Financial Frictions and Export Dynamics in Large Devaluations: Online Appendix

(For Online Publication)

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Contents

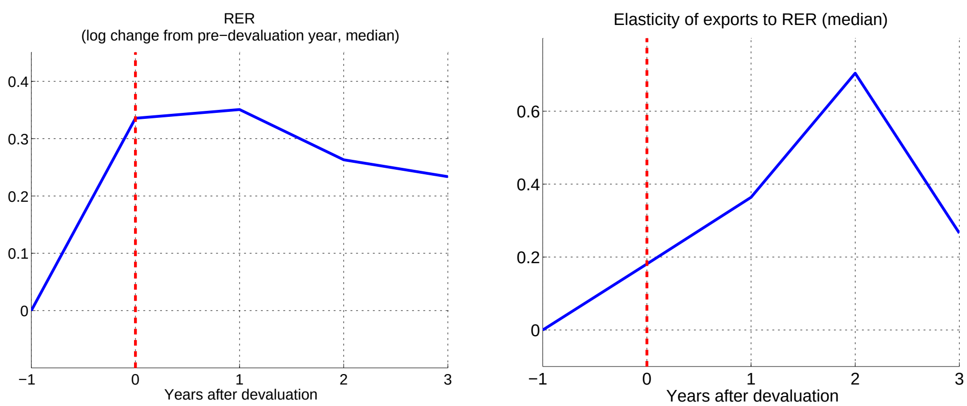
1 Cross-country empirical evidence

In this section, we document the facts that motivate our analysis using cross-country evidence. We first investigate the real exchange rate and aggregate exports dynamics in a sample of large devaluations over the past three decades. Next, we present evidence on the currency composition of debt at the firm level. Finally, we examine the extent to which firms are credit constrained in these economies.

1.1 Real exchange rate and export dynamics in large devaluations

We define the real exchange rate as the relative value of foreign to domestic prices measured in domestic units and we define large devaluations as year-to-year increases of the real exchange rate above 20%. We restrict our attention to the period between 1980 and 2013. Using this definition, we identify 12 episodes of large devaluations in our dataset: Argentina (2002), Brazil (1999), Iceland (2008), Indonesia (1998), South Korea (1998), Malaysia (1998), Mexico (1982, 1986, 1994), Turkey (2001), and Venezuela (2002, 2010).[1]

Figure 1: Aggregate dynamics of the RER and real exports



Panel A Panel B

Source: Multilateral effective real exchange rate (RER) from BIS; real exports data from the World Bank and the International Financial Statistics database published by the IMF.

In Figure 1, we plot the median log-change of the real exchange rate relative to its pre-devaluation level (Panel A) and the median elasticity of real exports to changes in the real exchange rate (Panel B).[2] We see that, following a devaluation, the median real exchange

[1] Our results are robust to defining large devaluations based on alternative thresholds as well as to using data at a quarterly frequency.

[2] More precisely, in Panel A we plot the median value of $\log(\xi_t/\xi_{-1})$, where ξ_t is the real exchange rate at time t and period -1 is the year before the devaluation. In Panel B, we plot $\log(X_t/X_{-1})/\log(\xi_t/\xi_{-1})$, where X_t denotes exports at time t. We detrend the log growth of exports in each country by subtracting its average log growth over the period.

rate increases by approximately 34%, and continues to increase slightly the year after before decreasing steadily over the following two years. However, even four years after a large devaluation, the median real exchange rate is 23% higher than its pre-devaluation level.

Panel B of Figure 1 shows that, despite the large change in the real exchange rate, real exports increase gradually following a devaluation. The exports elasticity increases steadily up to 0.7 three years after the devaluation, before dropping to 0.27. Moreover, the median export elasticity in the year of the devaluation is only 0.18, less than 25% of its peak value. Thus, as in Alessandria et al. (2015), Figure 1 shows that real exports increase slowly after sharp and sudden changes in the real exchange rate.

1.2 Currency composition of liabilities

In this section, we examine the currency composition of debt across manufacturing firms. To do so, we use the World Bank Enterprise Surveys (WBES) dataset, which contains data on firms' characteristics based on representative surveys of private firms conducted in 135 economies. Such surveys have been conducted since 2002 and cover a broad range of topics, including firms' financial position.[3] The dataset covers six of the nine countries that experienced a large devaluation according to our definition. Out of these, only the surveys conducted in Brazil, Indonesia, and Turkey contain information on the share of the firms' debt denominated in foreign and domestic currency. Thus, we limit our study of the currency composition of debt to these three economies.[4]

Table 1: Share of foreign-denominated debt at firm-level

		By export status			*By # of workers*		
	All firms	Non-exporters	Exporters	[0,25]	[26,100]	[101,250]	250+
Fraction of firms	0.25	0.13	0.48	0.12	0.24	0.38	0.57
Average share	0.59	0.59	0.59	0.45	0.55	0.62	0.62

Source: WBES data for Brazil (2003), Indonesia (2003), and Turkey (2005). We report average values across these countries. The average share of foreign-denominated debt is computed across manufacturing firms with foreign debt.

We report our results in Table 1. We observe that firms in our sample tend to have a significant amount of debt denominated in foreign currency, and reliance on such debt is substantially higher among exporters compared to non-exporters: 48% and 13%, respectively. Among firms that have a positive amount of foreign-currency-denominated debt, this debt constitutes on average 59% of their total debt stock both for exporters and non-exporters. Thus, while exporters are substantially more likely to have foreign-denominated debt than

[3]More details about the WBES data can be found at http://www.enterprisesurveys.org.

[4]All surveys were conducted within five years of the devaluation episodes. Results are very similar when computed for all countries for which there are data available on the currency composition of debt.

non-exporters, those that do so tend to issue a similar fraction of their debt in foreign currency. Finally, the last four rows of Table 1 present these statistics for firms of different sizes. We see that larger firms are more likely to have foreign-currency-denominated debt, although this relationship is not as stark for the fraction of debt these firms hold in foreign currency.

1.3 Share of credit-constrained firms

Given the prevalence of foreign-denominated debt documented in the previous subsection, large changes in real exchange rates may lead to substantial increases in the domestic value of the total stock of debt. However, to the extent that firms are not credit constrained, such increases in the debt burden are not likely to affect real outcomes. Thus, we conclude this section by documenting the extent to which firms are credit constrained in these episodes.

To do so, we restrict attention to manufacturing firms, using firm-level data collected by the WBES. Out of the devaluation countries identified above, only the surveys conducted in Brazil, Indonesia, Malaysia, and Turkey contain information on the share of credit-constrained firms. We focus on two questions asked by the survey. The first question asks managers to report the extent to which they find access to finance to be an obstacle for their operation and growth. The second question asks managers to classify the extent to which they find the cost of finance to be an obstacle for their operation and growth. They are given five options: no obstacle, minor obstacle, moderate obstacle, major obstacle, or very severe obstacle. We define firms to be credit constrained if they find access or cost of finance to be at least a moderate obstacle.

Table 2 reports the share of firms that find the access and cost of finance to be at least a moderate obstacle for their growth and operation. We find that a significant share of firms (53%) are credit constrained in their access to finance, while an even larger share (60%) find the cost of finance to be a significant constraint. Moreover, we find that this is also the case for both exporters and non-exporters, as reported in the second and third rows of the table: In fact, exporters appear to be more credit constrained than non-exporters.

Table 2: Share of credit-constrained firms

		By export status		*By # of workers*			
	All firms	Non-exporters	Exporters	[0,25]	[26,100]	[101,250]	250+
Access to finance	0.53	0.51	0.56	0.52	0.51	0.54	0.51
Cost of finance	0.60	0.56	0.65	0.55	0.59	0.65	0.63

Source: WBES data for Brazil (2003), Indonesia (2003), Malaysia (2002), and Turkey (2005). We report average values across these four countries. The averages for each country are computed across manufacturing firms.

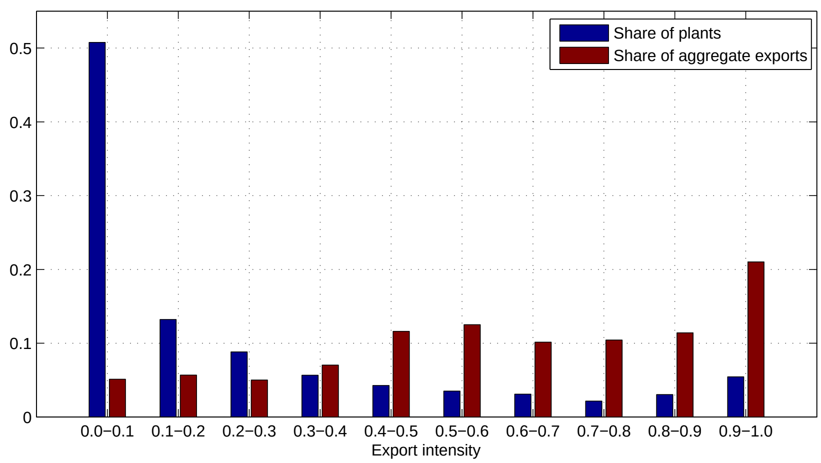
In the last four rows of the table, we report the share of credit-constrained firms across the size distribution, as measured by the number of workers. This table shows that the share of constrained firms is approximately constant and independent of firm size. Thus, while larger firms are more likely to hold foreign-currency debt, as shown in the previous subsection, they are also likely to be credit constrained in both the access to and cost of finance.[5]

This evidence suggests that credit frictions are important constraints on firms' growth and operation in the devaluation countries. Thus, we conclude that significant credit frictions were likely present when the devaluations took place, potentially affecting the dynamics of exports following these episodes.

2 Export intensity heterogeneity

In Section 4, we showed that the extent to which firms can increase exports by reallocating sales across markets depends on their initial export intensity. Therefore, in order to discipline the importance of this channel, we examine the degree of export intensity heterogeneity observed in the data across firms.

Figure 2: Export-intensity distribution in Mexico, 1994



We find that there is substantial heterogeneity in export intensity across firms. Figure 2 shows that, while export intensity is 0.23 on average (i.e., on average, exporters sell 23% of their sales to foreign markets), most exporters feature much lower export intensity and few of them sell most of their production to foreign markets. In particular, for approximately half of all exporters, their foreign sales constitute only 10% of total production, while almost 17% of exporters sell more than 50% of their output internationally.

[5] We also find firms that have debt denominated in foreign currency are slightly less constrained than firms that do not have, both in their access and cost of finance. Results are similar when we compute these statistics for all countries with WBES data available.

To discipline the extent to which sales reallocation across markets affects aggregate export dynamics, we extend the model to feature differences in export intensity across firms. We assume that there are two types of firms in the model: (i) a fraction ζ of firms that are subject to low iceberg export costs, τ_L, leading to high export intensity, and (ii) a fraction $1-\zeta$ of firms that face high iceberg export costs, τ_H, leading to low export intensity.

Table 3: Heterogeneity in export intensity in Mexico, 1994

Export intensity	Share of exports	Share of exporters	Avg. export intensity
0.0 - 0.6	0.47	0.87	0.13
0.6 - 1.0	0.53	0.13	0.84

We map these two types of exporters into the data by classifying them based on their export intensity. In particular, we divide exporters into low-export-intensity and high-export-intensity groups such that each category accounts for approximately half of aggregate exports. As shown in Table 3, the first group contains all firms that export less than 60% of their production, accounting for 47% of aggregate exports. It includes 87% of all exporters, and the average export intensity within this group is only 13%. The second group contains all firms with export intensity higher than 60% of their production, accounting for 53% of aggregate exports.[6]

3 Additional results

In this section, we show additional results and sensitivity analysis that complement those presented in the paper.

3.1 Shocks

In the main quantitative exercise of the paper we investigate the extent to which financial frictions and balance-sheet effects can account for the dynamics of aggregate exports observed in the data. Since many shocks might have hit Mexico during its large devaluation in 1994, we consider a broad array of shocks and use the data targets (real exchange rate, ratio of investment to output, and real GDP) to identify them.

To understand the role played by borrowing constraints and foreign-denominated debt in shaping the response of the economy, we contrast the dynamics implied by our baseline model with the dynamics implied by its frictionless counterpart. This alternative model is calibrated separately and is subject to an alternative sequence of shocks to $p_{m,t}$, r_t, and A_t, chosen to ensure that it also matches the dynamics of the real exchange rate, investment, and real GDP observed in the data.

[6]Defever and Riano (2017) use cross-country data to document that export intensity typically features "twin peaks," with some firms exporting a lot of their output and others a little.

Figure 3: Shocks to price of imports, aggregate productivity, and interest rate

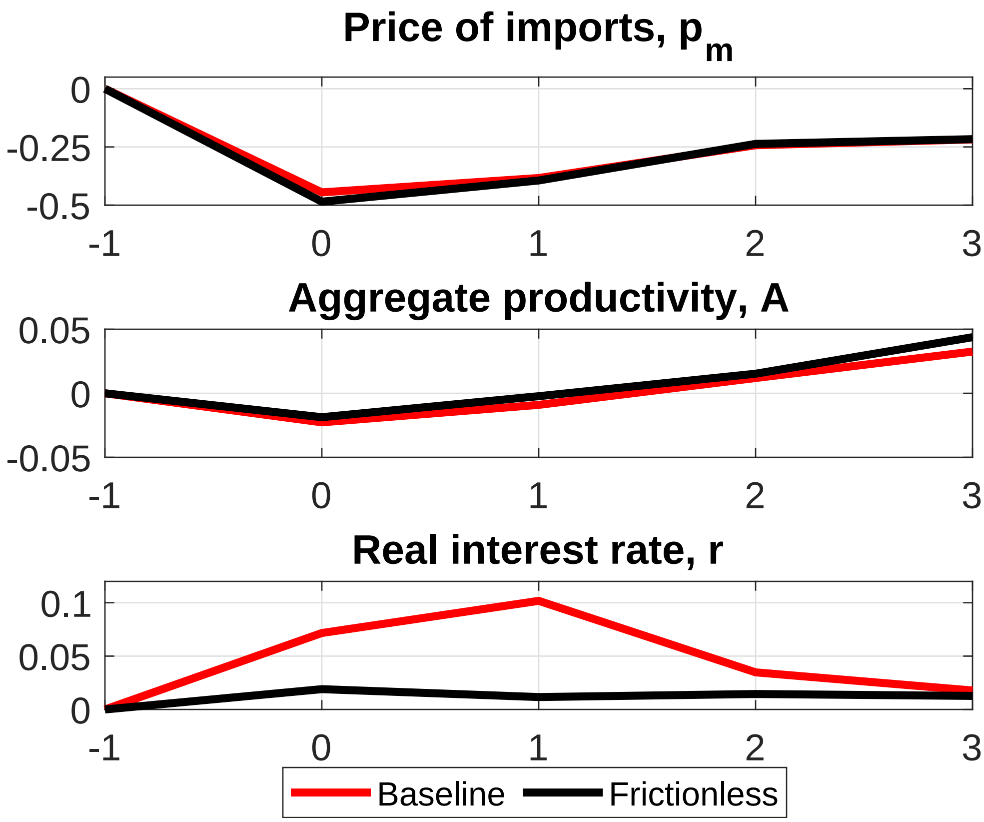
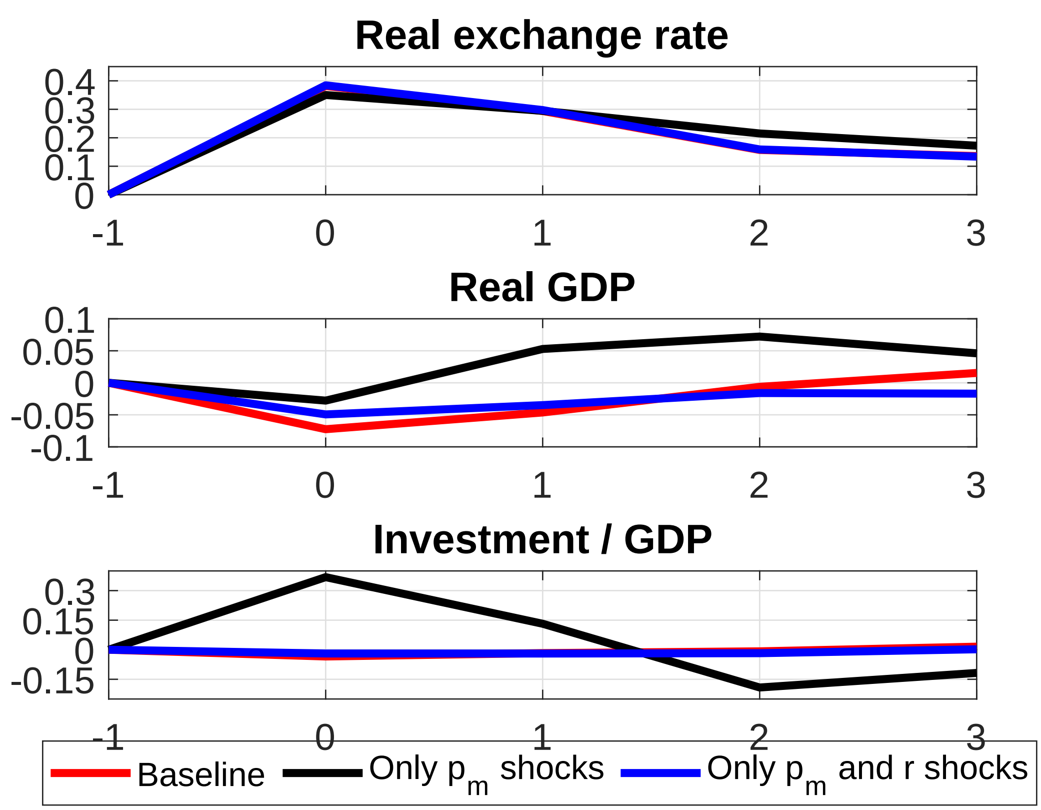


Figure 3 contrasts the sequence of shocks in the baseline economy to those in the frictionless setup. The figure plots, for each variable, the difference between its value at year t after the devaluation ($t = 0$ is the devaluation period) and its initial steady state value ($t = -1$). Since $p_{m,-1} = 1$ and $A_{-1} = 1$, these differences can be interpreted as percentage changes with respect to the initial steady state value. For the real interest rate, the figure reports the percentage points change with respect to the initial steady state value.

As shown in the figure, the shocks to the price of imports and productivity are almost identical in both economies. These shocks allow the models to match the large devaluation and output dynamics observed in the data. Finally, the increase in the real interest rate is much sharper in the baseline model than in the frictionless counterpart, but the differences are attenuated once one accounts for the effect of the real exchange rate on the effective interest rate faced by entrepreneurs in the baseline model.

3.2 Shocks decomposition

Figure 4: Shocks decomposition



In the quantitative analysis, we choose the shocks to the price of imports, aggregate productivity and real interest rate to match the joint dynamics of the real exchange rate, real GDP and investment observed in the data. In this section, we examine the individual importance of each of these shocks to match each of the targets in the baseline model.

Figure 4 plots the dynamics of the real exchange rate, real GDP and the ratio of investment to output in the model in response to three different sequence of shocks: (i) the ones we feed in the baseline calibration ("Baseline"); (ii) the same shocks to p_m and r as in the baseline calibration but keeping A fixed at its initial steady state value ("Only p_m and r shocks"); and (iii) the same shock to p_m as in the baseline calibration but keeping A and r fixed at their initial steady state values ("Only p_m shocks").

As can be seen in Figure 4, shocks to the price of imports play a key role in accounting for the dynamics of the real exchange rate observed in the data. The economy with just shocks to the price of imports can largely replicate the behavior of the real exchange rate. These might be interpreted as terms-of-trade shocks. However, the model with just shocks to the price of imports cannot reproduce the severity of the recession or the sharp drop in investment observed at the moment of the devaluation, and displays a counter-factually fast recovery of real GDP. Once we allow for shocks to the real interest rate in addition to the price of imports, the model is able to reproduce the dynamics of investment and the real exchange rate, although it displays a smoother behavior of real GDP than observed in the data. Finally, the baseline model, which also features aggregate productivity shocks to entrepreneurs, matches the three targets in the data almost exactly.

3.3 Same shocks in both models

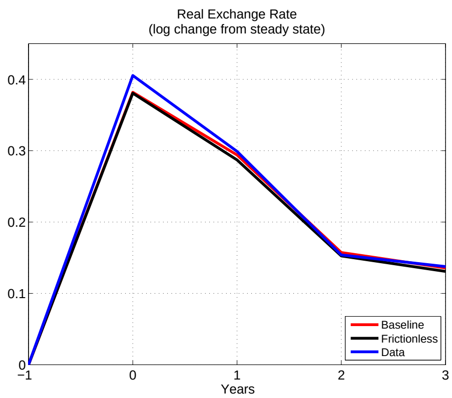
We believe that separately calibrating the shocks to match the dynamics of the empirical targets moments provides the sharpest comparison between the baseline and its frictionless counterpart, making them look as close as possible to each other from the lens of these moments. However, an alternative exercise would be to contrast the dynamics implied by these models when both are subject to the same shocks. Therefore, in Figure 5, we present the results of this alternative exercise, restricting attention to the set of shocks estimated to match the empirical targets in our baseline model.

Figure 5, Panel A, plots the percentage deviation of the real exchange rate from its pre-devaluation, steady-state level for the baseline economy, the frictionless economy, and the data. The figure shows that the same sequence of shocks does a good job in both models to match the observed dynamics in the data.

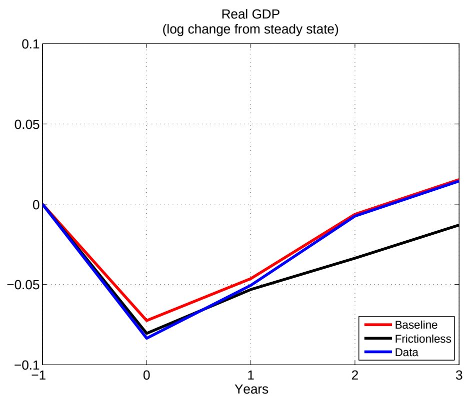
Similarly, Panel B of Figure 5 plots the percentage deviation of real GDP from its pre-devaluation, steady-state level for each of these economies. Consistent with the data, we measure real GDP as a Laspeyres quantity index, keeping prices fixed at their pre-devaluation levels and adjusting quantities over time. On the one hand, real GDP in the baseline model matches closely the dynamics observed in the data, by construction. On the other hand, real GDP in the frictionless economy matches closely the recession at the time of the large devaluation, but then recovers slower than in the data.

Finally, Panel C of Figure 5 shows the change in the investment-to-GDP ratio from its pre-devaluation level. Our baseline model with financial frictions and balance-sheet effects can

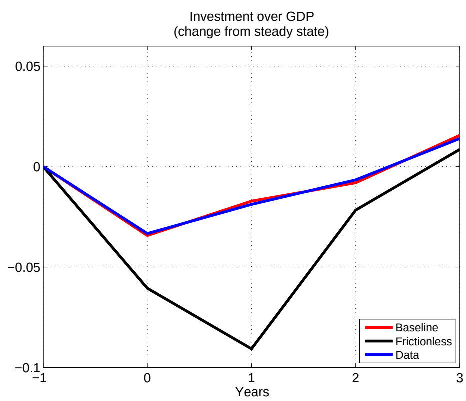
Figure 5: Real exchange rate, real GDP, investment, and exports elasticity



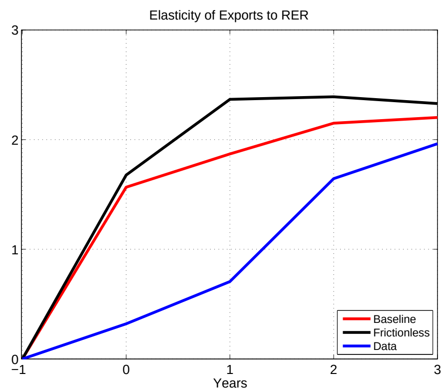
Panel A



Panel B



Panel C



Panel D

closely match the dynamics of the investment-to-GDP ratio observed in the data. However, the frictionless setup subject to the same shocks than the financial frictions economy displays a much sharper and prolonged drop in the investment-to-output ratio, and starts recovering only two years after the large devaluation. Notice that the real interest rate sequence that we feed into the frictionless setup is the same that we feed to the baseline economy, adjusted for the effect of the real exchange rate: the only difference between both models is that in the baseline economy, firms face an additional financing wedge due to financial frictions.

Panel D of Figure 5 shows the response of aggregate exports in the baseline and frictionless models. We find that the absolute percentage deviation between the exports elasticity implied by our baseline model and the data is only 17% lower than implied by the frictionless model, even less than what we found in the main experiment in the paper. Thus, financial frictions and balance sheet effects modestly improve the fit of the model along this dimension, suggesting that the slow growth of exports following a large devaluation is not significantly accounted by them. Thus, this finding is robust to either choosing shocks separately in both models to match the same targets, or choosing the same shocks in both models to match the empirical targets in the baseline economy.

3.4 Trade balance dynamics in the model and in the data

Given our focus on the dynamics of aggregate exports, a set of moments that may serve as validation are the dynamics of the current account after large devaluations. The current account is a key aggregate variable in general equilibrium models with preferences for intertemporal consumption smoothing and is one of the key variables observed by policy makers in response to aggregate shocks. In particular, the dynamics of the current account in our model capture changes in international borrowing and lending around this episode.

While the model features a rich export structure with endogenous choices both in the intensive and extensive margin, the import side is much more stylized. Notwithstanding this simplification, the model does a decent job at replicating the reversal of the current account in the data, as shown in Figure 6.

Figure 6: Trade dynamics in the model and in the 1994 Mexican devaluation

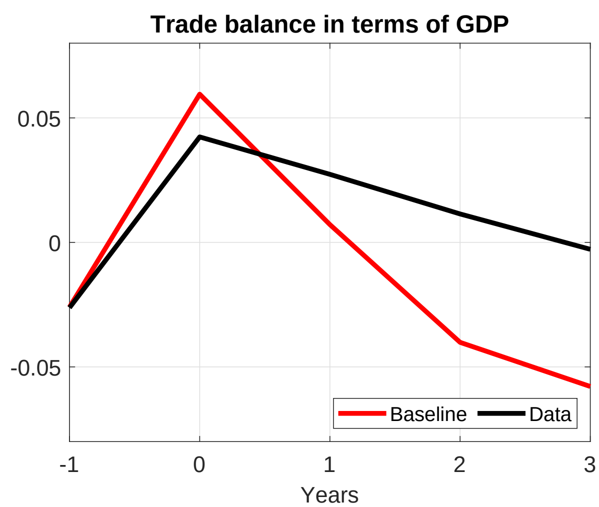


Figure 6 shows the dynamics of the trade balance, as a share of GDP, during Mexico's large devaluation both in the data and in the baseline model. Our measure of the trade balance in the data is the trade balance of goods in terms of GDP, in current US dollars, from the World Bank. Both in the data and in the model there is a trade balance reversal at the period of the devaluation, when the trade balance jumps from a deficit of 2.6% of the GDP to a surplus of 4.2% (6%) in the data (model). Thereafter the trade balance reverts to a deficit faster in the data than in the model. Nevertheless, the overall dynamics are qualitatively similar in the model and the data.

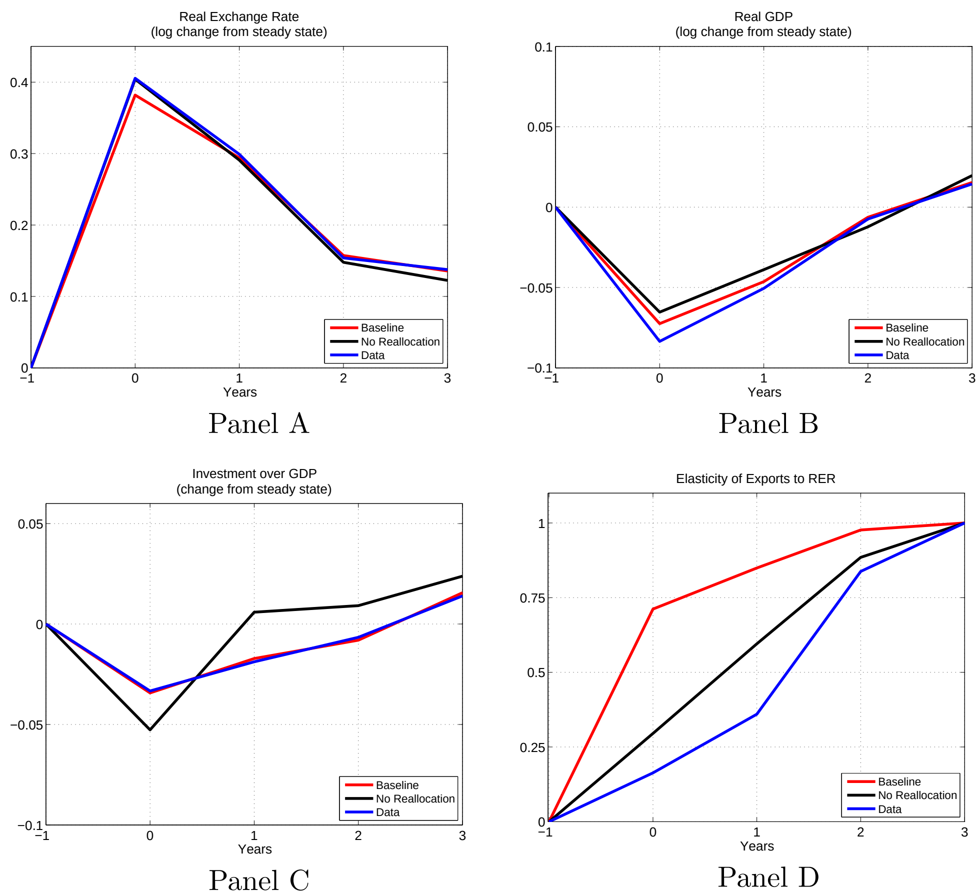
3.5 Baseline vs. No reallocation vs. One-type model

In Section 5 of the paper we report the aggregate export elasticity dynamics corresponding to two counter-factual economies: an economy in which exporters have no margin to reallocate sales across markets in response to shocks ("No reallocation"), and an economy with just one type of firms featuring a higher degree of reallocation than our baseline model ("One type").

We now complement the results presented in the paper by reporting the implications of these economies for the dynamics of the three series targeted in the estimation of the aggregate shocks: (*i*) the real exchange rate, (*ii*) real GDP, and (*iii*) the investment to GDP ratio.

Figures 7 and 8 report the dynamics of these variables corresponding to the economy with no reallocation and one type of firms, respectively. As in the baseline economy, in both of these counter-factual economies we are able to choose the shocks to the price of imports, aggregate productivity, and the interest rate to account well for the dynamics of the real exchange rate, real GDP, and the investment to GDP ratio.

Figure 7: Model with no reallocation



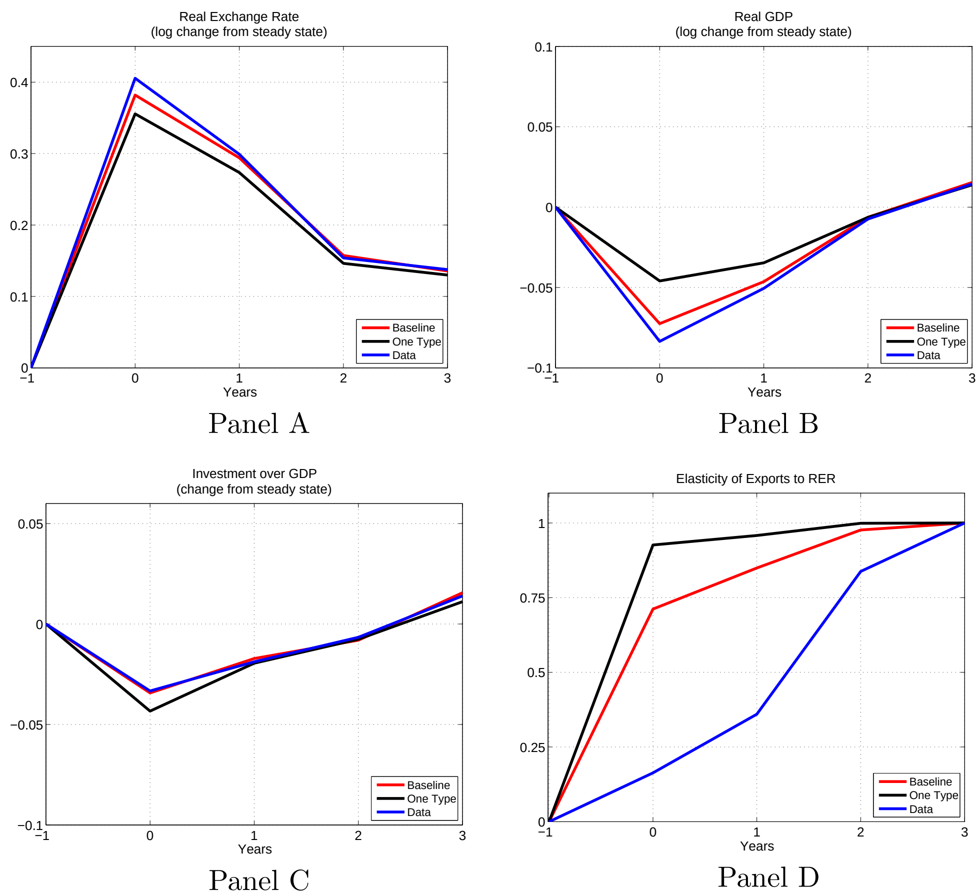
Panel A

Panel B

Panel C

Panel D

Figure 8: Model with one type of firms



4 Additional Evidence

4.1 Imported intermediates

As we discussed in Section 5.4 of the paper, there are alternative channels that could make exports react slowly to large devaluations. One such channel is the role of intermediate import goods. While large devaluations make exporting more attractive, they also make importing more costly. Thus, to the extent that exporters import a non-trivial fraction of their intermediate inputs (Bernard et al. 2007; Kugler and Verhoogen 2009), the higher cost of imports may slow down the adjustment of exports in these episodes. In this section, we evaluate the extent to which this might be the case using microdata on Mexican manufacturing establishments.

One key ingredient of this mechanism is the prevalence of imported intermediate inputs among exporters. If exporters make intensive use of imported intermediates, then large devaluations might not only slow down or depress the adjustment of exports due to credit frictions and foreign-denominated debt but also as a result of imported intermediates becoming more expensive. Table 4 shows that this key ingredient is indeed present in the data: Exporters make intensive use of imported intermediate inputs. First, we observe that the average exporter imports 30% of its intermediate inputs, while this value is only 14% among non-exporters. Second, we observe that high-export-intensity firms (with export intensity higher than 0.6, as defined in the paper) also import a higher fraction of its intermediates than their low-export-intensity counterparts: 35% vs. 29%, respectively. Finally, the second column of the table shows that these patterns of the data are also prevalent when examining imported intermediates relative to total sales rather than relative to total intermediates, which shows that imported intermediates are an important input in production among exporters (and more so among those with high-export-intensity).[7]

Table 4: Imported intermediates: Summary statistics

	Avg. $\frac{\text{Imported intermediates}}{\text{Intermediates}}$	Avg. $\frac{\text{Imported intermediates}}{\text{Sales}}$
Non-exporters	0.14	0.07
Exporters	0.30	0.13
Export intensity $\in (0, 0.6)$	0.29	0.13
Export intensity $\in [0.6, 1]$	0.35	0.18

Given the prevalence of imported intermediate inputs among exporters, we then investigate the extent to which these indeed slow down the adjustment of exports following a large

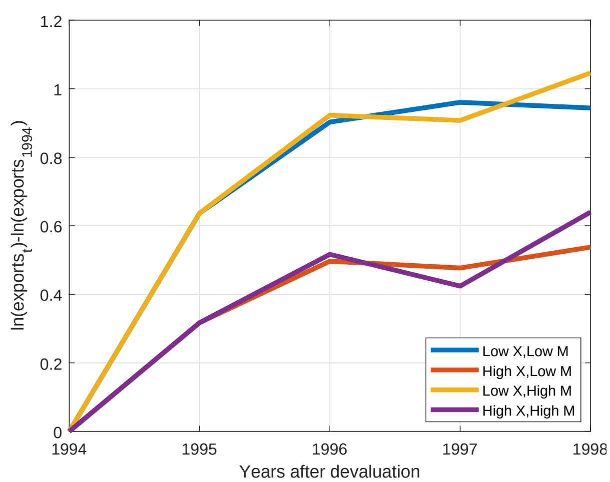
[7] These patterns of the data are also robust to controlling for plant-level sales, capital, wage bill, as well as to introducing industry fixed effects.

devaluation. To do so, we extend the empirical analysis conducted in Section 5.3.1 of the paper to examine the dynamics of exports growth (i) by export intensity while controlling for differences in the intensity on imported intermediates, as well as (ii) by import intensity controlling for differences in export intensity. Thus, this specification allows us to identify the extent to which differences in export or import intensity before the devaluation were associated with a differential response of exports after the devaluation. Then, we estimate the following specification:

$$\ln \frac{X_{i,t}}{X_{i,-1}} = \sum_{j=0}^{3} \left[\beta_j + \gamma_j \text{High initial export intensity}_{i,t}\right] \mathbb{I}_{\{t=j\}} + \sum_{k=0}^{3} \left[\widetilde{\beta}_k + \widetilde{\gamma}_k \text{High initial import intensity}_{i,t}\right] \mathbb{I}_{\{t=k\}} + \varepsilon_{i,t}$$

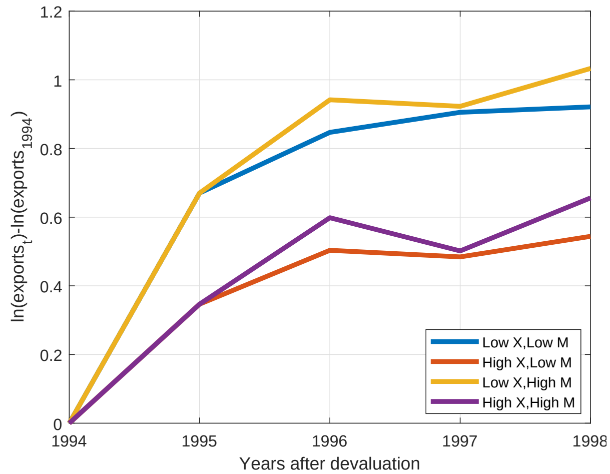
where High initial import intensity$_{i,t}$ is a dummy variable that is equal to one if firm i's import intensity is above the median in the pre-devaluation year and is zero otherwise. We also control for industry fixed effects as well as for inventories. All other variables and parameters are defined as in the paper.

Figure 9: Firm-level exports growth by $\frac{\text{exports}}{\text{sales}}$ and $\frac{\text{imported intermediates}}{\text{intermediates}}$



Figures 9 and 10 plot the dynamics of exports implied by the estimated specification for two alternative measures of import intensity: the ratio of imported intermediates to total intermediates and the ratio of imported intermediates to sales, respectively. Each of the figures plots the dynamics of exports corresponding to four alternative types of firms: (i) firms with low export and import intensity (low X, low M), (ii) firms with low export intensity and high import intensity (low X, high M), (iii) firms with high export intensity and

Figure 10: Firm-level exports growth by $\frac{\text{exports}}{\text{sales}}$ and $\frac{\text{imported intermediates}}{\text{sales}}$



low import intensity (high X, low M), and (iv) firms with high export and import intensities (high X, high M).

As in the paper, the figures show there is a substantial difference in the dynamics of exports between firms with high vs. low export intensity: low export-intensity firms exhibit faster growth of exports than high export-intensity firms. In contrast, the figures show that dynamics of exports for firms with high vs. low import intensities are very similar to each other. In particular, note that firms with high import intensity feature a higher increase of exports than firms with low import intensity (everywhere except in year 1997 in Figure 9), controlling for export intensity.

Thus, these findings suggest that the prevalence of imported intermediates among exporters are unlikely to have significantly slowed down the response of exports after the devaluation.

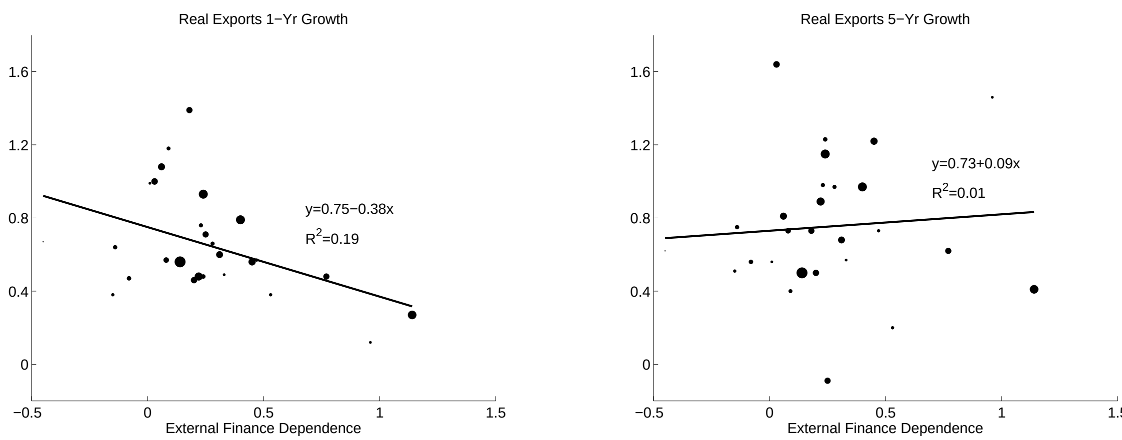
4.2 Industry-level adjustment and external finance dependence

The model above implies that financial frictions slow down the response of exports following large devaluations. To examine the extent to which financial frictions indeed slow down the adjustment of exports in the data, we now contrast the dynamics of exports across industries with differential degrees of dependence on external finance (Rajan and Zingales, 1998). Even though the model does not feature multiple industries, we exploit cross-industry variation in finance-intensity to identify the causal impact of financial constraints on the adjustment of exports.

To do so, we use Mexican firm-level data from 1994 to 1999 to compute the growth of exports across industries one year and five years after the devaluation. To the extent that financial frictions slow down the growth of exports, industries with lower external finance dependence should feature higher exports growth one year after the devaluation than their high-external-finance-dependence counterparts. Moreover, our mechanism also implies that exports growth five years after the devaluation should not differ systematically across industries based on their external finance dependence.

Figure 11 shows the implications of our mechanism are indeed observed in the data. Industries with high external finance dependence grow relatively less than their low-external-finance-dependence counterparts one year after the devaluation. However, as shown in Panel B, there is no systematic relationship between exports growth and finance-intensity five years after the devaluation. We interpret these findings as evidence in support of the qualitative impact of financial frictions on the dynamics of exports following large devaluations.

Figure 11: Industry-level exports growth and external finance dependence



Panel A

Panel B

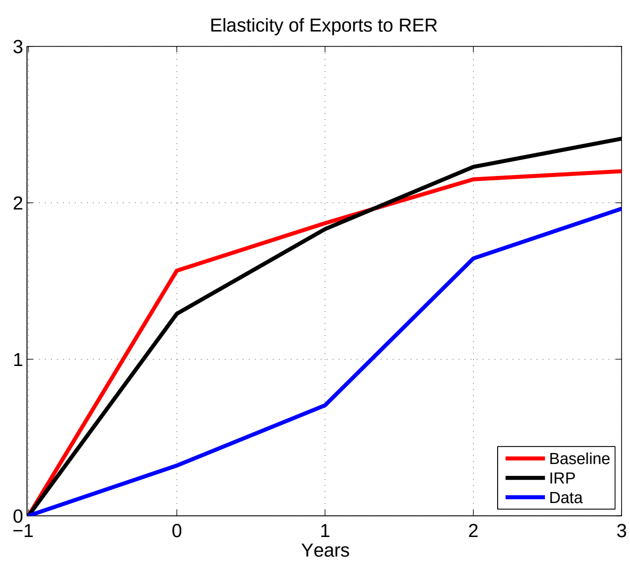
5 Sensitivity Analysis

In this section, we present several robustness exercises that complement the findings presented in the paper. We first investigate whether the non-fulfillment of the interest rate parity condition in the model is relevant for our results. We then study whether alternative assumptions on the distribution of foreign-denominated debt affect the main quantitative implications of the model. Finally, we investigate the response of the economy to shocks to the collateral constraint parameter that tighten the financial constraint and induce a "financial crisis."

5.1 Interest Rate Parity

In the first exercise, we re-do the exercise in section 4 in the paper under assumptions such that interest parity holds every period after the initial unexpected shock. To do so, we assume that after the devaluation hits in the baseline economy, all agents change their portfolios to just hold domestic debt (i.e. we set $\lambda = 1$ for all firms after period 2, as in the frictionless economy). Therefore, the only effect of foreign-denominated debt is in period 2 when agents are surprised by the devaluation; similarly, the only period where interest rate parity (IRP) does not hold is also period 2, when the shocks hit. We recalibrate the shocks for this economy, following the same approach as for the baseline economy.

Figure 12: Export Elasticity: Baseline vs. Baseline with IRP



As shown in Figure 12, this economy delivers qualitatively similar implications as the baseline economy. In particular, the exports elasticity increase is lower on impact, but higher two years after the large devaluation. This exercise allows us to disentangle the "balance-sheet" effects on the net worth of firms from the effects of the change in the real exchange rate through their effect on the cost of borrowing. In this case, while the "IRP" model displays a slower adjustment, it is still a modest change with respect to our baseline. Thus, we conclude that financial frictions and balance sheet effects have a small effect as a driver of the slow increase of the exports elasticity observed in the data.

5.2 Alternative Distribution of Foreign-Denominated Debt

We now investigate the extent to which alternative assumptions on the distribution of foreign-denominated debt may affect the model's implications for the dynamics of aggregate exports in large devaluations. We consider three alternative distributions of foreign-denominated debt: (*i*) an economy in which low-export-cost firms have more foreign-denominated debt (100% of the debt denominated in foreign units) than high-export-cost firms (50% of the debt denominated in foreign units);[8] (*ii*) an economy in which all debt is denominated in domestic units; and (*iii*) an economy in which all debt is denominated in foreign units. The implications of these alternative distributions of foreign-denominated debt for the export elasticity are presented in Figure 13, jointly with the one assumed under our baseline calibration (55% of the debt denominated in foreign units for both types of firms).

We find that the dynamics of the export elasticity is largely identical across the alternative debt distributions that we consider, suggesting that balance-sheet effects do not play a significant role in driving aggregate export dynamics. This finding is driven by the reallocation channel and by general equilibrium effects that operate through the labor market. In economies with a high share of foreign-denominated debt, devaluations lead to stronger negative balance-sheet effects, affecting non-exporters more than exporters. Therefore, non-exporters decrease labor demand relative to exporters, benefiting the latter via general equilibrium effects and offsetting the impact of foreign-denominated debt on exports.

5.2.1 Calibration of λ and λ_X

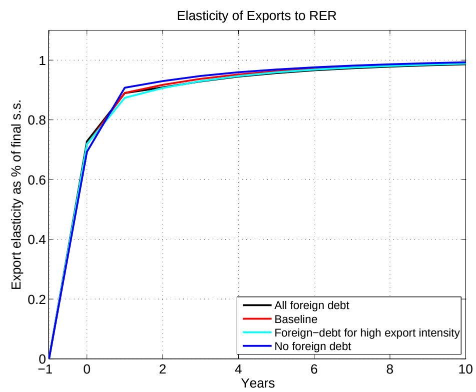
We now describe a calibration strategy for an economy with two potentially different compositions of debt for firms type 1 (λ_1) and type 2 (λ_2).

Since we don't have firm-level data on the currency composition of debt, we assume that this composition is fixed among firms in each of our two groups, but differ between groups. Recall that firms of type 1 are those with export intensity below a given threshold and the remaining firms are of type 2.

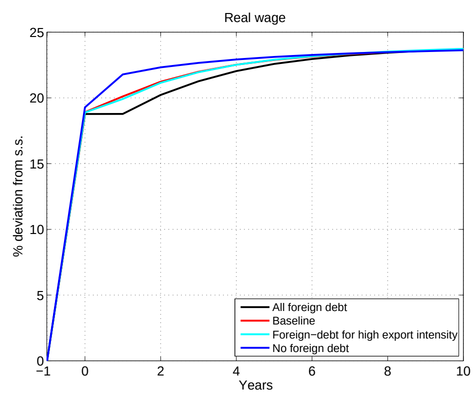
For industry J, the currency composition of debt between domestic currency debt (DC_J)

[8] These values are calibrated based on the joint distribution of the share of foreign-denominated debt in total debt and the share of firms with high export intensity across Mexican industries in 1994. See next section.

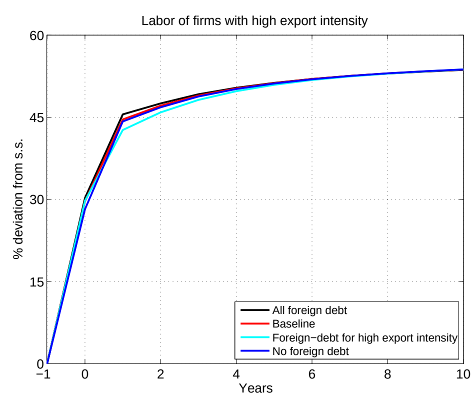
Figure 13: Alternative Distribution of Foreign-Denominated Debt



Panel A: Export elasticity



Panel B: Wages



Panel C: Employment in high-export intensity firms and foreign currency debt (FC_J) is given by:

$$\begin{aligned}
\frac{FC_J}{FC_J + DC_J} =& \sum_{i \in J} \frac{FC_i}{FC_i + DC_i} \frac{FC_i + DC_i}{FC_J + DC_J} \\
=& \sum_{i \in J_1} \frac{FC_i}{FC_i + DC_i} \frac{FC_i + DC_i}{FC_J + DC_J} + \sum_{i \in J_2} \frac{FC_i}{FC_i + DC_i} \frac{FC_i + DC_i}{FC_J + DC_J} \\
=& \lambda_1 \sum_{i \in J_1} \frac{FC_i + DC_i}{FC_J + DC_J} + \lambda_2 \sum_{i \in J_2} \frac{FC_i + DC_i}{FC_J + DC_J} \\
\frac{FC_J}{FC_J + DC_J} =& \lambda_1 \frac{(FC + DC)_{J_1}}{(FC + DC)_J} + \lambda_2 \left(1 - \frac{(FC + DC)_{J_1}}{(FC + DC)_J}\right)
\end{aligned}$$

where $\frac{(FC+DC)_{J_1}}{(FC+DC)_J}$ is the share of total debt among firms of type 1 (low export intensity), λ_1 is the average export intensity of firms type 1, and λ_2 is the average export intensity of firms type 2.

We have data on the currency composition of debt by sector, so if we had data on the share of debt among firms of type 1 out of total debt, we could find λ_1 and λ_2. Since there are two unknowns, we need at least data for two different sectors. With more than two sectors, we can find the two values that minimize the distance between the shares of foreign debt per industry implied by the formula above and the ones observed in the data.

Finally, since we don't have data on the share of debt for each group, we will instrument for these ratios. Notice that:

$$\begin{aligned}
\frac{(FC + DC)_{J_1}}{(FC + DC)_J} &= \frac{n_1^J (\overline{FC + DC})_{J_1}}{n_1^J (\overline{FC + DC})_{J_1} + n_2^J (\overline{FC + DC})_{J_2}} \\
\frac{(FC + DC)_{J_1}}{(FC + DC)_J} &= \frac{n_1^J}{n_1^J + n_2^J + n_2^J \left(\frac{(\overline{FC+DC})_{J_2}}{(\overline{FC+DC})_{J_1}} - 1\right)}
\end{aligned}$$

where n_1^J and n_2^J are the number of firms in each industry J with low and high export-intensity, respectively.

Finally, we assume that the ratio of average total debt of firms with high export-intensity to that of those with low export-intensity, can be approximated by the ratio of average sales. That is,

$$\frac{(\overline{FC + DC})_{J_2}}{(\overline{FC + DC})_{J_1}} = \frac{\overline{\text{sales}}_J^2}{\overline{\text{sales}}_J^1}$$

In our preferred calibration for λ and λ_X: We use total credit by commercial banks; we approximate the share of total debt by type 1 firms by their share of total sales; we choose lambdas to minimize the distance of each industry in our sample to a straight line, with the same weight to each industry; we use all 9 industries in our sample. We have done different robustness tests: (*i*) Using total credit by commercial and development banks; (*ii*) approximating total debt by type 1 by share of exports, share of domestic sales, and share of exporters; (*iii*) weighting industries by total sales, total credit, number of firms; including in our sample the total manufactuing, dropping industry B.4 (zero firms with high X/Y), dropping industries 3 (few firms), 4 (no firms with high X/Y) and 9 (few firms), or using just bottom and top industries by share of foreign currency credit. We obtain very similar results for most of these specifications.

5.2.2 Industry-level data

The data that we use to estimate λ_1 and λ_2 is presented in Table 5.

Table 5: Industry-level data

	Manuf.	B1	B2	B3	B4	B5	B6	B7	B8	B9	Av.	St.Dev.
						Total credit						
Commercial Banks	109894	22794	16112	3503	4147	15450	8327	9005	19485	11071	12210.4	6690.9
Comm. & Dev. Banks	120849	30067	16276	3573	4999	15659	8639	10381	19937	11318	13427.7	8194.1
						Foreign currency credit / Total credit						
Commercial Banks	0.55	0.55	0.61	0.31	0.44	0.71	0.44	0.82	0.44	0.46	0.53	0.16
Comm. & Dev. Banks	0.53	0.43	0.61	0.30	0.53	0.71	0.46	0.84	0.45	0.46	0.53	0.16
						Firm-level data (INEGI)						
Firms	4779	887	766	177	384	976	341	112	1083	53	531	399
Exporters	1547	208	202	25	88	385	87	53	475	24	172	162.5
Share of exporters	0.324	0.234	0.264	0.141	0.229	0.394	0.255	0.473	0.439	0.453	0.320	0.120
Exporters high X/Y / Exporters	0.043	0.063	0.025	0.045	0.003	0.031	0.041	0.071	0.061	0.057	0.044	0.022
Sales high X/Y / Sales	0.115	0.018	0.016	0.058	0.001	0.064	0.028	0.152	0.277	0.032	0.072	0.089
Exp. high X/Y / Exp.	0.529	0.384	0.186	0.513	0.049	0.339	0.289	0.630	0.602	0.208	0.356	0.197
Dom. high X/Y / Dom.	0.026	0.002	0.003	0.008	0.000	0.017	0.002	0.042	0.080	0.009	0.018	0.027

List of 1-digit industries: Manufacturing Industry (B1 to B9); B1. Nutritional Products, Drink and Tobacco; B2. Textiles, articles to dress and train of the Leather; B3. Industry of the Wood and Wood Products; B4. Paper, Products of Paper, Press and Editorials; B5. Chemical Substances derived form Petroleum, Rubber and Plastic; B6. Mineral Products Non-metallic Except Refuel and Coal; B7. Basic Metallic Industries; B8. Metallic Products, Machinery and Equipment; B9. Other Manufacturing Industries.

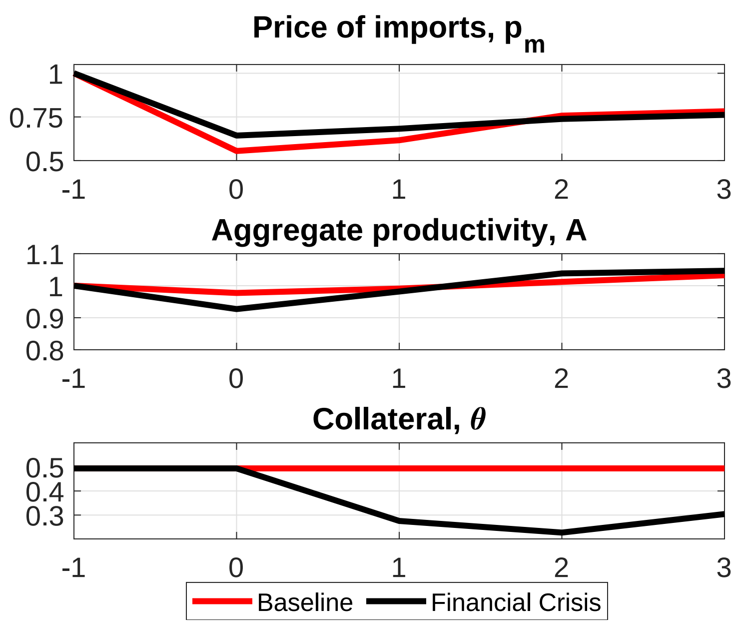
5.3 Financial Crisis Shock

Many of the large devaluation episodes analyzed in Section 2 have been accompanied by banking crises. A financial crisis could potentially worsen the economic consequences brought about by a large devaluation. In this section we examine the potential impact of this channel on our findings.

To do so, we re-compute our main experiment under an alternative set of shocks. In contrast to the baseline model, we consider shocks to the price of imports, aggregate productivity, and the fraction of collateralizable assets θ. We interpret shocks to θ as capturing the impact of a financial crisis on firms' decisions. Figure 14 shows the sequence of shocks that we estimate, and contrasts them with those estimated in the baseline experiment.

Figure 15 reports the implications of our model under these alternative shocks. We find that the model implies dynamics of the real exchange rate, real GDP, and investment that are close to the data. Moreover, we find that the implications of these shocks for aggregate export dynamics are close to those under the baseline experiment.

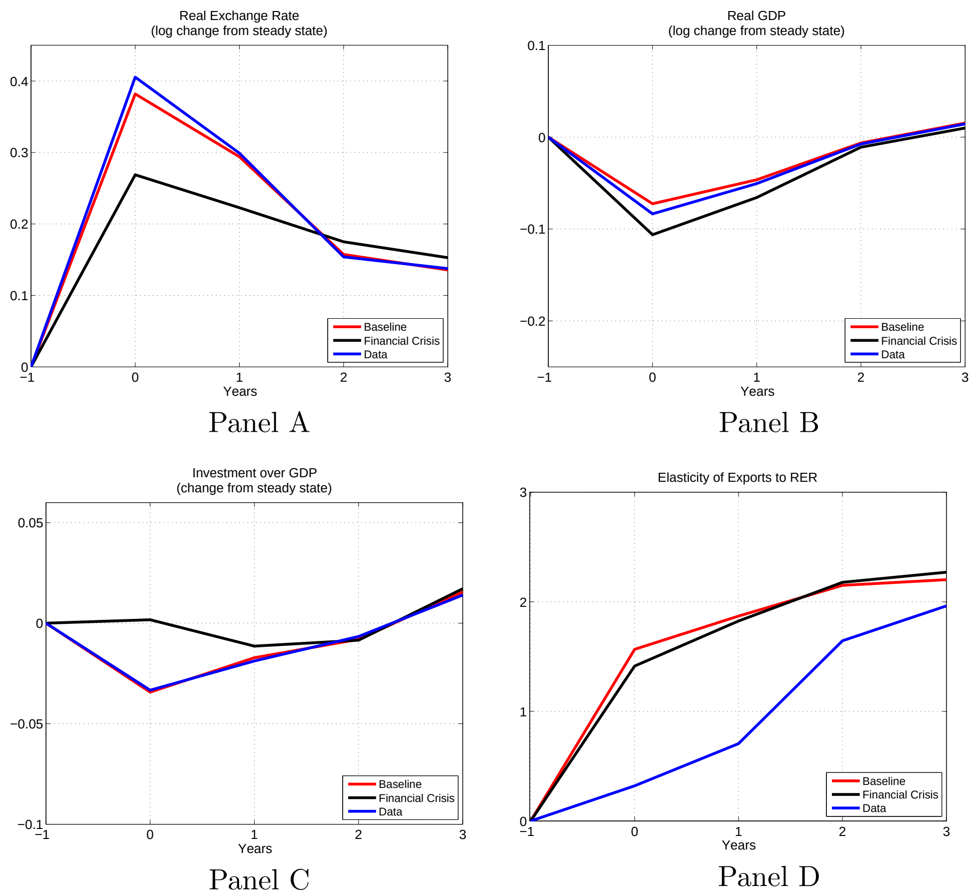
Figure 14: Shocks to price of imports, aggregate productivity, and collateral



5.4 Deviations from UIP and the Optimal Choice of Debt

In the model, for tractability, we assumed an exogenous portfolio composition. However, since we allow for deviation from UIP during the transition, we could allow entrepreneurs to switch to the cheaper debt after the initial shock. In particular, given the path of the real exchange rate after the initial depreciation, foreign denominated debt becomes more

Figure 15: Model with shocks to θ

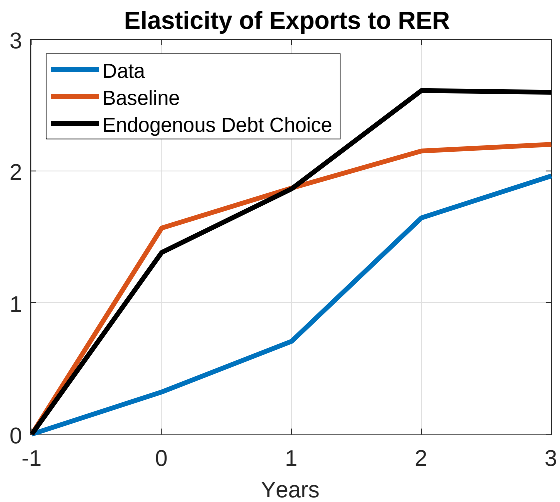


attractive as real exchange rate appreciate thereafter. Thus, if given a choice, in our model all entrepreneurs would switch to foreign debt. Below, we describe how this change to our model affects our results (see also Section ?? of the Appendix). Note that this simple exercise captures the spirit of endogenous currency choice considered Salomao and Varela (2018).[9]

As shown in Figure 16, the economy with financial frictions and endogenous choice of debt delivers qualitatively similar implications as our baseline model with financial frictions. In particular, as in our baseline model, the adjustment of exports is too rapid compared to the data. In addition, as you predicted in your report, allowing entrepreneurs to switch to choose debt optimally following devaluation leads to a substantially larger long-run elasticity

[9]In the stationary equilibrium of our model the interest rate parity holds since there are no shocks to exchange rate. Thus, in stationary equilibrium entrepreneurs are indifferent between holding foreign and domestic debt.

Figure 16: Export Elasticity: Baseline vs. Endogenous Debt Choice



than observed in the data. The reason is that domestic currency appreciation that follows the initial devaluation makes foreign debt very cheap effectively relaxing the borrowing constraint. As a consequence, exporters are able to expand their production more rapidly than in our baseline model.

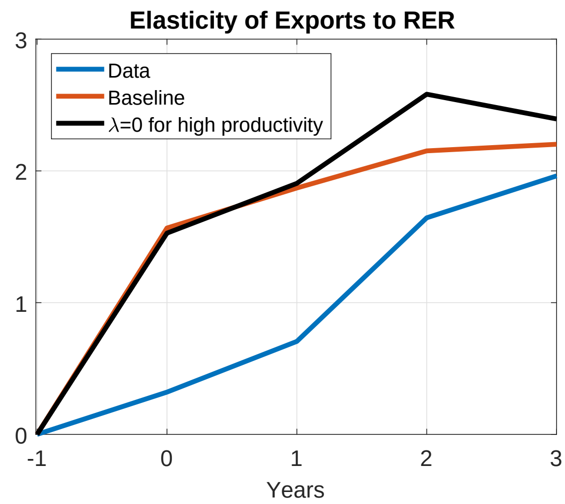
5.5 Composition of Debt as a Function of Productivity

In the paper, we assumed that all firms have the same fraction of debt denominated in foreign units. However, as reported in subsection 1.2, larger firms tend to hold more foreign debt (see also Salomao and Varela (2018)). Motivated by this observation. below we investigate if allowing more productive entrepreneurs to hold more foreign debt would change our results.

We assume that λ depends on entrepreneurs productivity z. For simplicity, we assume that if an entrepreneur receives a shock that belongs to the top quartile of the productivity distribution he will choose to hold only foreign-denominated debt ($\lambda = 0$). Otherwise, he will hold only domestic debt ($\lambda = 1$).

Again, we see that linking foreign debt to productivity does not affect our conclusion that financial frictions and balance sheet effects cannot explain the slow growth of export in large devaluation episodes. Moreover, the elasticity of export in periods 2 and 3 is actually larger than implied by our baseline model with financial frictions. This is driven by the fact that in this version of the model most of the exporters will be holding foreign debt (as these are the most productive firms in the economy). As such, these firms will be able to take advantage of the appreciation of the currency that follows the devaluation as such an appreciation makes foreign debt cheaper effectively relaxing their borrowing constraints. Thus, exporters are

Figure 17: Export Elasticity: Baseline vs. $\lambda = 0$ for high productivity



now able to expand their foreign sales by more than in the baseline model.[10]

[10]Not surprisingly, Figure 17 looks very similar to Figure 16 since the logic in these two cases is very similar. In particular, in both experiments the more productive firms are able to benefit from the appreciation of the currency that follows the initial depreciation.

6 Analytical Solution and Derivations

In this Section, we: (1) reformulate the entrepreneur's problem, (2) solve the static problem, (3) compute the firm-level elasticity of foreign sales to changes in the real exchange rate, (4) characterize firms' export-entry decisions, and (5) compute the aggregate elasticity of export to changes in the real exchange rate. This Section includes derivations of all the equations that appear in Section 3 of the paper.

6.1 Entrepreneur's Problem

6.1.1 Original problem

$$
\begin{aligned}
& v\left(k, d, z\right) = \max_{c, a' \geq 0} \frac{c^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_{z'}\left[g\left(a', z'\right)\right] \\
& \text{subject to} \\
& c + a' + d\left[\lambda + (1-\lambda)\frac{\xi}{\xi_{-1}}\right] = w + (1-\delta)k + \pi(k, z)
\end{aligned}
$$

where

$$
\begin{aligned}
& \pi\left(k, z\right) = \max_{p_h, y_h, p_f, y_f, n, e \in \{0,1\}} p_h y_h + e \xi_f y_f - wn - ewF \\
& \text{subject to} \\
& y_h + \tau y_f = A z k^{\alpha} n^{1-\alpha} \\
& y_h = \left(p_h\right)^{-\sigma} y \\
& y_f = \left(p_f\right)^{-\sigma} \bar{Y}^{*}
\end{aligned}
$$

where:

$$
\begin{aligned}
& g(a', z') = \max_{k', d'} v(k', d', z') \\
& \text{subject to} \\
& a' = k' - \frac{d'}{1+r} \\
& d'\left[\lambda + (1-\lambda)\frac{\xi'}{\xi}\right] \leq \theta k'
\end{aligned}
$$

6.1.2 Reformulated Problem

$$
\begin{aligned}
&g(a,z)=\max_{c,a'\geq 0}\frac{c^{1-\gamma}}{1-\gamma}+\beta\mathbb{E}_{z'}\left[g\left(a',z'\right)\right]\\
&\text{subject to}\\
&c+a'=w+\pi(a,z)+a(1+r)\left[\lambda+(1-\lambda)\frac{\xi}{\xi_{-1}}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
&\pi\left(a,z\right)=\max_{p_h,y_h,p_f,y_f,k,n,e\in\{0,1\}}p_hy_h+e\xi y_f-wn-ewF-k\left\{(1+r)\left[\lambda+(1-\lambda)\frac{\xi}{\xi_{-1}}\right]-(1-\delta)\right\}\\
&\text{subject to}\\
&y_h+\tau y_f=Azk^{\alpha}n^{1-\alpha}\\
&y_h=\left(p_h\right)^{-\sigma}Y_h\\
&y_f=\left(p_f\right)^{-\sigma}Y_f\\
&k\leq a\frac{(1+r)\left[\lambda+(1-\lambda)\frac{\xi}{\xi_{-1}}\right]}{(1+r)\left[\lambda+(1-\lambda)\frac{\xi}{\xi_{-1}}\right]-\theta}
\end{aligned}
$$

Reformulated Problem in frictionless model

In the case $\theta=\infty$ and $\lambda=1$, the reformulated problem is:

$$
\begin{aligned}
&g(a,z)=\max_{c,a'\geq 0}\frac{c^{1-\gamma}}{1-\gamma}+\beta\mathbb{E}_{z'}\left[g\left(a',z'\right)\right]\\
&\text{subject to}\\
&c+a'=w+\pi(z)+a(1+r)
\end{aligned}
$$

where

$$
\begin{aligned}
&\pi\left(z\right)=\max_{p_h,y_h,p_f,y_f,k,n,e\in\{0,1\}}p_hy_h+e\xi y_f-wn-ewF-k\left\{(1+r)-(1-\delta)\right\}\\
&\text{subject to}\\
&y_h+\tau y_f=Azk^{\alpha}n^{1-\alpha}\\
&y_h=\left(p_h\right)^{-\sigma}Y_h\\
&y_f=\left(p_f\right)^{-\sigma}Y_f
\end{aligned}
$$

6.2 Analytical Solution

6.2.1 Static Problem

$$
\begin{aligned}
\pi(a,z) = \max_{\{e',p_h,y_h,p_f,y_f,n,k\}} \; & p_h y_h + e'\xi p_f y_h - wn - (\widetilde{r}+\delta)k \\
& s.t. \\
& y_h = (p_h)^{-\sigma} Y_h \\
& y_f = (p_f)^{-\sigma} Y_f \\
& y_h + \tau y_f = z k^{\alpha} n^{1-\alpha} \\
& k \leq \frac{1+\widetilde{r}}{1+\widetilde{r}-\theta} a
\end{aligned}
$$

where, for simplicity, we assume that $A = 1$ for the rest of this section.

The Lagrangian associated with the above problem is given by

$$
L = y_h^{\frac{\sigma-1}{\sigma}} Y_h^{\frac{1}{\sigma}} + e'\xi y_f^{\frac{\sigma-1}{\sigma}} Y_f^{\frac{1}{\sigma}} - wn - (\widetilde{r}+\delta)k + \gamma(zk^{\alpha}n^{1-\alpha} - y_h - \tau y_f) + \mu\left(\frac{1+\widetilde{r}}{1+\widetilde{r}-\theta}a - k\right),
$$

where γ is the Lagrange multiplier on the technology constraint (i.e., production function) and μ is the Lagrange multiplier on the borrowing constraint.

At this point, we will need to consider separately four cases: (1) the case of an unconstrained exporter ($e' = 1$, $\mu = 0$), (2) the case of a constrained exporter ($e' = 1$, $\mu > 0$), (3) the case of an unconstrained non-exporter ($e' = 0$, $\mu = 0$), and finally (4) the case of a constrained non-exporter ($e' = 0$, $\mu > 0$).

Unconstrained Exporters The first-order conditions for the unconstrained exporters' problem are

$$
\frac{\sigma-1}{\sigma} y_h^{-\frac{1}{\sigma}} Y_h^{\frac{1}{\sigma}} - \gamma = 0 \tag{1}
$$

$$
\frac{\sigma-1}{\sigma} y_f^{-\frac{1}{\sigma}} Y_f^{\frac{1}{\sigma}} - \gamma = 0 \tag{2}
$$

$$
-w + \gamma(1-\alpha) z k^{\alpha} n^{-\alpha} = 0 \tag{3}
$$

$$
-(\widetilde{r}+\delta) + \gamma \alpha z k^{\alpha-1} n^{\alpha} = 0, \tag{4}
$$

and the associated complementary slackness conditions is

$$
\gamma(zk^{\alpha}n^{1-\alpha} - y_h - \tau y_f) = 0 \tag{5}
$$

To solve the above system of equations use F.O.C.s for n and k to express n in terms of k. Use this expression in the F.O.C. for k to obtain an expression for γ

$$
\gamma = \frac{1}{z}\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{\widetilde{r}+\delta}{\alpha}\right)^{\alpha} \tag{6}
$$

We can use this expression for γ in F.O.C.s for y_d, y_f to compute optimal domestic and foreign sales. It is then easy to show that

$$p_d y_d = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1} z^{\sigma-1}Y_h \tag{7}$$

$$p_f y_f = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1} z^{\sigma-1}\frac{\xi^{\sigma-1}}{\tau^{\sigma-1}}Y_f \tag{8}$$

Using the complementary slackness condition we can compute optimal k and then recover optimal n. Following these steps we obtain

$$k = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1}\frac{\alpha}{\widetilde{r}+\delta}z^{\sigma-1}\left(Y_h+\frac{\xi^{\sigma}}{\tau^{\sigma-1}}Y_f\right) \tag{9}$$

$$n = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1}\frac{1-\alpha}{w}z^{\sigma-1}\left(Y_h+\frac{\xi^{\sigma}}{\tau^{\sigma-1}}Y_f\right) \tag{10}$$

Finally, we can compute the total optimal profits of an unconstrained exporters.

$$\pi^x(a,z) = \frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1} z^{\sigma-1}\left(Y_h+\frac{\xi^{\sigma}}{\tau^{\sigma-1}}Y_f\right) \tag{11}$$

This completes the solution to the static profit maximization problem of an unconstrained exporter.

Constrained Exporters We now characterize the optimal choices of constrained exporters. In what follows we use μ_x to denote the Lagrange multiplier on the borrowing constraint of an exporter. The first-order conditions for the constrained exporters' problem are

$$\frac{\sigma-1}{\sigma}y_h^{-\frac{1}{\sigma}}Y_h^{\frac{1}{\sigma}}-\gamma=0 \tag{12}$$

$$\frac{\sigma-1}{\sigma}y_f^{-\frac{1}{\sigma}}Y_f^{\frac{1}{\sigma}}-\gamma=0 \tag{13}$$

$$-w+\gamma(1-\alpha)zk^{\alpha}n^{-\alpha}=0 \tag{14}$$

$$-(\widetilde{r}+\delta)+\gamma\alpha zk^{\alpha-1}n^{\alpha}-\mu_x=0, \tag{15}$$

and the associated complementary slackness conditions are

$$\gamma(zk^{\alpha}n^{1-\alpha}-y_h-\tau y_f)=0 \tag{16}$$

$$\mu_x\left(\frac{1+\widetilde{r}}{1+\widetilde{r}-\theta}a-k\right)=0 \tag{17}$$

Note that the F.O.C conditions for the problem of constrained exporter are almost identical to those of an unconstrained exporters. The only difference is that now in the F.O.C. for capital the Lagrange multiplier μ_x shows up. However, if we treat $\mu_x + \widetilde{r} + \delta$ as the rental rate of capital (instead of just $\widetilde{r} + \delta$) then we can follow the same steps as above to express $p_d y_d$, $p_f y_f$, k and n in terms of structural parameters and the Lagrange multiplier μ_x.

Following the same steps as in the case of unconstrained exporters we obtain

$$p_d y_d = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_x}\right)^{\alpha}\right]^{\sigma-1} z^{\sigma-1} Y_h \tag{18}$$

$$p_f y_f = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_x}\right)^{\alpha}\right]^{\sigma-1} z^{\sigma-1} \frac{\xi^{\sigma-1}}{\tau^{\sigma-1}} Y_f \tag{19}$$

$$k = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_x}\right)^{\alpha}\right]^{\sigma-1} \frac{\alpha}{\widetilde{r}+\delta+\mu_x} z^{\sigma-1}\left(Y_h + \frac{\xi^{\sigma}}{\tau^{\sigma-1}} Y_f\right) \tag{20}$$

$$n = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_x}\right)^{\alpha}\right]^{\sigma-1} \frac{1-\alpha}{w} z^{\sigma-1}\left(Y_h + \frac{\xi^{\sigma}}{\tau^{\sigma-1}} Y_f\right) \tag{21}$$

To find the expression for μ_x, note that if a firm is financially constrained then

$$k = \frac{1+\widetilde{r}}{1+\widetilde{r}-\theta} a$$

Thus, it has to be the case that the two expressions for k are the same, that is

$$\frac{1+\widetilde{r}}{1+\widetilde{r}-\theta} a = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_x}\right)^{\alpha}\right]^{\sigma-1} \frac{\alpha}{\widetilde{r}+\delta+\mu_x} z^{\sigma-1}\left(Y_h + \frac{\xi^{\sigma}}{\tau^{\sigma-1}} Y_f\right)$$

Solving the above equation for $\widetilde{r} + \delta + \mu_x$ we obtain

$$\widetilde{r} + \delta + \mu_x = \alpha\left[\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\sigma-1)} z^{\sigma-1} \frac{\left(Y_h + \frac{\xi^{\sigma}}{\tau^{\sigma-1}} Y_f\right)}{\frac{1+\widetilde{r}}{1+\widetilde{r}-\theta} a}\right]^{\frac{1}{\alpha(\sigma-1)+1}}, \tag{22}$$

The profits of a constrained exporters are given by

$$\begin{aligned} \pi^x(a,z) = {} & \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_x}\right)^{\alpha}\right]^{\sigma-1} z^{\sigma-1}\left(Y_h + \frac{\xi^{\sigma}}{\tau^{\sigma-1}} Y_f\right) \\ & \times \left[1 - \frac{\sigma-1}{\sigma}\left((1-\alpha) + \alpha\frac{\widetilde{r}+\delta}{\widetilde{r}+\delta+\mu_x}\right)\right] \end{aligned} \tag{23}$$

Unconstrained Non-Exporters We consider no unconstrained non-exporters. The first-order conditions for the constrained non-exporters' problem are

$$\frac{\sigma-1}{\sigma}y_h^{-\frac{1}{\sigma}}Y_h^{\frac{1}{\sigma}}-\gamma=0 \tag{24}$$

$$-w+\gamma(1-\alpha)zk^{\alpha}n^{-\alpha}=0 \tag{25}$$

$$-(\widetilde{r}+\delta)+\gamma\alpha zk^{\alpha-1}n^{\alpha}=0, \tag{26}$$

and the associated complementary slackness conditions is

$$\gamma(zk^{\alpha}n^{1-\alpha}-y_h)=0 \tag{27}$$

Following the same steps as in the case of unconstrained exporters, we find that the optimal domestic sales p_dy_d, optimal capital k, and the optimal labor n are given by

$$p_dy_d=\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1}z^{\sigma-1}Y_h \tag{28}$$

$$k=\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1}\frac{\alpha}{\widetilde{r}+\delta}z^{\sigma-1}Y_h \tag{29}$$

$$n=\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1}\frac{1-\alpha}{w}z^{\sigma-1}Y_h \tag{30}$$

The profits of an unconstrained non-exporter with net worth a and productivity z are given by

$$\pi^{nx}(a,z)=\frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1}z^{\sigma-1}Y_h \tag{31}$$

Constrained Non-Exporters Consider now constrained non-exporters and let μ_{nx} denote the Lagrange multiplier on the borrowing constraint of a non-exporter. The F.O.C.s for this case are given by:

$$\frac{\sigma-1}{\sigma}y_h^{-\frac{1}{\sigma}}Y_h^{\frac{1}{\sigma}}-\gamma=0 \tag{32}$$

$$-w+\gamma(1-\alpha)zk^{\alpha}n^{-\alpha}=0 \tag{33}$$

$$-(\widetilde{r}+\delta)+\gamma\alpha zk^{\alpha-1}n^{\alpha}-\mu_{nx}=0, \tag{34}$$

and the associated complementary slackness conditions is

$$\gamma(zk^{\alpha}n^{1-\alpha}-y_h)=0 \tag{35}$$

$$\mu_{nx}\left(\frac{1+\widetilde{r}}{1+\widetilde{r}-\theta}a-k\right)=0 \tag{36}$$

Following the same steps as in the case of constrained exporters we arrive at the following optimal choices for domestic sales, capital, and labor:

$$p_d y_d = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_{nx}}\right)^{\alpha}\right]^{\sigma-1} z^{\sigma-1}Y_h \tag{37}$$

$$k = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_{nx}}\right)^{\alpha}\right]^{\sigma-1} \frac{\alpha}{\widetilde{r}+\delta+\mu_{nx}} z^{\sigma-1}Y_h \tag{38}$$

$$n = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_{nx}}\right)^{\alpha}\right]^{\sigma-1} \frac{1-\alpha}{w} z^{\sigma-1}Y_h \tag{39}$$

It follows that the optimal profits of a constrained non-exporters are given by

$$\begin{aligned}\pi^{nx} = {} & \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_{nx}}\right)^{\alpha}\right]^{\sigma-1} z^{\sigma-1}Y_h \\ & \times\left[1-\frac{\sigma-1}{\sigma}\left((1-\alpha)+\alpha\frac{\widetilde{r}+\delta}{\widetilde{r}+\delta+\mu_{nx}}\right)\right]\end{aligned} \tag{40}$$

Finally, equating the expression for capital obtained above with the amount of capital implied by the borrowing constraint we obtain an expression for $\widetilde{r}+\delta+\mu_{nx}$:

$$\widetilde{r}+\delta+\mu_{nx} = \alpha\left[\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\sigma-1)} z^{\sigma-1}\frac{Y_h}{\frac{1+\widetilde{r}}{1+\widetilde{r}-\theta}a}\right]^{\frac{1}{\alpha(\sigma-1)+1}}, \tag{41}$$

6.2.2 Firm-level elasticity of foreign sales

Before we derive firm-level elasticity of foreign sales, it is convenient to compute the effect of a change in ξ on the borrowing constraints, the Lagrange multiplier μ, and the cost of capital $\widetilde{r}+\delta$.

First, we investigate how an increase in $\log\xi$ affects the borrowing constraint. We have:

$$\frac{\partial}{\partial\log\xi} log\left[\frac{1+\widetilde{r}}{1+\widetilde{r}-\theta}a\right] = -\theta\frac{(1-\lambda)(1+r)\frac{\xi}{\xi_{-1}}}{(1+\widetilde{r})(1+\widetilde{r}-\theta)} \tag{42}$$

Next, we consider the effect of an increase in $\log\xi$ on the cost of capital as captured by $\widetilde{r}+\delta$. We have:

$$\frac{\partial\log(\widetilde{r}+\delta)}{\partial\log\xi} = \frac{(1-\lambda)(1+r)\frac{\xi}{\xi_{-1}}}{\widetilde{r}+\delta} \tag{43}$$

Finally, the effect of an increase in ξ on $\widetilde{r} + \delta + \mu_x$ is given by

$$\frac{\partial \log\left(\widetilde{r} + \delta + \mu_x\right)}{\partial \log \xi} = \frac{\sigma}{\alpha(\sigma-1)+1} \frac{\frac{\xi^\sigma}{\tau^{\sigma-1}} Y_f}{Y_h + \frac{\xi^\sigma}{\tau^{\sigma-1}} Y_f} + \frac{\theta}{\alpha(\sigma-1)+1} \frac{(1-\lambda)(1+r)\frac{\xi}{\xi_{-1}}}{(1+\widetilde{r})(1+\widetilde{r}-\theta)} \tag{44}$$

where $\left(\frac{\xi^\sigma}{\tau^{\sigma-1}} Y_f\right) / \left(Y_h + \frac{\xi^\sigma}{\tau^{\sigma-1}} Y_f\right)$ is export intensity.

With these result in hand, we can now compute the elasticity of foreign sales for continuing exporters. Consider first unconstrained exporters. From Equation (8) we see that

$$\begin{aligned}\log(p_f y_f) =& (\sigma-1)\log\left(\frac{\sigma-1}{\sigma}\right) + (1-\alpha)(\sigma-1)\log\left(\frac{1-\alpha}{w}\right) + \alpha(\sigma-1)\log\alpha \\ &+ (\sigma-1)\log z - (\sigma-1)\log\tau + \log Y_f - \alpha(\sigma-1)\log(\widetilde{r}+\delta) + (\sigma-1)\log\xi\end{aligned}$$

Thus, it follows that the elasticity of foreign sales of an unconstrained exporter is given by

$$\begin{aligned}\frac{\partial \log p_f y_f}{\partial \log \xi} &= (\sigma-1) - \alpha(\sigma-1)\frac{\partial \log(\widetilde{r}+\delta)}{\partial \log \xi} \\ &= (\sigma-1) - \alpha(\sigma-1)\frac{(1-\lambda)(1+r)\frac{\xi}{\xi_{-1}}}{\widetilde{r}+\delta} \end{aligned} \tag{45}$$

Consider now constrained exporters. From Equation (19) we see that

$$\begin{aligned}\log(p_f y_f) =& (\sigma-1)\log\left(\frac{\sigma-1}{\sigma}\right) + (1-\alpha)(\sigma-1)\log\left(\frac{1-\alpha}{w}\right) + \alpha(\sigma-1)\log\alpha \\ &+ (\sigma-1)\log z - (\sigma-1)\log\tau + \log Y_f - \alpha(\sigma-1)\log(\widetilde{r}+\delta+\mu_x) + (\sigma-1)\log\xi\end{aligned}$$

Thus, it follows that the elasticity of foreign sales of a constrained exporter is given by

$$\begin{aligned}\frac{\partial \log p_f y_f)}{\partial \log \xi} &= (\sigma-1) - \alpha(\sigma-1)\frac{\partial \log(\widetilde{r}+\delta+\mu_x)}{\partial \log \xi} \\ &= (\sigma-1) - \alpha(\sigma-1)\left[\frac{\sigma}{\alpha(\sigma-1)+1} \times (\text{Export Intensity})\right] \\ &\quad - \alpha(\sigma-1)\left[\frac{\theta}{\alpha(\sigma-1)+1}\frac{(1-\lambda)(1+r)\frac{\xi}{\xi_{-1}}}{(1+\widetilde{r})(1+\widetilde{r}-\theta)}\right]\end{aligned} \tag{46}$$

Equations (45) and (46) correspond to Equations (5) and (6) in the paper. This completes the derivations of the firm-level foreign sales elasticities.

6.2.3 Export-entry decision

An entrepreneur decides to enter the export market if and only if this leads to higher total profits compared to producing only for the domestic market. In particular, an entrepreneur will export if and only if

$$\pi^x(a,z) \geq \pi^{nx}(a,z) + wF, \tag{47}$$

where $\pi^x(a,z)$ are the total profits of an exporter with assets a and productivity z, $\pi^{nx}(a,z)$ are the total profits of a non-exporters with assets a and productivity z, and wF is the fixed cost of exporting.

For each $a > 0$ we can find a productivity level, call it $\underline{z}(a)$, such that a firm will export if and only if $z \geq \underline{z}(a)$. We refer to firms that are just indifferent between exporting and producing only for domestic market, that is firms with productivity $a = \underline{z}(a)$, as marginal exporters.

We will need to consider separately three cases. In the first case, an entrepreneur who is indifferent whether to export can produce the optimal unconstrained amount if he decides to export. In the second case, an entrepreneur who is indifferent between exporting and not exporting cannot produce the optimal unconstrained amount if he decides to export (i.e., he is financially constrained if he exports), but can produce the unconstrained optimal amount if he produces only for the domestic market. Finally, we need to consider a case where an entrepreneur is constrained regardless of whether he exports or not.

Let $\overline{a}$ be the level of assets such that for all $a \geq \overline{a}$ marginal exporters are unconstrained both in the domestic and foreign market.[11] Then entrepreneurs with net worth $a > \overline{a}$ will decide to export if and only if $z \geq \underline{z}^u$, where $\underline{z}^u$ is the unique solution to

$$\frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1}(\underline{z}^u)^{\sigma-1}\frac{\xi^{\sigma}}{\tau^{\sigma-1}}Y_f = wF \tag{48}$$

Solving the above equation for $\underline{z}^u$ we obtain

$$\underline{z}^u = \left(\frac{\sigma}{\sigma-1}\right)\left[\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{\widetilde{r}+\delta}{\alpha}\right)^{\alpha}\right]\frac{\tau}{\xi^{\frac{\sigma}{\sigma-1}}}\left(\frac{\sigma wF}{Y_f}\right)^{\frac{1}{1-\sigma}} \tag{49}$$

In other words, for $a > \overline{a}$ we have $\underline{z}(a) = \underline{z}^u$.

Next, consider entrepreneurs with net worth $a < \overline{a}$. These entrepreneurs choose not to export if their idiosyncratic productivity is equal to $\underline{z}^u$ as financial constraints prevent them from attaining a scale that is required to make exporting profitable when $z = \underline{z}^u$. Nevertheless, entrepreneurs with $a < \overline{a}$ will export for sufficiently high z (i.e., if $z > \underline{z}(a)$) since a high z allows them to achieve high sales even with limited capital.

Here, we need to differentiate between two cases. In the first case, an exporter who is indifferent between exporting and not exporting is constrained if he chooses to export, but can produce optimal unconstrained quantity if he chooses not to export. Let $\underline{a}$ be the level of assets such that for all $a \in [\underline{a}, \overline{a})$ this is the case. In contrast, an entrepreneur with net

[11]To find $\overline{a}$ we compute first the entry decision of an unconstrained firm. This results in an export entry productivity threshold, call it $\underline{z}^u$, which is independent of a. We then find the minimum level of assets that is required to achieve the optimal scale at this threshold and call this level of assets $\overline{a}$. It follows that firms with productivity $z = \underline{z}^u$ and net worth $a \geq \overline{a}$ behave like unconstrained firms when deciding whether to export or not.

worth $a < \underline{a}$ and productivity $z = underbarz(a)$ is constrained regardless of whether he exports or not.

Consider first the case where $a \in [\underline{a}, \overline{a})$. In this case the export-entry productivity threshold is the unique solution to:

$$\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_x}\right)^{\alpha}\right]^{\sigma-1}\underline{z}(a)^{\sigma-1}\left(Y_h+\frac{\xi^{\sigma}}{\tau^{\sigma-1}}Y_f\right)$$
$$\times\left[1-\frac{\sigma-1}{\sigma}\left((1-\alpha)+\alpha\frac{\widetilde{r}+\delta}{\widetilde{r}+\delta+\mu_x}\right)\right]=$$
$$=wF-\frac{1}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta}\right)^{\alpha}\right]^{\sigma-1}\underline{z}(a)^{\sigma-1}Y_h$$

Rearranging the above equation, we obtain

$$\underline{z}(a)=\left(\frac{\sigma}{\sigma-1}\right)\left[\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{\widetilde{r}+\delta}{\alpha}\right)^{\alpha}\right]\left[\frac{wF}{\left(Y_h+\frac{\xi^{\sigma}}{\tau^{\sigma-1}}Y_f\right)\Delta_x^{\pi}-\frac{1}{\sigma}Y_h}\right]^{\frac{1}{\sigma-1}} \tag{50}$$

where

$$\Delta_x^{\pi}:=\left(\frac{\widetilde{r}+\delta}{\widetilde{r}+\delta+\mu_x}\right)^{\alpha(\sigma-1)+1}\left[1-\frac{\sigma-1}{\sigma}\left((1-\alpha)+\alpha\frac{\widetilde{r}+\delta}{\widetilde{r}+\delta+\mu_x}\right)\right]$$

Equation (50) defines implicitly the productivity export-entry threshold for all $a \in [\underline{a}, \overline{a})$. Note that $\underline{z}(a)$ is defined since a change in $\underline{z}(a)$ also affects the RHS of Equation (50) via its impact on $\widetilde{r}+\delta+\mu_x$.

Finally, we consider the export entry decision by entrepreneurs with assets $a < \underline{a}$. In this case, the export-entry threshold $\underline{z}(a)$ is defined implicitly by the following equation:

$$\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_x}\right)^{\alpha}\right]^{\sigma-1}\underline{z}(a)^{\sigma-1}\left(Y_h+\frac{\xi^{\sigma}}{\tau^{\sigma-1}}Y_f\right)$$
$$\times\left[1-\frac{\sigma-1}{\sigma}\left((1-\alpha)+\alpha\frac{\widetilde{r}+\delta}{\widetilde{r}+\delta+\mu_x}\right)\right]=$$
$$=wF+\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\frac{\alpha}{\widetilde{r}+\delta+\mu_{nx}}\right)^{\alpha}\right]^{\sigma-1}\underline{z}(a)^{\sigma-1}Y_h$$
$$\times\left[1-\frac{\sigma-1}{\sigma}\left((1-\alpha)+\alpha\frac{\widetilde{r}+\delta}{\widetilde{r}+\delta+\mu_{nx}}\right)\right]$$

Rearranging the above equation, we obtain:

$$\underline{z}(a)=\left(\frac{\sigma}{\sigma-1}\right)\left[\left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{\widetilde{r}+\delta}{\alpha}\right)^{\alpha}\right]\left[\frac{wF}{\left(Y_h+\frac{\xi^{\sigma}}{\tau^{\sigma-1}}Y_f\right)\Delta_x^{\pi}-Y_h\Delta_{nx}^{\pi}}\right]^{\frac{1}{\sigma-1}}, \tag{51}$$

where

$$\Delta^{\pi}_{nx} := \left(\frac{\widetilde{r}+\delta}{\widetilde{r}+\delta+\mu_{nx}}\right)^{\alpha(\sigma-1)+1}\left[1-\frac{\sigma-1}{\sigma}\left((1-\alpha)+\alpha\frac{\widetilde{r}+\delta}{\widetilde{r}+\delta+\mu_{nx}}\right)\right]$$

To sum up, the export-entry threshold is determined by Equation (49) if $a > \overline{a}$ by Equation (50) if $a \in [\underline{a}, \overline{a})$, and by Equation (19) if $a < \underline{a}$.

6.3 Aggregate Export Elasticity

In this Section, we derive the aggregate export elasticity reported in the paper (Equations (4) and (7)).

Recall that the aggregate exports X are given by

$$X = \int\limits_{a=0}^{\infty}\int\limits_{\underline{z}(a)}^{\infty} p_f(a,z)y_f(a,z)\,\mathrm{d}z\,\mathrm{d}a$$

Differentiating $\log X$ with respect to $\log\xi$ (and applying Leibniz integral formula), we obtain

$$\begin{aligned}\frac{\partial\log X}{\partial\log\xi} =& \frac{1}{X}\int\limits_{a=0}^{\infty}\int\limits_{\underline{z}(a)}^{\infty}\frac{\partial(p_f(a,z)y_f(a,z))}{\log\xi}\phi(a,z)\,\mathrm{d}z\,\mathrm{d}a \\ &+\frac{1}{X}\int\limits_{a=0}^{\infty}\frac{\partial\underline{z}(a)}{\partial\log\xi}\big(p_f(a,\underline{z}(a))y_f(a,\underline{z}(a))\big)\phi(a,\underline{z}(a))\,\mathrm{d}a\end{aligned}$$

Note that $\partial\log(f(x))/\partial x = (1/f(x))(\partial f(x)/\partial x)$. Therefore, we can write the above derivative as:

$$\begin{aligned}\frac{\partial\log X}{\partial\log\xi} =& \frac{1}{X}\int\limits_{a=0}^{\infty}\int\limits_{\underline{z}(a)}^{\infty}(p_f(a,z)y_f(a,z))\frac{\partial\log(p_f(a,z)y_f(a,z))}{\log\xi}\phi(a,z)\,\mathrm{d}z\,\mathrm{d}a \\ &+\frac{1}{X}\int\limits_{a=0}^{\infty}\frac{\partial\underline{z}(a)}{\partial\log\xi}\big(p_f(a,\underline{z}(a))y_f(a,\underline{z}(a))\big)\phi(a,\underline{z}(a))\,\mathrm{d}a,\end{aligned} \tag{52}$$

where the first term captures the elasticity of foreign sales by the continuing exporters (i.e., the contribution of the intensive margin) while the second term captures the contribution of export entry to aggregate elasticity.

Note that

$$
\begin{aligned}
&\frac{1}{X}\int_{a=0}^{\infty}\int_{\underline{z}(a)}^{\infty}(p_f(a,z)y_f(a,z))\frac{\partial \log(p_f(a,z)y_f(a,z))}{\log\xi}\phi(a,z)\,\mathrm{d}z\,\mathrm{d}a\\
&=\frac{1}{X}\iint_{\mathcal{S}^{const}}(p_f(a,z)y_f(a,z))\frac{\partial \log(p_f(a,z)y_f(a,z))}{\log\xi}\phi(a,z)\,\mathrm{d}z\,\mathrm{d}a\\
&+\frac{1}{X}\iint_{\mathcal{S}^{unconst}}(p_f(a,z)y_f(a,z))\frac{\partial \log(p_f(a,z)y_f(a,z))}{\log\xi}\phi(a,z)\,\mathrm{d}z\,\mathrm{d}a
\end{aligned}
$$

where $\mathcal{S}^{const}$ and $\mathcal{S}^{unconst}$ are sets of combinations of $\{a,z\}$ such that if $\{a,z\}\in\mathcal{S}^{const}$ then an entrepreneur is constrained and if $\{a,z\}\in\mathcal{S}^{unconst}$ then an entrepreneur is unconstrained. Next, note that $\partial(\log(p_f(a,z)y_f(a,z)))/\partial\log\xi$ does not depend on entrepreneur's state (a,z) regardless of whether the entrepreneur is constrained or not (see Equations (46) and (46)). Finally, define:

$$
\begin{aligned}
X^c &:= \iint_{\mathcal{S}^{const}}(p_f(a,z)y_f(a,z))\phi(a,z)\mathrm{d}z\,\mathrm{d}a\\
X^u &:= \iint_{\mathcal{S}^{unconst}}(p_f(a,z)y_f(a,z))\phi(a,z)\mathrm{d}z\,\mathrm{d}a,
\end{aligned}
$$

so that X^c and X^u are the total exports by constrained and unconstrained exporters, respectively. Then, using Equations (46) and (46), Equation (53) can be written as

$$
\begin{aligned}
\frac{\partial\log X}{\partial\log\xi} &= (\sigma-1)-\left[\alpha(\sigma-1)\frac{(1-\lambda)(1+r)\frac{\xi}{\xi-1}}{\widetilde{r}+\delta}\right]\frac{X^u}{X}\\
&-\alpha(\sigma-1)\left[\frac{\sigma}{\alpha(\sigma-1)+1}\times(\text{Export Intensity})\right]\\
&-\alpha(\sigma-1)\left[\frac{\theta}{\alpha(\sigma-1)+1}\frac{(1-\lambda)(1+r)\frac{\xi}{\xi-1}}{(1+\widetilde{r})(1+\widetilde{r}-\theta)}\right]\\
&+\frac{1}{X}\int_{a=0}^{\infty}\frac{\partial\underline{z}(a)}{\partial\log\xi}\big(p_f(a,\underline{z}(a))y_f(a,\underline{z}(a))\big)\phi(a,\underline{z}(a))\,\mathrm{d}a,
\end{aligned}
\tag{53}
$$

where the first four terms capture the elasticity of exports by continuing exporters. Finally, recall that for all $a>\overline{a}$ the export-entry productivity threshold is equal to $\underline{z}^u$. From Equation (49) we can see that

$$
\frac{\partial\underline{z}^u}{\partial\log\xi}=-\underline{z}^u\left[\frac{\sigma}{\sigma-1}-\alpha\frac{(1-\lambda)(1+r)\frac{\xi}{\xi-1}}{\widetilde{r}+\delta}\right],
$$

which is independent of a. Therefore, Equation (54) can be written as

$$\frac{\partial \log X}{\partial \log \xi} = (\sigma - 1) - \left[\alpha(\sigma-1)\frac{(1-\lambda)(1+r)\frac{\xi}{\xi_{-1}}}{\widetilde{r}+\delta}\right]\frac{X^u}{X}$$
$$- \alpha(\sigma-1)\left[\frac{\sigma}{\alpha(\sigma-1)+1}\times(\text{Export Intensity})\right]\frac{X^c}{X}$$
$$- \alpha(\sigma-1)\left[\frac{\theta}{\alpha(\sigma-1)+1}\frac{(1-\lambda)(1+r)\frac{\xi}{\xi_{-1}}}{(1+\widetilde{r})(1+\widetilde{r}-\theta)}\right]\frac{X^c}{X}$$
$$+ \frac{1}{X}\frac{\partial \underline{z}^u}{\partial \log \xi}\int\limits_{a=\overline{a}}^{\infty}\big(p_f(a,\underline{z}^u)y_f(a,\underline{z}^u)\big)\phi(a,\underline{z}^u)\,\mathrm{d}a$$
$$+ \frac{1}{X}\int\limits_{a=0}^{\overline{a}}\frac{\partial \underline{z}(a)}{\partial \log \xi}\big(p_f(a,\underline{z}(a))y_f(a,\underline{z}(a))\big)\phi(a,\underline{z}(a))\,\mathrm{d}a, \tag{54}$$

which corresponds to the Equation (7) in the main paper. Finally, setting $\lambda = 1$, $X^c = 0$ and $X^u = X$, we obtain the aggregate elasticity of exports in the frictionless economy with no foreign debt:

$$\frac{\partial \log X}{\partial \log \xi} = (\sigma-1) + \frac{1}{X}\frac{\partial \underline{z}^u}{\partial \log \xi}\int\limits_{a=0}^{\infty}\big(p_f(a,\underline{z}^u)y_f(a,\underline{z}^u)\big)\phi(a,\underline{z}^u)\,\mathrm{d}a \tag{55}$$

which corresponds to Equation (8) in the paper.

7 Numerical Solution Algorithm

7.1 Dynamic Problem

We find the optimal consumption and net worth choices numerically, through a value function iteration algorithm.

Setup To execute a value function iteration algorithm, we first discretize the state space and define a few useful solution objects:

- Net worth grid: $G_A = \{a_1, ..., a_{N_A}\}$
- Productivity grid: $G_Z = \{z_1, ..., z_{N_Z}\}$

where N_A and N_Z denote the number of net worth and productivity grid points, respectively. We approximate the autoregressive productivity process through a finite state Markov chain, following Tauchen (1986) to compute the associated transition matrix $P_Z : G_Z \times G_Z \to [0, 1]$.

Algorithm The goal of the algorithm is to find policy functions $c(a,z)$ and $a'(a,z)$ defined in the domain $G \equiv G_A \times G_Z$. To do so, we take all aggregate prices (ξ and w), as well as the aggregate demand for final goods Y_h[12], as given.[13] Given these prices, we begin by solving the static problem described above. We then proceed to the value function iteration algorithm:

1. We begin the algorithm by guessing an arbitrary value function $\widehat{g} : G \to \mathbb{R}$.

2. Given $\widehat{g}$, we solve the dynamic problem to find $c(a,z)$ and $a'(a,z)$ at every $(a,z) \in G$:

   (a) Evaluate the objective function $obj(a'; a, z)$ at every $(a'; a, z) \in G$:

   $$\begin{aligned} obj(a'; a, z) \equiv & \frac{1}{1-\gamma} \left[w + \pi(a,z) + a(1+r)\left[\lambda + (1-\lambda)\xi/\xi_{-1}\right] - a'\right]^{1-\gamma} \\ & + \beta \mathbb{E}_{z'}\left[\widehat{g}(a', z')\right] \end{aligned}$$

   (b) At every point in the state space $(a,z) \in G$, the optimal net worth decision is given by the value a' that solves

   $$a' = \text{argmax}_{\widetilde{a}' \in G_A} obj(a'; a, z)$$

   The collection of the optimal decisions at all points in the state space are combined into optimal policy function $a'(a,z)$.

   (c) Given the optimal net worth policy, compute the optimal consumption policy in state $(a,z) \in G$ as:

   $$c(a,z) = w + \pi(a,z) + a(1+r)\left[\lambda + (1-\lambda)\frac{\xi}{\xi_{-1}}\right] - a'(a,z)$$

3. Given $c(a,z)$ and $a'(a,z)$, we then derive the value function $g : G \to \mathbb{R}$ under these policies:

   $$g(a,z) = \frac{c(a,z)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_{z'}\left[g\left(a'(a,z), z'\right)\right]$$

4. Finally, we compare $g(a,z)$ with the initial guess of the value function $\widehat{g}(a,z)$.

   (a) If the difference between them (according to some metric) is below a given threshold, we then exit the loop and interpret c and a' as the solution to the entrepreneur's problem. In addition, to ensure the precision of the solution algorithm, we only exit the loop if the difference between the policy functions across consecutive iterations is below a given threshold.

[12]The aggregate demand for final goods affects the domestic demand for the entrepreneurs' varieties, which affects the solution to the static problem.

[13]We solve for their competitive equilibrium values through an algorithm described in the following section.

(b) If the difference between them is not low enough, we update our guessed value function with the function g computed in step 3, and then restart loop from step 2.

7.2 Stationary Equilibrium

To compute the stationary equilibrium outcomes of the economy, we proceed in two steps. First, we compute the stationary distribution of individuals across the state space following Heer and Maussner (2009).[14] This distribution allows us to evaluate the extent to which the market clearing conditions hold. In the second step, we present the algorithm to find the set of aggregate prices and quantities at which the market clearing conditions hold.

7.2.1 Stationary measure

The goal of the algorithm is to find the stationary distribution of individuals $\phi : G \to [0, 1]$, where $G = G_A \times G_Z$ denotes the entrepreneur's state space as defined in the previous section. To do so, the algorithm takes as given the aggregate prices and quantities, as well as the solution to the dynamic problem described above (characterized by optimal policy functions c and a'). Then, the algorithm consists of the following steps:

1. We begin the algorithm by guessing an arbitrary distribution $\widehat{\phi} : G \to [0, 1]$.

2. Given an initial distribution $\widehat{\phi}$, we use the optimal policy functions of the dynamic problem to compute the following period's distribution $\phi : G \to [0, 1]$:

   (a) We initialize ϕ by making it equal to zero at all points of the state space: $\phi(a, z) = 0$, for all $(a, z) \in G$.

   (b) We loop sequentially through each point of the state space $(a, z) \in G$ and assign the current mass of individuals at such state, $\widehat{\phi}(a, z)$, across their following-period states a' and z' as implied by the optimal policy functions and productivity distribution:

   - All individuals currently in state (a, z) choose $a'(a, z) \in G_A$. Therefore, in the following period, their net worth state is $a'(a, z) \in G_A$.
   - Moreover, for every z', only a fraction $P_Z(z, z')$ of the individuals in state (a, z) will transition to state z' in the following period, where P_Z is the transition matrix that approximates the autoregressive productivity process in the model.

[14] This approach provides significant gains in computational time relative to the popular Monte Carlo simulation alternative; moreover, it avoids the simulation error involved in implementing the latter.

- Therefore, we loop across all points of the state space $(a, z) \in G$ and all destination productivities $z' \in G_Z$ to compute $\phi : G \to [0, 1]$ by updating its value according to:

$$\phi(a'(a,z), z') = \phi(a'(a,z), z') + \widehat{\phi}(a,z) P_Z(z, z')$$

3. Finally, we compare $\phi : G \to [0, 1]$ with the initial distribution guessed in step 1.

   (a) If the difference between them is below a given threshold, then we exit loop and interpret $\phi : G \to [0, 1]$ as the stationary distribution of individuals across the state space for a given set of prices, quantities, and optimal policy functions.

   (b) If the difference between them is not low enough, I update my guessed distribution with function ϕ computed in step 2, and then I restart loop from step 2.

7.2.2 Equilibrium prices and quantities

In a stationary competitive equilibrium of the economy, prices (ξ and w) are such that all markets clear and the entrepreneurs' beliefs (about prices and aggregate domestic demand for final goods Y_h) are equal to their realized counterparts. We now describe the algorithm that we follow to compute these objects:

1. We begin the algorithm by guessing initial values of w, Y_h, and ξ.
2. Given the prices and quantities from step 1, we solve the entrepreneur's static problem.
3. Given the prices and quantities from step 1 and the solution to the entrepreneur's static problem, we solve the entrepreneur's dynamic problem.
4. Given the solution to the entrepreneur's dynamic problem, we compute the stationary distribution $\phi : G \to [0, 1]$ of entrepreneurs across the state space.
5. Given the prices from step 1, we solve the final good producer's problem.
6. Finally, we evaluate the extent to which the market clearing conditions hold as well as the extent to which some of the guessed objects differ from their realized counterparts.

   (a) We use the stationary distribution along with the enrepreneur's optimal policy functions from the static and dynamic problems, to evaluate the extent to which the market clearing conditions hold:

$$\int_{\mathcal{S}} \left[n(s) + F\mathbb{I}_{\{e(s)=1\}} \right] \phi(s) ds = 1$$

$$\int_{\mathcal{S}} \left[c(s) + x(s) \right] \phi(s) ds = Y_h$$

(b) We evaluate the extent to which the guessed values of Y_h differ from its realized counterpart:

$$Y_h = \left[\int_0^1 y_h(i)^{\frac{\sigma-1}{\sigma}} di + y_m^{\frac{\sigma-1}{\sigma}} \right]$$

(c) If the difference between the left- and right-hand-sides of the market clearing conditions is below a given threshold, and if the extent to which the guessed value of Y_h differ from its realized counterpart is sufficiently low, we then exit the loop. In that case, we interpret w, ξ, and Y_h as the general equilibrium prices and quantities. Otherwise, we use a nonlinear equation solver to update the guessed values of w, Y_h, ξ, and then restart the loop from step 2.[15]

7.3 Transitional Dynamics

We now describe our approach to computing the response of an economy that is originally in a stationary equilibrium to a one-time unexpected change in the economic environment. We assume that in period 0 the economy is in a stationary equilibrium that all agents believe will continue forever. In period 1, we assume that there is a one-time unexpected change in the economic environment. At that point, economic agents observe the sequence of a subset of structural parameters from period 1 up to the infinite future. We assume that all structural parameters remain constant from period $\widetilde{T}$ onwards, such that the economy eventually converges to a stationary equilibrium.

To simplify the exposition, here we restrict attention to a shock to the price of imports p_m.[16] In particular, we assume that the evolution of the imports price is characterized by a sequence $\{p_{m,t}\}_{t=0}^{\infty}$ such that $p_{m,t} = \overline{p_m}$ for $t \geq \widetilde{T}$. Our goal is to solve for the equilibrium path of prices $\{w_t, \xi_t\}_{t=0}^{\infty}$ and aggregate quantities $\{Y_{h,t}\}_{t=0}^{\infty}$ at which markets clear in every period $t = 0, ..., \infty$.

Step 1: Compute initial and final stationary equilibria The first step consists of computing the initial and final stationary equilibria of the economy following the methodology described in the previous sections.

Step 2: Compute transitional dynamics between initial and final stationary equilibria The second step consists of computing the transition dynamics between the initial and final stationary equilibria. The algorithm consists of the following steps:

1. Guess that the economy is in a final stationary equilibrium from period $T > \widetilde{T}$ onwards.

[15] We set up the nonlinear equation solver to equate the left- and right-hand side of the market clearing conditions, as well as to equate the guessed value of Y_h with its model counterpart.

[16] In the paper, we consider shocks to aggregate productivity A, the price of imports p_m, and the interest rate r.

2. Guess sequence of prices $\{w_t, \xi_t\}_{t=1}^T$ and aggregate quantities $\{Y_{h,t}\}_{t=1}^T$.

3. Solve for the sequence of the entrepreneur's optimal policy functions $\{c_t(a,z), a'_t(a,z)\}_{t=1}^T$

   (a) We solve for them iteratively, starting from period $t = T$.

   (b) Given the entrepreneur's value function g_{t+1} the entrepreneur's optimal policy is given by:

$$a'_t(a,z) = \text{argmax}_{\widetilde{a}' \in G_A} \frac{1}{1-\gamma} \left[w_t + \pi_t(a,z) + a(1+r)\left[\lambda + (1-\lambda)\xi_t/\xi_{t-1}\right] - \widetilde{a}'\right]^{1-\gamma} + \beta \mathbb{E}_{z'}\left[g_{t+1}(a',z')\right]$$

   i. For period $t = T$, $g_{t+1}(a',z')$ is equal to the value function in the final stationary equilibrium

   ii. For periods $t < T$, $g_{t+1}(a',z')$ is computed in previous iterations.

   (c) Given the optimal net worth policy, compute the optimal consumption policy in state $(a,z) \in G$ as:

$$c_t(a,z) = w_t + \pi_t(a,z) + a(1+r)\left[\lambda + (1-\lambda)\frac{\xi_t}{\xi_{t-1}}\right] - a'_t(a,z)$$

   (d) Given $c_t(a,z)$ and $a'_t(a,z)$, we then derive the value function $g_t : G \to \mathbb{R}$ under these policies:

$$g_t(a,z) =$$

   (e) We go back to step (b) but starting from period $t-1$.

4. Compute the sequence of distributions of entrepreneurs between periods 1 and T:

   (a) The distribution of entrepreneurs in period 0 is given by the distribution of entrepreneurs in the initial stationary equilibrium.

   (b) We solve for the sequence of distributions iteratively starting from period $t = 1$.

   (c) We initialize ϕ_t by making it equal to zero at all points of the state space: $\phi_t(a,z) = 0$, for all $(a,z) \in G$.

   (d) We loop sequentially through each point of the state space $(a,z) \in G$ and assign the current mass of individuals at such state, $\phi_{t-1}(a,z)$, across their following-period states a' and z' as implied by the optimal policy functions and productivity distribution:

      - All individuals currently in state (a,z) choose $a'(a,z) \in G_A$. Therefore, in the following period, their net worth state is $a'_{t-1}(a,z) \in G_A$.

- Moreover, for every z', only a fraction $P_Z(z, z')$ of the individuals in state (a, z) will transition to state z' in the following period, where P_Z is the transition matrix that approximates the autoregressive productivity process in the model.
- Therefore, we loop across all points of the state space $(a, z) \in G$ and all destination productivities $z' \in G_Z$ to compute $\phi : G \to [0, 1]$ by updating its value according to:

$$\phi_t(a'_{t-1}(a, z), z') = \phi_t(a'_{t-1}(a, z), z') + \phi_{t-1}(a, z)P_Z(z, z')$$

(e) If $t \neq T$, we go back to step (b) but starting from period $t + 1$.

5. Finally, we evaluate the extent to which the market clearing conditions hold as well as the extent to which some of the guessed objects differ from their realized counterparts.

(a) We use the stationary distribution along with the enrepreneur's optimal policy functions from the static and dynamic problems, to evaluate the extent to which the market clearing conditions hold in each period:

$$\int_{\mathcal{S}} \left[n_t(s) + F\mathbb{I}_{\{e_t(s)=1\}}\right] \phi_t(s)ds = 1 \quad \forall t = 1, ..., T$$
$$\int_{\mathcal{S}} \left[c_t(s) + x_t(s)\right] \phi_t(s)ds = Y_{h,t} \quad \forall t = 1, ..., T$$

(b) We evaluate the extent to which the guessed values of $Y_{h,t}$ differ from their realized counterpart:

$$Y_{h,t} = \left[\int_0^1 y_{h,t}(i)^{\frac{\sigma-1}{\sigma}} di + y_{m,t}^{\frac{\sigma-1}{\sigma}}\right] \quad \forall t = 1, ..., T$$

6. If the difference between the left- and right-hand-sides of the market clearing conditions is below a given threshold in every period, and if the extent to which the guessed value of $Y_{h,t}$ differ from its realized counterpart is sufficiently low in every period, we then exit the loop. In that case, we interpret w_t, ξ_t, and $Y_{h,t}$ as the general equilibrium prices and quantities. Otherwise, we use a nonlinear equation solver to update the guessed values of w_t, $Y_{h,t}$, ξ_t, and then restart the loop from step 2.[17]

7. Finally, if the difference between aggregate quantities and prices between period T (the last period of the transition) and $T + 1$ (the first period of the final stationary equilibrium) is small enough, we are done. Otherwise, we return to step 1 and increase T.

[17] We set up the nonlinear equation solver to equate the left- and right-hand side of the market clearing conditions, as well as to equate the guessed value of Y_h with its model counterpart.

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