

# Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis

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*This paper develops a computable general equilibrium model in which endogenous agency costs can potentially alter business-cycle dynamics. A principal conclusion is that the agency-cost model replicates the empirical fact that output growth displays positive autocorrelation at short horizons. This hump-shaped output behavior arises because households delay their investment decisions until agency costs are at their lowest—a point in time several periods after the initial shock. (JEL E32, E44)*

At least since Irving Fisher's (1933) "debt-deflation" explanation of the Great Depression, many economists have viewed financial factors, such as borrower net worth, as important elements of business-cycle fluctuations. The familiar story goes something like this. To engage in investment opportunities, entrepreneurs must partially rely on external finance. This borrowing is typically limited because of the agency costs involved. An aggregate shock that transfers wealth from entrepreneurs to lenders will lower aggregate investment because this wealth redistribution will increase the need for external finance and thus lead to greater agency costs. These shocks can then be propagated forward because the lower level of investment today tends to lead to lower levels of capital, output, and net worth tomorrow.

A seminal contribution to this line of research was made by Ben Bernanke and Mark Gertler (1989) (hereafter denoted as BG). BG developed a general equilibrium model in which agency costs arise endogenously. This is a nontrivial exercise because these agency problems arise only in a setting in which the

Modigliani-Miller theorem does not hold. An important insight of BG is the theoretical possibility that agency costs will enhance the propagation of productivity shocks. We borrow our title from BG because in this paper we build on their work by constructing a calibrated, computable general equilibrium model that can capture *quantitatively* the effects that BG analyze *qualitatively*. Our study thus also builds on Fuerst (1995), who provided a first attempt at quantitatively modeling the BG environment. As in BG, we will impose the agency costs on the creation of new capital. This is, of course, not the only possibility. Instead, it is a first step in a larger research program that will quantitatively investigate the role of agency costs in general equilibrium business-cycle models. The BG model is a natural place to begin this research program because of its prominence in the literature.

An important innovation in the current paper is to model the entrepreneurs as long lived.<sup>1</sup> BG ignore this issue by analyzing an overlapping generations model in which entrepreneurs make investment decisions in only one period. Although Fuerst (1995) assumes that households are infinitely lived, he follows BG by assuming that entrepreneurs live for only a single period. Allowing for long-lived entrepreneurs is potentially

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<sup>1</sup> The importance of long-lived entrepreneurs was also suggested by Gertler (1995) in his comments on Fuerst (1995).

difficult because the contracting problem between lenders and entrepreneurs then takes on the characteristics of a repeated game with moral hazard.<sup>2</sup> We ignore much of this difficulty by assuming that there is enough inter-period anonymity so that financial contracts can depend only on an entrepreneur's level of net worth, and not on his entire past history of debt repayment (although history, of course, plays a role in that it affects the current level of net worth).

Even with this simplification, we still have the problem of heterogeneity. At any point in time, there will be a great deal of net worth heterogeneity across entrepreneurs, and keeping track of the net worth distribution and of how it affects the aggregate economy is in general quite difficult. However, by assuming a linear investment and linear monitoring technology, we are able to exploit an aggregation result: Only the first moment of the distribution of entrepreneurial net worth has any effect on the aggregate economy. Keeping track of the mean is quite easy, and amounts to simply adding an additional state variable to the dynamic program.

A principal conclusion is that the agency-cost model replicates the empirical fact that output growth displays positive autocorrelation at short horizons. This hump-shaped output behavior arises because households delay their investment decisions until agency costs are at their lowest—a point in time several periods after the initial productivity shock. Agency costs fall with time because the productivity shock increases the return to internal funds, which in turn redistributes wealth from households to entrepreneurs. The hump in the aggregate variables thus mirrors the hump-shaped behavior of entrepreneurial net worth. The model's hump-shaped output response is of particular interest given the recent work of Timothy Cogley and James M. Nason (1995), who document this behavior in the data, and also demonstrate that standard real-business-cycle (RBC) models are inconsistent with this prediction.

A related attempt to model long-lived entrepreneurs is provided by Nobuhiro Kiyotaki and John H. Moore (1995). There are two distinct differences between the current paper and theirs. First, the underlying contracting environment in Kiyotaki and Moore is quite different. They build on the work of Oliver Hart and Moore (1994), which analyzes the contracting problem in an environment with *ex post* renegotiation and the inalienability of human capital. One implication of the Kiyotaki-Moore contract is that borrowing is so tightly constrained by the level of net worth that default never occurs in equilibrium. In contrast, we follow BG and adopt the costly state verification model of Robert M. Townsend (1979). Here, lending exceeds net worth, so that default is an equilibrium phenomenon. A second difference between this paper and Kiyotaki and Moore is that we attempt to quantify the effects of agency costs in an otherwise standard RBC model.

Section I of the paper develops the optimal financial contract in a partial equilibrium setting and demonstrates the aggregation result that is so important in the sequel. Section II lays out the complete general equilibrium environment. Section III discusses calibration, and Sections IV and V present our numerical results. We conclude in Section VI.

## I. The Financial Contract

In this section we consider the financial contract in a partial equilibrium setting. This financial contract generates an upwardly sloped supply curve for investment goods. In the next section we will embed this supply curve into an otherwise standard RBC model. We are able to separate consideration of the contract from the rest of the general equilibrium model because the contract is only one period in length—it is negotiated at the beginning of a period and resolved by the end of that same period. General equilibrium issues affect the contract through the level of entrepreneurial net worth,  $n > 0$ , and through the aggregate price of capital,  $q > 0$ . For the purposes of this section, we will take  $n$  and  $q$  parametrically.

The contract consists of two parties: an entrepreneur with net worth  $n > 0$ , and a lender with resources that he may wish to lend to

<sup>2</sup> See Gertler (1992) for a theoretical analysis of an agency-cost model in which entrepreneurs write two-period contracts.

the entrepreneur. Both are assumed to be risk neutral.<sup>3</sup>

The entrepreneur has access to a stochastic technology that contemporaneously transforms  $i$  consumption goods into  $\omega i$  units of capital. The random variable  $\omega$  is i.i.d. across time and across entrepreneurs, with distribution  $\Phi$ , density  $\phi$ , a nonnegative support, and a mean of unity. Agency issues are introduced into the environment by assuming that  $\omega$  is privately observed by the entrepreneur. Others can privately observe  $\omega$  only at a monitoring cost of  $\mu i$  capital units, i.e., the attempt to monitor the project results in the destruction of  $\mu i$  units of capital. This informational asymmetry creates a moral hazard problem because, absent monitoring, the entrepreneur may wish to misreport the true value of  $\omega$ . The optimal contract will be structured in such a way that the entrepreneur will always truthfully report the  $\omega$  realization. Note that the capital production and monitoring technologies each exhibit constant returns to scale. This assumed linearity is the source of the aggregation result below.

To make the asymmetric information problem relevant, assume that net worth is sufficiently small so that entrepreneurs would like to receive some external financing from firms. Douglas Gale and Martin Hellwig (1985) and Stephen Williamson (1987a) have demonstrated that in environments of this type, the optimal contract between lenders and entrepreneurs is risky debt.<sup>4</sup> The contract will be characterized by an interest rate  $r^k$ . An entrepreneur who borrows  $(i - n)$  consumption

goods agrees to repay  $(1 + r^k)(i - n)$  capital goods to the lender. The entrepreneur will default if the realization of  $\omega$  is "low," i.e., if  $\omega < (1 + r^k)(i - n)/i \equiv \bar{\omega}$ . The lender will monitor the project outcome only if the entrepreneur defaults, in which case it will confiscate all the returns from the project. Note that the contract is completely defined by the pair  $(i, \bar{\omega})$ , and that it is convenient to consider the optimization problem over these two arguments. Once the optimal  $(i, \bar{\omega})$  have been found, one can then back out the implied lending rate of interest,  $(1 + r^k) \equiv \bar{\omega}i/(i - n)$ . Under the contract (with  $q$  denoting the end-of-period price of capital), expected entrepreneurial income is given by

$$q \left[ \int_{\bar{\omega}}^{\infty} \omega i \Phi(d\omega) - (1 - \Phi)(1 + r^k)(i - n) \right].$$

Using the definition of  $\bar{\omega}$ , this can be simplified to

$$\begin{aligned} qif(\bar{\omega}) \\ = qi \left\{ \int_{\bar{\omega}}^{\infty} \omega \Phi(d\omega) - [1 - \Phi(\bar{\omega})] \bar{\omega} \right\}, \end{aligned}$$

where  $f(\bar{\omega})$  is interpreted as the fraction of the expected net capital output received by the entrepreneur. Similarly, the expected income of the lender on such a contract is given by

$$\begin{aligned} q \left[ \int_0^{\bar{\omega}} \omega i \Phi(d\omega) - \Phi \mu i \right. \\ \left. + (1 - \Phi)(1 + r^k)(i - n) \right], \end{aligned}$$

or

$$\begin{aligned} qig(\bar{\omega}) \equiv qi \left\{ \int_0^{\bar{\omega}} \omega \Phi(d\omega) - \Phi(\bar{\omega}) \mu \right. \\ \left. + [1 - \Phi(\bar{\omega})] \bar{\omega} \right\}, \end{aligned}$$

<sup>3</sup> In the next section, risk-averse households will be the source of loanable funds to the entrepreneurs. However, in terms of the financial contract, they will be effectively risk neutral because: (1) there will be no aggregate uncertainty over the duration of the contract, and (2) they will carry out their lending through a capital mutual fund (CMF). By funding a large number of entrepreneurs, the CMF will take advantage of the law of large numbers to eliminate idiosyncratic entrepreneurial uncertainty and guarantee a sure return to the households.

<sup>4</sup> In addition, we must assume that a commitment device exists, and that stochastic monitoring is impossible. See, for example, Townsend (1979) and Edward Prescott and Townsend (1984). In contrast, BG allow for the possibility of stochastic monitoring so that the optimal contract cannot be interpreted as risky debt. Recent work by John H. Boyd and Bruce D. Smith (1994) suggests that (in a quantitative sense) there is little loss of generality in restricting the contract to pure strategies.

where  $g(\bar{\omega})$  is interpreted as the fraction of the expected net capital output received by the lender. Note that

$$f(\bar{\omega}) + g(\bar{\omega}) = 1 - \Phi(\bar{\omega})\mu,$$

so that on average,  $\Phi(\bar{\omega})\mu$  of the produced capital is destroyed by monitoring, and the remainder is split between the entrepreneur [ $f(\bar{\omega})$ ] and lender [ $g(\bar{\omega})$ ].

The optimal contract is given by the  $(i, \bar{\omega})$  pair that maximizes the entrepreneur's expected return subject to the lender being indifferent between loaning the funds and retaining them.<sup>5</sup> More precisely, the optimal contract is given by the solution to

$$\max qif(\bar{\omega}), \text{ subject to } qig(\bar{\omega}) \geq (i - n).$$

An additional constraint guarantees the participation of the entrepreneurs, namely,  $qif(\bar{\omega}) \geq n$ , which will always be satisfied below. It is also straightforward to show that the entrepreneur will always want to invest all of his net worth in his own project. The first-order conditions to the problem include

$$\begin{aligned} (1) \quad & q\{1 - \Phi(\bar{\omega})\mu \\ & + \phi(\bar{\omega})\mu[f(\bar{\omega})/f'(\bar{\omega})]\} = 1; \\ (2) \quad & i = \{1/[1 - qg(\bar{\omega})]\}n. \end{aligned}$$

Equation (1) defines an implicit function  $\bar{\omega}(q)$ , with  $\bar{\omega}$  increasing in  $q$ . Substituting this function into (2), we have the implicit function  $i(q, n)$ , which represents the amount of consumption goods placed into the capital technology. The expected capital output is then given by  $I^S(q, n) \equiv i(q, n)\{1 - \mu\Phi[\bar{\omega}(q)]\}$ , which, given the infinite number of entrepreneurs, can also be interpreted as the investment (or new-capital) supply function. Since (1) pins down  $\bar{\omega}$  uniquely, the linearity of (2) implies that this supply function aggregates:

gates: aggregate investment (summing  $I^S$  across all entrepreneurs) depends only on the economywide price of capital  $q$  and aggregate net worth (summing  $n$  across all entrepreneurs). It is this aggregation result that we exploit below.<sup>6</sup>

Before proceeding to the general equilibrium model, it is instructive to review some of the comparative statics of the contracting problem. It is straightforward to show that  $I_1^S(q, n) > 0$  and  $I_2^S(q, n) > 0$  (see the Appendix). The positive slope to the capital supply curve ( $I_1^S > 0$ ) is isomorphic to models that assume some increasing cost to adjusting the capital stock. Here, this positive slope is a natural result of the agency problem. For a given level of net worth, increases in capital production are possible only with a greater reliance on external funds, and these external funds are subject to greater agency costs. As for net worth, it can be viewed as a supply-curve shifter because increases in net worth lower agency costs and thus boost the level of capital production for a given price of capital ( $I_2^S > 0$ ).

Another variable that will be useful below is the expected return to internal funds. This is given by  $qf(\bar{\omega})i/n = qf(\bar{\omega})/[1 - qg(\bar{\omega})]$ . (Intuitively, net worth of size  $n$  is leveraged into a project of size  $i$ , entrepreneurs keep share  $f(\bar{\omega})$  of the capital produced, and capital is priced at  $q$  consumption goods.) Because we have assumed that all economic rents on the contract flow to the entrepreneur, this return is strictly increasing in  $q$ .

## II. The General Equilibrium Model

The goal of this section is to embed the contracting problem of Section I into an otherwise standard RBC model. The standard model assumes that new capital is created at the end of the period using consumption goods, with a nonstochastic one-to-one transformation rate

<sup>5</sup> We are assuming that the economic rents generated by the contract flow to the entrepreneur—an assumption that is quite plausible given that entry into lending is more likely than entry into entrepreneurial activity. Also, since these loans are intraperiod, the opportunity cost of the funds is simply  $(i - n)$ .

<sup>6</sup> The linearity in the optimal contract has obvious modeling conveniences. There are of course limitations, the foremost being the empirical implausibility of the implication that the bankruptcy probability is the same across entrepreneurs (all with differing levels of net worth). Whether this limitation at the micro-level has any aggregative consequences is a topic for future work.

(implying an investment supply curve that is perfectly elastic at unity). This newly produced capital then comes "on line" in the next period. Below, we will utilize this same timing, but replace the one-to-one transformation assumption with the contracting problem outlined in Section I. In particular, if a household wishes to purchase capital, it must fund entrepreneurial projects, and these projects are subject to agency problems. Table 1 summarizes the sequence of events in a given period. We will now turn to the specifics of the model.

The model economy consists of a continuum of agents with unit mass. The agents are of two types: households (fraction  $1 - \eta$ ) and entrepreneurs (fraction  $\eta$ ). As discussed in the previous section, the entrepreneurs are involved in producing the investment good. Entrepreneurs receive their external financing from households via intermediaries that we will refer to as capital mutual funds (CMFs). The economy is also populated with numerous firms producing the single consumption good. We follow BG and assume that these consumption-producing firms are not subject to any agency problems, so that we need not be specific about how they are financed. Because their activities are somewhat standard, we will first discuss the behavior of households and firms. We will then turn to the entrepreneurs and the CMFs.

Households are infinitely lived, with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - L_t),$$

where  $E_0$  denotes the expectation operator conditional on time-0 information,  $\beta \in (0, 1)$  is the personal discount factor,  $c_t$  is time- $t$  consumption,  $L_t$  is time- $t$  labor, and the leisure endowment is normalized to unity. In the course of any given period, households sell their labor input to consumption-producing firms at a wage rate of  $w_t$ , rent their previously accumulated capital holdings to these firms at rental rate  $r_t$ , purchase consumption from these firms at a price of unity (i.e., consumption is the numeraire), and purchase new capital goods at a price of  $q_t$ . Capital goods are purchased at the end of the period with the assistance of CMFs (to which we will return

below). Household choices are summarized in the labor supply curve

$$U_L(t)/U_C(t) = w_t,$$

and in the dynamic capital-demand relationship

$$q_t U_c(t) = \beta E_t U_c(t+1) [q_{t+1}(1 - \delta) + r_{t+1}],$$

where  $\delta$  is the rate of depreciation on capital. Each household also owns an equal equity share in each of the firms.

The firms in this economy produce the consumption good utilizing a standard constant-returns-to-scale production function. In the aggregate, this technology is given by

$$Y_t = \theta_t F(K_t, H_t, H_t^e),$$

where  $Y_t$  denotes aggregate output of the consumption good,  $\theta_t$  denotes the stochastic productivity parameter,  $K_t$  denotes the aggregate capital stock (including entrepreneurial capital),  $H_t$  denotes the aggregate supply of household labor, and  $H_t^e$  denotes the aggregate supply of entrepreneurial labor. Competition in the factor market implies that wage and rental rates are equal to their respective marginal products:  $r_t = \theta_t F_1(t)$ ,  $w_t = \theta_t F_2(t)$ , and  $x_t = \theta_t F_3(t)$ , where  $x_t$  is the wage rate for entrepreneurial labor. The assumption of entrepreneurial labor income is necessary because it ensures that each entrepreneur always has a nonzero level of net worth. This is important because the financial contracting problem is not well defined for zero levels of net worth. Below, we will assume that this source of net worth is quite small but nonzero.

We will now turn to entrepreneurial behavior. As noted earlier, a key innovation in the paper is to model entrepreneurs as long-lived. The aggregation result of Section I suggests that this is quite tractable. The contracting problem assumes risk neutrality, so we maintain that assumption here. Agency costs imply that the return to internal funds is greater than the return to external funds. This creates a problem: Absent some additional assumption on behavior, entrepreneurs will postpone consumption and quickly accumulate enough

TABLE 1—THE SEQUENCE OF EVENTS IN A GIVEN TIME PERIOD

1. The current aggregate productivity shock is realized ( $\theta_t$ ).
2. Firms hire labor and rent capital from households and entrepreneurs. These inputs are used to produce the consumption good,  $Y_t = \theta_t F(K_t, H_t, H_t^e)$ .
3. Households decide how much of their labor and capital income to consume immediately, and how much to use to purchase the investment good. For each unit of investment that the household wishes to purchase, it gives  $q_t$  consumption goods to the capital mutual fund (CMF).
4. The CMFs use the resources obtained from households to provide loans to an infinite number of entrepreneurs utilizing the optimal financial contract derived in Section I.
5. Entrepreneurs borrow resources from the CMF and place all of these resources (along with their entire net worth) into their capital-creation technology.
6. The idiosyncratic technology shock of each entrepreneur is realized,  $\omega^j$ , where  $j$  indexes the infinite number of entrepreneurs. If  $\omega^j \geq \bar{\omega}$ , the loan from the CMF is repaid; otherwise, the entrepreneur declares bankruptcy and is monitored by the CMF.
7. Those entrepreneurs who are still solvent make their consumption decision.

capital so that they are completely self-financed ( $i = n$ ) and agency costs disappear. There are several ways to deal with this problem. Essentially, we need to make sure that entrepreneurial consumption occurs to such an extent that self-financing does not arise. Here, we take the most direct route. We will assume that entrepreneurs discount the future more heavily than do households.<sup>7</sup> Formally, we will assume that entrepreneurs maximize the intertemporal objective

$$E_0 \sum_{t=0}^{\infty} (\beta\gamma)^t c_t^e,$$

where  $c_t^e$  denotes time- $t$  consumption and  $\gamma \in (0, 1)$  denotes the additional rate of discounting. To raise internal funds, the entrepreneur rents his capital and inelastically supplies his unit endowment of labor to firms. The entrepreneur then sells his remaining undepreciated capital to a CMF for consumption goods (recall that the capital-creating technology uses consumption goods as the input). After these transactions are finished, the net worth of the entrepreneur (in consumption units) is given by

$$n_t = x_t + z_t[q_t(1 - \delta) + r_t],$$

<sup>7</sup> In the earlier working paper version of this paper, we assumed that a certain fraction of the entrepreneurs died each period, and sold their accumulated capital stock to households. In terms of consumption, this is equivalent to assuming that the entrepreneurs consume a constant fraction of their capital holdings each period.

where  $z_t$  denotes the capital holdings of the entrepreneur at the beginning of period  $t$ . The entrepreneur uses this net worth as the basis for the loan agreement that he will enter into with the lender. Risk neutrality and the high internal return imply that the entrepreneur will always choose to pour his entire net worth into the loan contract. As noted in the introduction, we sidestep any repeated game aspects of the financial contract by assuming that the contract can be based solely on this net worth level, and not on past contractual outcomes. At the end of the period, those entrepreneurs who are still solvent make their consumption decision, trading off the benefit of current consumption with the future return on internal funds. Assuming an interior solution, this internal calculus implies the following Euler equation:

$$q_t = E_t \beta \gamma [q_{t+1}(1 - \delta) + r_{t+1}] \\ \times \{q_{t+1} f(\bar{\omega}_{t+1}) / [1 - q_{t+1} g(\bar{\omega}_{t+1})]\}.$$

The modeling advantages of our linearity assumptions are quite evident in this Euler equation—individual net worth does not appear so that this condition holds for all solvent entrepreneurs. The term in braces is greater than one and is the gross expected return on internal funds (see Section I). It is this additional return that encourages entrepreneurs to accumulate capital even though they discount the future more heavily than do households ( $\beta\gamma$  vs.  $\beta$ ). To avoid the self-financing out-

come mentioned above, our calibration below will select  $\gamma$  to exactly offset the steady-state internal return:  $\gamma q f/[1 - qg] = 1$ . Aggregating across the entrepreneurs' budget constraints we have:

$$\begin{aligned} Z_{t+1} = & \{ \eta x_t + Z_t [q_t(1 - \delta) + r_t] \} \\ & \times \{ f(\bar{\omega}_t)/[1 - q_t g(\bar{\omega}_t)] \} \\ & - \eta c_t^e/q_t, \end{aligned}$$

where  $Z_t$  denotes the aggregate entrepreneurial capital stock, and, in a slight abuse of notation,  $c_t^e$  now denotes average entrepreneurial consumption.

The lenders in this economy are ultimately the households that wish to purchase capital. As in Douglas Diamond (1984) and Williamson (1986), there is a clear role for CMFs to intermediate these purchases between households and entrepreneurs. By providing resources to an infinite number of entrepreneurs, the CMF can ensure a certain return to the household, i.e., an expenditure of  $q_t$  consumption goods guarantees one capital good. The law of large numbers, and the assumption that there is no aggregate uncertainty over the duration of the contract, implies that the CMF will be, as assumed in Section II, risk neutral. The CMF is thus a type of cooperative by which capital can be efficiently purchased. A household that turns over  $q_t$  consumption goods to the CMF receives in return one capital good. This capital comes from three distinct sources. First, in advance of the loan contract, entrepreneurs transform their accumulated capital into consumption goods by selling off this capital to CMFs. Second, after the outcome of the loan contract, CMFs receive newly created capital as loan repayment from the entrepreneurs. Third, those entrepreneurs who are still solvent sell off a portion of their newly created capital to finance end-of-period entrepreneurial consumption.

To close the model, we need only state the market-clearing conditions. There are four markets in this economy: two labor markets, a consumption-goods market, and a capital-goods market. The respective clearing conditions are given by

$$H_t = (1 - \eta)L_t$$

$$H_t^e = \eta$$

$$(1 - \eta)c_t + \eta c_t^e + \eta i_t = Y_t$$

$$K_{t+1} = (1 - \delta)K_t + \eta i_t [1 - \Phi(\bar{\omega}_t)\mu].$$

A recursive competitive equilibrium is defined by decision rules for  $K_{t+1}$ ,  $Z_{t+1}$ ,  $H_t$ ,  $q_t$ ,  $n_t$ ,  $i_t$ ,  $\bar{\omega}_t$ ,  $c_t^e$ , and  $c_t$ , where these decision rules are stationary functions of  $(K_t, Z_t, \theta_t)$  and satisfy the following:

$$(3) \quad U_L(t)/U_C(t) = \theta_t F_2(K_t, H_t, \eta);$$

$$(4) \quad q_t U_C(t) = \beta E_t U_C(t+1) [q_{t+1}(1 - \delta) + \theta_{t+1} F_1(K_{t+1}, H_{t+1}, \eta)];$$

$$(5) \quad K_{t+1} = (1 - \delta)K_t + \eta i_t [1 - \Phi(\bar{\omega}_t)\mu];$$

$$(6) \quad (1 - \eta)c_t + \eta c_t^e + \eta i_t = Y_t;$$

$$(7) \quad q_t = 1/\{1 - \Phi(\bar{\omega}_t)\mu + \phi(\bar{\omega}_t)\mu[f(\bar{\omega}_t)/f'(\bar{\omega}_t)]\};$$

$$(8) \quad i_t = \{1/[1 - q_t g(\bar{\omega}_t)]\} n_t;$$

$$(9) \quad n_t = \theta_t F_3(K_t, H_t, \eta) + (Z_t/\eta)[q_t(1 - \delta) + \theta_t F_1(K_t, H_t, \eta)];$$

$$(10) \quad Z_{t+1} = \eta n_t \{ f(\bar{\omega}_t)/[1 - q_t g(\bar{\omega}_t)] \} - \eta c_t^e/q_t;$$

$$(11) \quad q_t = E_t \beta \gamma [q_{t+1}(1 - \delta) + r_{t+1}] \times \{ q_{t+1} f(\bar{\omega}_{t+1}) \div [1 - q_{t+1} g(\bar{\omega}_{t+1})] \}.$$

In the Appendix, we demonstrate that if we hold net worth constant, then equilibrium conditions (3)–(8) are isomorphic to a standard RBC model with costs of adjusting the capital stock.<sup>8</sup> It is in this sense that the agency-cost model can be seen as a particular way of endogenizing adjustment costs. One unique characteristic of these adjustment costs is that they are affected by the level of net worth—increases in net worth lower agency costs and thus make it easier to expand the capital stock. The remaining three equations (9)–(11) track the dynamic behavior of this net worth variable.

### III. Calibration

The model is parameterized at the nonstochastic steady state to roughly match empirical counterparts.

Household preferences are given by  $U(c, 1 - L) = \ln(c) + \nu(1 - L)$ , where the constant  $\nu$  is chosen so that  $L = 0.3$ . We set  $\beta = 0.99$ , thus implying a 4-percent annual real rate of interest. The value of  $\eta$  is simply a normalization.

The consumption production technology is assumed to be Cobb-Douglas with a capital share of 0.36, a household labor share of 0.6399, and an entrepreneurial labor share of 0.0001. Recall that this last share needs to be positive to ensure that each entrepreneur always has at least some net worth. We set it arbitrarily small so that the model with  $\mu = 0$  essentially collapses to the standard RBC model. The capital depreciation rate is set to  $\delta = 0.02$ .

As for the monitoring technology, there is a great deal of controversy within the empirical literature on this number. One perspective is that it should entail only the direct costs of bankruptcy. For example, Jerald Warner (1977) examines the railroad industry and comes up with a bankruptcy cost estimate of about 4 percent. However, in our view, this parameter should also include indirect costs of financial distress, such as lost sales and lost profits, because these costs can be viewed as

deadweight losses from keeping capital idle for some period of time. One study that includes indirect costs estimates the sum of direct and indirect bankruptcy costs at about 20 percent of total firm assets (Edward I. Altman, 1984).

In our model, bankruptcy can be viewed as the entrepreneur being closed and his assets being liquidated. This suggests that another measure of bankruptcy costs could be obtained by comparing the value of the firm as a going concern with the liquidation value of the firm (absent any other direct or indirect costs of bankruptcy). Using data from Chapter 11 proceedings, Michael J. Alderson and Brian L. Betker (1995) estimate the internal and external value of the firm (where the former is the firm's value as a going concern, and the latter is the value if its assets were liquidated). Using these estimates, they calculate that liquidation costs are equal to approximately 36 percent of firm assets.

For our benchmark results, we set  $\mu = 0.25$  (at the low end of the 0.2 to 0.36 range). Below we provide some sensitivity analysis to help assess the importance of this choice.

As for the distribution  $\Phi$ , we assume that it is lognormal with a mean of unity and a standard deviation of  $\sigma$ .

We are thus left with two parameters:  $\sigma$  and  $\gamma$ . We treat these two variables as unobservable, and instead choose them indirectly to uniquely match two measures of measured default risk: (1) the bankruptcy rate, and (2) the risk premium. The model's bankruptcy rate is given by  $\Phi(\bar{\omega}_t)$ . As for the risk premium, a loan of one consumption good implies a risky return of  $1 + r_t^k$  capital goods, or  $q_t(1 + r_t^k)$  consumption goods, so that the model's risk premium is given by  $[q_t(1 + r_t^k) - 1]$ .

Jonas D. M. Fisher (1994) reports a quarterly bankruptcy rate of 0.974 percent (using the Dun & Bradstreet data set for 1984–1990). The average spread between the prime rate and the three-month commercial paper rate is an annual risk premium of 187 basis points (for the period April 1971 to June 1996). Matching these two empirical measures provides the last two identifying restrictions: we set  $\sigma = 0.207$  and  $\gamma = 0.947$ .<sup>9</sup> These

<sup>8</sup> With  $\mu = 0$ , (7) implies that  $q = 1$ , so that the model further collapses to the standard RBC model with no adjustment costs.

<sup>9</sup> Interestingly, this standard deviation is comparable to the corresponding empirical standard deviations reported



imply an internal financing percentage of  $n/i = 38$  percent.<sup>10</sup> Other steady-state statistics include: entrepreneurial consumption of  $c^e/n = 6.7$  percent; an internal rate of return of  $[qf/(1 - qg)] = 1/\gamma = 1.056$ , or 5.6 percent; an entrepreneurial share of the loan contract equal to  $f(\bar{\omega}) = 0.39$ ; and the price of capital,  $q = 1.024$ .

#### IV. Simulation

We are now ready to turn to a numerical analysis of the model. The methods are familiar. The equilibrium conditions (3)–(11) are linearized about the steady state, and linear decision rules are then computed using the method of undetermined coefficients.

Figures 1 and 2 report the results of two experiments. For both experiments, we compute the impulse responses for the model with agency costs ( $\mu = 0.25$ ) and for a model without agency costs ( $\mu = 0$ ). The latter is essentially the standard RBC model.<sup>11</sup> The steady states of the two models differ, since the capital stock is slightly higher (3.6 percent) in the RBC model than in the agency-cost model. For this reason, we report the behavior of all variables relative to their steady-state values.

##### A. A Wealth Shock

The first experiment we consider is a one-time shock to the distribution of wealth in the two economies. This shock will be a one-time transfer of capital from households (lenders)

to entrepreneurs (borrowers). This experiment is useful for considering the effects of various shocks to the economy that might redistribute wealth from households to entrepreneurs. For example, in Irving Fisher's (1933) debt-deflation story, surprise increases in the price level shift wealth from lenders to borrowers. Another shock might be the standard productivity shock in RBC models. (The next section will consider the effect of such a productivity shock.) Since productivity shocks will also (indirectly) cause a wealth redistribution, it is instructive to examine a pure wealth shock in order to help understand the second, more complicated experiment.

Figure 1 presents the economy's response to a one-time redistribution of capital from households to entrepreneurs. The redistribution is 0.1 percent of the steady-state capital stock. This trivial reduction in household capital is actually a relatively large increase (13 percent) in entrepreneurial net worth.

In the frictionless RBC model, the source of investment financing is irrelevant, so that the decline in the need for external finance has no effect on the aggregate economy. As for direct wealth effects, they are so small as to be imperceptible (the impulse responses are just flat lines). Hence, we do not report the RBC model.

Matters are much different in the economy with agency costs. Here, increases in entrepreneurial net worth lower the need for external financing and thus reduce the agency costs of investment. This increase in net worth shifts the investment supply curve to the right (as discussed in Section II), thus boosting investment and lowering the equilibrium price of capital. This increased investment entails lower household consumption, which in turn motivates households to increase their labor input, which raises output. In particular, investment increases by 5.5 percent, household consumption declines by 0.8 percent (although aggregate consumption actually rises), household labor increases by 2.2 percent, and output increases by 1.4 percent. After the initial shock, the economy returns to the steady state as entrepreneurs consume their excess capital holdings.

##### B. A Productivity Shock

The second experiment we consider is a shock to aggregate productivity. To be

by Boyd and Smith (1994) in their analysis of debt contracts in an agency-cost framework.

<sup>10</sup> We could have calibrated the model by matching an empirical measure of this aggregate internal financing percentage. We chose not to follow this route because the model makes no clear prediction on this percentage. Remember, there are two different types of firms in our model—investment firms and consumption-producing firms. Although there is a clear prediction for this ratio for the investment firms, the form of capital financing for the consumption-producing firms is indeterminate. This indeterminacy is just the Modigliani-Miller theorem.

<sup>11</sup> For the case of  $\mu = 0$ , the higher discount rate implies that entrepreneurs hold no capital in the steady state. Hence, the only important difference between our model with  $\mu = 0$  and the RBC model of say, Robert G. King et al. (1988), is the small share of labor income flowing to entrepreneurs.

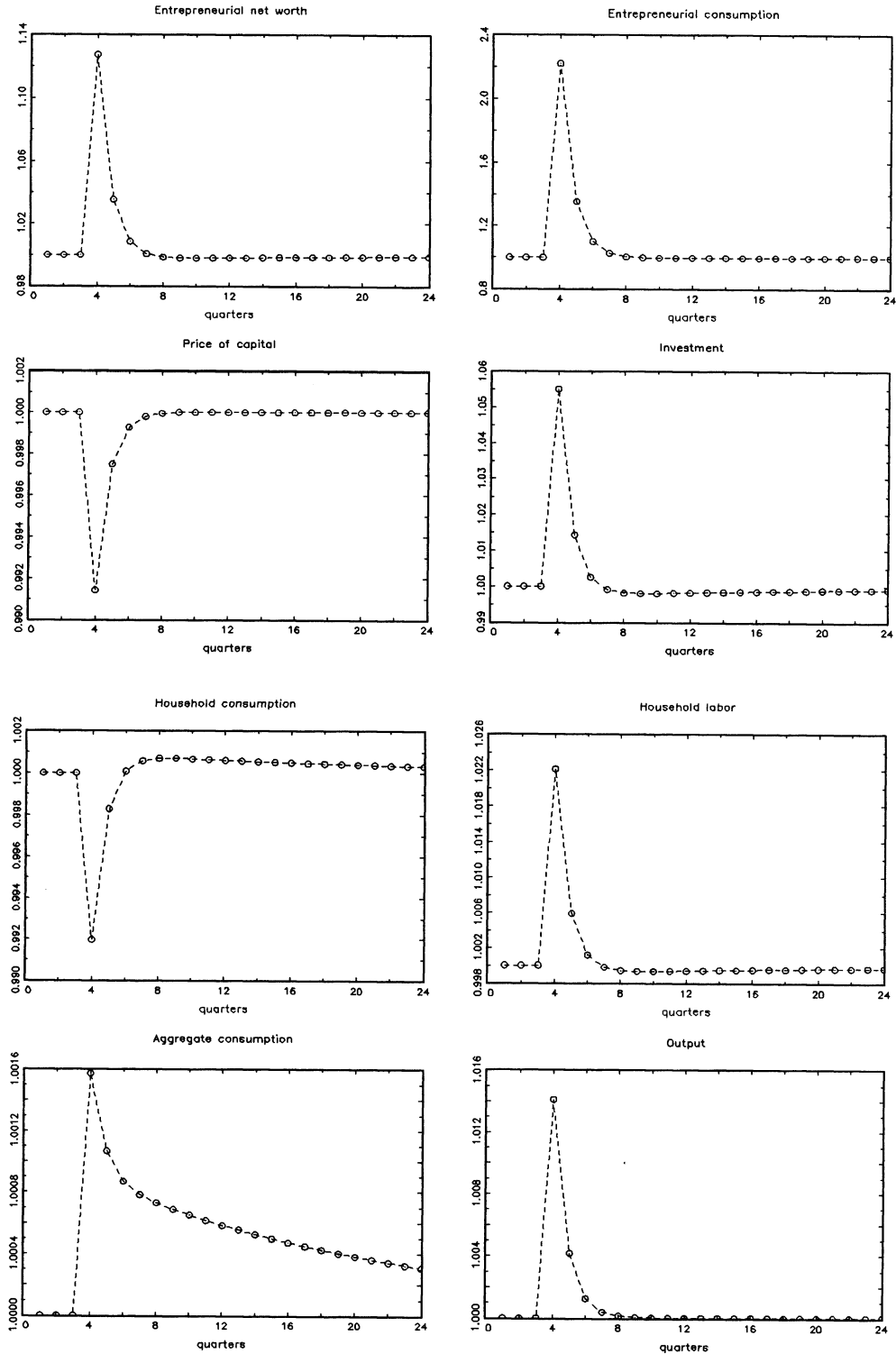


FIGURE 1. THE RESPONSE TO A WEALTH SHOCK IN THE AGENCY-COST MODEL

precise, the technology process is assumed to follow

$$\theta_t = (1 - \rho) + \rho\theta_{t-1} + \nu_t,$$

where  $\nu_t$  is a serially uncorrelated shock,  $\rho$  is the autocorrelation coefficient, and the nonstochastic steady state of  $\theta$  is unity. Following the typical RBC calibration methodology we set  $\rho = 0.95$ . The shock is  $\nu = 0.01$ . Although this is a one-time shock, because technology is autocorrelated, productivity will stay above trend for several quarters.

The results are presented in Figure 2. Each figure contains the dynamics of three different models. The first model sets agency costs to zero and is thus the standard RBC model. The second model holds net worth in the relevant agency-cost model constant and is thus isomorphic to a standard cost-of-adjustment model (see the Appendix). The third model is an economy with agency costs.

The RBC dynamics are familiar. There is a spike in investment, hours, and output as productivity increases, then each series slowly returns to normal as productivity starts declining back to its steady state. As Cogley and Nason (1993, 1995) demonstrate, the dynamics of investment, hours, and output are all inherited from the autocorrelation structure of the technology shock. Capital adds little propagation to these variables in and of itself.

The cost-of-adjustment model resembles the RBC model except that the initial impulse for investment, hours, and output is muted. Initial investment increases by much less as the rise in the price of capital serves to choke off investment demand. This muted response of investment amplifies the initial increase in household consumption. The increase in household consumption shifts back the labor supply curve, thus muting the responses of both hours and output. After the initial impulse, investment, hours, and output all start returning to their steady state. The dynamics closely resemble those of the RBC model.

In the agency-cost model, the dynamics in the early periods are quite different because of the behavior of net worth. On impact, net worth increases slightly as the technology shock boosts entrepreneurs' wage and rental income. However, entrepreneurial capital is initially fixed, limiting net worth's rise. Sub-

sequently, the share of entrepreneurial capital picks up rapidly as the increased demand for capital pushes up the price of capital, thus sharply driving up the return to internal funds  $[qf/(1 - qg)]$ .<sup>12</sup> Along with its direct effect on net worth, this high internal return also increases net worth by leading the risk-neutral entrepreneurs to sharply reduce their consumption (an initial decline of 50 percent). Although the model is highly stylized, this entrepreneurial behavior tends to mimic the behavior of internal funds over the actual business cycle. In particular, the presence of fixed costs implies that during expansions firms see their internal funds rise relative to their fixed obligations, thus freeing up more internal resources for financing.<sup>13</sup> Returning to the model, note that net worth peaks (at about 5.8 percent above steady state) in period six, two periods after the shock. At this point, the price of capital has returned to its steady-state level, and the model's dynamics hereafter mirror the RBC dynamics.

The important difference between the agency-cost model and either the adjustment-cost or the standard RBC model is the hump-shaped response function for investment. The hump shape in investment leads to a "reverse hump" in household consumption after its initial increase. The decline in household consumption (after its initial increase) raises household labor supply, which, when coupled with the increase in labor demand (due to the technology shock), results in a hump-shaped response for hours worked. This hump is pronounced enough to lead to a hump shape in output as well.<sup>14</sup>

To better understand the model's investment behavior, it is instructive to consider a supply/demand analysis of the end-of-period

<sup>12</sup> Recall from Section I that this return is increasing in the price of capital.

<sup>13</sup> We thank an anonymous referee for suggesting these comments.

<sup>14</sup> In the earlier working paper version of this paper, the hump in both investment and output was extremely small. The difference is that the earlier version (essentially) assumed that the entrepreneurs consumed a constant fraction of their capital each period (see footnote 7). Hence, entrepreneurial consumption immediately rose in response to a technology shock so that entrepreneurial net worth did not respond as sharply to the shock.

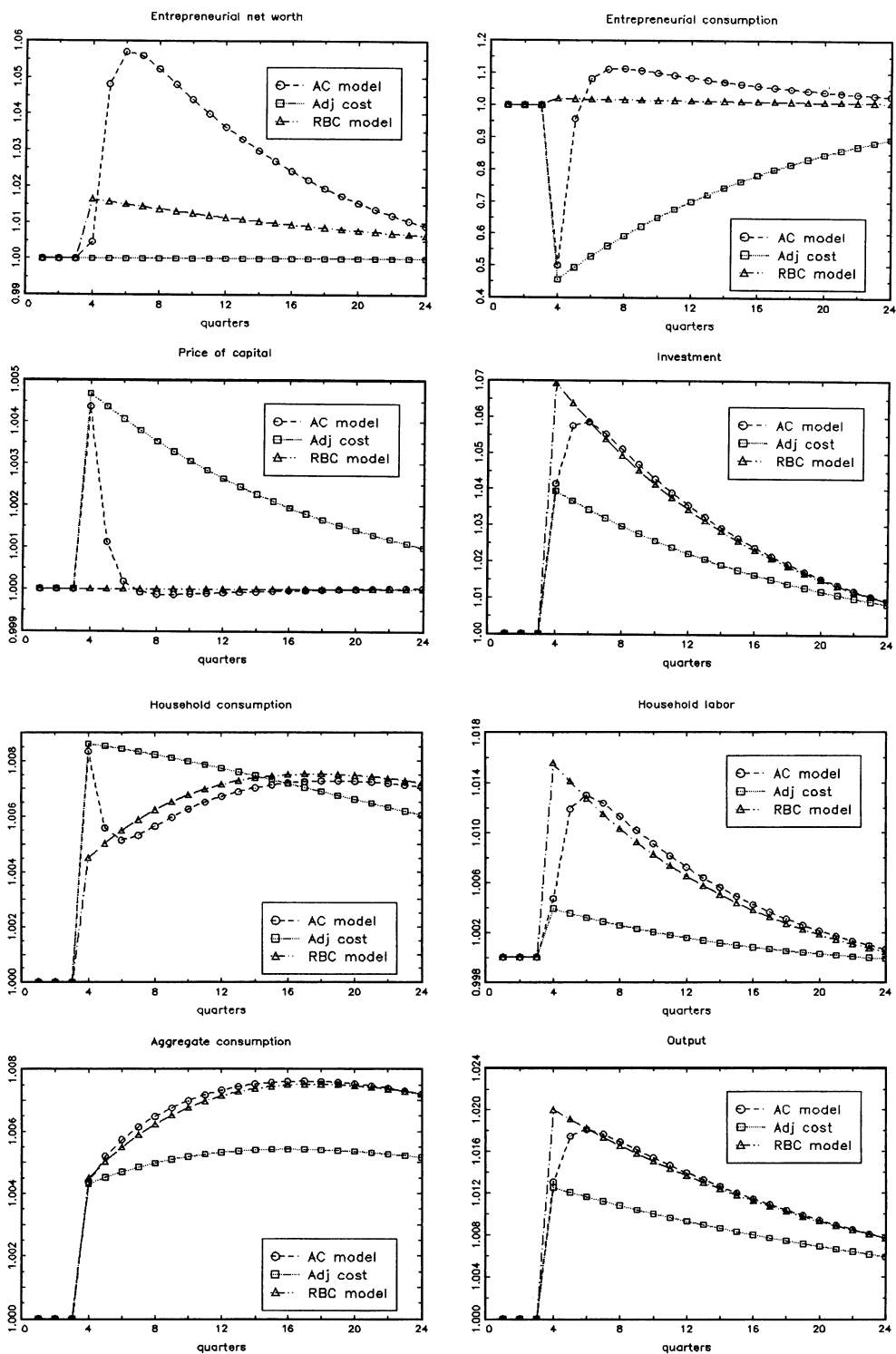


FIGURE 2. THE RESPONSE TO A PRODUCTIVITY SHOCK IN THE AGENCY-COST MODEL, ADJUSTMENT-COST MODEL, AND REAL-BUSINESS-CYCLE MODEL

market for capital goods. One can think of this as a market in which households are demanding capital goods, and in which the CMFs are supplying capital via their intermediation services with the entrepreneurs. The household's demand curve for capital is given by equation (4). The supply curve is given by

$$\eta I^s(q_t, n_t) = [Z_{t+1} - (1 - \delta)Z_t],$$

where the second term is deducted because it represents the entrepreneurs' net capital acquisitions. The technology shock shifts out the investment demand curve. Subsequently, investment demand starts moving slowly back to normal as time progresses. This movement is largely driven by the autocorrelation coefficient,  $\rho$ . The productivity shock increases the return to internal funds, thereby causing entrepreneurial capital and, hence, net worth to rise. By boosting entrepreneurial net worth, the productivity shock shifts the investment supply function to the right. This continues for two periods as net worth continues to grow. It is this dynamic behavior that generates the hump in the investment response and that leads to a similar hump in hours and output. This does not occur in either the standard RBC model (where the supply curve is completely elastic at unity) or a cost-of-adjustment model (where the supply curve remains stationary).<sup>15</sup>

The model's hump-shaped impulse response is of particular interest given the recent work of Cogley and Nason (1995). Their study documents that: (1) a hump-shaped response of output to a transient shock is consistent with U.S. time series, and (2) standard RBC models are unable to deliver this hump-shaped behavior. In the agency-cost model, this hump shape is a natural outcome of the dynamic behavior of net worth in response to a technology shock.

<sup>15</sup> In the adjustment-cost model, the supply curve remains stationary when adjustment costs are assumed to be a function of investment only. If instead these costs depend on the investment-capital ratio, then the supply curve also shifts out as capital begins to grow. This, however, does not lead to a hump-shaped investment response since households internalize the effect and increase their initial investment in anticipation. With agency costs, investment supply shifts out as net worth grows, but since net worth is exogenous from the household's standpoint, this shift is not internalized.

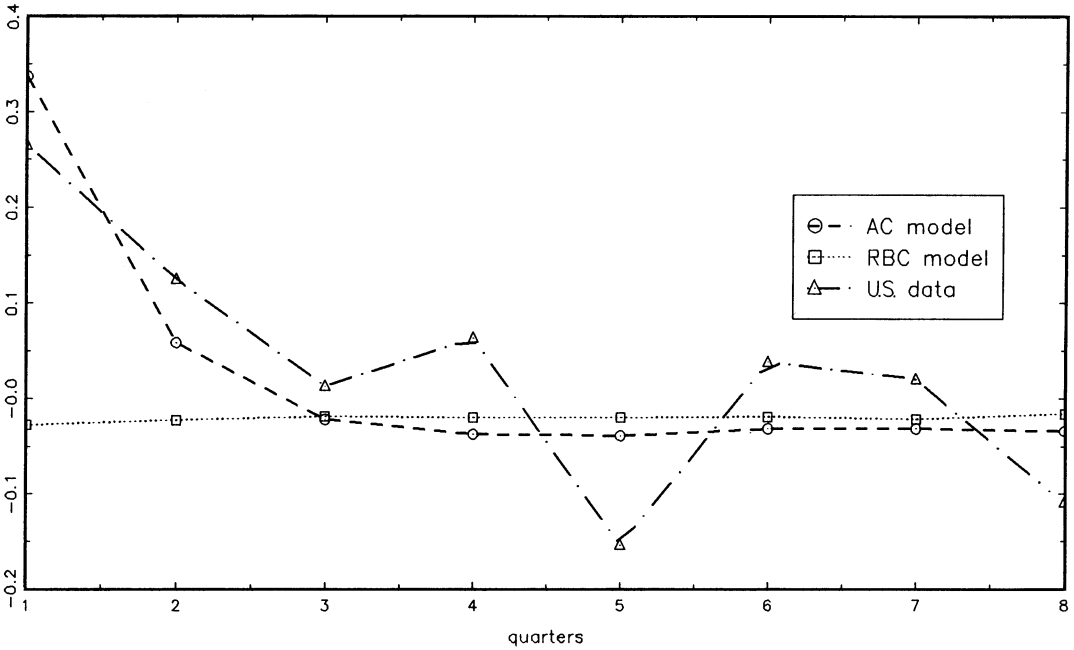
As a metric for this hump shape, Cogley and Nason (1995) suggest the autocorrelation function (ACF) for output growth. Figure 3 graphs the ACF for quarterly GDP and investment growth in the data (1954–1995). The hump shape corresponds to a positive autocorrelation for the first few quarters, and then turns negative for the later quarters. Figure 3 also graphs the ACF for the agency-cost model with  $\mu = 0$  and  $\mu = 0.25$ , where the former is just the RBC model.<sup>16</sup> As noted by Cogley and Nason (1995), the RBC model does remarkably poorly along this dimension.<sup>17</sup> In contrast, the agency-cost model represents a clear improvement to the standard model. Figure 3 also documents the same improvement for the ACF for investment growth.

Cogley and Nason (1995) note that the labor-hoarding model of Craig Burnside et al. (1993), also improves upon the RBC model's ability to match the ACF for output growth. Similarly, the recent contributions by Monika Merz (1995) and David Andolfatto (1996) demonstrate that adding labor market search to the RBC model has the same positive effect. There is a similarity between these results and the results of this paper. These labor papers generate positive output growth autocorrelation by introducing a delayed response to a persistent productivity shock. The delay arises because of the assumed inability to quickly

<sup>16</sup> The model's ACF was calculated by averaging this correlation over 500 model simulations. Each simulation was 300 periods in length, and the statistics were calculated only over the last 200 periods. For the standard deviation of the aggregate technology shock we used 0.005. This is somewhat smaller than the standard 0.007, as we wished to avoid bumping into the nonnegativity constraint on entrepreneurial consumption. We have also calculated the "standard" RBC second-moment statistics after HP-filtering the model's data (the data was not filtered for the ACF growth statistics). In comparison to the RBC model, the agency-cost model behaves like a model with adjustment costs—consumption is slightly more variable, while output and investment are somewhat less so. None of these results are particularly surprising, and so, in the interest of space, we do not report these second-moment statistics here (although, as suggested by a referee, this lack of surprise may be of interest in itself).

<sup>17</sup> Although for space limitations we do not report it here, the adjustment-cost model does no better in this regard.

ACF for output growth



ACF for investment growth

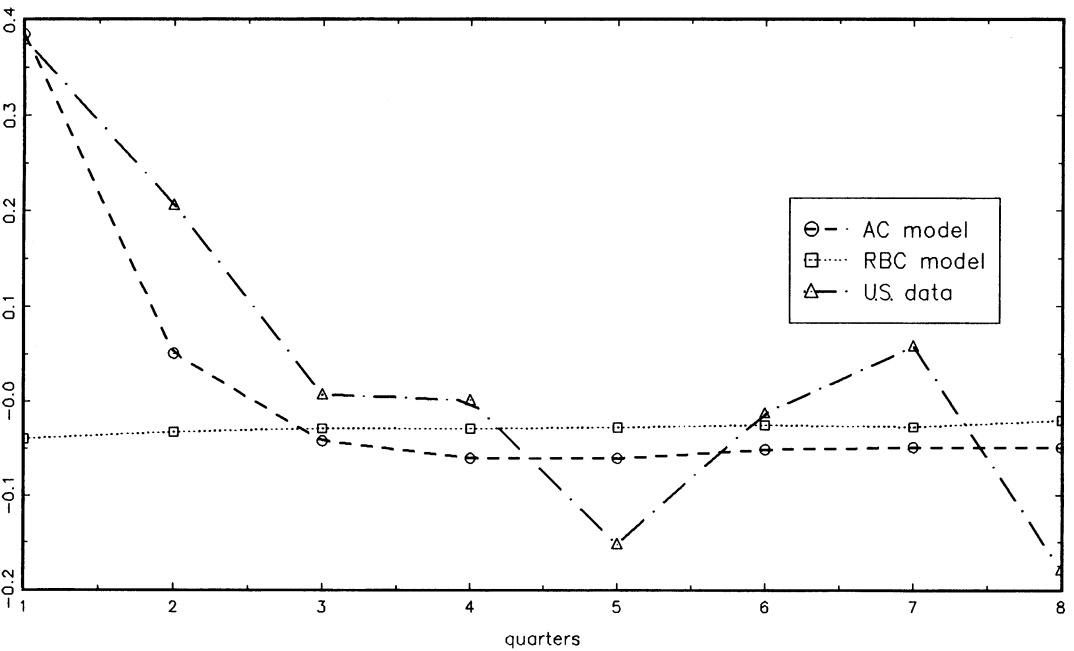


FIGURE 3. AUTOCORRELATION FUNCTIONS FOR OUTPUT AND INVESTMENT GROWTH IN THE AGENCY-COST MODEL, REAL-BUSINESS-CYCLE MODEL, AND U.S. DATA

adjust employment.<sup>18</sup> After this one- or two-period delay, the model's dynamics revert to the RBC model. But this short delay is enough to aid in matching the data's ACF for output growth. There is a similar delay in the agency model, caused by the sluggish behavior of entrepreneurial net worth. Because net worth is primarily accumulated capital, it takes some time for this to sufficiently respond to the technology shock. As with the labor models, this delay allows the model to more closely match the data's output dynamics. This discussion also makes clear the importance of persistent shocks in generating the hump shape in all of these models. With i.i.d. shocks, a one-period delay is one period too long so that the peak output response occurs contemporaneously.

Before turning to some sensitivity analysis, we should point out some of the model's failings. The foremost problem is the cyclical behavior of bankruptcy rates and the risk premia. Because of our linearity assumptions, these variables are functions solely of the aggregate price of capital. Hence, the increase in the price of capital that occurs with a positive technology shock also leads to an increase in bankruptcy rates and risk premia. These rates then move back to steady state as the price of capital moves back to the steady state (by about the third period after the shock). From a theoretical perspective this behavior is not surprising: The supply curve for capital is upward sloped because of agency costs, so that a demand-induced movement up this curve must imply an increase in risk premia. This effect could be overcome if entrepreneurial net worth would rise more sharply in the period of the shock. Modeling efforts along this line are worth pursuing in future work.

## V. Sensitivity Analysis

A key quantitative conclusion of the model is the shape of the ACF function. In this sec-

tion we carry out some sensitivity analysis to examine how this result is affected by varying the degree of agency costs in the model. The model's agency costs arise from the unobserved, idiosyncratic shocks faced by entrepreneurs. Hence, two parameters are particularly important: the degree of idiosyncratic uncertainty ( $\sigma$ ), and the cost of state observability ( $\mu$ ). We will report sensitivity results on these two parameters.

As a first experiment, we considered two lower values for the monitoring cost,  $\mu = 0.15$  and  $\mu = 0.04$ . A  $\mu$  of 0.15 is at the low end of Altman's (1984) estimates of bankruptcy costs inclusive of both direct and indirect costs, while  $\mu = 0.04$  is comparable to Warner's (1977) estimate of only the direct costs of bankruptcy. For these experiments, we held the remainder of the calibration fixed, i.e., we varied  $\sigma$  and  $\gamma$  so that they remained consistent with a risk premium of 187 basis points, and a bankruptcy rate of 0.974 percent. For the  $\mu = 0.04$  case, this implied  $\sigma = 0.562$  and  $\gamma = 0.992$ , while for the  $\mu = 0.15$  case we have  $\sigma = 0.37$  and  $\gamma = 0.973$ .

For our second experiment, we varied  $\sigma$  to trace out differing levels of the risk premium. We report results for a risk premium of 157 basis points and 260 basis points. The former is reported by Fisher (1994), while the latter is the risk premium between the prime rate and the three-month T-bill rate. As before, we allowed  $\gamma$  to vary to keep the bankruptcy rate at 0.974 percent. Hence, for the 157 basis-point risk premium we have  $\sigma = 0.145$  and  $\gamma = 0.935$ , while for the 260 basis-point case we have  $\sigma = 0.335$  and  $\gamma = 0.954$ .

The results of these two experiments are reported in Tables 2 and 3. The ACFs move in the expected way. One interesting observation places our monitoring cost parameter in proper perspective: the benchmark results would be essentially replicated with a lower monitoring cost and a higher risk premium.

## VI. Conclusion

The principal contribution of this paper is to demonstrate a tractable way of modeling and quantifying the role of agency costs in the business cycle. One quantitative conclusion warrants restatement: The agency-cost model replicates the empirical fact that output growth

<sup>18</sup> There are actually two employment adjustment costs at work here. All three labor models assume that it is infinitely costly to contemporaneously adjust employment. Burnside et al. (1993) assume that this cost drops to zero in subsequent periods. The labor market search models of Merz (1995) and Andolfatto (1996) imply a finite, non-zero cost of subsequent employment adjustment.

TABLE 2—ACF FOR QUARTERLY OUTPUT GROWTH

	Lag 1	Lag 2	Lag 3	Lag 4
U.S. data	0.265	0.125	0.013	0.063
RBC	-0.028	-0.023	-0.018	-0.020
Benchmark	0.337	0.058	-0.022	-0.037
$\mu = 0.04$	0.220	0.075	0.014	-0.017
$\mu = 0.15$	0.349	0.106	0.011	-0.028
$rp = 157$	0.290	0.036	-0.022	-0.040
$rp = 260$	0.413	0.113	0.007	-0.032

Notes: The autocorrelation functions (ACF) for the U.S. data are for quarterly growth rates for the period 1954–1995. The RBC statistics are for the standard RBC model. The benchmark results are for the calibration in Section III. The sensitivity analyses are for two different monitoring cost levels and for two different levels of the risk premium (see Section V).

displays positive autocorrelation at short horizons. This hump-shaped output behavior arises because households delay their investment decisions until agency costs are at their lowest—a point in time several periods after the initial shock. Agency costs fall with time because the productivity shock increases the return to internal funds, which in turn redistributes wealth from households to entrepreneurs. The hump in the aggregate variables thus mirrors the hump-shaped behavior of entrepreneurial net worth. The model's hump-shaped output response is of particular interest given the recent work of Cogley and Nason (1995), who document this behavior in the data, and also demonstrate that standard real-business-cycle models are inconsistent with this prediction.

Another contribution of the paper is to demonstrate the linkages between explicit models of agency costs and adjustment-cost models, which assume that there are increasing costs to producing capital. Holding net worth fixed, this paper's agency-cost model closely resembles an adjustment-cost model in that both deliver an upwardly sloped capital supply curve. The paper thus delivers an endogenous model of capital adjustment costs. As part of this endogeneity, the model also demonstrates how the capital supply curve is shifted by movements in entrepreneurial net worth.

There are several natural extensions of the current work. First, the model is easily amenable to considering other shocks to the econ-

TABLE 3—ACF FOR QUARTERLY INVESTMENT GROWTH

	Lag 1	Lag 2	Lag 3	Lag 4
U.S. data	0.379	0.206	0.007	0.001
RBC	-0.040	-0.033	-0.029	-0.029
Benchmark	0.385	0.051	-0.042	-0.060
$\mu = 0.04$	0.054	-0.001	-0.020	-0.031
$\mu = 0.15$	0.318	0.080	-0.012	-0.047
$rp = 157$	0.352	0.028	-0.044	-0.063
$rp = 260$	0.468	0.112	-0.012	-0.056

Notes: The autocorrelation functions (ACF) for the U.S. data are for quarterly growth rates for the period 1954–1995. The RBC statistics are for the standard RBC model. The benchmark results are for the calibration in Section III. The sensitivity analyses are for two different monitoring cost levels and for two different levels of the risk premium (see Section V).

omy. For example, Fisher (1994) and Fuerst (1994, 1995) examine the effect of monetary shocks in related agency-cost models. Similarly, Williamson (1987b) considers shocks to the variance of the entrepreneur's technology in an overlapping generations model. Second, a wide variety of assumptions can deliver differing models of agency costs. In this paper, agency costs arise in the creation of new capital and thus affect the investment supply curve. One obvious alternative is to construct a model in which agency costs arise in the consumption sector, and thus affect the investment demand curve. Finally, several authors have stressed that only a subset of firms are constrained by agency issues, while many other firms are so large that these agency issues are relatively unimportant (for a recent survey and discussion, see Bernanke et al. [1996]). An interesting extension of this work would be to construct a model that captures these types of asymmetries.

## APPENDIX

In this Appendix, we demonstrate how the agency-cost model developed in the paper is isomorphic to a model in which there are costs to adjusting the capital stock. A standard cost-of-adjustment model (Fumio Hayashi, 1982) assumes that capital evolves according to

$$K_{t+1} = (1 - \delta)K_t + \psi(I_t),$$

where  $\psi > 0$  is increasing and concave. Max-



imizing lifetime utility subject to this constraint yields

$$(A1) \quad q_t U_c(t) = \beta E_t U_c(t+1) [q_{t+1}(1-\delta) + \theta_{t+1} F_1(K_{t+1}, H_{t+1})];$$

$$(A2) \quad q_t = 1/\psi'(I_t),$$

where  $q_t$  corresponds to the price of installed capital (or Tobin's  $q$ ). Equation (A1) corresponds to equation (4) in the agency-cost model. Equation (A2) represents the supply curve for newly installed capital,  $I_t \equiv S(q_t)$ , with  $dS/dq_t > 0$ . In the agency-cost model, we have a supply curve for new capital given by  $I^s(q_t, n_t) \equiv i(q_t, n_t) \{1 - \mu \Phi(\bar{\omega}(q_t))\}$ . Analogous to the adjustment-cost model, we have  $I_1^s > 0$ . In contrast to the adjustment-cost model, we also have that the supply curve is shifted by net worth. Specifically, increases in net worth shift the supply curve to the right,  $I_2^s > 0$ . This effect is a central issue in the agency-cost model. Note in particular that if net worth is held constant, the agency-cost model is isomorphic to the adjustment-cost model.

It is a straightforward application of comparative statics to show that  $I_1^s > 0$ , and  $I_2^s > 0$ . Define the Lagrangean

$$L(i, \bar{\omega}, \lambda) \equiv q i \bar{\omega} + \lambda [q i \bar{\omega} - i + n].$$

We then have the following comparative statics:

$$di/dq = [ig L_{\omega\omega} L_{i\lambda} + L_{\omega\lambda}^2 (f + \lambda g)] / \Delta > 0$$

$$d\bar{\omega}/dq = -(f + \lambda g) L_{\omega\lambda} L_{i\lambda} / \Delta > 0$$

$$di/dn = L_{\omega\omega} L_{i\lambda} / \Delta > 0$$

$$d\bar{\omega}/dn = 0,$$

where  $\Delta \equiv -L_{i\lambda}^2 L_{\omega\omega} > 0$  is the second-order condition. Since  $\bar{\omega}$  does not vary with  $n$ , it is now obvious that  $I_2^s > 0$ . As for  $I_1^s$ , the optimal contract also maximizes  $i\bar{\omega} = i[1 - \Phi(\bar{\omega})\mu - g(\bar{\omega})]$ . An increase in  $q$  relaxes the

constraint on the lender's return, so that  $i\bar{\omega}$  must be increasing in  $q$ . Since  $ig(\bar{\omega})$  is increasing in  $q$ , we then have that  $i[1 - \Phi(\bar{\omega})\mu] \equiv I^s$  is also increasing in  $q$ .

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