings. At any given time, the CME currently has trading in the next 12 monthly expirations and also December expirations for a number of years into the future. Panel B of figure of figure 8 plots average returns for crude oil straddles with maturities between 1 and 11 months (not 12 because of how we interpolate to construct the monthly portfolios); panel C reports Sharpe ratios. The figure shows that the behavior at longer maturities remains similar, and returns continue to rise slightly beyond the five months examined in the main analysis (though they eventually flatten). When we calculate the IV portfolio using the 11- instead of the five-month maturity, we also obtain similar results.

5 Linear factor models

The evidence presented on the pricing of implied and realized volatility risk relies on the Black—Scholes model to give an approximation for the risk exposures of the portfolios. While table A.3 (and also figure 5) show that those predictions are a reasonably accurate description of the data, our findings are not actually dependent on the Black—Scholes holding with perfect accuracy. To estimate the price of risk for realized and implied volatility purely empirically, with no appeal to a theoretical model, we now estimate standard factor specifications and combine them across markets using a random effects model.

5.1 Models based on aggregate factors

We begin by attempting to explain the cross-section of straddle returns based on a small number of factors. First, we consider a specification based on S&P 500 returns, which can be viewed as an extension of the standard CAPM. In addition to the return on the S&P 500 itself, since straddles have highly nonlinear exposures, we also include the square of the S&P 500 return, scaled by the previous implied volatility (as in the Black–Scholes model, since the price of a straddle scales with volatility; we obtain similar results without this scaling). Finally, to account for the potential pricing of shocks to uncertainty, our third factor is the change in the at-the-money implied volatility of 5-month options on the S&P 500, again scaled by previous implied volatility.

The time series model of returns is

$$r_{i,n,t} = a_{i,n} + \beta_{i,n}^{S\&P} \frac{r_{S\&P,t}}{IV_{S\&P,t-1}} + \beta_{i,n}^{S\&P^2} \left(\frac{r_{S\&P,t}}{IV_{S\&P,t-1}}\right)^2 + \beta_{i,n}^{\Delta IV_{S\&P}} \frac{\Delta IV_{S\&P,t}}{IV_{S\&P,t-1}} + \varepsilon_{i,n,t}$$
(12)

where $r_{i,n,t}$ is the return on the straddle with underlying i at maturity n on date t. We specify risk premia as

$$E[r_{i,n,t}] = \gamma^{S\&P} \beta_{i,n}^{S\&P} + \gamma^{S\&P^2} \beta_{i,n}^{S\&P^2} + \gamma^{\Delta IV_{S\&P}} \beta_{i,n}^{\Delta IV_{S\&P}} + \alpha_{i,n}$$
(13)

We use standard methods to estimate the model; the appendix gives the details. We exclude the S&P 500 straddles from the estimation, since they are almost guaranteed to be priced, at least mechanically, by these factors.

The first column of appendix table A.4 reports the results of this model. It explains almost none of the cross-sectional variation in average returns. The cross-sectional R^2 for the mean returns is only 15 percent, none of the risk prices are individually significant, and a χ^2 test of the hypothesis that they are all equal to zero yields a p-value of 0.39. In addition, factors used in the first stage have almost no explanatory power for the time series of returns: the average R^2 in the time series regressions is only 1.8 percent.

As a second alternative, instead of using the S&P 500 alone to construct the factors, we examine factors based on the entire cross-section of underlyings. We replace $r_{S\&P,t}$ with the average return on the underlying futures contracts across the 19 markets on date t (our panel of futures returns is not balanced, so on each date we average across all available returns). The second factor is then that return scaled by implied volatility and squared. Finally, the third factor is the average across all available underlyings of the scaled change in 6-month at-the-money implied volatility (i.e. of $\Delta IV_{i,t}/IV_{i,t-1}$).

The appendix shows that the model based on averages across all the underlyings performs somewhat better than that based on the S&P 500 returns, but the cross-sectional R^2 for average returns is still only 37 percent, leaving the vast majority of the variation in returns across markets and maturities unexplained.

The fact that models based on aggregate factors explain only a minority of the cross-sectional variation in average straddle returns suggests that more pricing factors are needed. The most

natural place to look for those factors is in the individual markets themselves (similar to how equities and bonds are typically priced, for example). It is possible that the aggregate factors by themselves do not price the straddles because volatility in each of the individual markets is an undiversifiable risk that is priced on its own. Another possibility is that investors are not fully diversified across markets (e.g. because financial intermediaries such as market makers or investment funds specialize in particular sectors; see Bates (2003) for motivation and Garleanu and Pedersen (2009) for a fully developed model).

In either case, our next step is to price straddles using factors associated with the individual underlyings. The market-specific factor models will have the advantage of both fitting the cross-section of average returns well and also providing formal estimates of the market-specific risk prices of exposure to realized and implied volatility.

5.2 Market-specific factors

5.2.1 Specification

Our market-specific models mimic the specification used with the S&P 500 factors above, but now using the futures returns from the individual markets. Specifically, the time series model is now

$$r_{i,n,t} = a_{i,n} + \beta_{i,n}^f \frac{f_{i,t}}{IV_{i,t-1}} + \beta_{i,n}^{f^2} \frac{1}{2} \left(\frac{f_{i,t}}{IV_{i,t-1}} \right)^2 + \beta_{i,n}^{\Delta IV} \frac{\Delta IV_{i,t}}{IV_{i,t-1}} + \varepsilon_{i,n,t}$$
(14)

where $f_{i,t}$ is the futures return for underlying i and $\Delta IV_{i,t}$ is the change in the five-month at-themoney implied volatility for underlying i. Including the underlying futures return allows us to control for any exposure of the straddles to the spot, though we would expect that effect to be small.

Much more important is the fact that straddles have a nonlinear exposure to the futures return. Including $(f_{i,t}/IV_{i,t-1})^2$ allows us to capture that nonlinearity and estimate the price of risk associated with it. Consistent with our construction and interpretation of the RV portfolio, $\beta_{i,n}^{f^2}$ will be interpreted as the exposure of the straddles to realized volatility, since realized volatility is calculated based on squared returns of the underlying.

Finally, the third factor we include is the change in the at-the-money implied volatility for the specific market at the five-month maturity. Since the IVs may be measured with error, we construct this factor by regressing available implied volatilities on maturity for each underlying and date and then taking the fitted value from that regression at the five-month maturity. The inclusion of the implied volatility factor allows us to understand the relative pricing of realized volatility – through

⁹There are obviously numerous closely related specifications of that second term that could be substituted. We obtain similar results when the second factor is the absolute value of the futures return instead of its square, for example, or when it is measured as the sum of squared daily returns over the return period (recall that the straddle returns cover two weeks, so the factor in that case is the two-week daily realized volatility). We focus on the squared return because it can be interpreted as a second-order term in the pricing kernel and also because it allows a direct link to the gamma of the straddles.

the exposure to the squared return – compared to the pricing of uncertainty about the future.

To estimate the risk premia on the three factors, we use a standard linear specification for expected returns,

$$E[r_{i,n,t}] = \gamma_i^f \beta_{i,n}^f + \gamma_i^{f^2} \beta_{i,n}^{f^2} + \gamma_i^{\Delta IV} \beta_{i,n}^{\Delta IV} + \alpha_{i,n}$$
(15)

$$E\left[f_{i,t}\right] = \gamma_i^f \tag{16}$$

This model of risk premia can be interpreted in two ways. One is that the γ coefficients represent the risk premia that are earned by investments that provide direct exposure to the factors. That is, the γ 's are estimates of what the mean returns on the factors would be if it were possible to invest in them directly (neither $f_{i,t}^2$ nor $\Delta IV_{i,t}$ is an asset return that one can directly purchase). When describing the estimated prices of risk, then, we will generally describe them as the return earned for exposure to the various risks. They thus provide a formalization of the results in the previous section. Whereas the returns of the RV and IV portfolios had to be interpreted through the lens of the Black–Scholes model, which gives predictions for their risk exposures, here we simply measure risk exposures (β 's) directly and then calculate the associated premia.

An alternative way to interpret the coefficients is to transform them to represent coefficients in a linear specification for the pricing kernel (Cochrane (2005)),

$$1 = E_{t-1}[M_{i,t}r_{i,n,t}] (17)$$

$$M_{i,t+1} = 1 + \lambda_i^f \frac{f_{i,t}}{IV_{i,t-1}} + \lambda_i^{f^2} \frac{1}{2} \left(\frac{f_{i,t}}{IV_{i,t-1}} \right)^2 + \lambda_i^{\Delta IV} \frac{\Delta IV_{i,t}}{IV_{i,t-1}}$$
(18)

 $\lambda_i^{f^2}$ then measures the curvature of the pricing kernel in market i with respect to the underlying futures return and $\lambda_i^{\Delta IV}$ measures how changes in implied volatility affect the pricing kernel. the γ 's are related to the λ 's by multiplying by the covariance matrix of the factors.

5.2.2 Results

We estimate the model in two steps using standard linear methods – estimating the time series regressions first and then the cross-sectional model (15–16) taking the estimates of loadings as given. The standard errors are constructed as in Cochrane (2005) to account for estimation error in the factor loadings when calculating the standard errors for the risk premia. As noted above, since we use overlapping two-week returns, we correct the standard errors by using the Hansen–Hodrick (1980) estimator of the long-run covariance with a two-week lag window. The standard errors also allow for correlation across maturities and assets in the residuals $\varepsilon_{i,n,t}$.

We begin by examining the estimated loadings of the straddle returns on the factors. The Black–Scholes approximation used above (equation 3) gives a prediction for what the factor loadings of the straddles should be: $\beta_{i,n}^f = 0$, $\beta_{i,n}^{f^2} = 1/n$, and $\beta_{i,n}^{\Delta IV} = 1$. To test that prediction, figure 5 plots the actual estimated factor loadings from time series regressions. Across the panels, the predictions

hold surprisingly accurately. The loadings on the spot return are all near zero, if also generally slightly positive. The loadings on the change in implied volatility are all close to 1, with little systematic variation across maturities. And the loadings on the squared spot return tend to begin near 1 (though sometimes biased down somewhat) and then decline monotonically, consistent with the predicted n^{-1} scaling.

To help interpret the risk prices, we standardize all of the factors in what follows to have unit annualized variances (though note they were not standardized in calculating the loadings in figure 5). The γ coefficients can then be interpreted as the annualized Sharpe ratio of a portfolio that is perfectly correlated with the related factor.

Figure 6 plots the estimated risk premia across the various markets along with 95-percent confidence bands. For the squared futures return, 13 of 19 estimated premia are negative (9 statistically significantly), while for the change in implied volatility, 18 of 19 are positive (8 statistically significantly). Those results are consistent with what we observe in figure 4: investors have historically earned negative returns on assets that yield exposure to squared futures returns, while they earn positive returns on portfolios designed to isolate implied volatility.

The magnitude of the estimated risk prices is generally in line with priors – they are typically bounded in absolute value by 1, and the majority are much smaller. As with the RV portfolio, the largest risk premium we estimate is for realized volatility for the S&P $500 - \gamma_{S\&P500}^{f^2} = -1.36$. That finding is consistent with the well-known variance risk premium. In terms of magnitude, this is similar to what Dew-Becker et al. (2017) find for the Sharpe ratio for one-month S&P 500 variance swaps and what Andries et al. (2015) report for short-maturity S&P 500 straddles.

5.2.3 Random effects model for risk premia

To quantify the average behavior of risk premia across markets, we estimate a random effects model similar to that used above for mean returns and Sharpe ratios. The specification for market-specific risk premia is

$$\gamma_i \equiv \left[\gamma_i^f, \gamma_i^{f^2}, \gamma_i^{\Delta IV}\right]' \tag{19}$$

$$\gamma_i \sim N(\mu_{\gamma}, \Sigma_{\gamma})$$
 (20)

As with means, the sample estimates of the risk premia, denoted $\hat{\gamma}_i$, are asymptotically normally distributed, and we denote the variance matrix as

$$\hat{\gamma}_I \equiv \left[\hat{\gamma}_1', \hat{\gamma}_2', \dots \right]' \tag{21}$$

$$\hat{\gamma}_I \sim N(\gamma_I, \Sigma_{\hat{\gamma}})$$
 (22)

which then yields the full likelihood,

$$\hat{\gamma}_I \sim N\left(\left[\mu_{\gamma}', \mu_{\gamma}', \ldots\right]', \Sigma_{\hat{\gamma}} + \Sigma_{\gamma}\right)$$
 (23)

As before, we also allow the mean, μ_{γ} , to differ between financial and nonfinancial underlyings.

The results of the random effects model for risk premia are reported in table 7. Consistent with the market-specific results and the results for the RV and IV portfolios, the high-level mean risk premium for the squared futures return is statistically and economically significantly negative (-0.36), while the mean risk premium for the change in implied volatility is significantly positive (0.49). The magnitude of the premia is similar to what we observe for the RV and IV portfolios, though here we find a somewhat larger premium for implied volatility.

The estimated cross-sectional standard deviations of the premia are again large, implying that there is substantial variation in the risk premia across markets. While there is a negative variance risk premium in many markets, there is not a negative variance risk premium in every market – our estimates imply that 73 percent of markets will have negative variance risk premia, while the remainder will be positive. Similarly, our estimates imply that the premium for implied volatility is positive in 90 percent of markets and negative in the remaining 10 percent.

Finally, the left-hand side of table 7 examines the difference in the premia between the financial and nonfinancial underlyings. In this case we have less power than we did with the RV and IV portfolios (presumably because the factor loadings are estimated in this section, instead of being imposed theoretically). While the differences are similar to what we observed before – near zero for the squared futures return, and a smaller premium for implied volatility in financials – they are not statistically significant in this case.

Similar results hold when looking at risk prices for these shocks, i.e. the estimated loadings of the SDF onto the three risk factors. Appendix table A.5 reports the results, which show similar qualitative and quantitative patterns.

6 Conclusion

This paper studies the pricing of realized and implied volatility across a broad array of options on financial and commodity futures. We show that realized and implied volatility in the futures markets that we study are strongly related to a range of measures of macroeconomic uncertainty, including uncertainty about real quantities, goods prices, and the state of financial markets.

Exposure to realized volatility yields a negative estimated price of risk on average. Investors have historically paid premia – associated with Sharpe ratios averaging -0.36 – for protection against large movements in futures prices. Moreover, when one holds a portfolio that sells protection against realized volatility in all markets simultaneously (essentially selling straddles in all markets), that Sharpe ratio rises to 0.9. Our findings here reinforce and extend past results on the variance risk premium, showing that it appears robustly across many commodity markets and helps explain the cross-section of straddle returns.

We draw a distinction between realized volatility and uncertainty. Shocks to uncertainty represent changes in how large people expect shocks to be in the future, rather than the magnitude of the shock that the economy just received. Empirically, we link changes in uncertainty to changes in

option implied volatility. The hypothesis that investors are averse to high uncertainty should then imply that shocks to implied volatility carry negative risk premia. For financial underlyings we find that implied volatility robustly earns no risk premium, while for nonfinancials, implied volatility shocks in fact earn statistically and economically *positive* premia. The simplest piece of evidence for that fact is that five-month straddles for the S&P 500 have returns near zero, while those for crude oil and other physical commodities are significantly positive.

Whereas protection against shocks to realized volatility is costly on average, protection against shocks to implied volatility has historically earned investors money. Buying such protection – essentially by buying long-term straddles – yields Sharpe ratios of 0.13 for financial underlyings and 0.48 for nonfinancials. Moreover, a strategy that holds long-term straddles in all markets simultaneously earns a Sharpe ratio of 0.76. Investors have historically earned positive returns when hedging shocks to implied volatility.

So across many markets, it is realized rather than implied volatility that investors have paid to hedge. One potential explanation for that result is that periods of high innovation, when growth is high (and marginal utility might be low) are often periods of high volatility. Another possibility is that the pattern of returns could be explained by microstructure factors, but the fact that the patterns even appear for crude oil, which is a large and efficient market, argues against technical factors being dominant.

The conclusion we then draw is that the joint evidence from the largest commodity and financial futures markets suggests that investors have not historically been averse to fluctuations in uncertainty about the future.

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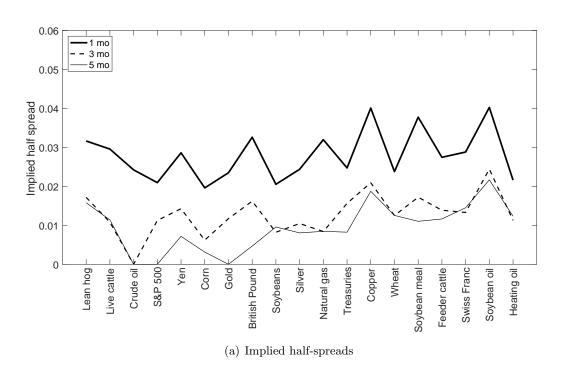
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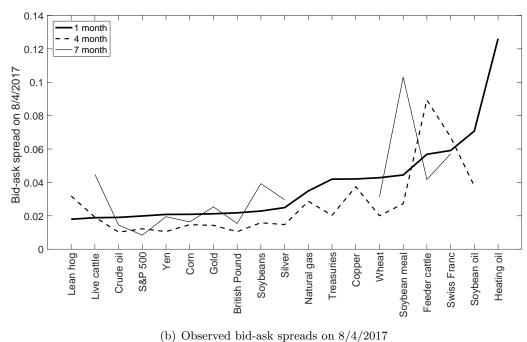
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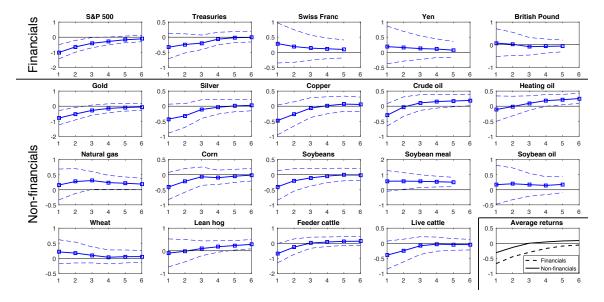
Figure 1: Bid-ask spreads





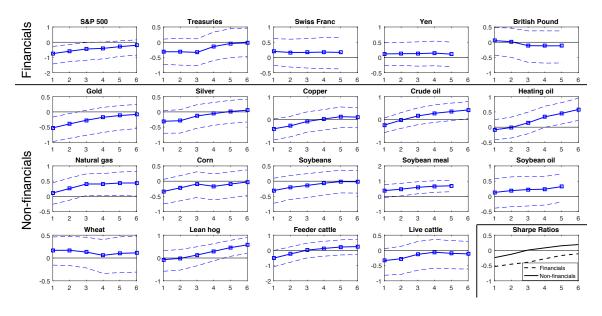
Note: The top panel plots for each market the effective half-spread computed from observed option returns, calculated as in Roll (1984). The bottom panel reports posted bid-ask spreads for at-the-money straddles obtained from Bloomberg on of August 4, 2017.

Figure 2: Average returns



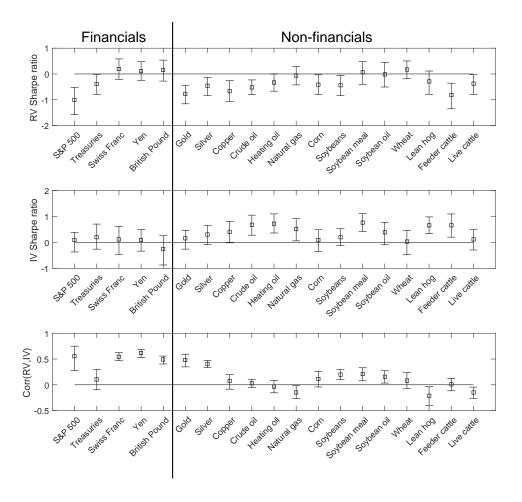
Note: Average returns of at-the-money straddles of different maturities across markets. The returns plotted are two-week holding period returns, annualized. The dotted lines are 95-percent confidence intervals calculated with a block bootstrap. The bottom-right corner plots average returns for all financial and nonfinancial straddles.

Figure 3: Sharpe ratios



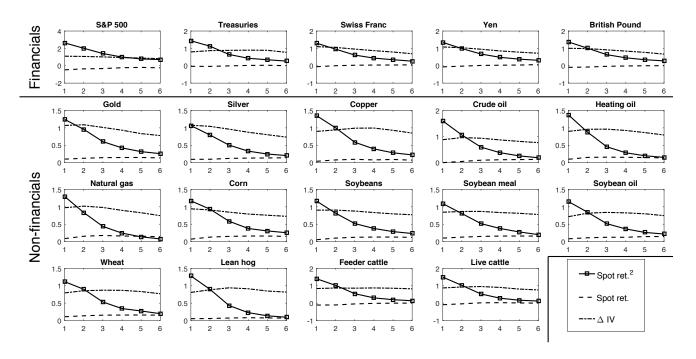
Note: Sharpe ratios corresponding to the returns plotted in figure 2.

Figure 4: RV and IV portfolio returns



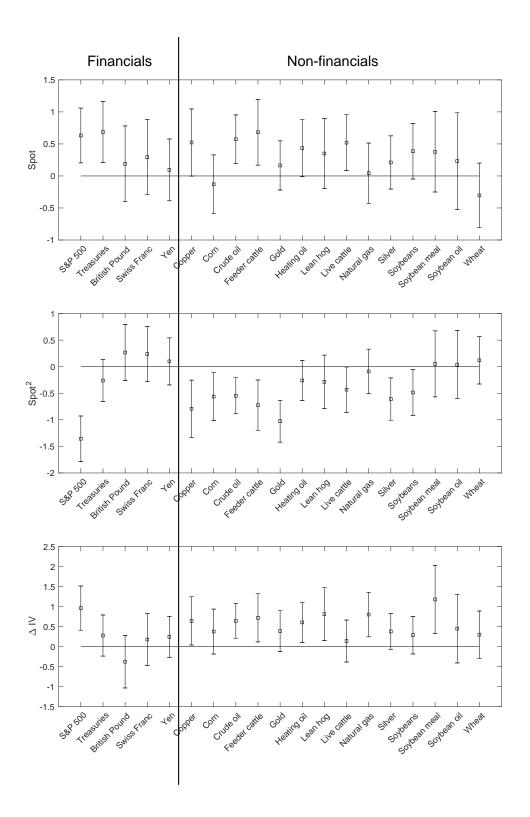
Note: Squares are point estimates and vertical lines represent 95-percent confidence intervals from a block bootstrap.

Figure 5: Factor loadings



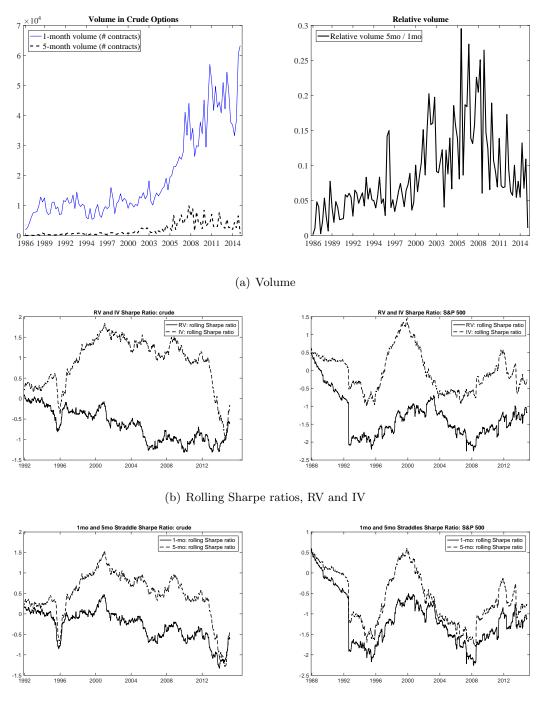
 $\bf Note:$ Loadings of two-week straddle returns on the three risk factors.

Figure 6: Factor risk premia across markets



Note: Estimated risk premia for the three market-specific factors. Squares are point estimates and lines represent 95-percent confidence intervals calculated through Hamiltonian Monte Carlo.

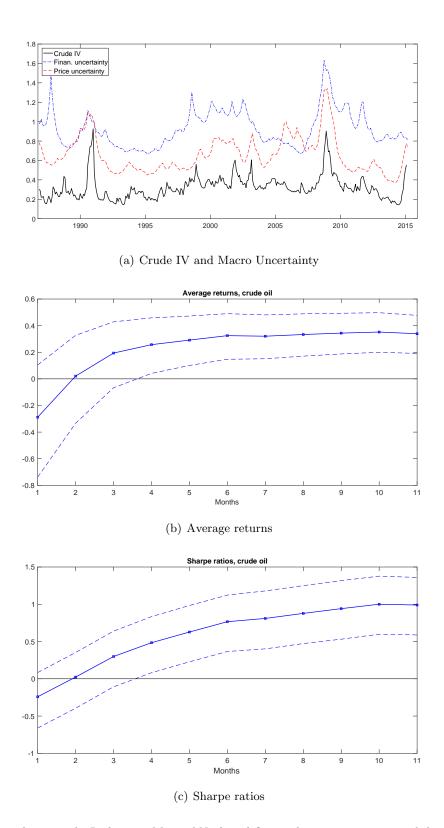
Figure 7: Case study: crude oil (I)



(c) Rolling Sharpe ratios, 1mo and 5mo straddles

Note: The top panel reports the volume in number of contracts for the 1-month and the 5-month straddles (left), and the ratio of the 5 month to the 1 month volume (right) for crude oil. The middle panel reports rolling sharpe ratios for the RV and IV portfolios, for crude oil (left) and for the S&P 500 (right). The bottom panel reports rolling Sharpe ratios for the 1-month and 5-month straddles, for crude oil (left) and for the S&P 500 (right).

Figure 8: Case study: crude oil (II)



Note: The top panel reports the Ludvigson, Ma, and Ng (2015) financial uncertainty series and the macroeconomic price uncertainty series together with the implied volatility for crude oil. The middle and bottom panels plot average returns and Sharpe ratios along with block-bootstrapped 95-percent confidence intervals.

Table 1: Descriptive statistics for straddle returns

Contract		1mo	2mo	3mo	4mo	5mo	6mo	Contract		1mo	2mo	3mo	4mo	5mo	6mo
S&P 500	Mean	-1.22	-0.79	-0.49	-0.35	-0.20	-0.08	Treasuries	Mean	-0.41	-0.27	-0.2	-0.04	-0.01	-0.06
(1983 - 2015)	Std. dev.	1.23	0.98	0.80	0.65	0.57	0.57	(1982 - 2015)	Std. dev.	1.15	0.91	0.66	0.56	0.49	0.4
	Skewness	6.93	8.06	10.15	9.09	8.95	10.47		Skewness	2.19	2.49	2.44	2.75	3.15	2.46
	# years	29.20	30.60	31.20	31.10	29.70	27.90		# years	26.8	30.1	31.4	30.4	28.1	21.8
British Pound	Mean	-0.34	-0.23	-0.18	-0.04	-0.05		Gold	Mean	0.09	0.07	0.11	0.22	0.12	
(1988 - 2015)	Std. dev.	1.26	1.03	0.77	0.64	0.53		(1986 - 2015)	Std. dev.	1.41	1.1	0.83	0.68	0.57	
	Skewness	2.29	2.60	2.76	3.08	2.64			Skewness	3.49	2.73	3.32	2.51	2.52	
	# years	25.80	26.00	25.80	24.00	19.90			# years	26.7	28.3	27	23.6	18.8	
Silver	Mean	-0.13	-0.06	0.00	0.11	0.10		Swiss Franc	Mean	-0.38	-0.16	-0.03	0.02	0.08	
(1986 - 2015)	Std. dev.	1.49	1.18	0.91	0.75	0.67		(1985 - 2015)	Std. dev.	1.2	1.01	0.81	0.68	0.59	
,	Skewness	3.10	3.39	3.25	3.07	3.31		,	Skewness	2.39	2.64	2.98	2.93	2.85	
	# years	27.80	28.10	28.10	27.90	26.40			# years	17.9	19.7	19.4	18.7	18.4	
Yen	Mean	-0.47	-0.24	-0.18	-0.17	-0.12		Copper	Mean	-0.33	-0.04	0.09	0.14	0.16	0.2
(1986 - 2015)	Std. dev.	1.19	0.95	0.74	0.61	0.55		(1988 - 2015)	Std. dev.	1.24	0.94	0.69	0.57	0.51	0.46
,	Skewness	2.09	2.63	2.92	2.79	2.69		, ,	Skewness	1.95	2.16	2.15	2.1	2.16	2.04
	# years	24.40	28.30	29.10	29.10	29.10			# years	27.5	27.5	27.5	27.5	27.5	27.3
Corn	Mean	-0.56	-0.12	0.05	0.09	0.11	0.12	Crude Oil	Mean	-0.79	-0.39	-0.2	-0.09	-0.04	-0.01
(1985 - 2015)	Std. dev.	1.42	1.08	0.78	0.65	0.58	0.54	(1986 - 2015)	Std. dev.	1.52	1.39	1.09	0.91	0.77	0.69
	Skewness	2.71	2.72	2.46	2.46	2.34	2.45		Skewness	5.38	7.26	7.87	8.03	7.81	7.12
	# years	21.50	26.10	27.30	27.30	27.30	25.40		# years	26.7	27.9	27.9	27.9	27.9	27.9
Feeder Cattle	Mean	-0.18	-0.09	0.09	0.20	0.24	0.25	Heating Oil	Mean	-0.06	-0.01	0.14	0.19	0.22	0.22
(1987 - 2015)	Std. dev.	1.38	1.03	0.71	0.58	0.50	0.44	(1988 - 2015)	Std. dev.	1.42	0.98	0.73	0.58	0.5	0.48
	Skewness	3.31	3.44	2.44	2.58	2.16	2.00		Skewness	2.62	2.91	2.82	3.03	2.85	2.91
	# years	26.30	26.30	24.90	23.00	21.70	19.90		# years	23.7	29.1	29.2	29.2	29	27.9
Lean Hog	Mean	-0.39	-0.08	0.02	0.03	0.01	0.01	Live Cattle	Mean	0.08	0.18	0.22	0.19	0.2	0.19
(1985 - 2015)	Std. dev.	1.20	0.86	0.67	0.54	0.45	0.41	(1984 - 2015)	Std. dev.	1.37	1.01	0.72	0.56	0.49	0.43
,	Skewness	2.70	2.73	2.59	2.53	2.29	2.10	,	Skewness	2.34	2.31	2.34	2.2	2.07	1.79
	# years	24.30	29.40	29.40	29.40	29.40	29.40		# years	21.7	21.7	21.7	21.7	21.4	19.9
Natural Gas	Mean	-0.41	-0.23	0.00	0.06	0.10	0.08	Soybeans	Mean	-0.41	-0.29	-0.14	-0.06	-0.02	0
(1992 - 2015)	Std. dev.	1.43	1.17	0.92	0.72	0.63	0.54	(1984 - 2015)	Std. dev.	1.29	0.93	0.76	0.65	0.58	0.53
, ,	Skewness	2.42	2.61	3.15	2.70	2.71	2.43	,	Skewness	2.38	2.37	2.52	2.58	2.07	2.03
	# years	25.80	27.40	27.80	27.80	27.80	25.80		# years	27	29.4	29.4	29.4	29.3	28.7
Soybean Meal	Mean	0.43	0.46	0.44	0.44	0.54		Soybean Oil	Mean	0	0.11	0.15	0.16	0.29	
(1987 - 2015)	Std. dev.	1.47	1.16	0.93	0.80	0.72		(1987 - 2015)	Std. dev.	1.27	1.05	0.82	0.68	0.61	
•	Skewness	2.51	2.68	2.74	2.95	2.70			Skewness	2.01	2.51	2.86	2.51	2.41	
	# years	24.20	27.00	27.00	25.70	23.10			# years	23.8	27	26.8	24.4	19.5	
Wheat	Mean	0.13	0.17	0.09	0.06	0.05	0.03								
(1986 - 2015)	Std. dev.	1.30	1.04	0.79	0.66	0.59	0.50								
	Skewness	2.22	2.64	2.68	2.63	2.64	2.02								
	# years	22.60	26.70	27.50	27.40	27.30	24.60								

Note: Annualized descriptive statistics for the returns of at-the-money straddles in 19 option markets. Returns are for two-weed holding periods. The sample period depends on the contract and is reported in the table.

Table 2: Regressions of IV onto macroeconomic uncertainty measures

IV	Financial Unc.	Macro Unc.	R2	Financial Unc.	Macro Unc. (Real)	Macro Unc. (prices)	R2
S&P 500	0.80	0.04	0.71	0.83	-0.05	0.07	0.72
T-bonds	0.40	0.19	0.31	0.48	0.02	0.09	0.30
GBP	-0.03	0.50	0.26	0.11	0.20	0.19	0.18
CHF	0.10	0.23	0.10	0.22	-0.07	0.15	0.08
JPY	0.38	0.16	0.27	0.43	0.09	0.00	0.26
Copper	-0.12	0.61	0.37	-0.09	0.39	0.33	0.39
Corn	0.28	0.17	0.18	0.24	0.32	-0.08	0.22
Crude oil	0.28	0.44	0.46	0.33	0.07	0.47	0.54
Feeder cattle	0.07	0.25	0.10	0.12	0.26	-0.07	0.10
Gold	0.22	0.46	0.41	0.22	0.34	0.24	0.45
Heating oil	0.32	0.43	0.51	0.37	0.04	0.47	0.58
Lean hog	0.41	-0.05	0.15	0.40	-0.07	0.03	0.16
Live cattle	0.04	0.24	0.07	0.10	0.04	0.18	0.07
Natural gas	0.12	0.27	0.16	0.13	-0.10	0.52	0.29
Silver	0.06	0.38	0.19	0.05	0.45	0.00	0.23
Soybeans	0.14	0.31	0.17	0.16	0.26	0.06	0.17
Soybean meal	0.16	0.28	0.18	0.16	0.23	0.12	0.19
Soybean oil	0.02	0.38	0.16	0.11	0.03	0.30	0.15
Wheat	0.23	0.27	0.22	0.22	0.39	-0.07	0.26
Average of the IVs	0.36	0.44	0.54	0.42	0.22	0.23	0.53

Note: The left panel reports, in each row, the coefficients from regressions of each implied volatility onto the financial and macroeconomic uncertainty measures developed in Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (2017). The last row regresses the average of all 19 IVs on the LMN uncertainty measures. All variables are normalized to have unit standard deviations prior to the regressions. The right panel repeats the exercise dividing the macro uncertainty measure in one constructed only real quantities, and one constructed using prices.

Table 3: Regressions of RV onto macroeconomic uncertainty measures

RV	Financial Unc.	Macro Unc.	R2	Financial Unc.	Macro Unc. (Real)	Macro Unc. (prices)	R2
S&P 500	0.72	0.23	0.72	0.75	0.08	0.13	0.71
T-bonds	0.36	0.34	0.35	0.41	0.07	0.24	0.33
GBP	0.02	0.43	0.22	0.13	0.15	0.25	0.19
CHF	0.02	0.13	0.02	0.03	0.08	0.03	0.01
JPY	0.11	0.19	0.07	0.15	0.07	0.07	0.05
Copper	0.18	0.46	0.36	0.32	0.02	0.39	0.35
Corn	0.05	0.37	0.15	0.09	0.10	0.25	0.13
Crude oil	0.01	0.25	0.07	0.02	0.10	0.19	0.08
Feeder cattle	0.05	0.17	0.04	0.08	0.01	0.13	0.03
Gold	0.11	0.39	0.22	0.14	0.23	0.20	0.21
Heating oil	0.14	0.20	0.08	0.16	0.09	0.15	0.09
Lean hog	0.11	0.02	0.02	0.12	-0.09	0.10	0.03
Live cattle	0.11	0.16	0.06	0.14	-0.02	0.14	0.05
Natural gas	-0.04	0.12	0.01	-0.03	0.10	0.05	0.02
Silver	0.11	0.32	0.16	0.15	0.11	0.18	0.13
Soybeans	0.03	0.34	0.13	0.07	0.09	0.25	0.11
Soybean meal	0.03	0.27	0.09	0.06	0.05	0.20	0.07
Soybean oil	0.03	0.47	0.25	0.08	0.15	0.30	0.20
Wheat	0.02	0.25	0.07	0.06	0.00	0.22	0.07
Average of the RVs	0.22	0.50	0.42	0.28	0.14	0.35	0.37

Note: The left panel reports, in each row, the coefficients of a regression of each realized volatility onto the financial uncertainty measure and the macro uncertainty measures developed in Jurado, Ludvigson, and Ng (2015) and Ludvigson, Ma, and Ng (2017). The last row regresses the average of all 19 RVs on the LMN uncertainty measures. Each row also reports the R^2 of the regression. All variables are standardized. The right panel repeats the exercise separating the macro uncertainty measure in one constructed only real variables, and one constructed using prices.

Table 4: Estimates from random effects model for mean returns

	1 mo	2 mo	3 mo	4 mo	5 mo	RV port.	IV port.
Mean, nonfin.	-0.388	-0.188	-0.025	0.039	0.072	-0.460	0.187
	[-0.608,-0.169]	[-0.368,-0.005]	[-0.165,0.114]	[-0.077,0.155]	[-0.032,0.175]	[-0.607,-0.313]	[0.097,0.276]
Mean, fin.	-0.483	-0.335	-0.234	-0.148	-0.110	-0.373	-0.017
	[-0.839,-0.093]	[-0.630,-0.015]	[-0.462,0.008]	[-0.336,0.047]	[-0.276,0.062]	[-0.611,-0.118]	[-0.157,0.127]
Difference	-0.095	-0.147	-0.208	-0.187	-0.182	0.087	-0.204
	[-0.504,0.352]	[-0.480,0.212]	[-0.464,0.062]	[-0.398,0.035]	[-0.368,0.014]	[-0.188,0.378]	[-0.363,-0.041]
Std dev	0.326	0.263	0.203	0.172	0.154	0.233	0.380
	[0.202,0.512]	[0.156,0.420]	[0.124,0.320]	[0.112,0.263]	[0.103,0.232]	[0.162,0.333]	[0.240,0.589]

Note: Estimation results for the random effects model for mean returns. The first row reports the estimated high-level mean of the average returns across nonfinancial underlyings, for each maturity from 1 to 5 months, and for the RV and IV portfolios; the second row reports the high-level mean of the average returns for financial underlyings; the third row reports the difference between the two high-level means, and the last row reports the estimated cross-sectional standard deviation of means. 95-percent confidence intervals reported in brackets are calculated through Hamiltonian Monte Carlo.

Table 5: Estimates from random effects model for Sharpe ratios

	1 mo	2 mo	3 mo	4 mo	5 mo	RV port.	IV port.
Mean, nonfin.	-0.191	-0.072	0.080	0.176	0.243	-0.363	0.478
	[-0.358,-0.019]	[-0.246,0.104]	[-0.101,0.267]	[-0.011,0.367]	[0.054,0.433]	[-0.586,-0.133]	[0.290,0.660]
Mean, fin.	-0.196	-0.138	-0.090	-0.034	-0.005	-0.235	0.131
	[-0.473,0.086]	[-0.422,0.154]	[-0.383,0.208]	[-0.333,0.272]	[-0.308,0.303]	[-0.610,0.156]	[-0.195,0.448]
Difference	-0.006	-0.066	-0.170	-0.209	-0.247	0.128	-0.346
	[-0.315,0.312]	[-0.378,0.260]	[-0.493,0.161]	[-0.538,0.133]	[-0.580,0.098]	[-0.302,0.576]	[-0.688,-0.010]
Std dev	0.224	0.227	0.233	0.241	0.246	0.425	0.239
	[0.128,0.345]	[0.121,0.360]	[0.116,0.377]	[0.126,0.385]	[0.135,0.386]	[0.271,0.632]	[0.108,0.406]

Note: Results similar to table 4, but for Sharpe ratios.

Table 6: Combining straddles across markets

Panel A: Sharpe ratios

	RV	IV	RV+IV	RV+IV, risk parity
All underlyings	-0.90	0.76	1.34	1.26
Nonfinancials	-0.82	0.75	1.16	1.12
Financials	-0.51	0.12	0.63	0.36

Panel B: Skewness

	RV	IV	RV+IV	RV+IV, risk parity
All underlyings	1.12	1.73	-0.69	0.94
Nonfinancials	1.26	1.49	-0.92	0.72
Financials	1.83	3.33	-0.77	3.27

Note: Sharpe ratios (panel A) and and the skewness (panel B) of different portfolios combining straddles across markets. For each panel, the first row reports a portfolio constructed using straddles from all available markets on each date, the second row using only nonfinancial underlyings, the third row only financial underlyings. Each column corresponds to a different portfolio. The first column is an equal-weighted RV portfolio, the second is an equal-weighted IV portfolio, the third is an equal-weighted long-short IV minus RV portfolio, and the last is the same long short portfolio but weighted by the inverse of the variance.

Table 7: Random effects model for factor risk premia

Panel A	Spot	Spot ²	ΔΙV
Mean	0.32	-0.36	0.49
	[0.13,0.50]	[-0.61,-0.09]	[0.26,0.71]
Std.dev.	0.30	0.59	0.38
	[0.15,0.50]	[0.38,0.89]	[0.20,0.64]

Panel B	Spot	Spot ²	ΔΙV
Mean, nonfin	0.28	-0.42	0.55
	[0.06,0.49]	[-0.71,-0.11]	[0.29,0.81]
Mean, fin	0.40	-0.20	0.31
	[0.06,0.72]	[-0.69,0.29]	[-0.12,0.72]
Diff	0.12	0.22	-0.24
	[-0.26,0.50]	[-0.35,0.79]	[-0.73,0.22]
Std. dev.	0.31	0.60	0.40
	[0.15,0.53]	[0.37,0.91]	[0.20,0.67]

Note: Estimation results of the random-effects model for the risk premia of the three market-specific factors (spot innovation, squared spot innovation, and change in IV), estimated on the cross-section of straddle returns of the 19 markets. Panel A reports the high-level mean and standard deviation estimate, with 95-percent confidence intervals in brackets. Panel B also reports the high-level means of risk premia for financial and nonfinancials underlyings, and their difference.

A.1 Calculating risk prices with unbalanced panels and correlations across markets

In estimating the factor models, we have two complications to deal with: the sample length for each underlying is different, and returns are correlated across underlyings. This section discusses how we deal with those issues.

We have the model

$$E_{T_i}[R_i] = \lambda_i \beta_i + \alpha_i \tag{A.1}$$

where E_{T_i} denotes the sample mean in the set of dates for which we have data for underlying i, R_i is the vector of returns of the straddles, λ_i is a vector of risk prices, β_i is a vector of risk prices, and α_i is a vector of pricing errors. Note that these objects are all true values, rather than estimates. In order to calculate the sampling distribution, we need to know the covariance of the pricing errors. Note that there is also a cross-sectional regression with

$$E_{T_i}[R_i] = a_i + \beta_i E_{T_i}[f_i] + E_{T_i}[\varepsilon_i]$$
(A.2)

where ε_i is a vector of residuals and f_i is a vector of pricing factors. That formula can be used to substitute out returns and obtain

$$\alpha_i = a_i + \beta_i E_{T_i} [f_i] + E_{T_i} [\varepsilon_i] - \lambda_i \beta_i \tag{A.3}$$

Since a_i , λ_i , and β_i are fixed in the true model, the distribution of α_i depends only on the distributions of the sample means $E_{T_i}[f_i]$ and $E_{T_i}[\varepsilon_i]$. Denoting the long-run (e.g. Newey-West or Hansen-Hodrick) covariance matrix of f_i as Σ_{f_i} and that of ε_i as Σ_{ε_i} , we have

$$var\left(\alpha_{i}\right) = \beta_{i}T_{i}^{-1}\Sigma_{f}\beta_{i}' + T_{i}^{-1}\Sigma$$
(A.4)

Since the λ_i are estimated from a regression, if we denote their estimates as $\hat{\lambda}_i$, we obtain the usual formula for the variance of $\hat{\lambda}_i - \lambda_i$

$$var\left(\hat{\lambda}_i - \lambda_i\right) = \left(\beta_i'\beta_i\right)^{-1} \beta_i'var\left(\alpha_i\right) \beta_i \left(\beta_i'\beta_i\right)^{-1}$$
(A.5)

$$= \Sigma_f + (\beta_i'\beta_i)^{-1} \beta_i' \Sigma_{\varepsilon_i} \beta_i (\beta_i'\beta_i)^{-1}$$
(A.6)

Beyond the variance of $\hat{\lambda}_i$, we also need to know the covariance of any pair of estimates, $\hat{\lambda}_i$ and $\hat{\lambda}_j$. Using standard OLS formulas, we have

$$\begin{bmatrix} \hat{\lambda}_i - \lambda_i \\ \hat{\lambda}_j - \lambda_j \end{bmatrix} = \begin{bmatrix} (\beta_i' \beta_i)^{-1} \beta_i' \alpha_i \\ (\beta_j' \beta_j)^{-1} \beta_j' \alpha_j \end{bmatrix}$$
(A.7)

$$= \begin{bmatrix} (\beta_i'\beta_i)^{-1} \beta_i' (\beta_i E_{T_i} [f_t] + E_{T_i} [\varepsilon_{j,t}]) \\ (\beta_j'\beta_j)^{-1} \beta_j' (\beta_2 E_{T_j} [f_t] + E_{T_j} [\varepsilon_{j,t}]) \end{bmatrix}$$
(A.8)

The covariance between $\hat{\lambda}_i$ and $\hat{\lambda}_j$ is then

$$\frac{T_{12}}{T_1 T_2} \left(\Sigma_{f,i,j} + \left(\beta_1' \beta_1 \right)^{-1} \beta_1' \Sigma_{\varepsilon,i,j} \beta_2 \left(\beta_2' \beta_2 \right)^{-1} \right) \tag{A.9}$$

where $\Sigma_{f,i,j}$ and $\Sigma_{\varepsilon,i,j}$ are now long-run covariance matrices (again from the Newey–West or Hansen–Hodrick method). Using these formulas, we then have estimates of risk prices in each market individually along with a full covariance matrix of all the estimates.

A.2 Calculating the covariance of the sample mean returns

There are two features of our data that make calculating covariance matrix of sample means difficult: we have an unbalanced panel and the covariance matrix is either singular or nearly so. We deal with those issues through the following steps.

1. For each market, we estimate the two largest principal components, therefore modeling straddle returns for underlying i and maturity n on date t as

$$r_{i,n,t} = \lambda_{1,i,n} f_{1,i,t} + \lambda_{2,i,n} f_{2,i,t} + \theta_{i,n,t}$$
(A.10)

where the λ are factor loadings, the f are estimated factors, and θ is a residual that we take to be uncorrelated across maturities and markets (it is also in general extremely small).

2. We calculate the long-run covariance matrix of all $J \times 2$ estimated factors. The covariance matrix is calculated using the Hansen–Hodrick method to account for the fact that the returns are overlapping. The elements of the covariance matrix are estimated based on the available nonmissing data for the associated pair of factors. That means that the covariance matrix need not be positive semidefinite. To account for that fact, we set all negative eigenvalues of the estimated covariance matrix to zero.

Given the estimated long-run covariance matrix of the factors, denoted Σ_f , and given the (diagonal) long-run variance matrix of the residuals θ , denoted Σ_{θ} , the long-run covariance matrix of the returns is then

$$\Sigma_r \equiv \Lambda \Sigma_f \Lambda' + \Sigma_\theta \tag{A.11}$$

where Λ is a matrix containing the factor loadings λ .

3. Finally, it is straightforward to show that the covariance matrix of the sample mean returns is

$$\Sigma_{\hat{r}} = M \odot \Sigma_r \tag{A.12}$$

where \odot denotes the elementwise product and M is a matrix where the element for a given return pair is equal to the ratio of the number of observations in which both returns are available to the product of the number of observations in which each return is available individually (if all returns had the same number of observations T, then we would obtain the usual T^{-1} scaling). We then have the approximation that

$$\hat{r} \Rightarrow N\left(\bar{r}, \Sigma_{\hat{r}}\right)$$
 (A.13)

where \hat{r} is a vector that stacks the \hat{r}_i and \bar{r} stacks the \bar{r}_i and \Rightarrow denotes convergence in distribution.

A.3 Data filters and transformations

We focus primarily on data on futures and option prices from the CME. The observed option prices very often appear to have nontrivial measurement errors. This section describes the various filters we apply and then proceeds to provide more information about the specifics of the data transformations we apply. The full details of the implementation are too numerous to describe here. The code is the definitive record and is available on request.

First, we note that the price formats for futures and strike prices for many of the commodities change over time. That is, they will move between, say, 1/8ths, 1/16ths, and pennies. We make the prices into a consistent decimal time series for each commodity by inspecting the prices directly and then coding by hand the change dates.

We then remove all options with the following properties

- 1. Strikes greater than 5 times the spot
- 2. Options with open interest below the 5th percentile across all contracts in the sample
- 3. Price less then 5 ticks above zero
- 4. Maturity less than 9 days
- 5. Maturity greater than 7 years
- 6. Volume equal to zero or missing
- 7. Options with prices below their intrinsic value (the value if exercised immediately)

We then calculate implied volatilities using the Black formula, treating the options as though they are European. We have also replicated the analysis using American implied volatilities and find nearly identical results (the reason is that in most cases we ultimately end up converting the IVs back into prices, meaning that any errors in the pricing formula are largely irrelevant – it is just a temporary data transformation, rather than actually representing a volatility calculation).

The data are then further filtered based on the IVs:

- 1. Eliminate all zero or negative IVs
- 2. All options with IV more than 50 percent (in proportional terms) different from the average for the same underlying, date, and maturity
- 3. We then filter outliers along all three dimensions, strike, date, and maturity, removing the following:
 - (a) If the IV changes for a contract by 15 percent or more on a given day then moves by 15 percent or more in the opposite direction in a single day within the next week, and if it moves by less than 3 percent on average over that window, for options with maturity greater than 90 days (this eliminates temporary large changes in IVs that are reversed that tend to be observed early in the life of the options).
 - (b) If the IV doubles or falls by half in either the first or last observation for a contract
 - (c) If, looking across maturities at a given strike on a given date, the IV changes by 20 percent or more and then reverses by that amount at the next maturity (i.e. spikes at one maturity). This is restricted to maturities within 90 days of each other.
 - (d) If the last, second to last, or third to last IV is 40 percent different from the previous maturity.
 - (e) If, looking across strikes at a given maturity on a given date, the IV changes by 20 percent and reverses at the next strike (for strikes within 10 percent of each other).
 - (f) If the change in IV at the first or last strike is greater than 20 percent, or the change at the second or second to last option is greater than 30 percent.

At-the-money (ATM) IVs are constructed by averaging the IVs of the options with the first strike below and above the forward price. The ATM IV is not calculated for any observation where we do not have at least one observation (a put or a call) on either side of the forward price.

We next fit the SVI model of IVs from Gatheral and Jacquier (2011, 2014). the SVI model is a five-parameter curve fit to the IVs that typically fits IVs well. We use it for the purpose of interpolating and extrapolating when calculating model-free implied volatilities (which are an integral over all strikes). In fitting the SVI model to a particular date/maturity combination, we also include data from the two dates just before and after the current date, weighting them by a multiple of 1/8 compared to the target date. This helps provide the estimation more information, but at a cost of causing some blurring. The weights are set so that 75 percent of the weight in the estimation is from the target date and 25 percent from the neighboring dates. We eliminate

from this part of the estimation any options with strikes more than three ATM-implied standard deviations away from the spot.

We calculate the VIX from the IVs fit by the SVI curve. Since the IVs were originally calculated from the observed option prices using the Black model, we go back to prices (which are the inputs to the VIX formula) using the Black model again. We also, for robustness, calculate straddle prices based on the IVs from the SVI fit.

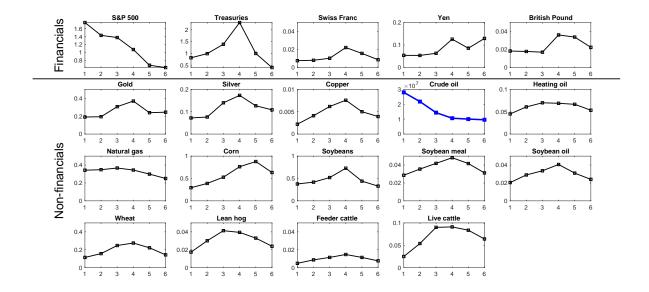
To calculate ATM straddle returns, we first construct returns for straddles with all observable strikes. This is done both using the IV observed at each strike and also, for robustness, using the IV fitted by SVI. We calculate ATM straddle returns by averaging across the two closest strikes above and below the current spot price as long as they are less than 0.5 ATM standard deviations from the spot. Denote the returns on the four straddles in order of increasing strike as R_1 to R_4 , with associated strikes S_1 to S_4 . The interpolated return is then

$$\frac{1}{2} \left(R_2 \frac{S_3 - F}{S_3 - S_2} + R_3 \frac{F - S_2}{S_3 - S_2} \right) + \frac{1}{2} \left(R_1 \frac{S_4 - F}{S_4 - S_1} + R_1 \frac{F - S_1}{S_4 - S_1} \right) \tag{A.14}$$

That is, we linearly interpolate pairwise through R_2 and R_3 and then R_1 and R_4 and average across those two interpolations. The reason to use four straddles instead of two is to try to reduce measurement error. The linear interpolation ensures that the portfolio has an average strike equal to the forward price F. If there is only one straddle available on either side of the forward price, we then interpolate using just a single pair of options, the nearest to the money on either side of the forward price.

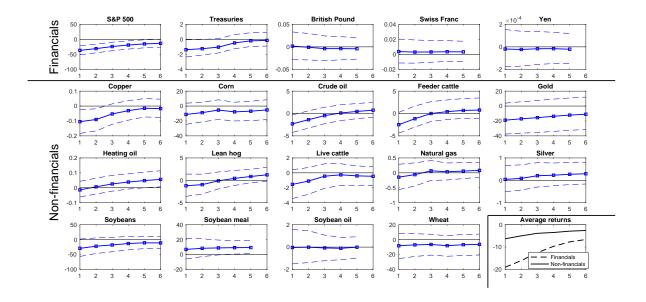
To calculate returns at standardized maturities, we again interpolate. If there are options available with maturities on both sides of the target maturity and they both have maturities differing from the target by less than 60 days, then we linearly interpolate. If options are not available on both sides of the target, then we use a single option if it has a maturity within 35 days of the target. This does mean that the maturity of the option used for a portfolio at a desired maturity can deviate from the target.

Figure A.1: Volume across markets and maturities



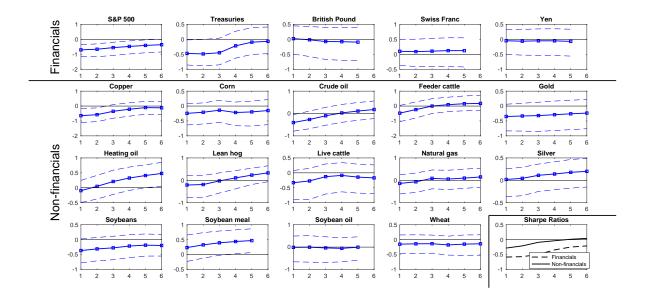
Note: Average daily volume of options in different markets. The panel corresponding to crude oil reports values in dollars. All other panels show values relative to the volume in the crude oil market, matched by maturity.

Figure A.2: Average returns (unscaled)



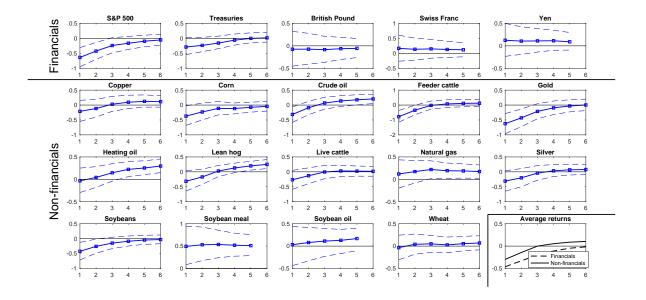
Note: Same as Figure 2, but using unscaled returns.

Figure A.3: Sharpe ratios (unscaled)



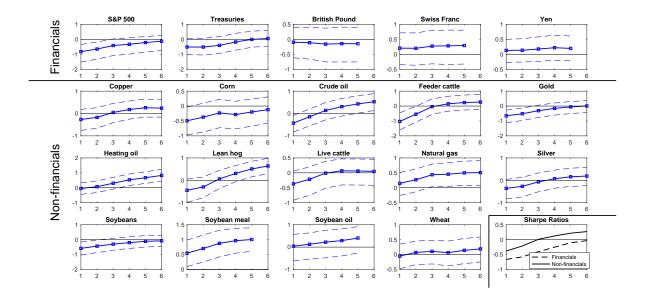
Note: Same as Figure 3, but using unscaled returns.

Figure A.4: Average returns (delta-hedged)



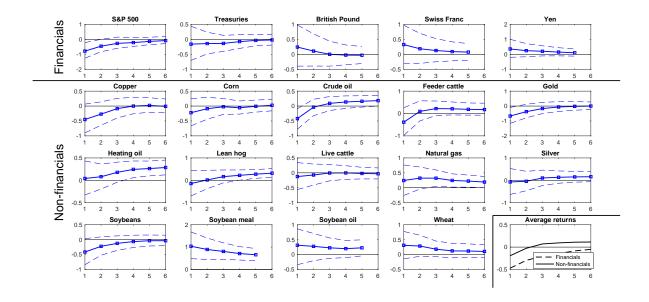
Note: Same as Figure 2, but using delta-hedged returns.

Figure A.5: Sharpe ratios (delta-hedged)



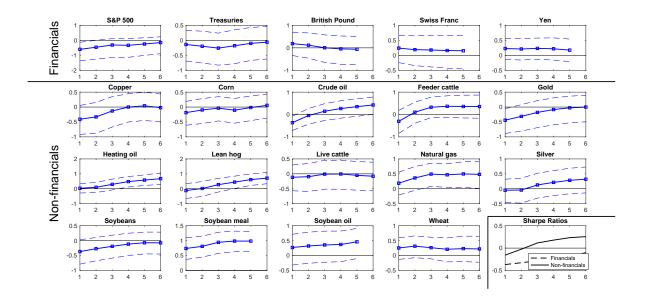
Note: Same as Figure 3, but using delta-hedged returns.

Figure A.6: Average returns (7-day holding period)



Note: Same as Figure 2, but using a 7-day holding period.

Figure A.7: Sharpe ratios (7-day holding period)



Note: Same as Figure 3, but using a 7-day holding period.

Table A.1: Correlations of realized volatilities across markets

RV	Treasuries	S&P 500	Swiss Franc	Yen	British Pound	Plog	Silver	Copper	Crude oil	Heating oil	Natural gas	Corn	Soybeans	Soybean meal	Soybean oil	Wheat	Lean hog	Feeder cattle
S&P 500	0.63																	
Swiss Franc	0.17	0.12																
Yen	0.31	0.32	0.15															
British Pound	0.43	0.36	0.24	0.31														
Gold	0.44	0.47	0.15	0.24	0.31													
Silver	0.42	0.43	0.15	0.22	0.27	0.65												
Copper	0.51	0.51	0.11	0.24	0.43	0.50	0.52											
Crude oil	0.24	0.24	0.13	0.20	0.20	0.32	0.14	0.24										
Heating oil	0.20	0.22	0.04	0.14	0.15	0.30	0.11	0.15	0.91									
Natural gas	0.03	0.08	0.04	-0.04	0.00	0.05	-0.06	0.00	0.08	0.18								
Corn	0.33	0.35	0.04	0.09	0.27	0.37	0.40	0.49	0.12	0.03	-0.04							
Soybeans	0.33	0.30	0.03	0.16	0.30	0.33	0.35	0.40	0.11	0.05	-0.07	0.74						
Soybean meal	0.33	0.25	0.03	0.19	0.19	0.31	0.32	0.30	0.08	0.02	-0.06	0.68	0.94		_			
Soybean oil	0.48	0.43	0.11	0.21	0.41	0.40	0.41	0.51	0.17	0.12	-0.04	0.67	0.88	0.72				
Wheat	0.30	0.24	0.02	0.08	0.11	0.31	0.34	0.33	0.11	0.04	-0.08	0.63	0.51	0.47	0.47			
Lean hog	0.12	0.12	0.08	0.20	-0.03	0.00	0.00	0.05	0.10	0.09	0.11	0.06	0.11	0.12	0.11	0.12		
Feeder cattle	0.22	0.20	0.03	0.04	0.06	0.10	0.16	0.30	0.10	0.07	0.12	0.35	0.32	0.32	0.27	0.22	0.26	
Live cattle	0.41	0.24	0.13	0.11	0.11	0.17	0.24	0.28	0.07	0.07	0.09	0.22	0.22	0.27	0.30	0.23	0.28	0.63

Note: Pairwise correlations of monthly realized volatility across markets. Darker areas indicate larger absolute-value t-statistics for the correlation.

Table A.2: Correlations of implied volatilities across markets

IV	Treasuries	S&P 500	Swiss Franc	Yen	British Pound	Plog	Silver	Copper	Crude oil	Heating oil	Natural gas	Corm	Soybeans	Soybean meal	Soybean oil	Wheat	Lean hog	Feeder cattle
S&P 500	0.65																	
Swiss Franc	0.46	0.35																
Yen	0.39	0.57	0.49															
British Pound	0.46	0.42	0.76	0.48														
Gold	0.56	0.54	0.20	0.32	0.36													
Silver	0.46	0.34	0.21	0.33	0.34	0.79												
Copper	0.43	0.46	0.09	0.39	0.33	0.76	0.79											
Crude oil	0.47	0.61	0.27	0.44	0.30	0.52	0.33	0.48										
Heating oil	0.44	0.61	0.20	0.38	0.20	0.51	0.31	0.51	0.93									
Natural gas	0.11	0.38	-0.02	0.07	0.05	0.22	0.00	0.34	0.45	0.58								
Corn	0.32	0.33	-0.02	0.15	0.16	0.48	0.54	0.61	0.23	0.20	0.02							
Soybeans	0.29	0.29	-0.01	0.18	0.18	0.45	0.47	0.63	0.29	0.28	0.10	0.84						
Soybean meal	0.32	0.29	-0.08	0.18	0.05	0.53	0.49	0.63	0.28	0.28	0.12	0.79	0.91					
Soybean oil	0.33	0.26	0.13	0.16	0.27	0.45	0.48	0.61	0.25	0.27	0.10	0.70	0.87	0.80				
Wheat	0.44	0.38	0.04	0.20	0.13	0.61	0.62	0.63	0.33	0.33	0.07	0.81	0.72	0.71	0.59			
Lean hog	0.29	0.39	-0.01	0.33	-0.05	0.24	0.13	0.36	0.40	0.49	0.37	0.26	0.37	0.36	0.34	0.34		
Feeder cattle	0.46	0.36	0.15	0.20	0.10	0.41	0.53	0.58	0.28	0.33	0.11	0.51	0.46	0.52	0.46	0.55	0.37	
Live cattle	0.44	0.26	0.18	0.18	0.07	0.35	0.40	0.49	0.32	0.41	0.23	0.31	0.35	0.42	0.44	0.42	0.46	0.78

Note: Pairwise correlations of monthly 1-month implied volatilities across markets. Darker areas indicate larger absolute-value t-statistics for the correlations.

Table A.3: Risk exposures of RV and IV portfolios

RV portfolio					IV portfolio				
	Spot	Spot ²	ΔΙV	\mathbb{R}^2		Spot	Spot ²	ΔΙV	\mathbb{R}^2
S&P 500	-0.06	1.31	0.05	0.71	S&P 500	0.04	0.24	0.90	0.71
T-bonds	-0.01	0.80	-0.02	0.81	T-bonds	-0.11	1.17	0.80	0.62
GBP	-0.02	0.73	0.06	0.84	GBP	0.07	0.39	0.70	0.78
CHF	-0.02	0.68	0.07	0.83	CHF	0.07	0.48	0.70	0.85
JPY	-0.02	0.70	0.06	0.81	JPY	0.02	0.34	0.69	0.74
Copper	-0.01	0.75	-0.01	0.62	Copper	0.16	0.31	0.77	0.66
Corn	-0.02	0.62	0.04	0.73	Corn	0.14	0.16	0.73	0.80
Crude oil	-0.02	0.97	0.01	0.75	Crude oil	0.10	0.08	0.93	0.70
Feeder cattle	-0.02	0.83	0.00	0.74	Feeder cattle	0.15	-0.25	0.78	0.64
Gold	-0.01	0.66	0.05	0.71	Gold	0.16	-0.30	0.80	0.58
Heating oil	-0.01	0.84	0.01	0.78	Heating oil	0.17	-0.52	0.78	0.52
Lean hog	0.00	0.81	0.00	0.76	Lean hog	0.18	0.32	0.69	0.69
Live cattle	-0.02	0.95	0.01	0.74	Live cattle	0.15	0.24	0.76	0.76
Natural gas	-0.01	0.83	0.03	0.81	Natural gas	0.17	0.16	0.79	0.70
Silver	-0.01	0.59	0.06	0.77	Silver	0.18	0.27	0.79	0.68
Soybeans	-0.02	0.63	0.03	0.73	Soybeans	0.17	0.22	0.83	0.75
Soybean meal	-0.02	0.57	0.02	0.71	Soybean meal	0.07	-0.49	0.83	0.48
Soybean oil	-0.02	0.62	0.00	0.76	Soybean oil	0.02	-0.31	0.85	0.68
Wheat	-0.01	0.61	0.00	0.77	Wheat	0.03	-0.55	0.79	0.62
Average	-0.02	0.76	0.03	0.76	Average	0.10	0.10	0.79	0.68

Note: The table reports regression coefficients of the RV portfolios (left panel) and IV portfolios (right panel) for each market onto three market-specific factors: the spot return, the squared spot return, and the change in IV.

Table A.4: Factor model estimation with S&P 500 and average factors

	(1) S&P 500 factors	(2) Average factors
Spot risk price	0.057	0.991 *
	(0.039)	(0.574)
Spot ² risk price	0.003	-0.576 ***
	(0.005)	(0.212)
ΔIV risk price	0.029	0.616 **
	(0.019)	(0.307)
Cross-sectional R ²	0.152	0.372
Avg. time-series R ²	0.018	0.146

Note: The table reports the estimation results for two different specifications of a three-factor model (spot innovation, squared spot innovation, and change in IV), on the cross-section of straddle returns on the 19 markets. The left column uses the three factors specific to the S&P 500, and it excludes the S&P 500 contract from the set of test assets. The right column constructs each of the three factors by averaging the 19 market-specific factors. The table reports the risk premia estimates and its standard error, as well as the cross-sectional \mathbb{R}^2 achieved by the model and the average time-series \mathbb{R}^2 for each test asset.

Table A.5: Random effect model for factor risk prices (SDF coefficients)

Panel A	Spot	Spot ²	ΔΙV
Mean	0.44	-0.80	0.82
	[0.22,0.65]	[-1.18,-0.42]	[0.47,1.17]
Std.dev.	0.41	0.91	0.74
	[0.23,0.65]	[0.59,1.37]	[0.44,1.14]

Panel B	Spot	Spot ²	ΔΙV
Mean, nonfin	0.40	-0.90	0.93
	[0.13,0.65]	[-1.35,-0.45]	[0.52,1.32]
Mean, fin	0.50	-0.48	0.49
	[0.09,0.90]	[-1.23,0.27]	[-0.19,1.13]
Diff	0.11	0.42	-0.44
	[-0.38,0.57]	[-0.44,1.30]	[-1.20,0.31]
Std. dev.	0.43	0.94	0.75
	[0.24,0.70]	[0.59,1.42]	[0.45,1.19]

Note: The table reports the estimation results of the random-effect model for the risk prices (coefficients of the SDF) of the three market-specific factors (spot innovation, squared spot innovation, and change in IV), estimated on the cross-section of straddle returns of the 19 markets. Panel A reports the high-level mean and standard deviation estimate, with 90% confidence intervals. Panel B also reports the high-level means of risk prices for financial and nonfinancials underlyings, and their difference.