Calibration notes for Endogenous Labor OG Model

Richard W. Evans and Jason DeBacker

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1 Two methods (but really just one)

There are two methods to calibrate the χ^n_s parameters of the OG model with endogenous labor. The first is to use the S initial-period labor supply Euler equations. The second is to use GMM to match average labor supply moments from the model steady-state to average labor supply moments from the data. The first method is orders of magnitude more simple, more intuitive, and more tractable.

1.1 Initial period labor supply Euler equations

In the model from Chapter 7 of the textbook, the general form of the labor supply Euler equation is the following.

$$w_t\left(c_{s,t}\right)^{-\sigma} = \chi^n_s\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1-\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}} \quad \forall s,t \tag{1}$$

We have already estimated the elliptic utility parameter values of b and υ, and we have calibrated the value for σ from other studies.

All the consumption $c_{s,t}$ and wage w_t values in the set of equations represented by (1) are given in consumption units (consumption is the numeraire good). So w_t represents the units of consumption that a worker is paid for each unit of labor supplied. And the implicit price of one unit of consumption is one consumption unit. This can be seen from the budget constraint.

$$c_{s,t} + b_{s+1,t+1} = (1+r_t)b_{s,t} + w_t n_{s,t} \tag{2}$$

For this reason, $c_{s,t}$ also represents the total individual consumption expenditure of age-s household at in period t.

One may identify the χ^n_s via a method of moments estimation that uses Equation 1 as the set of moment conditions and data on consumption, wages, and labor supply. However, the estimates of χ^n_s will depend upon the units of measurement for wages and consumption. Thus we could only identify the χ^n_s from the model up to a scale. This is clearly seen if we rearrange Equation 1 to isolate χ^n_s on the left hand side:

$$\chi^n_s = \frac{w_t \left(c_{s,t}\right)^{-\sigma}}{\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1-\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}} \quad \forall s,t \tag{3}$$

1.2 Scaling

We have assumed that $\tilde{l}=1$. Regardless of the value, the appearance of $\frac{n_{s,t}}{\tilde{l}}$ denominator on the right-hand-side of (3) is a unit-free percent of total time endowment. However, both consumption $c_{s,t}$ and wages w_t on the right-hand-side of (3) are in model units (consumption units).

To overcome this identification issue, consider a scaling that relates model units to data units. Call this parameter $factor_t$ and define it as:

$$factor_t = \frac{\bar{y}^{data}_t}{\bar{y}^{model}_t} \quad \forall t \quad \Longrightarrow \quad \bar{y}^{data}_t = \bar{y}^{model}_t \times factor_t \tag{4}$$

In other words, $factor$ scales model units to data units for those variables measured in consumption units in the model and nominal amounts in the data. Thus we also have the relations

$$\begin{aligned} w^{model}_t \times factor_t &= w^{data}_t \\ c^{model}_{s,t} \times factor_t &= c^{data}_{s,t} \end{aligned} \tag{5}$$

With this, we can return to Equation 3. Let's write two versions of this equation. One that identifies the χ^n_s in the model and one that identifies it's data counterpart. To be clear, let χ^n_s be the model version and $\hat{\chi}^n_s$ represent the χ^n_s to be identified from the data. Thus we have:

$$\chi^n_s = \frac{w^{model}_t \left(c^{model}_{s,t}\right)^{-\sigma}}{\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1-\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}} \quad \forall s,t \tag{6}$$

and

$$\hat{\chi}^n_s = \frac{w^{data}_t \left(c^{data}_{s,t}\right)^{-\sigma}}{\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1-\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}} \quad \forall s,t \quad \text{where} \quad \hat{\chi}^n_s \equiv factor_t^{1-\sigma}\chi^n_s \tag{7}$$

Note that for brevity, we do not have *data* or *model* superscripts on the labor supply terms. This is because, as noted above, labor supply is always divided by labor endowment and so the ratio is in percentages both in the data and the model.

Now let's do some algebra with Equation 6:

$$
\begin{aligned}
\chi^n_s &= \frac{w_t^{model}\left(c_{s,t}^{model}\right)^{-\sigma}}{\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1-\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}} \quad \forall s,t \\
\implies factor_t^{1-\sigma}\chi^n_s &= \frac{factor_t^{1-\sigma}w_t^{model}\left(c_{s,t}^{model}\right)^{-\sigma}}{\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1-\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}} \quad \forall s,t \\
\implies factor_t^{1-\sigma}\chi^n_s &= \frac{\left(factor_t w_t^{model}\right)\left(factor_t c_{s,t}^{model}\right)^{-\sigma}}{\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1-\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}} \quad \forall s,t \qquad (8)\\
\implies factor_t^{1-\sigma}\chi^n_s &= \underbrace{\frac{w_t^{data}\left(c_{s,t}^{data}\right)^{-\sigma}}{\left(\frac{b}{\tilde{l}}\right)\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon-1}\left[1-\left(\frac{n_{s,t}}{\tilde{l}}\right)^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}}}}_{=\hat{\chi}^n_s} \quad \forall s,t \\
\implies \hat{\chi}^n_s &\equiv factor_t^{1-\sigma}\chi^n_s \quad \forall s,t
\end{aligned}
$$

Thus, by estimating $\hat{\chi}^n_s$ using the data on wages, consumption, and labor supply, one finds the model parameters up to a scale. That scale is function of the model scale parameter, $factor_t$.

1.3 Changes to Model Solution Algorithm

In theory, one wants to use the factor from model period t, where model period t corresponds to they year of your data (e.g., if the data are from 2017 and your initial period in the time path of your model is 2017, then you'd want determine the factor as $factor_0 = \frac{\bar{y}^{data}_{2017}}{\bar{y}^{model}_0}$). Because it depends on mean model income, the factor is endogenous and depends upon χ^n_s. Therefore, there is the need for some fixed point algorithm: guess a $factor$, use that to determine χ^n_s, solve the model and see if mean income in the data divided by mean income in the model returns the factor you guess, if not, update and do again. To compute the time path at each step in this fixed point algorithm would be very expensive. We therefore make a simplifying assumption. In particular, we assume that the factor is determined as the ratio of income in data from year t and from the model's steady state. That is,

$$
factor_t = factor = \frac{\bar{y}^{data}_{2017}}{\bar{y}^{model}_{SS}} \ \forall t \qquad (9)
$$

While not a perfect mapping, this means that at each iteration of the fixed point algorithm that solves for the model $factor$ only the steady state needs to be computed.

Also note that the model income represents real, stationarized income. So growth and inflation are not affecting this measurement, which helps this approximation be more accurate.

The modification to the algorithm to solve the steady state in Chapter 7 need only be modified to include the guess of $factor$ as one of our outer-loop variables along with $\bar{r}$ in the steady-state computational approach. Given the guess for $\bar{r}$ and $factor$ one can use the relation in Equation 8 to transform the $\hat{\chi}^n_s$ into the model-scaled parameter. With these χ^n_s values, we can solve for the steady-state household decisions $\{\bar{n}_s\}_{s=1}^S$ and $\{\bar{b}_s\}_{s=2}^S$. From these decisions, we can compute the corresponding steady-state interest rate $\bar{r}$ and average household income in the model $\bar{y}$. We update $\bar{r}$ and $factor$ until the interest rate and factor implied by steady state equilibrium equal the guesses of $\bar{r}$ and $factor$ at that iteration. Once the steady state is solved and $factor$ is determined, then this same factor is applied over the time path, so no adjustment is needed for that solution method.

1.3.1 A Note About Initial the Initial Guess for $factor$

Because the difference between model units and real world units might be multiple orders of magnitude, it is helpful to get the initial guess for $factor$ near its true value. If one has trouble finding an initial guess that will not break the model, a good strategy is to compute the steady-state of the model assuming that $\chi^n_s = 1$ for all s. This means that only $\bar{r}$ is in the outer loop. Use the resulting $\bar{y}$ to derive an initial guess for $factor$ according to (4). This should get your initial guess in the neighborhood of the final value.