Adding Heterogenous Earnings Processes to the OG Model

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Heterogeneity in earnings

- Allows model to general a distribution of income and wealth (within cohort)
- Now you can do distributional analysis across and within generations
- Since labor is endogenous, we want heterogeneity in hourly earnings (not annual earnings)

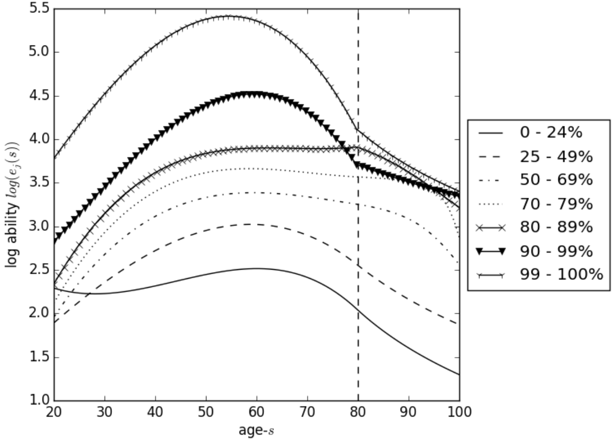
How to introduce heterogeneity in hourly earnings?

Three (of many) ways:

1. Permanent and constant differences in lifetime earnings among types
2. Differences in the lifecycle earnings profiles of the types
3. Stochastic income (perhaps with different lifecycle earnings profiles)

We go with option (2), which gives us a great deal of realism, with lower computational cost than stochastic income.

Lifecycle Profiles of Hourly Earnings



How do we estimate these profiles?

- Start with panel data that contains hourly earnings equivalent to the model unit
- Here we are modeling households, so we use panel data on hourly earnings of households (actually “tax filing units", but they are close)
- Panel will allow us to get the lifecycle profiles

Defining ability groups

- Want to define ability groups
- These can’t be based on total earnings, since that is endogenous (a function of labor supply)
- We use “potential lifetime income"
- Essentially, households are grouped according to what they would earning if they worked full time from ages 21 to 80

Defining lifetime income

$$LI_i = \sum_{t=21}^{80} \left(\frac{1}{1+r} \right)^{t-21} (w_{i,t} * 4000)$$

- Panel isn't balanced and don't observe anyone from age 21 to 80 (where age defined by age of primary filer in tax unit/primary earner of household)
- So we impute wages for each year of life:

$$ln(w_{i,t}) = \alpha_i + \beta_1 age_{i,t} + \beta_2 age_{i,t}^2 + \beta_3 * age_{i,t}^3 + \varepsilon_{i,t}$$

- These imputed wages are used to determine potential lifetime income for each household

Descriptive statistics

Lifetime Income Category:	1	2	3	4	5	6	7	All
Percentiles	0-25	25-50	50-70	70-80	80-90	90-99	99-100	0-100
Observations	65,698	101,484	74,253	33,528	31,919	24,370	2,129	333,381
Fraction Single								
Females	0.30	0.24	0.25	0.32	0.38	0.40	0.22	0.28
Males	0.18	0.22	0.30	0.35	0.38	0.37	0.20	0.26
Fraction Married								
Female Head	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.02
Male Head	0.45	0.53	0.45	0.32	0.23	0.23	0.57	0.39
Mean:								
Age, Primary	51.72	44.15	38.05	34.09	31.53	30.79	40.17	39.10
Hourly Wage	11.60	16.98	20.46	23.04	26.06	40.60	237.80	21.33
Annual Wages	25,178	44,237	54,836	57,739	61,288	92,191	529,522	51,604
Lifetime Income	666,559	1,290,522	1,913,029	2,535,533	3,249,287	5,051,753	18,080,868	2,021,298

* CWHS data, 1991-2009, all nominal values in 2005$.

Estimating lifecycle profiles by group

- We partition the households into lifetime groups based on their percentile in the distribution of lifetime income
  - We break the sample into 7 groups:

    $$[0-25\%, 25-50\%, 50-70\%, 70-80\%, 80-90\%, 90-99\%, 100\%]$$

  - One could do more/less groups, different percentiles
- Then for each lifetime income group, we estimate the lifecycle profile of earnings as:

$$ln(w_{i,t}) = \alpha + \beta_1 age_{i,t} + \beta_2 age^2_{i,t} + \beta_3 * age^3_{i,t} + \varepsilon_{i,t}$$

Ages 80-100

- We model individuals up until age 100, but our data contain few observations over age 80
- We thus interpolate our data out through these final 20 years
- We do this with an arctangent function:

$$y = \left(\frac{-a}{\pi}\right) * arctan(bx + c) + \frac{a}{2}$$

Ages 80-100

- In the function

$$y = \left(\frac{-a}{\pi}\right) * arctan(bx + c) + \frac{a}{2}$$

- The variable x is age
- The parameters a, b, and c are found by finding those that best fit of the function given:
  1. The value of the function should match the value of the data at age 80
  2. The slope of the arctan should match the slope of the data at age 80
  3. The value of the function should match the value of the data at age 100 times a constant.
     - This constant is .5 for all lifetime income groups, except the 2nd highest ability is .7
     - Otherwise, the 2nd highest has a lower income than the 3rd highest ability group in the last few years.

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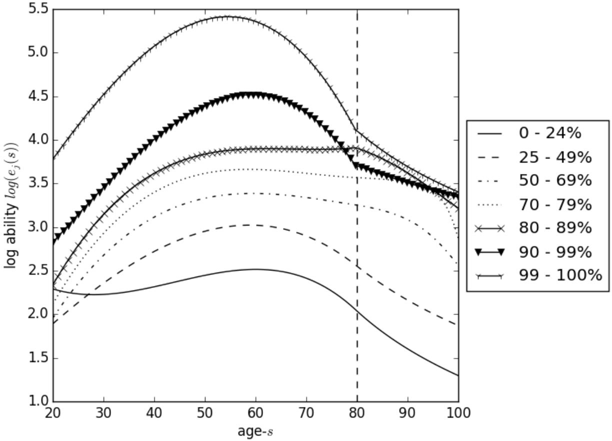


What's the model analogue to the data?

- Model thus far has labor supply (hours) and hourly wage rate
- Data are wages per hour worked
- Consider the transformation:
    - Let the wage rate be the rate per effective hour worked
    - E.g. the person with average ability earns w_t per unit of labor supplied
    - Then wages per hour in the data are $w_t \times$ effective hours

Modeling ability

- Let e_s be effective labor hours per unit of labor supplied by an age s household
- Let j represent the lifetime income group (we'll call this the ability group)
- Then $e_{j,s}$ are effective labor hours per unit of labor supplied by a household of age s and ability group j
- So total labor income in period t for this household is $w_t * e_{j,s} * n_{j,s,t}$

Filling in $e_{j,s}$

- The matrix of effective labor units (ability) is $J \times S$
- Recall we estimated the lifecycle earnings profiles for each type J
- Next step: Find implied values at each age, s

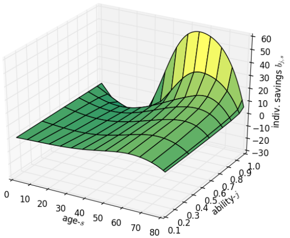
One more piece...

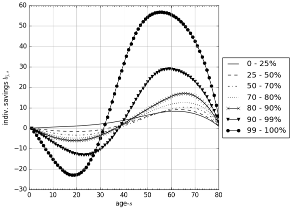
- How many households of each type?
- The way we estimated this, it's clear by the percentile groups
- Divide the unit measure born in each cohort into these groups
- Let λ_j give the measure of households of type j

$$\boldsymbol{\lambda} = [0.25, 0.25, 0.20, 0.10, 0.10, 0.09, 0.01]$$

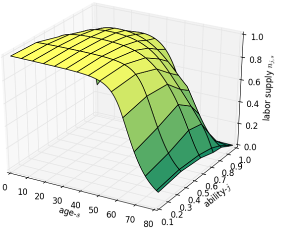
Results

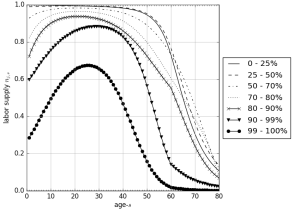
Lifecycle Profiles of Savings





Lifecycle Profiles of Labor Supply





Lifecycle Profiles of Consumption