Standard Errors in Matlab

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Outline

- Importance of Standard Errors
- Recovering SEs in GMM models
- Recovering SEs in MLE models
- Recovering SEs in other models

Why all the fuss over standard errors?

- Empiricists are in the business of estimating models and assigning meaning to those model estimates
- Coupled with point estimates, standard errors are the most crucial piece of an empiricists findings
- If the standard errors are wrong, then the results will be wrong, too, because false inferences will be made
- This is why so much energy is spent deriving correct standard errors under a variety of assumptions (e.g. Hayashis textbook)

Standard errors

Economists prefer to have models with parameter estimates that are *consistent* and *asymptotically normal*, i.e.

$$\sqrt{n}\left(\hat{\beta}-\beta\right) \xrightarrow{d} N\left(0, \text{Avar}\right)$$

(Hayashi, p. 113)

- We want to know the variance of our parameter estimates so that we can tell whether or not we got a lucky draw, or if they are actually different from some number (typically zero)
- The standard errors of the $\hat{\beta}$ vector are embedded in the $\widehat{\text{Avar}}$ matrix (our estimate of the true Avar matrix)
- In the next few slides, we'll go over what the formula is for $\widehat{\text{Avar}}$ in various common econometric models

Standard errors for GMM models (Hayashi pp. 125, 213)

$$\widehat{\text{Avar}} = \left(\mathbf{Z}'\mathbf{X}\widehat{\mathbf{S}}^{-1}\mathbf{X}'\mathbf{Z}\right)^{-1}$$

where the optimal weighting matrix (plim $\widehat{\mathbf{W}} = \mathbf{S}^{-1}$) is used, and

$$\begin{aligned}\widehat{\mathbf{S}} &= \sum_{i=1}^{n} \hat{\varepsilon}_i^2 \mathbf{x_i x_i'} \\ &= \mathbf{X}'\mathbf{B}\mathbf{X}\end{aligned}$$

where

$$\mathbf{B} = \begin{bmatrix} \hat{\varepsilon}_1^2 & & \\ & \ddots & \\ & & \hat{\varepsilon}_n^2 \end{bmatrix}$$

so

$$\widehat{\text{Avar}} = \left(\mathbf{Z}'\mathbf{X}\left(\mathbf{X}'\mathbf{B}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{Z}\right)^{-1}$$

Standard errors for GMM models

From the previous slide:

$$\widehat{\text{Avar}} = \left(\mathbf{Z}'\mathbf{X}\left(\mathbf{X}'\mathbf{B}\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{Z}\right)^{-1}$$

where

- $\mathbf{X}$ is a matrix of instruments
- $\mathbf{Z}$ is a matrix of endogenous regressors
- $\mathbf{B}_{i,i} = \left(y_i - x_i\hat{\beta}\right)^2$
- Note: for simple OLS, substitute $\mathbf{X}$ for $\mathbf{Z}$ in the above formula

The standard errors are then

$$\text{SE}_{\hat{\beta}_k} = \sqrt{\widehat{\text{Avar}}_{k,k}}$$

Standard errors for MLE models (Hayashi pp. 475-476)

$$\widehat{\text{Avar}} = -\left\{ \sum_{i=1}^{n} \mathbf{H}\left(\mathbf{x_i}; \widehat{\beta}\right) \right\}^{-1}$$

where

$$\mathbf{H}\left(\mathbf{x_i}; \widehat{\beta}\right) = \frac{\partial^2 \ell\left(x_i, \beta\right)}{\partial\beta\partial\beta'}$$

where $\ell\left(x_i, \beta\right)$ is the log likelihood function

Standard errors for MLE models

Recall in Matlab that `fminunc` and `fmincon` can return the numerical hessian of the objective function:

```
[x,fval,exitflag,output,gradient,hessian] = fminunc('fun',x0,options,varargin)
```

Assuming that all regularity conditions are met, the standard errors of `x` are

```
se = sqrt(diag(inv(hessian)));
```

since Matlab returns $-\left\{\sum_{i=1}^{n}\mathbf{H}\left(\mathbf{x_i};\widehat{\beta}\right)\right\}$ in the `hessian` output

Standard errors for other models

- All classical econometric models can be considered as either GMM or MLE. (Furthermore, GMM and MLE are asymptotically equivalent in some situations)
- Most structural econometric models, however, incorporate simulation methods, multiple estimation stages, and/or contraction mapping/fixed point algorithms (in addition to a main GMM/MLE optimization)
- In these more complex estimation algorithms, it's unclear how the standard errors of the parameter estimates are affected by the use of previous stage estimates and/or simulation draws
- The solution to this problem is bootstrapping, which we will cover in the second Matlab module